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FREE CONVECTION OF A RADIATION-ABSORBING FLUID

by

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ABSTRACT - An analysis has been made to determine the effect of thermal radiation upon the free-convection boundary layer on a vertical flat plate. Laminar free convection of an absorbing-emitting gas is considered, and second-order interaction effects between the convection and radiation processes are evaluated for $Pr = 1.0$. The plate surface is assumed to be isothermal and black, while the gas is assumed to be a gray absorber and emitter.

NOMENCLATURE

a,	absorption coefficient
c,	constant, $(g\beta\Delta T/4\nu^2)^{\frac{1}{4}}$
c_p,	specific heat at constant pressure
e,	black body emissive power, σT^4
$E_n(t)$,	exponential integral, $\int_0^1 \mu^{n-2}\exp(-t/\mu)d\mu$
f,	dimensionless stream function, $\psi/4\nu cx^{3/4}$
g,	acceleration of gravity
Gr,	Grashof number, $g\beta\Delta Tx^3/\nu^2$
H_o,	function defined by equation (30)
H_1,	function defined by equation (31)
k,	thermal conductivity
Nu,	convective Nusselt number, $q_{cw}x/k(T_w-T_\infty)$
Pr,	Prandtl number, ν/α
q,	local heat transfer rate per unit area
t,	dummy variable of integration
T,	absolute temperature
ΔT,	T_w-T_∞
u,	velocity component in x-direction
v,	velocity component in y-direction
x,	coordinate along plate surface
y,	coordinate normal to plate surface
α,	thermal diffusivity
β,	coefficient of thermal expansion
γ,	temperature ratio, T_w/T_∞
Γ,	$\left[(\gamma^4-1)/(\gamma-1)\right]^{1/3}$

δ, boundary layer thickness

ζ, $ax^{\frac{1}{4}}/c$

η, $cy/x^{\frac{1}{4}}$

θ, dimensionless temperature

ν, kinematic viscosity

ξ, $4\sigma a T_{\infty}^{3} x^{\frac{1}{2}}/\rho c_p (g\beta\Delta T)^{\frac{1}{2}}$

ρ, density

σ, Stefan-Boltzmann constant

τ, optical distance, ay

ψ, stream function

Subscripts

c, convection

r, radiation

w, plate surface

δ, outer edge of boundary layer

∞, ambient

INTRODUCTION

Convection phenomena involving fluids which absorb and emit thermal radiation is an area which has recently attracted considerable investigation with respect to forced convection. There have not, however, been any studies made concerning combined radiation and free convection heat transfer. Since radiation interaction is most pronounced when convection heat transfer is small, it would appear that radiation could play a significant role in free convection problems involving absorbing fluids.

The present investigation considers laminar free convection of an absorbing-emitting fluid from a vertical flat plate. The method of solution is patterned after that presented in references [1] and [2] for forced convection across a flat plate, and this assumes that the convection boundary layer is optically thin. Second-order interaction effects between free convection and radiation heat transfer are considered. The purpose of the investigation is primarily to illustrate how such interaction effects arise, and thus a number of simplifying assumptions, such as a black plate and a gray fluid, are employed. Admittedly, assuming the absorbing-emitting medium to be gray is often far from realistic.

ANALYSIS

Theoretical Model

The physical model and coordinate system are illustrated in Fig. 1 for a heated plate, while for a cooled plate the y-coordinate is reversed. Laminar free convection of a constant property fluid is assumed, and the surface temperature of the plate is taken to be uniform. It is further assumed that thermal conduction and viscous effects are restricted to a thin region of thickness δ adjacent to the plate surface, which is simply the conventional free-convection boundary layer, and that this boundary layer is optically thin. The optically thin boundary layer, however, represents only a portion of the entire temperature and velocity fields within the fluid. This is due to the fact that radiation emitted by the plate will pass virtually unattenuated through the boundary layer, such that temperature and velocity fields are established outside of this layer. It is therefore necessary to consider an adjacent radiation layer which is not optically thin but within which temperature and velocity gradients, and thus conduction and viscous effects, are assumed to be small.

In other words, it has been assumed that thermal conduction and viscous shear are restricted to a boundary layer whose thickness is much less than the radiation penetration length. Adjacent to this boundary layer is a radiation layer whose thickness is large in comparison with the boundary layer thickness, and within which thermal conduction and viscous shear are negligible. In carrying out the solution, the temperature and velocity profiles within the radiation layer must first be determined. From these the temperature and velocity at the outer edge of the boundary layer are obtained, and the boundary layer solution follows.

Several assumptions will now be made regarding the radiation transfer process:

1. The fluid is gray, nonscattering, a diffuse absorber and emitter, and has an index of refraction of unity.
2. The plate surface is black.
3. Net radiation in the x-direction within the fluid is negligible.

A discussion of the first assumption is given in [2], while an order of magnitude analysis concerning the third assumption is discussed in [1] for forced convection. If the same procedure is applied to free convection, it is found that radiation transfer in the x-direction may be neglected providing the condition

$$\left[\frac{\rho a c_p x}{\sigma T^3}\left(\frac{\sigma a T_\infty^3}{\rho c_p}\right) g\beta \Delta T x^2\right]^{1/3} >> 1$$

is satisfied.

The equations expressing conservation of mass, momentum and energy may now be written, respectively, as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T-T_\infty) \qquad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \qquad (3)$$

where q_r denotes the radiation flux within the fluid. From [1] the quantity $-\partial q_r/\partial\tau$, with τ representing the optical distance ay, is given by

$$-\frac{\partial q_r}{\partial\tau} = 2e_w E_2(\tau) + 2\int_0^\infty e(x,t)E_1(|\tau-t|)dt - 4e(x,\tau) \qquad (4)$$

As previously discussed, it is first necessary to obtain velocity and temperature solutions for the radiation layer.

Radiation Layer

Equations (1) to (4) apply to the radiation layer providing the terms $\nu\partial^2u/\partial y^2$ and $\alpha\partial^2T/\partial y^2$ are deleted in equations (2) and (3), respectively. Eliminating equation (1) in the usual manner through use of the stream function ψ, defined as

$$u = \frac{\partial\psi}{\partial y} \quad , \quad v = \frac{\partial\psi}{\partial x}$$

then equations (2), (3) and (4) yield

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} = g\beta(T-T_\infty) \qquad (5)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \frac{2\sigma a}{\rho c_p}\left[T_w^4E_2(\tau) + \int_0^\infty T^4(x,t)E_1(|\tau-t|)dt - 2T^4\right] \qquad (6)$$

with the boundary conditions

$$T = T_\infty, \ \frac{\partial\psi}{\partial y} = 0 \ ; \ x = 0$$

$$T = T_w, \ \frac{\partial\psi}{\partial y} = 0 \ ; \ y = \infty$$

$$\frac{\partial \psi}{\partial x} = 0 \; ; \; y = 0$$

Dimensionless quantities will now be defined as

$$\xi = \frac{4\sigma a T_\infty^3 x^{\frac{1}{2}}}{\rho c_p (g\beta\Delta T)^{\frac{1}{2}}} \quad , \; \gamma = \frac{T_w}{T_\infty} \; , \; \Gamma = \left(\frac{\gamma^4 - 1}{\gamma - 1} \right)^{1/3}$$

and solutions of equations (5) and (6) are assumed of the form

$$\psi = \Gamma \left(\frac{g\beta\Delta T x}{a^2} \right)^{\frac{1}{2}} \left[F_o(\tau)\xi^{1/3} + F_1(\tau)\xi + \ldots \right] \tag{7}$$

$$\frac{T}{T_\infty} = 1 + (\gamma - 1)\Gamma^2 \left[G_o(\tau)\xi^{2/3} + G_1(\tau)\xi^{4/3} + \ldots \right] \tag{8}$$

Upon substituting equations (7) and (8) into equations (5) and (6) and collecting like powers of ξ, one obtains the ordinary differential equations

$$\left.\begin{aligned} F_o F_o'' - (F_o')^2 &= -\frac{3}{2} G_o \\ 2F_o G_o' - F_o' G_o &= -\frac{3}{2} E_2(\tau) \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} F_o F_1'' - \frac{5}{2} F_o' F_1' + \frac{3}{2} F_o'' F_1 &= -\frac{3}{2} G_1 \\ F_o G_1' - F_o' G_1 &= \frac{1}{2} F_1' G_o - \frac{3}{2} F_1 G_o' \\ &+ \frac{2}{\Gamma} \left[2G_o - \int_0^\infty G_o(t) E_1(|\tau - t|)\,dt \right] \end{aligned}\right\} \tag{10}$$

The boundary conditions are in turn

$$F_o(0) = F_1(0) = 0$$

$$G_o(\infty) = G_1(\infty) = F_o'(\infty) = F_1'(\infty) = 0$$

Since the boundary layer is assumed to be optically thin ($\tau_\delta = a\delta << 1$), then $F_o(\tau_\delta) \simeq F_o(0)$, $G_o(\tau_\delta) \simeq G_o(0)$, etc. Letting u_δ and T_δ denote the velocity and temperature at the outer edge of the boundary layer, equations (7) and (8) give

$$U_\delta = \Gamma(g\beta\Delta Tx)^{\frac{1}{2}} \left[F_o'(0)\xi^{1/3} + F_1'(0)\xi + \ldots \right] \tag{11}$$

$$\frac{T_\delta}{T_\infty} = 1 + (\gamma-1)\Gamma^2 \left[G_o(0)\xi^{2/3} + G_1(0)\xi^{4/3} + \ldots \right] \tag{12}$$

These expressions thus constitute the appropriate boundary conditions to be employed at the outer edge of the free convection boundary layer.

Boundary Layer

In considering the solution of equations (2) and (3) for the optically thin boundary layer, additional dimensionless quantities will be defined as

$$\eta = \frac{cy}{x^{\frac{1}{4}}} , \quad c = \left(\frac{g\beta\Delta T}{4\nu^2} \right)^{\frac{1}{4}}$$

$$F(\xi,\eta) = \frac{\psi(x,y)}{4\nu c x^{3/4}} , \quad \theta(\xi,\eta) = \frac{T(x,y)}{T_\infty}$$

where η is the conventional similarity variable for free convection from a flat plate in the absence of radiation. In terms of $f(\xi,\eta)$ and $\theta(\xi,\eta)$, equations (2) and (3) transform to

$$\frac{\partial^3 f}{\partial\eta^3} + 3f\frac{\partial^2 f}{\partial\eta^2} - 2\left(\frac{\partial f}{\partial\eta}\right)^2 + 2\xi\frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial\xi} - 2\xi\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta} = \frac{1-\theta}{\gamma-1} \qquad (13)$$

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + 3f\frac{\partial\theta}{\partial\eta} + 2\xi\frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta} - 2\xi\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi} = \frac{\xi}{2\sigma T_\infty^4}\frac{\partial q_r}{\partial\tau} \qquad (14)$$

while the velocity components are given by

$$u = \frac{\partial\psi}{\partial y} = 4\nu c^2 x^{\frac{1}{2}}\frac{\partial f}{\partial\eta} \qquad (15)$$

$$v = -\frac{\partial\psi}{\partial x} = \nu c x^{-\frac{1}{4}}\left(\eta\frac{\partial f}{\partial\eta} - 3f - 2\xi\frac{\partial f}{\partial\xi}\right) \qquad (16)$$

It remains to simplify the absorption-emission term, $\partial q_r/\partial\tau$, in accord with the assumption that the boundary layer is optically thin. Recalling that $\tau_\delta = a\delta$, and splitting the integral in equation (4) into two parts, one may write

$$-\frac{\partial q_r}{\partial\tau} = 2e_w E_2(\tau) + 2\int_0^{\tau_\delta} e(x,t)E_1(|\tau-t|)dt$$
$$+ 2\int_{\tau_\delta}^{\infty} e(x,t)E_1(|\tau-t|)dt - 4e(x,\tau) \qquad (17)$$

The emissive power occuring in the second integral corresponds to the temperature distribution within the radiation layer. Thus, from equation (8)

$$e(x,\tau) = e_\infty\left[1 + 4(\gamma-1)\Gamma^2 G_o(\tau)\xi^{2/3} + \ldots\right] \tag{18}$$

for $\tau > \tau_\delta$. Upon substituting this into the second integral in equation (17), noting that $\tau_\delta << 1$, and evaluating the integral for $\tau < \tau_\delta$, one has

$$\int_{\tau_\delta}^{\infty} e(x,t)E_1(|\tau-t|)dt = 2e_\infty + O(\xi^{2/3})$$

The first integral in equation (17) is of order τ_δ and may thus be neglected. With these simplifications, equation (17) reduces to

$$-\frac{\partial q_r}{\partial \tau} = 2\sigma T_\infty^4(\gamma^4 + 1 - 2\theta^4) + O(\xi^{2/3}) \tag{19}$$

which is the form applicable for use in equation (14).

Solutions of equations (13) and (14) will now be assumed as

$$f(\xi,\eta) = f_o(\eta) + f_1(\eta)\Gamma\xi^{1/3} + f_2(\eta)\Gamma^2\xi^{2/3} + \ldots \tag{20}$$

$$\theta(\xi,\eta) = 1 + (\gamma-1)\left[\theta_o(\eta) + \theta_1(\eta)\Gamma\xi^{1/3} + \theta_2(\eta)\Gamma^2\xi^{2/3} + \ldots\right] \tag{21}$$

Upon substituting equations (19), (20) and (21) into equations (13) and (14) and collecting like powers of ξ, it is found that

$$\left.\begin{aligned} f_o''' + 3f_o f_o'' - 2(f_o')^2 &= -\theta_o \\ \frac{1}{Pr}\theta_O'' + 3f_o\theta_o' &= 0 \end{aligned}\right\} \tag{22}$$

$$\left.\begin{aligned} & f_1''' + 3f_o f_1'' - \frac{14}{3} f_o' f_1' + \frac{11}{3} f_o'' f_1 = -\theta_1 \\ & \frac{1}{Pr}\theta_1'' + 3f_o\theta_1' - \frac{2}{3} f_o'\theta_1 = -\frac{11}{3} f_1\theta_o' \end{aligned}\right\} \quad (23)$$

$$\left.\begin{aligned} & f_2''' + 3f_o f_2'' - \frac{16}{3} f_o' f_2' + \frac{13}{3} f_o'' f_2 = -\theta_2 + \frac{8}{3}(f_1')^2 - \frac{11}{3} f_1 f_1'' \\ & \frac{1}{Pr}\theta_2'' + 3f_o\theta_2' - \frac{4}{3} f_o'\theta_2 = -\frac{11}{3} f_1\theta_1' - \frac{13}{3} f_2\theta_o' + \frac{2}{3} f_1'\theta_1 \end{aligned}\right\} \quad (24)$$

To obtain boundary conditions for these equations, it may be noted that the original boundary conditions are

$$y = 0 : u = v = 0 \,, \quad T = T_w$$

$$y = \infty : u = u_\delta \,, \quad T = T_\delta$$

Consequently, from equations (11), (12), (15), (16), (20) and (21) one finds that

$$f_o(0) = f_1(0) = f_2(0) = f_o'(0) = f_1'(0) = f_2'(0) = 0$$

$$\theta_o(0) = 1, \; \theta_1(0) = \theta_2(0) = 0$$

$$f_o'(\infty) = 0, \; f_1'(\infty) = \tfrac{1}{2}F_o'(0), \; f_2'(\infty) = 0$$

$$\theta_o(\infty) = \theta_1(\infty) = 0, \; \theta_2(\infty) = G_o(0)$$

As would be expected, the functions $f_o(\eta)$ and $\theta_o(\eta)$ correspond to free convection in the absence of radiation, while the expansion

$$\left.\begin{aligned} & f_1''' + 3f_of_1'' - \frac{14}{3} f_o'f_1' + \frac{11}{3} f_o''f_1 = -\theta_1 \\ & \frac{1}{Pr}\theta_1'' + 3f_o\theta_1' - \frac{2}{3} f_o'\theta_1 = -\frac{11}{3} f_1\theta_o' \end{aligned}\right\} \quad (23)$$

$$\left.\begin{aligned} & f_2''' + 3f_of_2'' - \frac{16}{3} f_o'f_2' + \frac{13}{3} f_o''f_2 = -\theta_2 + \frac{8}{3}(f_1')^2 - \frac{11}{3} f_1f_1'' \\ & \frac{1}{Pr}\theta_2'' = 3f_o\theta_2' - \frac{4}{3}f_o'\theta_2 = -\frac{11}{3} f_1\theta_1' - \frac{13}{3} f_2\theta_o' + \frac{2}{3}f_1'\theta_1 \end{aligned}\right\} \quad (24)$$

To obtain boundary conditions for these equations, it may be noted that the original boundary conditions are

$$y = 0 : u = v = 0 \ , \ T = T_w$$

$$y = \infty : u = u_\delta \ , \quad T = T_\delta$$

Consequently, from equations (11), (12), (15), (16), (20) and (21) one finds that

$$f_o(0) = f_1(0) = f_2(0) = f_o'(0) = f_1'(0) = f_2'(0) = 0$$

$$\theta_o(0) = 1, \ \theta_1(0) = \theta_2(0) = 0$$

$$f_o'(\infty) = 0, \ f_1'(\infty) = \tfrac{1}{2}F_o'(0), \ f_2'(\infty) = 0$$

$$\theta_o(\infty) = \theta_1(\infty) = 0, \ \theta_2(\infty) = G_o(0)$$

As would be expected, the functions $f_o(\eta)$ and $\theta_o(\eta)$ correspond to free convection in the absence of radiation, while the expansion parameter ξ appearing in equations (20) and (21) constitutes a measure of the importance of radiation versus convection. Before proceeding, it is of interest to discuss the separate mechanisms by which radiation alters the free convection boundary layer, and to illustrate how each of these enter into the expansions in powers of $\xi^{1/3}$ of equations (20) and (21). There are three such mechanisms, and these are as follows:

1. The induced velocity $u_\delta(x)$ at the outer edge of the boundary layer. From equation (11) it is seen that this is a first-order radiation effect, since it initially appears as a term of order $\xi^{1/3}$.

2. The variable temperature $T_\delta(x)$ imposed at the outer edge of the boundary layer. Since this first occurs as a $\xi^{2/3}$ term, as can be noted from equation (12), it is consequently a second-order radiation effect.

3. The absorption and emission of radiant energy within the boundary layer. From equation (19) and the right side of equation (14), it is found that this effect is first present through terms of order ξ in equations (20) and (21). The absorption-emission process within the boundary layer is therefore a third-order effect and does not appear in the present second-order analysis.

It is worth noting that the temperature ratio $\gamma = T_w/T_\infty$ is not present in equations (23) and (24), but enters into the boundary layer solution only through the quantity Γ in equations (20) and (21). If, however, third-order terms were retained in equations (20) and (21), the resulting appearance of the nonlinear absorption-emission term, equation (19), would introduce γ into the differential equations describing $f_3(\eta)$ and $\theta_3(\eta)$.

Numerical Results

The present second-order boundary layer analysis requires the integration of equations (9), (23) and (24), and this has been accomplished numerically on an IBM 1620 electronic computer. In the case of equation (9), a backward integration was employed since two of the three boundary conditions are located at infinity. The backward integration was additionally necessary since equations (9) indicate a possible singularity at the origin*. Fig. 2 illustrates F_o' and G_o, and the quantities $F_o'(0)$ and $G_o(0)$ which appear in the boundary conditions for equations (23) and (24), were found to have the values

$$F_o'(0) = 1.23 \quad , \quad G_o(0) = 1.01$$

It should be noted that the solution of equations (9) is independent of Prandtl number.

Equations (23) and (24) have been solved numerically for $Pr = 1.0$ using a conventional forward integration together with the tables of $f_o(\eta)$ and $\theta_o(\eta)$ given in reference [3]. The quantities f_o', f_1' and f_2' are illustrated in Fig. 4. In addition, the values

$$\theta_o'(0) = -0.567, \; \theta_1'(0) = -0.072, \; \theta_2'(0) = 0.091 \qquad (25)$$

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* It can be shown that G_o is logarithmic at the origin.

will be required in the following section.

HEAT TRANSFER RESULTS

As in the forced convection analysis of [1], it will be convenient to consider separately the convection and radiation heat transfer at the plate surface, and the net radiation transfer between the plate and the absorbing fluid will be considered first. In this respect, it will be necessary to introduce a new dimensionless variable ζ defined as

$$\zeta = \frac{ax^{\frac{1}{4}}}{c}$$

Recalling that the boundary layer thickness for pure free convection is proportional to $x^{\frac{1}{4}}/c$, then ζ is essentially a measure of the optical thickness of the free convection boundary layer. Thus, it is required that ζ be small. Actually, the assumption of an optically thin boundary layer is equivalent to stating that the analysis is first-order in ζ. The present solution therefore contains first-order terms in ζ and second-order in ξ.

With the assumption that the plate surface is black, the radiation heat transfer between the plate and the absorbing fluid is given by [1]

$$q_{rw} = (e_w - e_\infty) - 2 \int_0^\infty (e - e_\infty) E_2(\tau) d\tau \tag{26}$$

Upon combining equations (8) and (21), the temperature distribution throughout the entire gas may be expressed by

$$\frac{T}{T_\infty} = 1 + (\gamma-1)\left[\theta_o(\eta) + \theta_1(\eta)\Gamma\xi^{1/3} + \theta_2(\eta)\frac{G_o(\tau)}{G_o(0)}\Gamma^2\xi^{2/3} + \ldots\right] \tag{27}$$

and this in turn gives

$$\frac{e}{e_\infty} = \left[1 + (\gamma-1)\theta_o\right]^4 + 4(\gamma-1)\left[1 + (\gamma-1)\theta_o\right]^3 \theta_1\Gamma\xi^{1/3} + O(\xi^{2/3}) \tag{28}$$

Upon substituting equation (28) into equation (26), noting that $E_2(\tau) \simeq 1$ where θ_o and θ_1 are nonzero, and neglecting terms of order $\zeta\xi^{2/3}$, there is obtained

$$\frac{q_{rw}}{e_w - e_\infty} = 1 - H_o\zeta - H_1\xi^{1/3}\zeta + \ldots \tag{29}$$

where

$$H_o = \frac{2}{\gamma^4-1} \int_o^\infty \left\{\left[1 + (\gamma-1)\theta_o\right]^4 - 1\right\} d\eta \tag{30}$$

$$H_1 = 8\left(\frac{\gamma-1}{\gamma^4-1}\right)^{2/3} \int_o^\infty \left[1 + (\gamma-1)\theta_o\right]^3 \theta_1 d\eta \tag{31}$$

The quantities H_o and H_1 have been evaluated by numerical integration, and the results are given in Table 1 and Fig. 5.

Table 1. The Quantities H_o and H_1

γ	H_o	H_1
1/4	3.61	-1.38
1/2	3.08	-1.21
1	2.20	-.940
2	1.39	-0.706
4	0.997	-0.712

The first term in equation (29) simply represents radiation exchange between the plate surface and an infinite isothermal gas at temperature T_∞, since the emissivity of an infinite isothermal gas is unity. The second term in equation (29), $-H_o\zeta$, represents a first-order correction due to the fact that the gas is actually nonisothermal. In particular, this correction is based upon the zero-order boundary-layer temperature profile $\theta_o(\eta)$, and it is always negative since the gas within the boundary layer is at a temperature which is closer to the plate temperature than T_∞. In other words, the radiation heat transfer between the plate surface and the gas will always be less than for an isothermal gas, since the temperature difference between the plate and the boundary-layer gas is less than $(T_w - T_\infty)$.

Considering next the convective heat transfer, and defining the Nusselt number in the conventional manner

$$Nu = \frac{q_c x}{k(T_w - T_\infty)}$$

then

$$Nu = -\frac{x}{(T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

and from equations (21) and (25) the convective heat transfer from the plate surface may be expressed in dimensionless form by

$$\frac{Nu}{(Gr)^{\frac{1}{4}}} = 0.401 + 0.051\Gamma\xi^{1/3} - 0.064\Gamma^2\xi^{2/3} + \ldots \qquad (32)$$

As would be expected, the first term in this expression corresponds to free convection in the absence of any radiation interaction. The first-order interaction term, $0.051\Gamma\xi^{1/3}$, is the result of the induced motion at the outer edge of the free convection boundary layer, and this "forced-convection" effect results in an increase in convection heat transfer. Conversely, it is seen that the second-order interaction term, $-0.064\Gamma^2\xi^{2/3}$, acts to reduce convection heat transfer. This is partially due to the fact that this term includes the first influence of the variable temperature at the outer edge of the boundary layer, $T_\delta(x)$. The net result of this variable temperature is that the temperature difference across the boundary layer decreases with increasing x. From the similarity solutions of Sparrow and Gregg [4], it is found that this type of behavior leads to a reduction in convective heat transfer.

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FIGURE CAPTIONS

Fig. 1. Physical Model and coordinate system.

Fig. 2. The functions F_o and G_o.

Fig. 3. The functions f_o', f_1' and f_2' for Pr = 1.0.

Fig. 4. The functions θ_o, θ_1 and θ_2 for Pr = 1.0.

Fig. 5. The radiation heat transfer functions H_0 and H_1 for $Pr = 1.0$.

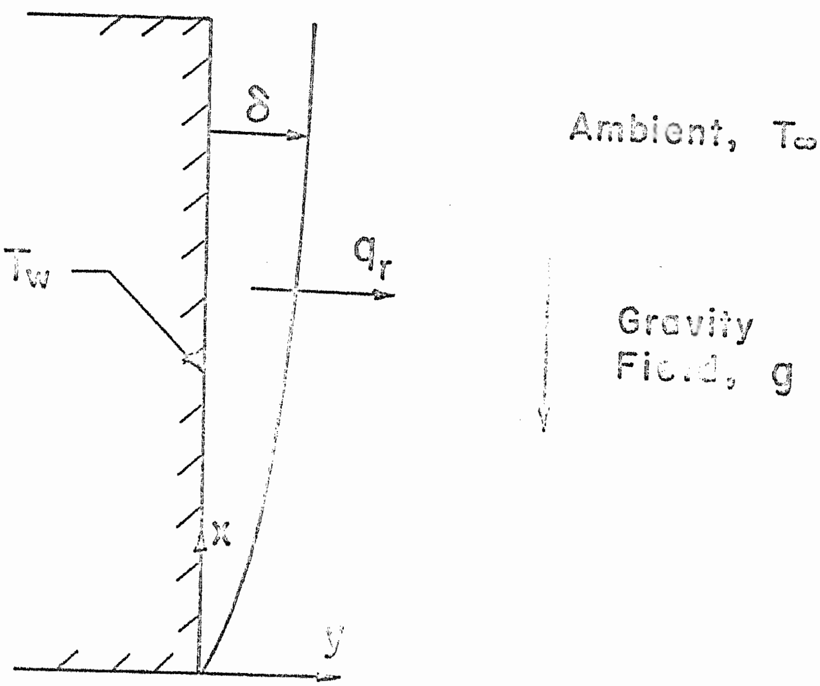

Fig. 1

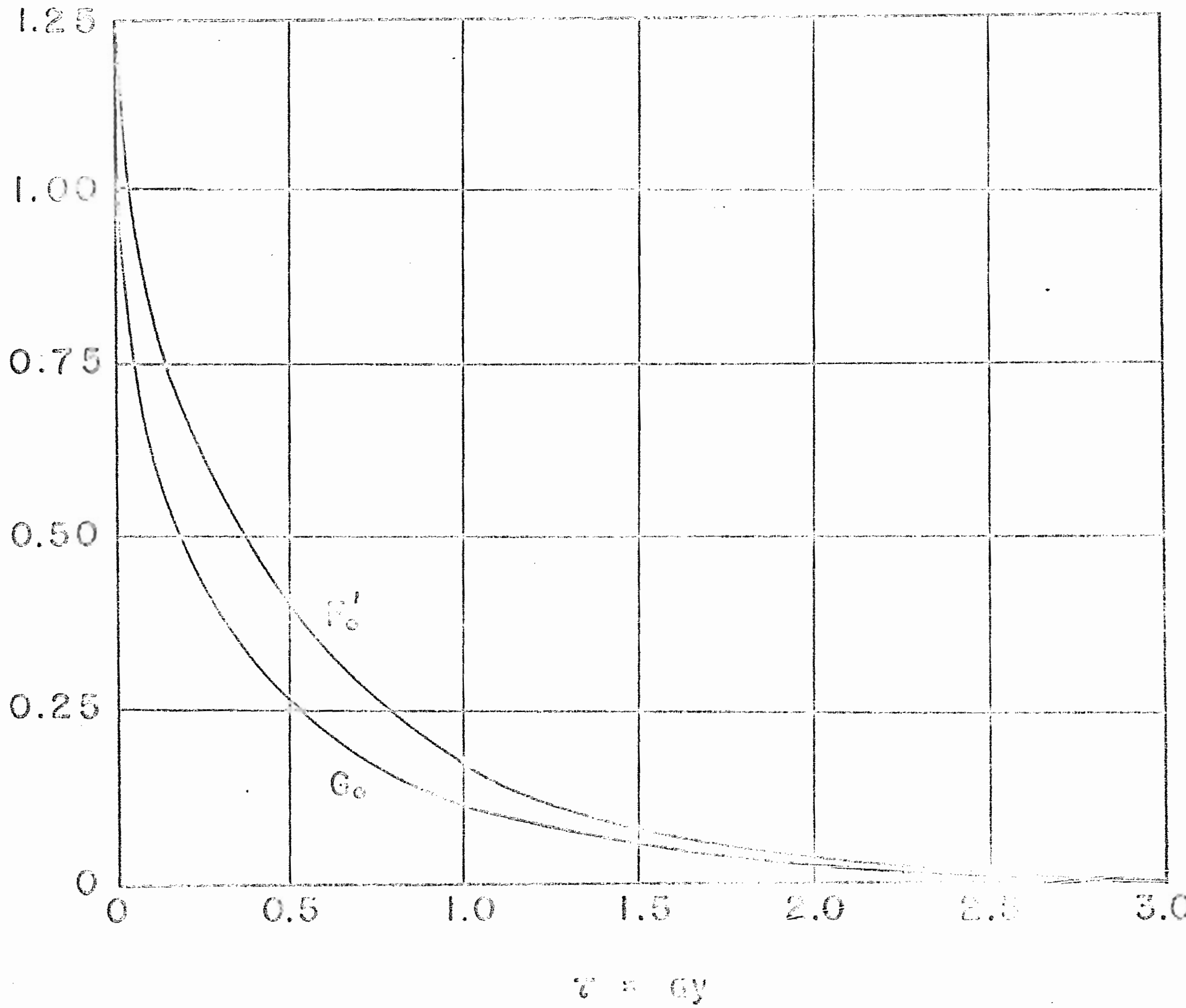

Fig.

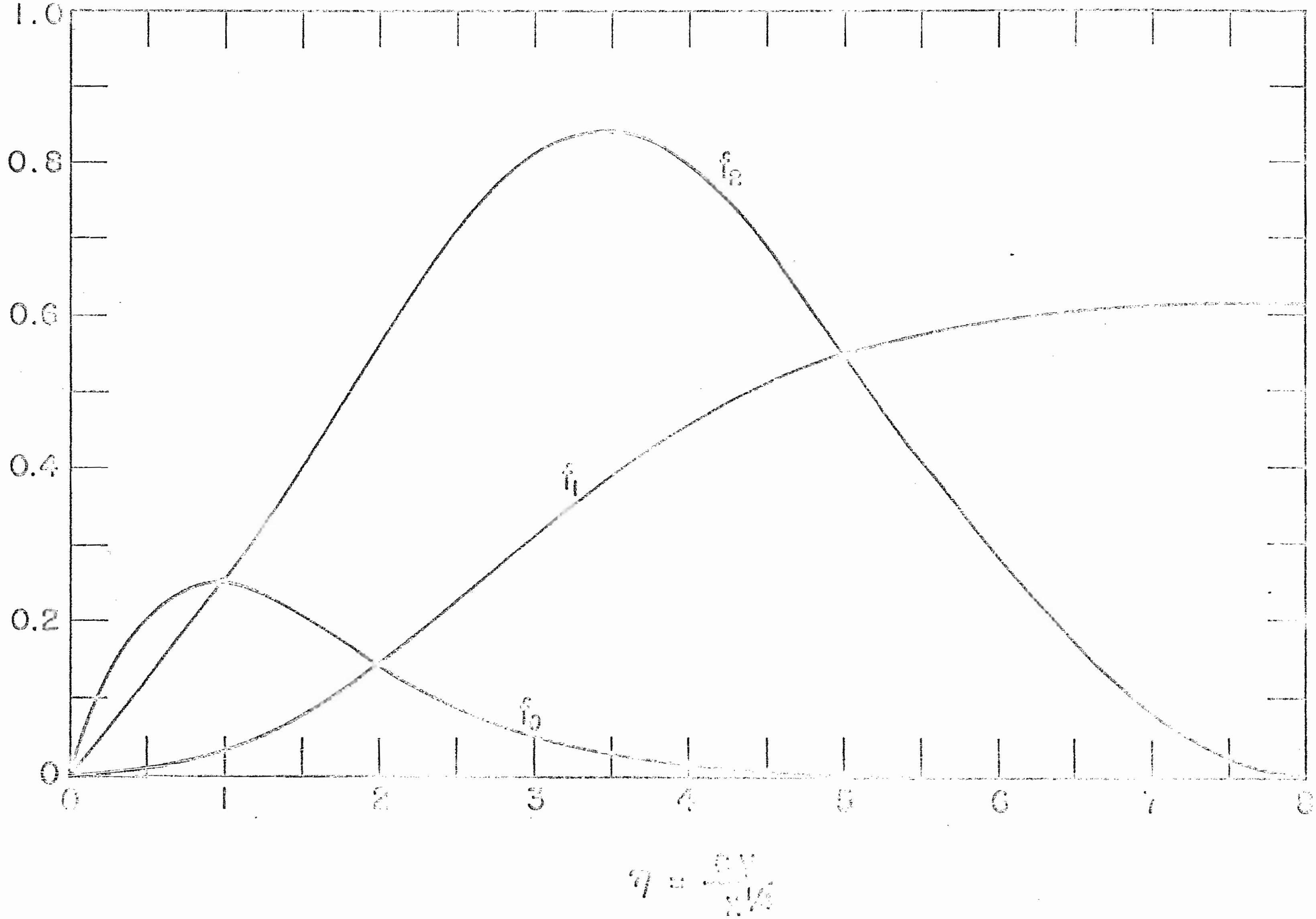

Fig. 5

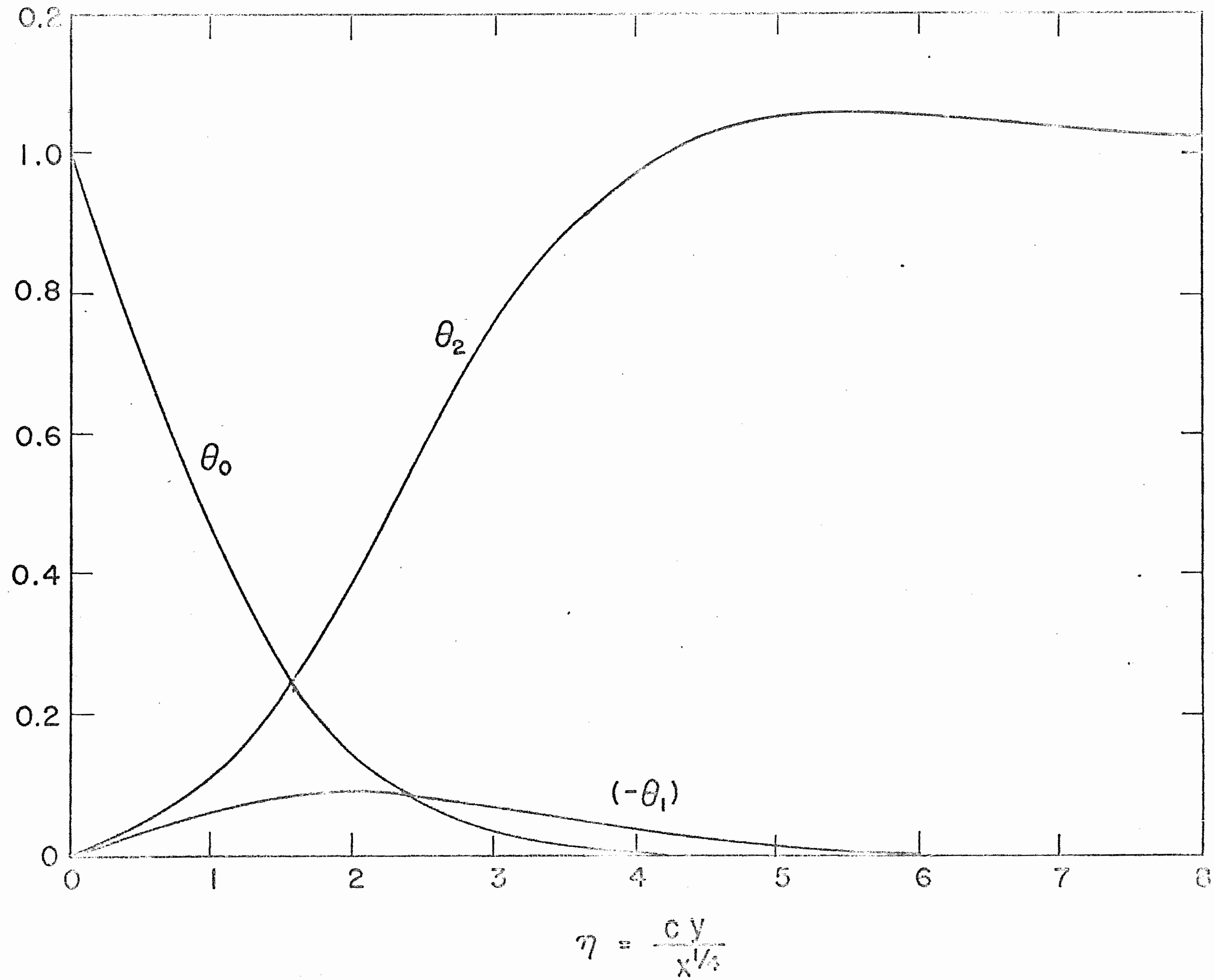

Fig. 4

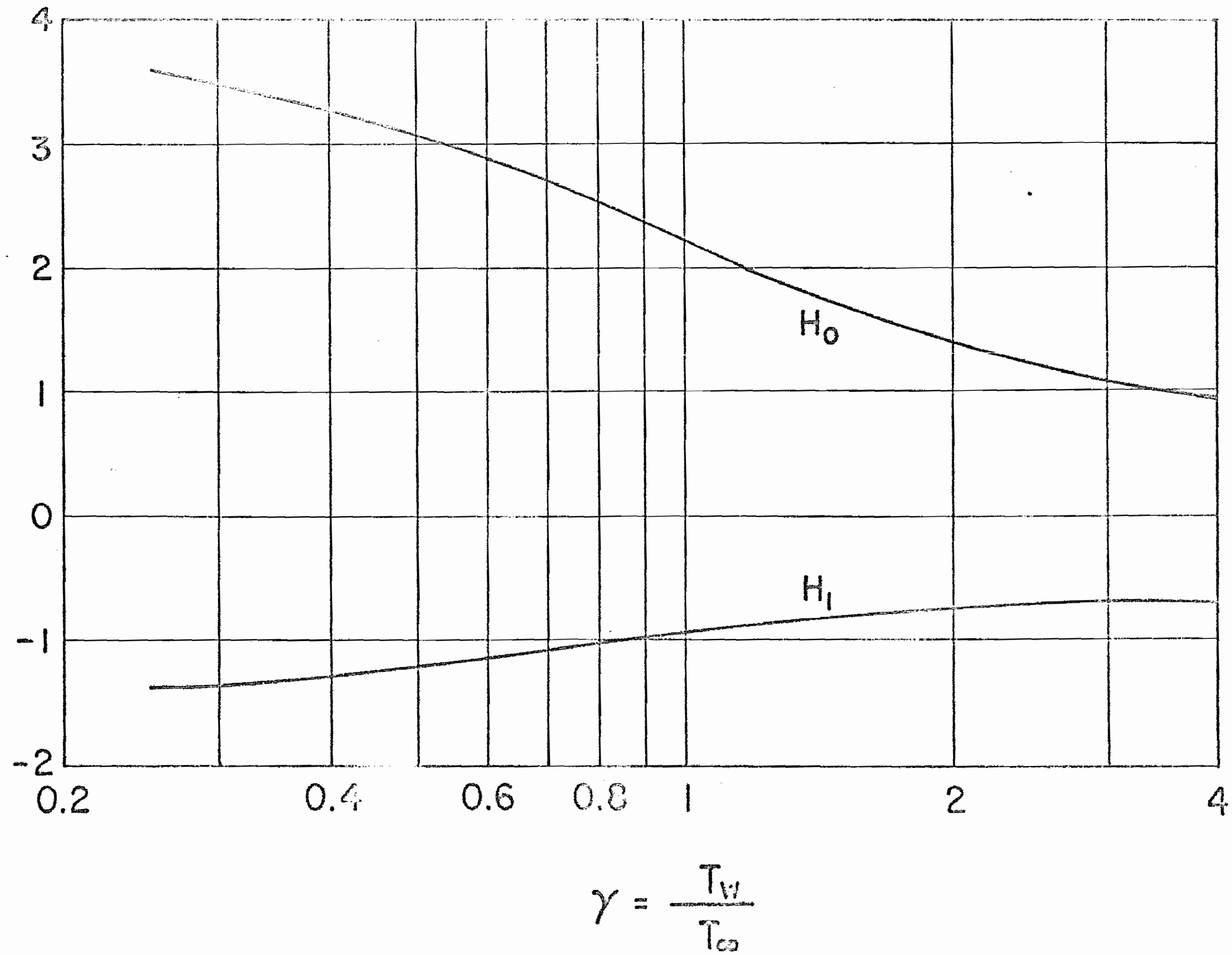

Fig. 5