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THE ACTIVE RENAL COUNTERFLOW SYSTEM

by

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ABSTRACT

A general formulation in terms of the mass transport equation is provided for the loop of Henle mechanism which takes into account transport through the peritubular space and diffusion. It is shown that as one approximation the results of Kuhn and Ramel are obtained but with a term that depends on the ratio of the sodium permeabilities of the tubes.

THE ACTIVE RENAL COUNTERFLOW SYSTEM

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The countercurrent mechanism of the loop of Henle has been discussed by Kuhn and Ramel (1) who formulated their classic analysis in terms of certain ad hoc thermodynamic assumptions about steady state transport through membranes. These authors also neglect the effect of diffusion on the transport of ions. The purpose of the present brief report is that of showing how a more rigorous and straightforward development is possible proceeding directly from the classical mass transport equations, the appropriate boundary conditions and the assumption of a linear ion pump in the ascending tubule. Transport in the peritubular space is also included in the analysis given here. While a general analytical solution to the resulting equation is not given, it is shown that as one approximation the basic expression of Kuhn and Ramel result. However, the present formulation emphasizes the importance of the ratio of the sodium permeabilities of the ascending to that of the descending tubule.

A model of the loop of Henle in the vicinity of the bend is shown in Figure 1. For purposes of geometric simplicity the tubule of length h , width w and thickness q is assumed to have a rectangular cross section. The distance between

the descending and ascending tubule is d . The urinary flow velocity in the loop of Henle, \bar{v} , may to a good approximation be considered as constant in magnitude with a component only in the y direction, but with a positive sign in the descending loop and a negative sign in the ascending loop.

For the present purposes we shall only consider the transport of sodium ion since it is understood that the anion will move passively in such a way as to achieve charge neutrality. The concentration of sodium ion will be represented by $C_i(x,y,z)$ where the subscript, i , may assume the values 1, 2, and 3 designate respectively the concentration in the descending tubule, the peritubular space and the ascending tubule. Transport is considered to occur only at the interior facing surfaces at $x = q$ and $x = q + d$.

In general, for a sodium flux J taken positive in the direction of the outwardly drawn normal, we have under steady state conditions

$$\nabla \cdot \bar{J} = 0 \quad (1)$$

The flux vector associated with the tubule must take into account transport due to flow and diffusion so that

$$\bar{J} = C\bar{v} - D\nabla C \quad (2)$$

where D is the diffusion constant regarded as independent of position. The diffusion equation, which must be solved for each of the three regions, is

$$v \frac{\partial C}{\partial y} - D\nabla^2 C = 0 \quad (3)$$

with $v = 0$ in the peritubular space. The three solutions

are related through the boundary conditions which state that diffusion to the boundary equals transport through the boundary. If P_1 represents the permeability of the passive descending tubule then at $x = q$

$$-D \left[\frac{\partial C_1}{\partial x} \right] = P_1 (C_1 - C_2) \quad (4)$$

$$-D \left[\frac{\partial C_2}{\partial x} \right] = P_1 (C_1 - C_2) \quad (5)$$

where the negative sign results from the sign convention for the flux. Active transport is considered to occur across the wall of the ascending tubule. To a first approximation the flux due to the sodium ion pump is taken to depend linearly on the sodium ion concentration with a proportionality constant K so that $x = q + d$

$$D \frac{\partial C_2}{\partial x} = P_3 (C_3 - C_2) + K_3 C_3 \quad (6)$$

$$D \frac{\partial C_3}{\partial x} = P_3 (C_3 - C_2) + K_3 C_3 \quad (7)$$

As indicated the transport at the other surfaces is taken as zero so that at these surfaces $\partial C / \partial x = 0$.

Since we are interested in the variation of $C(x, y, z)$ in the y direction, and in any case, do not expect much change in the x and z directions, we integrate (2) over x , z in each region using the boundary conditions (4) - (7) and assume that at the boundaries $\partial C / \partial x$ is independent of z .

Introducing the average of C over x and z , namely

$$\bar{C}_1(y) = \frac{1}{q_w} \int_0^w \int_0^q C_1(x, y, z) dx dz \quad (8)$$

with similar definitions for the other two regions, we have in terms of this average,

$$D \frac{d^2 \bar{C}_1}{dy^2} - v \frac{d \bar{C}_1}{dy} - \frac{P_1}{q} (\bar{C}_1 - \bar{C}_2)_q = 0 \quad (9)$$

$$D \frac{d^2 \bar{C}_3}{dy^2} + v \frac{d \bar{C}_3}{dy} - \left[\frac{P_3}{q} (\bar{C}_3 - \bar{C}_2) + \frac{K_3}{q} \bar{C}_3 \right]_{q+d} = 0 \quad (10)$$

$$D \frac{d^2 \bar{C}_2}{dy^2} + \frac{1}{d} \left[P_3 (\bar{C}_3 - \bar{C}_2)_{q+d} + P_1 (\bar{C}_1 - \bar{C}_2)_q + K_3 (\bar{C}_3)_{q+d} \right] = 0 \quad (11)$$

The straightforward solution to (9) - (11) results in a sixth order secular equation. However, one useful approximation, leading to the results of Kuhn and Ramel, may be obtained in the case where the diffusion terms are small in comparison to the flow and wall transport terms so that the second order terms in (9) - (11) can be neglected. Assuming $\bar{C}_2(q) = \bar{C}_2(q + d)$ it follows from (11) that

$$\bar{C}_2 = \frac{P_1 \bar{C}_1}{P_1 + P_3} + \frac{(P_3 + K_3) \bar{C}_3}{P_1 + P_3} \quad (12)$$

Introducing (12) into (9) and (10) (with the diffusion terms

neglected) gives the result, with boundary condition $C_1 = C_3$ at the bend,

$$\bar{C}_1(y) = \bar{C}_3(y) = \bar{C}_1(0) \exp \left[\frac{K_3 P_1 y}{vq(P_1 + P_3)} \right] \quad (13)$$

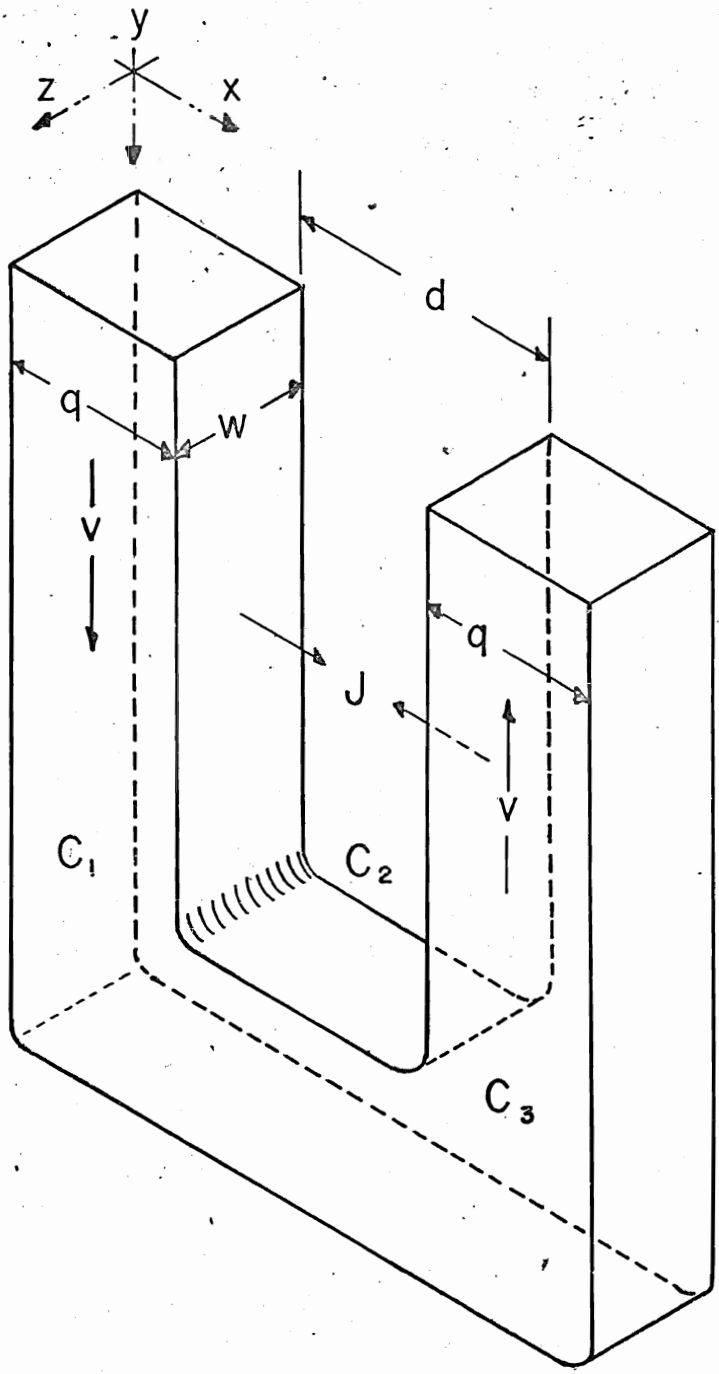
where $C(0)$ is $C_1(y)$ at $y=0$.

This result is in the form obtained by Kuhn and Ramel, showing that the increase in sodium concentration will occur if there is active transport across the ascending tubule wall. This increase also depends on the flow velocity in the tubules as well as on the ratio of the sodium permeability of the descending to that of the ascending tubule. However it is worthy of if $P_1/P_3 \ll 1$ then according to (13) there should be little dependency on tubule permeabilities.

References

- (1) Kuhn, W. and A. Ramel, Helvetica Chimica Acta, 42, 1959, 628.

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Model of Loop of Henle System

Figure 1