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AXISYMMETRICAL TURBULENT SWIRLING NATURAL CONVECTION PLUME

PART I - THEORETICAL INVESTIGATION

by

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ABSTRACT

A theoretical investigation is made of the behavior of an axisymmetrical turbulent swirling natural convection plume in an otherwise motionless ambient fluid. With the introduction of the assumption of similar axial and swirling velocity profiles and similar buoyancy profile, and the assumption of lateral entrainment of ambient fluid, the order of the governing differential equations is reduced by one after the initial integration following the Karman integral method. The behavior of the swirling plume is found to depend solely on two physical parameters associated with the source characteristics, the source Froude number and the source swirling velocity parameter. A series solution developed in the vicinity of the source of the swirling plume is obtained for any values of these two physical parameters. Numerical solutions for extended range of axial distance from the source of the swirling plume and for wide ranges of selected representative values of these two physical parameters are obtained with the use of a digital electronic computer. The behavior of the swirling plume is found to approach that of a non-swirling jet, a swirling jet, or a non-swirling plume when these two physical parameters are assigned values approaching those designating each of the aforementioned simpler, boundary situations.

INTRODUCTION

A swirling natural convection plume embodies the very interesting combination of the motion of a swirling jet and the motion of a natural convection plume, each of which represents a very complex problem by itself. The situation is further complicated by the fact that under most circumstances the flow field becomes turbulent. Consequently, the problem has never been attempted with much success.

For the simpler problem of the motion of a turbulent natural convection plume, most of the previous successful theoretical works employ the method of the lateral entrainment assumption on the ambient otherwise stationary air first introduced by Taylor (1945) [1]. The assumption states that the time-mean velocity of entrained surrounding air at a certain level is proportional to a certain characteristic time-mean velocity of the turbulent plume at the same level.

Morton, Taylor and Turner (1956) [2] investigated the simple two-dimensional case of the plume from an idealized mathematical line source and the simple axisymmetrical case of the plume from an idealized mathematical point source of infinitesimal physical size, infinite buoyancy intensity and zero mass and momentum fluxes. Their results check very well with the experimental findings of Rouse, Yih and Humphreys (1952) [3] for plumes above a very small gas flame and a line of very small gas flames designed to simulate the idealized point and line sources.

Morton (1959) [4, 5] investigated the axisymmetrical case of the plume from an axisymmetrical source of finite mass, momentum and buoyancy fluxes. His results depend on the solution for the case of plume from a ficticious point source of finite buoyancy and momentum fluxes but zero mass flux at a lower level. Furthermore, in his analysis, no description has been made about the physical size of the source and its possible influence on the behavior of the plume.

Lee and Emmons (1961) [6] investigated theoretically the two-dimensional case of the plume from a finite-size strip source of finite mass, momentum and buoyancy fluxes. Their results brought out the significance of a source Froude number F. A quadrature solution was obtained for each of two separate ranges of the Froude number, $F < 1$ or $F > 1$. In neither of these cases the finite-size strip source can be accurately represented by an equivalent mathematical line source at a lower level. Only the special case, $F = 1$, can be so represented and its solutions, with a shift of reference coordinates, check with the line source solutions obtained by Morton, Taylor and Turner (1956) [2].

Lee and Emmons (1961) [6] also investigated experimentally the behavior of the plume of hot gases above a diffusion flame of liquid fuel burned in a long finite-size channel burner. Their measurements check closely with the results of their theoretical investigation of the two-dimensional case of the plume from a finite-size source of finite mass, momentum and buoyancy.

For the simpler problem of the motion of a turbulent swirling jet, on the other hand, most previous attempts simply extended the solutions obtained for the case of a laminar swirling jet to the case of a turbulent swirling jet by the rather unjustified argument of the constancy of an apparent kinematic viscosity.

Loitsyanskii (1953) [7] studied the axisymmetrical laminar swirling jet with both the axial and radial pressure variations taken into consideration. He obtained series solutions for the velocity components in ascending powers of the inverse of the axial distance from a virtual point source for the swirling jet. The coefficient of each term of each of the power series is found to be a function of a sole dimensionless similarity variable which involves both the axial and the radial distances. The first terms of these series represent the solutions of velocity components of a laminar jet without swirl. The author extended his analysis to the case of an axisymmetrical turbulent swirling jet by the use of the Prandtl's mixing length and momentum transfer theorems. He pointed out that if the kinematic viscosity of the laminar case could be replaced by a constant apparent kinematic viscosity of the turbulent case, the two cases would become identical and, therefore, the solutions for the laminar swirling jet could be used for the turbulent swirling jet. His argument on the constancy of the apparent kinematic viscosity is based entirely on the assumption that the product of the maximum axial velocity and a characteristic radius of the swirling jet remains constant at all axial distances. This could be true if only the first terms be retained in the series solutions for the velocity field. In other words, the assumed constancy of the apparent kinematic viscosity can be justified only for case of an axisymmetrical turbulent jet with no swirl which contradicts with the very physical model of an axisymmetrical turbulent jet with swirl under investigation.

Görtler (1954) [8] studied the axisymmetrical laminar jet with very weak swirl. He completely ignored the pressure variations in the flow field and, as a consequence, the momentum equation governing the axial velocity field was set independent of the swirling velocity field. His solution for the axial velocity field for a laminar jet with weak swirl is naturally identical with that for a laminar jet with no swirl at all and the extent of validity of this solution is evidently questionable. His solution for the swirling velocity field is given by a series of eigenfunctions. Each of the eigenfunctions is a function of the axial distance and a dimensionless similarity variable which involves both the axial and the radial distances. He also extended his solutions for an axisymmetrical laminar jet with very weak swirl to the case of an axisymmetrical turbulent jet with very weak swirl by the same argument of the constancy of an apparent kinematic viscosity. This extension seems well justified since the analysis of the main flow is just that of an axisymmetrical jet, laminar in one case and turbulent in the other, with no swirl. However, the inherited questionable extent of validity of the solutions remains unaltered.

Steiger and Bloom (1960) [9] applied the Karman integral method to the axisymmetrical laminar swirling jet problem. Non-similar solutions, in closed form, for incompressible and compressible flow were derived by assuming specific functional forms for the velocity distributions. In particular, these solutions depend on the assumption of the non-vanishing radial gradient of the swirling velocity along the axis of the swirling jet. No where in their paper have they made any justification for such an assumption. They have not made any claim of being able to extend their solutions for the laminar case to cover the turbulent case by assuming the constancy of the apparent kinematic viscosity. It should be noted that the apparent kinematic viscosity could not be made constant because of the functional forms of their solutions.

The only experimental work on axisymmetrical swirling jet existing in the literature was reported by Rose (1962) [10] who studied a swirling round turbulent jet of air generated by flow issuing from a rotating pipe into a reservoir of motionless air. He used a constant-temperature hot-wire anemometer to measure the velocity field of the swirling jet extending from the pipe discharge out to a distance of fifteen pipe diameters. From his measurements, it can be observed that very nearly similar Gaussian profiles exist for the axial velocity at all measured axial stations from one and one half diameters on. From the same measurements, it can also be observed that the distribution of the swirling velocity at all measured axial stations assumes the same similar profile which is related to the corresponding similar profile for the axial velocity distribution at the same axial station through a characteristic radius of the swirling jet. He also specifically reported the decays of the maximum axial and the maximum swirling velocities and the effect of swirl on jet spray and found that they apparently did not compare satisfactorily, over the range of axial distance covered, with some of the simple conclusions drawn from the afore-mentioned existing theoretical solutions.

More recently, Lee (1965) [11] investigated the case of an axisymmetrical turbulent swirling jet issuing from a circular source into a semi-infinite otherwise motionless ambient fluid. He succeeded in obtaining a simple closed form solution by introducing the assumptions of similar axial and swirling velocity profiles and lateral entrainment of ambient fluid into the integrated governing equations. Results for the decays of the axial and swirling velocities and the spray of the jet agree closely with the experimental findings on the swirling round turbulent jet of air reported by Rose (1962) [10].

In view of the success in obtaining a satisfactory solution for turbulent natural convection plumes by the introduction of the lateral entrainment assumption and for an axisymmetrical turbulent swirling jet by the same lateral entrainment assumption respectively, it is, therefore, only logical to attempt the solution of the combined complex problem of a swirling natural convection plume by the introduction of the afore-mentioned lateral entrainment assumption.

ANALYSIS

Let us assume that the flow field be fully turbulent and as a consequence the molecular effects can be considered negligible in comparison with the turbulent effects and that the plume covers a very narrow region in the direction of its axis of symmetry and therefore the usual boundary layer approximations can be made. Let us also assume that the local density variations are everywhere small in comparison with some reference density in the flow field. Even though a source of heated fluid is causing the natural convection, it is the buoyancy rather than the thermal properties of the flow which is fundamental to the phenomenon. Therefore, although the buoyancy force due to density difference is sufficiently great to contribute to vertical acceleration, the corresponding variation in the mass density of the fluid undergoing acceleration is sufficiently small, in comparison with the density itself, to be neglected in the governing continuity and energy equations and the inertia terms of the governing momentum equations.

If we let u, v, and w be the components in the x-(axial), r-(radial), and O-(tangential) directions respectively of the time-mean velocity of the fluid at a point A inside the plume flow field as shown in the definition sketch of Figure 1, the governing equations for the axisymmetrical swirling plume are then as follows:

Continuity Equation:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{1}$$

x-direction Boundary-Layer Momentum Equation:

$$\frac{\partial}{\partial x}(ru^2) + \frac{\partial}{\partial r}(ruv) = -\frac{r}{\rho_1}\frac{\partial p}{\partial x} - \frac{\partial}{\partial r}(r\overline{u'v'}) + \frac{r\Delta\gamma}{\rho_1} \tag{2}$$

r-direction Boundary-Layer Momentum Equation:

$$\frac{w^2}{r} = \frac{1}{\rho_1}\frac{\partial p}{\partial r} \tag{3}$$

Θ-direction Boundary-Layer Momentum Equation:

$$\frac{\partial}{\partial x}(r^2 uw) + \frac{\partial}{\partial r}(r^2 vw) = -\frac{\partial}{\partial r}(r^2 \overline{v'w'}) \qquad (4)$$

Boundary-Layer Energy Equation:

$$\frac{\partial}{\partial x}(ru\Delta T) + \frac{\partial}{\partial r}(rv\Delta T) = -\frac{\partial}{\partial r}(r\overline{v'\Delta T'}) \qquad (5)$$

where ρ_1 = density of the undisturbed ambient fluid

p = local time-mean pressure

$\Delta\gamma = g\Delta\rho = g(\rho_1 - \rho)$ = local buoyancy

ρ = local time-mean density

$-\rho_1\overline{u'v'}$, $-\rho_1\overline{v'w'}$ = Reynold's stresses

$-\rho_1 Cp\,\overline{v'\Delta T'}$ = eddy heat transfer

u' = fluctuation velocity in the x-direction

v' = fluctuation velocity in the r-direction

w' = fluctuation velocity in the Θ-direction

$\Delta T = T - T_1$ = local time-mean temperature increment

T = local time-mean temperature

T_1 = temperature of the undisturbed ambient fluid

$\Delta T'$ = fluctuation temperature increment

Cp = specific heat at constant pressure

If we further assume that the local temperature increment is small as compared to some reference temperature, say T_1, and that the local pressure change is small enough not to cause any significant change of the thermodynamic properties of the fluid but large enough to influence the dynamic behavior of the fluid, the equation of state of an ideal gas reduces to the following:

$$\frac{\Delta T}{T_1} = \frac{\Delta \rho}{\rho_1} = \frac{\Delta \gamma}{g\rho_1}$$

and the boundary-layer energy equation, Equation (5), reduces to

$$\frac{\partial}{\partial x}(ru\Delta\gamma) + \frac{\partial}{\partial r}(rv\Delta\gamma) = -\frac{\partial}{\partial r}(r\overline{v'\Delta\gamma'}) \tag{6}$$

Let us assume the following similar profiles for the axial and swirling velocities and the local buoyancy at all axial stations:

$$u(x, r) = u(x) \exp(-r^2/b^2) \tag{7}$$

$$w(x, r) = w(x) f(r/b) \tag{8}$$

$$\Delta\gamma(x, r) = \Delta\gamma(x) \exp(-r^2/\lambda^2 b^2) \tag{9}$$

where $b = b(x)$ is the value of r at which the magnitude of the axial velocity is $1/e$ of that of the maximum axial velocity, $u(x)$, along the axis and $f(r/b)$ is the profile distribution of the swirling velocity, $w(x, r)$, which assumes the maximum value $w(x)$ at the maximum of $f(r/b)$ which has the magnitude of unity. The functional form of $f(r/b)$ is to be determined experimentally. $\Delta\gamma(x) = \Delta\gamma(x, o)$ is the time-mean buoyancy along the plume axis. The axial velocity profile is characterized by a length scale $b(x)$ while the local buoyancy profile is of the same shape but is characterized by a slightly different length scale $\lambda b(x)$ where λ is an universal constant.

With the swirling velocity profile of Equation (8) introduced, Equation (3) can be readily integrated from the plume axis, $r = 0$, to the edge of the plume, $r = r_e$, to give

$$p(x, r) = p_1 - \rho_1 w^2(x) I(r/b) \tag{10}$$

where

p_1 = pressure of the undisturbed ambient fluid

and

$$I(r/b) = \int_{(r/b)}^{(r_e/b)} \frac{[f(r/b)]^2}{(r/b)} d(r/b) \tag{11}$$

Since $f(r/b)$ usually decays rapidly toward the edge of the plume, the upper limit of integration of Equation (11) can be lifted from the edge of the plume, r_e/b, to ∞ without introducing much error. Therefore, Equation (11) can be replaced by the following expression:

$$I(r/b) = \int_{(r/b)}^{\infty} \frac{[f(r/b)]^2}{(r/b)} \, d(r/b) \tag{12}$$

Integrate Equation (2) from 0 to ∞ with respect to r, with Equations (7), (9) and (12) introduced. The radial velocity $v(x, r)$ and the Reynold's stress vanish at $r = 0$ due to symmetry and Reynold's stress vanish at $r = \infty$ in the ambient fluid. The resulting equation takes the form:

$$\frac{d}{dx}\left(u^2 b^2 - 4kw^2 b^2\right) = \frac{2\lambda^2 b^2 \Delta\gamma}{\rho_1} \tag{13}$$

where

$u = u(x)$

$b = b(x)$

$w = w(x)$

$\Delta\gamma = \Delta\gamma(x)$

and

$$k = \int_0^{\infty} I(r/b) \cdot (r/b) \, d(r/b) \tag{14}$$

The value of the swirling velocity profile constant k depends solely on the shape of the swirling velocity profile $f(r/b)$. For instance, for the swirling velocity profile of the swirling turbulent round jet of air, obtained by Rose (1962) [10], a value of 0.208 for k was evaluated.

Integrate Equation (4) from 0 to ∞ with respect to r, with Equations (7) and (8) introduced. The radial and swirling velocities, $v(x, r)$ and $w(x, r)$, and the Reynold's stress vanish at $r = 0$ due to symmetry and $w(x, r)$ and the Reynold's stress vanish at $r = \infty$ in the ambient fluid. The resulting equation takes the form:

$$\frac{d}{dx}\left(uwb^3\right) = 0 \tag{15}$$

Integrate Equation (1) from 0 to ∞ with respect to r, with Equation (7) and the condition $v(x, 0) = 0$ introduced, we have

$$\frac{d}{dx}(ub^2) = -2(rv)\Big|_{r \to \infty} \tag{16}$$

which, when multiplied by the constant factor $(\pi\rho_1)$ on both sides, states that the increase of the mass flux in the axial direction is supplied by the mass entrainment from the ambient fluid in the radial direction at the edge of the plume.

In order to attempt to get a solution for the problem, some additional information about this entrainment mass flux must be introduced at this point. Taylor (1945) [1] studied the turbulent natural convection plume released from an instant point heat source. He succeeded in obtaining similarity solutions by the introduction of the assumption of uniform axial velocity and buoyancy profiles across the plume and an entrainment assumption based on a dimensional argument. His entrainment assumption relates the horizontal entrainment velocity at the edge of the plume, $[-v_{edge}]$, to the axial velocity within the plume at the same height, $\bar{u}$, in the form

$$[-v_{edge}] = \alpha\bar{u} \tag{17}$$

where α is the entrainment coefficient and is a constant for the case under study. This entrainment assumption was modified by Morton, Taylor and Turner (1956) [2] to take the form

$$[-(rv)\Big|_{r \to \infty}] = \alpha b(x)u(x) \tag{18}$$

for the case of an axisymmetrical non-swirling plume released from a maintained point buoyancy source, with similar Gaussian axial velocity and buoyancy profiles assumed, such as those expressed by Equation (7) and Equation (9) for $\lambda = 1$ respectively. In Equation (18), α again is the entrainment coefficient and is also a constant for this case under study but a different constant from the one introduced in Equation (17). For the present problem of an axisymmetrical turbulent swirling plume, the entrainment assumption of Equation (18) still seems well justified as it can be reasoned from Equation (16) that the radial entrainment velocity is solely responsible for making up the change of the axial mass flux and the swirling velocity does not come into the picture of the gross continuity consideration over the cross section of the plume. By the introduction of the entrainment assumption of Equation (18), Equation (16) becomes

$$\frac{d}{dx}(ub^2) = 2\alpha bu \tag{19}$$

Integrate Equation (6) from 0 to ∞ with respect to r, with Equations (7) and (9) introduced. The radial velocity $v(x, r)$ and the eddy heat transfer vanish at $r = 0$ due to symmetry, and the local buoyancy and the eddy heat transfer vanish at $r = \infty$ in the ambient fluid. The resulting equation takes the form:

$$\frac{d}{dx}(u\Delta\gamma b^2) = 0 \tag{20}$$

Now, the governing equations for the axisymmetrical turbulent swirling plume problem are summarized as follows:

$$\frac{d}{dx}(ub^2) = 2\alpha bu \tag{19}$$

$$\frac{d}{dx}(u^2b^2 - 4kw^2b^2) = \frac{2\lambda^2b^2\Delta\gamma}{\rho_1} \tag{13}$$

$$\frac{d}{dx}(uwb^3) = 0 \tag{15}$$

$$\frac{d}{dx}(u\Delta\gamma b^2) = 0 \tag{20}$$

The boundary conditions can be assumed to be:

$$u = u_o, \quad w = w_o, \quad \Delta\gamma = \Delta\gamma_o, \text{ and } b = b_o \text{ at } x = 0 \tag{21}$$

which state physically that a swirling source with the velocity and buoyancy profiles specified by the similar distribution of Equations (7), (8), and (9), and characteristics u_o, w_o, $\Delta\gamma_o$, and b_o, is assumed to exist at the starting level of $x = 0$ of the plume.

Let us introduce the following transformations to eliminate the constant coefficients from the governing equations:

$$X = 2\alpha x/(b_o F^{4/7} G^{3/7})$$

$$B = b/(B_o F^{4/7} G^{3/7})$$

$$U = uF^{6/7} G^{1/7}/u_o \qquad (22)$$

$$W = wF^{6/7} G^{8/7}/w_o$$

$$P = (\Delta\gamma/\gamma_1) F^{2/7} G^{5/7}/(\Delta\gamma_o/\gamma_1)$$

where

$$F = \alpha^{1/2} u_o \,/\, [\lambda\, g^{1/2} b_o^{\,1/2} (\Delta\gamma_o/\gamma_1)^{1/2}]$$

is the source Froude number,

and

$$G = 2k^{1/2} w_o/u_o$$

is the source swirling velocity parameter which has the form of the reciprocal of a source Rossby number. The source Froude number F describes the nature of the source. The extreme value of $F = 0$ indicate a pure buoyancy source while the extreme value of $F = \infty$ indicate a pure momentum source. Therefore, small values of F always associate with sources of a restrained nature and, on the other hand, large values of F always associate with sources of an impelled nature. The source swirling velocity parameter G describes the relative amount of swirling of the fluid at the source. For cases of turbulent plumes with very large swirl where, say, $w_o > u_o$, the Gaussian axial velocity and buoyancy profiles probably would no longer prevail at all axial stations as assumed throughout the analysis. Therefore, with a value of k, say, in the neighborhood of 0.2 as evaluated from the existing experimental results on a turbulent swirling jet, we could fairly safely say that the present analytical formulation would be valid for cases with $G < 1$ or, with some uncertainty, up to $G = 1$ which, however, already corresponds to fairly large swirl.

With these transformations, Equations (22), the governing equations, Equations (19), (13), (15), and (20) become

$$\frac{d}{dX}(UB^2) = UB \tag{23}$$

$$\frac{d}{dX}(U^2B^2 - W^2B^2) = PB^2 \tag{24}$$

$$\frac{d}{dX}(UWB^3) = 0 \tag{25}$$

$$\frac{d}{dX}(UPB^2) = 0 \tag{26}$$

and the corresponding boundary conditions, Equations (21), become

$$\left.\begin{aligned} U &= F^{6/7}G^{1/7} \\ W &= F^{6/7}G^{8/7} \\ P &= F^{2/7}G^{5/7} \\ B &= F^{-4/7}G^{-3/7} \end{aligned}\right\} \quad \text{at } X = 0 \tag{27}$$

Equations (25) and (26) can be immediately integrated to give

$$UWB^3 = 1 \tag{28}$$

and

$$UPB^2 = 1 \tag{29}$$

respectively.

With W and P eliminated by the use of Equations (28) and (29) respectively, Equation (24) can be written as

$$\frac{d}{dX}\left(U^2B^2 - \frac{1}{U^2B^4}\right) = \frac{1}{U} \tag{30}$$

Introducing the additional transformations

$$M = UB \qquad N = UB^2 \tag{31}$$

into Equations (23) and (30) respectively, we have

$$\frac{dN}{dX} = M \tag{32}$$

$$\frac{d}{dX}\left(M^2 - \frac{1}{N^2}\right) = \frac{N}{M^2} \tag{33}$$

and the corresponding boundary conditions, Equations (27), become

$$M = F^{2/7}G^{-2/7} \qquad N = F^{-2/7}G^{-5/7} \qquad \text{at } X = 0 \tag{34}$$

Equation (33) can be simplified with the use of Equation (32) to take the form

$$\frac{dM}{dX} = -\frac{1}{N^3} + \frac{N}{2M^3} \tag{35}$$

First, power series solution developed around the origin $X = 0$ are attempted for both M and N. Let us assume the following series solutions:

$$N = a_o + a_1 X + a_2 X^2 + a_3 X^3 + a_4 X^4 + \dots$$
$$M = c_o + c_1 X + c_2 X^2 + c_3 X^3 + c_4 X^4 + \dots \qquad (36)$$

Substituting Equations (36) into Equation (32), we have

$$c_o = a_1, \; c_1 = 2a_2, \; c_2 = 3a_3, \; c_3 = 4a_4, \; \dots \qquad (37)$$

The boundary conditions of Equations (34) require that

$$a_o = F^{-2/7} G^{-5/7}$$

$$a_1 = c_o = F^{2/7} G^{-2/7}$$

Substituting the series of N and M, Equations (36), into Equation (35) and equating the coefficients of the successive terms of powers of X to zero, we have after some manipulation

$$a_2 = c_1/\,2 = \frac{1}{4} F^{-8/7} G^{1/7} (1 - 2F^2 G^2)$$

$$a_3 = c_2/\,3 = \frac{1}{24} F^{-18/7} G^{4/7} (-3 + 2F^2 + 6F^2 G^2 + 12F^4 G^2)$$

$$a_4 = c_3/\,4 = \frac{1}{192} F^{-4} G\,(21 - 22F^2 - 16F^4 - 12F^2 G^2 + 20F^4 G^2 - 6F^4 G^4 - 96F^6 G^2 - 156F^6 G^4)$$

....

However, since the power series solutions for N and M have been developed around the origin X = 0, they will be valid for sufficiently small values of X. These power series solutions will then only give a description of the behavior of the axisymmetrical turbulent swirling plume up to an axial distance not too far from the source. For more general description of the behavior of the plume in the flow field extending from the level of the source, X = 0, to any unrestricted axial station along the X-axis, numerical schemes on Equations (32) and (35) with boundary conditions, Equations (34), have been attempted and computations have been performed on a digital electronic computer. The computor used is an IBM 1410 digital computer and the program is expressed in terms of the Fortran machine language. The number of pairs of the initial values of M and N used in the computation is seventy-seven since there are seven selected values of F and eleven selected values of G employed. The selected values of F are 0.001, 0.01, 0.1, 1, 10, 100, and 1000. The selected values of G are 0.001, 0.005, 0.01, 0.025, 0.05, 0.075, 0.1, 0.25, 0.5, 0.75, and 1. The results for the dimensionless maximum axial velocity, U, and the dimensionless reciprocal of maximum buoyancy, 1/P, for different values of the source swirling velocity parameter, G, but the same value of the source Froude number, F, are plotted against the dimensionless axial distance above the source, X, in Figures 2-1 through 2-7. The corresponding results for the dimensionless maximum swirling velocity, W, and the dimensionless characteristic plume radius, B, are plotted in Figures 2-8 through 2-14.

CONCLUSION

It has been found that the behavior of an axisymmetrical turbulent swirling natural convection plume is determined solely by two physical parameters associated with the source characteristics, namely, the source Froude number F and the source swirling velocity parameter G. Over wide ranges of values of F and G, solutions for the problem have been found for such quantities as the dimensionless maximum axial velocity U, the dimensionless reciprocal of the maximum local buoyancy 1/P, and the dimensionless characteristic radius B of the swirling plume, each as a function of the dimensionless axial distance above the source X.

The general behavior of a swirling plume, as far as such quantities as U, 1/P, and B are concerned, seems to be similar in a qualitative way to that of a non-swirling plume except for some quantitative modifications caused by the amount of swirling as characterized by the source swirling velocity parameter G. For small values of the source Froude number F, U will first increase to a maximum and then will start to decrease, 1/P will increase slowly, and B will first decrease to a minimum and then will start to increase. The appearance of the swirling plume will show a bottle neck at the axial station where B has a minimum. An increase of the value of G will tend to make the initial increase of U faster, the occurrence of the minimum of B sooner, but will not have much significant effect on the behavior of 1/P. For large values of the source Froude number F, U will decrease first rather quickly and then slowly, 1/P and B both will increase almost linearly at a rather steep slope. An increase of the value of G will tend to make the initial decrease of U faster, the slope of the increase of 1/P less steep, and the magnitude of B smaller but increasing at more or less the original slope.

The dimensionless maximum swirling velocity W in a swirling plume behaves in a qualitative way very mucy like the dimensionless maximum axial velocity U. For small values of the source Froude number F, W will first increase to a maximum and then will start to decrease. An increase of the value of G will tend to make the initial increase of W faster. For large values of the source Froude number F, W will decrease first rather quickly and then slowly. An increase of the value of G will tend to make the initial decrease of W faster.

Finally, it can be said that the behavior of the swirling plume will approach that of a non-swirling jet, a swirling jet, or a non-swirling plume when the proper values of the two parameters, F and G, are assigned. For very large values of F and very small values of G, the behavior of U and B approaches that of a non-swirling jet. For very large values of F, the behavior of U, B, and W approaches that of a swirling jet. And, for very small values of G, the behavior of U, B, and 1/P approaches that of a non-swirling plume.

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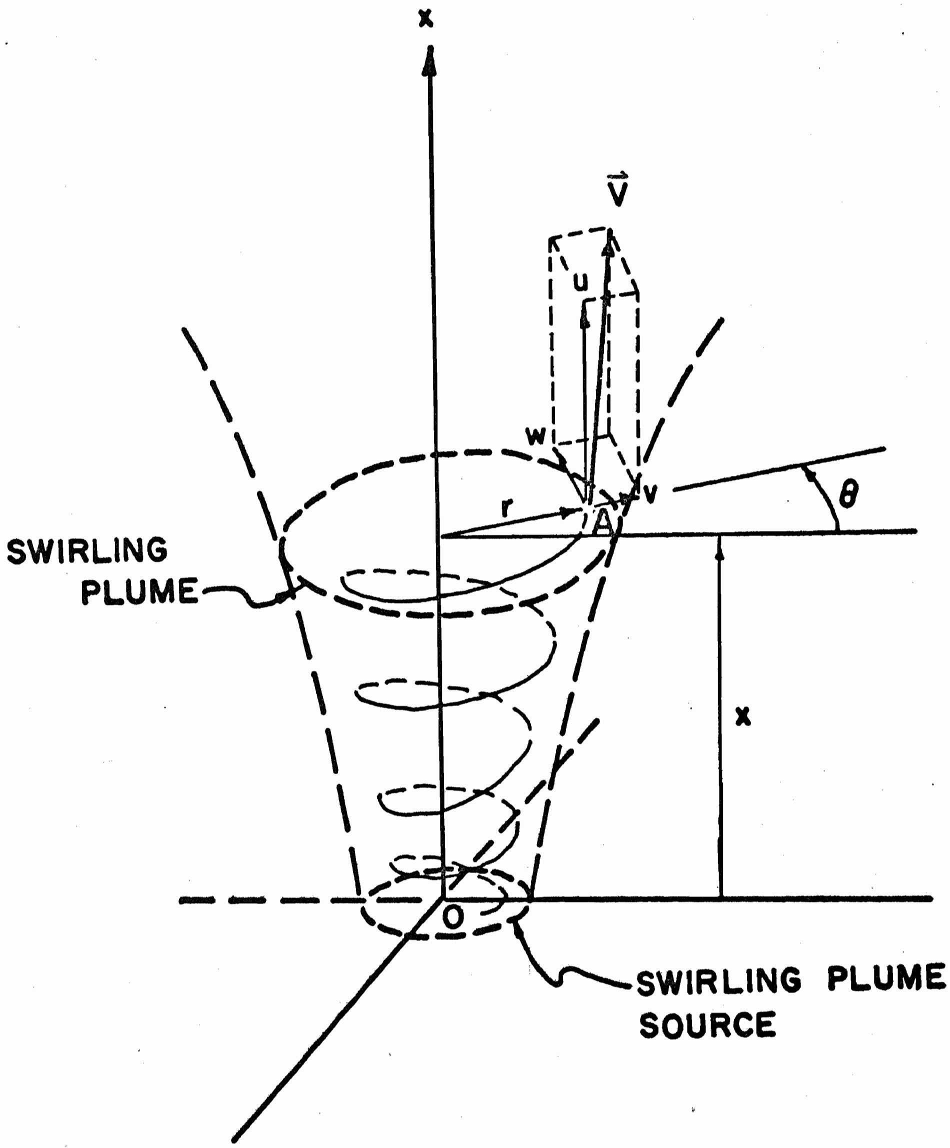

FIGURE 1. DEFINITION SKETCH

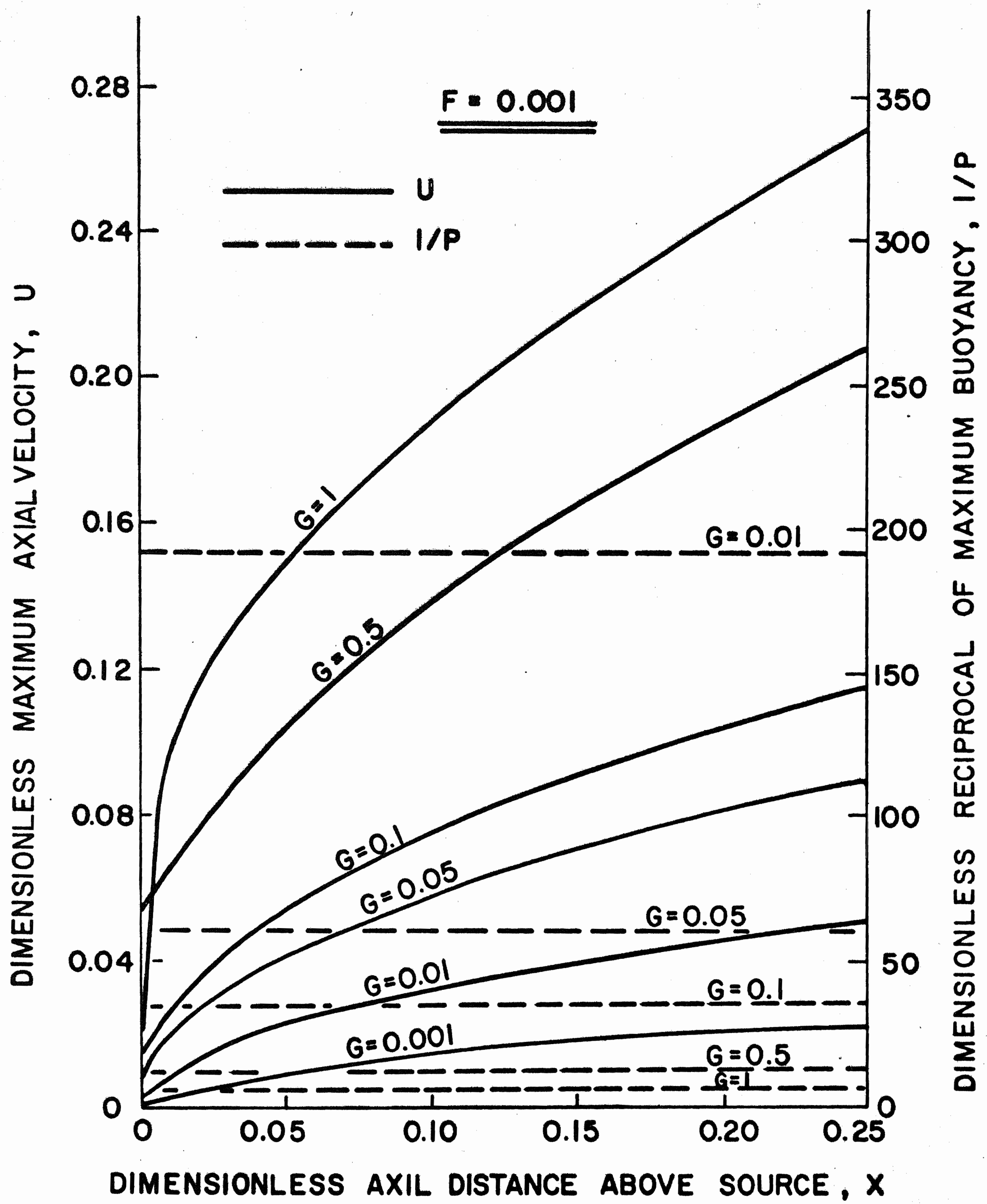

FIGURE 2-1. RESULTS OF MAXIMUM AXIAL VELOCITY AND MAXIMUM BUOYANCY (F = 0.001)

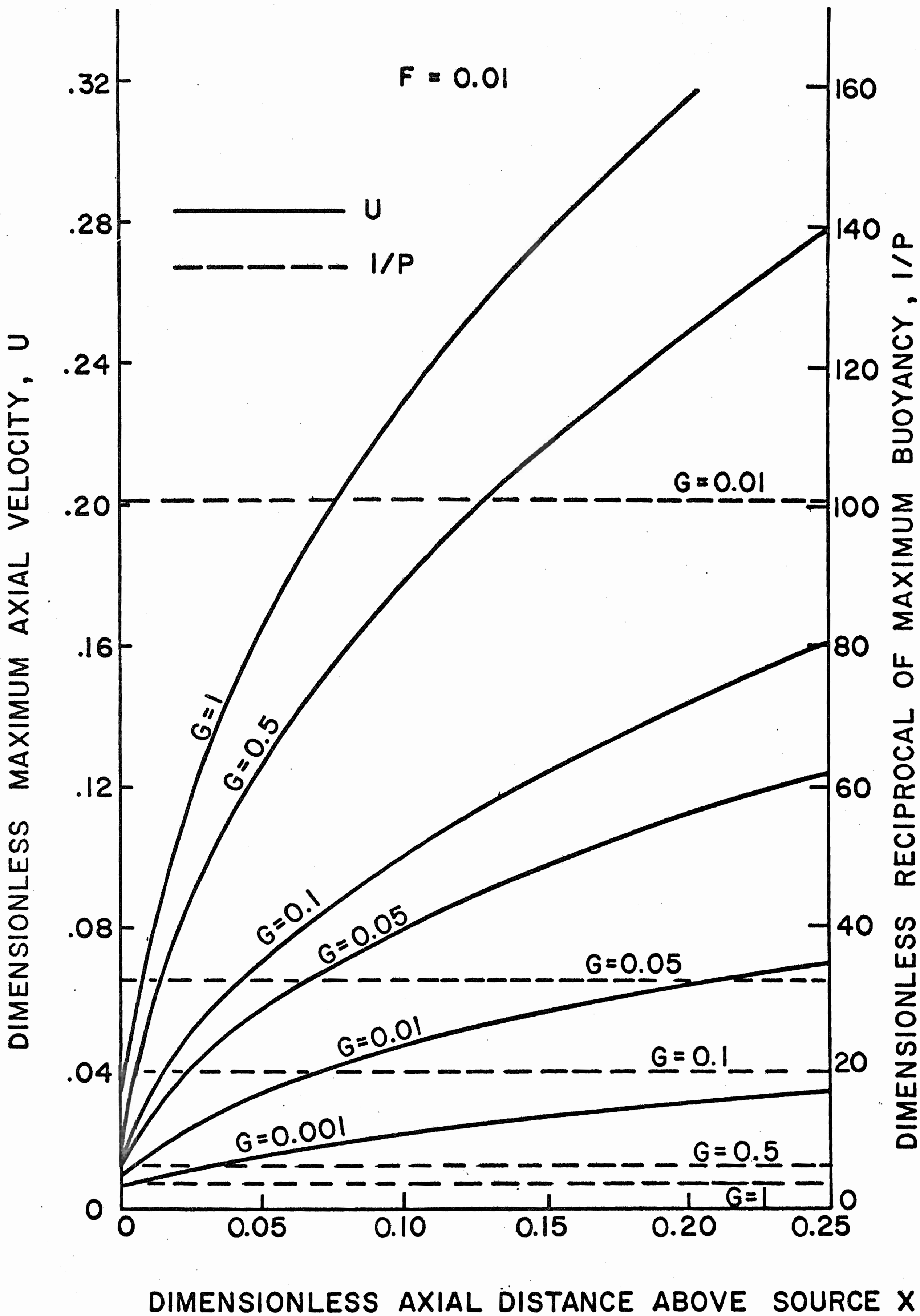

FIGURE 2-2. RESULTS OF MAXIMUM AXIAL VELOCITY AND MAXIMUM BUOYANCY (F = 0.01)

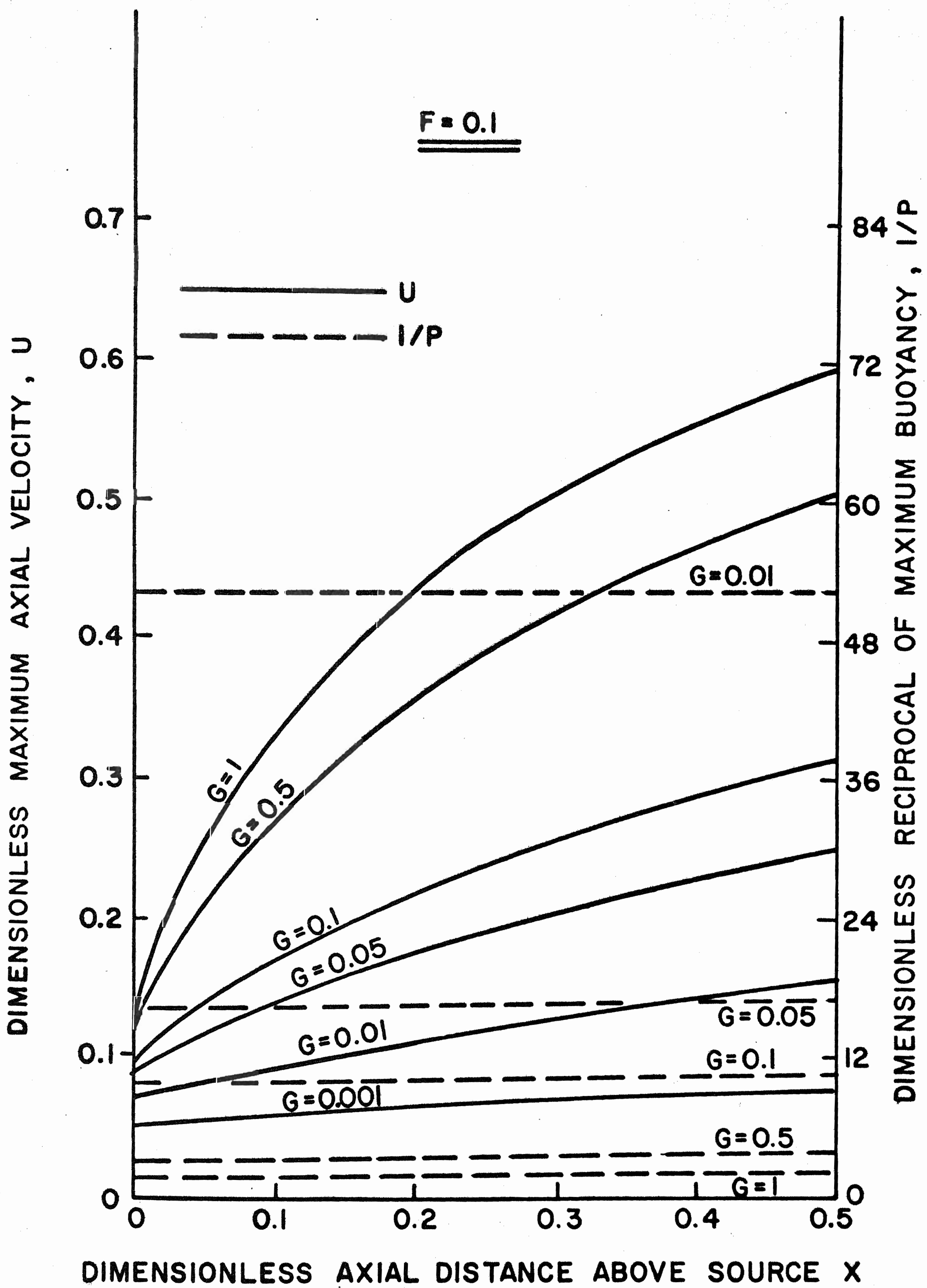

FIGURE 2-3. RESULTS OF MAXIMUM AXIAL VELOCITY AND MAXIMUM BUOYANCY (F = 0.1)

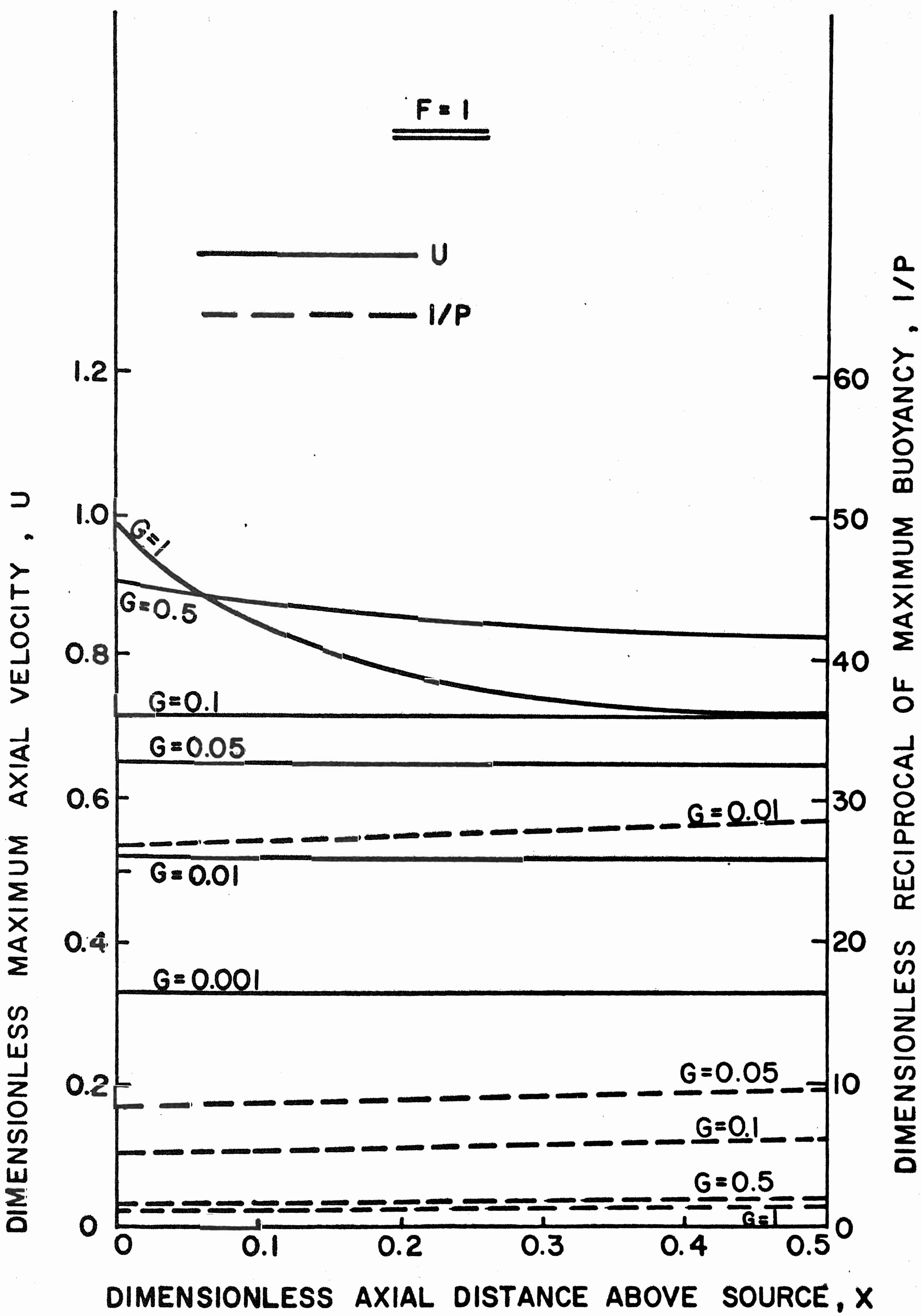

FIGURE 2-4. RESULTS OF MAXIMUM AXIAL VELOCITY AND MAXIMUM BUOYANCY (F = 1)

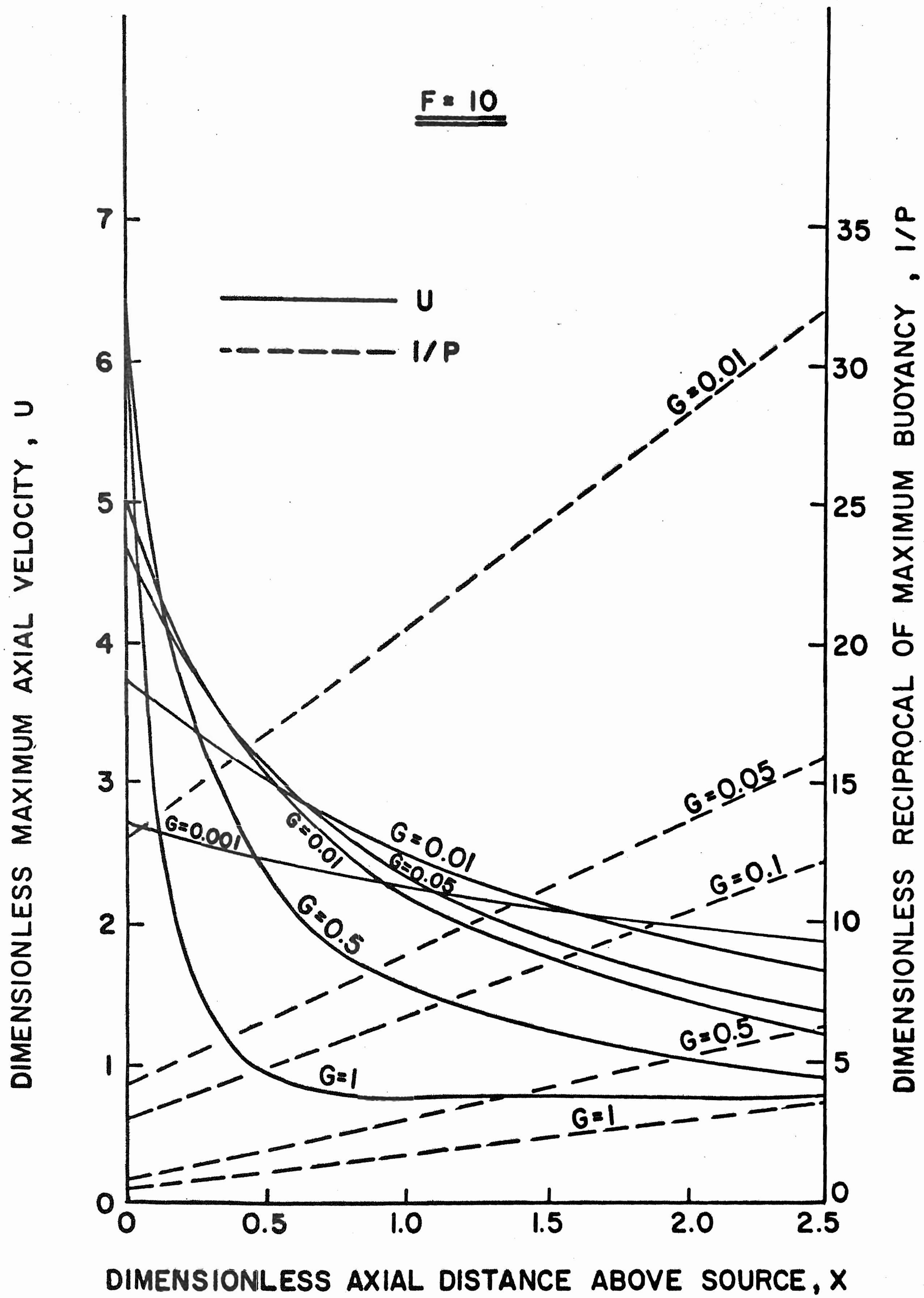

FIGURE 2-5. RESULTS OF MAXIMUM AXIAL VELOCITY AND MAXIMUM BUOYANCY (F=10)

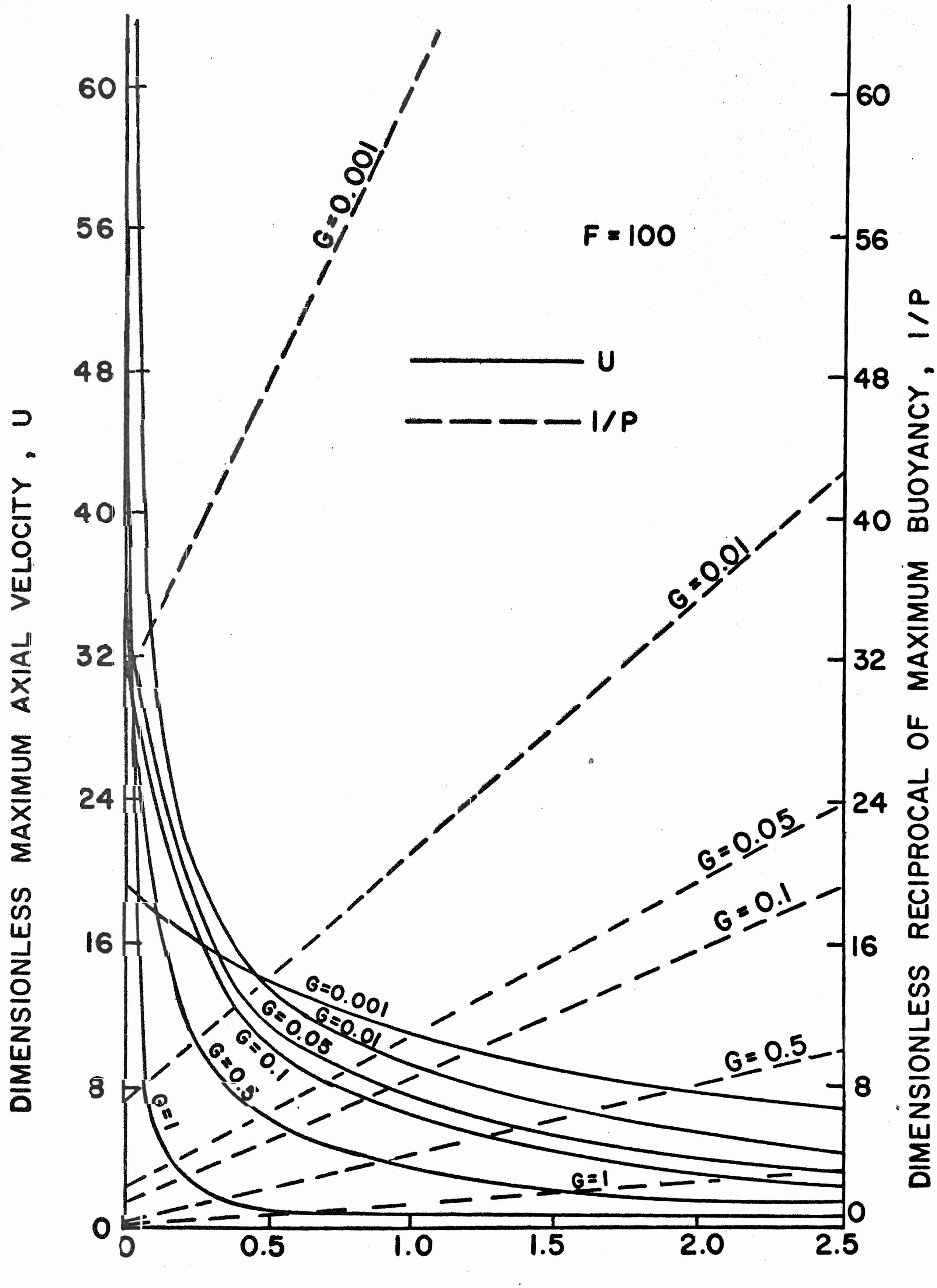

FIGURE 2-6. RESULTS OF MAXIMUM AXIAL VELOCITY AND MAXIMUM BUOYANCY (F = 100)

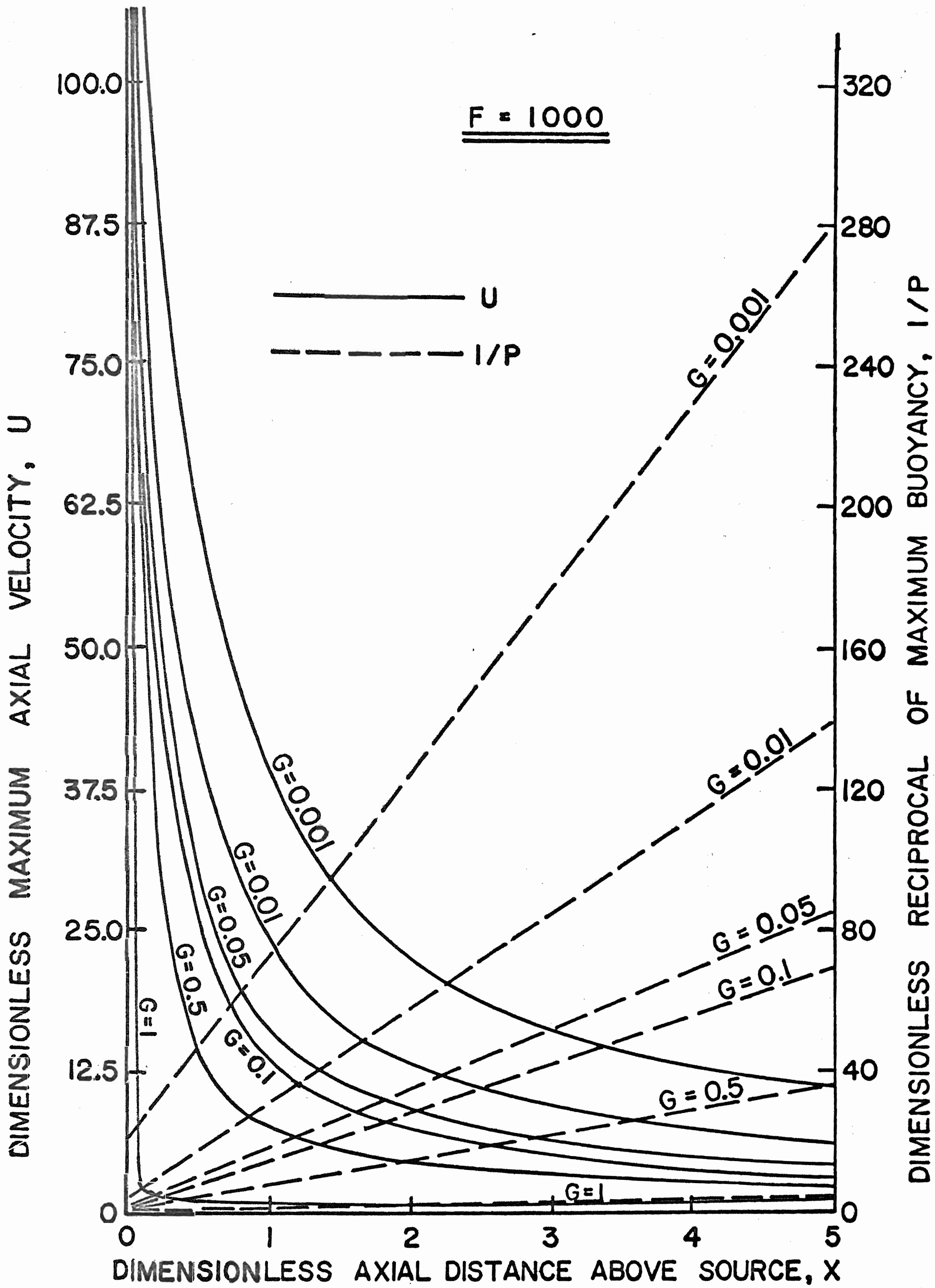

FIGURE 2-7. RESULTS OF MAXIMUM AXIAL VELOCITY AND MAXIMUM BUOYANCY (F=1000)

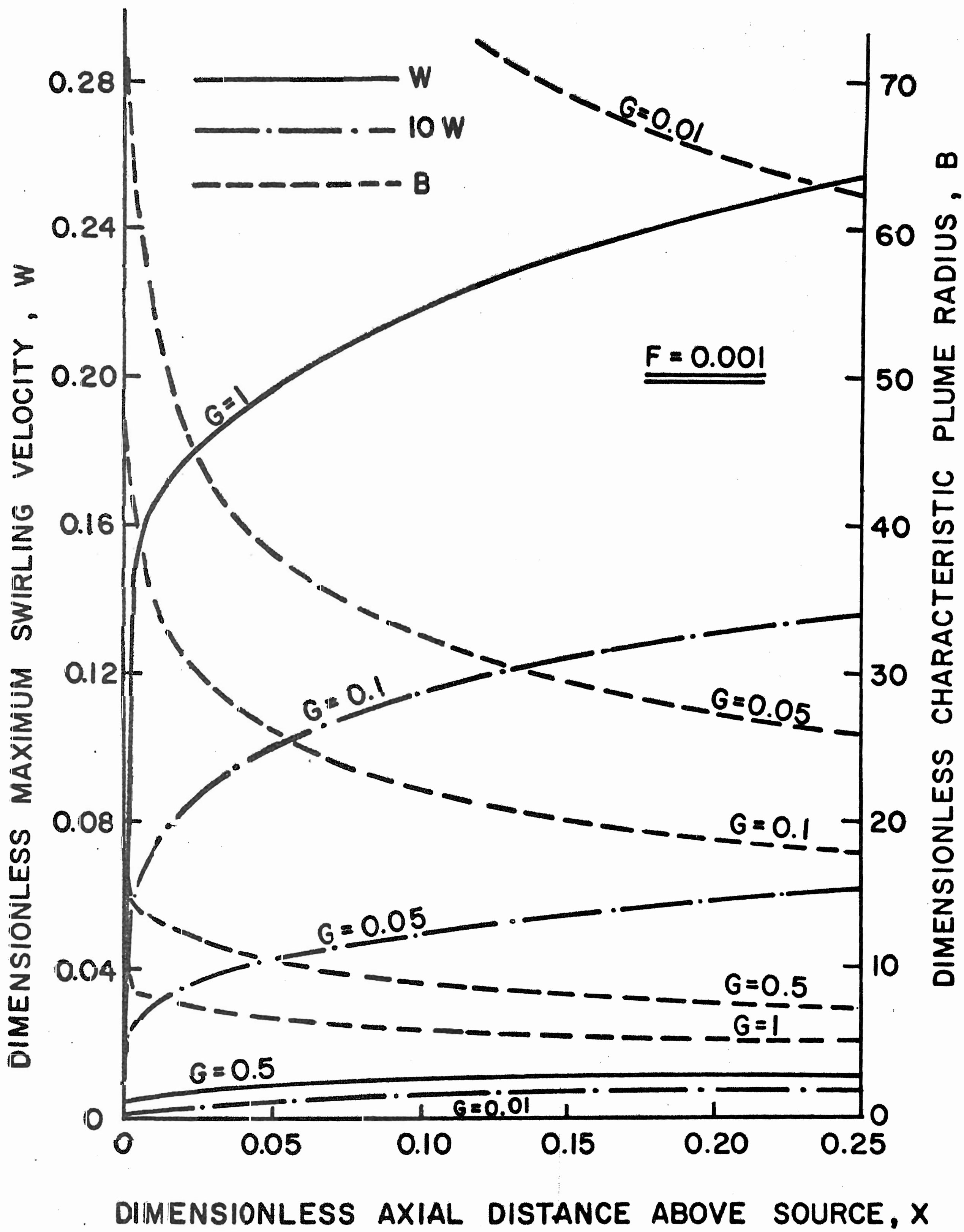

FIGURE 3-1. RESULTS OF MAXIMUM SWIRLING VELOCITY AND CHARACTERISTIC PLUME RADIUS (F=0.001)

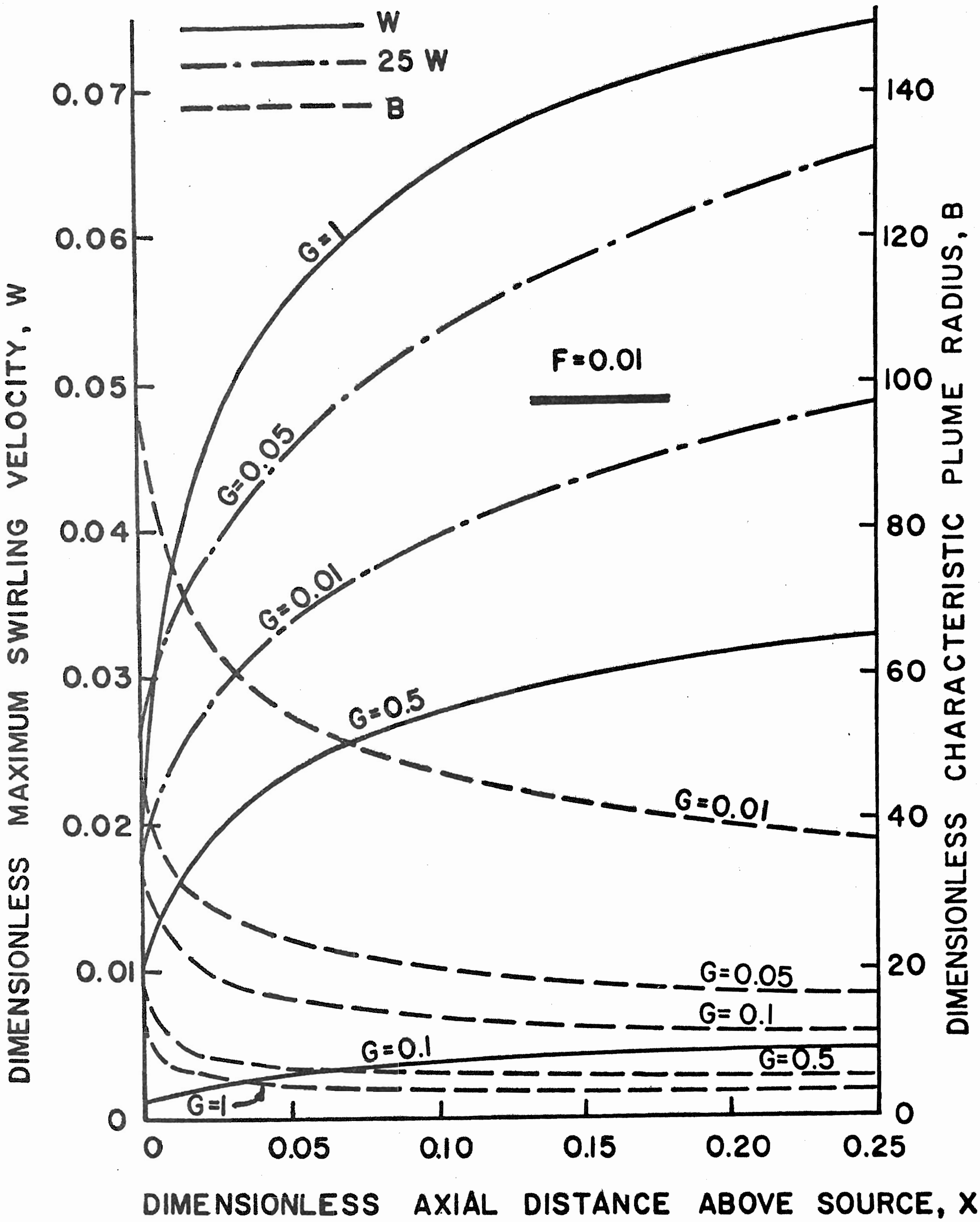

FIGURE 3-2. RESULTS OF MAXIMUM SWIRLING VELOCITY AND CHARACTERISTIC PLUME RADIUS (F=0.01)

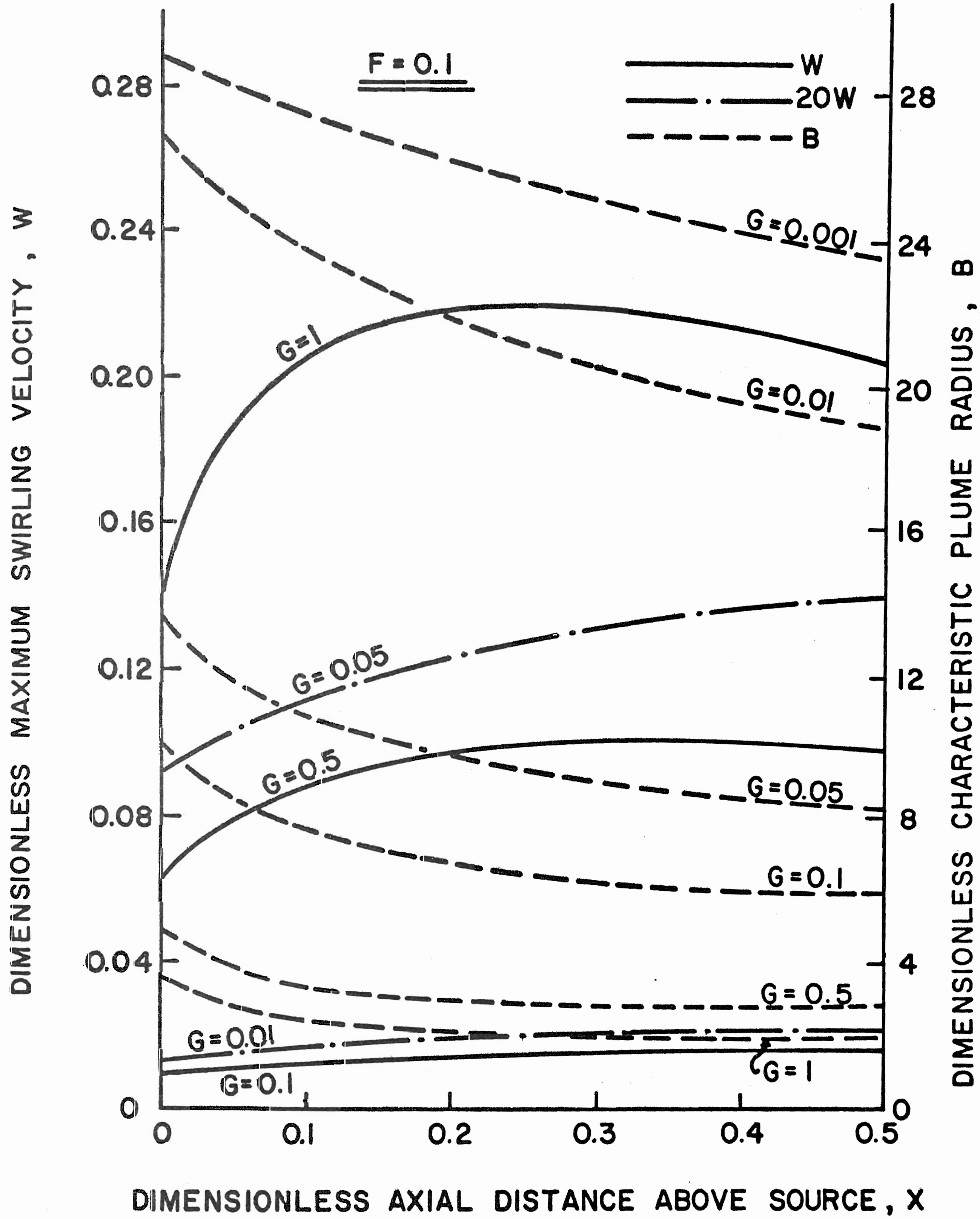

FIGURE 3-3. RESULTS OF MAXIMUM SWIRLING VELOCITY AND CHARACTERISTIC PLUME RADIUS (F = 0.1)

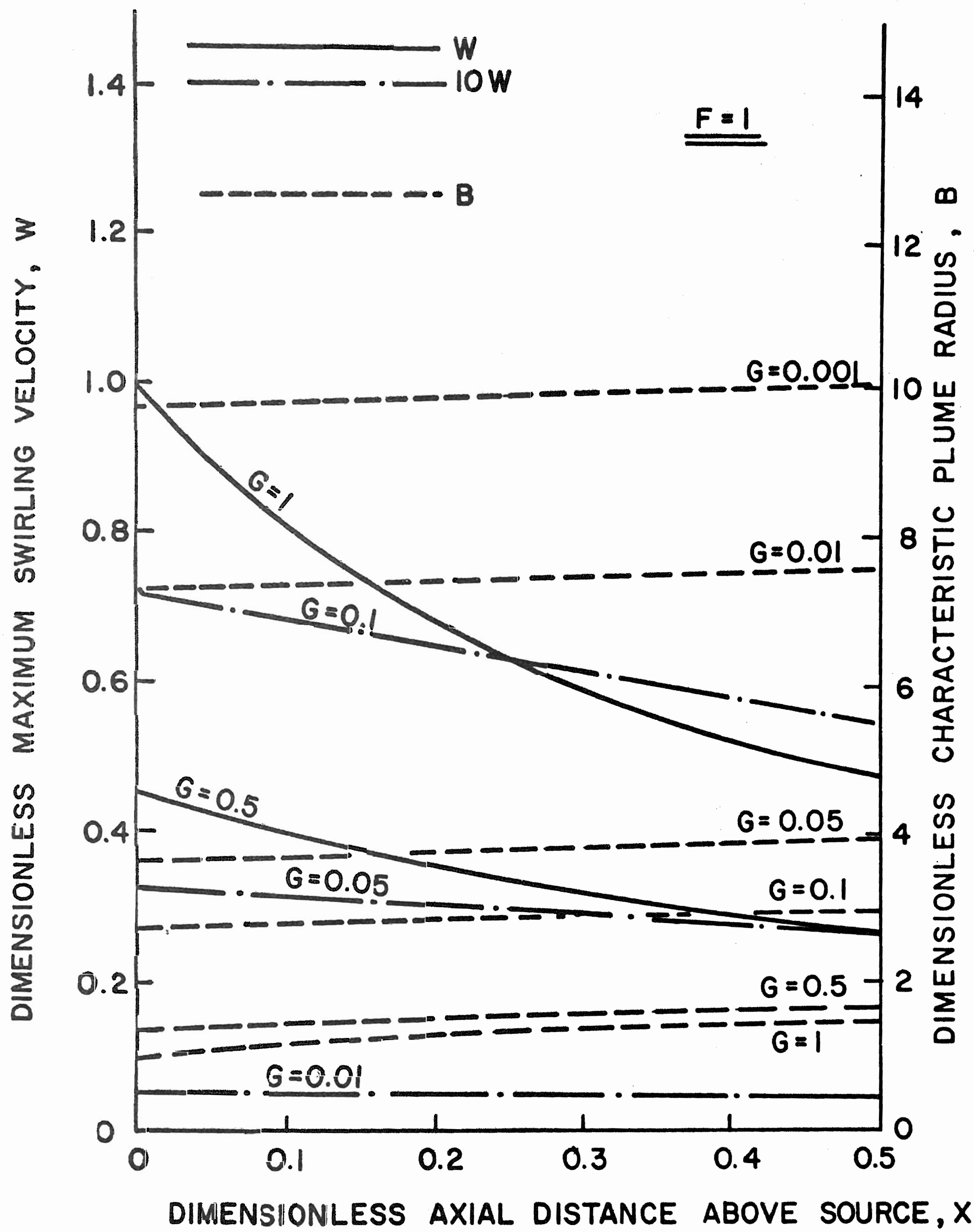

FIGURE 3-4. RESULTS OF MAXIMUM SWIRLING VELOCITY AND CHARACTERISTIC PLUME RADIUS (F = 1)

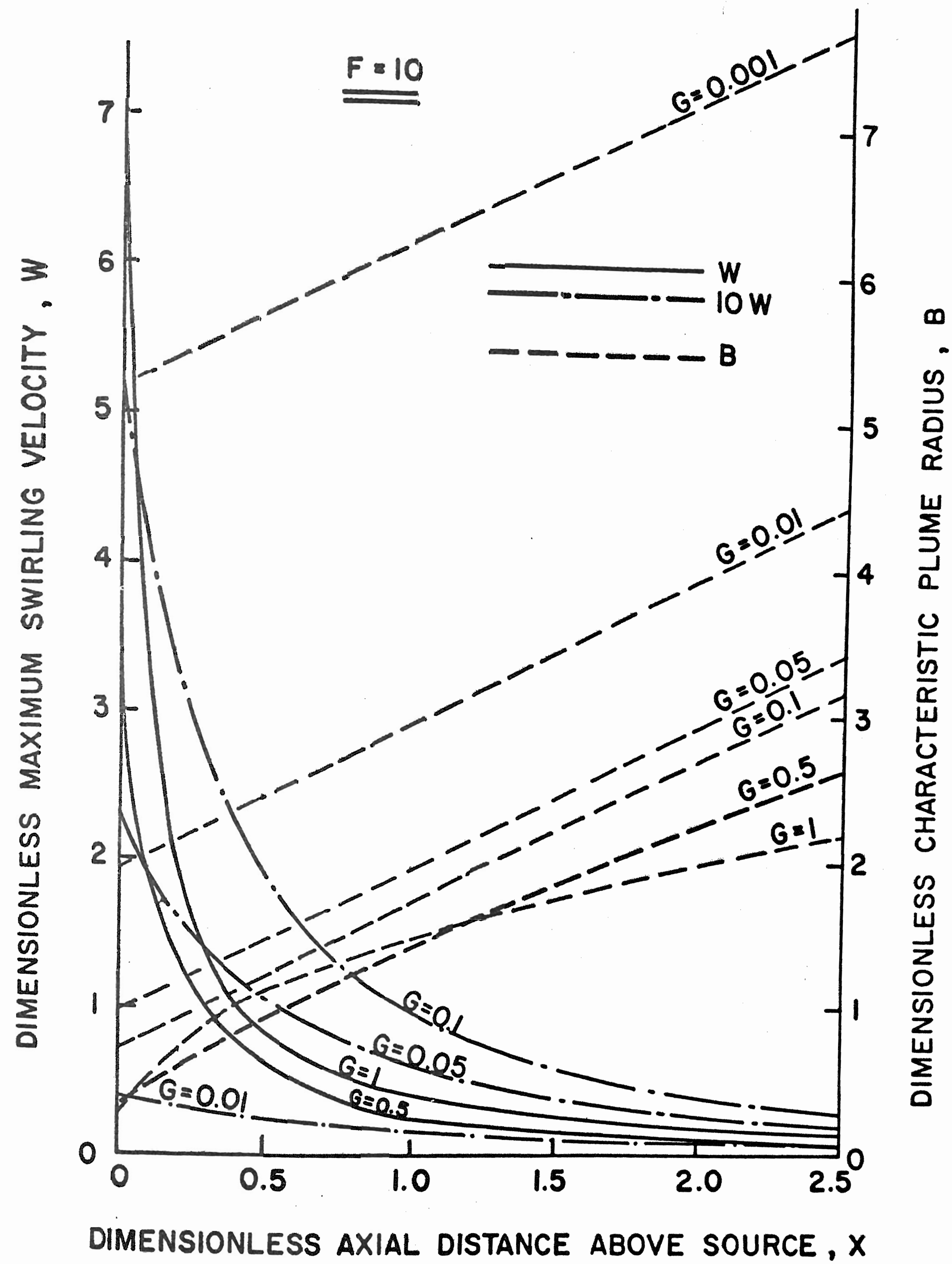

FIGURE 3-5. RESULTS OF MAXIMUM SWIRLING VELOCITY AND CHARACTERISTIC PLUME RADIUS (F = 10)

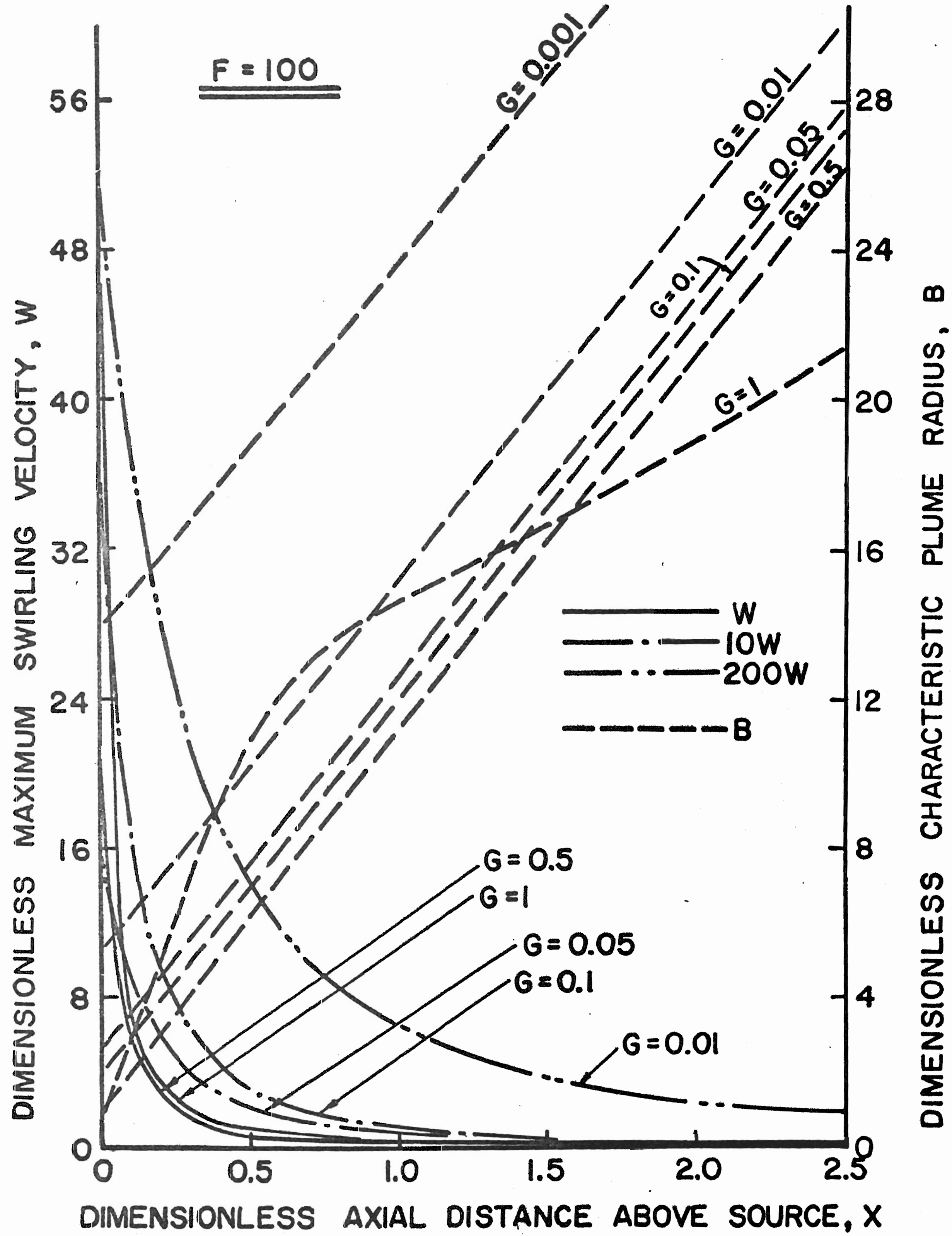

FIGURE 3-6. RESULTS OF MAXIMUM SWIRLING VELOCITY AND CHARACTERISTIC PLUME RADIUS (F=100)

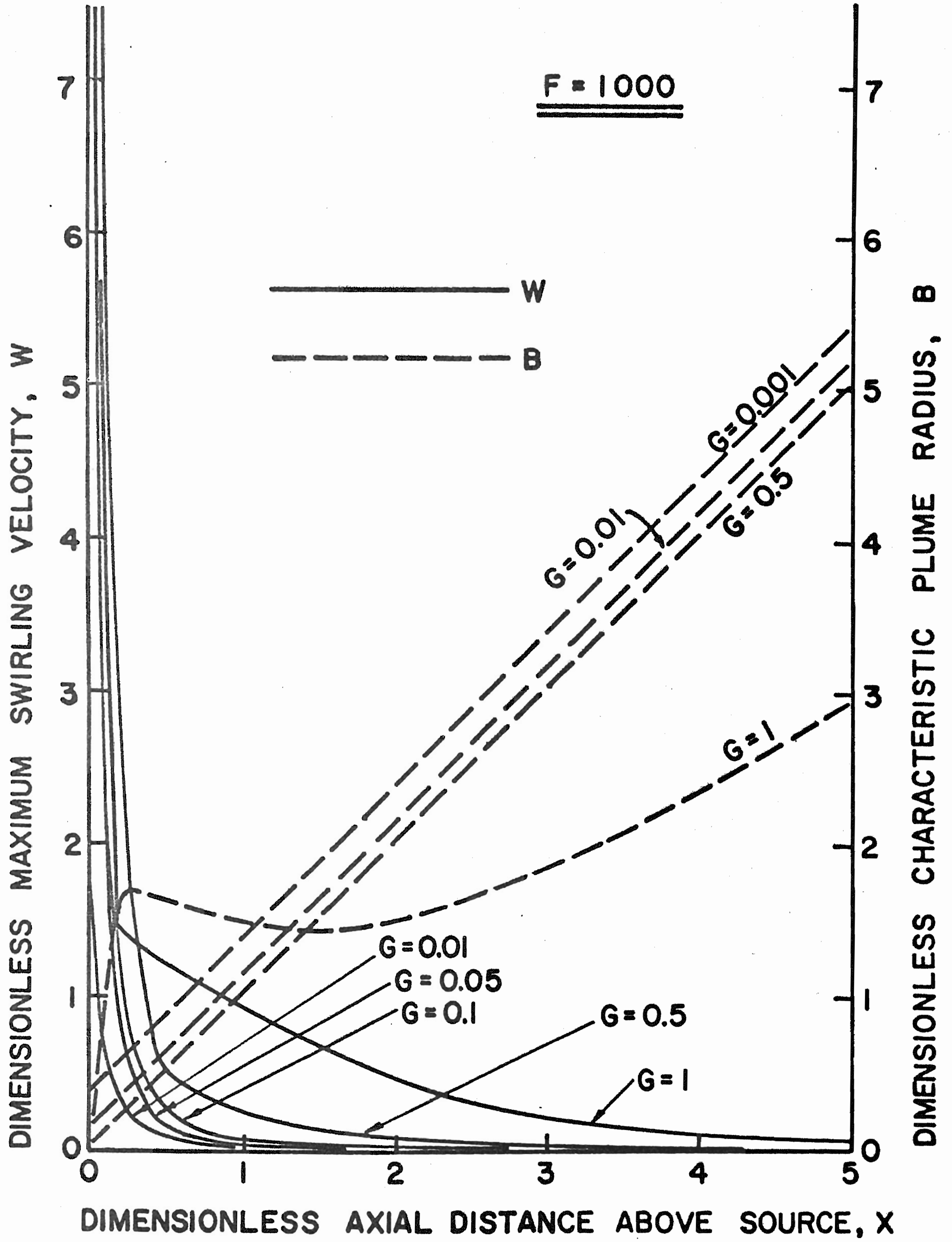

FIGURE 3-7. RESULTS OF MAXIMUM SWIRLING VELOCITY AND CHARACTERISTIC PLUME RADIUS (F = 1000)