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ANALYSIS AND SYNTHESIS OF APERTURE FIELDS
OF LOG PERIODIC ANTENNAS

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Abstract

This paper establishes the relationship between the variation of field intensity along the surface of a log-periodic (L.P.) antenna and the propagation constant of its uniformly periodic prototype. The prototype is then studied to determine the effect of modifications of the structure on the aperture field of the L.P. antenna. It is proposed that the directivity of an L.P. antenna may be improved by designing it for multi-mode operation with multiple active regions from which radiation occurs.

In order to evaluate the coupling of modes in the active regions of an L.P. dipole array a simple coupled mode periodic transmission line model has been employed. Between active regions the characteristic impedances of the two dominant modes are badly mismatched and little coupling occurs. In an active region the modes are strongly coupled, and the resulting perturbation of the propagation constants is evaluated by coupled-mode theory.

Finally, the effect of variation of several parameters of the structure to bring about desired modifications of the dispersion curve is investigated.

*The research reported here was supported by Air Force Cambridge Research Laboratories, Office of Aerospace Research, USAF, under Contract No. AF 19(628)-4144.

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1.0 Introduction

Since the discovery of the log-periodic antenna principle by DuHamel in 1955 (DuHamel and Isbell, 1957) these novel devices have been intensively investigated until today the principles of their operation are quite well understood and adequate design criteria have been established. A summary of the development of log-periodic antennas is given in a recent review article by Jordan, Deschamps, Dyson and Mayes (1964). As these authors have pointed out, one of the challenging questions which remains to be answered is whether or not it is possible to design log-periodic or other frequency-independent antennas to have very high gain. It is basically this question which has prompted the research reported here.

If we try to discover the reason that the directivity of a log-periodic antenna is so limited, it becomes apparent that it is due to the fact that only a small portion of the antenna is effective in producing radiation at any specified frequency. This so-called "active region" of the antenna is limited to those elements whose characteristic dimensions put their resonant frequencies near the frequency of operation. Some improvement in the directivity may be achieved by lengthening the active region of a successful design through increasing the scaling parameter τ to nearly unity. However, there is a limit as to how much the length of the active region can be increased in

this way. Since a log-periodic antenna must operate in a backfire mode, energy which is radiated from the end of the active region nearest the apex of the structure never illuminates the rest of the antenna. Thus the amplitude of the current distribution in the active region is tapered rapidly away from the feed end, further limiting the effective length.

An alternate proposal for increasing the effective aperture length of a log-periodic antenna is to operate the antenna so that multiple active regions are effective in producing radiation at each frequency. It has already been demonstrated (Mayes, 1963) that backfire radiation can be obtained from active regions in which the element lengths correspond to higher order resonances. However, it has not been determined whether or not it is possible to permit part of the energy to pass the initial active region, so as to excite additional radiating regions, without destroying the frequency independent character of the antenna.

This paper treats several topics which have bearing on the question just raised, as well as contributing to a better understanding of log-periodic antennas in general.

1.1 Aperture Field Analysis by Means of Dispersion Curves

It has been recognized for some time that the field intensity variation along a log-periodic (L.P.) structure can be analyzed by reference to a dispersion curve similar to that for a periodic structure. For a uniformly periodic structure, as shown schematically

in Fig. 1, the dispersion curve is a plot of the complex propagation constant, $\gamma = \alpha + j\beta$, versus the free-space wavenumber, k_0 . The propagation constant and wavenumber can be normalized to the length d of each unit cell, so that γd is plotted against $k_0 d$.

For an L.P. structure, as shown in Fig. 2, the cell length varies with the distance z_n from the apex of the structure to the center of each cell according to

$$d_n = z_n \frac{1 - \tau}{\tau^{1/2}}$$

Thus the plot of γd vs. $k_0 d$ for an L.P. structure is a plot of the propagation constant per cell vs. electrical cell length at constant frequency. It has been postulated (Mitra, 1962; Oliner, 1963) that this curve for an L.P. structure should be very nearly identical to the dispersion curve of the prototype periodic structure. The prototype is a uniformly periodic structure from which the given L.P. geometry can be generated by scaling the linear dimensions of each succeeding cell by the scaling factor τ . Since the immediate environment of each cell of the L.P. structure differs very little from that of a cell of the prototype structure of the same electrical length, it is reasonable to suppose that the local fields, and therefore the complex phase shift per cell, will be nearly the same for the two structures.

With this application in mind, Mitra and Jones (1964) have investigated in detail the dispersion curve of a uniformly periodic dipole array, each of whose elements is connected across a uniform

transmission line. This type of structure provides a good model for analysis, and its behavior is typical of that of the class of structures which are periodic prototypes of successful L.P. antennas. Without reviewing the paper of Mittra and Jones, we may summarize some of their conclusions. a) The dispersion curve depends on the local fields to such an extent that inclusion of only the mutual impedance between nearest neighboring dipoles provided good agreement with experiment. b) Unlike the case of a closed periodic structure which exhibits stop bands and pass bands, a complex propagating wave can exist on the dipole-loaded transmission line at all frequencies. c) Radiation occurs when the length of the dipole elements approaches one-half wavelength, and is not necessarily accompanied by "fast" wave propagation ($|\beta_n| < k_0$ for some space harmonic) as was originally predicted by Oliner (1963) from heuristic arguments.

We now turn our attention to three fundamental questions which are of great importance in extending the theory of uniformly periodic structures to apply to L.P. structures. These are

- 1) What is the effect on the L.P. dispersion curve of perturbing the periodic boundary conditions,
- 2) What is the behavior in the vicinity of the resonant elements, i.e. in the active region of an antenna, and
- 3) can the dispersion relation be controlled by design to make possible L.P. antennas with higher directivity than is presently available?

2.0 Mode Coupling on an L.P. Structure

It has been shown (Kieburz, 1965) that simple integration of the dispersion relation for the periodic prototype provides a phase-integral approximation for the complex phase variation along an L.P. structure which is valid when certain conditions are fulfilled. One of these conditions is that the rate of change of the propagation constant with distance is not too great,

$$\frac{d}{dz} \frac{\gamma d}{2} \ll \frac{2\pi}{d} \quad (1)$$

Then the complex phase shift along the L.P. structure is approximately given by

$$\phi(z) = j \int_0^z \gamma(z') dz'$$

Upon inspection of the dispersion curves for a periodic dipole array, given in Fig. 3, it is seen that the condition (1) is violated in the neighborhood of the dipole resonances. Since these are also the regions of the diagram for which the radiation from a periodic array is greatest, these will correspond to the active regions of the L.P. antenna.

In Fig. 3 are displayed the dispersion curves for two modes of the periodic dipole array. These are obtained by solving the approximate dispersion relation

$$\cosh \gamma d = \cos \beta_0 d + j \frac{Z_0 \sin \beta_0 d}{2Z_{11} - 4Z_{12} \cosh \gamma d} \quad (2)$$

in which

- γ is the complex propagation constant for the voltage along the period dipole array,
- β_0 is the phase constant of the unloaded transmission line,
- Z_0 is its characteristic impedance,
- Z_{11} is the self impedance of each dipole, and
- Z_{12} is the mutual impedance between nearest neighboring dipole elements.

The minus sign in front of Z_{12} is introduced to account for the reversal of the phase of excitation between neighboring elements. Formulas for Z_{11} and Z_{12} are given by Uda and Mushiake (1954). Curves similar to Fig. 3 have been given by Mittra and Jones (1964) except that those authors have shown only the propagation constant for the least-attenuated mode at each frequency. It is this mode which will be measured on a semi-infinite periodic structure at a distance of several wavelengths from the point of excitation.

On an L.P. structure which is excited at one end, it may be expected that a single one of the modes (labeled 1 and 2 in Fig. 3) once excited will perpetuate itself in propagating along the structure, unless conditions are favorable for a coupling of the two modes. The most likely regions of the L.P. structure for coupling to occur, if at all, are the active regions, where the corresponding complex values of γ_1 and γ_2 for the periodic prototype are nearly equal at some frequency.

There will be an important difference in the operation of the L.P. antenna, depending on whether sufficiently strong mode coupling

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occurs or not. If no coupling occurs, the portion of the antenna beyond the first active region will operate in a highly attenuated mode analogous to a cutoff mode or stop band of a closed periodic structure. It has been postulated (Jordan, Deschamps, Dyson, and Mayes, 1964) that this type of behavior, in which energy is reflected from the region just beyond the active region, is characteristic of at least many of the frequency-independent L.P. antennas. However, if mode coupling does occur, then any energy which is not radiated in the first active region will propagate beyond it in a mode of low attenuation until the wave encounters either the end of the antenna or a second active region. It is postulated here that this second type of behavior, in which mode coupling occurs at the active regions, is characteristic of most of the L.P. dipole arrays and probably many other L.P. antennas as well.

2.1 Coupled Mode Theory for Aperiodic Structures

In order to evaluate the effect of mode coupling on an L.P. structure we shall employ coupled-mode theory, first developed by Pierce (1954). The modes we shall consider will be those of the periodic dipole array. Coupling occurs because the boundary conditions on the fields at the ends of a cell of the L.P. structure deviate from those of the uniformly periodic prototype. Ideally, these boundary conditions should insure continuity of the electric and magnetic field components transverse to an entire plane perpendicular to the axis of the structure. However, it is recognized that the power propagating from one cell to the next away from the feed end of the structure will be carried principally along the two-wire

transmission line. Therefore, as an approximation to the actual boundary conditions on the total fields, we shall impose the impedance boundary condition on the transmission line joining two cells.

The transmission line representation of the L.P. dipole array is as shown in Fig. 4. Suppose the field incident from the left is entirely that of mode 1 of the periodic prototype. Due to the change in iterative impedance in passing from cell C to cell C' we must assume that the fields in cell C' are now a linear combination of the fields of the two modes. Thus

$$V_1(d) = \sigma_{11} V_1^i(0) + \delta_{12} V_2^i(0) \quad (3)$$

$$I_1(d) = \sigma_{11} I_1^i(0) + \delta_{12} I_2^i(0) \quad (4)$$

With the iterative impedances defined at the cell boundaries as

$$Z_i = \frac{V_i(0)}{I_i(0)} = \frac{V_i(d)}{I_i(d)}, \quad i = 1, 2 \quad (5)$$

equation (3) becomes

$$Z_1 I_1(d) = \sigma_{11} Z_1^i I_1^i(0) + \sigma_{12} Z_2^i I_2^i(0) \quad (6)$$

The impedances Z_1 and Z_1^i may be expected to differ since the electrical dimensions of cells C and C' differ by a factor of τ . If the modal voltages and currents are normalized to unit power,

$$\text{Re}[V_i I_i^*] = 1,$$

then (4) and (6) can be solved simultaneously for the modal transmission and modal coupling coefficients,

$$\sigma_{11} = \frac{Z_2^i - Z_1}{Z_2^i - Z_1^i} \frac{\text{Re}[Z_1^i]}{\text{Re}[Z_1]} \quad 1/2 \quad (7)$$

$$\delta_{12} = \frac{Z_1' - Z_1}{Z_1' - Z_2'} \frac{\text{Re}[Z_2']^{1/2}}{\text{Re}[Z_1']} \quad (8)$$

Similarly, the modal transmission coefficient σ_{22} for mode 2 and the coefficient of coupling from mode 2 to mode 1 are found to be

$$\sigma_{22} = \frac{Z_1' - Z_2}{Z_1' - Z_2'} \frac{\text{Re}[Z_2']^{1/2}}{\text{Re}[Z_2']} \quad (9)$$

and

$$\delta_{21} = \frac{Z_2' - Z_2}{Z_2' - Z_1'} \frac{\text{Re}[Z_1']^{1/2}}{\text{Re}[Z_2']} \quad (10)$$

The modal currents in the two cells may then be related by a coupling matrix

$$\begin{bmatrix} I_1(d) \\ I_2(d) \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \delta_{12} \\ \delta_{21} & \sigma_{22} \end{bmatrix} \begin{bmatrix} I_1'(0) \\ I_2'(0) \end{bmatrix} \quad (11)$$

It is easily verified that power is conserved under this transformation.

To employ coupled mode theory, we now relate the fields at the input end of each cell, requiring that a certain linear combination of these modal currents, which constitutes a new mode of the coupled structure, be related by a constant

$$\begin{bmatrix} I_1(0) \\ I_2(0) \end{bmatrix} = e^{\gamma d} \begin{bmatrix} I_1'(0) \\ I_2'(0) \end{bmatrix} \quad (12)$$

Since for the original modes, the currents at the two ends of each cell are related by a propagation constant

$$I_i(d) = e^{-\gamma_i d} I_i(0) \quad i = 1, 2. \quad (13)$$

We combine (11), (12) and (13) to get

$$e^{\gamma d} \begin{bmatrix} I_1^{\nu}(0)e^{\gamma_1 d} \\ I_2^{\nu}(0)e^{\gamma_2 d} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \delta_{12} \\ \delta_{21} & \sigma_{22} \end{bmatrix} \begin{bmatrix} I_1^{\nu}(0) \\ I_1^{\nu}(0) \end{bmatrix} \quad (14)$$

From (14) is obtained the determinantal equation for the propagation factor $e^{\gamma d}$,

$$e^{2\gamma d} - (\sigma_{11}e^{\gamma_1 d} + \sigma_{22}e^{\gamma_2 d})e^{\gamma d} + \Delta e^{(\gamma_1 + \gamma_2)d} = 0 \quad (15)$$

where Δ is the determinant of the coupling matrix. Solving (15), we obtain the propagation constants for the new modes of cell C of the L.P. structure,

$$\gamma d = \log_e \left\{ \frac{1}{2}(\sigma_{11}e^{\gamma_1 d} + \sigma_{22}e^{\gamma_2 d}) \pm \frac{1}{4}(\sigma_{11}e^{\gamma_1 d} + \sigma_{22}e^{\gamma_2 d})^2 - e^{(\gamma_1 + \gamma_2)d} \right\} \quad (16)$$

2.2 Coupled Modes on the L.P. Dipole Antenna

The propagation constants obtained from (16) are plotted in the L.P. dispersion curve of Fig. 5 for a value of the scaling parameter $\tau = 0.95$. In Figs. 6, 7 and 8 the active region in which coupling occurs is shown on an expanded scale for $\tau = 0.90, 0.95$ and 0.990 . It is seen that for $\tau = 0.90$ and 0.95 , coupled mode theory predicts a propagation constant which in the active region makes a transition between the values for modes 1 and 2 of the prototype structure. Thus a wave propagating on the L.P. structure is strongly attenuated in the active region due to energy spent in backfire radiation, but does not encounter a cutoff region beyond. Instead, energy which remains with the structure past the active region will be propagated with little loss up to the next

active region of the antenna. This means also that for an antenna with only one active region, frequency-independent operation is achieved only when the coupling to radiating elements is so strong that virtually no energy passes the active region.

There is another feature of the results of applying coupled-mode theory which is worth mentioning here. Notice from Figs. 6-8 that the attenuation constant jumps suddenly from a small to a large value at the beginning of the active region. The ramification of this fact in antenna design is that it does not appear to be possible to design an L.P. dipole antenna having a long active region in which the attenuation constant per cell is initially small and increases slowly. Thus the electrical length of the radiating aperture necessary to achieve high gain is apparently not to be achieved with a single active region.

3.0 Aperture Field Synthesis

Let us now direct our attention to possible means of controlling the amplitude and phase variations of the fields along an L.P. antenna. It is not necessary to examine directly the currents on the antenna elements; instead one can determine the relative intensity of radiation which occurs from each region of the antenna from the attenuation constant for the voltage on the transmission line. The relative phase shift can also be found from the phase constant for the voltage. Each active region of the antenna may be regarded as a separate element having a pattern whose maximum is in the backfire direction.*

The relative phase shift between active regions may be controlled by varying the apex angle of the antenna. For instance, when $\tan \alpha/2 = 1/N$, where n is an integer the phase shift between active regions will be very nearly $2N\pi$ radians. If $\tan \alpha/2 = 1/(N + \frac{1}{2})$ the phase shift will be $(2N + 1)\pi$ radians. Since the broadside pattern factors of dipoles operating at successive resonances alternate in sign, the first condition $\tan \alpha/2 = 1/N$ will favor backfire radiation, while the condition $\tan \alpha/2 = 1/(N + \frac{1}{2})$ favors broadside radiation.

The relative power radiated by each active region can be estimated by integrating the attenuation constant over the length of the region,

$$A_n = \int_{D_n} \text{Re}[\gamma_n(Z)]$$

As an approximation to the actual attenuation curves obtained by coupled-mode theory, one can take the attenuation curve formed

*Mayes, (1963) has shown that the pattern maximum of an L.P. antenna whose elements radiate at the $\frac{3\lambda}{2}$ and $\frac{5\lambda}{2}$ dipole resonances can be directed back by inclining the dipole arms into a V.

by the portions of attenuation curves of each of the two modes of the prototype which give the smaller attenuation at each frequency. The problem is then to reduce the relative area under the first attenuation peak in order to permit an appreciable fraction of the energy to pass to the second active region, but to maintain or increase the relative area under the second attenuation peak in order to prevent appreciable energy from reaching the base end of the antenna.

3.1 Effect of Element-to-Transmission Line Impedance Ratio

It may be expected that the relative amount of radiation from an array of nearly resonant dipoles coupled to a transmission line will depend on the coupling network linking the dipoles to the line. Tight coupling of the elements to the feed line should produce a high rate of radiation, hence a high attenuation of the primary wave traveling down the line. Loose coupling may cause more gradual radiation, since the current on each element will be less, and a correspondingly smaller attenuation constant. In the simplest case the dipoles are directly coupled to the transmission line, and the degree of element coupling is determined solely by the ratio of transmission line characteristic impedance to the radiation resistance of each element in the array. In Figs. 9 and 10 are compared the attenuations produced when dipoles with L/ρ ratio of 177 are coupled to transmission lines whose characteristic impedances are 100 ohms and 25 ohms, respectively. As expected, the transmission line of lower impedance couples less energy to each dipole than does the 100-ohm line. The relative powers radiated in

the first active region of each antenna are approximately 0.989 and 0.779 respectively, for $\tau = 0.95$. A value of τ nearer unity would increase the relative power radiated from each active region by lengthening the region. Apparently, then, the amount of energy radiated from each active region can be controlled by varying the element coupling.

3.2 Element Coupling Networks

Since the relative power radiated from each active region can be controlled by properly adjusting the element coupling, it may be possible to control the power radiated from successive active regions of the same antenna by the use of log-periodic coupling networks. There should be reactive networks since it is not desirable to dissipate energy in ohmic losses on the antenna. The network should not introduce additional regions of high attenuation into the dispersion relation, and should produce tighter coupling of elements in successive active regions.

The simplest type of element coupling network which meets these requirements consists simply of a capacitance in series with each dipole element at the point of connection to the transmission line. These capacitances must satisfy the L.P. scaling relation, $C_{n+1} = C_n/\tau$. The capacitive reactance then limits the current amplitude on dipoles at half wave resonance in the first active region, but has less effect in subsequent active regions. In Fig. 11 is the dispersion curve for a periodic dipole array with series capacitive reactive $X_C = \frac{\pi}{2.5 k_0 d}$ and $Z_0 = 100$ ohms. It is seen by comparing Figs. 9 and 11

that the attenuation in the first active region is appreciably reduced by introducing series capacitance. The relative powers radiated from the first and second active regions of an L.P. antenna generated from this periodic prototype will be approximately 0.791 and 0.125 respectively; the relative powers for an antenna having the same parameters except for the capacitance would be approximately 0.989 and 0.0075.

The insertion of suitable coupling networks to give loose coupling of elements in the first active region and tighter coupling in subsequent regions can therefore be used as a means of allowing some fraction of the energy delivered to an L.P. antenna to pass the first active region. In order to achieve frequency-independent operation, however, this energy must be radiated from one or more subsequent active regions before reaching the base end of the antenna. Several possible means of increasing the radiation from the second active region have been investigated. One of the simplest and most promising appears to be to reduce the phase velocity on the transmission line until the second active region occurs at just the point that the propagation constant $\beta_0 d$ for the $n = -1$ space harmonic crosses the $\beta = -k_0$ line. This condition is illustrated in Fig. 12. As a result of this condition the corresponding attenuation peak is greatly broadened. It is interesting to note that the attenuation in the first active region is not significantly altered by this change in the structure in spite of the fact that $\beta > k_0$ for all space harmonics in the vicinity of the first active region; that is, all space harmonics

are slow leaky waves. For an L.P. antenna corresponding to the dispersion curve of Fig. 12, the relative power radiated in the first and second active regions would be 0.989 and 0.010.

If the two modifications of the basic dipole array are now combined, using a transmission line whose phase velocity is 0.44286 with dipoles having series capacitance at the base, the dispersion curve of Fig. 13 results. Here the radiation in the first active region is partially suppressed, while the attenuation peak in the second active region is broadened. The corresponding L.P. antenna would have relative attenuations in the two active regions of 0.804 and 0.156.

4.0 Conclusions

It has been seen that coupled-mode theory affords a means of obtaining the behavior of fields on an L.P. antenna from the dispersion curve for its periodic prototype. The coupled mode calculations indicate that the first active region of an L.P. dipole antenna is followed by another low-loss transmission region, then a second active region, etc. No cutoff region of high attenuation has been found; rather, the attenuation is always associated with an active, radiating region.

The fact that multiple active regions on an L.P. antenna appear to exist raises the possibility that L.P. antennas having higher gain than has previously been available may be developed utilizing

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FIG. 1 UNIFORMLY PERIODIC
DIPOLE ARRAY WITH
PHASES OF ALTERNATE
ELEMENTS REVERSED.

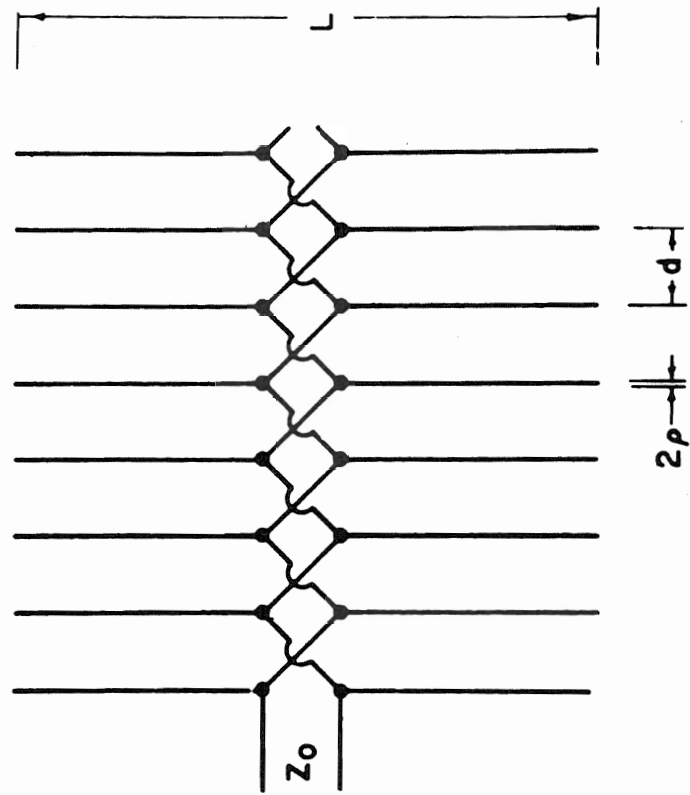


FIG. 2

LOG-PERIODIC DIPOLE
 ARRAY. $\frac{L_n}{d_n}$ IS CONSTANT,
 EQUAL TO L/d OF THE
 PERIODIC PROTOTYPE.

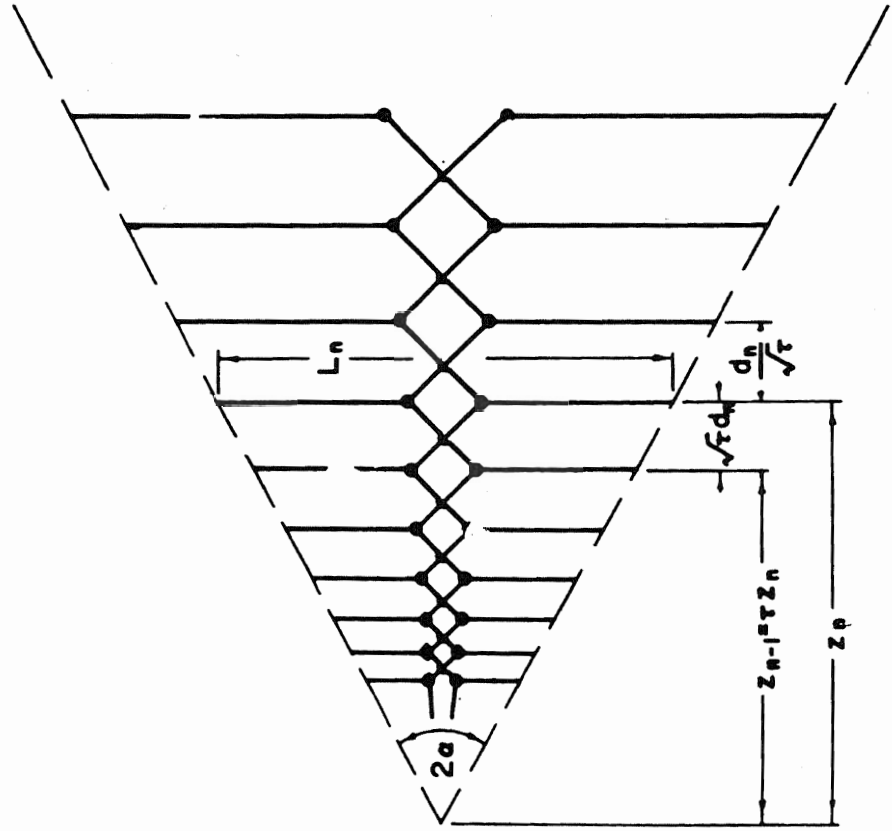


FIG. 3 DISPERSION CURVE FOR A PERIODIC DIPOLE ARRAY.

$Z_0 = 50$ OHMS , $d/L = 0.1$, $L/\rho = 177$

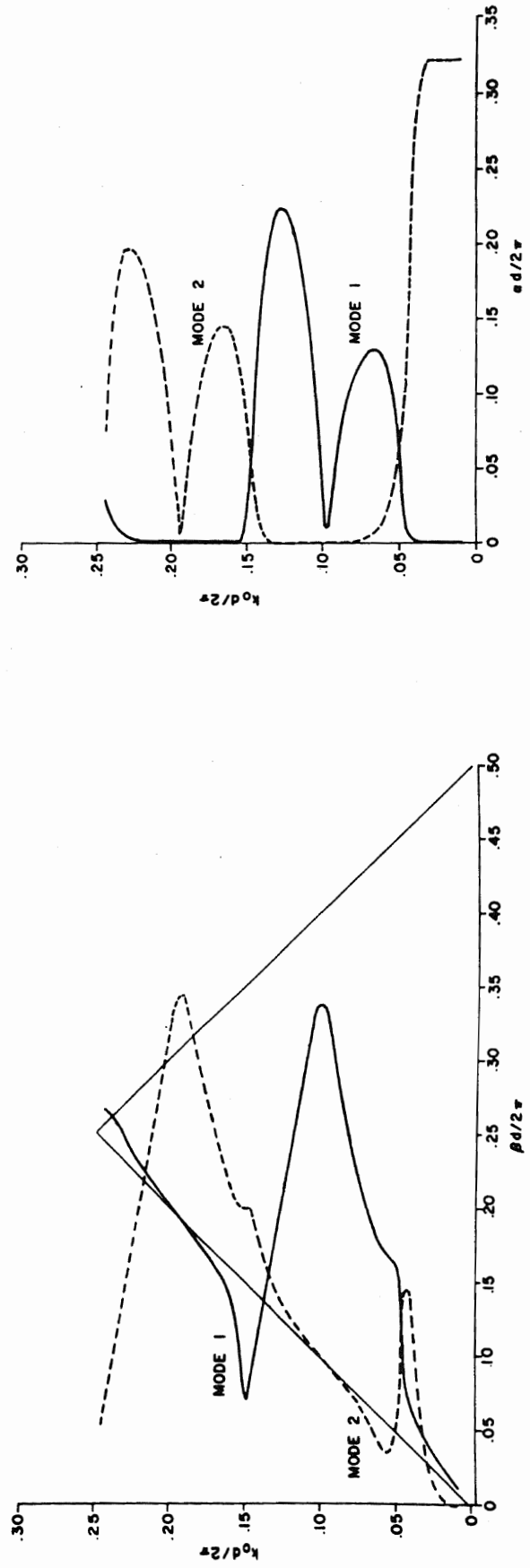
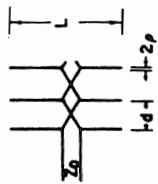


FIG. 4
 TRANSMISSION LINE
 REPRESENTATION OF CELLS
 OF THE L.P. STRUCTURE

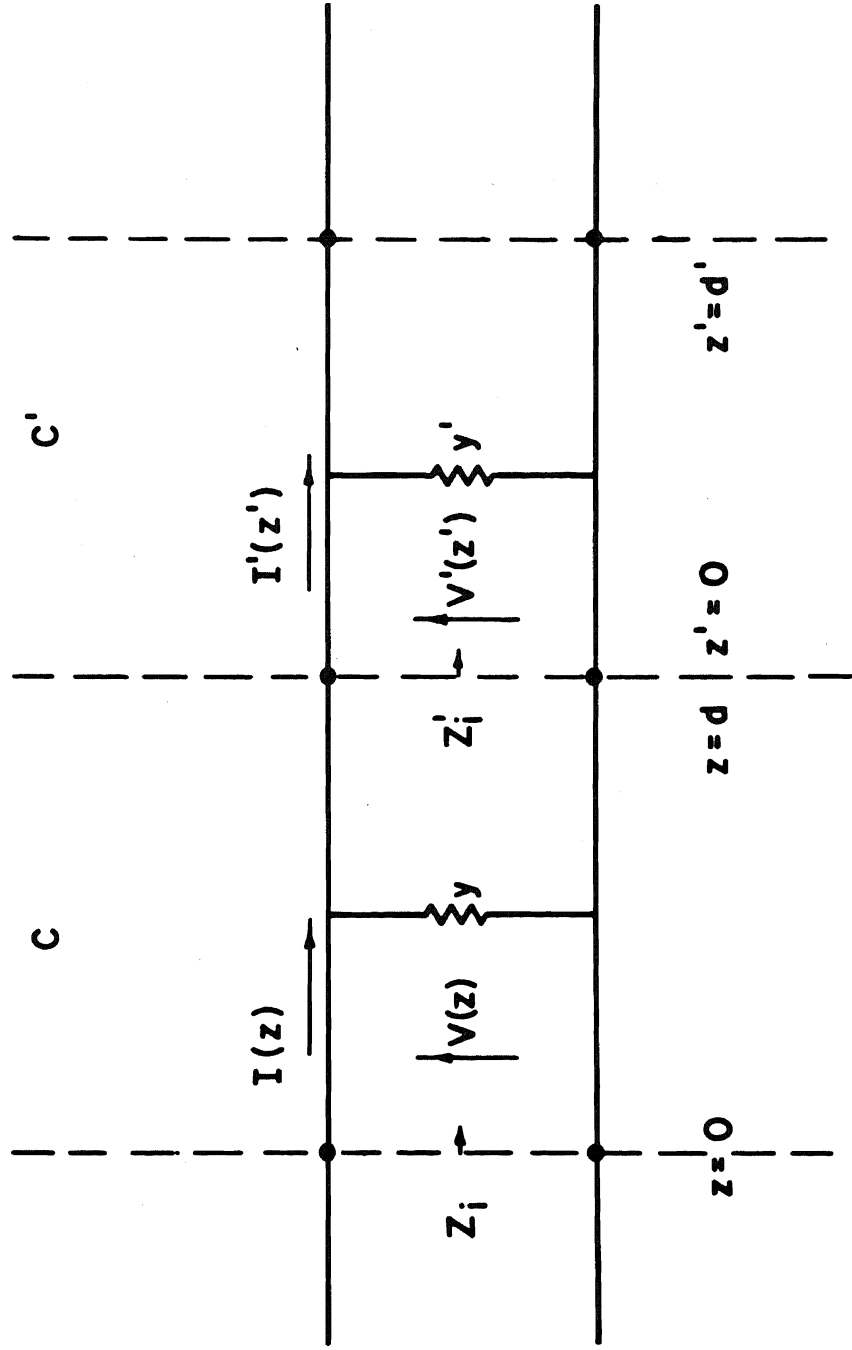


FIG. 5 DISPERSION RELATION FOR LOG-PERIODIC DIPOLE ANTENNA.

$Z_0 = 50$ OHMS

$\tau = 0.95$

$d/L = 0.1$

$L/\rho = 177$

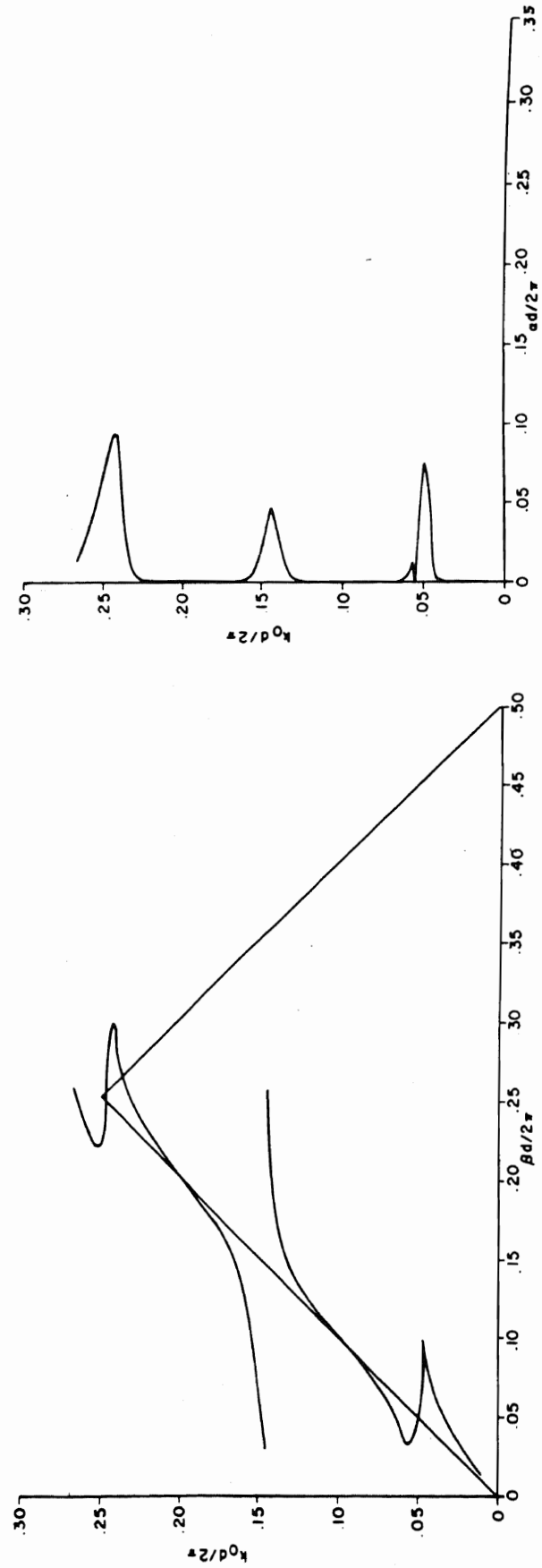


FIG. 6 MODE COUPLING REGION ON THE L.P. ANTENNA.

$Z_0 = 50$ OHMS $r = 0.900$

THE LIGHT LINES ARE THE MODES OF THE PERIODIC PROTOTYPE.
HEAVY LINES INDICATE MODES OF THE L.P. ANTENNA OBTAINED FROM COUPLED MODE THEORY.

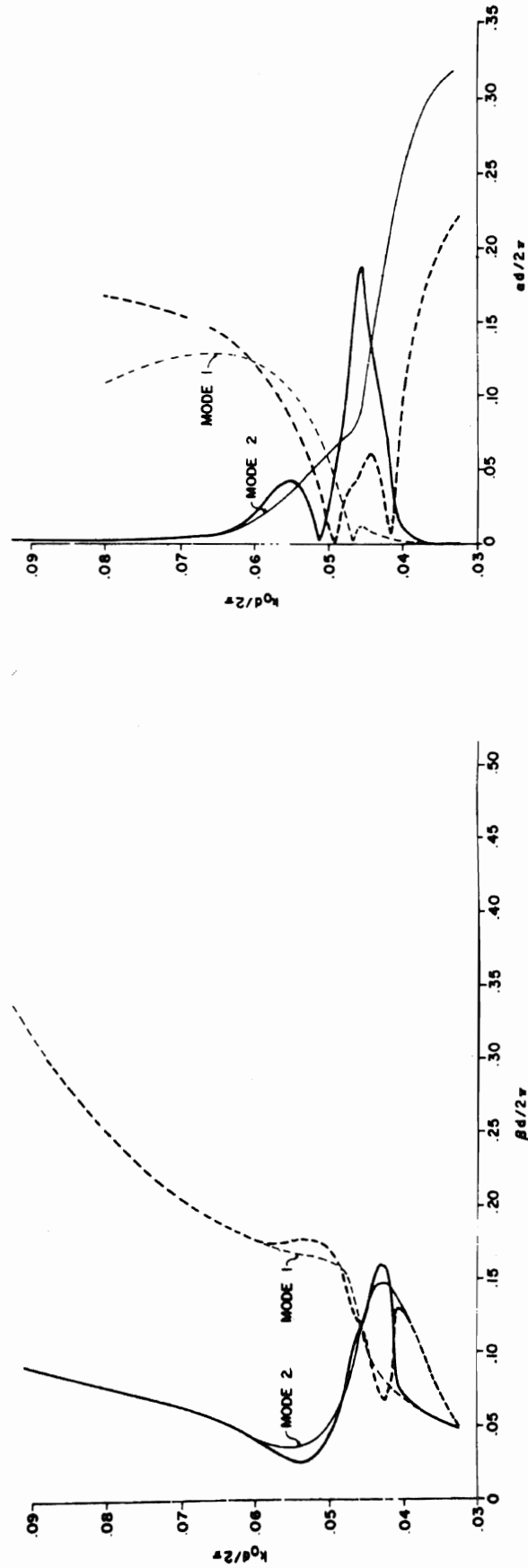


FIG. 7 MODE COUPLING REGION ON THE L.P. ANTENNA.

$Z_0 = 50$ OHMS $r = 0.95$

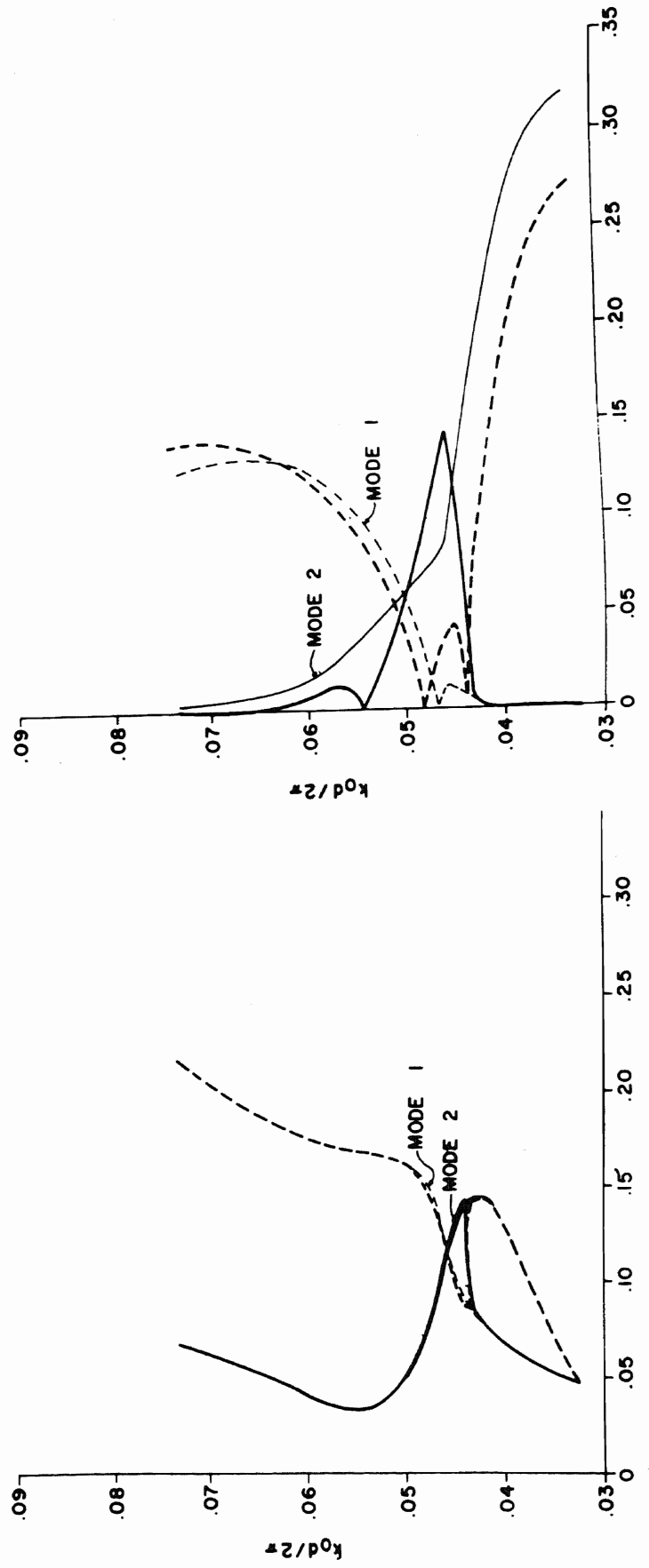


FIG. 8 MODE COUPLING REGION OF L.P. DISPERSION CURVE

$Z_0 = 50$ OHMS $\tau = 0.990$
 $d/L = 0.1$ $L/\rho = .177$

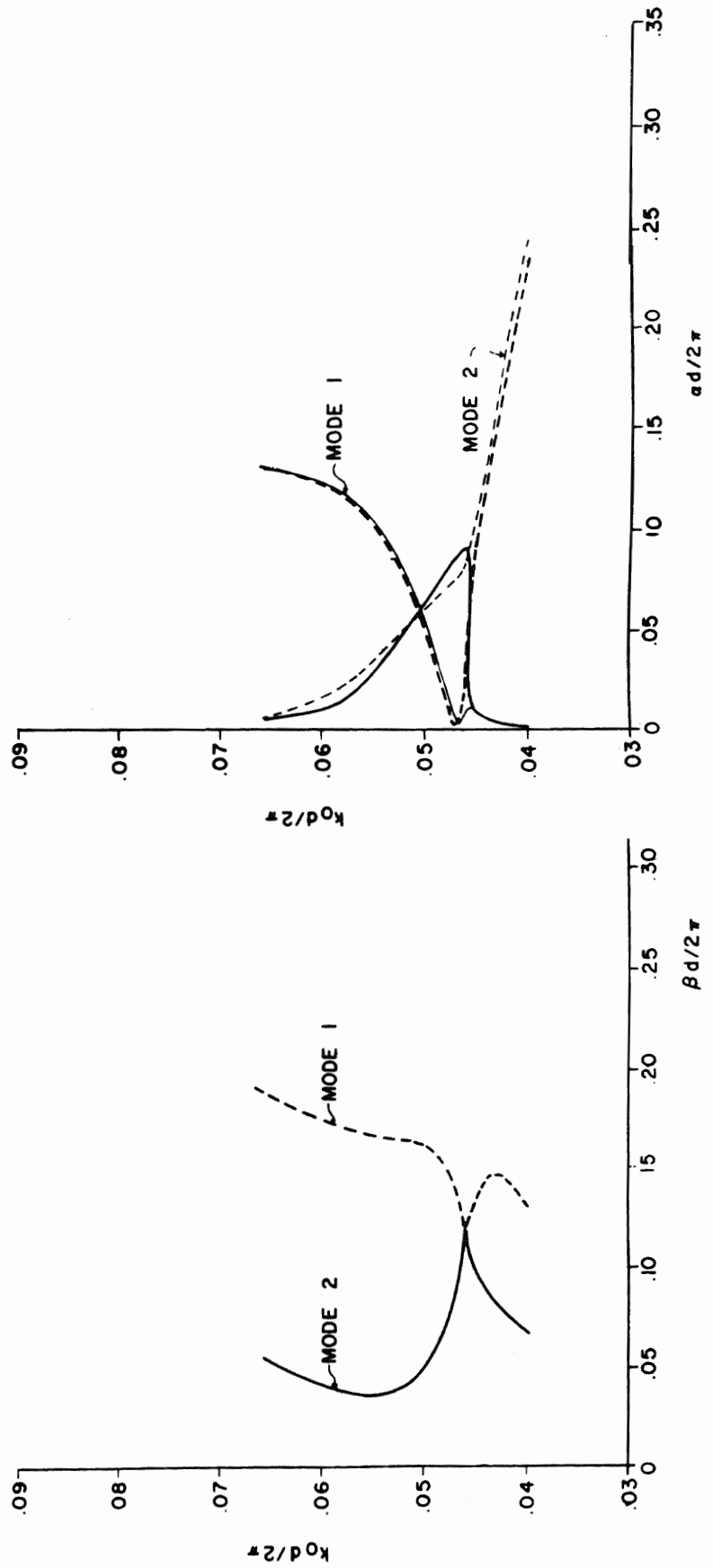


FIG. 9 DISPERSION CURVE ($K-\beta$ DIAGRAM) FOR A PERIODIC DIPOLE ARRAY, ALTERNATE ELEMENTS PHASE-REVERSED.

$Z_0 = 100$ OHMS, $D/L = 0.1$, $L/\rho = 177$

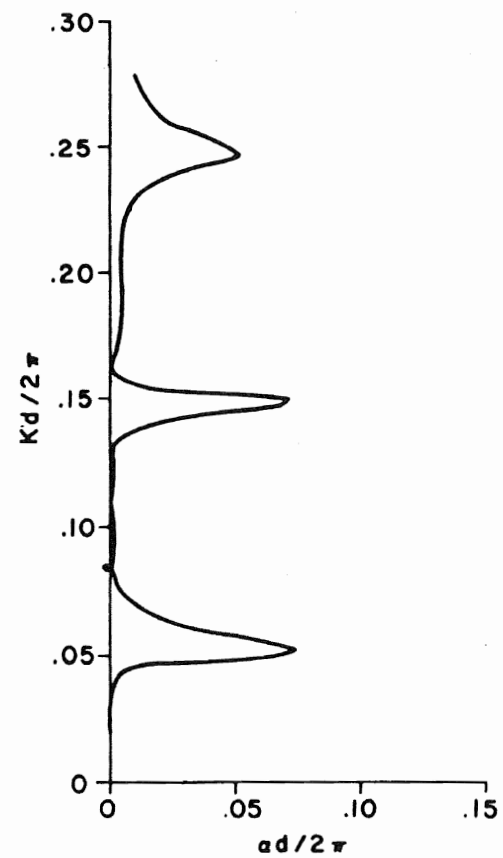
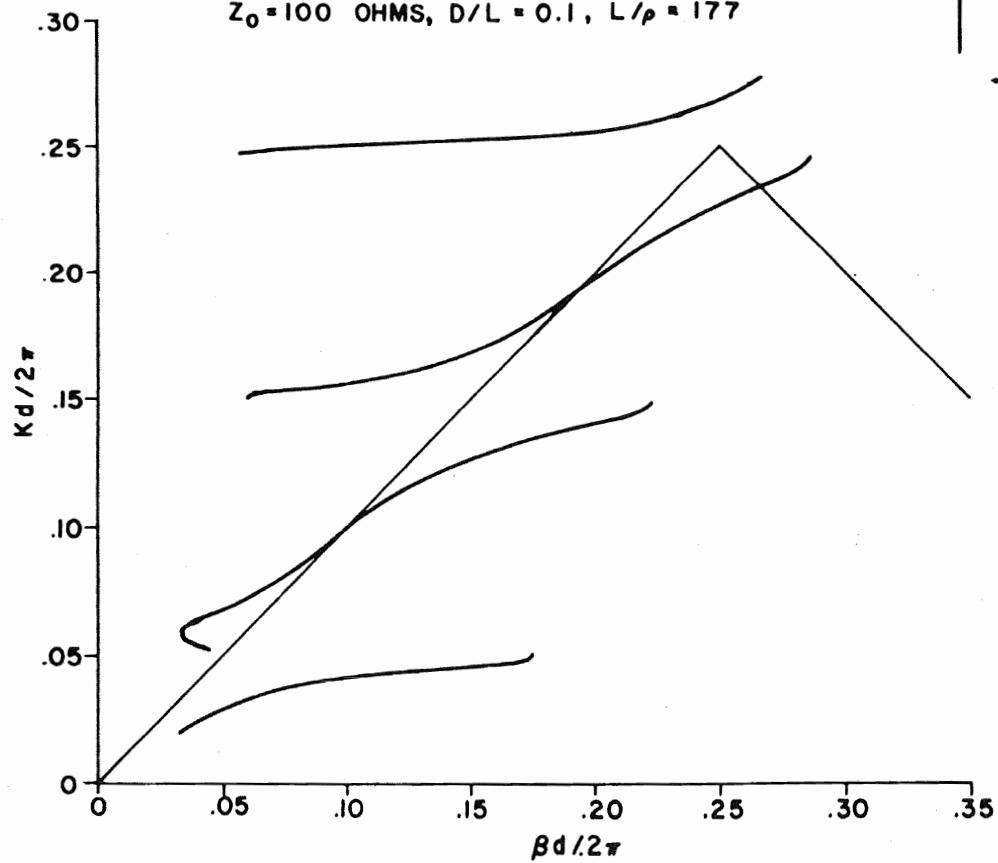
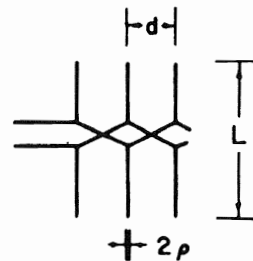


FIG. 10 DISPERSION CURVE ($K-\beta$ DIAGRAM) FOR A PERIODIC DIPOLE ARRAY, ALTERNATE ELEMENTS PHASE-REVERSED.

$Z_0 = 25$ OHMS, $D/L = 0.1$, $L/\rho = 177$

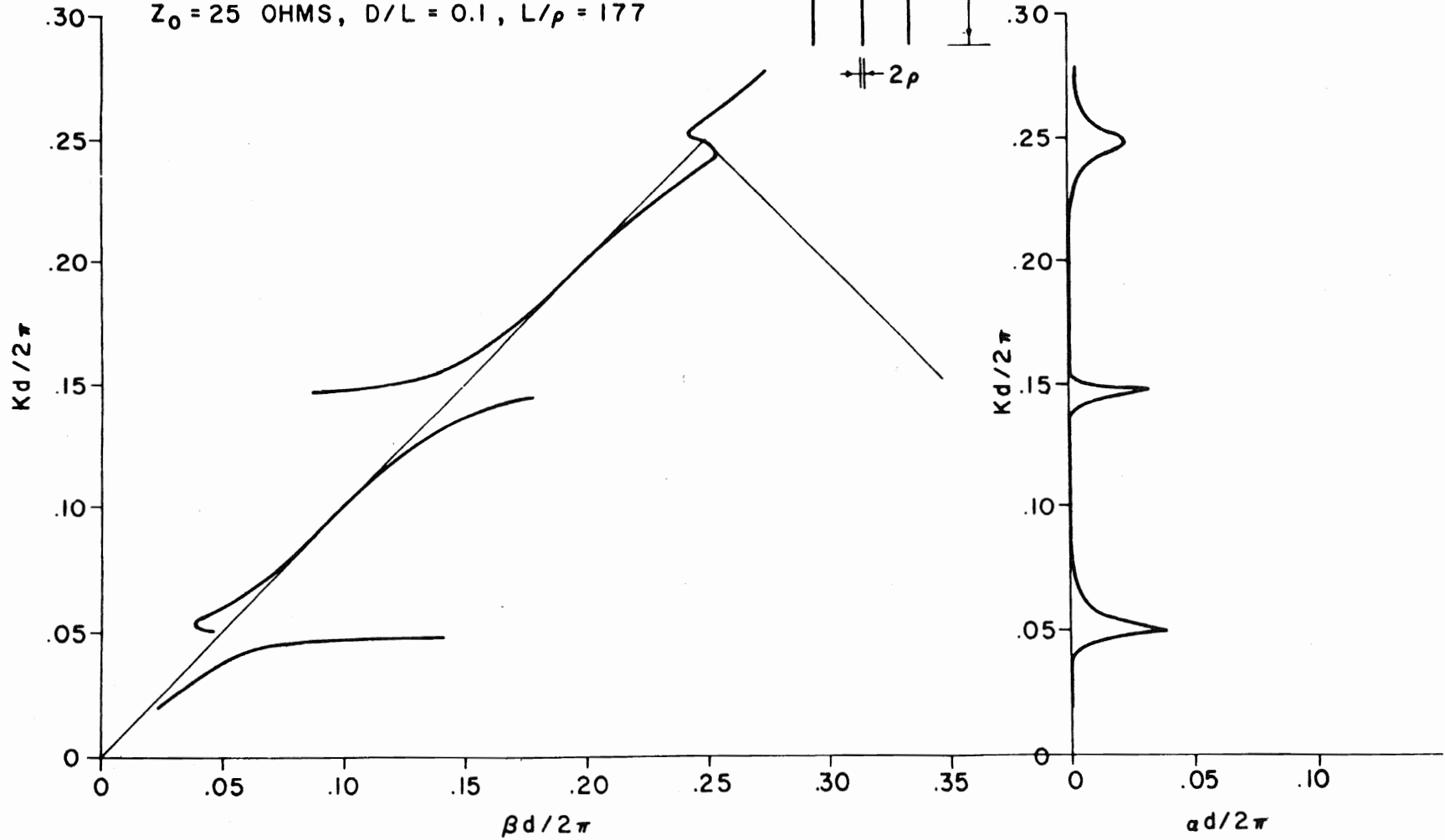
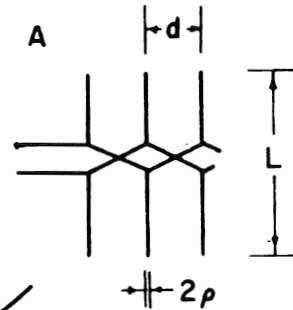


FIG. II DISPERSION CURVE FOR A PERIODIC DIPOLE ARRAY, ELEMENTS CAPACITIVELY COUPLED.

$Z_0 = 100$ OHMS, $D/L = 0.1$, $L/\rho = 177$

$$X_c = \frac{\pi}{2.5K_0d}$$

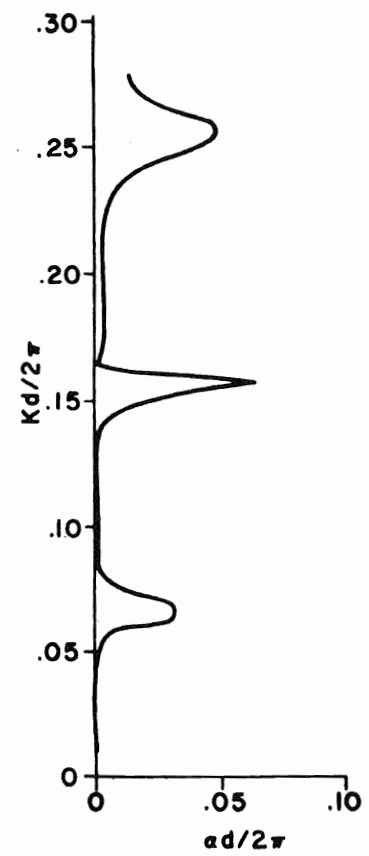
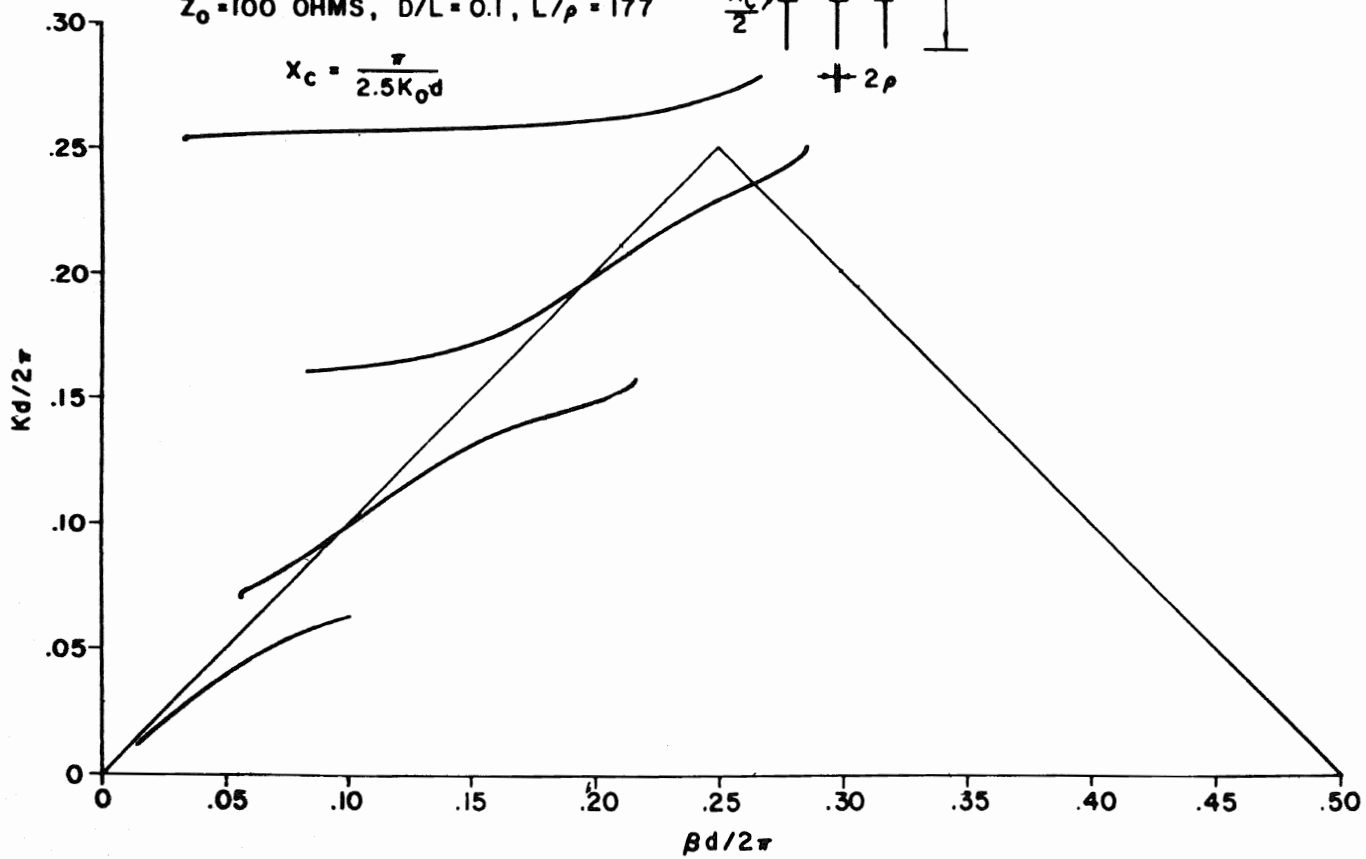
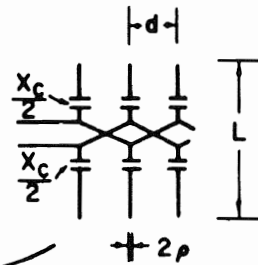


FIG. 12 DISPERSION CURVE FOR A PERIODIC
 DIPOLE ARRAY, TRANSMISSION LINE
 PHASE VELOCITY = 0.44286 c.
 $Z_0 = 100$ OHMS, $D/L = 0.1$, $L/\rho = 177$

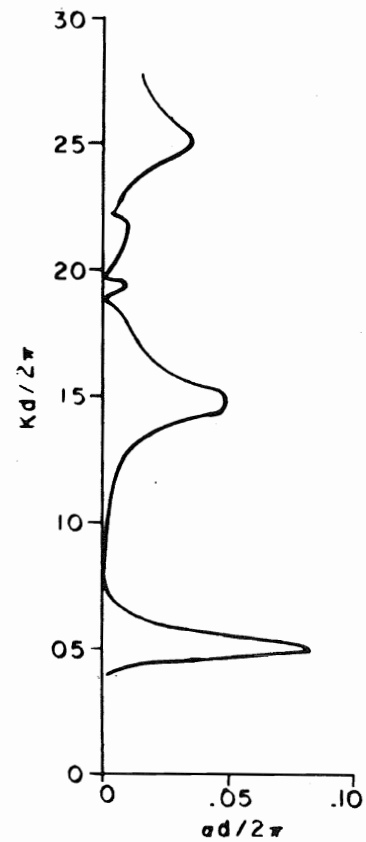
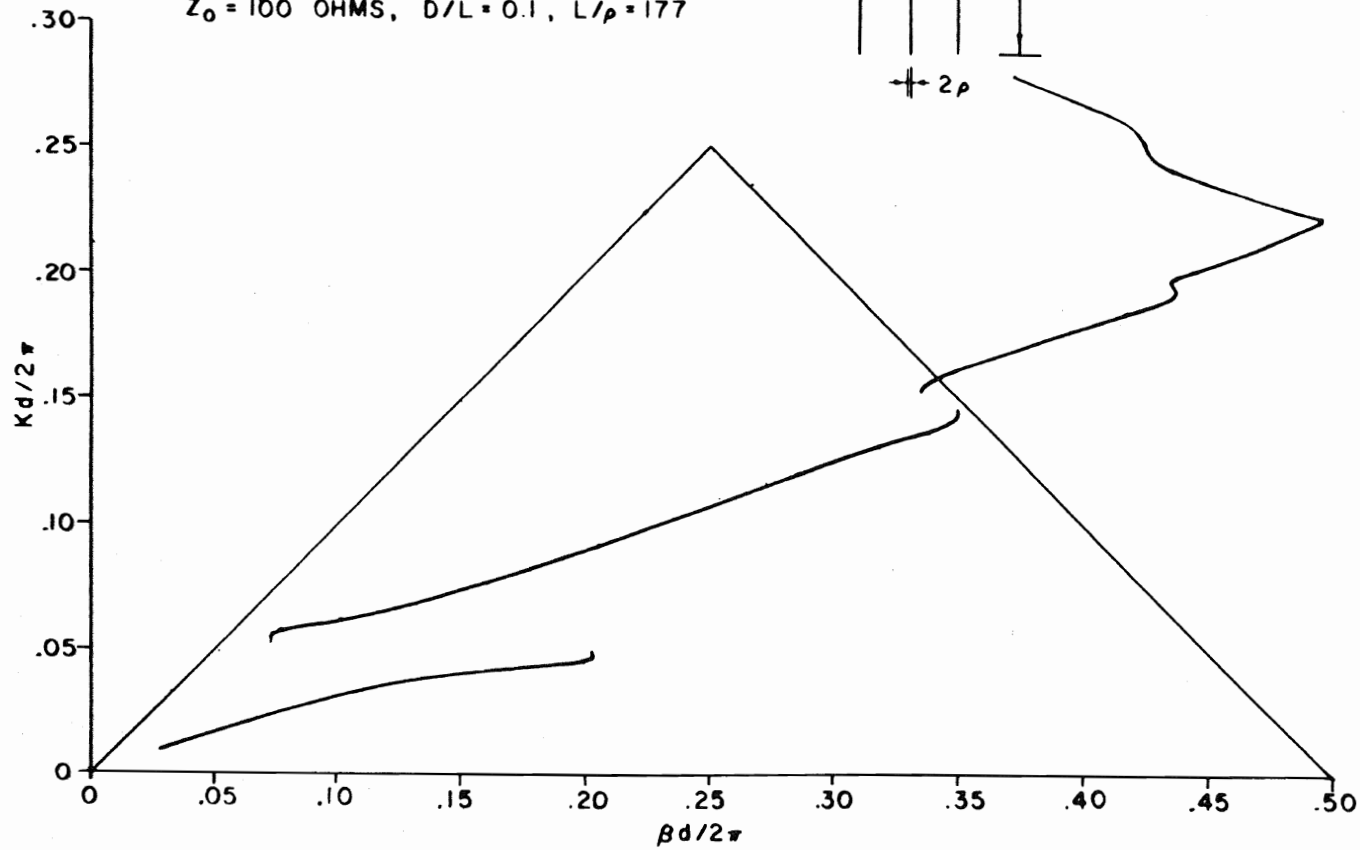
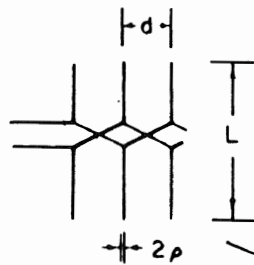


FIG. 13 DISPERSION CURVE FOR A
PERIODIC DIPOLE ARRAY, ELEMENTS
CAPACITIVELY COUPLED

$Z_0 = 100$ OHMS, $D/L = 0.1$, $L/\rho = 177$

$$X_c = \frac{\pi}{2.5 K_0 d}$$

