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The Joint Probabilistic Data Association Filter (JPDAF) for Multi-Target Tracking

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Abstract

This report is about the issue of tracking problem in detection, classification, tracking problems. JPDAF is explained and related with particle filtering.

0.1 Introduction

JPDAF is popular approach to tracking multiple moving objects. This method is also based on the Bayesian estimate. JPDAF computes a Baysian estimate of the correspondence between features detected in the sensors and many targets to be tracked. In other words, JPDAF computes the probabilities of association of the set of measurements Y_t to the various targets, where $Y_t \triangleq \{\mathbf{y}_{i,t}\}_{i=1}^{m_t}, m_t$ is the number of measurements at time t. If we write the state equation again here,

$$\begin{aligned} \mathbf{X}_t &= f(\mathbf{X}_{t-1}) + u_t \\ \mathbf{Y}_t &= h(\mathbf{X}_t) + v_t \end{aligned} \tag{1}$$

Necessary definitions can be made before it proceeds as follows,

- The set of measurements at time t, where m_t is the number of measurements at time t.

  $$\mathbf{Y}_t \triangleq \{\mathbf{y}_{i,t}\}_{i=1}^{m_t} \tag{2}$$

  If $m_t = 0$ (no detections fell inside the gate), then the set $\mathbf{Y}_t$ is empty.

- The total number of measurements in the set $\mathbf{Y}^t$ is

  $$N \triangleq \sum_{j=1}^{t} m_j \tag{3}$$

- The cumulative set of measurements up to time t is

  $$\mathbf{Y}^t \triangleq \{\mathbf{Y}_j\}_{j=1}^{t} \tag{4}$$

- *Validation region* or *gate*
  It is the ellipse (or ellipsoid) of probability concentration-the region of minimum volume that contains a given probability mass (see Figure 1). Measurements that lie inside that gate are considered valid; those outside are discarded.

- *Maneuvers*
  Unpredictable changes in target motion.

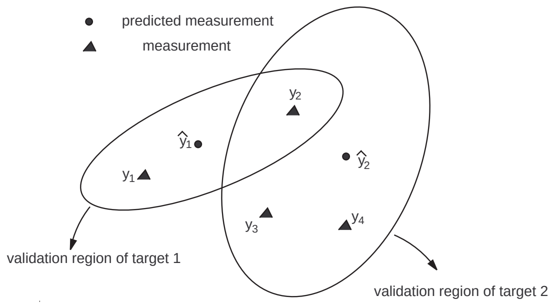

Figure 1: Two targets with a common measurement.

- *Clutter*
  Refers to detections or returns from nearby objects,weather, electromagnetic interference, false alarms, etc. that are generally random in number, location and intensity. This leads to the occurrence of (possibly) several measurements in the validation region (gate) of a target. This set of validated measurements consists of:

  1. The correct measurement (if it has been detected and it fell in the gate).
  2. Incorrect measurements (originating from clutter/false alarms).

0.1.1 Two targets in Clutter

The situation is depicted in Figure 1, where the validation region is an ellipse centered at $\hat{y}_i(t+1|t)$, the predicted measurement. All the measurements in a validation region are not " too unlikely " to have originated from the target of interest, even though only one is assumed to be the true one. Tracking several targets in the same vicinity, as well as dealing with clutter, is significantly more complicated than the single-target-in-clutter case. Measurement y_1 could have originated from target 1 or clutter, y_2 from either 1 or 2 or clutter and y_3 and y_4 from either 2 or clutter. Furthermore, if y_2 originated from 2 then it is quite likely that y_1 originated from 1. This illustrates the interdependence of measurement association when " persistent interference

" (a neighboring target) is present in addition to the " random interference " (clutter). It is assumed that a measurement could have originated from either target 1, or target 2, or clutter. However, in view of the fact that any signal processing system has an inherent resolution threshold, an additional possibility should be considered: measurement y_2 could be the result of the " merging " of the measurements from the two targets. This constitutes a fourth origin hypothesis for a measurement that lies in the intersection of two validation regions. In our case, this merged two targets are under the consideration always. We also assume that there is no false measurements nor change of number of targets.

0.2 The Joint Probabilistic Data Association Filter

When there are more than one target to be tracked, each target has a dynamic and a measurement model as in (0.1). The state models for the various targets do not have to be the same.

Consider the problem of tracking K targets. $\mathbf{X}_t = \{\mathbf{x}_{1,t}, \dots, \mathbf{x}_{K,t}\}$. And, let $\mathbf{Y}_t = \{\mathbf{y}_{1,t}, \dots, \mathbf{y}_{m_t,t}\}$ denote a measurement at time t. The key question when tracking multiple objects is how to assign the observed measurements to the individual targets. Let us define necessary things first here,

- *A joint association event* $\boldsymbol{\theta}$;

$$\boldsymbol{\theta} = \bigcap_{j=1}^{m_t} \theta_{jk_j} \tag{5}$$

  where θ_{jk} means measurement j is originated from target k and $j = 1, \cdots, m_t$; $k = 0, 1, \cdots, K$. And k_j is the index of the target to which measurement j is associated in the event under consideration.

- Θ_{ji} denote the set of all valid joint association events that assign measurement j to target k.

- *Validation matrix* ;

$$\boldsymbol{\Omega} \triangleq [\omega_{jk}], \quad j = 1, \dots, m_t; \quad k = 0, \dots, K \tag{6}$$

with binary elements ω_{jk} that indicate 1 if measurement j lies in the validation region of target k. The index $k = 0$ stands for " no target " and the corresponding column of $\mathbf{\Omega}$ has all units, since each measurement could have originated from clutter. A typical validation matrix for two targets and four measurements looks as follows:

$$\mathbf{\Omega} = \quad \begin{array}{c} \begin{array}{ccc} {\scriptstyle 0} & {\scriptstyle 1} & {\scriptstyle 2} \end{array} \\ \left[\begin{array}{ccc} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{array}\right] \end{array} \begin{array}{c} \\ {\scriptstyle 1} \\ {\scriptstyle 2} \\ {\scriptstyle 3} \\ {\scriptstyle 4} \end{array} \tag{7}$$

This corresponds to the situation depicted in Figure 1. A joint association event $\boldsymbol{\theta}$ can be represented by the matrix

$$\hat{\mathbf{\Omega}}(\boldsymbol{\theta}) = [\hat{\omega}_{jk}(\boldsymbol{\theta})] \tag{8}$$

where,

$$\hat{\omega}_{jk}(\boldsymbol{\theta}) = \begin{cases} 1 & \text{if } (j,k) \in \boldsymbol{\theta} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

- We call a measurement *false alarm* when it is not caused by any of the targets.
- *Target detection indicator* ;

$$\delta_k(\boldsymbol{\theta}) \triangleq \sum_{j=1}^{m_t} \hat{\omega}_{jk}(\boldsymbol{\theta}), \quad k = 1, \ldots, K \tag{10}$$

- *Measurement Association indicator* ;

$$\tau_j(\boldsymbol{\theta}) \triangleq \sum_{k=1}^{K} \hat{\omega}_{jk}(\boldsymbol{\theta}), \quad j = 1, \ldots, m_t \tag{11}$$

- The number of false measurements in event $\boldsymbol{\theta}$;

$$\phi(\boldsymbol{\theta}) = \sum_{j=1}^{m_t} [\tau_j(\boldsymbol{\theta}) - 1] \tag{12}$$

- *The marginal association probability* (*Assignment probability*)
  This is the posterior probability that measurement j is caused by target k. It is obtained from the joint probabilities by summing over all the joint events in which the marginal event of interest occurs.

$$\beta_{jk} \triangleq P(\theta_{jk}|\mathbf{Y}^t) = \sum_{\boldsymbol{\theta} \in \Theta_{jk}} P(\boldsymbol{\theta}|\mathbf{Y}^t)\hat{\omega}_{jk}(\boldsymbol{\theta}), \quad j = 1, \cdots, m_t; \quad k = 0, 1, \cdots, K \tag{13}$$

  This marginal probabilities are mutually exclusive , hence

$$\sum_{j=1}^{m_t} \beta_{jk} = 1, \quad \text{for} \ \ k = 1, \cdots, K \tag{14}$$

  This means that only *one* or *none* of m_t measurements is related with each target. It is important that one measurement could be related with none or more than one measurements but one target is not related with more than one measurement.

0.3 Particle Filtering Based on JPDAF (PF-JPDAF)

Particle filtering approximate posterior filtered density $P(\mathbf{X}_t|\mathbf{Y}^t)$. $P(\mathbf{X}_t|\mathbf{Y}^t)$ is the same as $P(\mathbf{X}_t|\mathbf{Y}_1, \mathbf{Y}_2, \cdots, \mathbf{Y}_t)$ as specified in (4). In PF-JPDAF we are interested in the posterior filtered density under certain joint association event $\boldsymbol{\theta}$. Suppose targets (hidden states) are not correlated. If we use *sampling importance resampling* (SIR) particle filtering, likelihood function will be the weight, but likelihood function is slightly different in PF-JPDAF. Because we are not sure about observation (measurement) in JPDAF situation so that we need to find likelihood function for PF-JPDAF. For each target in multi-target situation, we have referent measurement to each target. We need to find that referent measurement with a certain probabilities. If targets are not correlated each other, the weight is computed for each target as follows,

$$w_{k,t}^i \propto \sum_{j=1}^{m_t} \beta_{jk} P(\mathbf{y}_{j,t}|\mathbf{x}_{k,t}^i) \tag{15}$$

,where

- i : i^{th} particle.
- k : target k.
- t : at time t.
- m_t : the number of measurements at time t.
- $\mathbf{x}_{k,t}^i$: i^{th} particle for target k at time t.
- $\mathbf{y}_{j,t}$ j^{th} : measurement at time t.

So total weight at each time sequence will be

$$w_t^i \propto \prod_k^K w_{k,t}^i \tag{16}$$

0.4 Problem Statement

It is assumed that the number of target is known and the initial location of targets are known. Two targets are moving with random accelerations to any directions at starting points (0,0) and (100,100). Sensors are distributed in random over the sensor field. Sensors are receiving power of signal from each target that depends on the distance and might change with time. Our objective is to track the trajectories of those two targets using Particle Filtering method based on JPDAF (PF-JPDAF). In the standard JPDAF, it is generally assumed that the underlying densities are Gaussians and linear, and Kalman filtering is applied. It is not linear anymore in our problem here. The prior density is chosen as the importance density and resampling is applied at every time step (SIR Particle Filter).

0.4.1 Received Signal Strength (RSS) Model

Acoustic energy model that is an example of of RSS model is used. In [1], the sensors measure the power of the received signal in dB(dBm). Thus, the received power signal at sensor n at time t is expressed as follows,

$$y_{n,t} = \sum_{k=1}^{K} \left[P_0 - 10\alpha \log_{10} \left(\frac{|\mathbf{r}_n - \mathbf{l}_{t,k}|}{d_0} \right) \right] + v_{n,t}, \quad n = 1, \cdots, N \tag{17}$$

where P_0 is the power received by the sensor at a reference distance d_0, $\mathbf{r}_n$ is the position of the nth sensor, $\mathbf{l}_{t,k}$ is the location of the source target k at time t, α is an attenuation parameter that depends on the transmission medium, $v_{n,t} \sim \mathcal{N}\ (0,\ \sigma_v^2)$ is a Gaussian noise process with known variance σ_v^2, N is the total number of sensors used, and K is the total number of source targets. This expresses the received power signal at nth sensor at time t. So our observation matrix will be as follows,

$$\mathbf{y}_t = [y_{1,t}\ \ y_{2,t}\ \ \cdots\ \ y_{N,t}]^T \tag{18}$$

But in JPDAF, we have m_t measurements and we still need to know the measurement that is caused by targets.

0.4.2 State Equation

$$\boldsymbol{\alpha}_{k,t} = \boldsymbol{\alpha}_{k,t-1} + \mathbf{w}_{k,t} \tag{19}$$

$$\mathbf{u}_{k,t} = \mathbf{u}_{k,t-1} + \boldsymbol{\alpha}_{k,t-1}T \tag{20}$$

$$\mathbf{l}_{k,t} = \mathbf{l}_{k,t-1} + \mathbf{u}_{k,t-1}T + \frac{1}{2}\boldsymbol{\alpha}_{k,t-1}T^2 \tag{21}$$

where $\boldsymbol{\alpha}_{k,t}$ is the acceleration of target k at time t and is the *hidden state*. $\mathbf{u}_{k,t}$ is the velocity of the target k, $\mathbf{l}_{k,t}$ is the location of target k at time t which are related with $\boldsymbol{\alpha}_{k,t}$ by the law of classical mechanics. $\mathbf{w}_k$ is distributed uniformly between W_{max} and W_{min}, T is sampling time.

0.4.3 Solution by PF-JPDAF

We are interested in the posterior filtered density $P(\mathbf{X}_t|\mathbf{Y}^t)$ under the joint association event $\boldsymbol{\theta}$. In this report, the situation of problem is assumed that we always have two merged targets and no false measurements. Under that assumption, our validation matrix will be as follows according to (7),

$$\hat{\boldsymbol{\Omega}} = \left[\begin{array}{ccc} 1 & 1 & 1 \end{array}\right] \tag{22}$$

And β_{jk} can be computed according to (13) and will be $\sum_{j=1}^{1}\beta_{jk} = 1$ every time in this problem. Therefore the weight will be

$$w_t^i = w_{1,t}^i \cdot w_{2,t}^i = P(\mathbf{y}_{1,t}|\mathbf{x}_{1,t}^i)P(\mathbf{y}_{2,t}|\mathbf{x}_{2,t}^i) = P(\mathbf{y}_{1,t}|\mathbf{x}_{1,t}^i, \mathbf{x}_{2,t}^i) \tag{23}$$

In this case two targets can not be resolved and we should find out proper solution to apply this method.

Bibliography

[1] N. Patwari, A.O. Hero III, M. Perkins N. S., Correal, and R. J. O'Dea, "Relative location estimation in wireless sensor networks," *IEEE Trans. Signal Processing, Special Issue on Signal Processing in Networks*, vol. 51, no. 8, pp. 2137–2148, Nov 2003.