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Heat Flux And Temperature Variation
At A Wavy Water-Air Interface

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Abstract

The solution for the temperature distribution between two constant pressure surfaces of a Gerstner wave is used to calculate the local instantaneous heat flux and surface temperature variation at a wavy water-air interface. Radiation within the water is accounted for by the Rosseland diffusion approximation. The heat flux is found to be only weakly dependent on wave steepness, whereas the surface temperature variation is proportional to the third power of wave steepness and inversely proportional to the first power of wave period. RMS values of the surface temperature variations are far smaller than experimentally measured values. This seems to suggest that conduction in the thin thermal layer near the ocean surface is not the mechanism that produces these surface temperature fluctuations.

Introduction

It is now generally recognized that there exists a thin thermal layer near the surface of the ocean across which there is an appreciable difference in fluctuating as well as in mean temperature. Ewing and McAlister [1960] found a mean difference between surface and subsurface of $-.6^{\circ}\text{C}$ and a thermal layer thickness of less than one millimeter, whereas Schooley [1967] measured mean and RMS fluctuating differences of only $-.086^{\circ}\text{C}$ and $.025^{\circ}\text{C}$ respectively over a thickness of 10 cm. Experimental data collected by Hasse [1963] show that mean differences of 1.5°C and -1.0°C are possible. These wide differences indicate the varied conditions under which the data was taken. Saunders [1967] has proposed a simple theory which relates the mean temperature difference to the total heat flux and wind shear stress at the surface. A great deal of interest in these surface phenomena has been generated in recent years as a result of improved radiometric techniques for measuring surface temperatures.

The processes taking place at an air-ocean interface are extremely complicated and in order to fully describe the temperature distribution in the ocean one would have to account for absorbed solar and atmospheric radiation as well as emission from the water, heat transfer between air and water, evaporation and surface shape as well as convective, conductive and radiative heat transfer processes within the water. In this analysis some of these processes will be neglected while others can be accounted for in special ways.

For example, the rate of energy absorption by the water per unit time and volume at a distance y below the ocean surface is given by

$$\int_{\lambda} H_{\lambda} (1 - \rho_{\lambda}) K_{\lambda} e^{-K_{\lambda} y} d\lambda$$

where H_{λ} is the monochromatic radiation incident on the surface, ρ_{λ} and K_{λ} are the monochromatic reflectance and absorption coefficient for water. However, since the mean free path of solar radiation in water is very long compared to the thermal layer thickness, the small amount of energy absorbed gets absorbed uniformly across the layer. Therefore the main effect of solar radiation is to help determine the mean temperature level of the thermal layer. The same role is also played by evaporation, conductive heat transfer between air and ocean, and radiative emission from the water surface under quasi-steady conditions. Consequently the above processes need not be explicitly considered if the mean surface temperature as well as the mean subsurface temperature are assumed known. Absorption of radiation from the atmosphere and radiation initially emitted by the water but subsequently reflected back again by clouds will also be neglected.

The thermal layer is considered to lie between two constant pressure surfaces of a Gerstner wave. This wave [Lamb, 1945] is a simple finite amplitude progressive wave which probably describes very closely, except for their drift velocity, the motion of particles in large slope gravity waves. With the additional simplification that radiative transfer in the ocean can be approximated by the Rosseland diffusion equation, the local surface heat flux and surface temperature fluctuations can be calculated from O'Brien [1967].

Radiative heat flux and energy equation

Since water is optically thick to infrared radiation, the radiative heat flux q_R is given by the Rosseland approximation [Sparrow and Cess, 1966] as:

$$q_R = \frac{4 \nabla e_b}{3 K_R}$$

where K_R = Rosseland mean absorption coefficient and $e_b = n^2 \sigma T^4$ under the assumption that the index of refraction n of water is independent of wave length. n is not a constant however and varies between 1.1 and 1.7 from $.3\mu$ to 50μ [Goody, 1964]; but for computational purposes a mean n of 1.4 can be used. Note that q_R is strictly the radiative flux within the water and is not equal to the radiation emitted to the atmosphere which is $\epsilon \sigma T_1^4$.

If $\delta T = T_1 - T_2$ is the difference between the mean surface and subsurface temperatures, then $\frac{\delta T}{T_1} = O(10^{-3})$ and q_R can be linearized to give

$$q_R = \frac{16 T_1^3 \sigma n^2 \delta T \nabla \theta}{3 K_R}$$

where $\theta = \frac{T_1 - T}{T_1 - T_2}$. The energy equation therefore takes the form

$$\frac{D\theta}{Dt} = D_1 \nabla^2 \theta \quad (1)$$

where $D_1 = \left(\frac{16 T_1^3 \sigma n^2}{3 K_R \rho C_p} + D \right)$ and D is the thermal diffusivity of water.

Since O'Brien's results are valid when $\frac{2\pi D}{\lambda C} \ll 1$ (λ and C are the wave length and wave speed) which for water at S.T.P. is no greater than 10^{-4} ;

it is also necessary that $\frac{32 \pi T_1^3 \sigma n^2}{3 K_R \rho C_p \lambda C} \ll 1$. No information could be found on K_R for water. However, K_R is very unlikely to be less than $10^{-1} K_p$, where K_p is the Plank mean absorption coefficient which at

S.T.P. is 0(.1 mm). With these assumptions it can be shown that $\frac{16 T_1^3 \kappa \sigma n^2}{3 K_R C_p \rho C}$ has an upper bound no greater than 10^{-4} . Where $\kappa = \frac{2\pi}{\lambda}$.

Heat flux at the surface

The average temperature of a fluid particle is obtained from (1) [O'Brien, 1967] to be

$$T = T_1 - \delta T \ln \frac{[\cosh \beta_1 / \cosh \beta]}{[\cosh \beta_1 / \cosh \beta_2]} \quad (2)$$

where $\beta = \kappa b$ and b is the mean vertical position of the particle. β_1 and β_2 identify the surface and subsurface. The mean thickness of the thermal layer is

$$(\beta_1 - \beta_2) / \kappa = \delta \beta / \kappa \equiv \delta_0$$

Note also that $\beta_2 < \beta_1 < 0$ and that $\beta_1 = 0$ is not allowed.

The normal heat flux at any location on the surface due to the mean temperature field (2) can be calculated to be

$$q_s = 2\pi \rho C_p D_1 [1 + e^{\beta_1} \cos \kappa(a + ct)] [1 + s^2]^{\frac{1}{2}} \frac{dT}{d\beta} \quad (3)$$

where a is the mean horizontal position of a fluid particle, and s the local slope of the wave. s is given by

$$s = \frac{e^{\beta} \sin \kappa(a + ct)}{1 + e^{\beta} \cos \kappa(a + ct)}$$

and

$$\frac{dT}{d\beta} = \frac{\delta T (1 - e^{2\beta_1})}{\ln[\cosh \beta_1 / \cosh \beta_2] (1 + e^{2\beta_2})}$$

After some algebra (3) becomes

$$\bar{q}_s = - \frac{2\pi \rho C_p D_1 \delta T [1 + 2\pi \bar{s} \cos k(a + ct) + \pi^2 \bar{s}^2]^{1/2}}{\lambda [1 + \pi^2 \bar{s}^2] \phi} \quad (4)$$

where the wave steepness \bar{s} is

$$\bar{s} = \frac{\text{wave height}}{\text{wave length}} = \frac{e^{\beta}}{\pi} < \frac{1}{\pi}$$

and

$$\phi = [\frac{1}{2}(1 - \tanh^2(\ln \pi \bar{s})) \delta \beta^2 - \delta \beta \tanh(\ln \pi \bar{s})]$$

also

$$\lim_{\bar{s} \rightarrow 0} \phi \rightarrow \delta \beta$$

$$\lim_{\bar{s} \rightarrow \frac{1}{\pi}} \phi \rightarrow \frac{1}{2} \delta \beta^2$$

Furthermore, in ϕ it has been assumed that $\delta \beta = \frac{2\pi}{\lambda} \delta_0 \ll 1$

The RMS of (4) over one wave period is

$$(\overline{q_s^2})^{1/2} = \frac{2\pi \rho C_p D_1 |\delta T|}{\lambda [1 + \pi^2 \bar{s}^2]^{3/2} \phi} \quad (5)$$

A comparison of (4) and (5) shows that q_s is more sensitive to wave steepness \bar{s} than $(\overline{q_s^2})^{1/2}$.

Temperature fluctuation at surface

The first order temperature variation in $\frac{2\pi D_1}{\lambda c}$ at the surface is given by O'Brien as

$$T' = - \frac{2\pi D_1 L_2^{(1)} T(\beta) \sin k(a + ct)}{\lambda c}$$

where

$$\begin{aligned}
 L_2^{-1} T(\beta) &= -e^\beta (1 - e^{2\beta})^{-2} \left[\frac{d^2}{d\beta^2} + 2e^\beta (1 - e^{2\beta})^{-1} \frac{d}{d\beta} \right] T(\beta) \\
 &= \frac{-2e^{3\beta} \delta T}{(1 - e^{2\beta})(1 + e^{2\beta})^2 \ln[\cosh \beta_1 / \cosh \beta_2]} \\
 &= \frac{2\pi^3 \bar{s}^3 \delta T}{(1 - \pi^2 \bar{s}^2)(1 + \pi^2 \bar{s}^2)^2 \phi}
 \end{aligned}$$

and

$$T' = \frac{-4\pi^4 \bar{s}^3 D_1 \delta T \sin \kappa(a + ct)}{\lambda c (1 - \pi^2 \bar{s}^2)(1 + \pi^2 \bar{s}^2)^2 \phi} \quad (6)$$

From (5)

$$\frac{|\delta T|}{\lambda \phi} = \left(\frac{q_s^2}{\rho_s}\right)^{\frac{1}{2}} \frac{[1 + \pi^2 \bar{s}^2]^{3/2}}{2\pi \rho c_p D_1}$$

and substituting into (6) we have

$$T' = \pm \frac{2\pi^3 \bar{s}^3 \left(\frac{q_s^2}{\rho_s}\right)^{\frac{1}{2}} \sin \kappa(a + ct)}{\rho c_p [1 - \pi^2 \bar{s}^2][1 + \pi^2 \bar{s}^2]^{\frac{1}{2}} c} \quad (7)$$

where + is for $\delta T < 0$ i.e. upward heat flux and - is for $\delta T > 0$ i.e. downward heat flux.

From the relationships for deep water gravity waves

$$c^2 = \frac{\lambda g}{2\pi}, \quad \lambda = \frac{g P^2}{2\pi}$$

where P is the wave period, (7) becomes

$$T' = \pm \frac{4\pi^4 \bar{s}^3 \left(\frac{q_s^2}{\rho_s}\right)^{\frac{1}{2}} \sin \kappa(a + ct)}{P g \rho c_p [1 + \pi^2 \bar{s}^2]^{\frac{1}{2}} [1 - \pi^2 \bar{s}^2]} \quad (8)$$

Alternatively as $\bar{s} \rightarrow 0$ $\phi \rightarrow \delta\beta$ and (6) can be written

in terms of the mean layer thickness δ_0 as

$$T' = \frac{-4\pi^4 \bar{s}^3 D_1 \delta T \sin \kappa(a + ct)}{P g [1 - \pi^2 \bar{s}^2][1 + \pi^2 \bar{s}^2]^2 \delta_0} \quad (9)$$

A plot of $\sin \kappa(a + ct)$ and the wave shape is given in figure 1. It can be seen that T' is 72° out of phase with wave shape. This phase lag is not dependent on the details of the Gerstner wave kinematics but is

solely a consequence of the relatively slow diffusion process and it should therefore be a general property of traveling waves on a water surface. Furthermore, from (9) we see that $T' \propto \bar{S}^3/\rho$. Neither of these results is in agreement with Watering [1968] who found T' and wave shape in phase and $T' \propto \bar{S} \rho^{1/2}$. However, it must be remembered that the Gerstner solution is for $\frac{L}{\lambda} \rightarrow \infty$ where L is the water depth, whereas the linear relationship that Watering finds from his experiment holds only for wave periods greater than one second. Most of the waves he considers (those with wave period between 1.5 and 3.5 second) are somewhere between shallow gravity waves and deep water waves. For periods less than one second (these are definitely deep water waves) he finds a smaller T' than predicted by a linear dependence on \bar{S} indicating a stronger dependence on \bar{S} . An evaluation of (9) with $\bar{S} = .1$, $D_1 = 1.4 \text{ cm}^2/\text{sec}$, $\delta T = 1^\circ\text{C}$ and $\delta_0 = 1\text{mm}$ gives $[(T')^2]^{1/2} = 10^{-5} \text{ }^\circ\text{C}$. This is far smaller than $2.5 \times 10^{-2} \text{ }^\circ\text{C}$ recorded by Schooley, and laboratory measurements made by Watering. This suggests that conduction and radiation in the thin thermal layer near the ocean surface is not responsible for the observed temperature fluctuations.

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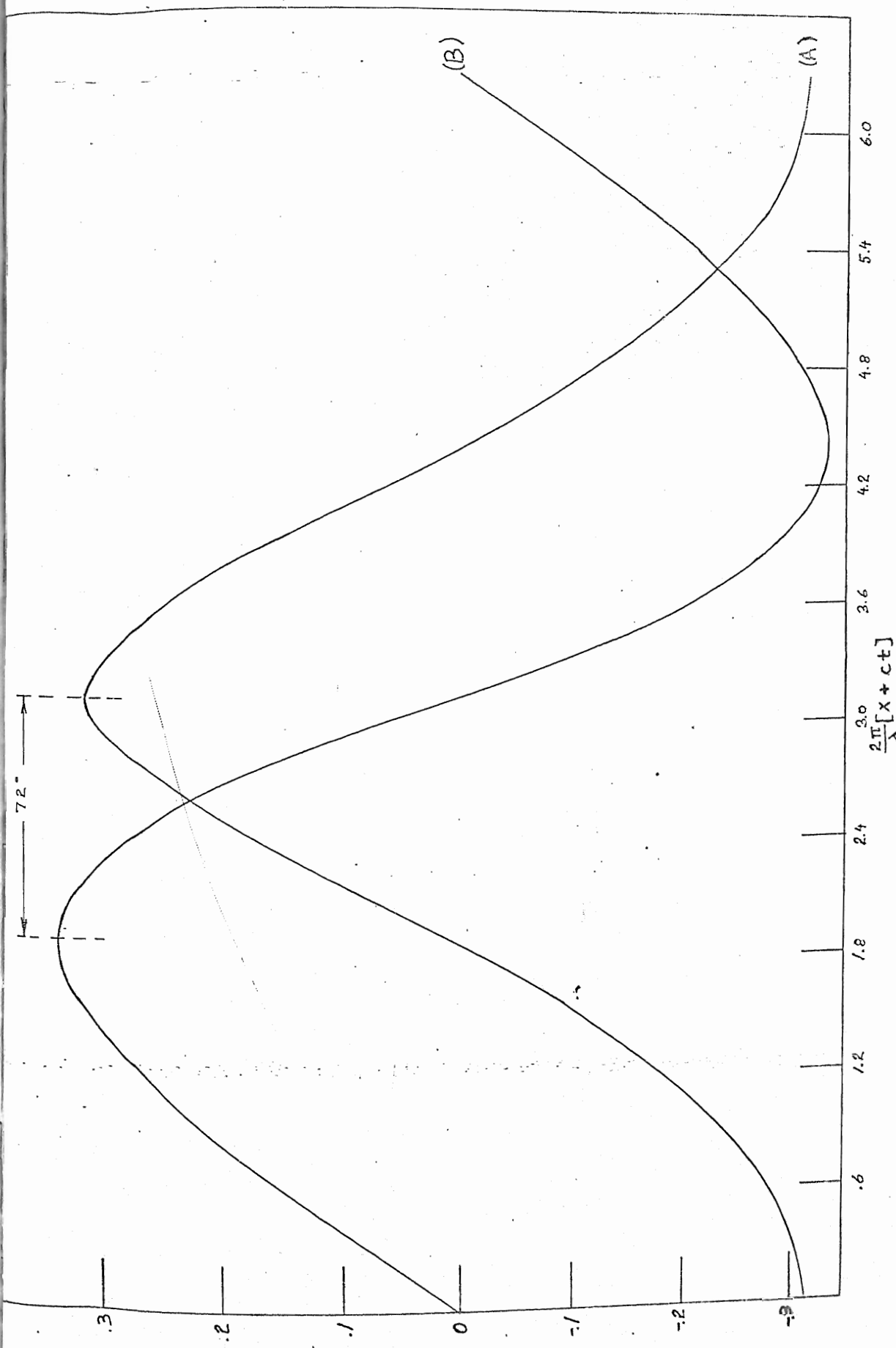


FIGURE 1. (A), The Gerstner wave ($\bar{S}=1$). (B), $\frac{2\pi}{\lambda} \sin \frac{2\pi}{\lambda} (\alpha + ct)$.

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