ON THE EXTENSION OF PHOTOELASTICITY TO PLANE ELASTIC PROBLEMS WITH STATE OF STRESS DEPENDING ON POISSON'S RATIO

by

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Introduction

Photoelasticity is definitely one of the most powerful and widely used methods in experimental stress analysis. However the fact that photoelastic models have to be made of certain types of transparent material with elastic constants quite different from that of the prototype has raised questions from time to time as to the validity of the experimental result. From theory of elasticity we know that if the problem is either plane stress or plane strain and the boundary conditions are such that they are specified in terms of surface tractions, then in the absence of body force, the state of stress of the body is independent of both elastic constants, provided that the body occupies a singly connected region. For multiply connected bodies it is necessary to have the additional condition that the sum of all forces on each and every boundary is zero. Other than this rather restricted case, the effect of elastic constants is always present in the solution of a elastostatic problem.

In model scaling the effect of Young's modulus E can be matched by introducing non-dimensional quantity such as σ/E [1];* the effect of Poisson's ratio ν, being non-dimensional itself, however, can only be matched by using the same value as that of the prototype. Since it has been unsuccessful to change the ν of a photoelastic material, efforts have been directed to finding out specifically what effect Poisson's ratio has on the state of stress and how to correct it. Numerous papers are devoted to this subject. For example, papers by Bickly [2], Clutterbuck [3], Fessler and Lewin [4], and Kenny [5] among others.

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*Numbers in brackets denote references at the end of paper.

The common conclusion from these investigations have been that the effect of ν on state of stress is less than 10% and is of the same order of magnitude as that of experimental error; hence it is customary to neglect the Poisson's effect.

However, in 1966 Sanford [6] showed that the effect of Poisson's ratio in general cannot be neglected. For the particular case studied by him, the effect could be as large as 86%.

Indeed it is not difficult to generate some problems for which the effect is of significant importance. For example, if a tension bar is connected to a frame made of the same material as shown in Fig. 1, the stress at section AA or BB of the horizontal connecting bar is approximately equal to νP if the frame is made relatively rigid compared to the connecting bars. The magnitude of the stress can then range from zero to 1/2 P depending upon whether ν is zero or a half, respectively. It is obvious that the effect is even more pronounced if there are holes or notches present.

One method to correct the Poisson's effect on stress distribution is to introduce a Volterra dislocation to a second model as proposed by Filon [7,8]. The stress due to dislocation is then added to the stress due to load. While the method is theoretically sound, the extreme experimental difficulty involved in producing Volterra dislocation has practically prevented it from being used by experimentalists.

In 1967 Dundurs [9] published a very interesting paper showing that the dependence on Poisson's ratio in plane elasticity can be derived explicitly and that the state of stress of a body of any Poisson's ratio can be obtained if it is known for two models of different Poisson's ratio.

The possible impact of this result on photoelasticity is obvious and it is the intent of this paper to provide some experimental evidence to the analysis and to explore the practical aspect of its adaptability to photo-elasticity.

Scope of the Study and Experimental Procedures

While Dundurs' result applies to both plane stress and plane strain problems, the former was chosen for the study because of the difficulties involved in producing plane strain condition for models of various materials. For plane stress case it was shown by Dundurs that

$$\sigma_{ij} = q[\Psi_{ij}(x,y) + \nu\psi(x,y)] \tag{1}$$

in which σ_{ij} is the stress tensor, q the loading, and Ψ_{ij} and ψ_{ij} are functions of position only. It is seen therefore that if the state of stress is known for two models with different ν, Ψ_{ij} and ψ_{ij} can be solved to yield the state of stress for any Poisson's ratio. Experimentally the state of stress should be plotted against ν, and the result should be a straight line. Stresses for other Poisson's ratios can then be obtained by linear inter-or extrapolation. The adaptability of this technique depends on whether or not there are two photoelastic materials with Poisson's ratios different enough so that accurate inter-or extrapolations can be made. It is obvious that if the data points from two photoelastic models are too close together the straight line connecting them may not represent the actual direction because of experimental errors.

In this study four different materials: CR-39, epoxy, urethane rubber, and aluminum were chosen as model materials. Their corresponding Poisson's ratios are listed in the following:

Material	Poisson's Ratio
Urethane Rubber	0.456
CR-39	0.428
Epoxy (Hysol 4290)	0.377
Aluminum	0.361

The reason for choosing these materials is mainly the availability to the author at the time of experiment. Tests were performed first for the three photoelastic models to see if the data points form a straight line and whether it is sufficient to render accurate inter-or extrapolations. The test on the aluminum model was performed last to check the accuracy of extrapolation.

The geometry of the model is shown in Fig. 2 together with the loading device. Under the force so applied the state of stress at the central horizontal bar is clearly a function of Poisson's ratio. The circular notches are introduced to increase the fringe order (thus the effect of Poisson's ratio). Each model is loaded at four different levels of loading and the stress at the tip of notch (averaged over the two notches) plotted against the loading to reduce experimental errors. A typical phtotelastic pattern of a CR-39 model under loading is shown in Fig. 3. Stress-optic coefficients are calibrated using circular disks under diametrical compression, and Poisson ratios are obtained using tension bars. For measuring the longitudinal and transverse strains of a tension bar, moiré method was used for the case of urethane rubber whereas strain gages were used for the rest of them. Strain gages were also used for measuring the strains at the tip of notches of the aluminum model.

Results and Discussions

The normalized stresses at the tip of notch in different models are plotted against Poisson's ratio ν as shown in Fig. 4. Normalization of stress is done by dividing the stress by the applied average pressure P_{ave}, which is defined to be the total applied force divided by the area of application. It is seen from Fig. 4 that the state of stress is indeed a linear function of Poisson's ratio and that accurate inter-or extrapolations could be made from the straight line connecting the three photoelastic data points. The fact that the data point from aluminum model falls below the predicted position, however, is due to the averaging effect of a strain gage of finite size at the tip of notch.

While the photoelastic material chosen here are that of urethane rubber, CR-39 and room-temperature epoxy, there is no reason of course to restrict oneself to the selected three. Judging from the accuracy shown in Fig. 4 it is sufficient to use only two models with data points as widely separated as possible. The fact that epoxy at critical temperature has a Poisson's ratio of approximately 1/2 renders itself as the best choice for one of the two needed points. The other point may be obtained either from a Homolite 100 model ($\nu \cong 0.36$) or a room-temperature epoxy model ($\nu \cong 0.34$-0.38). Other photoelastic materials with relatively low Poisson's ratio of course could also be used. As mentioned before the present choice of materials was mainly for the convenience of the author; the use of high-temperature epoxy, however, was deliberately excluded since it needed four models for four different levels of loading as compared to one for the rest of the materials.

Conclusions

The experimental evidence provided in the paper shows that in a multiple connected body with non-zero surface tractions on internal boundaries the state of stress depends linearly on Poisson's ratio. More important it shows that there are photoelastic materials available (epoxy at critical temperature with $\nu \cong 05$ and Homolite 100 with $\nu = 0.36$, for example) with Poisson's ratio different enough that accurate linear inter- or extrapolation can be made to material of any Poisson's ratio. The theoretical result of Dundurs and the experimental result presented in this paper open up a wide class of problems that can be attacked by photoelasticity with confidence. It is therefore no longer necessary, at least in plane elasticity, to guess the effect of Poisson's ratio.

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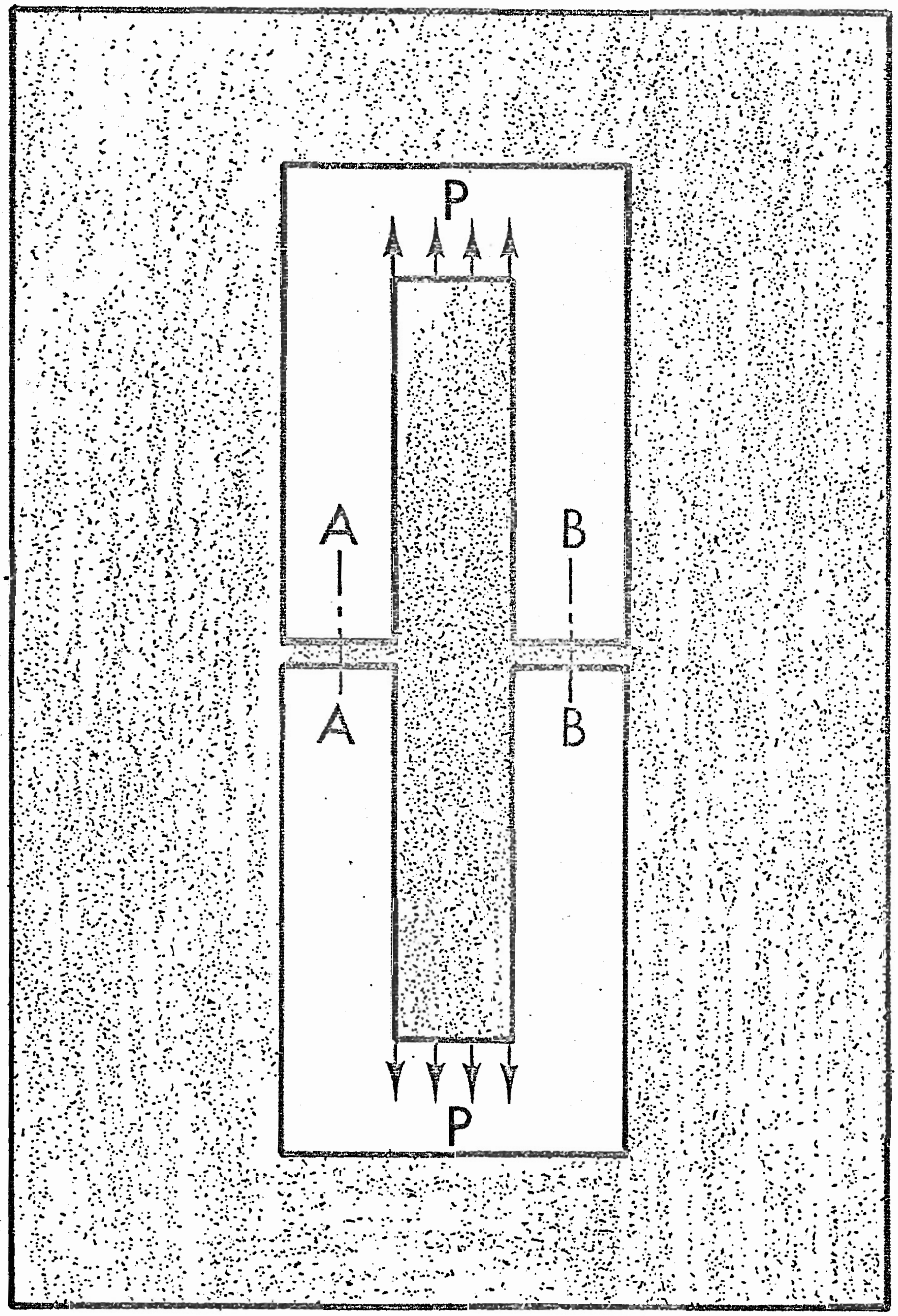

Fig. 1 A Body under Loading Showing the Dependence of Stress on Poisson's Ratio at Sections A-A and B-B

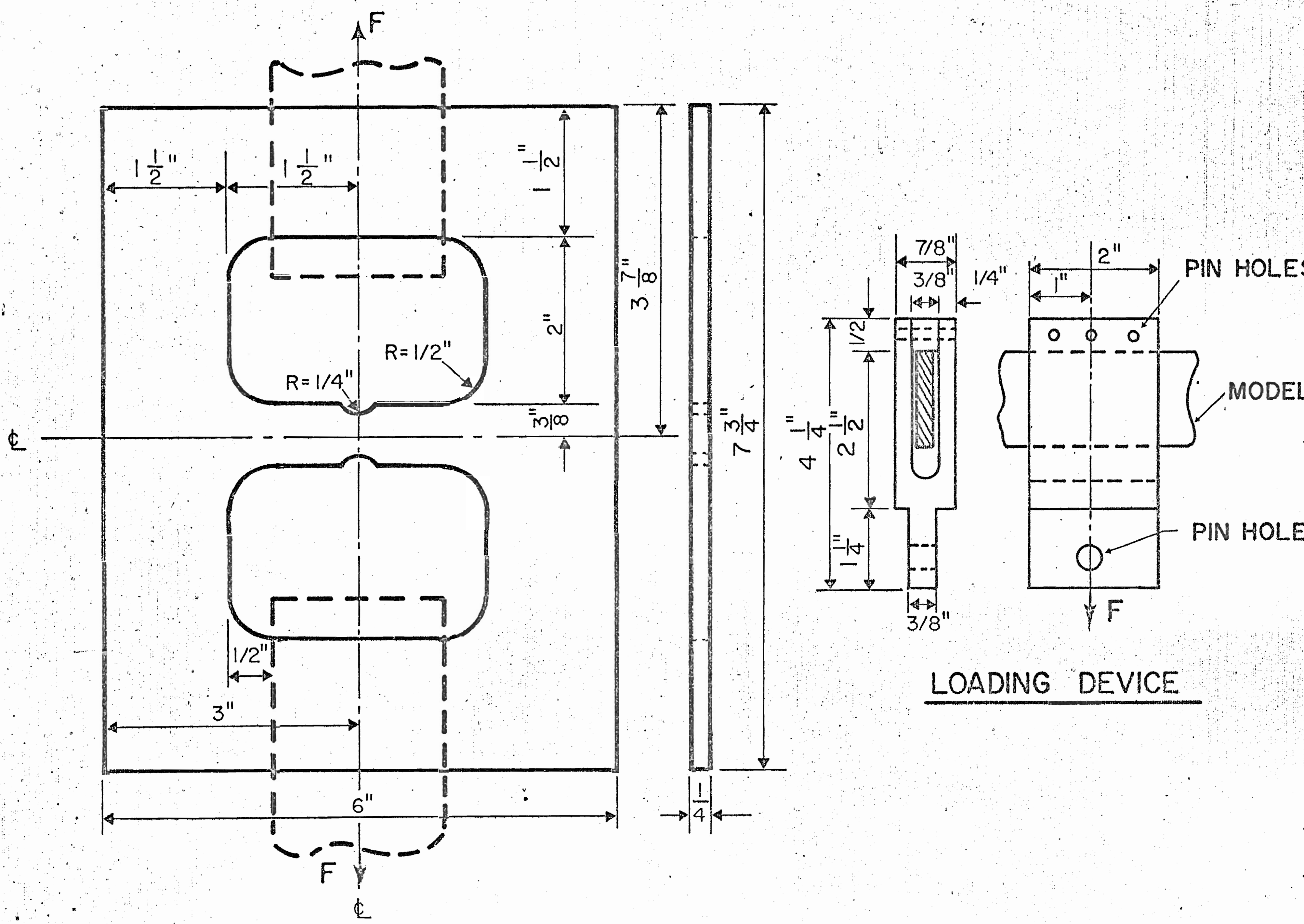

Fig. 2 Geometry of Model Used for the Tests and the Loading Device

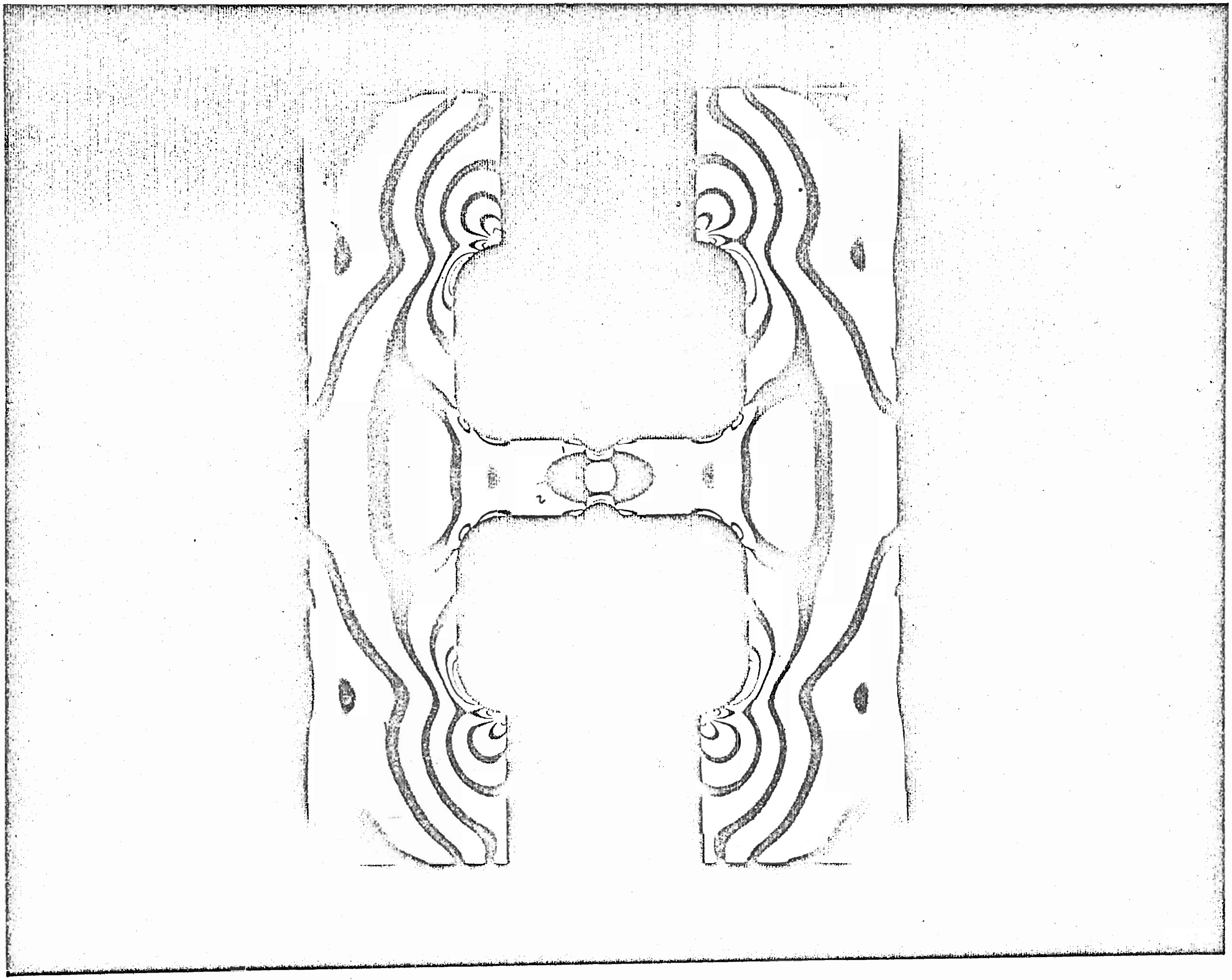

Fig. 3 Typical Photoelastic Pattern of a CR-39 Model Under Load

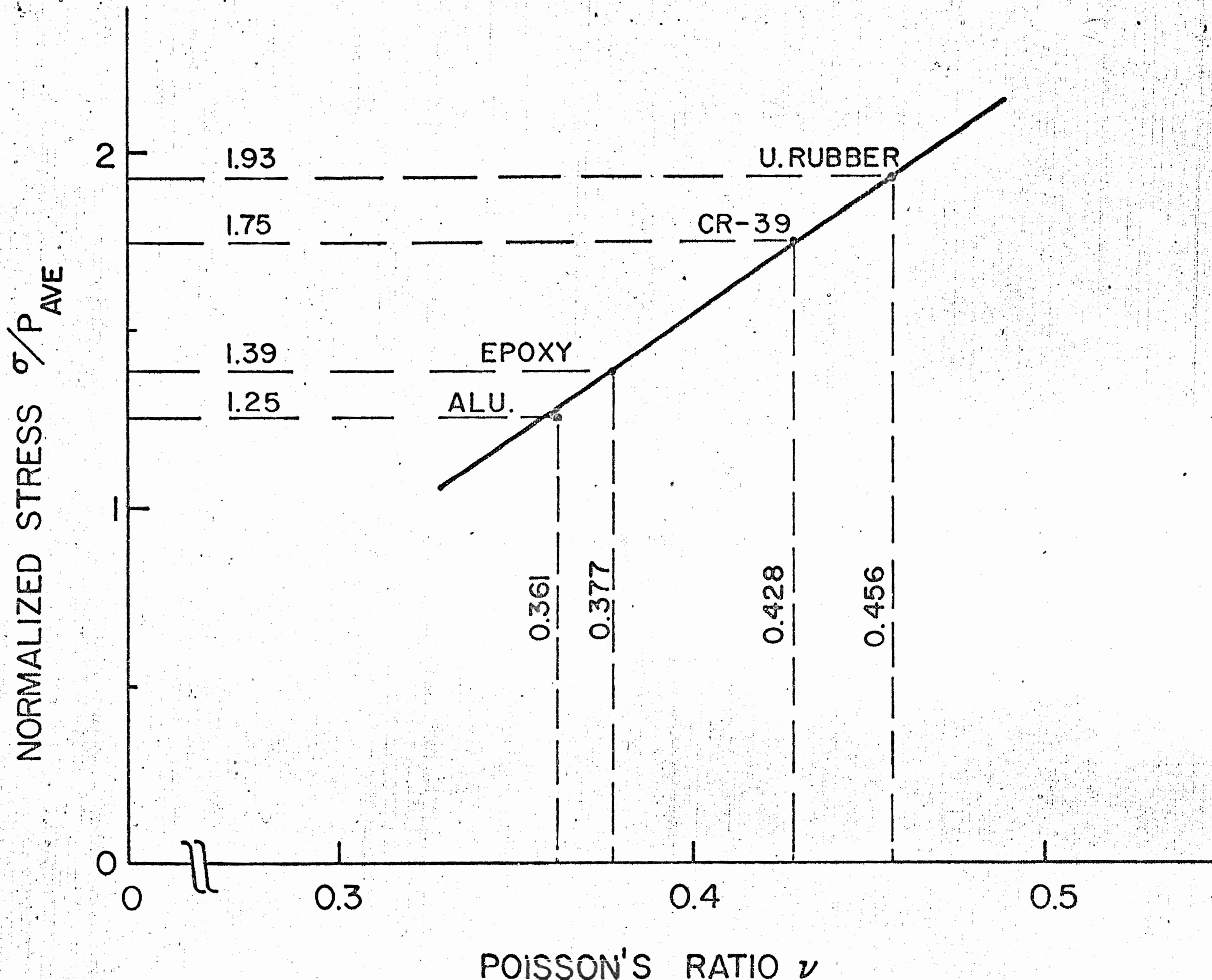

Fig. 4 Relation Between Stress at Notch and ν $P_{ave} = \frac{F}{A}$, F = Total Force Applied, $A = \frac{1}{-}$, sq. in. - area of contact