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ABSTRACT

The radiant heat transfer at any given cross section of a non-circular duct can be an important part of the total heat transfer. This paper investigates the influence of radiation on convective heat transfer in a flat, semi-infinte duct for slug, laminar and turbulent flows. Various ratios of the heat generated in the two walls are assumed in order to model the non-uniform heat flow distribution in the cross section of non-circular ducts. It is shown that if the radiation effects are neglected, serious errors in the interpretation of experimental data or in practical engineering calculations can result.

INTRODUCTION

Recent technological developments have fostered a renewed interest in heat exchange equipment which is both compact and capable of operating at high temperature levels. Ducts with non-circular cross sections are often used. This departure from circular symmetry introduces peripheral distributions of temperature and velocity, thus in fact, adding a new dimension to the heat transfer problem.

A previous investigation (ref. 1) examined the temperature field in wedge-shaped passages under the conditions of fully developed laminar flow for a fluid with constant properties, and with a constant rate of heat addition in the flow direction. This analysis showed that the peripheral temperature distribution has a marked influence on the average Nusselt number at any cross section. For a 15° isosceles triangle, the Nusselt number with constant peripheral wall temperature, was ten times greater than for the case where the wall temperature distribution was obtained by prescribing constant peripheral heat input in the fluid at any section. This Nusselt number was formed by the difference between the average wall temperature and the fluid bulk temperature. In both cases the heat addition per unit length was constant.

This difference in average Nusselt number has a physical interpretation. The case of constant peripheral wall temperature could be obtained if the wall had infinite conductivity in the peripheral direction and the heat generation rate was uniform. In this way, heat may flow without temperature degradation to that position in the cross section where it can enter the fluid most easily by convection. If, however, the wall is visualized as having zero thermal conductivity in the peripheral direction, the heat is forced to enter the fluid at the wall location where it is generated regardless of how poor the local convection conditions may be. Thus, wall temperature differences in the cross section will occur thereby reducing the average Nusselt number.

If the fluid flowing through a duct is transparent to thermal radiation, a situation similar to that described above will exist. The heat generated in the walls may, rather than enter the fluid at the location where it is generated, transfer by radiation to a more advantageous location from a convection standpoint. Thus, radiation supresses peripheral wall temperature differences in the same manner as wall conduction with a consequent increase in the average Nusselt number at the cross section. Although the effects of cross-section radiation and peripheral wall conduction are the same, the cross section radiation improves the average heat transfer without the penalty of additional weight and space.

A recent analysis by Keshock and Siegel (ref. 2) has examined the parallel plate duct for the turbulent flow case where all of the heat is generated in one wall. In their paper the radiation was considered as taking place between one wall element and all elements of the other wall as well as out the ends of the tube. In the present study the radiation exchange is confined to each given cross-section. The advantage of our simpler formulation is a reduction in the number of free parameters in the problem (Keshock and Siegel had six free parameters.) Also the purpose of the present study is to point out the effect of radiation on convection and to give some quantitative results on when this radiation effect might be expected to be important. Results are presented for slug, laminar and turbulent flows. The present paper is an extension of an analysis presented previously by one of the authors (ref. 3).

ANALYSIS

The semi-infinite duct and the coordinate system are illustrated in Fig. 1. The upper wall is designated as wall (1) with temperature T_{W_1} while the temperature of the lower wall is T_{W_2}. The two walls are infinite in extent in the x direction and are separated from each other by a distance 2a. The Z direction is the flow direction.

Throughout the analysis the properties of the fluid are assumed constant and it is specified that the velocity field is fully developed.

For fully developed turbulent flow the flow field is described by the following dimensionless differential equation (ref. 4).

$$\frac{d}{dy'}\left[\left(1+\frac{\varepsilon_m}{\nu}\right)\frac{dw'}{dy'}\right] = \frac{Re}{4}\ \frac{dp'}{dz'} \tag{1}$$

The boundary condition for Eq. (1) is that $w' = 0$ at $y' = \pm 1$

Since $\frac{dp'}{dz'}$ is a function of the Reynolds number, the solution to Eq. (1) has the following form

$$w' = w'\left(Re, \frac{\varepsilon_m}{\nu}, y'\right) \tag{2}$$

In fully developed flow, $\frac{\varepsilon_m}{\nu}$ cannot be regarded as a free parameter since if the walls are smooth, it will be determined by internal flow processes. Thus, the dimensionless point velocity will only depend upon location and Reynolds number.

In laminar flow, the diffusivity $\varepsilon_m = 0$ and $\frac{dp'}{dZ'} = \frac{24}{Re}$ so that

$$w' = w'(y') \tag{3}$$

For slug flow, w' is a constant.

Referring again to Fig. 1, the heat transfer problem will now be discussed. Equal amounts of heat are generated per unit wall area for each wall but the heat generation rates will, in general, be different for the two walls. The external wall surfaces are adiabatic.

When an internal convection problem is solved under the boundary conditions of heat generation in the duct walls, if wall heat conduction affects the heat transfer to the fluid, then the energy equation in the wall and the fluid must be solved simultaneously. This difficulty is suppressed in the present analysis since no temperature differences will exist in the x direction and there will be no wall conduction in this direction. In addition, since equal amounts of heat are generated in the flow direction, the wall temperatures will vary in a linear manner in the Z direction and thus there will be no wall conduction effects in that direction. Thus, heat which is generated in the wall must leave the inner wall surface by either convection or radiation at the generation location.

For turbulent fully developed flow, the temperature field is described by the relation

$$\frac{\partial}{\partial y'}\left[\left(1+\frac{\varepsilon_q P_r}{\nu}\right)\frac{\partial \theta}{\partial y'}\right] = \frac{R_e P_r}{4}\, w'\, \frac{\partial \theta}{\partial Z'} \tag{4}$$

In Eq. (4), θ is a dimensionless temperature given by $\theta = \frac{T}{(c_1+c_2)d}$ where the heat generated per unit area of the upper and lower walls are kc_1 and kc_2 respectively. Since this heat must enter the fluid by either convection or radiation, the boundary conditions on Eq. (4) are

$$y = a, \quad \frac{\partial T}{\partial y} = c_1 - \frac{\mathcal{F}\sigma}{k}\left(T_{w_1}^4 - T_{w_2}^4\right) \tag{5a}$$

$$\frac{\partial T}{\partial y} = - c_2 - \frac{\xi \sigma}{k}(T_{w_1}^4 - T_{w_2}^4) \qquad (5b)$$

ξ is an interchange factor which is equal to unity if the walls are isothermal and black to thermal radiation. If ϵ is the emissivity of the inner surface of either wall, then

$$\frac{1}{\xi} = \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1$$

The boundary conditions 5(a) and 5(b) are true only when the two walls are at constant temperature. This is not the situation here as the wall temperatures vary in the flow direction causing a radiation exchange between wall elements in this direction. In the present analysis this effect is neglected so that radiation is considered as taking place only in the plane of each cross section. Thus, the same amount of heat as is generated at a cross section enters the fluid at this location. This assumption is reasonable when the temperature gradient $\frac{\partial \theta}{\partial Z'}$, is small compared to the mean temperature level and is further improved if the wall spacing is small compared with the wall length in the flow direction. It is shown in Ref. 2 that the latter condition prevails if the ratio of duct length to distance between plate is greater than 50.

Also it will be specified that at every location in the flow direction the temperature field has the same shape as a fully developed convective temperature field having the same ratio of heat fluxes entering the fluid. The temperature profile is therefore changing at an equal rate in the flow direction and the temperature gradient $\frac{\partial \theta}{\partial Z'}$, is a constant.

If, in addition, the temperature difference $(T_{w_1} - T_{w_2})$ is small compared with the temperature level, Eq. (5a) may be written:

$$\frac{dT}{dy} = C_1 - \frac{\xi \sigma \bar{T}_w^3 d}{K} \cdot \frac{4(T_{w_1} - T_{w_2})}{d} \tag{6}$$

The dimensionless parameter $\frac{\xi \sigma \bar{T}_w^3 d}{K}$ will be written ϕ. It dictates the influence of radiation on the convective heat transfer.

Equations (5a) and (5b) now become in dimensionless form:

$$y' = 1, \qquad \frac{d\theta}{dy'} = \frac{1}{4}\left[\frac{1}{1+\lambda} - 4\phi(\theta_{w_1} - \theta_{w_2})\right] \tag{7a}$$

$$y' = -1, \qquad \frac{d\theta}{dy'} = \frac{1}{4}\left[\frac{-\lambda}{1+\lambda} - 4\phi(\theta_{w_1} - \theta_{w_2})\right] \tag{7b}$$

where λ represents the ratic of heat generation rates, C_2/C_1.

Equations (4) and (7) indicate that the dimensionless temperature distribution in the flow will be given by a relation of the following form:

$$\theta = \theta\left(P_r, Re, \lambda, \phi, \frac{\varepsilon_m}{\nu}, \frac{\varepsilon_q}{\nu}, y'\right) \tag{8}$$

Once again for smooth walls ε_m and ε_q will be determined by internal flow processes and cannot be regarded as independent parameters.

For laminar flow the diffusivities ε_m and ε_q equal zero. Furthermore, since the heat added to the fluid at every cross section and the temperature gradient $\frac{d\theta}{dz'}$, are specified as constant, a heat balance in the axial direction yields

$$\frac{d\theta}{dz'} = \frac{1}{2 P_r Re} \tag{8a}$$

Hence, with constant properties Eq. (4) simplified to

$$\frac{d^2\theta}{dy'^2} = \frac{w'}{\gamma} \tag{9}$$

and Eq. (8) becomes

$$\theta = \theta(\lambda, \phi, y') \tag{10}$$

A similar expression is obtained in the case of slug flow. An average heat transfer coefficient is defined by

$$\bar{q} = 2\bar{h}(T_{W1} - T_B) \tag{11}$$

or in dimensionless form

$$\overline{Nu} = \frac{1}{2(\theta_{W1} - \theta_B)} \tag{12}$$

where θ_B is the dimensionless bulk temperature.

Actually, when the wall heat fluxs are specified as in the present case the Nusselt number is not of great value in describing the heat transfer situation. Of greater interest is the difference between the wall and fluid temperatures. However, the Nusselt number defined above may be thought of being inversely proportional to the difference between the upper wall and fluid bulk temperatures.

The drivations of expressions for the temperature field, wall temperature difference and Nusselt number are presented in detail in the Appendix. Only the results are given below.

SLUG FLOW SOLUTIONS

Integration of Eq. (9) and the application of Eq. (7) is straightforward and gives the following results:

$$\theta - \theta_{w2} = \frac{y'^2}{16} + \left[\frac{1-\lambda}{8(1+\lambda)} - \frac{\phi\cdot(1-\lambda)}{4(1+\lambda)(1+2\phi)}\right] y' + \left[\frac{\lambda-3}{16(1+\lambda)} + \frac{(1+\phi)(1-\lambda)}{4(1+2\phi)(1+\lambda)}\right] \tag{13}$$

$$\theta_{w1} - \theta_{w2} = \frac{1-\lambda}{4(1+\lambda)(1+2\phi)} \tag{14}$$

$$\bar{N}u = \frac{6(1+\lambda)}{(2-\lambda) - \dfrac{3(1-\lambda)}{\frac{1}{\phi}+2}} \tag{15}$$

LAMINAR FLOW SOLUTIONS

$$\theta - \theta_{w2} = -\frac{y'^4}{64} + \frac{3y'^2}{32} + \left[\frac{1-\lambda}{8(1+\lambda)} - \frac{\phi(1-\lambda)}{4(1+\lambda)(1+2\phi)}\right] y' + \left[\frac{3\lambda-13}{64(1+\lambda)} + \frac{(1+\phi)(1-\lambda)}{4(1+2\phi)(1+\lambda)}\right] \tag{16}$$

$$\theta_{w1} - \theta_{w2} = \frac{1-\lambda}{4(1+\lambda)(1+2\phi)} \tag{17}$$

It is interesting to note that Eqs. (17) and (14) are identical.

$$\bar{N}u = \frac{2(1+\lambda)}{\dfrac{26-9\lambda}{35} - \dfrac{1-\lambda}{\frac{1}{\phi}+2}} \tag{18}$$

TURBULENT FLOW SOLUTIONS

The following two equations were derived by extending the analysis in Ref. (5) where values of η_∞ and $Nu(0)$ are tabulated for a wide range of Prandtl and Reynolds numbers.

$$\theta - \theta_{w2} = \frac{(1-\lambda)(\eta_\infty + 1)}{(1+\lambda)\{\eta_\infty Nu(0) + 8\phi(\eta_\infty + 1)\}} \tag{19}$$

$$\overline{Nu} = \frac{\eta_\infty Nu(0)\frac{1}{2}(1+\lambda)\{\eta_\infty Nu(0) + 8\phi(\eta_\infty + 1)\}}{Nu(0)\eta_\infty(\eta_\infty - \lambda) + 4\phi(1+\lambda)(\eta_\infty^2 - 1)} \tag{20}$$

Equations (19) and (20) are equivalent to Eqs. (17) and (18) for laminar flow taking $\eta_\infty = 2.89$ and $Nu(0) = 5.385$, and to Eqs. (14) and (15) for slug flow taking $\eta_\infty = 2.0$ and $Nu(0) = 6.0$.

DISCUSSION OF RESULTS

Figure 2 shows the effect of radiation on the dimensionless wall temperature difference in slug and laminar flow for values of λ from 0 to 10. It is clear that if the radiation parameter exceeds 10, the wall temperature difference is almost completely suppressed; the effect is essentially the same as if equal heat generation rates had been prescribed in the two walls. It may also be noted that for values of ϕ greater than 0.1, the influences of radiation is appreciable and should be considered. The resulting increase in the average Nusselt number is illustrated in Figs. 3 and 4 and is seen to be considerable. For example, at $\lambda = 0$ in the slug flow case, the Nusselt number is doubled by increasing ϕ from 0 to 1·0. The effect is of the same order of magnitude in laminar flow.

Similar curves to the above can be plotted for turbulent flow for each combination of Reynolds and Prandtl numbers. Figures 5 and 6 show the results in summarized form for Pr = 1·0. and $\lambda = 0$. It is evident that the effect of radiation on the Nusselt number and on the dimensionless wall temperature difference is less in turbulent flow but remains significant with ϕ greater than ten over most of the Reynolds number range investigated.

In Fig. 7 the results for laminar flow are compared with those for turbulent flow at three Reynolds numbers for Pr = 1·0. To simplify direction comparison, the ordinate scale is expressed as the ratio of the wall temperature difference for any value of ϕ and λ to the wall temperature difference for $\lambda = 0$ and $\phi = 0$. Notice that for $\lambda = 0$ a value of the radiation parameter $\phi = 10{\cdot}0$ suppresses the wall temperature difference by 95% in the case of laminar flow. This value decreases to 69%, 27% and 5.5% in turbulent flow as the Reynolds number is increased to 15,000, 150,000, and 1,500,000 respectively.

The high convective heat transfer rates associated with high Reynolds numbers diminish the radiation effect. Figure 8 illustrates this for the Nusselt number, also expressed as a ratio for ease of comparison. For laminar flow, with $\lambda = 0$, the re-distribution of heat brought about by radiation with $\phi = 10{\cdot}0$ causes the Nusselt number to increase to 86% of its value for $\lambda = 1{\cdot}0$. In turbulent flow the percentage increase of $\overline{Nu}$ becomes less, though the absolute value is increased as shown in Fig. 6. With Re = 150,000 and $\lambda = 0$, $\phi = 10{\cdot}0$ only gives an increment of 13% of the value at $\lambda = 1{\cdot}0$. Figure 9 shows these results in summarized form for $\lambda = 0$.

NUMERICAL EXAMPLE FOR A TYPICAL DUCT WITH AIR AS THE CONVECTIVE FLUID

d = 0.4 in (2a = 0.2 in.)

ξ = 1.0 (assuming surfaces black to radiation)

$\bar{T}_w$ = 2060°R

$\frac{dT}{dZ}$ = 1000°R per ft.

K = 0.0488 BTU/hr.ft.°F

Under the above conditions, ϕ is equal to 10.44. References to Figures 2 and 7 show that for λ = 0, the effect of radiation is to decrease the dimensionless wall temperature difference by the following amounts.

Slug flow - Dimensionless wall temperature difference reduces by factor of 20 from 1.0 to 0.05.

Laminar Flow - Same as for slug flow.

Turbulent Flow -	Reynolds Number	$\frac{(Tw_1 - Tw_2)_{\phi, \frac{c_2}{c_1} = 0}}{(Tw_1 - Tw_2)_{\phi = 0, c_1/c_1 = 0}}$
	1.5×10^4	From 1.0 to 0.32
	1.5×10^5	From 1.0 to 0.75
	1.5×10^6	From 1.0 to 0.95

SUMMARY AND CONCLUSIONS

The influence of radiation on convection in a flat duct has been studied for fully developed flows in which the radiation effects are confined to any given cross-section of the duct, slug, laminar, and turbulent flows have been investigated. The conditions have been illustrated under which radiation markedly influences the convection process. Under these conditions, extreme care should be used in applying classical convection solutions. It has also been shown that the internal radiation effect may be used to increase the heat transfer performance of non-circular ducts without having to expend additional space, weight and cost to increase the heat conduction within the duct walls.

APPENDIX

DERIVATION OF SOLUTIONS FOR LAMINAR FLOW

The temperature distribution for laminar flow fully developed thermally and hydrodynamically, assuming constant properties, is given by the solution to the following differential equation.

$$\frac{d^2\theta}{dy'^2} = \frac{w'}{4} Re\, Pr \frac{d\theta}{dZ'} \tag{21}$$

The velocity field in laminar flow is given by

$$w' = \frac{Re}{16} \frac{dp'}{dZ'}(1 - y'^2)$$

where

$$\frac{dp'}{dZ'} = \frac{24}{Re}$$

so that

$$w' = 1.5(1 - y'^2) \tag{22}$$

Assuming that $\frac{d\theta}{dZ'}$ is a constant, the heat balance on the fluid gives

$$\frac{d\theta}{dZ'} = \frac{1}{2\, Pr\, Re} \tag{23}$$

Substituting Eqs. (23) and (22) in Eq. (21) and integrating gives

$$\theta = \frac{3}{32}\left(y'^2 - \frac{y'^4}{6}\right) + Ay' + B$$ where A and B are constants of integration.

The boundary conditions applicable are:

$y' = 1$ $$\frac{d\theta}{dy'} = \frac{1}{4}\left[\frac{1}{1+\lambda} - 4\phi(\theta_{w_1} - \theta_{w_2})\right]$$

and

$$y' = -1 \quad , \quad \theta = \theta_{w2}$$

Thus the temperature distribution is given by:

$$\theta - \theta_{w2} = \frac{-y'^4}{64} + \frac{3y'^2}{32} + y'\left[\frac{1-\lambda}{8(1+\lambda)} - \frac{\phi(1-\lambda)}{4(1+\lambda)(1+2\phi)}\right] + \left[\frac{3\lambda - 13}{64(1+\lambda)} + \frac{(\phi+1)(1-\lambda)}{4(1+2\phi)(1+\lambda)}\right] \tag{24}$$

When $y' = 1$, $\theta = \theta_{w1}$ so that

$$\theta_{w1} - \theta_{w2} = \frac{1-\lambda}{4(1+\lambda)(1+2\phi)} \tag{25}$$

An average Nusselt number is defined by:

$$\bar{N}u = \frac{1}{2(\theta_{w1} - \theta_B)} \tag{26}$$

where the dimensionless bulk temperature is given by:

$$\theta_B = \frac{3}{4}\int_{-1}^{1} \theta(1-y'^2)\,dy' \tag{27}$$

Evaluation of Eq. (27) finally gives for the average Nusselt number

$$\bar{N}u = \frac{2(1+\lambda)}{\dfrac{26-9\lambda}{35} - \dfrac{1-\lambda}{\frac{1}{\phi}+2}} \tag{28}$$

DERIVATION OF SOLUTIONS FOR SLUG FLOW

In slug flow the velocity is constant across the duct, and using the same assumption as in the previous case, the temperature distribution is given by the solution to Eq. (21).

In this case, $\omega' = 1$, and applying the heat balance expressed by Eq. (23) gives:

$$\frac{d^2\theta}{dy'^2} = \frac{1}{8} \tag{29}$$

Integrating and using the same boundary conditions as in the laminar flow case gives Eq. (13).

DERIVATION OF SOLUTIONS FOR TURBULENT FLOW

The solutions were obtained by extending the analysis presented by R. P. Stein in Ref. 5. In this report, Stein treats of the effect of the ratio of the heat fluxes from the opposing walls of annular ducts, on the Nusselt number and temperature distribution for heat convection only. Flow between parallel planes for laminar, slug and turbulent flow are treated as limiting cases of annular flow. For turbulent flow, the universal velocity profile of Deissler (Ref. 6) was assumed. The effective conductivity in the fluid was obtained by applying the analogy between momentum and heat transfer.

Stein presents his results in terms of the parameter Nu(0) and η_∞ , where Nu(0) is the local Nusselt number when heat is entering the fluid from one side of the duct only, and η_∞ is the ratio of the heat fluxes at which one wall temperature becomes equal to the fluid bulk temperature. All the heat generated in a wall enters the fluid by convection from that wall.

Stein's results were utilized by interpreting his value for heat entering the fluid, as composed of a part from heat generated in the wall and the remainder from the radiation contribution.

Thus, the ratio of the heat fluxes is given by:

$$\eta = \frac{1-4(1+\lambda)\phi(\theta_{w1}-\theta_{w2})}{-\lambda-4(1+\lambda)\phi(\theta_{w1}-\theta_{w2})} \tag{30}$$

Where η is the ratio of the heats entering the fluid by convection, as used in Stein's analysis; and λ is the ratio of the heats generated in the wall.

The analysis in Ref. 3 can be used to give the following equations:

$$\bar{N}u = \frac{Nu(0)(1+\eta)\eta_\infty}{2(\eta_\infty-\eta)}$$

$$(\theta_{w1}-\theta_{w2}) = \frac{(\eta_\infty+1)(1-\eta)}{\eta_\infty Nu(0)(1+\eta)}$$

and finally

$$\bar{N}u = \frac{\eta_\infty Nu(0)\frac{1}{2}(1+\lambda)\left[\eta_\infty Nu(0)+8\phi(\eta_\infty+1)\right]}{Nu(0)\eta_\infty(\eta_\infty-\lambda)+4\phi(1+\lambda)(\eta_\infty^2-1)} \tag{31}$$

$$(\theta_{w1}-\theta_{w2}) = \frac{(1-\lambda)(\eta_\infty+1)}{(1+\lambda)\left[\eta_\infty Nu(0)+8\phi(\eta_\infty+1)\right]} \tag{32}$$

Values of η_∞ and Nu(0) are given in Ref. 3 for a wide range of Prandtl numbers. However, in the present report only the results for Pr = 1·0 are presented.

FIGURE CAPTIONS

NOMENCLATURE

a	half of duct width (Fig. 1)
C	constant proportional to rate of heat generation
d	duct hydraulic diameter
h	local heat transfer coefficient (Ref. 3)
$\bar{h}$	average heat transfer coefficient
k	thermal conductivity
l	half of duct length
p	pressure
p'	dimensionless pressure
q	heat flow from wall surface into fluid by convection per unit time and area
$\bar{q}$	total heat flow into fluid at any cross section per unit time and area
Q	radiation heat exchange (Appendix II)
T	absolute temperature
$\bar{T}$	arithmetic average temperature
W	velocity in direction of Z coordinate (Fig. 1)
$\bar{W}$	mean velocity in Z direction
W'	dimensionless velocity $\frac{W}{\bar{W}}$
x,y,z	coordinate distances
x', y', z'	dimensionless coordinate distances $\frac{x}{a}$, $\frac{y}{a}$, $\frac{z}{a}$,

Nomenclature (cont'd)

Nu	local Nusselt number hd/k
$\overline{Nu}$	average Nusselt number $\overline{h}d/k$
Pr	Prandtl number
Re	Reynolds number $\overline{W}d/v$
ϵ	emissivity of duct wall
ε_m	turbulent diffusivity for mementum
ε_q	turbulent diffusivity for heat
η	ratio of heat entering the fluid by convection $\frac{q_1}{q_2}$
η_∞	value of η at which one wall temperature is equal to the fluid bulk temperature
θ	dimensionless temperature $\frac{T}{(C_1 + C_2) d}$
β	ratio of duct width to length $\left(\frac{a}{\ell} \right)$
ν	Kinematic viscosity
ξ	radiation interchange factor
λ	ratio of wall heat fluxes $(\lambda = \frac{C_2}{C_1})$
ρ	mass density
σ	Stephan - Boltzmann constant
ϕ	radiation parameter

SUB-SCRIPTS

1	upper wall
2	lower wall
B	bulk or average fluid conditions
W	wall

BIBLIOGRAPHY

1. Eckert, E. R. G., Irvine, T. F., Jr., and Yen, J. T.; "Local Laminar Heat Transfer in Wege-Shaped Passages"; Trans. ASME, Vol. 80, 1958, p. 1433.

2. Keshock, E. G. and Siegel R.; "Combined Radiation and Convection in an Asymmetrically Heated Parallel Plate Flow Channel:, To be published, Journal of Heat Transfer, Feb. 1964.

3. Irvine, T. F., Jr.; "The Influence of Radiation on Convection in Non-Circular Ducts"; ASME Paper No. 58-A-155, 1958.

4. Eckert, E. R. G. and Drake, R. M., Jr.; "Heat and Mass Transfer"; McGraw Hill, 1959.

5. Stein, R. P.; "The Dependence of the Heat Transfer Coefficient on the Ratio of the Heat Fluxes from the Walls of Parallel Plane Flow Channels"; Reactor Heat Transfer Conference of 1956, TID-7529 (pt. 1) Book 2. United States Atomic Energy Commission.

6. Deissler, R. G.; "Investigations of Turbulent Flow and Heat Transfer in Smooth Tubes Including the Effects of Variable Properties"; ASME Trans., Vol. 73, 1951, p. 101.

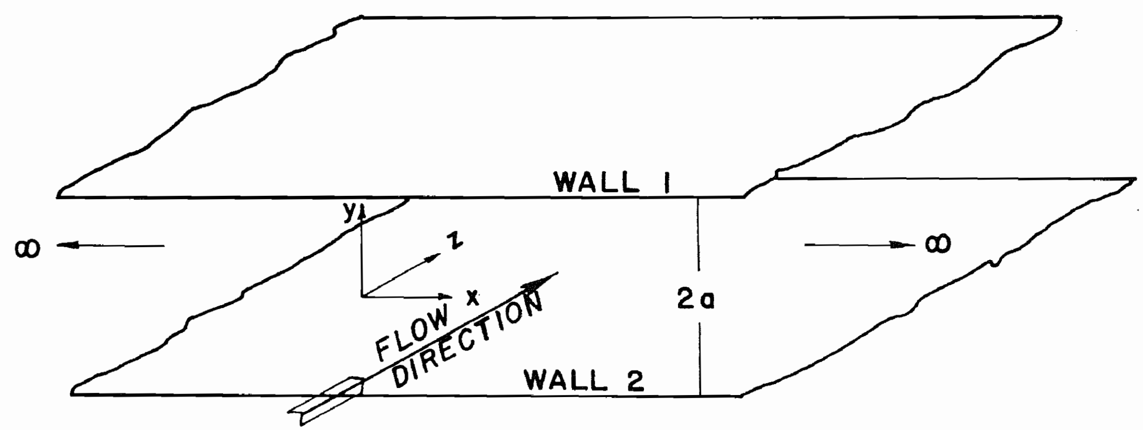


Fig. 1

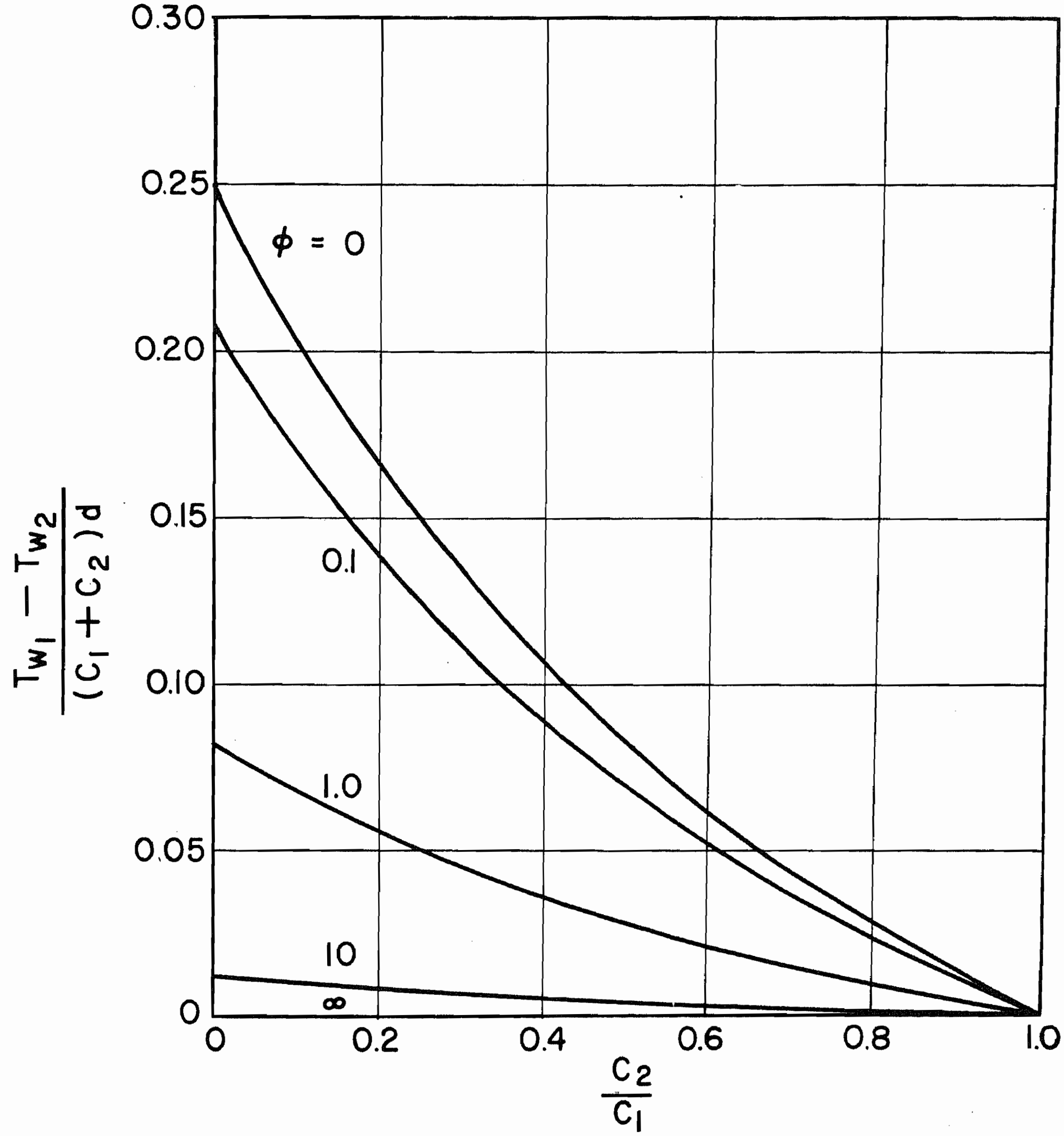
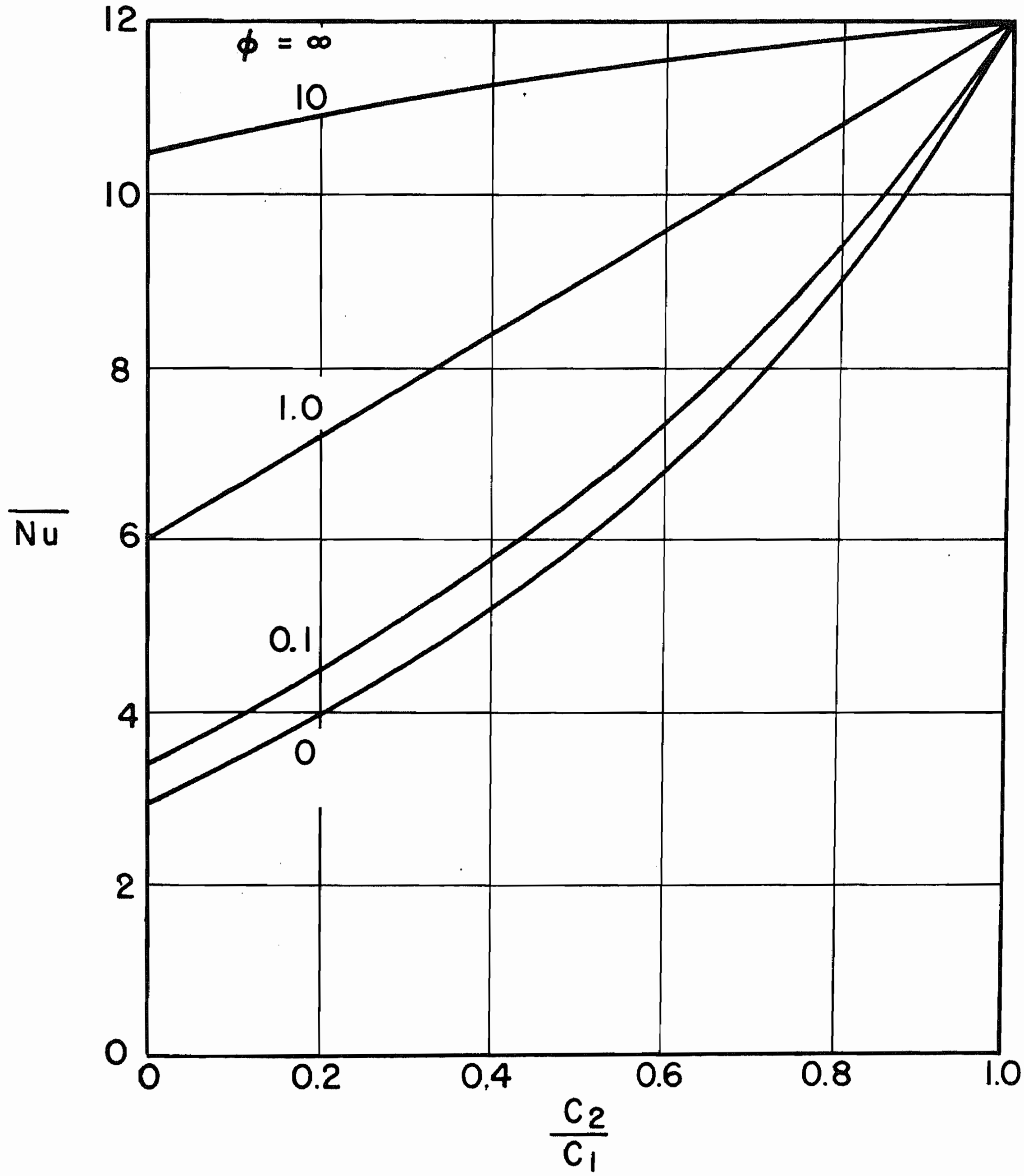
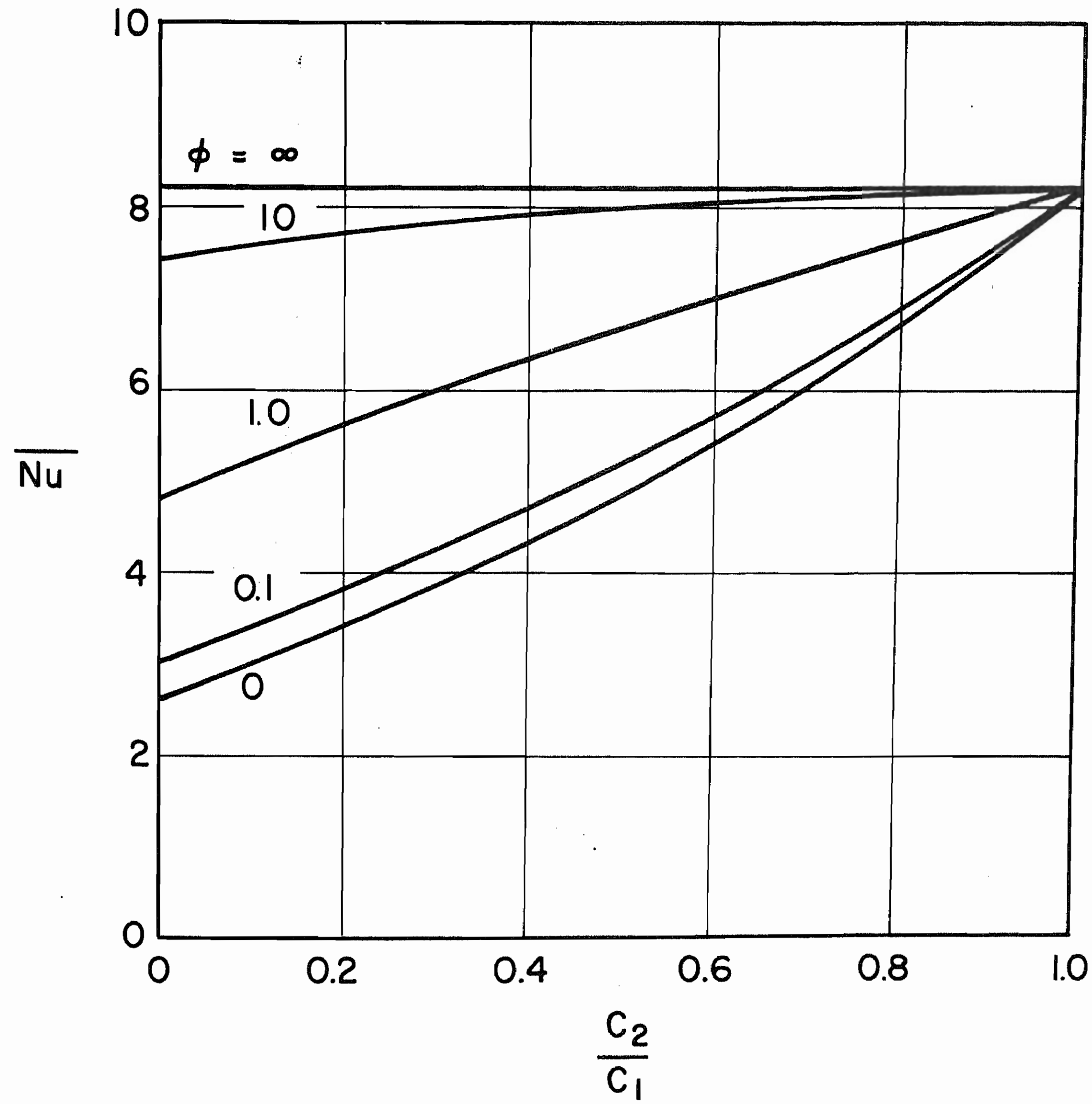

0.30
0.25
0.20
0.15
0.10
0.05
0
$\frac{T_{w_1} - T_{w_2}}{(C_1 + C_2)d}$
$\phi = 0$
0.1
1.0
10
∞
0
0.2
0.4
0.6
0.8
1.0
$\frac{C_2}{C_1}$

Fig. 2



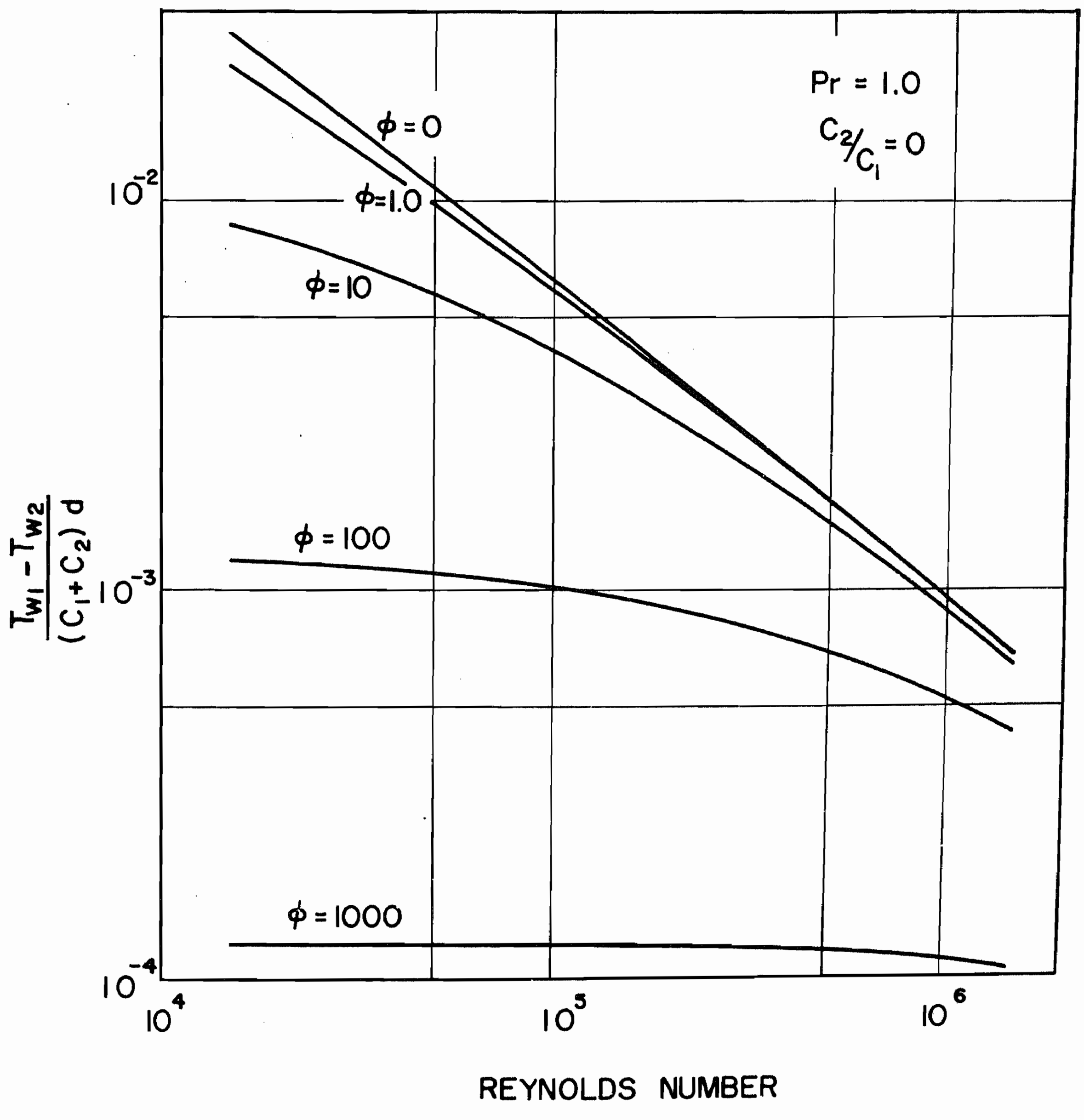
12
10
8
6
4
2
0
$\phi = \infty$
10
1.0
0.1
0
$\overline{Nu}$
0
0.2
0.4
0.6
0.8
1.0
$\frac{C_2}{C_1}$

Fig. 3



10
8
6
4
2
0
0
0.2
0.4
0.6
0.8
1.0
$\phi = \infty$
10
1.0
0.1
0
$\overline{Nu}$
$\frac{C_2}{C_1}$

Fig. 4



Pr = 1.0
C2/C1 = 0
φ = 0
φ = 1.0
φ = 10
φ = 100
φ = 1000
10^-2
10^-3
10^-4
10^4
10^5
10^6
(T_W1 - T_W2) / ((C1 + C2) d)
REYNOLDS NUMBER

Fig. 5

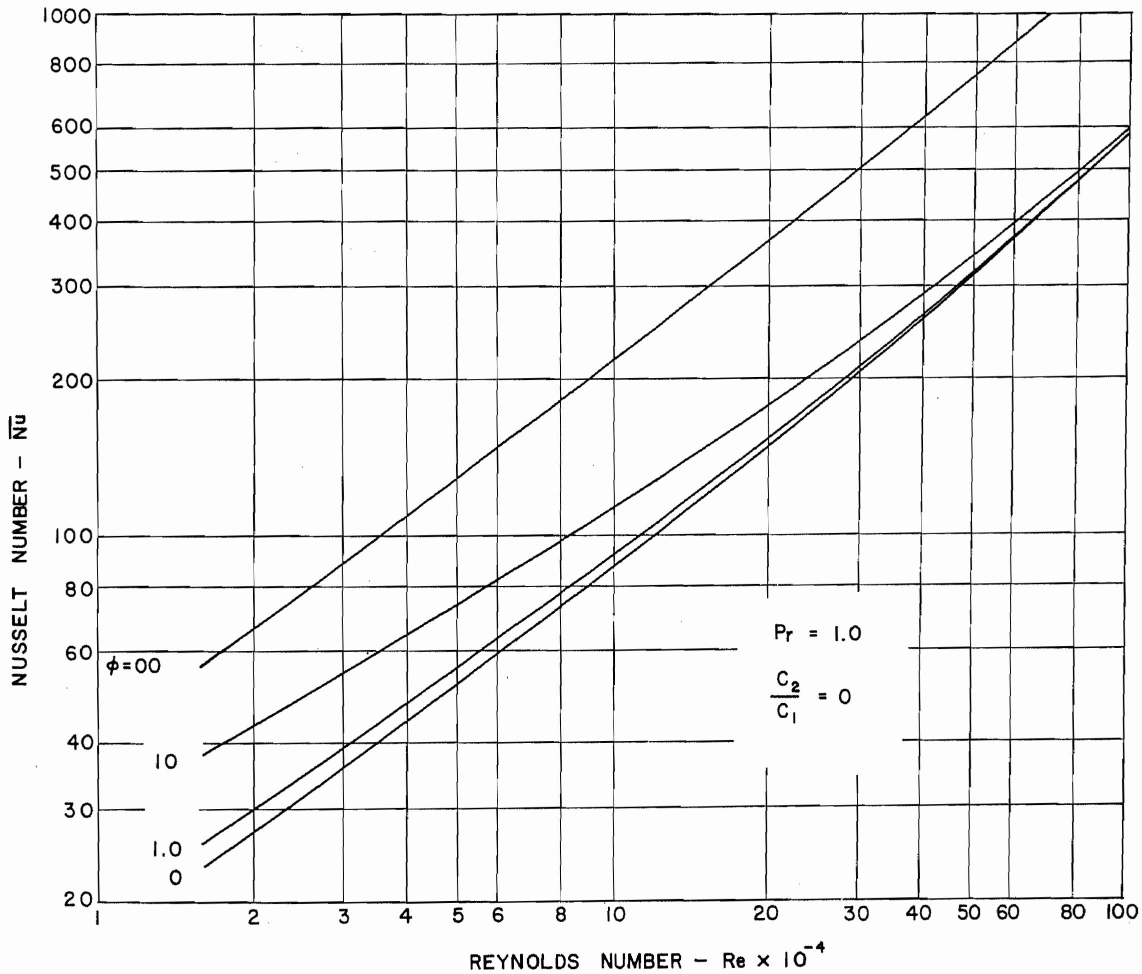
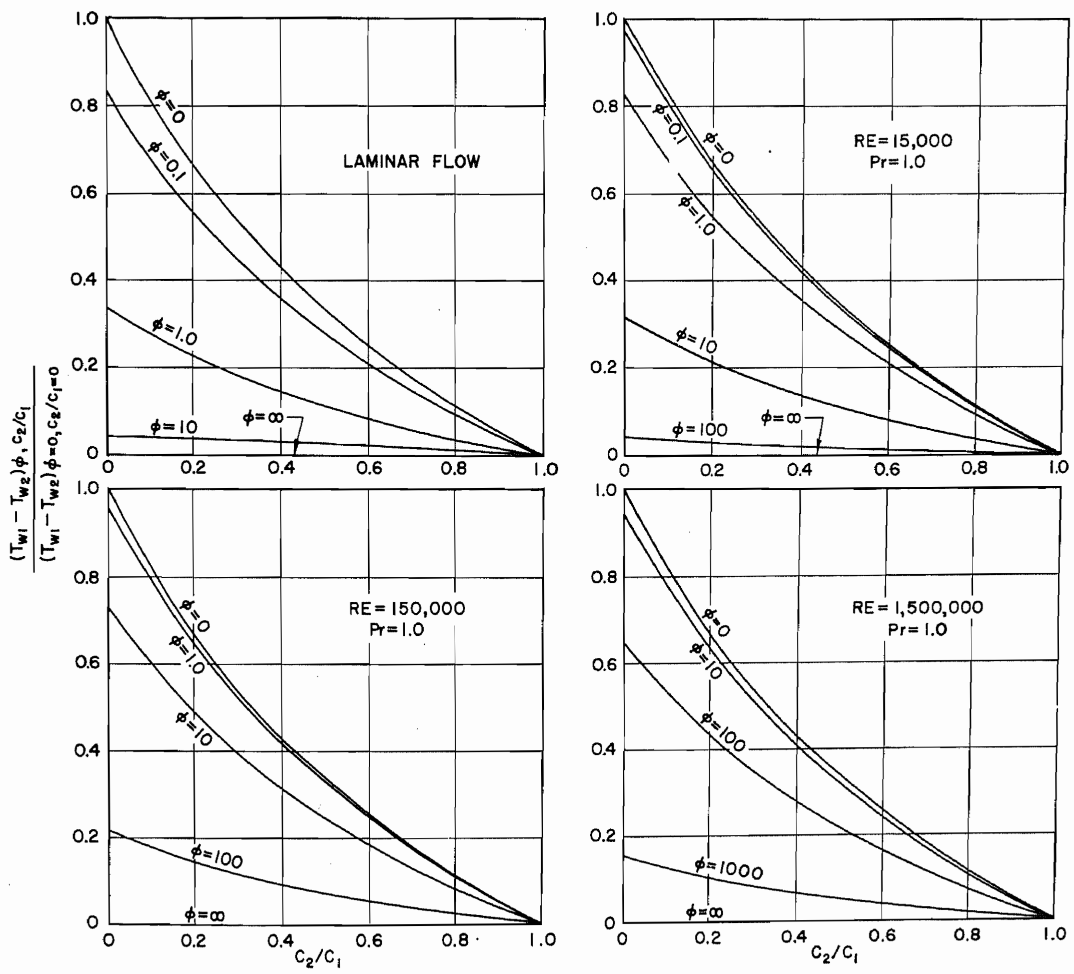

1000
800
600
500
400
300
200
100
80
60
40
30
20
NUSSELT NUMBER – $\overline{Nu}$
φ=00
10
1.0
0
Pr = 1.0
$\frac{C_2}{C_1} = 0$
1
2
3
4
5
6
8
10
20
30
40
50
60
80
100
REYNOLDS NUMBER – Re × 10^{-4}

Fig. 6



LAMINAR FLOW
RE= 15,000
Pr= 1.0
RE= 150,000
Pr= 1.0
RE= 1,500,000
Pr= 1.0
φ=0
φ=0.1
φ=1.0
φ=10
φ=100
φ=1000
φ=∞
(T_W1 − T_W2)φ, C2/C1 / (T_W1 − T_W2)φ=0, C2/C1=0
C2/C1
0
0.2
0.4
0.6
0.8
1.0

Fig. 7

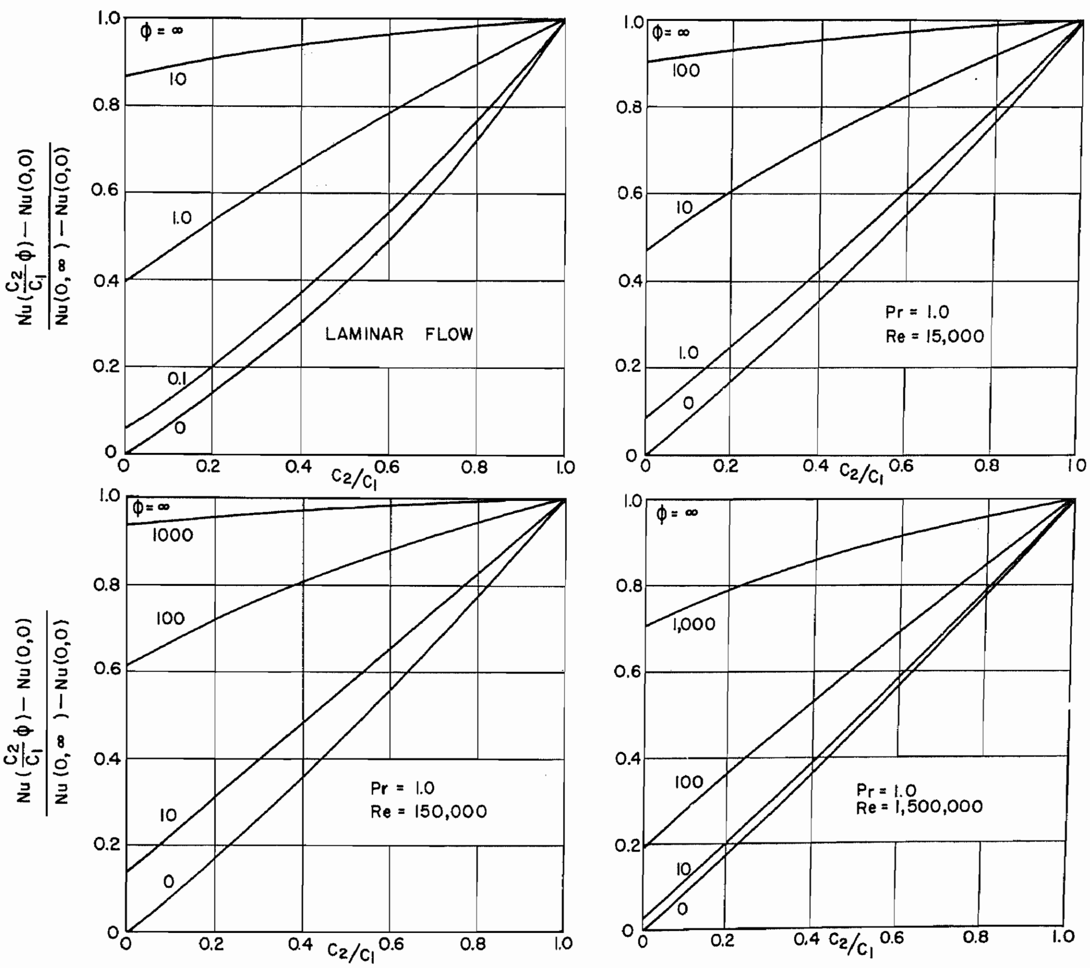
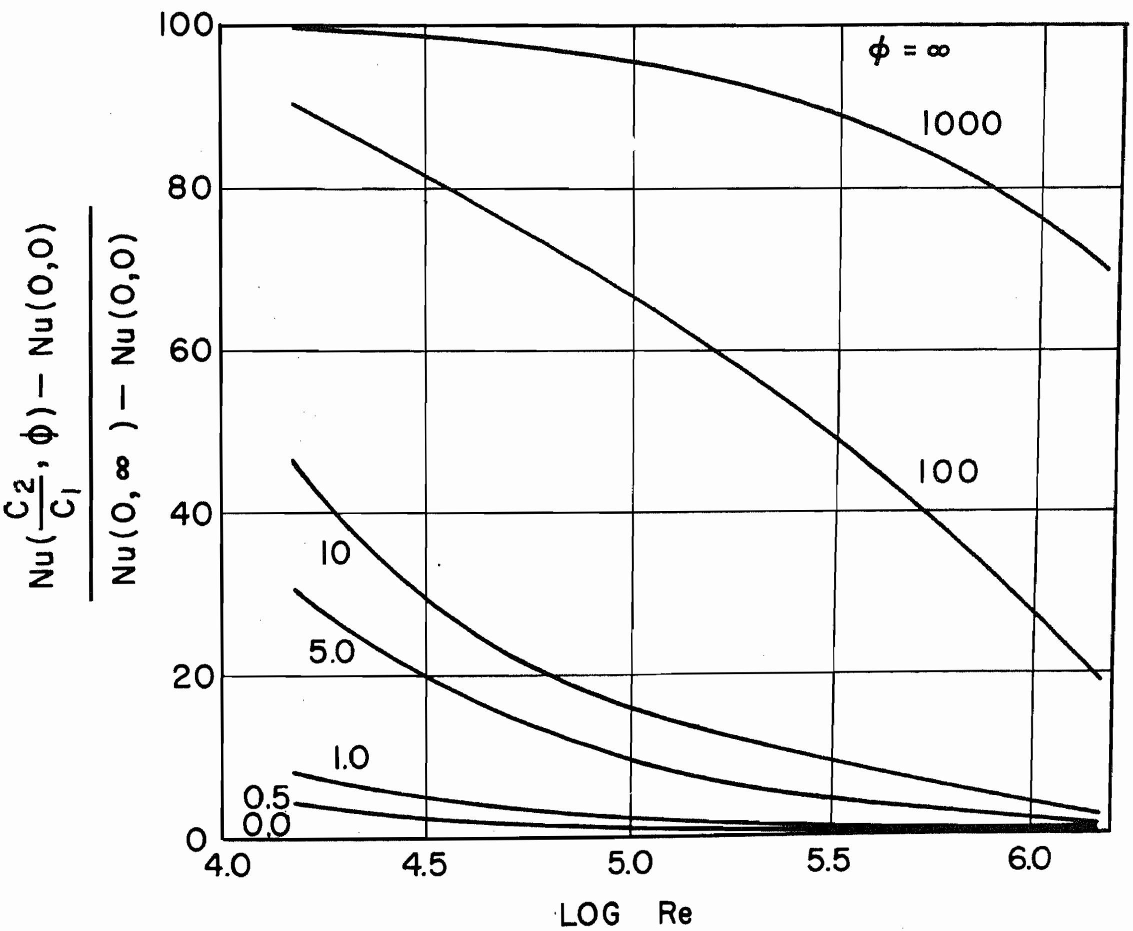

LAMINAR FLOW
Pr = 1.0
Re = 15,000
Pr = 1.0
Re = 150,000
Pr = 1.0
Re = 1,500,000
C2/C1
Nu(C2/C1 φ) − Nu(0,0) / Nu(0,∞) − Nu(0,0)
φ = ∞

Fig. 8



100
80
60
40
20
0
4.0
4.5
5.0
5.5
6.0
LOG Re
$\phi = \infty$
1000
100
10
5.0
1.0
0.5
0.0
$\frac{Nu(\frac{C_2}{C_1}, \phi) - Nu(0,0)}{Nu(0,\infty) - Nu(0,0)}$

Fig. 9