MOIRÉ ROSETTE METHOD FOR STRAIN ANALYSIS

by

Fu-pen Chiang
Assistant Professor
Department of Mechanics
College of Engineering
State University of New York at Stony Brook
Stony Brook, New York

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ABSTRACT

A new approach to the use of moiré fringes for the analysis of strain is proposed in the paper. Moiré patterns representing displacements along three different directions are obtained from crossed gratings through optical spatial filtering and the state of strain determined by the use of strain rosette equations. The method has the advantage over the conventional method in that it is less time consuming and more accurate.

INTRODUCTION

The conventional way of using moiré fringes for strain analysis [1,2] is the following: a fine grid of orthogonal lines (crossed grating) forming a x-y coordinate system is first printed on the surface of a model, and the model is then loaded. A reference (or master) grating with identical density of lines (but only one directional) is superimposed on the deformed model grating with the direction of lines aligned with y-direction to yield the moiré fringes representing the displacements in x-direction (u-field fringes); and then the reference grating is rotated 90° to yield the fringe pattern representing displacements along y-direction (v-field). Displacement curves are plotted from moiré patterns along the sections of interest with the interpretation that each fringe represents a displacement of a p (pitch of grating) normal to the grating lines. Graphical or numerical differentiation of displacement curve along x- and y-directions gives $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$, respectively from the u-field fringes, and $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ from the v-field fringes. These four derivatives are, of course, sufficient for the complete determination of a two dimensional state of strain through the use of the following relations:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \quad , \quad \varepsilon_{yy} = \frac{\partial v}{\partial y}$$
$$\gamma_{xy} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \qquad (1)$$

In obtaining the two sets of moiré fringes it is extremely important to align the master grating properly. If the rotation of the master grating is not exactly 90°, an apparent shearing strain is introduced into the result with an annoyingly large magnitude [3]. The error introduced in the normal strain is, however, negligible. The fact this being true can be easily seen from the phenomenon that if two identical gratings are superimposed with a small rotational mismatch (difference in orientation), a set of parallel fringes will be formed with direction approximately perpendicular to grating lines. While it is conceivable to build a precision device to give an accurate rotation, it is also possible to eliminate this possible error by recording both the u- and v-field moiré patterns simultaneously using a crossed grating as master as proposed by Post [3]. It can be shown that in so doing the errors due to rotational mismatch will have opposite effects on the two families of fringes and therefore the two cross derivatives $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$. As a result they cancel out each other. This method, however, suffers from one drawback in that sometimes the two families of moiré fringes tend to tangle together and that it becomes difficult to identify fringes with their respective families. Chiang [4] developed a method to eliminate this difficulty by optically separating the two families of fringes by spatial filtering (either before or after they are recorded by film).

The method presented herein uses a different approach to obtain the state of strain through moiré fringes. It utilizes the strain rosette concept commongly employed in strain gage techniques. It will be shown that normal strains along three non-parallel directions (0°-, 45°-, and 90°-strains) can be obtained from a pair of crossed model and master gratings. The advantage of this method over the conventional one is also discussed.

The Strain Rosette

It is well known that the Cartesian components ε_{xx}, ε_{yy}, and γ_{xy} of a two dimensional state of strain can be obtained if normal strains along any three non-parallel directions are known, from the strain transformation formula

$$\varepsilon_\Theta = \varepsilon_{xx}\cos^2\Theta + \varepsilon_{yy}\sin^2\Theta + \gamma_{xy}\sin\Theta\cos\Theta \qquad (2)$$

in which ε_Θ is the normal strain along the direction Θ from x-axis. It will be shown in the next section that the moiré fringe patterns will give strains along directions for $\Theta = 0^\circ$, 45° and 90°. It is clear then that

$$\varepsilon_{0^\circ} = \varepsilon_{xx} \qquad (3)$$

$$\varepsilon_{45^\circ} = \tfrac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy} + \gamma_{xy}) \qquad (4)$$

$$\varepsilon_{90^\circ} = \varepsilon_{yy} \qquad (5)$$

which give

$$\gamma_{xy} = 2\varepsilon_{45^\circ} - \varepsilon_{0^\circ} - \varepsilon_{90^\circ} \qquad (6)$$

The principal strains ε_1 and ε_2 can also be expressed in terms of these three strains as in the following:

$$\varepsilon_1 = \tfrac{1}{2}(\varepsilon_{0^\circ} + \varepsilon_{90^\circ}) + \tfrac{1}{2}\sqrt{[(\varepsilon_{0^\circ} - \varepsilon_{90^\circ})^2 + (2\varepsilon_{45^\circ} - \varepsilon_{0^\circ} - \varepsilon_{90^\circ})]} \qquad (7)$$

$$\varepsilon_2 = \tfrac{1}{2}(\varepsilon_{0^\circ} + \varepsilon_{90^\circ}) - \tfrac{1}{2}\sqrt{[(\varepsilon_{0^\circ} - \varepsilon_{90^\circ})^2 + (2\varepsilon_{45^\circ} - \varepsilon_{0^\circ} - \varepsilon_{90^\circ})]} \qquad (8)$$

The Moiré Rosette

It is well known that when two gratings (one deformed model grating, one reference grating) are superimposed one against another (either directly or through an optical system) a set of fringes appears. These fringes represent displacement of the model along the direction perpendicular to the lines of the reference grating (master). When two crossed gratings are superimposed, a set of crossed fringes will appear. They are the combination of two families of fringes representing displacements in two orthogonal directions. In the following it will be shown that by using a coherent optical system through optical filtering three families of fringes representing displacements in three different directions can be obtained. Therefore normal strains along these directions can, in term, be obtained, and eqs. (3), (5) and (6) used to yield the Cartesian components of the state of strain.

For this technique a coherent optical system forming image in two stages, as shown in Fig. 1, is necessary. The coherent light from a laser is first expanded into a diverging beam by a microscopic objective and then collimated by a collimator. A lens L 1 collects the light and forms an image of the light source at plane P2 (the first image plane). A second lens L 2 receives the light from P2 to form an image of the source at plane P3 (the second image plane). The relative positions of the lenses are as shown in Fig. 1. If an information carrying transparent object is inserted into the light path at the front focal plane of the lens L 1, its diffraction spectrum will be displayed at plane P2 and an inverted image of the object will be formed at plane P3.

It can be shown [5] that under coherent illumination the relationship between the light disturbances at the front and back focal planes of a lens is that of a Fourier transform. Indeed if $f(x,y)$ denotes the complex amplitude of the light flux at plane P1, the complex amplitude of the light flux at plane P2 is given by

$$F(p,q) = \int_{-\infty}^{\infty}\int f(x,y)e^{i(px+qy)}dxdy \qquad (9)$$

in which (x,y) and (p,q) are the coordinates of the front and back focal planes of lens L 1, respectively. This relation gives an easy access to mathematical analysis as well as visual representation of the diffraction spectra of many objects.*

In order to have a true image of the object at plane P3, it is necessary to have the whole diffraction spectrum collected by the lens L 2, and if only a portion of spectrum is allowed to go through the optical system, only the portion (or the component) of the object which gives rise to the partial diffraction spectrum will be imaged at plane P3.

When a moiré grating is inserted in field of a coherent light at plane P1, it will behave like a diffraction grating. The mechanism of the diffraction may be simply illustrated as shown in Fig. 2, where an impinging plane wave is disturbed by the presence of a grating. As a result, cylindrical wave-lets will be generated at the slits. These wave-lets will be reinforcing one another at some directions given by the angle Θ_n where

$$\Theta_n = \sin^{-1} \frac{n\lambda}{P} \qquad (10)$$

in which P = pitch of the grating, λ = wave length, and $n = 0, \pm 1, \pm 2, \ldots$.

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* For example: the Fourier transform of a unit function is a delta function, which corresponds to the case of a plane wave being focused into a point source; and the Fourier transform of a cosine function is a pair of delta functions situated equal distances away from the optical axis. With the help of Fourier series it is possible to "see" diffraction spectra of more complicated functions through their Fourier components.

In any other directions the wave-lets interfere with one another destructively. These diffracted directions are called the orders of interference. When these orders are collected by the lens L 1, they form the diffraction spectrum as a series of equally spaced bright spots (diffraction orders) along a straight line perpendicular to the direction of grating lines, as shown in Fig. 3*. The distance d between any two neighboring orders, as can be easily shown, is given by the following equation:

$$d = \frac{f_1 \lambda}{p} \tag{11}$$

in which f_1 is the focal length of lens L 1.

If the light disturbance immediately after the grating plane is represented by a Fourier series, it can be shown by using eq. (9) that the zero order diffraction corresponds to the constant term of the Fourier expansion, the two first orders are from the fundamental harmonic, and the rest higher harmonics. Therefore, if a mask is made in such a way that only the 0, ±1 orders of the spectrum are allowed to be collected by lens L 2, the image form at plane P3 will be a grating of the same periodicity as the original but with a different detailed structure.

Now if a cross grating is placed at plane P1, the diffraction spectrum will be that of a crossed orders as shown in Fig. 4, which is the actual diffraction spectrum of a 1000 lines per inch crossed grating. As before, to preserve the basic two dimensional periodicity of the grating, it is only necessary to let pass the following combined orders: (0,0), (1,0), (0,1), (-1,0), (0,-1), (1,1), (-1,-1), (-1,1), and (1,-1). The resulting image will be essentially the same as the original grating. If,

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* The actual phenomenon is, of course, more complicated than as indicated. There are many secondary maximum intensity positions in between any two bright spots, and the intensities among the bright spots also vary from one to the other. For a detailed explanation see, for example, reference [5].

now, an additional mask is made to block all but the horizontal array of orders [(1,0), (0,0), and (-1,0)], only the vertical lines of the crossed grating will be imaged at plane P3. Similarly if all but the vertical array of orders [(0,1), (0,0), and (0,-1)] are blocked, the image will be that of a set of horizontal lines. Furthermore, if the mask allows only one diagonal array of orders, for example (1,1), (0,0), and (-1,-1), to go through the lens, the image formed at P3 will be that of a grating with lines diagonally oriented, 90° from the direction of the orders. The pitches of the horizontal and vertical gratings so formed are the same as that of the original grating but the diagonal grating has a pitch of $\frac{P}{\sqrt{2}}$, where P is the pitch of original grating. The reason for this being so is easily seen from the spacing of the orders and eq. (10). The filtered orders and their corresponding grating images are shown in Fig. 5.

It is now evident that if a crossed grating identical to that of the model grating at plane P1 is erected at plane P3 and optical filterings are performed so as to form the horizontal, vertical and diagonal grating images at P3, the interference of this grating with the three images, respectively, will give three families of moiré fringes representing displacements in three different directions.

An example is given in Fig. 6 where the three separated families of moire' fringes are shown for a model under load. In order to obtain the state of strain at a point of interest, it is necessary to draw displacement curves through this point along the direction normal to the grating lines from each of these three moiré patterns, and to differentiate the displacement curves graphically at the point to yield the following

$$\varepsilon_{0^\circ} = \frac{\partial u_{0^\circ}}{\partial r_{0^\circ}} , \quad \varepsilon_{45^\circ} = \frac{\partial u_{45^\circ}}{\partial r_{45^\circ}} , \quad \& \quad \varepsilon_{90^\circ} = \frac{\partial u_{90^\circ}}{\partial r_{90^\circ}} \qquad (12)$$

In so doing, it is important to bear in mind that the pitches for the horizontal, diagonal, and vertical gratings are P, $\frac{1}{\sqrt{2}}P$, and P, respectively.

After this is done, the three normal strains can then be substituted either into eqs. (3), (5), and (6), to give the complete Cartesian components of the state of strain, or into eqs. (7) and (8) to yield the two principal strains. The process has to be repeated if it is to determine the state of strain at other points.

Comparison between the Conventional and the Rosette Methods

It may appear at the first glance that this method is more tedious than the conventional, because it requires three photographs instead of two. Actually it is much simpler than the conventional method. It may be recalled that the most time consuming part of the moiré method for strain analysis is not the taking of pictures; it is the plotting of displacement curves and the painstaking point by point graphical differentiation of displacement curves. In the conventional moire method, it requires four displacement curves (one each for the four derivatives) and four differentiations to obtain the state of strain at a generic point, whereas the moiré rosette method only needs three displacement curves and three differentiations. For example, as shown in Fig. 6, it is desired to determine the state of strain at point P. For the conventional method displacement curves along sections A-A and B-B in picture (a) have to be drawn to obtain $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$, respectively, and displacement curves along sections C-C and D-D in picture (b) have to be drawn to yield $\frac{\partial v}{\partial y}$ and $\frac{\partial v}{\partial x}$, respectively; whereas for the rosette method, it is only necessary to draw the displacement curves along section A-A (from picture (a)), C-C (from picture (b)), and E-E (from picture (c)) to yield ε_0, ε_{90°, and ε_{45°, respectively.

Therefore, it is evident that if there are many points whose states of strain are to be determined, the moiré rosette method presented herein can save many hours of painstaking curve plotting and differentiation. Furthermore as mentionned at the beginning of the paper the conventional method suffers from the fact that a large shear error is introduced if the rotation of master grating is not exact; the rosette method, however, does not have this difficulty*. It is easy to see that at points of symmetry the

* It may be noted that with the optical system presented herein if one chooses to use the conventional moiré method for analysis, he only needs to obtain by filtering, the 0° and 90° patterns, and in so doing also eliminates the possible error caused by the rotation of the master grating.

conventional and present methods are identical.

CONCLUSION

It may be concluded that the moiré rosette method, which utilizes three different fringe patterns obtained from crossed gratings through optical spatial filtering, offers a new approach to the use of moiré fringes for experimental strain analysis. It has advantages over the conventional moiré method in that it is less time consuming and more accurate.

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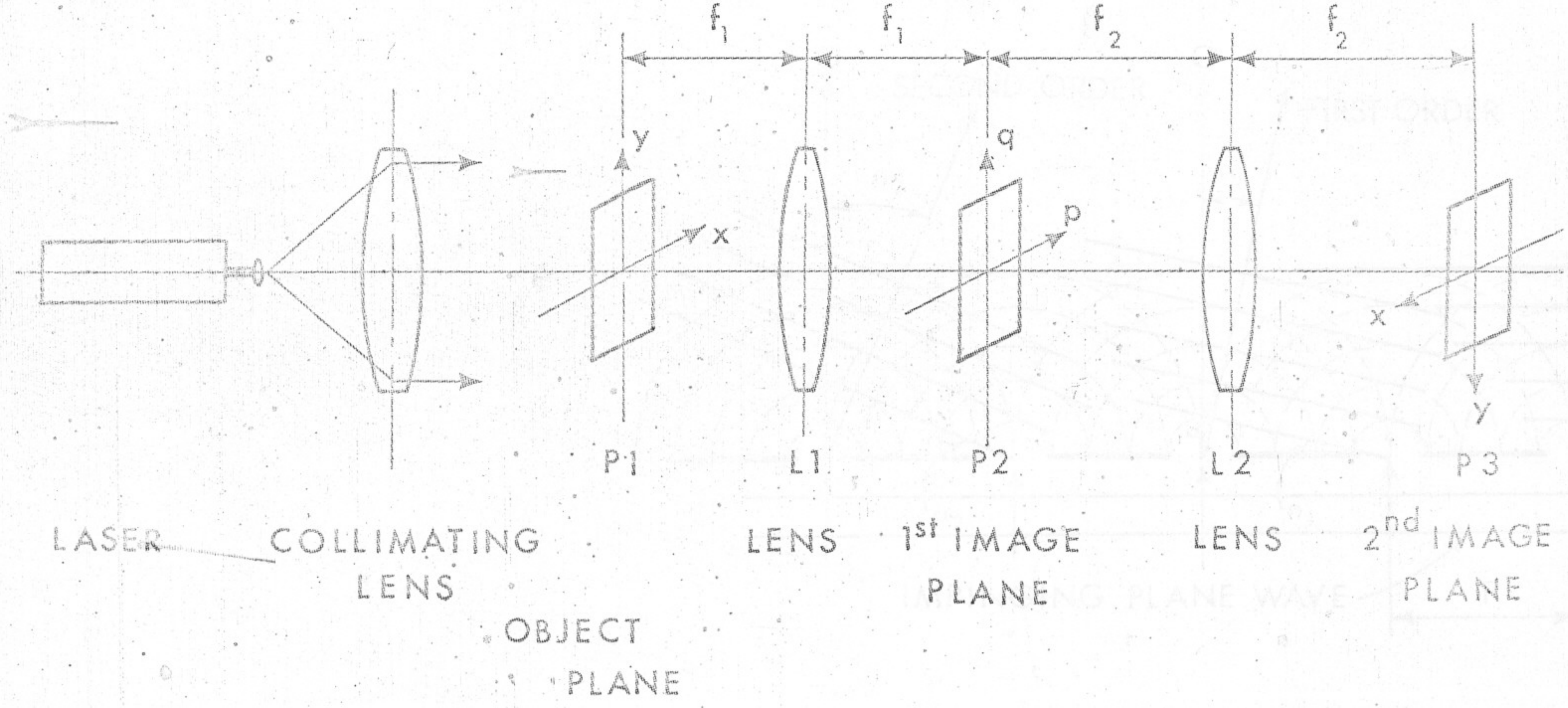

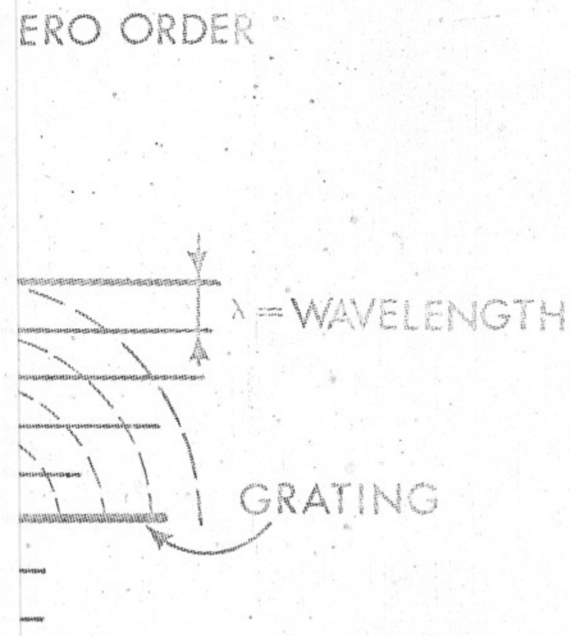

FIG. 1 ARRANGEMENT OF THE COMPONENTS OF AN OPTICAL SPATIAL FILTERING SYSTEM

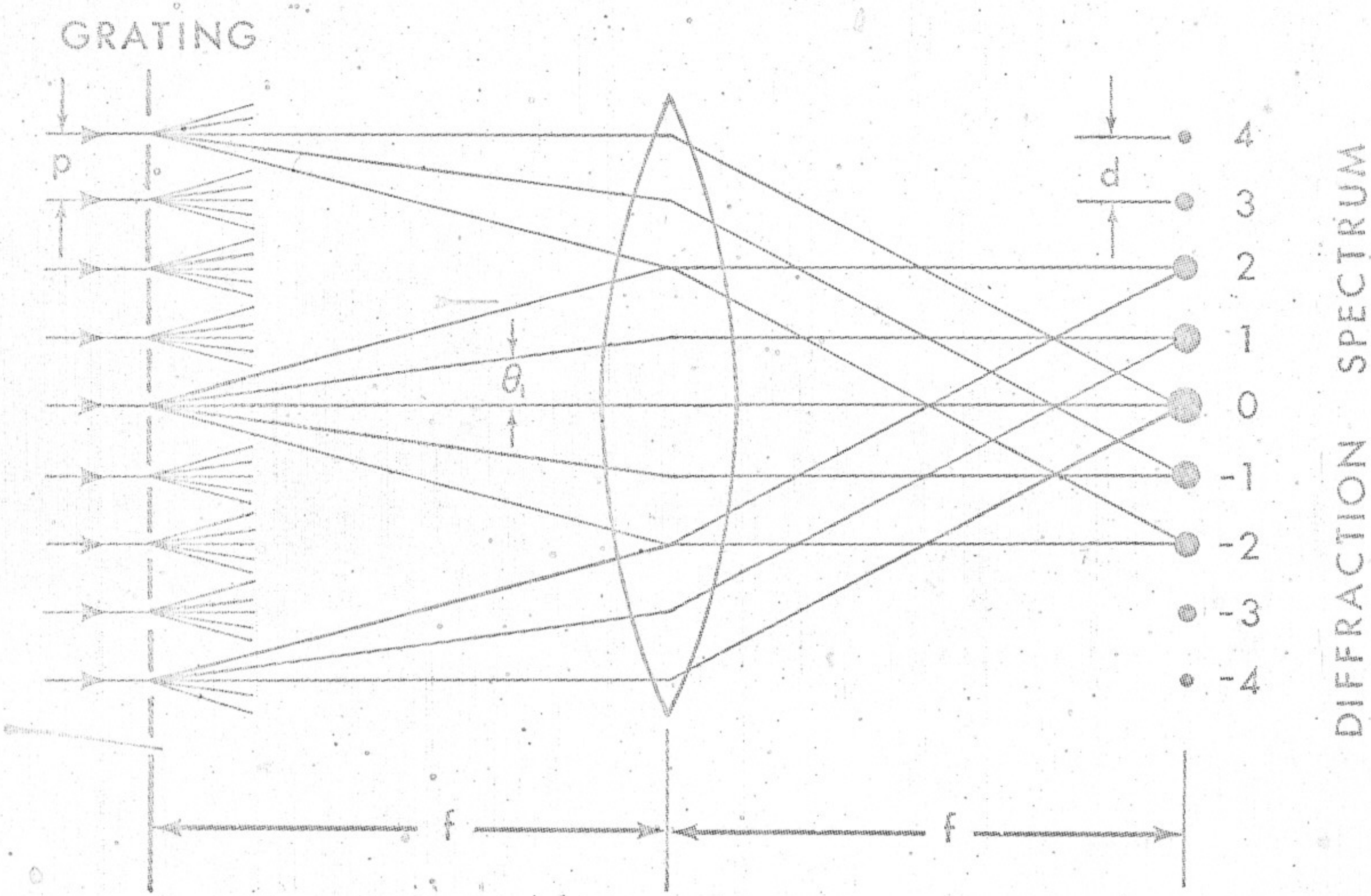

FIG. 3 IMAGE FORMATION MECHANISM OF THE DIFFRACTION SPECTRUM OF A GRATING

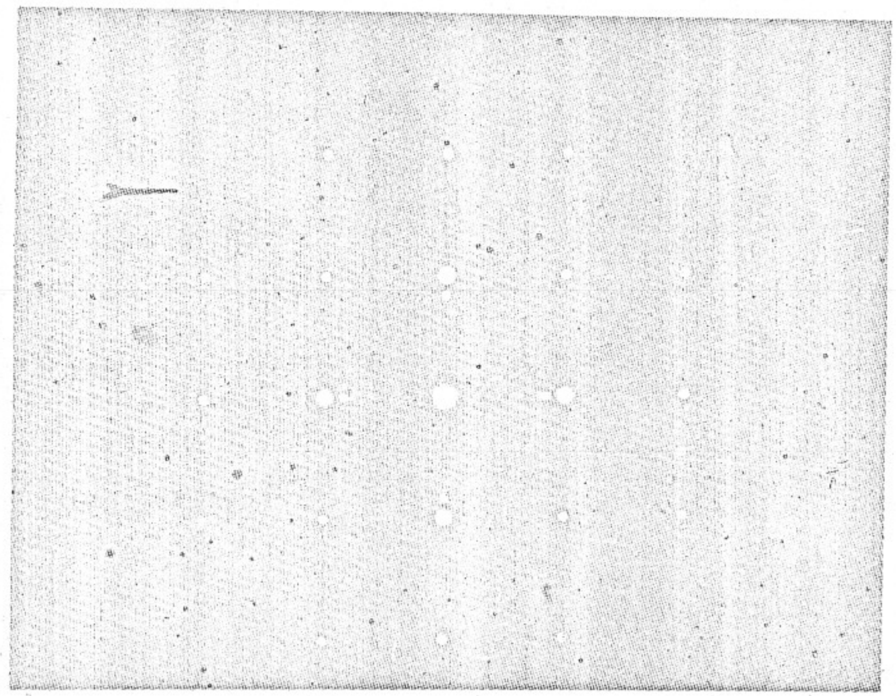

FIG. 4 PHOTOGRAPH OF THE DIFFRACTION SPECTRUM OF A CROSS GRATING (1000 LINES PER INCH)

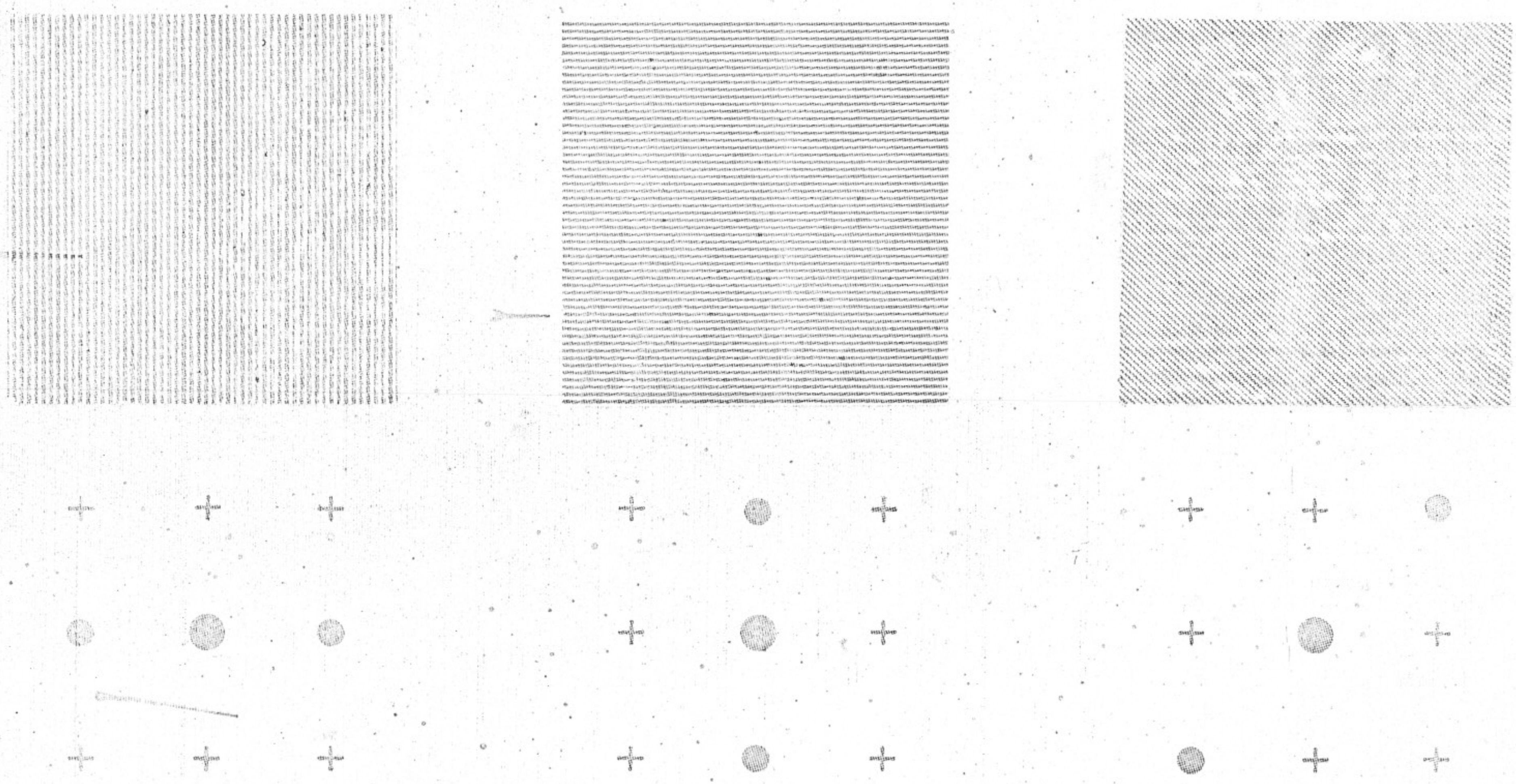

Fig. 5 FILTERED DIFFRACTION SPECTRA AND THEIR CORRESPONDING GRATING IMAGES

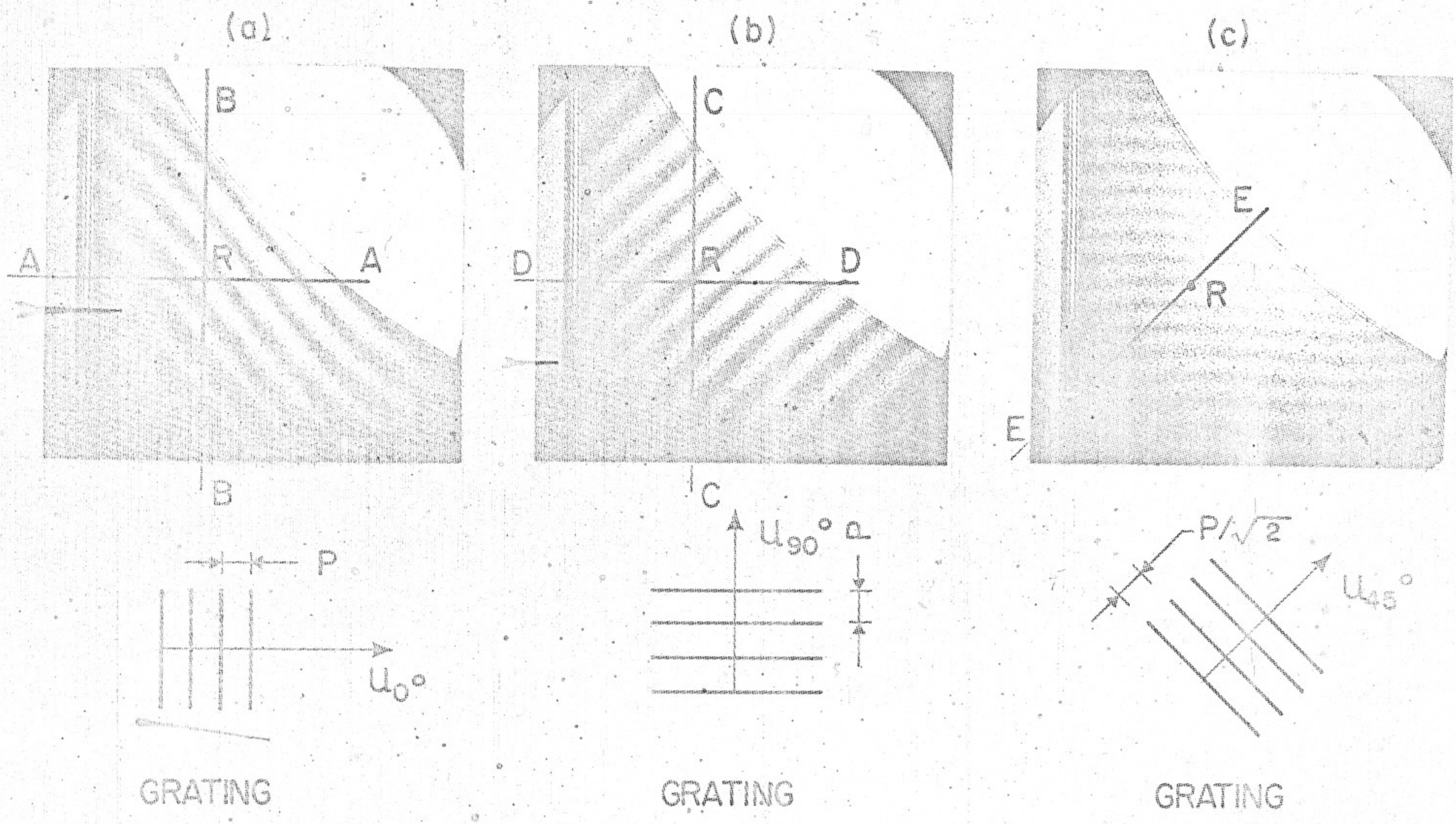

Fig. 6 FILTERED MOIRE' PATTERNS AND THEIR CORRESPONDING GRATING ORIENTATIONS