COLLEGE OF ENGINEERING

REPORT # 10

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Analysis of Limiting Thermal Conditions Encountered by a Manned Space Suit in Orbit

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Abstract

In this study, three thermal problems have been examined which occur in the design of space suits to be used when personnel are outside the parent vehicle. The first concerned the time-temperature variation of an infinite thermal conductivity suit when exposed to extreme conditions of heating and cooling. The second was related to temperature differences which may occur from the top to the bottom of the suit thereby causing physiological discomfort. Finally the scheme was examined whereby these temperature differences might be ameliorated by circulating a fluid in passages behind the suit material.

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Introduction

One of the difficult problems of protecting a man in space while free of a shielding vehicle or structure is to preserve him from the adverse thermal environment. Without adequate temperature control, a man would readily freeze when shaded from the sun or burn when exposed to the sun. In order to provide a satisfactory temperature regimen, the parameters involved in this thermal problem must be defined and investigated. A general analysis of the problem is complicated by the changing types of thermal radiation fields to which he may be exposed and the influence of his orientation both with respect to the solar system and nearby objects.

In addition to his orientation and form, man himself by reason of his physiological processes influences the problem through body heat generation and respiration and perspiration processes. Thus, the number of free parameters in the problem become very large, and there is some doubt whether a completely general analysis which accounts for all of these factors is feasible.

An approach which appears to offer more immediate progress is to divide the total problem into separate and manageable parts while remaining conscious of the interrelations among these various parts. This paper concentrates on three of these parts, which, other than metabolic heating, are independent of all physiological considerations. Thus, the conclusions are valid for determining the feasibility of the suit thermal design approach.

In the first part an analysis is made of the equilibrium temperature history of a space suit assuming that the thermal conductivity of the suit is infinite and that no temperature control system exists. In the second part an analysis is made of the temperature difference which might be found from the top to the bottom of the suit assuming a finite thermal conductivity of the suit material. In the third part, the effect of circulating a liquid coolant is examined in its ability to reduce the top to bottom temperature differences calculated in the second part.

General Orbit Consideration

The reasons for the choice of orbits for the present study may be illustrated by a brief discussion of the motion of the earth in the solar system. Figure 1(a) shows the important aspects of the earth's motion around the sun looking down on the ecliptic plane (the plane containing the earth's path around the sun).

The four important references locations of the orbit, the summer and winter solstices and the vernal and autumnal equinoctial points are also shown in figure 1(a). Figure 1(b) is a side view looking parallel to the ecliptic showing only the solstice points. It is important to note two things about the polar axis of the earth. First, the polar axis is not perpendicular to the ecliptic but is tipped at an angle of about 23 degrees from this perpendicular. Second, the polar axis remains fixed in its orientation in space as the earth travels around the sun. Thus, at the winter solstice the North polar axis points away from the sun and at the summer solstice it points toward the sun.

A close-up view of the earth at winter solstice (Fig. 2) is useful in our discussion of satellite orbits. If the satellite is moving in an orbit perpendicular to the plane of the paper, then only two numbers are needed to define its location with respect to the solar system. The satellite altitude, h, and the angle, β, between the perpendicular to the ecliptic and the radius vector to the satellite are convenient choices for this problem. For example, if $\beta = \gamma$ and $h = H$ (Fig. 2) the satellite is traveling in a polar orbit at altitude, H, above the earth's surface.

Equilibrium satellite temperatures can be calculated for a wide variety of orbits. However, for our purposes we will examine only the extreme cases to which any design may be subjected. These cases are when the satellite is continuously exposed to both the solar and earth's radiation fields ($\beta = 0$) and when the satellite is shielded from the solar field by being in the shadow of the earth. These may be called the "hot" and "cold" cases and will be described in more detail in later sections.

Human Geometry

The orbital space suit in its final evolution may well bear a close resemblance to the general outlines of a man. The choice of the geometry for the purposes of this calculation has an important influence on the complexity of the calculations and the applicability of the final results, i.e., the more nearly man-like the geometry, the more difficult the calculations and the more useful the results. Since the final configuration of the space suit to be worn by a man in orbit is at present unknown, no attempt was made at this time to consider the geometry of a man wearing a space suit with its accompanying life-support, stabilization, and locomotion equipment.

Possible geometric choices of space suit configurations have been investigated by Charles Clauser, Aerospace Medical Division of the Wright Air Development Division. Some of the results of his investigations are shown in Figs. 3 and 4. These geometric forms are representative of the general size of a 50th percentile (height and weight) nude Air Force male. Total body surface area for such a man would be approximately 21 square feet. An important decision in the present investigation was whether to consider geometrics which were singly cylindrical and everywhere plane or convex, such as in Fig. 4 or to include other geometrics such as Fig. 3 which would allow radiation exchange between various parts. The geometry described in Fig. 4 was chosen so as not to obscure the consideration of the thermal problem with the additional complicated geometric details of the analysis if the geometry in Fig. 3 had been used.

PART I

INFINITE THERMAL CONDUCTIVITY SUIT

At any instant in time, a man wearing a space suit will be under the influence of a number of factors which will determine his thermal environment and consequently the average temperature of his thermodynamic system. Among these factors are the thermal radiation fields from the sun, earth, and nearby objects, the thermal properties of the suit, his own metabolic heat generation and other biological processes, and the previous history or variations of all these factors.

A great deal of useful information and background knowledge should accrue from an analysis of a part of this complex system. For example, the temperature history of a space suit without a man inside (but with energy generation within the suit to model the metabolic heat rate) may be examined with a fair degree of sophistication. Such information, coupled with the present knowledge of man's physiological processes, serves as an excellent starting place for determining additional modifications necessary to a space suit before a man is placed inside.

The energy balance for determining the temperature-time history was based on the suit scheme in Fig. 5. Additional specifications were an adiabatic inside surface and a very high thermal conductivity for the space suit material so that no heat-transfer processes inside the cylinder would require consideration and no temperature differences would exist in the suit material.

Under the conditions described above, the energy balance on the suit assumes the following form as applied to a unit of time:

$$Q_{\alpha R} + Q_G = Q_{\epsilon R} + \frac{dQ}{dt} \qquad (1)$$

The first term, $Q_{\alpha R}$, consists of a number of terms representing the separate radiation fields absorbed by the suit. These fields include direct and reflected solar radiation, earth radiation, and radiation from nearby objects, e.g., the parent satellite or space ship. The second term allows for the internal generation of heat within the suit or on its boundaries. The third term describes the amount of thermal radiation which leaves the suit and the last describes the influence of the heat capacity of the suit. Implicitly, it has been assumed that any radiation leaving the suit does not return by reason of a reflection process.

Details of the energy balance are given in Appendix I and the final equation in a dimensionless form is

$$N_S + N_V + N_G + N_E = \theta^4 + 3.78\, t_c^{\Delta} \frac{d\theta}{dt^{\Delta}} \tag{2}$$

In Eq. (2) N_s and N_v are the dimensionless values of the absorbed solar and nearby vehicle radiation fields, N_G the dimensionless internal heat generation, and N_E the dimensionless absorbed earth radiation. The symbol θ represents a dimensionless suit temperature referred to the average temperature of the earth, i.e., $\theta = T/T_E$. The term $3.78\, t_c^{\Delta}$, developed in Appendix I, is a dimensionless measure of the energy retention capacity of the suit material and t^{Δ} is a dimensionless time referred to the period of orbit of the suit.

Eq. (2) is a first order, nonlinear differential equation which requires the specification of one boundary condition. The condition which has been chosen for all calculations is

$$\text{When} \quad t^{\Delta} = 0, \quad \theta = 1.2 \tag{3}$$

Equation (3) states that the space suit temperature at the beginning of the calculation is $80^{\circ}F$. The calculation then predicts whether the temperature of the suit increases or decreases under its environmental influences and how much.

Calculation Results

The basic energy equation discussed above was solved on a computer for two extreme conditions. The first condition was when $\beta = 0$ and the space suit at $80^{\circ}F$ was suddenly placed in the sunlight and remained there during subsequent time. Parametric values were chosen so that the solar and earth-radiation inputs as well as the radiation input from the nearby vehicle were maximum. The internal energy generation rate was chosen to approximate that of a man engaged in minor activity. Thus, this calculation could be expected to predict the maximum temperatures that might be found in a space suit. Details of the choice of parametric values are given in Appendix II.

As an illustration of the temperature control factors available to the designer, the calculation was then repeated for two additional cases. In one, the suit properties were chosen so that a minimum amount of outside radiation was absorbed and the internal heat generation rate the same. The second condition considered the space suit at $80^{\circ}F$, suddenly placed in the earth's shadow (in a 300 mile orbit in the plane of the ecliptic) so that the only incident radiation field was the earth's. To represent the "coldest" case, the internal heat generation rate was taken at zero.

The result of the calculations for these two conditions is discussed in detail below.

Case I - Hot (Maximum Radiation Input and Heat Generation Rate)

Figure 6 presents the computed variation of the nondimensional suit temperature with time, measured in multiples of the orbital period, for three values of $\frac{\alpha_s}{\epsilon}$. The extreme equilibrium values of θ = 3.23 and 1.13 or T = 993°F and 48°F reveal the wide variation of suit temperatures permitted by the choice of the suit's spectral properties. The parameter, t_c^{Δ}, the heat retention parameter, is defined as

$$t_c^{\Delta} = \frac{1}{3.78} \left[\frac{\rho c V}{\alpha_E A_T \sigma T_E^3 t_P} \right] = \frac{1}{3.78} \left[\frac{\rho c \tau}{\epsilon \sigma T_E^3 t_P} \right]$$

Thus, for a fixed orbital period, t_c^{Δ} is directly proportional to the specific heat per unit volume (ρC) and the suit material thickness, τ. Conversely t_c^{Δ} is inversely proportional to the suit's emissivity, ϵ.

Figure 6 indicates that the suit designer must limit the external heat inputs by choosing a low $\frac{\alpha_s}{\epsilon}$ value to maintain comfortable suit temperatures. It may also be noted that the choice of a sufficiently large value for the heat retention parameter will lengthen the time in which the suit reaches its equilibrium temperature.

Case II - Cold (Earth Heating Only)

Figure 7 shows the results of the calculations for the coldest case. Judiciously chosen suit properties will inhibit rapid temperature changes when the suit is being heated by earth radiation alone. For example, when

$\overset{\Delta}{t}_c$ is equal to 12, the suit temperature drops only 25°F from its original value of 80°F in one equivalent period (90 minutes). This slow decrease is interesting since the equilibrium (long time) temperature for these conditions is -100°F and the suit can stay in the earth's shadow for only 34 minutes in a 300-mile orbit.

Thus, controlling the thermal environment of a man in space under conditions of earth heating only through the selection of proper suit properties should be considered. This is true whether control is desired on the unsteady state temperature, the final equilibrium temperature, or both.

For an application of the calculations the following "suit problem" will be examined.

Design Requirements:

A. Passive Heat Control

B. θ max = 1.22 or 90°F

θ min = 1.17 or 60°F

C. Exposure time: one 300 mile orbit

D. Lightweight suit material

E. Thin suit material to permit mobility

To be determined:

A. Surface coating ($\frac{\alpha_s}{\epsilon}$)

B. Suit material (C)

C. Suit's total mass (ρV)

D. Material thickness (τ)

A white $PbCo_3$ paint ($\frac{\alpha_s}{\epsilon} = 0.13$) for the surface coating will maintain the maximum suit temperature below the required 90°F since $\frac{\alpha_s}{\epsilon}$ is only slightly larger than the 0.11 value plotted in Fig. 6 At low $\hat{t}_c$ value and a large ρC value must be selected if a small suit thickness is required. Thus, it is necessary for the material to have a high specific heat (c) to obtain a low total mass suit. If, for example, a water filled five-eights-inch thick shell is chosen for the suit material then:

$$\rho C = 60, \quad \hat{t}_c = 4$$

This value of $\hat{t}_c$ will limit the minimum suit temperature to $\theta = 1.16$ or 62°F for a 300-mile orbit. The minimum temperature requirement of 60°F will thus be met. The resulting total mass of the suit is then a not unreasonable 64 pounds.

PART II

SUIT WITH FINITE THERMAL CONDUCTIVITY

The preceding analysis assumed an infinite thermal conductivity of the space suit material so that no temperature gradients could exist in the suit. This restriction will now be relaxed to allow different suit thicknesses and thermal conductivities in order that these temperature gradients may be examined. Since the maximum temperature difference which might be encountered is of primary interest, the heat input and the outgoing radiation have been chosen so that extreme temperature gradient conditions will be encountered. Fig. 7 illustrates the model which was used. From Fig. 7 it is seem that the suit is to be subjected to a solar heat flux, q_s , only at one end and the remaining two surfaces of the suit are allowed to radiate into space at zero degrees absolute. In addition the total heat absorbed by the end area is assumed to be concentrated uniformly around the circumference of the shell at its initial boundary $x = 0$. The metabolic heat is uniformly distributed only along the inner cylindrical surface, and the entire heat-transfer process is assumed to be at steady-state.

With these assumptions and the suit geometry of Fig. 7, the heat balance for an elemental length of the cylindrical shell is expressed by:

$$\frac{dQ}{dx} + \frac{Q_m}{L} = \frac{Q_r}{L} \tag{4}$$

Substituting the defined expressions:

$$Q = -k \pi D \tau_2 \frac{dT}{dx}$$

$$Q_r = \sigma \epsilon \pi D L T^4$$

$$Q_m = \pi D L q_m$$

for the various heating rates, and nondimensionalizing:

$$\frac{d^2\theta}{d\eta^2} + A\theta^4 - \bar{q} = 0 \tag{5}$$

The boundary conditions for this ordinary second order differential equation can be determined by considering the heat balance with the external environment at each end:

$$\text{at } \eta = 0, \; Q = -K\pi D\tau_2 \frac{T_a}{L}\frac{d\theta}{d\eta} = \frac{\pi D^2 \alpha_s q_s}{4} \qquad (6)$$

$$\text{or } \left.\frac{d\theta}{d\eta}\right|_{\eta=0} = \frac{L D \alpha_s q_s}{4K\tau_2 T_a} = -B$$

$$\text{at } \eta = 1, \; Q = -K\pi D\tau_2 \frac{T_a}{L}\frac{d\theta}{d\eta} = \sigma\epsilon \frac{\pi D^2}{4} T_L^4 \qquad (7)$$

$$\text{or } \left.\frac{d\theta}{d\eta}\right|_{\eta=1} = -\frac{L D T_a^3 \sigma \epsilon}{4K\tau_2}\theta_L^4 = -C\theta_L^4$$

Therefore, the solution of Eq. (5) subject to the boundary condition, Eqs. (6) and (7) will provide the temperature distribution along the cylindrical portion of the suit for various suit materials and thicknesses. The maximum temperature difference will then simply be the difference between the temperatures at the two ends.

Equation (5) was solved by a numerical integration scheme to provide the required longitudinal temperature distribution. The numerical work is greatly reduced by the use of the relations for the various parameters:

$$A = \frac{4\sigma T_a^4 \epsilon L}{q_s \alpha_s D} B, \quad B = \frac{L D \alpha_s q_s}{4 T_a K \tau_2}, \quad C = \frac{\sigma T_a^4 \epsilon}{\alpha_s q_s} B$$

The following constants used in Part I were also selected:

$$L = 5.75 \text{ ft.}$$
$$D = 1.04 \text{ ft.}$$
$$\alpha_s = 0.12$$
$$\epsilon = 0.89$$
$$Q_m = 800 \text{ Btu/hr}$$
$$A_T = 20.5 \text{ ft}^2$$

The number of arbitrary parameters is then reduced to one, the initial slope B, which is a function of the material thickness and thermal conductivity.

The required suit shell thickness and mass can then be determined from the relations

$$\tau_2 = \frac{0.1397}{KB} \tag{8}$$

$$M = \frac{2.86}{B} \quad \frac{\rho}{K} \tag{9}$$

In the first part the suit temperature variation with time was determined from the balance of the metabolic heating with the external environment as influenced by the heat retention capacity of the material and the surface radiation properties. The shell thickness (τ_1) was determined as a function of the heat retention parameter, which was chosen as four in the sample problem, and the surface radiation properties to limit the minimum suit temperature to 62°F for a 300-mile orbit:

$$\tau_1 = \frac{\epsilon\, t_c^{\Delta}}{1.132\,\rho C} \tag{10}$$

Therefore, with the various assumed constants, the ratio of the thicknesses or masses is a function of the material thermal diffusivity and the initial slope B:

$$\frac{\tau_1}{\tau_2} = \frac{M_1}{M_2} = 22.5\,\alpha B \tag{11}$$

When this ratio is greater than one, the material thickness as required by the first analysis must be chosen to provide a proper thermal environment. Since this thickness is greater than that required by the present analysis, smaller equilibrium maximum temperature differences will exist. If the ratio is less than one, then the thickness computed in Eq. (11) must be chosen to minimize the maximum temperature difference. This thickness is greater than that computed with Eq. (10) and this provides a greater thermal capacity which will lengthen the time for the suit to reach thermal equilibrium with its external environment. This ratio is useful only for the above type comparison since it compares the results of the time-dependent analysis of the first part with the results of this non-time-dependent study.

Finite Thermal Conductivity Results

Equation (5), as restricted by the boundary conditions and simplifying relations, was integrated numerically with the Runge-Kutta method on a digital computer for a range of initial slopes, B. Agreement with pre-computed slopes at $\eta = 1$ was obtained to seven significant figures.

Figure 9 and 10 present in graphical form the numerical results for the temperature variation along the suit and the maximum temperature difference from x = 0 to x = L. The strong dependence of both the temperature and maximum temperature difference on the parameter B can be seen from the figures. A small value of B is desirable in any suit design problem. It may be seen, however, that small values of B come at the expense of increased shell thickness and mass for a given shell material. The shell thickness increases as the ratio $\frac{\rho}{K}$ increases and therefore, it is desirable to have this ratio low. Aluminum and Copper have the lowest ratio for metals while the ratios for fabrics are approximately 100 times as large.

The interlocking considerations of both the infinite and finite thermal conductivity analyses can be illustrated by the following example:

Given: Maximum Tolerable $\Delta T = 30^{\circ}F$

To be determined: Material type, thickness, mass, and resulting maximum ΔT.

Solution:

1. With Fig. 9 and $\Delta T = 30^{\circ}F$, $B_2 = 0.043$
2. From Eq. (9) and $B_2 = 0.043$, $M_2 = 95 lb_m$ for aluminum
3. From Eq. (8) and $B_2 = 0.043$, $\tau_2 = 0.33$ in for aluminum
4. From Eq. (11) $\frac{\tau_1}{\tau_2} = \frac{M_1}{M_2} = 3.7$ for aluminum

5. Since $\frac{\tau_1}{\tau_2} > 1$, then the thickness τ_1, and mass M_1, must be used. Therefore,

$$\tau_1 = 3.7 \times 0.33 = 1.22 \text{ in.}$$

$$M_1 = 3.7 \times 95 = 352 \text{ lbm}$$

6. With Eq. (9) and $M_1 = 352$ lb_m: $B_1 = 0.011$ for aluminum
7. With Fig. (9) and $B_1 = 0.011$: $\Delta T = 5.8^{\circ}F$

Thus a suit constructed with aluminum shell 1.22 inches thick would have a mass of 352 lb_m and a $5.8^{\circ}F$ maximum equilibrium temperature difference.

Since the thermal diffusivity of most light metals is in the range of 3.6 to 4.4 ft^2/hr, the thickness ratio will be greater than one for $B > 0.01$. Therefore, most light-metal suit thicknesses computed with the results of the first analysis will develop small equilibrium temperature difference and be very thick and heavy. With the general conclusion the 5/8-inch-thick, water-filled shell suggested previously becomes attractive since it would have a mass of 64 lb_m. The very large equilibrium and non-equilibrium temperature difference that would exist, since $\left.\frac{\rho}{K}\right|_{H_2O} = 186$ could be reduced by circulating the water. This possibility will be examined in the next section.

PART III

THE EFFECT OF CIRCULATING WATER

In order to examine the effect on the shell temperature difference of circulating water in passages just inside the outer wall, the model used in the second analysis will have to be modified as shown in Fig. 10. Since maximum temperature differences are of primary interest, the mathematical model is chosen to produce these extreme conditions. Thus the solar flux is again assumed to be incident on one end of the cylinder while the other two surfaces are radiating to a zero temperature environment. The solar flux is also assumed to be concentrated uniformly around the circumference of the outer shell at its initial boundary. In addition, the metabolic heating is uniformly distributed along the inner shell surface and the suit is taken to be in thermal equilibrium with its environment.

If the fluid flowing through the passage is specified as being incompressible, laminar and fully developed and having constant physical properties, the energy equation describing the temperature field in the fluid may be written as

$$u \frac{\partial T}{\partial x} = \alpha_w \frac{\partial^2 T}{\partial y^2} \tag{12}$$

The temperature boundary conditions are specified as

Solution $$T = T(x, y)$$

Boundary Conditions

$$T(0, y) = T_0 \tag{13a}$$

$$T(x, r) = a e^{-\ell x} \tag{13b}$$

$$\frac{\partial T}{\partial y}(x, -r) = \frac{Q_m}{k} \tag{13c}$$

Condition 13b, as is explained more fully below, is taken from the results of the second analysis to give the maximum possible temperature difference along the inner shell.

Equation (12) may be put into dimensionless form by the following substitutions $\eta = \frac{x}{L}$, $\xi = \frac{y}{r}$, $\theta = \frac{T}{T_0}$

$$\frac{u_{max}}{u}\frac{\partial^2 \theta}{\partial \xi^2} - a\frac{\partial \theta}{\partial \eta} = 0 \tag{14}$$

where $a = r^2 u_{max}/\kappa_w L$. The boundary conditions become

$$\theta(0, \xi) = 1$$
$$\theta(\eta, 1) = e^{-B\eta}$$
$$\frac{\partial \theta}{\partial \xi}(\eta, -1) = -\lambda$$

where $\lambda = q_m r / K_w T_0$ and the heat conducted along the inner wall is neglected.

In the above, it has been assumed that the solar heat is conducted only through the outer shell and is not influenced by the flowing fluid. The resulting temperature difference in the outer shell is then the one computed in the second analysis and is applied in this analysis as the boundary condition on that surface. Under these conditions the flow rates computed will be maximum since the flowing water would clearly produce a more moderate temperature distribution in the outer shell. Details of the solution of Eq. (14) are given in Appendix III. The results are discussed below.

Temperature differences in the inner shell and over the passage length are shown in Fig. 11. They are plotted against the dimensionless flow rate of water (a) and for two values of the dimensionless metabolic heating parameter (λ).

Figure 2 illustrates the decrease in temperature difference with increasing water flow rate. This decrease is shown quantitatively in Table I for particular values of the suit and water passage geometry, i.e., $L = 5.76$ ft, $r = 5/16$ in, $\alpha_w = 5.5 \times 10^{-3}$ ft^2/hr. The flow rates are for water flow around the complete suit circumference.

Table I - Water Flow Rates

ΔT °F	$\lambda = 0.0$ Flow Rate (Gals./min)	$\lambda = 0.006$ Flow Rate (Gals. min)
0	2.02	1.618
5	1.416	1.214
10	1.112	1.012
15	0.950	0.808
20	0.808	0.728
25	0.606	0.668

Figure 2 also indicates that changes in the metabolic heating rate do not have a great influence on the water flow required. The two values given on the curves are for zero metabolic heat rate and for a value of $\lambda = 0.006$ which corresponds to the metabolic heat rate found in a normal man.

It may be concluded from the above that temperature differences in a space suit under the conditions specified may be kept to reasonable values without excessive flow rates of the cooling fluid. Also the variations in the distribution of metabolic heating over the body surface and those due to differences in physical activity do not appear to be important in their influence on the required flow rate for the cooling fluid.

SUMMARY & CONCLUSIONS

An appropriately chosen analytical space suit model subjected to environmental extremes has been examined to provide knowledge of the dominant heat transfer processes and to determine the suit temperature control requirements. Analytical results demonstrate that a wide range of temperatures may be produced by variation of the surface spectral properties and the external heating sufficiently limited by selection of low α_s/ϵ ratios. High heat capacity materials are required to inhibit rapid temperature changes and prevent freezing temperatures when passing through the earth's shadow. A water liner is recommended as the highest capacity per pound suit construction.

Intolerable temperature differences can exist over the suit surface depending on its material and thickness. However, circulation of the water at small flow rates will prevent their occurrence.

Finnally, a water jacketed space suit appears practical and when modified to satisfy the requirement of an astronaut's physiological processes should result in a garment capable of protecting him outside a vehicle that is located in an orbit about the earth.

APPENDIX I

Details of Orbital Suit Energy Balance

Figure 5 illustrates the model chosen for the orbital suit. Application of the energy relation, Eq. 1, to the variety of radiation field shown in the figure yields

$$\alpha_S q_S A_{\alpha S} + \alpha_V q_V A_{\alpha,V} + \alpha_S q_S \rho_{ES} R A_{\alpha,R} + K_D q_E a_E A_{\alpha E} + q_G A_T$$
$$= A_T \sigma \epsilon T^4 + \rho c V \frac{dT}{dt} \qquad \text{(I-1)}$$

In the above equation, the symbol, q , represents a heat flux (btu/hr ft) with an appropriate identifying subscript. If the equation is divided by $\epsilon A_T q_E$, after rearrangement, it may be written as follows:

$$\left[\frac{A_{\alpha,S}}{A_T} + \rho_{E,S} R \frac{A_{\alpha,R}}{A_T}\right]\frac{\alpha_S}{\epsilon}\frac{q_S}{q_E} + \left[\frac{\alpha_V}{\epsilon}\frac{A_{\alpha V}}{A_T}\right]\frac{q_V}{q_E} + \left[\frac{1}{\epsilon}\right]\frac{q_G}{q_E} + \left[K_D\frac{A_{\alpha E}}{A_T}\right] \qquad \text{(I-2)}$$
$$= \frac{\sigma T^4}{q_E} + \left[\frac{\rho c V}{\epsilon A_T q_E}\right]\frac{dT}{dt}$$

The reasonable assumption has been made that since the object in orbit will have the same order of magnitude of absolute temperature as the earth, then

$$\epsilon = \alpha_E \qquad \text{(I-3)}$$

The independent and dependent variables may now be made dimensionless by the relations

$$t^{\Delta} = \frac{t}{t_P} \quad \text{and} \quad \theta = \frac{T}{T_E}. \tag{I-4}$$

These relations and the expressions for the earth's radiation field at its surface, $q_E = \sigma T_E^4$ permit rewriting of the entire equation:

$$N_S + N_V + N_G + N_E = \theta^4 + N_C \frac{d\theta}{dt^{\Delta}} \tag{I-5}$$

where

$$N_S = \left[\frac{A_{\alpha,S}}{A_T} + \rho_{ES} R \frac{A_{\alpha,R}}{A_T}\right] \frac{\alpha_S}{\epsilon} \frac{q_S}{q_E}$$ Influence of Direct and Reflected Solar Radiation

$$N_V = \left[\frac{\alpha_V}{\epsilon} \frac{A_{\alpha,V}}{A_T}\right] \frac{q_V}{q_E}$$ Influence of Nearby Radiation Fields

$$N_G = \left[\frac{1}{\epsilon}\right] \frac{q_G}{q_E}$$ Influence of Internal Heat Generation in the Suit

$$N_E = \left[K_D \frac{A_{\alpha,E}}{A_T}\right]$$ Influence of Earth Radiation

$$N_C = \left[\frac{\rho c V T_E}{\epsilon A_T q_E t_P}\right]$$ Influence of Suit Heat Capacity

At this point Eq. I-5 may be rewritten with the aid of the solution to a simple case. If the suit were suddenly placed in space and not under the influence of any external radiation fields or internal heat generation, its temperature would begin to decrease at a rate dependent upon the thermal properties of the suit. Specifically, the parameters N_S, N_y, N_G, and N_E, would equal zero and Eq. I-5 would reduce to:

$$\frac{d\theta}{dt^{\Delta}} = \frac{-\theta^4}{Nc} \tag{I-6}$$

If the suit temperature at zero time were $80^{\circ}F$, the boundary condition applicable to the above equation would be

$$\text{when} \qquad t^{\Delta} = 0 , \quad \theta = 1.2 . \tag{I-7}$$

The solution to Eq. I-6 with the boundary condition I-7, can be obtained by direct integration yielding

$$\theta = \left[\frac{1}{\frac{3t^{\Delta}}{Nc} + \frac{1}{(1.2)^3}} \right]^{\frac{1}{3}} \tag{I-8}$$

A time constant for the suit may be defined as the time at which the suit temperature drops to 0.9 of its original value of $26^{\circ}F$ and may be designated by the symbol, t_c^{Δ} with the value

$$t_c^{\Delta} = \frac{Nc}{3.78} \tag{I-9}$$

The basic equation, I-5, may now be written in the form as shown in Eq. 2

$$N_S + N_V + N_G + N_E = \theta^4 + 3.78\, t_c^{\Delta} \frac{d\theta}{dt^{\Delta}} \tag{I-10}$$

APPENDIX II

Selection of Parametric Values

The maximum and minimum values of the parameters in the computer calculation were based on the following data which are applicable to the case of a circular orbit 300-miles in altitude.

q_S = 420 Btu/hr

q_E = 71.5 Btu/hr

T_E = 450°R

A_T = 20.54 ft^2

K_D = 0.86 , $\left[K_D = \left(\frac{R_E}{R_E + h} \right)^2 \right]$

t_p = 1.15 hrs.

R = 2 (Assumes diffuse reflection from a flat earth.)

ρ_{ES} = 0.4

For the calculation of the maximum suit temperature the suit orientation must be such that

$$A_{\alpha,S} = A_{\alpha,R} = A_{\alpha,V} = A_{\alpha,E} = \frac{A_T}{2}$$

thus permitting specification of maximum and minimum values for the parameters. The details are given below

a. $$N_S = \left[\frac{A_{\alpha,S}}{A_T} + \rho_{ES} R \frac{A_{\alpha,R}}{A_T} \right] \frac{\alpha_S}{\epsilon} \frac{q_S}{q_E}$$

Consider a maximum value of α_S/ϵ to be 10: then

$$N_S \Big|_{max} = 53 .$$

Actual values taken were

$$N_S\Big|_{max} = 50,$$

$$N_S\Big|_{min} = 0.$$

b.
$$N_V = \left[\frac{\alpha_V}{\alpha_E}\ \frac{A_{\alpha,V}}{A_T}\right]\frac{q_V}{q_E}.$$

When $\alpha_V = \alpha_E$ and q_V, max = 1000 btu/hr ft^2,

$$N_V\Big|_{max} = 7$$

Actual values taken were

$$N_V\Big|_{max} = 10,$$

$$N_V\Big|_{min} = 0.$$

e.
$$N_G = \left[\frac{1}{\alpha_E}\right]\frac{q_G}{q_E}$$

When α_E, min = 0.1 and q_G = 39.1 btu/hr ft^2 (Based on a total body heat production rate of 800 btu/hr), then

$$N_G\Big|_{max} = 5.47$$

Actual values taken were

$$N_G\Big|_{max} = 10$$

$$N_G\Big|_{min} = 0.$$

d.
$$N_E = \left[K_D \frac{A_{\alpha,E}}{A_T}\right]$$

This parameter has a unique value since the suit is always under the influence of the earth's radiation field. The calculated value is

$$N_E = 0.43$$

e. $$t_c^{\Delta} = \frac{1}{3.78}\left[\frac{\rho c\, V T_E}{\epsilon A_T q_E t_p}\right]$$

for rubber or wood ρc has the approximate value of 25 Btu/ft^2 °F. Taking a minimum value of ϵ as o.1 and a maximum thickness as 0.5 inches

$$t^{\Delta}_{c\ max} = 11.55$$

Actual value taken was

$$t_c^{\Delta} = 16$$

Using the parametric values calculated abot, Eq. I-10 in Appendix I may be written

$$3.78\, t_c^{\Delta} \frac{d\theta}{dt^{\Delta}} + \theta^4 = 70.43\,, \quad \theta_{(0)} = 1.2 \qquad \text{(II-1)}$$

This hottest case together with the results of a more moderate case are shown in Fig. 6. The range of possible temperatures is illustrated by another case (also shown in Fig. 6) where the choice of the suit spectral properties limited the external heat inputs was computed by Eq. II-2;

$$3.78\, t_c^{\Delta} \frac{d\theta}{dt^{\Delta}} + \theta^4 = 1.62\,, \quad \theta_{(0)} = 1.2 \qquad \text{(II-2)}$$

This solution predicts the lowest temperatures to be expected in a 300-mile orbit normal to the ecliptic plane.

For the low temperature case, all incident radiation fields except the earth's as well as the internal heat generation rate were zero. The resulting equation was

$$3.78\, t_c^{\Delta} \frac{d\theta}{dt^{\Delta}} + \theta^4 = 0.43\,, \quad \theta(0) = 1.2 \qquad \text{(II-3)}$$

These results are shown in Fig. 7.

APPENDIX III

Solution of the Cooling Fluid Equations

If the laminar velocity profile is inserted in the energy equation, Eq. (14), the equation to be solved is

$$\frac{\partial^2 \theta}{\partial \xi^2} - \alpha(1-\xi^2)\frac{\partial \theta}{\partial \eta} = 0 \qquad \text{(III-1)}$$

where $\theta(0,\xi) = 1$, $\theta(\eta,1) = e^{-B\eta}$, $\frac{\partial \theta}{\partial \xi}(\eta,-1) = -\lambda$

Since Eq. (III-1) is linear the solution can be expressed as a sum of solutions. Thus, the variable boundary conditions can be satisfied by a careful choice of particular solutions and the general solution will have zero boundary conditions. The general solution can then be expressed as an infinite series of functions that are orthogonal to each other and the weighting function $(1-\xi^2)$.

Thus, assume that the form of the temperature function is

$$\theta(\eta,\xi) = \lambda(1-\xi) + G(\xi)e^{-B\eta} + \sum_{n=1}^{\infty} C_n e^{-\delta_n \eta}\phi_n(\xi) \qquad \text{(III-2)}$$

Therefore

$$\theta(0,\xi) = 1 = \lambda(1-\xi) + G(\xi) + \sum_{n=1}^{\infty} C_n \phi_n(\xi)$$

$$\theta(\eta,1) = e^{-B\eta} = G(1)e^{-B\eta} + \sum_{n=1}^{\infty} C_n e^{-\delta_n \eta}\phi_n(1)$$

$$\frac{\partial \theta}{\partial \xi}(\eta,-1) = -\lambda = -\lambda + \frac{\partial G}{\partial \xi}(-1)e^{-B\eta} + \sum_{n=1}^{\infty} C_n e^{-\delta_n \eta}\frac{\partial \phi_n}{\partial \xi}(-1)$$

and to satisfy the boundary conditions, let

$$G(1)=1,\ \frac{\partial G}{\partial \xi}(-1)=0,\ \phi_n(1)=0,\ \frac{\partial \phi}{\partial \xi}(-1)=0$$

$$\sum_{n=1}^{\infty} C_n \phi_n(\xi) = 1-\lambda(1-\xi)-G(\xi)$$

$$C_n = \frac{\int_{-1}^{1}(1-\xi^2)\left[1-\lambda(1-\xi)-G(\xi)\right]\phi_n(\xi)\,d\xi}{\int_{-1}^{1}(1-\xi^2)\,\phi_n^2(\xi)\,d\xi}$$

Substitution of the temperature function, Eq. III-2 into Eq. III-1 reduces it to two ordinary differential equations, Eq. III-3 and Eq. III-4. The prime notation represents ordinary differentiation with respect to ξ .

$$G''(\xi) - E(1-\xi^2)G(\xi) = 0 \qquad \text{(III-3)}$$

where $$G(1) = 1,\quad G'(-1)=0$$

and $$\phi''(\xi) - S_n(1-\xi^2)\phi_n(\xi) = 0 \qquad \text{(III-4)}$$

where $$\phi_n(1) = 0\ ,\quad \phi'_n(-1) = 0$$

To solve Equation III-2 let:

$$G(\xi) = K_0 \sum_{n=1}^{\infty} E^{n-1} f_n(\xi)\ ,\quad f_n(\xi) = \sum_{j=1}^{4n-3} K_{n,j}\,\xi^{j-1}$$

and it becomes:

$$f_n'' = (1-\xi^2) f_{n-1} = (1-\xi^2)\sum_{j=1}^{4n-7} K_{n-1,j}\,\xi^{j-1} \qquad \text{(III-5)}$$

Now $f_n^1(-1) = 0$ and by taking $f_1(\xi) = 1$ and setting $f_n(-1) = 0$, for $n \geq 2$, then the series for $G(\xi)$ will converge for all finite values of E. The condition $G(1) = 1$ can be met by setting

$$K_0 = \left[\sum_{n=1}^{\infty} E^{n-1} f_n(1) \right]^{-1}$$

The solution to Eq. III-5 is then:

$$f_n(\xi) = \sum_{j=1}^{4n-7} \left[\frac{-K_{n-1,j} + K_{n-1,j+2}}{(j+2)(j+3)} \right] \xi^{j+3} + \frac{K_{n-1,2}}{6}\xi^3 + \frac{K_{n-1,1}}{2}\xi^2 + K_{n,2}\xi + K_{n,1}$$

where

$$K_{n,1} = K_{n,2} - \frac{K_{n-1,1}}{2} + \frac{K_{n-1,2}}{6} + \sum_{j=1}^{4n-7} (-1)^j \left[\frac{-K_{n-1,j} + K_{n-1,j+2}}{(j+2)(j+3)} \right]$$

$$K_{n,2} = K_{n-1,1} - \frac{K_{n-1,2}}{2} - \sum_{j=1}^{4n-7} (-1)^j \left[\frac{-K_{n-1,j} + K_{n-1,j+2}}{(j+2)} \right]$$

Eq. III-4 can also be solved by letting:

$$\phi_n(\xi) = \sum_{k=1}^{\infty} S_n\, g_k(\xi), \quad g_k(\xi) = \sum_{j=1}^{4k-3} D_{n,j}\, \xi^{j-1}$$

and it becomes:

$$g_k''(\xi) = (1-\xi^2)\, g_{k-1} = (1-\xi^2) \sum_{j=1}^{4k-7} D_{n-1,j}\, \xi^{j-1} \qquad \text{(III-6)}$$

where $g'(-1) = 0$ and by setting $g_1(\xi) = 1$ and $g_k(-1) = 0$, for $k \geq 2$, the series for $\phi_n(\xi)$ will converge for all finite values of S_n.

The solution to Eq. III-6 is then:

$$g_k(\xi) = \sum_{j=1}^{4k-7} \left[\frac{-D_{k-1,j} + D_{k-1,j+2}}{(j+2)(j+3)} \right] \xi^{j+3} + \frac{D_{k-1,2}}{6}\xi^3 + \frac{D_{k-1,1}}{2}\xi^2 + D_{k,2}\xi + D_{k,1}$$

where

$$D_{K,1} = D_{K,2} - \frac{D_{K-1,1}}{2} + \frac{D_{K-1,2}}{6} + \sum_{j=1}^{4K-7} (-1)^j \left[\frac{-D_{K-1,j} + D_{K-1,j+2}}{(j+2)\,(j+3)} \right]$$

$$D_{K,2} = D_{K-1,1} - \frac{D_{K-1,2}}{2} - \sum_{j=1}^{4K-7} (-1)^j \left[\frac{-D_{K-1,j} + D_{K-1,j+2}}{(j+2)} \right]$$

and the eigenvalues, S_n, are determined from $\sum_{K=1}^{\infty} S_n^{K-1}\, g_K(1) = 0$. The first four are shown in Table I and are sufficient to provide acceptable accuracy for α' s less than ten.

TABLE I

$S_1 = -0.91138802$, $S_2 = -8.8465043$, $S_3 = -24.8104112$, $S_4 = -48.944780$

Since the maximum temperature difference is required, let

$$\Delta\theta = \theta(0,-1) - \theta(1,-1)$$

and $\Delta\theta$ is the temperature difference on the inner surface between the suit ends.

Thus,

$$\Delta\theta = (1-e^{-B})\frac{\sum_{n=1}^{\infty} E^{n-1} f_n(-1)}{\sum_{n=1}^{\infty} E^{n-1} f_n(1)} + \sum_{n=1}^{\infty} C_n(1-e^{S_n/a})\left[\sum_{k=1}^{\infty} S_n^{k-1} g_k(-1)\right] \qquad \text{(III-7)}$$

However, since $f_1(\xi) = 1$, $f_n(-1) = 0$ for $n \geq 2$ and $g_1(\xi) = 1$, $g_k(-1) = 0$ for $k \geq 2$

the Eq. III-7 becomes

$$\Delta\theta = \frac{(1-e^{-B})}{\sum_{n=1}^{\infty} E^{n-1} f_n(1)} + \sum_{n=1}^{\infty} C_n(1-e^{S_n/a}) \qquad \text{(III-8)}$$

Since $\Delta\theta$ is a function of the paramenter a it is a function of the water flow rate, U, which can be computed as follows.

$$U = 2wru_{av}, \quad \text{where} \quad u_{av} = u_{max}\int_0^1 (1-\xi^2)\,d\xi = \frac{2}{3}u_{max}$$

and

$$u_{max} = \frac{\alpha_w L}{r^2}a, \quad \text{thus} \quad U = \frac{4}{3}\left(\frac{w\alpha_w L}{r}\right)a \qquad \text{(III-9)}$$

FIGURES

Figure 1 (a)	Earth-Sun Relation Looking Normal to Ecliptic Plane
Figure 1 (b)	Earth-Sun Relation Looking Parallel to Ecliptic Plane
Figure 2	Relation Between Polar Axis and Ecliptic Plane
Figure 3	Geometric Model of Man, All Cylinders
Figure 4	Geometric Model of Man, Cylinder
Figure 5	Model Chosen to Analyze Time-Temperature Variation in Orbit Suit
Figure 6	Variation of Suit Temperature with Time-Hottest Case
Figure 7	Variation of Suit Temperature with Time Coldest Case, Earth Heating Only
Figure 8	Analytical Suit Model for Part II
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Figure 10	Maximum Suit Temperature Difference
Figure 11	Analytical Suit Model for Part III
Figure 12	Nondimensional Flow Rate

LIST OF SYMBOLS

A	=	$4\sigma T_a^4 \epsilon L B / q_s \alpha_s D$
A_T	=	Total outside area of man - - ft^2
	=	Area associated with absorption of energy from energy field - - ft^2
a	=	$r^2 u_{max} / \alpha_w L$
B	=	$L D \alpha_s q_s / 4 T_a k \tau_2$
C	=	$\sigma T_a^2 \epsilon B / \alpha_s q_s$
C_n	=	Coefficients defined in Part III
c	=	specific heat of suit - $Btu/lb_m\,^{\circ}F$
D	=	Outside diameter of suit - ft
$D_{n,j}$	=	Coefficients defined in Part III
E	=	-Ba
f_n	=	Functions defined in Part III
G	=	Function defined in Part III
g_k	=	Functions defined in Part III
H	=	A given satellite altitude
h	=	Altitude of satelite
K_D	=	Diminution factor of earth's radiation to suit location
$K_{n,j}$	=	Coefficients defined in Part III
k	=	Thermal conductivity of suit material - $\frac{Btu\ ft}{hr\ ft^2\ ^{\circ}F}$
L	=	Overall suit length - ft
l	=	B/L

M	=	Mass of suit - lb_m
N	=	Dimensionless heat parameter
Q	=	Heating rate btu's/hr
q	=	Heat flux - Btu/hr ft^2
q_x	=	Radiation heat flux associated with location, x, Btu/hr ft^2
$\bar{q}$	=	Nondimensional heat flux, $\bar{q} = q_m L/K \tau_2 T_a$
R	=	Reflection factor (fraction of solar flux which is incident on suit if the earth's reflectivity were 100%)
r	=	Channel half width - ft
S_n	=	$\delta_n a$
T	=	Temperature - degrees Rankine
T	=	Maximum temperature difference - $^\circ R$
t	=	Time - hours
t	=	t/tp
U	=	Volume flow rate-gal/min
u	=	Flow velocity - ft/sec
V	=	Volume of suit - ft^3
W	=	Width of passage - ft
x	=	Coordinate parallel to suit axis - ft
y	=	Coordinate perpendicular to suit axis - ft
α	=	Total absorptivity to incident radiation
θ	=	Angle between satellite and normal to ecliptic
γ	=	Angle of inclination of earth's axis measured clockwise from normal to ecliptic
δ_n	=	Eigenvalues defined in Part III
ϵ	=	Total hemispherical emissivity

η	=	Nondimensional coordinate parallel to suit axis, $\eta = x/L$
θ	=	T/T_E Part I, T/T_a Part II, T/T_o Part III
λ	=	Nondimensional heat flux, $\lambda = q_m \tau / K_w T_o$
ξ	=	Nondimensional coordinate perpendicular to suit axis, $\xi = y/r$
ρ	=	Density of suit - lbm/ft^3
ρ_{ES}	=	Reflectivity of earth and atmosphere to solar radiation
σ	=	Stefan-Boltzman Constant, $\sigma = 0.173 \times 10^{-8}$ Btu/ft^2 Hr $^{o}F^4$
τ	=	Thickness of suit material - - ft
ϕ	=	Functions defined in Part III

Subscripts

a	=	Reference - $540^{o}R$
c	=	Heat capacity effects
E	=	Earth
G	=	Generated energy
L	=	Value at x = L
m	=	Metabolic heating
p	=	period of orbit
R	=	Reflected energy
r	=	Radiated heat
S	=	Sun
V	=	Nearby vicinity
w	=	Water
0	=	Reference - 510 ^{o}R
1	=	From Part I
2	=	From Part II

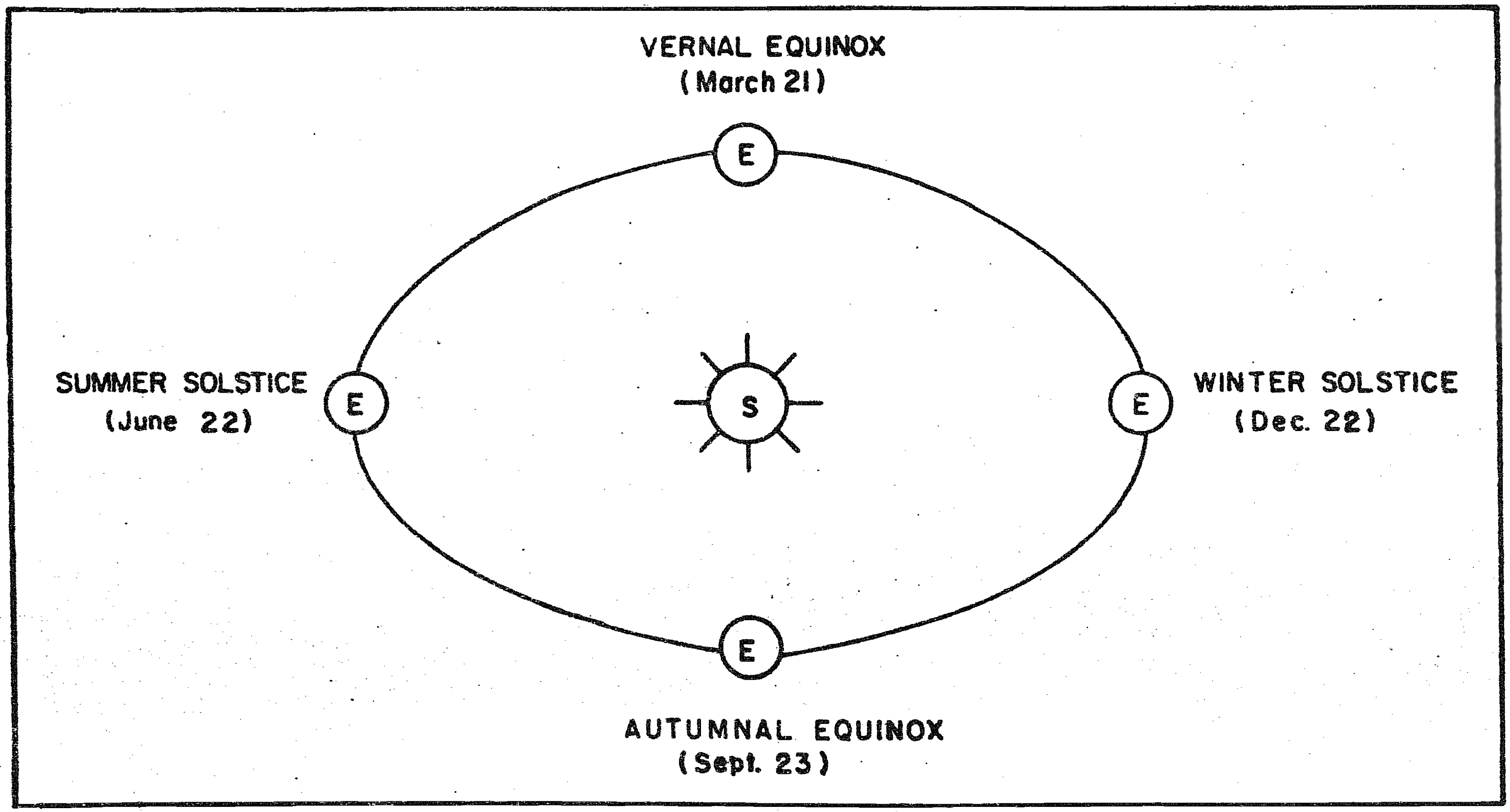

VERNAL EQUINOX
(March 21)
SUMMER SOLSTICE
(June 22)
E
S
WINTER SOLSTICE
(Dec. 22)
AUTUMNAL EQUINOX
(Sept. 23)

FIGURE 1(a)

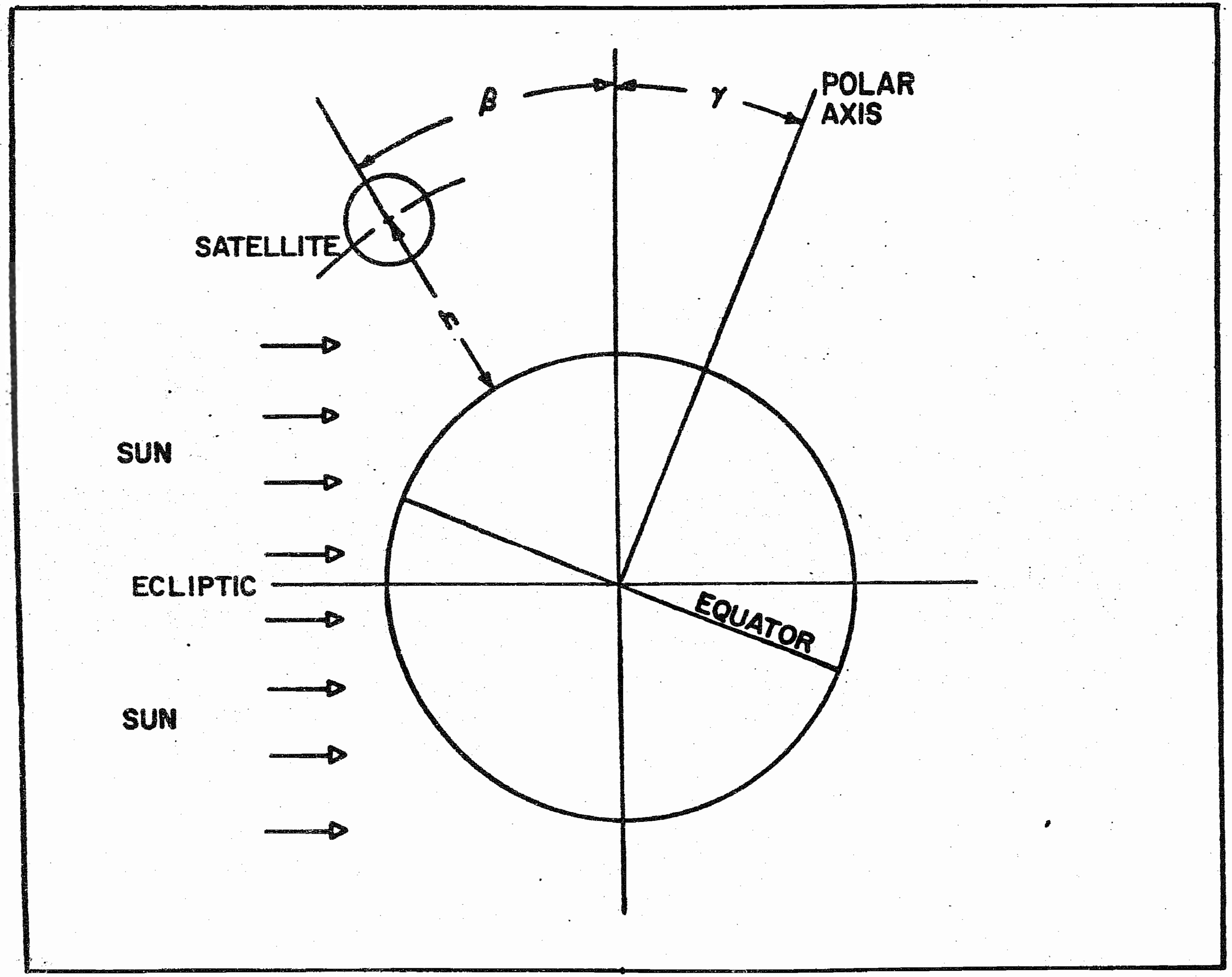


FIGURE 2

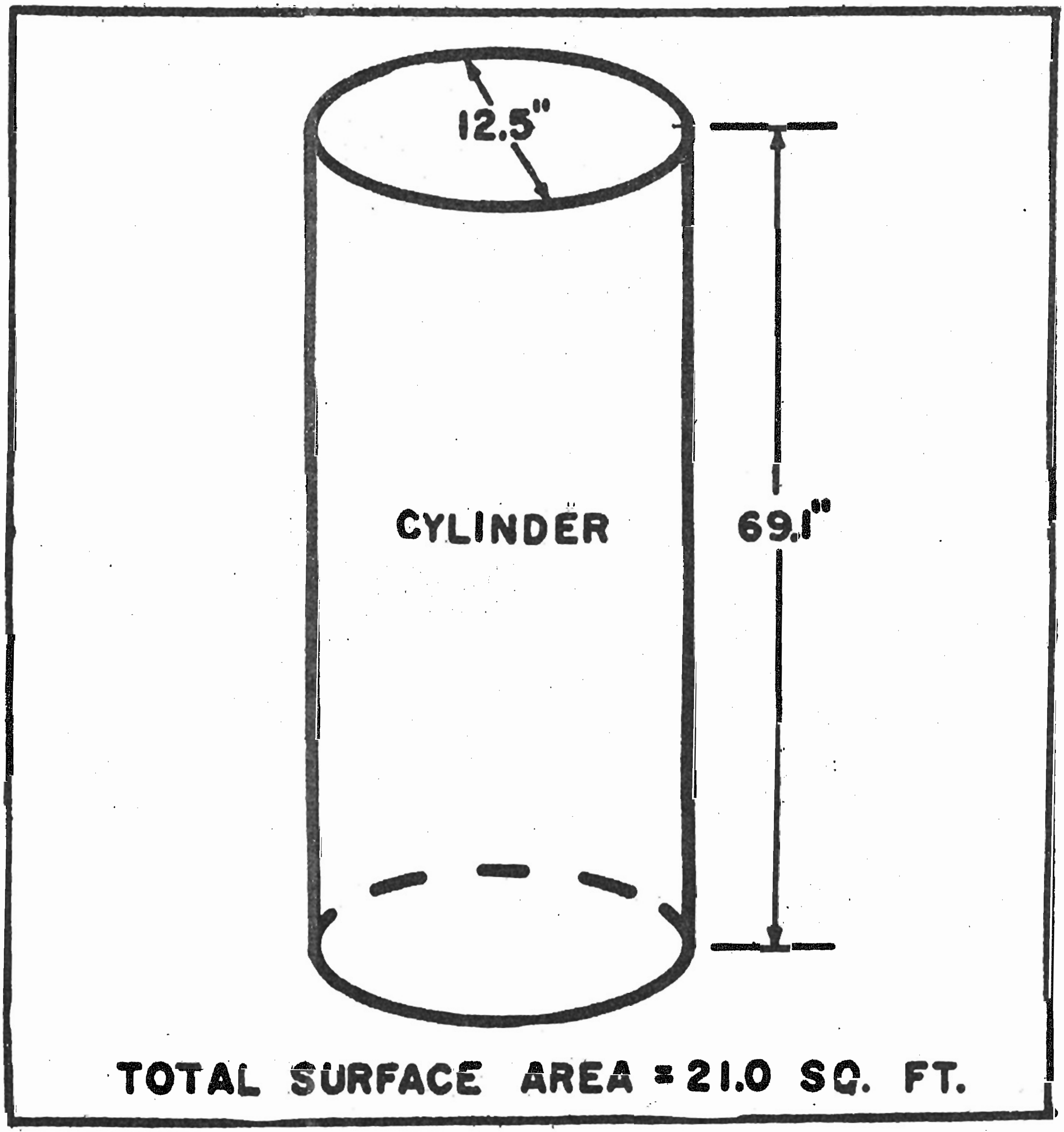
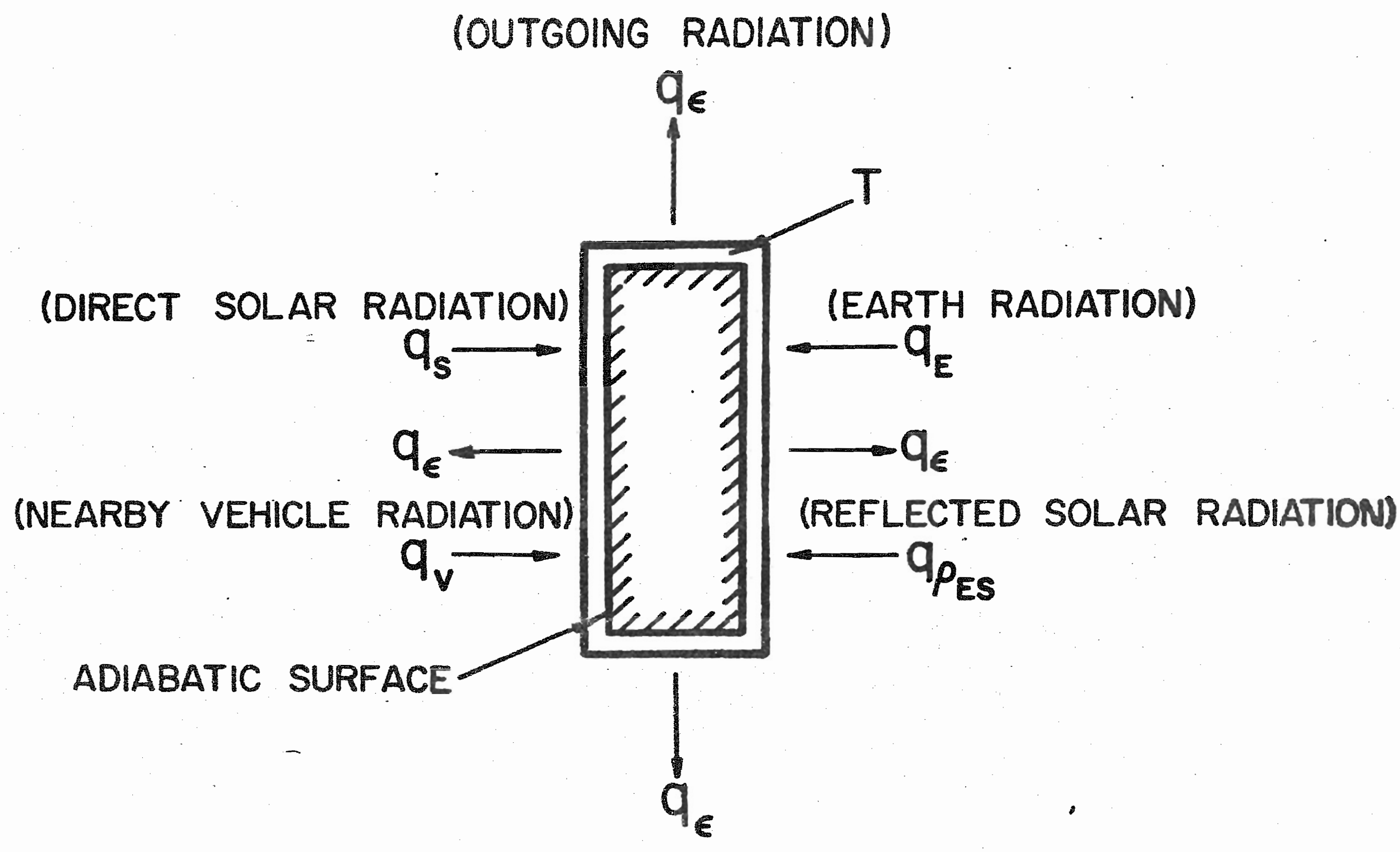
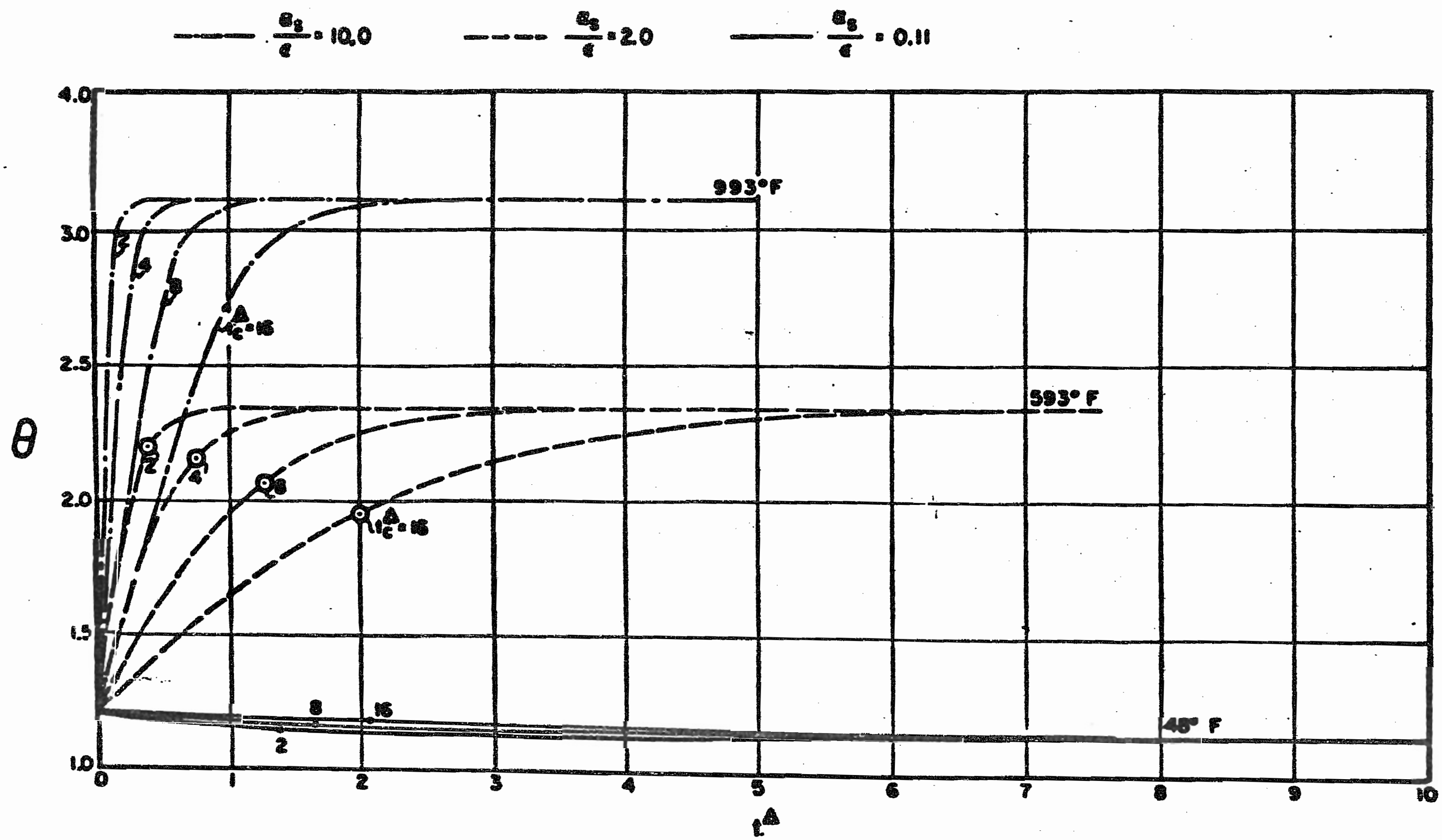


FIGURE 4



(OUTGOING RADIATION)
q_ϵ
T
(DIRECT SOLAR RADIATION)
q_S
(EARTH RADIATION)
q_E
q_ϵ
q_ϵ
(NEARBY VEHICLE RADIATION)
q_V
(REFLECTED SOLAR RADIATION)
$q_{\rho_{ES}}$
ADIABATIC SURFACE
q_ϵ

FIGURE 5



$\frac{a_s}{\epsilon} = 10.0$
$\frac{a_s}{\epsilon} = 2.0$
$\frac{a_s}{\epsilon} = 0.11$
993°F
593° F
45° F
θ
t^A

FIGURE 6

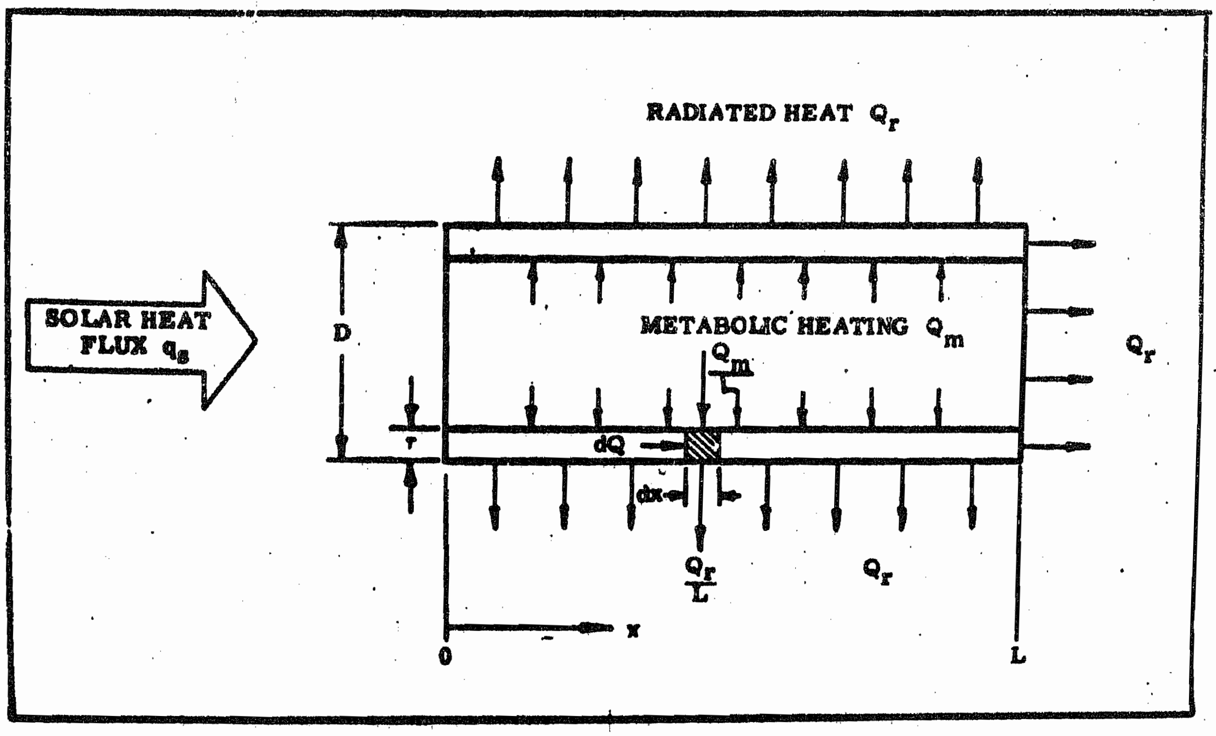

RADIATED HEAT Q_r
SOLAR HEAT FLUX q_s
D
METABOLIC HEATING Q_m
$\frac{Q_m}{L}$
Q_r
dQ
dx
$\frac{Q_r}{L}$
Q_r
x
0
L

FIGURE 8

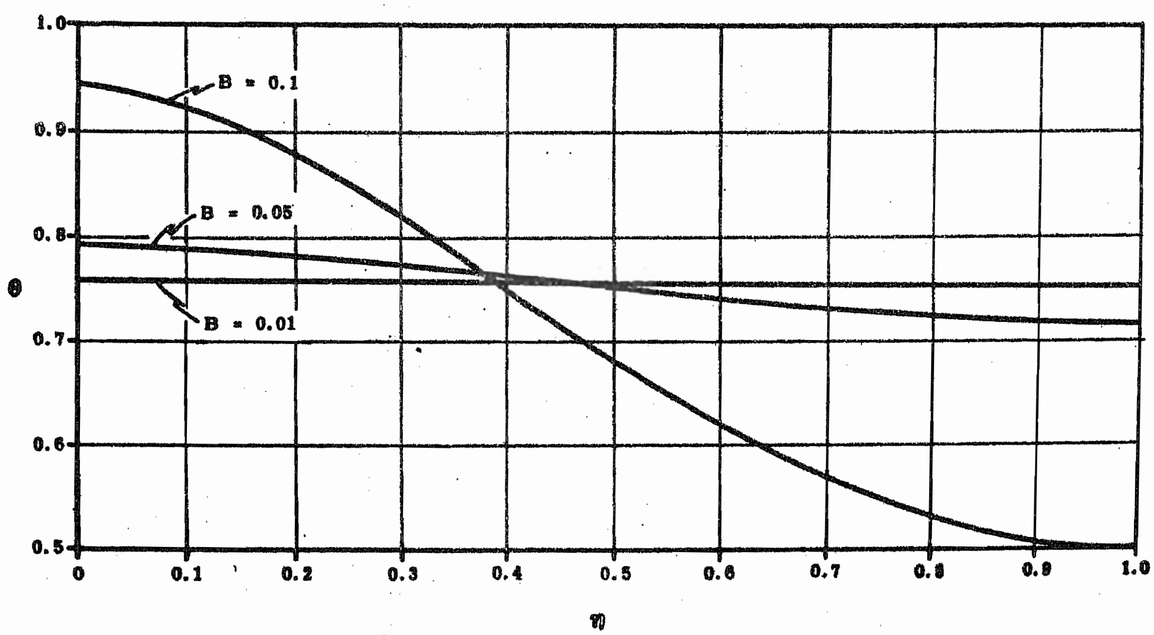


FIGURE 9

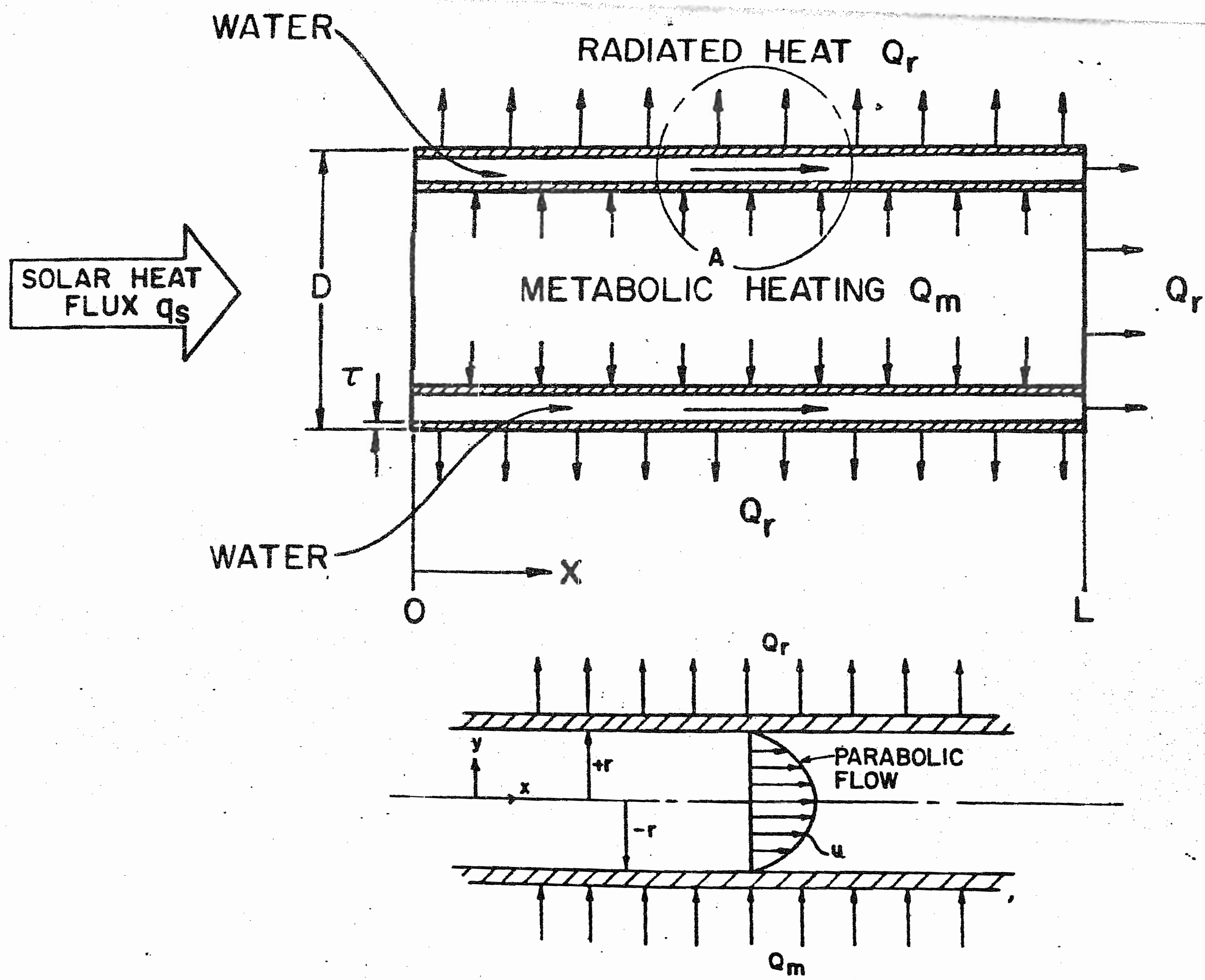


FIGURE 11

SECTION "A"