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SOME STEADY STATE SOLUTIONS OF THE DIFFUSION EQUATION IN THE IRROTATIONAL FLOW PAST OBSTACLES

by

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Abstract

The steady state diffusion equation for irrotational flow is transformed to that of Schröedinger. The temperature (or solute's concentration) distribution in the flow past a cylinder of arbitrary cross-section is then evaluated. Various boundary conditions are considered. Perturbation type of solution is then constructed for the case of flow past an isothermal sphere. Availability of these solutions in terms of tabulated functions demonstrates the advantages of the proposed new approach to the problem.

Introduction

In analyzing the temperature (or solute concentration) in the potential flow past a heat (or mass) emitting body it is very often assumed that the amount of heat (or mass) conducted upstream is negligible[1,2]. Further assumptions are made when the temperature distribution is other than two-dimensional[3,4]. Under these assumptions many cases can be solved, even in a closed form, especially when use is made of Boussinesq's[4] transformation. However, under the assumption of negligible upstream conduction the diffusion equation is trunkated and is therefore no longer elliptic. Thus, its solutions represent a process which is, in certain qualitative respects, different from that of heat or mass diffusion. For example, according to these solutions the temperature of the fluid upstream of the front stagnation point is unaffected by the heat emitting body. Hence though solutions of the trunkated equation are acceptable on design engineers they leave something to be desired.

There are available exact solutions for the two dimensional diffusion equation which represent heat source in an irrotational flow. Attempts were also made to solve the boundary value problems at hand in terms of suitable sources and sinks distributions[5]. The resulting expressions for the temperature are in the form of definite integrals which are often difficult to evaluate. Furthermore, this method has not been shown to be applicable either to three dimensional cases or to those in which the general linear conditions hold, i.e. when a linear combination of the temperature and the flux is prescribed at the obstacle surface.

This survey shows not only the shortcomings of the various approaches mentioned but also the lack of a more universal one. In an attempt to overcome this inadequacy the problems under consideration are reformulated in a form which can be tackled systematically. It is shown that for both two and three dimensional cases the governing diffusion equation can be reduced to that of Schröedinger. Standard techniques are thus employed in the solution for the temperature distribution in the unseparated flow past an isothermal cylinder of arbitrary cross section. This solution is then extended to the case of circular cylindrical obstacle on which the general linear condition holds. Finally, the temperature in the irrotational flow past a sphere is evaluated using a perturbation technique. Thus, unlike any of the existing ones the proposed formulation and approach is applicable to a variety equation are acceptable on design engineers they leave something to be desired.

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Transformation of the Diffusion Equation

With T as the temperature or concentration of solute and u_i $(i = 1, 2, 3)$ as the components of velocity of the fluid in the directions of the cartesian co-ordinate x_i , the diffusion equation can be written thus:

$$P_e \sum_{j=1}^{3} u_j \frac{\partial T}{\partial x_j} = \sum_{j=1}^{3} \frac{\partial^2 T}{\partial x_j^2} \quad . \tag{1}$$

The variables x_i and u_i are non-dimensionalized with respect to a characteristic length L and the speed of the stream when unobstructed U . The non-dimensional Peclet number is defined by

$$P_e \equiv UL/k$$

where k is the diffusivity of the fluid. It is therefore a measure of the relative strength of convection with respect to conduction.

For irrotational flow the following hold

$$u_i = -\partial\varphi/\partial x_i$$

where the potential function φ is a known function of x_i and $(\partial\varphi/\partial x_1)$ is unity far from the solid body. Therefore the variable Θ , defined by

$$\Theta \equiv T \exp(Pe\,\varphi/2)$$

satisfies the equation

$$\sum_{j=1}^{3}\left[\frac{\partial^2}{\partial x_j^2} - \left(\frac{Pe}{2}\right)^2\left(\frac{\partial\varphi}{\partial x_j}\right)^2\right]\Theta = 0 \; . \tag{2}$$

Though this transformation appears in the literature on heat transfer(5) very few attempts have been done to analyze temperature distribution by solving equation (2). This formulation is advantageous not only because the governing equation (2) is fairly well known but also because Θ satisfies conditions which are no more complicated than those imposed on T . The general linear condition is taken to be

$$M\,T + N(\partial T/\partial n) = T_0\, f(s) \, , \tag{3}$$

where n is the normal to the obstacle surface, $f(s)$ is a function of position s on it and T_0 is a characteristic temperature (or concentration). When one of the non-dimensional constants M and N vanish either T or the flux $(\partial T/\partial n)$ is prescribed, but in general both constants are non-zero. Since $(\partial\varphi/\partial n)$ vanishes on the obstacle surface, condition (3) reduces to

$$M\,\Theta + N(\partial\Theta/\partial n) = T_0\, f(s)\, \exp(Pe\,\varphi/2) \; . \tag{4}$$

Two Dimensional Cases

Under Boussinesq's[4] transformation equation (2) reduces to

$$\left[\partial^2/\partial\varphi^2 + \partial^2/\partial\psi^2 - (Pe/2)^2\right]\Theta = 0 \qquad (2')$$

where ψ is the Lagrange's stream function, and the complex conjugate of φ . If the flow past the cylinder is either symmetric with respect to x_2 = const. or one which does not form a wake, the origin of the co-ordinates (φ, ψ) can be so chosen that the cross-section of the wet surface is $\psi = \pm o$, $-a < \varphi < a$. In terms of the co-ordinates (ξ, η) defined by

$$\varphi = a\cosh\xi\cos\eta, \qquad \psi = a\sinh\xi\sin\eta, \qquad (5)$$

this boundary is $\xi = o$. Solutions for Θ in terms of Mathieu and related function of η and ξ , respectively, are available for a large class of boundary conditions.

Consider the elementary case of an isothermal cylinder, so that $\beta(s)$ and M are equal to unity and N vanishes. It is further assumed that the temperature of the stream at infinity, T_∞, is also uniform. The solution of equation (2') has the following form

$$\Theta = (T_o - T_\infty)\sum_{p=o}^{\infty} a_p\, Fek_p(\xi, -(Pe/2)^2)\, ce_p(\eta, -(Pe/2)) + T_\infty \exp(Pe\,\varphi/2). \qquad (6)$$

Using Ince-Goldstein notation, the functions ce_p and Fek_p are defined by:

$$
\left.
\begin{aligned}
ce_{2q}(\eta, -(Pe/2)^2) &= (-1)^q \sum_{i=0}^{\infty} (-1)^i A_{2i}^{(2q)} \cos(2i\eta) \\
ce_{2q+1}(\eta, -(Pe/2)^2) &= (-1)^q \sum_{i=0}^{\infty} (-1)^i B_{2i+1}^{(2q+1)} \cos((2i+1)\eta) \\
Fek_{2q}(\xi, -(Pe/2)^2) &= (-1)^q \sum_{i=0}^{\infty} (-1)^i A_{2i}^{(2q)} K_{2i}(Pe \cosh \xi) \\
Fek_{2q+1}(\xi, -(Pe/2)^2) &= (-1)^q \sum_{i=0}^{\infty} (-1)^i B_{2i+1}^{(2q+1)} K_{2i+1}(Pe \cosh \xi)
\end{aligned}
\right\} \quad (7)
$$

where K_p is modified Bessel Function of the second kind. The constants $A_{2i}^{(2q)}$ and $B_{2i+1}^{(2q+1)}$ satisfy well known recurrence relationships and are normalized thus

$$
\left(A_0^{(2q)}\right)^2 + \sum_{i=0}^{\infty}\left(A_{2i}^{(2q)}\right)^2 = \sum_{i=0}^{\infty}\left(B_{2i+1}^{(2q+1)}\right)^2 = 1 \qquad (8)
$$

Since the functions Fek_p approach zero as ξ is increased indefinitely the assumed form satisfies the condition of isothermal stream at infinity. The conditions at the solid surface yields

$$
\exp\left(Pe\, a(\cos\eta)/2\right) = \sum_{p=0}^{\infty} a_p \, Fek_p(0, -(Pe/2)^2)\, ce_p(\eta, -(Pe/2)^2) \qquad (9)
$$

The constant coefficients a_p are thus found to be given in terms of the modified Bessel Functions of the first kind by

$$a_{2q} = [\, Fek_{2q}(0, -(Pe/2)^2)]^{-1} 2(-1)^q \sum_{i=0}^{\infty} (-1)^i A_{2i}^{(2q)} I_{2i}(Pe\,a/2)$$

$$a_{2q+1} = [\, Fek_{2q+1}(0, -(Pe/2)^2)\, 2(-1)^q \sum_{i=0}^{\infty} (-1)^i B_{2i+1}^{(2q+1)} I_{2i+1}(Pe\,a/2) \qquad (10)$$

In deriving these, use is made of the orthogonality of the Mathier Functions, and the identity

$$I_p(z) = (2\pi)^{-1} \int_{-\pi}^{\pi} exp(z\eta) \cos(p\eta)\, d\eta \qquad (11)$$

More complicated integrals are encountered when $f(s)$ is not uniform. The resulting distribution $T(\varphi, \Psi)$ for $Pe = 1$ is plotted in Figure 1.

While the solution just obtained holds for isothermal cylinder of arbitrary shape it is impossible to obtain such general solution for cases in which M, N and $f(s)$ do not vanish. In terms of (ξ, η) equation (4) reads

$$M\Theta + N\,(a \sin\eta)^{-1} (\partial\Psi/\partial n)(\partial\Theta/\partial\xi) = T_0 f(s) \cdot exp(Pe(\cos\eta)/2). \quad (4')$$

The term $(\partial\Psi/\partial n)$ is equal to the velocity tangential to the obstacle surface which vary from one cross section to another. The applicability of the proposed methods to the cases in which the general linear condition hold is therefore demonstrated by considering the case of circular cylinders. For unseparated flow the potential and stream functions are given by

$$\varphi = (\rho + \rho^{-1}) \cos\gamma \qquad \Psi = (\rho - \rho^{-1}) \sin\gamma \qquad (12)$$

where (ρ, γ) are polar co-ordinates and L is so chosen that $\rho = 1$ is the cylindrical surface. Comparison of equations (5) and (12) implies that the following identities hold

$$a = 2 \qquad \ln \rho = \xi \qquad \gamma = \eta \qquad (\partial\psi/\partial n)_{\rho=1} = 2\sin\eta$$

Therefore if $f(s)$ is again unity the constants a_p differ from those given by equation (10) only in their being proportional to $[\,M\,\mathrm{Fek}_p(0, -(Pe/2)^2) \;+\; N\,\mathrm{Fek}_p'(0, -(Pe/2)^2)\,]^{-1}$ rather than to $[\,\mathrm{Fek}_p(0, -(Pe/2)^2)\,]^{-1}$.

It is finally remarked that the assumed form (6) holds whenever the temperature (or mass) distribution in the flow field is symmetric with respect to $\psi = 0$. This condition is satisfied when the temperatures at infinity and at the solid surface are uniform. Nevertheless, the flow pattern and the geometry of the cross section need not be symmetric with respect to any straight line in the (x_1, x_2) plane. For example, when the temperature at infinity and at the solid surface are uniform the form (6) holds for unseparated flow past almost any airfoil. If there is separation this form holds if there is no transfer across the resulting wake $\psi = 0$ $\varphi < -a$, and the pattern is symmetric with respect to the line x_2 = const. In such cases the zero flux condition is satisfied because $ce_p'(\pi, -(Pe/2)^2)$ vanish. In such cases T and Θ appear to be continuous across $\eta = \pi$ but physically the two branches of this curve enclose a region which is not part of the domain under discussion. The more general cases, when T and Θ are not symmetric, can be solved by generalizing the expansion (6) to include also products of Mathieu and related functions which are anti-symmetric with respect to $\eta = 0$ and $\eta = \pi$.

Temperature in the Flow Past an Isothermal Sphere

Consider a sphere of uniform temperature T_0. Let the temperature of the flow past it be uniform at infinity so that the following hold

$$\Theta = T_0 \exp(3/4\, Pe \cos\theta), \qquad r = 1, \tag{13}$$

$$\Theta = T_0 \exp(1/2\, Pe \cos\theta), \qquad r \to \infty. \tag{14}$$

Spherical coordinates are used so that the solid surface is defined by $r=1$ and the governing equation reads

$$\left\{ \frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} - \left(\frac{Pe}{2}\right)^2 \left[1 + \right.\right.$$

$$\left.\left. + \frac{1}{r^3}(1 - 3\cos^2\theta) + \frac{1}{4}\frac{1}{r^6}(1 + 3\cos^2\theta)\right]\right\} \Theta = 0 \tag{2'}$$

Solution of the following form is assumed

$$\Theta = (T_0 - T_\infty)\sum_{i=0}^{\infty} \Theta_i (Pe/2)^{2i} + T_\infty \exp[(Pe/2)(r + 1/2\, r^{-2})\cos\theta] \tag{15}$$

where the multipler of $(Pe/2)$ in the argument of the exponent is equal to φ. The functions Θ_i for $i \geq 0$ are made to satisfy

$$\left[\frac{1}{r}\frac{\partial^2}{\partial r^2}r + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} - \left(\frac{Pe}{2}\right)^2\right]\Theta_i = \left[\frac{1}{r^3}\left(1-3\cos^2\theta\right)+\frac{1}{4}\frac{1}{r^6}\left(1+3\cos^2\theta\right)\right]\Theta_{i-1} \qquad (16_i)$$

The functions Θ_i for $i > 0$ are homogeneous at the two boundaries so that equations (13) and (14) are satisfied by prescribing the appropriate conditions on Θ_0 .

In view of equation (16_0) (and the inexistance and hence the vanishing of Θ_{-1}) Θ_0 can be expressed by the form

$$\Theta_0 = \sum_{p=0}^{\infty} C_p \, r^{-1/2} K_{p+1/2}(Pe\, r/2)\, P_p(\cos\theta) \qquad (17)$$

where $P_p(\cos\theta)$ are Legendre Polynonials. The constants C_p are obtained from the boundary conditions at the solid surface which yield

$$\sum_{p=0}^{\infty} C_p K_{p+1/2}(Pe/2)\, P_p(\cos\theta) = \exp\left(3/4\, Pe \cos\theta\right)$$

By expressing P_p in terms of $\cos\left[(p-2k)\theta\right]$ and utilizing equation (11) these constants are found to be given by

$$C_p = \frac{(2p+1)}{K_{p+1/2}(Pe/2)} \Bigg\{ \delta_{p0} I_0(3/4\, Pe) - \sum_{i=1}^{\infty} I_i(3/4\, Pe) \times$$

$$\times \sum_{p=0}^{t} \frac{\varepsilon}{2}\left(1 + (-1)^{p+2k+i}\right)\left[\left((p+i+2k)^2-1\right)^{-1} + \left((p-i-2k)^2-1\right)^{-1}\right] \times$$

$$\times \frac{\langle 2p-2k-1\rangle\langle 2k-1\rangle}{\langle 2p-2k\rangle\langle 2k\rangle}\Bigg\} . \qquad (18)$$

where δ_{pq} is the Kroneker delta. The upper limit t is an integer which is equal either to $p/2$ or $(p-1)/2$ and ε is equal to 2 and 1 when $2k \neq p$ and $2k = 0$, respectively. The bracketed quantities are defined thus:

$$\langle 2n \rangle \equiv n!\,2^n \qquad \langle 2n-1 \rangle = (2n)!/\langle 2n \rangle$$

The function Θ_1 is solved by expressing the right hand side of equation (16$_1$) as a sum of terms of the form $r^{-5/2} K_{\nu+1/2}(P_e r/2)\, P_\mu(\cos\theta)$. Once this is done the solution for Θ_1 can be expressed as the sum of functions $\chi_{\nu+1/2,\mu}$ which satisfy the equation

$$\left[\frac{1}{r}\frac{\partial^2}{\partial r^2} r + \frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} - \left(\frac{P_e}{2}\right)^2\right]\chi_{\nu+1/2,\mu} = r^{-5/2} K_{\nu+1/2}(P_e r/2)\, P_\mu(\cos\theta)$$

are finite at infinity and vanish on $r = 1$. These are given by

$$\chi_{\nu+1/2,\mu} = r^{-1/2}\left[\nu(\nu+1) - \mu(\mu+1)\right]^{-1}\left(K_{\nu+1/2}(P_e r/2)\, K_{\mu+1/2}(P_e/2) - K_{\nu+1/2}(P_e/2)\, K_{\mu+1/2}(P_e r/2)\right) P_\mu(\cos\theta). \quad (20)$$

and when $\nu = \mu$ $\chi_{\nu+1/2,\mu}$ is given by the limit approached as $\nu \to \mu$.

In rearranging the right hand side of equation (16$_1$) use is made of the relationship

$$\cos^2\theta\, P_p(\cos\theta) = \sum a_\alpha^p\, P_{p+2-2\alpha}(\cos\theta) \qquad (21)$$

The coefficients a_α^p are given by

$$a_\alpha^p = \sum_{j=0}^{\alpha,t} (-1)^j \frac{(2p+5-4\alpha)\,\langle 2p-2j-1\rangle\ (p+2-2j)!}{(p-2j)!\,\langle 2j\rangle\,\langle 2p+5-2\alpha-2j\rangle\,\langle 2\alpha-2j\rangle} \tag{22}$$

where the upper limit is α or t , whichever is smaller. Using the well known recurrance relationship the following can easily be shown to hold

$$r^{-7/2} K_{p+1/2}(p_e r/2) = r^{-5/2}\left(b_{p,1}\, K_{p+3/2}(p_e r/2) + b_{p,-1}\, K_{p-1/2}(p_e r/2)\right),$$

$$r^{-13/2} K_{p+1/2}(p_e r/2) = r^{-5/2} \sum_{i=-2}^{2} C_{p,2i}\, K_{p+2i+1/2}(p_e r/2), \tag{23}$$

where

$$b_{p,1} = -b_{p,-1} = (2p+1)^{-1}$$

$$C_{p,\pm 4} = (2p+1)^{-1}(2p+1\pm 2)^{-1}(2p+1\pm 4)^{-1}(2p+1\pm 6)^{-1}$$

$$C_{p,\pm 2} = -\frac{1}{(2p+1)\,(2p+1\pm 2)^2}\left[\frac{4p+2}{(2p+1)(2p+1\pm 2)} + \frac{4p+2\pm 8}{(2p+1\pm 4)\,(2p+1\pm 6)}\right] \tag{24}$$

$$C_{p,0} = \frac{1}{(2p+1)}\left[\frac{1}{(2p+5)\,(2p+3)^2} + \frac{(4p+2)^2}{(2p+3)^2(2p+1)(2p-1)^2} + \frac{1}{(2p-1)^2(2p-3)}\right]$$

The solution for Θ_1 is therefore

$$\Theta_1 = \sum_{p=0}^{\infty} C_p \left\{ \sum_{\alpha=0}^{t+1} (\delta_{\alpha 1} - 3a_\alpha^p)\left[b_{p,1}\, \chi_{p+3/2,\, p+2-2\alpha} + b_{p,-1}\, \chi_{p-1/2,\, p+2-2\alpha}\right]\right.$$

$$\left. + \frac{1}{4} \sum_{\alpha=0}^{t+1} (\delta_{\alpha 1} + 3a_\alpha^p) \sum_{i=-2}^{2} C_{p,2i}\, \chi_{p+2i+1/2,\, p+2-2\alpha} \right\}. \tag{25}$$

Functions Θ_i for $i > 0$ will not be evaluated within the framework of this treatise. The foregoing solution nevertheless outlines one of the methods one could use. The general scheme implied by equations (15) and (16_i) is fairly well known. Its convergence and the other methods for solving Θ_i are discussed by Morse and Feshbach[6].

Discussion

Unlike any of the available solutions those obtained or outlined here are not restricted to any range of Pe and hold when it is small. Thus for very ineffective convection the equation (15), (17) and (18) reduces to

$$T = (T_0 - T_\infty)\, r^{-1} + T_\infty + O(Pe^1) \qquad (26)$$

The contribution to the right hand side which is of $O(Pe^0)$ is the spherically symmetric distribution of temperature or concentration in a stagnant medium. Understandably it is the effect of convection, as reflected by terms of $O(Pe^k)$, $k > 0$, which gives rise to asymmetry with respect to $\theta = \pi/2$. The situation is somewhat dissimilar in the case of flow past an isothermal cylinder. When Pe is small $A_{2i}^{(2q)}$ and $B_{2i+1}^{(2q+1)}$ are negligible unless $i = q$. Therefore, in view of equation (10) again the first term in expansion (6) is the predominant and the distribution is essentially η-independent. Terms which represent η variations are smaller by an order of magnitude. However, in this case one obviously cannot carry the limiting process to the extreme, $Pe = 0$, because there is no two-dimensional solution for the diffusion in a stagnant infinitely wide medium.

The features of the solutions just discussed have interesting counterparts in the theory of laminar flow past obstacles. In such cases Reynolds number, Re, is the measure of the effectiveness of momentum transfer by convection as compared to its diffusion by viscosity. When Re vanish there is no solution for the flow past a cylinder but there is for flow past a sphere. Just as equations (15) (17) and (18) reduces to equation (26) when $Pe \rightarrow 0$ so is the Stokes flow past a sphere believed to be the limit of the viscous flow solution when $Re \longrightarrow 0$. However, it is impossible to analyze the viscous flow or temperature distribution around a sphere by perturbations, with the zeroth perturbation as the "convectionless" (zero Re or Pe) solution.

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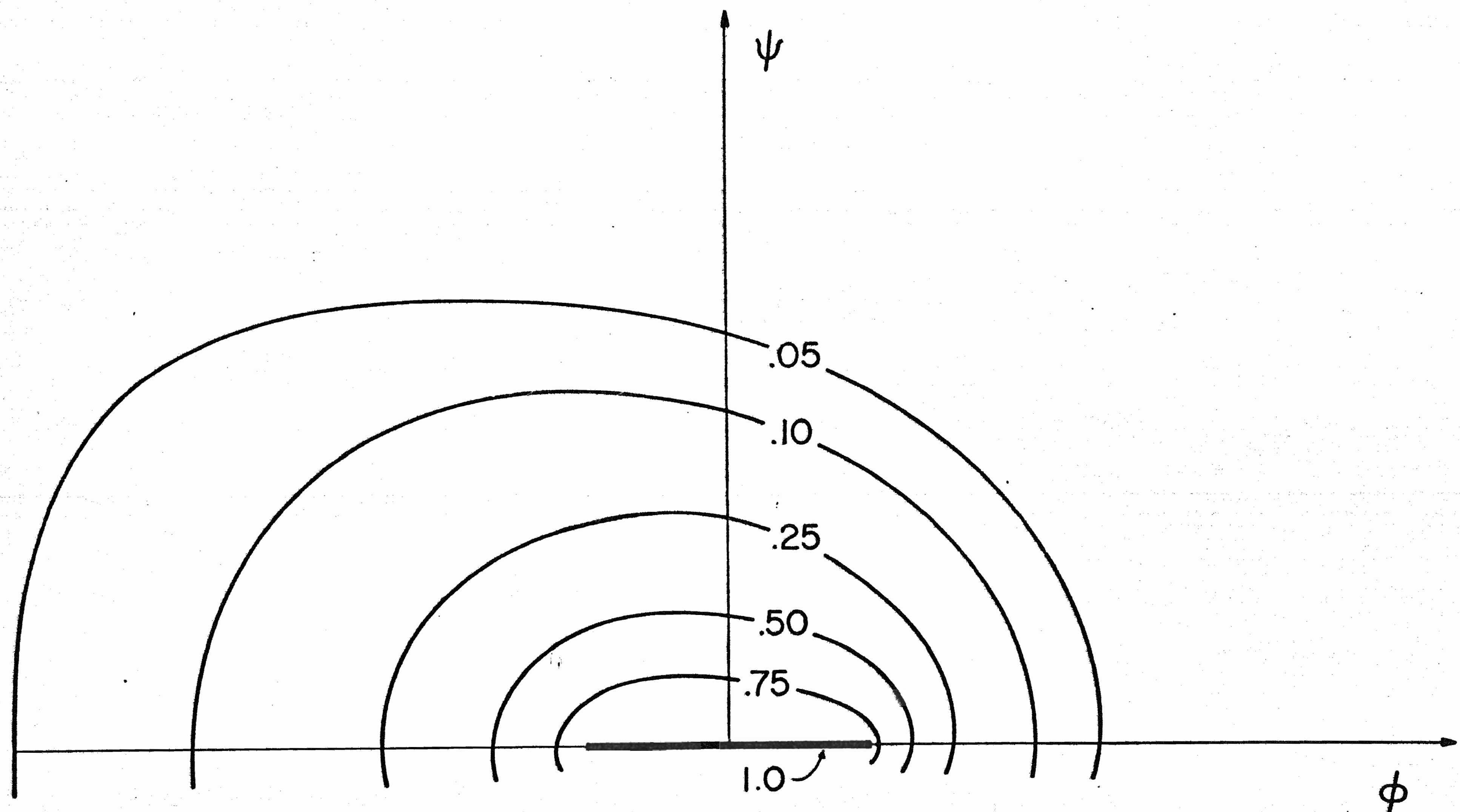

Fig. 1. Temperature Distribution $T(\varphi, \psi)$ in the Flow Past an Isothermal Two dimensional Obstacle. $T_o = 1$, $T_\infty = 0$