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COLLEGE OF
ENGINEERING

Report No. 134

SOME EXPERIMENTAL EVIDENCE FOR THE VALIDITY OF MOIRE GAP EQUATION

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INTRODUCTION

In a recent paper by Sciammarella and Chiang [1]* the effect of a gap between two gratings with no rotational mismatch under the illumination of incoherent diffused light was examined and an equation derived, which gives the fictitious strain as a function of gap:

$$\varepsilon_r = \frac{\partial u}{\partial r} = \frac{\Delta z}{z} + \left(\frac{r}{z}\right) \frac{\partial \Delta z}{\partial r} \quad **$$

in which εr = apparent strain from the moiré pattern in the direction of r.

r = distance measured from the optical axis of the camera in the direction normal to the grating lines.

u = displacement in the direction of r.

z = distance between camera lens and master grating.

Δz = gap between master and model gratings.

Figure (1) gives the schematic diagram of the experimental set-up. Experimental evidence for the validity of the equation was furnished in the paper for a special case where the gap is constant and therefore the second term on the right side of equation (1) drops out automatically. The purpose of the present paper is to test the validity of the equation for cases where the second term is not zero. Examples chosen for the study are linearly varying gaps. The fact that r appears in the equation, implies that the position of optical axis of the camera should play an important role in the interpretation of patterns.

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* Numbers in brackets designate references at the end of the paper.

** In reference [1], x instead of r is used.

APPROACH TO THE PROBLEM AND EXPERIMENTAL SET-UP

In order to avoid the inaccuracy usually associated with graphical or numerical differentiation, another form of equation (1) is chosen for the analysis, i.e.

$$u = r \frac{\Delta z}{z} \quad \text{or} \quad \frac{u}{r} = \frac{\Delta z}{z} \qquad (2)$$

which, by differentiation, gives equation (1). It should be noted that r - the distance measured from the optical axis normal to the grating lines* - is always a positive quantity because of the axially symmetrical property of a lens.

The experimental set-up as shown in Fig. 1 is the following: an eight inch by ten inch glass plate master grating of 300 lines per inch is erected perpendicular to the optical axis of the camera at a certain distance z away. It can be seen from equation (1) that as z approaches infinity the fictitious strain approaches zero. Therefore, usually in practice z should be kept as large as possible in order to reduce the fictitious strain. For this experiment, however, z is kept relatively small so that there are more fringes in the field to render an accurate analysis. The camera is focused on the plane of the master grating. An identical second grating (also of eight inch by ten inch glass plate) is then erected with lines parallel to the master but slightly inclined. The first set of experiments were run with the gap varying from zero on one end to 0.238 inch on the other (a slope of 0.0238). The moiré pattern thus formed consists of a set of parallel fringes with a variable spacing. In Fig. 2, the moiré patterns of the same arrangement photographed with optical axis of the camera at three different

*Actually r could be along any direction; it is because that moiré fringes measure displacements in the direction normal to the grating lines that r is so defined herein.

positions are shown: case I, optical axis near maximum gap; case II, optical axis near center; and case III, optical axis near minimum gap. As can be seen, the effect of the position of optical axis on the appearance of the moiré patterns is very pronounced. It should be noted, however, that the displacements of the optical axis are only along the direction of r. With the present arrangement of gratings displacements along the direction parallel to the grating lines will not change the moiré pattern, because there is no perspective effect in this direction.

DATA ANALYSIS AND RESULTS

The displacement curves corresponding to the moiré patterns of Fig. 2 are plotted in Fig. 3 from which u/r is computed to yield $\Delta z/z$. It should be noted in ordering fringes that in this study it is the absolute displacement represented by the fringes that are of interest. From equation (2), it is seen that u must be zero when r is zero. While it would be ideal to have a fringe located at the point where the optical axis intersects the pattern, in most cases the point of intersection is somewhere between two fringes*. It is, therefore, necessary to shift the abcissa in order to accommodate the condition (i.e. $u=0$ at $r=0$). Successively increasing orders should be assigned to the fringes as r increases until, when there is a change of sign [2] as in cases I and II of Fig. 3. When the optical axis is outside the field it is also necessary to consider the boundary condition. For example, in case III of Fig. 3, in addition to $u=0$ at $r=0$, it is necessary to have $u=0$ at $x=0$, because where Δz is zero.

The experimental results of $\Delta z/z$ obtained from Fig. 3 using eq. 2, are plotted in the lower part of Fig. 4, and compared with the measured $\Delta z/z$.

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* It is always possible of course to either shift the fringe [3] or the camera in such a way that the optical axis intersects a zero order fringe.

The agreement is very good. It is also interesting to note that while the position of optical axis does influence the appearance of moiré pattern, it has no effect on the end results, i.e. $\Delta z/z$, computed from the pattern. However, from the point of view of having more data points for plotting the displacement curve it is advantageous to place the optical axis at the minimum gap as in case III of Fig. 3.

Another method to increase the fringe density of a moiré pattern is to introduce an equivalent mismatch. This is accomplished by superposing a uniform gap throughout the field. An example is given in Fig. 5 where a uniform gap of 0.233 inch is added to case II of Fig. 3 where the optical axis is near the center. It is seen that the number of fringes is considerably increased. The corresponding displacement curve is also plotted in Fig. 5, whereas the computed $\Delta z/z$ together with the actual one is shown in the upper part of Fig. 4. This "mismatch" technique of increasing fringe density is useful, because in most cases the moiré pattern occupies the whole field of camera back and it is impractical to place the optical axis at the optimum position.

CONCLUSIONS AND DISCUSSIONS

It may be concluded that the validity of the moiré gap equation is further strengthened by the additional experimental evidence presented herein. The usefulness of moiré gap equation is threefold: it can be used to estimate the error caused by the out of plane deformation of a model under plane stress; it can be applied to take into account the effect of a constant gap when in some cases the master grating has to be placed some distance away from the model [4]; and it renders itself, in the form of eq. (2), as a new means to study the deflection of plates and related problems.

Acknowledgment

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References

1. Sciammarella, C.A. and Chiang, Fu-pen, "Gap Effect on Moiré Patterns", Zeitschrift fur agewandte Mathematik und Physik, Vol. 19, Fasc. 2, 1968.

2. Chiang, Fu-pen, "On the Determination of Signs in the Moiré Method", Technical Report No. 128, College of Engineering, State University of New York at Stony Brook, April, 1969.

3. Chiang, Fu-pen, Parks, V.J. and Durelli, A.J., "Moiré-fringe Interpolation and Multiplication by Fringe Shifting", EXPERIMENTAL MECHANICS Vol. 8, No. 12, Dec. 1968.

4. Sciammarella, C.A. and Chiang, Fu-pen, "Dynamical Stresses and Strains in Propellant Grains", ICRPG Mechanical Behavior Working Group 6th Meeting, Dec. 1967, JPL, Pasadena, California. CPIA Publication No. 158, Vol. 1, Oct. 1967.

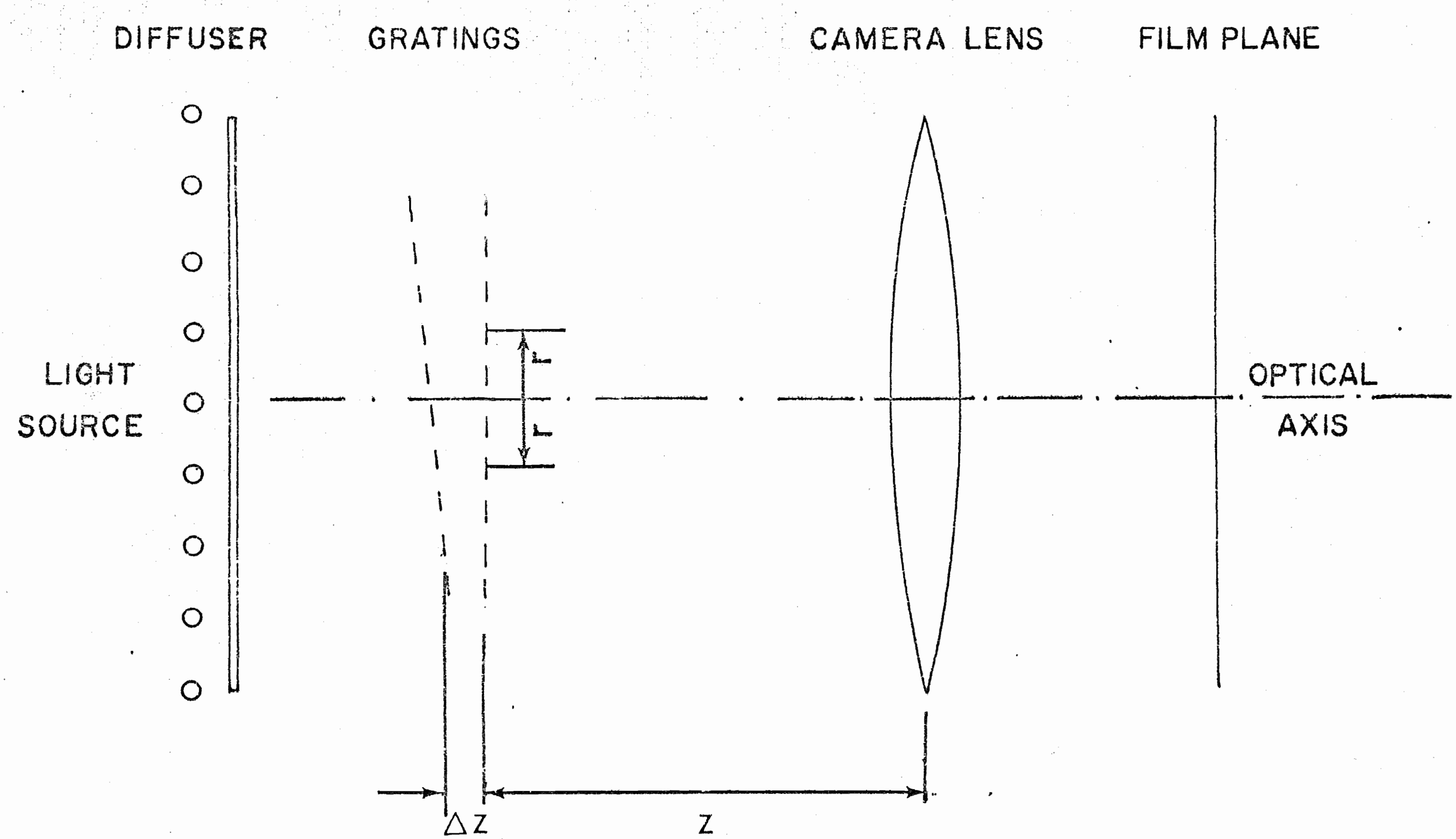

Fig. 1 Schematic Arrangement of the Experimental Set-up

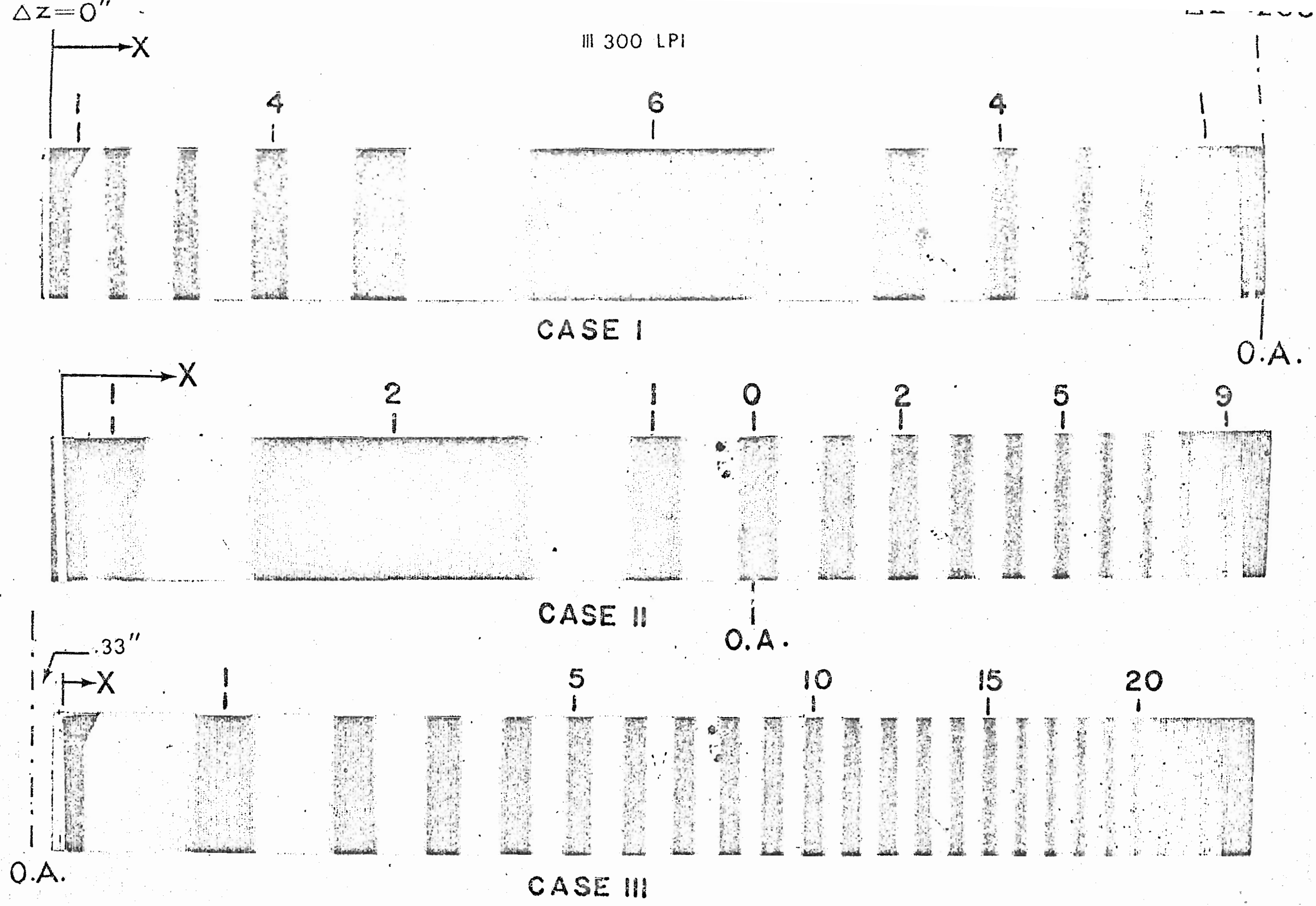

Fig. 2 Moire Patterns of Two Gratings with a Linearly Varying Gap in Between showing Effect of Position of Optical Axis of Camera

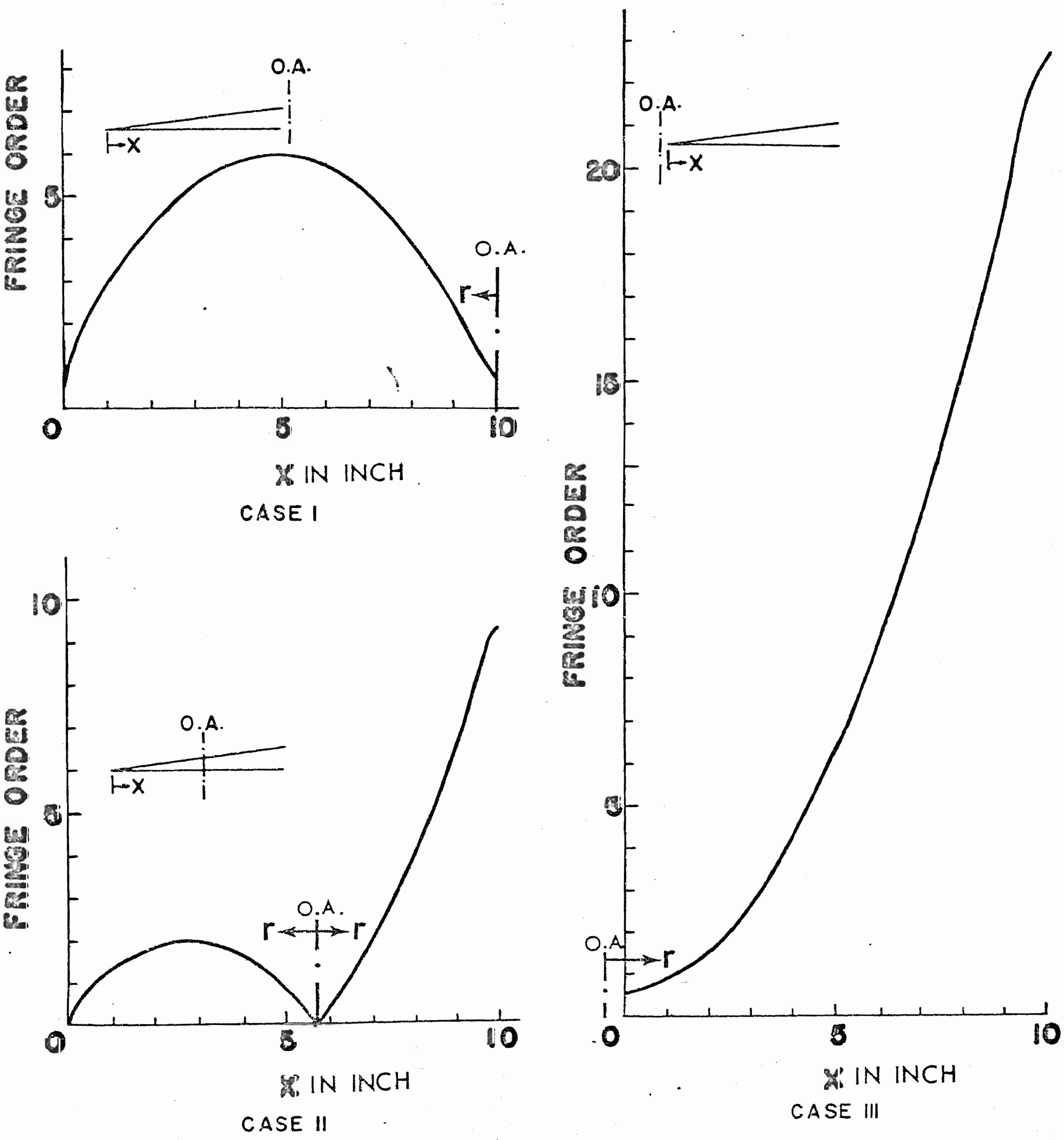

Fig. 3 Displacement Curve Plotted from Patterns in Fig. 1

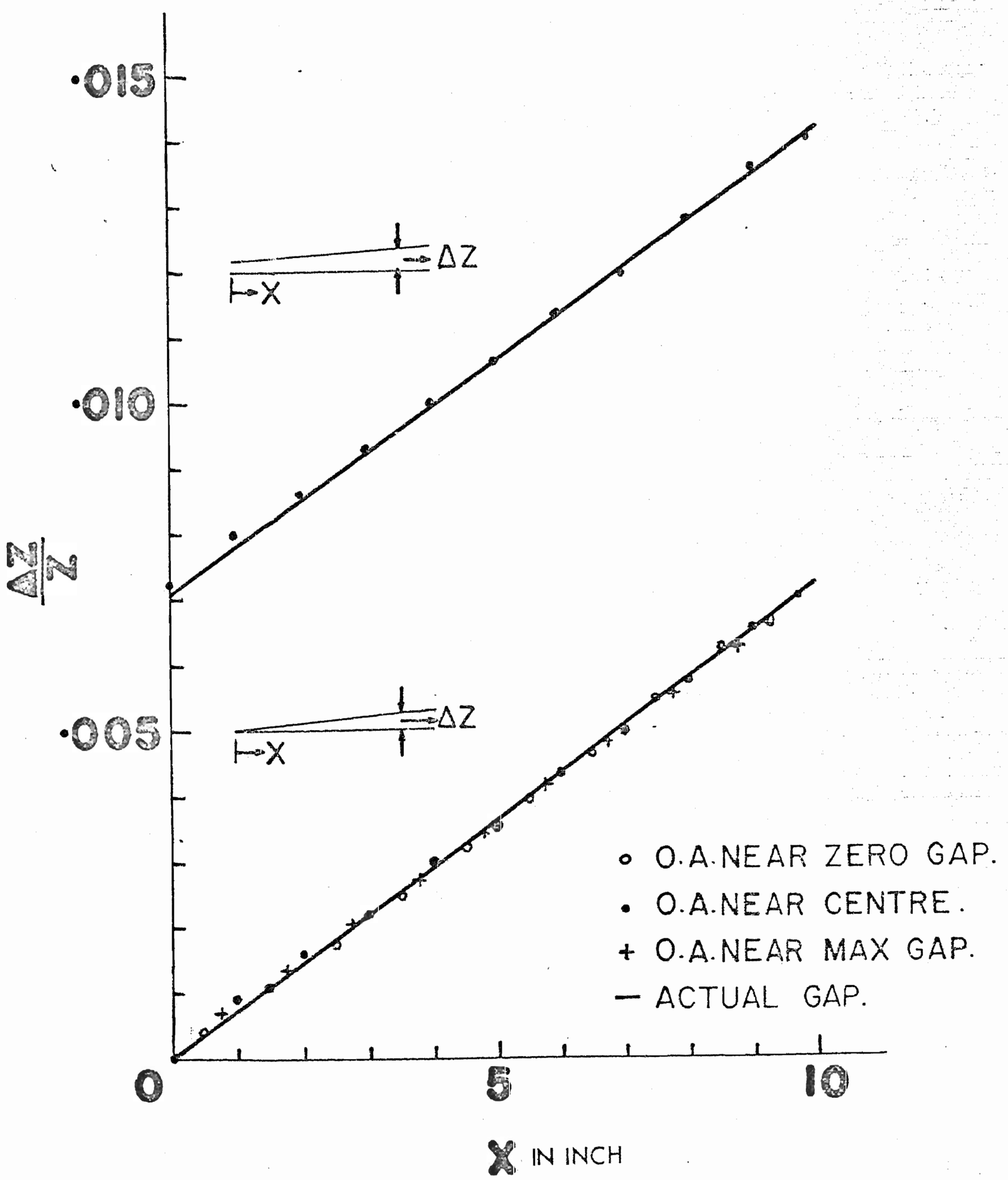

Fig. 4 Comparison of the Experimental Data Points and the Actual Gap

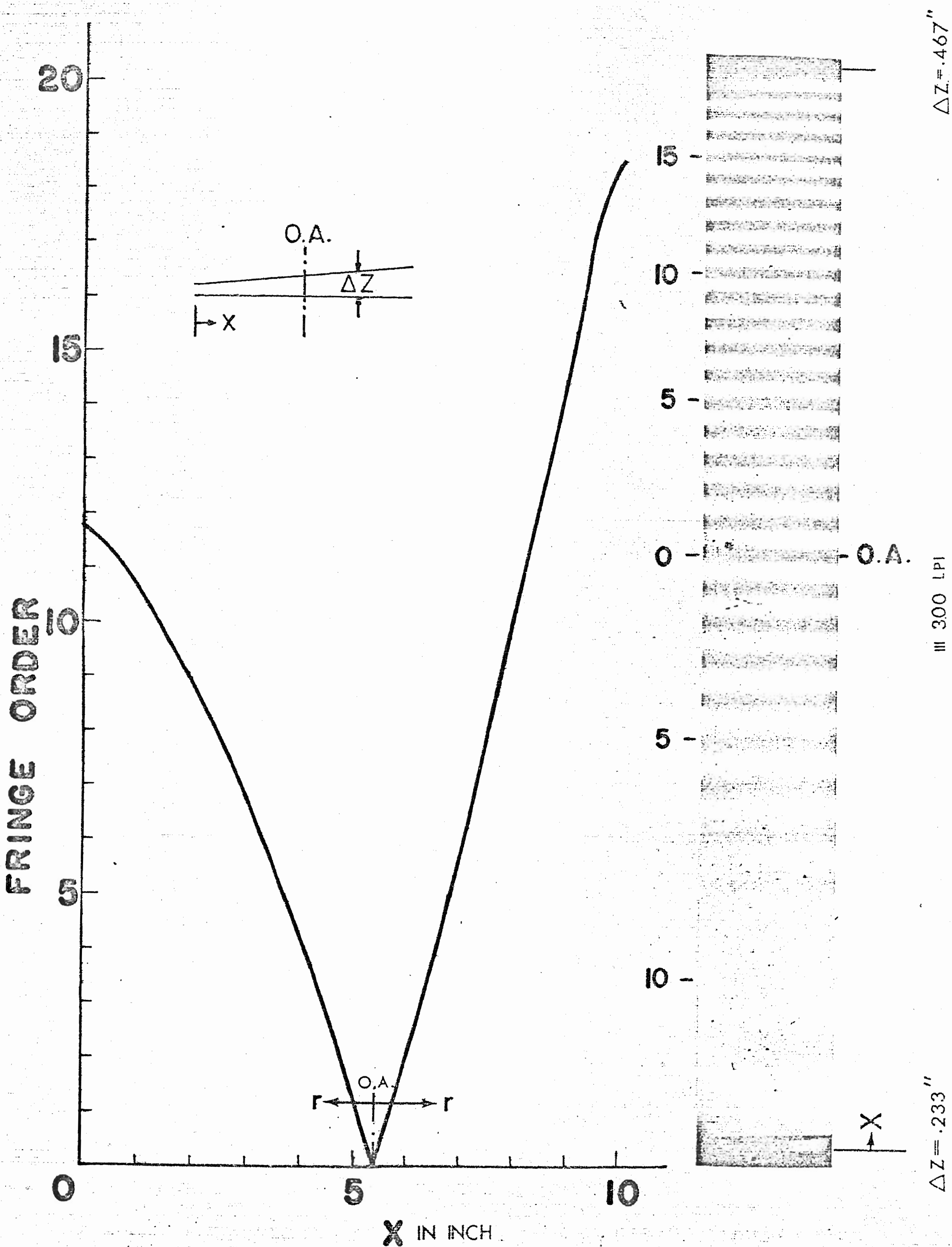

Fig. 5 Moire Pattern and Corresponding Displacement Curve for Two Gratings with Linearly Varying Gap