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A Multibeam Medium Acess Scheme for Multiple Services in Wirelsss Cellular Communications

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Abstract

Multibeam cellular communication systems in which multiple services are of interest are considered. A sectorized multibeam medium access scheme which can support multiple services with different CIR requirements is proposed. Dynamic channel Assignment (DCA) is used to improve channel reuse and to allocate different CIR quality channels within a channel layout which incorporates several frequency reuse patterns. Channels with higher CIR levels are primarily for services that require higher quality communication links, such as data calls, and channels with lower CIR levels are only for services that require lower quality links, such as voice calls. Services that require lower quality links can also access channels with higher CIR levels if channels of lower quality groups are not available. We develop a tractable analytical model for the system using multidimentional birth-death processes with an appropriate state characterization. Theoretical traffic performance characteristics such as blocking probability, forced termination probability and carried traffic are determined.

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1. Introduction

Multiple integrated services in wireless communications is an important trend. A wide range of services for users, such as data, voice, multimedia, etc. will coexist in a system for next generation wireless personal communications [1]. These services have distinct requirements on bit error rate, time delay and channel bandwidth. For instance, data services can tolerate certain time delay but require more reliable transmission and thus lower bit error probability. Voice services, on the other hand, can permit higher bit error probability but demand real-time communications and therefore less time delay. Multimedia services need high speed transmission so that more channel bandwidth is needed. In this paper, we consider the issue that distinct services require different BER. This can be achieved with channels having different CIR levels. According to the CIR quality that they require, the services are classified into several types. A multibeam medium access scheme with a channel layout having different frequency reuse patterns is used to allow distinct services operating at different CIR levels. Channels are divided into several groups, each of which is reused according to one of the frequency reuse patterns. As a result, channels of different groups have different CIR qualities. Depending on the CIR quality which they demand, calls of each service type can be served by the channels of the corresponding group. This resembles reuse partitioning (RP) [2], in which channels of different groups are reused at different reuse distances. But unlike RP, the proposed scheme uses DCA to improve channel reuse and employs directional multibeam antennas to reduce co-channel interference. In RP, the CIR levels of channels of all groups are maintained on the same level.

Wireless channels sometimes suffer high BER due to wireless environment such as fading, co-channel interference, etc. Space Division Multiple Access (SDMA) is a technique which can reduce the fading and co-channel interference by using directional multibeam antennas [3], [4]. This allows reduced frequency reuse distances for the same quality of communication links and results in higher system capacity. One application of SDMA in cellular communications is to use switched multibeam scheme at the base station sites. Multiple beams are used to cover the service area of a base station and the beam with the strongest signal power is selected to serve the user [5]. DCA is a technique which can improve channel reuse in cellular communications [6]. Channels are dynamically assigned to wireless users subject to the allowable co-channel interference constraints. Channels of different groups have different co-channel interference constraints. DCA is made so that channels of different groups satisfy their CIR levels. We propose a multibeam medium access scheme in which distinct services can operate at different

CIR levels to meet their BER requirements. This scheme combines the advantages of multibeam schemes and DCA.

An example problem with two types of services -- voice and data, is considered. Channels are partitioned into two groups, L and H. A DCA scheme is used so that channels of group H provide higher CIR quality channels than channels of group L. In the proposed scheme, channels of group H are primarily for data calls and channels of group L are only for voice calls. Voice calls can also access channels of group H if channels of group L are not available. Data calls are restricted to access channels of group H only since they require a higher CIR level. Models to compute fundamental traffic performance measures for the proposed scheme are devised. These include call blocking probability, forced termination probability and carried traffic. Multidimensional birth-death processes are discussed in [7], [8], [9]. The global balance equations are determined and solved for the state probabilities, using the framework developed in earlier work [7], [8], [9]. Performance characteristics are found from these state probabilities.

2. *Model Description*

2.1. *System model*

We consider a cellular system that services large number of wireless platforms, such as vehicles and pedestrians. Each platform can generate either a voice or a data call, but at most one call can be supported on a platform at any given time. A cell is divided into 3 sectors and each sector is covered by two directional beams. This provides a total of six beams per cell as shown in Figure 1. The beam in the counterclockwise direction is called the beam 1 and the other is called the beam 2. The services are classified into two types – high and low CIR quality.

Channels are partitioned into two groups, L and H, and are reused in two different reuse patterns. Channels of group L are reused in a cluster size of seven with three subgroups in a cell. That is, channels of group L are divided into twenty-one subgroups and channels of each subgroup are allocated to each sector. Channels of group H are reused in a cluster size of seven with six subgroups in a cell. That is, channels of group H are divided into forty-two subgroups and channels of each subgroup are allocated to each beam.

2.2. *Channel assignment*

Let C_L and C_H denote the numbers of channels of group L and H allocated to each beam respectively. That is, there are $2C_L$ channels of group L allocated to each sector. Data calls are restricted to access only channels of group H. Voice calls can access channels of group L as well as channels of group H. The channel assignment to voice calls is made so that the voice calls first attempt to search channels of group L. Voice calls can access channels of group H if channels of group L are not available. DCA is used on channels of group L. Channels of group L of a sector are dynamically assigned to wireless users of the sector who are eligible to access L-channels without regard to the beam through which they communicate. That is, the voice calls in any beam can access $2C_L$ channels of group L. The use of DCA improves the channel reuse of L-channels and allows H-channels have higher CIR level than L-channels. The lower CIR level of L-channels is due to the interference contributed by two wireless users in the co-channel sectors that are assigned the same channel simultaneously. Nevertheless L-channels are only used by voice calls, which can tolerate lower CIR level compared to data calls. DCA is made as long as the CIR requirement of L-channels are satisfied.

2.3. Teletraffic model

The infinite population model is used. It is assumed that the number of non-communicating platforms is much larger than the number of channels in a beam so that the call generation rate generated in a beam does not depend on the number of calls in progress. The new call origination rate from a non-communicating platform is denoted as Λ. The number of non-communicating platforms in a beam is denoted as ν. Therefore the total call origination rate in a beam, Λ_n, is $\Lambda_n = \Lambda \cdot \nu$. It is assumed that Λ is very small and ν is very large so that Λ_n remains the same without regard to the number of calls in progress. The average new call origination rate of voice and data calls can be different and are modeled as follows. Let the fraction of total traffic that is voice call be *f*. The fraction that is data call is *1-f*. The new call origination of voice and data calls in a beam are modeled as Poisson point process having mean arrival rates Λ_{nv} and Λ_{nd} respectively.

The mobility of platforms was characterized by dwell time in earlier work [7], [8], [9]. The dwell time is the amount of time that a wireless platform is within communicating range of a given gateway. It is modeled by a random variable with a negative exponential distribution (n.e.d). The dwell time of a wireless platform in a beam is modeled as a n.e.d. random variable having a mean $T_D = 1/\mu_D$. The unencumbered call session duration of voice and data calls can be different and are modeled as n.e.d. random variables having means $T_{Cv} = 1/\mu_{Cv}$ and $T_{Cd} = 1/\mu_{Cd}$ respectively.

3. State Description

For the problem under consideration (infinite population model, single platform type and two call types), six state variables are needed to describe the status of each sector. Because of the homogeneous property of the system, the behavior of any sector in statistical equilibrium is the same as any other. We use v to denote the number of voice calls being served and d for the number of data calls being served. We use the subscript L and H to denote the channels of group L and H respectively and subscript i to denote the beam i. We define the state of the sector by a sequence of nonnegative integers, v_{L1}, v_{L2}, v_{H1}, v_{H2}, d_1, d_2. In this sequence, the state variables, v_{Li}, $i = 1,2$, is the number of voice calls served by L-channels in beam i of the sector, v_{Hi}, $i = 1,2$, is the number of voice calls served by H-channels in beam i of the sector and, d_i, $i = 1,2$, is the number of data calls served (by H-channels) in beam i of the sector. Then, for convenience, we order the states using an index s=0,1,2,......., $S_{\max}$. Thereafter, v_{Li}, v_{Hi} and d_i, $i = 1,2$, can be shown as explicitly dependent on the state. That is, $v_{Li} = v_L(s,i)$, $v_{Hi} = v_H(s,i)$, and $d_i = d(s,i)$, $i = 1,2$, in which $v_L(s,i)$ is the number of voice calls served by L-channels in beam i of the sector when the sector is in state, s, $v_H(s,i)$ is the number of voice calls served by H-channels in beam i of the sector when the sector is in state, s, and $d(s,i)$ is the number of data calls served (by H-channels) in beam i of the sector when the sector is in state, s. Let C denotes the number of channels per beam, C_L the number of L-channels per beam and C_H the number of H-channels per beam. We can specify the constraints on permissible states as

$$\sum_{i=1}^{2} v_{Li} \le 2C_L. \tag{1}$$

$$v_{Hi} + d_i \le C_H, \qquad i = 1,2. \tag{2}$$

$$v_{Li} \ge 0,\ v_{Hi} \ge 0,\ d_i \ge 0, \quad i = 1,2. \tag{3}$$

4. Driving Processes and State Transition Flow

The state probabilities, *p(s)*, in statistical equilibrium are required to determine the performance measures of interest. To calculate the state probabilities, the state transitions and corresponding transition rates must be identified and calculated. There are eight relevant driving processes. These are: {nv} the generation of new voice calls; {nd} the generation of new data calls; {cv} the completion of voice calls; {cd} the completion of data calls; {hv} the hand-off arrival of communicating voice calls; {hd} the hand-off arrival of communicating data calls; {dv} the hand-off departure of communicating voice calls; {dd} the hand-off departure of communicating data calls. The state transitions due to these driving processes and their corresponding transition rates are explained in Appendix.

5. Flow Balance Equations

Once the transition rates due to each driving process are calculated, the total transition flow into *s* from any permissible predecessor state *x* can be found using

$$q(s,x)=\sum_{i=1}^{4}[\gamma_{ni}(s,x)+\gamma_{hi}(s,x)]+\sum_{i=1}^{6}[\gamma_{ci}(s,x)+\gamma_{di}(s,x)+\gamma_{hdi}(s,x)] \qquad (4)$$

in which $s \neq x$, and flow into a state has been taken as a positive quantity. The total flow out of state *s* is denoted as $q(s,s)$, and is given by

$$q(s,s)=-\sum_{\substack{k=0\\k\neq s}}^{S_{\max}} q(k,s) \qquad (5)$$

To find the statistical equilibrium state probabilities for a sector, we write the flow balance equations for the states. These are a set of $S_{\max}+1$ simultaneous equations for the unknown state probabilities $p(s)$. They are of the form

$$\sum_{j=0}^{S_{\max}} q(i,j)p(j)=0\,, \qquad i=0,1,2,\ldots\ldots,S_{\max}-1\,.$$

$$\sum_{j=0}^{S_{\max}} p(j)=1\,. \qquad (6)$$

In which, for $i \neq j$, $q(i,j)$ represents the net transition flow into state *i* from state *j*, and $q(i,i)$ is the total transition flow out of state *i*. These equations express that in statistical equailibrium, the net probability flow into any state is zero, and the sum of the probabilities is unity.

6. Performance Measures

There are four performance measures of interest: 1) call blocking probability, 2) hand-off failure probability, 3) forced termination probability, and 4) carried traffic. Once the statistic equilibrium state probabilities are found, the required performance measures can be calculated.

6.1 Blocking probability

The blocking probability for a call is the average fraction of new calls that are denied access to a channel. Blocking of new calls occurs if there are no channels to serve the call.

1) Voice call blocking probability

Blocking of a voice call in beam 1 occurs when the channels of both group L and H are all occupied. Due to symmetry, the blocking probability of voice calls in beam 1 is the same as that in beam 2. We define the following set of states

$$B_v = \{s: v_L(s,1) + v_L(s,2) = 2C_L, v_H(s,1) + d(s,1) = C_H\}$$

Then the blocking probability of voice calls in a beam is

$$P_{Bv} = \sum_{s \in B_v} p(s) \tag{7}$$

2) Data call blocking probability

Blocking of a data call in beam 1 occurs when channels of group H are all occupied. The blocking probability of data calls in beam 1 is the same as that in beam 2 due to symmetry. We define the following set of states

$$B_d = \{s: v_H(s,1) + d(s,1) = C_H\} \tag{8}$$

Then the blocking probability of data calls in a beam is

$$P_{Bd} = \sum_{s \in B_d} p(s) \tag{9}$$

6.2 Hand-off failure probability

The hand-off failure probability for calls is the average fraction of hand-off attempts that are denied a channel.

1) Voice call hand-off failure probability

We define the following sets of states, in which hand-off attempts of voice calls will fail.

$$H_v = \{s: v_L(s,1) + v_L(s,2) = 2C_L, v_H(s,1) + d(s,1) = C_H\} \tag{10}$$

Then the hand-off failure probability of voice calls due to call hand-offs across sectors can be written as

$$P_{Hv_1} = \sum_{s \in H_v} p(s) \tag{11}$$

and the hand-off failure probability of voice calls due to call hand-offs within beams of the same sector can be written as

$$P_{Hv_2} = \sum_{s \in Hv} p(s) \frac{v_H(s,2)}{v_L(s,2) + v_H(s,2)} \tag{12}$$

Let F_1 denote the fraction of hand-off departures from a beam of a sector to other sectors and F_2 denote the fraction of hand-off departures from a beam to the other beam of the same sector. The average hand-off failure probability of voice calls is

$$P_{Hv} = F_1 \cdot P_{Hv_1} + F_2 \cdot P_{Hv_2} \tag{13}$$

2) Data call hand-off failure probability

We define the following set of states as

$$H_d = \{s: v_H(s,1) + d(s,1) = C_H\} \tag{14}$$

Then the hand-off failure probability of data calls is

$$P_{Hd} = \sum_{s \in H_d} p(s) \tag{15}$$

6.3 Forced termination probability

The forced termination probability is defined as the probability that a call that is not blocked is interrupted due to hand-off failure during its lifetime. Let p_v be the probability that a non-blocked voice call satisfactorily completes before the hand-off attempt occurs, and q_v be the probability that hand-off attempt occurs first. Similarly, let p_d be the probability that a non-blocked data call satisfactorily completes before the hand-off attempt occurs, and q_d be the probability that hand-off attempt occurs first. Because of the negative exponential assumption, we have

$$p_v = \mu_{Cv} / (\mu_{Cv} + \mu_D) \tag{16}$$

$$q_v = \mu_D / (\mu_{Cv} + \mu_D) \tag{17}$$

$$p_d = \mu_{Cd} / (\mu_{Cd} + \mu_D) \tag{18}$$

$$q_d = \mu_D / (\mu_{Cd} + \mu_D) \tag{19}$$

1) Voice call forced termination probability

The probability that a non-blocked voice call is forced to terminate on its k^{th} hand-off attempt is

$$Y_v(k) = P_{Hv} \cdot q_v^{\,k} \cdot (1 - P_{Hv})^{k-1} \tag{20}$$

The forced termination probability of voice calls is therefore

$$P_{FTv} = \sum_{k=1}^{\infty} Y_v(k) \tag{21}$$

This can be compactly written in closed form as

$$P_{FTv} = \frac{q_v \cdot P_{Hv}}{1 - q_v \cdot (1 - P_{Hv})} \tag{22}$$

2) Data call forced termination probability

Similarly, the probability that a non-blocked data call is forced to terminate on its k^{th} hand-off attempt is

$$Y_d(k) = P_{Hd} \cdot {q_d}^k \cdot (1 - P_{Hd})^{k-1} \tag{23}$$

The forced termination probability of data calls is therefore

$$P_{FTd} = \sum_{k=1}^{\infty} Y_d(k) \tag{24}$$

This can be compactly written in closed form as

$$P_{FTd} = \frac{q_d \cdot P_{Hd}}{1 - q_d \cdot (1 - P_{Hd})} \tag{25}$$

6.4 Carried traffic

The carried traffic in a beam is the average number of channels occupied by the calls in that beam.

1) Voice call carried traffic

The carried traffic of voice calls per beam is

$$A_v = \sum_{s=0}^{S_{max}} (v_L(s,1) + v_H(s,1)) \cdot p(s) \tag{26}$$

2) Data call carried traffic

The carried traffic of data calls per beam is

$$A_d = \sum_{s=0}^{S_{max}} d(s,1) \cdot p(s) \tag{27}$$

7. Numerical Results and Discussions

For all the figures, a mean unencumbered call duration of 100 seconds was assumed for both voice and data calls. Channel limit per beam is 14 and H-channel limit per beam is 8. The total number of platforms per beam is 300. The mean dwell time of platforms is 100 seconds. The symbol α is defined as the ratio of voice to data new call origination rate.

Figure 2 shows the comparison of blocking probability between DCA and fixed channel assignment (FCA). When DCA on L-channels is used, the blocking probability of voice calls is reduced significantly. This is because DCA directly improves the channel reuse of L-channels, which are used only by voice calls. The use of DCA improves the overall channel usage efficiency. Data calls also benefit from DCA. Because L-channels can accommodate more voice calls due to the DCA so that less voice calls need to access H-channels, which reduces the competition of H-channels from voice calls and allows more H-channels available for data calls. This results in the decrease of blocking probability of data calls.

The comparison of forced termination probability between DCA and FCA is shown in Figure 3. Forced termination probability of both voice and data calls is smaller in DCA case when compared with the FCA case. This means that the wireless users will less likely experience the interruption of calls during the call lifetime when DCA is used. A bigger reduction of forced termination probability for voice calls is expected since voice calls benefit directly from the use of DCA in comparison with the data calls. From the subscribers' point of view, voice calls are more sensitive to the interruption of calls since voice calls usually need real time communication, while data calls can be put into the buffers for retransmission with certain acceptable time delay.

Figure 4 to Figure 6 are plotted for different ratios of voice to data call origination. The traffic of new call origination per platform is fixed, however the fraction of voice and data traffics generated is variable. Figure 4 shows the blocking probability. When the ratio is decreased, it means that data call traffic is increased and voice call traffic is decreased, however the total traffic is constant. In this case, blocking probability of data calls is increased. This is because data calls have more traffic and need to meet specific requirement (H-channel limitation). On the other hand, the blocking probability of voice calls is decreased since voice call traffic is decreased and blocking of data calls is sacrificed to accommodate voice calls. As the ratio goes to a large value, it means that voice calls dominate the overall traffic and the blocking probability of voice calls is increased. But no matter how heavy the voice call traffic is, the blocking probability of voice calls is always smaller than the blocking probability of data calls. Voice calls can access more channel resources than data calls since the voice calls can access both L- and H-channels, while the data calls can only access H-channels.

The similar situation occurs on forced termination probability shown in Figure 5. Forced termination of a call is due to the lack of channel availability of the target base station when a call attempts to hand-off to that base station during its lifetime. As the ratio increases, voice call traffic is increased. More voice calls compete with limited channel resource as they hand-off, which results in a higher probability of forced termination of calls. On the other hand, the data traffic is decreased, so does the forced termination probability of data calls.

Figure 6 shows the dependence of carried traffic on demand. As the ratio is increased, the carried traffic of voice calls is increased and the carried traffic of data calls is decreased as expected and vice versa. The carried traffic of both voice and data calls is increased with the increase of new call origination rate.

10. Conclusions

The proposed multibeam medium access scheme can support multiple integrated services that demand different CIR qualities of channels. Two type of services, data and voice, are investigated. A 120^0-sectorized multibeam cellular system with two beams in each sector is considered. DCA is used to improve channel reuse of L-channels and to allocate different CIR quality channels within a channel layout incorporating two frequency reuse patterns. The use of DCA on L-channels significantly reduces the blocking of new voice calls in comparison with the system which uses fixed channel assignment. The blocking probability of data calls is also reduced due to the improvement in the overall channel usage efficiency achieved with DCA. Forced termination of both voice and data calls is reduced. It means that both voice and data calls will less likely experience the interruption of calls during their call lifetime in the system which use DCA compared with FCA. Voice calls have access to more channel resources than data calls due to the H-channel limitation for data calls. The blocking and forced termination probabilities of data calls are larger than those of voice calls. Blocking and forced termination probabilities of data calls are reduced as the data traffic generated is decreased, or alternatively as H-channel limitation for data calls is relaxed. A tractable analytical model using a state description and multidimensional birth-death processes is developed for the proposed scheme and can be used to calculate the theoretical teletraffic performance characteristics.

REFERENCE

[1] M. Umehira, M. Nakura, H. Sato and A. Hashimoto, "ATM Wireless Access for Mobile Multimedia: Concept and Architecture," IEEE Personal Communications, October 1996, pp. 39-48.

[2] S. W. Halpern, "Reuse Partitioning in Cellular Systems", IEEE Vehicular Technology Conference, pp. 322-327, 1983.

[3] S. C. Swales, M. A. Beach, D. J. Edwards, and J. P. McGeehan, "The Performance Enhancement of Multibeam Adaptive Base-station Antennas for Cellular Land Mobile Radio Systems", IEEE Trans. on Veh. Technol., vol. 39, Feb. 1990, pp. 56-67.

[4] A. F. Naguib and T. Kailath, "Capacity Improvement with Base-Station Antenna Arrays in Cellular CDMA", IEEE Transactions on Vehicular Technology, vol. 43, no. 3, August 1994, pp. 691-698.

[5] W. C. Y. Lee, "An Optimum Solution of the Switching Beam Antenna System", 47th Vehicular Technology Conference, 1997, pp.170-172.

[6] M. Zhang and T. -S. P. Yum, "Comparisons of Channel Assignment Strategies in Cellular Mobile Telephone Systems", IEEE Transactions on Vehicular Technology, vol. 38, no. 4, November 1989, pp. 211-215.

[7] S. S. Rappaport, "Blocking, Hand-off and Traffic Performance for Cellular Communication Systems with Mixed Platforms", IEE (UK) Proceedings, Part I, Communications, Speech and Vision, October 1993, Vol. 140, no5, pp. 389-401.

[8] S. S. Rappaport, "The Multiple-Call Hand-off Problem in High-Capacity Cellular Communications Systems," IEEE Transaction on Vehicular Technology, August 1991, Vol. VT-40, no. 3, pp.546-557.

[9] J. L. Pan, P. M. Djuric and S. S. Rappaport, "Multibeam Cellular Communication Systems with Dynamic Channel Assignment," to appear in Proc. of IEEE Vehicular Technology Conference, Ottawa, Canada, May 1998.

APPENDIX

The state transitions and their corresponding transition flows due to the driving processes are explained as follows.

1. New Call Arrivals

1.1 New voice call arrivals in beam 1 of the sector

A transition into state s due to a new voice call arrival in beam 1 of the sector when the basic element is in state x_n, will cause the state variable $v_L(x_n,1)$ to be incremented by 1. We note that the L-channels in the sector can be dynamically assigned to voice calls without regard to the beam through which they communicate. A new voice call can be served in beam 1 of the sector only if the number of L-channels in use in the sector does not exceed $2C_L$. Thus a permissible state x_n is a predecessor state of s for new call arrivals in beam 1 of the sector, if $v_L(x_n,1) + v_L(x_n,2) < 2C_L$, and the state variables are related by

$$\begin{aligned} &v_L(x_n,1) = v_L(s,1) - 1, \\ &v_L(x_n,2) = v_L(s,2), \\ &v_H(x_n,i) = v_H(s,i), && i = 1,2, \\ &d(x_n,i) = d(s,i), && i = 1,2. \end{aligned} \tag{1}$$

Let Λ_{nv} denote the average arrival rate of new voice calls in a beam. The flow into state s from x_n due to new voice call arrivals in the beam 1 of the sector is

$$\gamma_{n1}(s,x_n) = \Lambda_{nv}, \qquad \text{if } v_L(x_n,1) + v_L(x_n,2) < 2C_L. \tag{2}$$

We also note that voice calls can access H-channels if L-channels are not available. In this case, a transition into state s due to a new voice call arrival in beam 1 of the sector when the basic element is in state x_n, will cause the state variable $v_H(x_n,1)$ to be incremented by 1. A new voice call can be served in beam 1 of the sector only if the number of H-channels in use in the sector does not exceed C_H. Thus a permissible state x_n is a predecessor state of s for new call arrivals in beam 1 of the sector, if $v_L(x_n,1) + v_L(x_n,2) = 2C_L$ and $v_{H1} + d_1 < C_H$, and the state variables are related by

$$v_L(x_n,i) = v_L(s,i), \qquad i = 1,2$$

$$v_H(x_n,1) = v_H(s,1) - 1,$$
$$v_H(x_n,2) = v_H(s,2),$$
$$d(x_n,i) = d(s,i), \qquad i = 1,2. \qquad (3)$$

The flow into state s from x_n due to new voice call arrivals in the beam 1 of the sector is

$$\gamma_{n1}(s,x_n) = \Lambda_{nv}, \qquad \text{if } v_L(x_n,1) + v_L(x_n,2) = 2C_L \text{ and } v_{H1} + d_1 < C_H. \qquad (4)$$

1.2 New voice call arrivals in the beam 2 of the sector

A transition into state s due to a new voice call arrival in beam 2 of the sector when the basic element is in state x_n, will cause the state variable $v_L(x_n,2)$ to be incremented by 1. We note that the L-channels in the sector can be dynamically assigned to voice calls without regard to the beam through which they communicate. A new voice call can be served in beam 1 of the sector only if the number of L-channels in use in the sector does not exceed $2C_L$. Thus a permissible state x_n is a predecessor state of s for new call arrivals in beam 1 of the sector, if $v_L(x_n,1) + v_L(x_n,2) < 2C_L$, and the state variables are related by

$$v_L(x_n,1) = v_L(s,1),$$
$$v_L(x_n,2) = v_L(s,2) - 1,$$
$$v_H(x_n,i) = v_H(s,i), \qquad i = 1,2,$$
$$d(x_n,i) = d(s,i), \qquad i = 1,2. \qquad (5)$$

The flow into state s from x_n due to new voice call arrivals in the beam 1 of the sector is

$$\gamma_{n2}(s,x_n) = \Lambda_{nv}, \qquad \text{if } v_L(x_n,1) + v_L(x_n,2) < 2C_L. \qquad (6)$$

We also note that voice calls can access H-channels if L-channels are not available. In this case, a transition into state s due to a new voice call arrival in beam 1 of the sector when the basic element is in state x_n, will cause the state variable $v_H(x_n,2)$ to be incremented by 1. A new voice call can be served in beam 1 of the sector only if the number of H-channels in use in the beam 1 does not exceed C_H. Thus a permissible state x_n is a predecessor state of s for new call arrivals in beam 1 of the sector, if $v_L(x_n,1) + v_L(x_n,2) = 2C_L$ and $v_{H2} + d_2 < C_H$, and the state variables are related by

$$v_L(x_n,i) = v_L(s,i), \qquad i = 1,2$$
$$v_H(x_n,1) = v_H(s,1),$$
$$v_H(x_n,2) = v_H(s,2) - 1,$$

$$d(x_n,i) = d(s,i), \qquad i = 1,2. \qquad (7)$$

The flow into state s from x_n due to new voice call arrivals in the beam 1 of the sector is

$$\gamma_{n2}(s,x_n) = \Lambda_{nv}, \qquad \text{if } v_L(x_n,1) + v_L(x_n,2) = 2C_L \text{ and } v_{H2} + d_2 < C_H. \qquad (8)$$

1.3 New data call arrivals in the beam 1 of the sector

A transition into state s due to a new call arrival in the beam 1 of the sector when the basic element is in state x_n, will cause the state variable $d(x_n,1)$ to be incremented by 1. A new data call can be served in the beam 1 of the sector only if the number of channels in use in beam 1 does not exceed C_H. Thus a permissible state x_n is a predecessor state of s for new call arrivals in the beam 1 of the sector, if $v_H(x_n,1) + d(x_n,1) < C_H$ and the state variables are related by

$$\begin{aligned} &v_L(x_n,i) = v_L(s,i), && i = 1,2 \\ &v_H(x_n,i) = v_H(s,i), && i = 1,2, \\ &d(x_n,1) = d(s,1) - 1, && \\ &d(x_n,2) = d(s,2). && (12) \end{aligned}$$

The flow into state s from x_n due to new call arrivals in the beam 1 of the sector is

$$\gamma_{n3}(s,x_n) = \Lambda_{nd}, \quad \text{if } v_H(x_n,1) + d(x_n,1) < C_H. \qquad (13)$$

1.4 New data call arrivals in the beam 2 of the sector

A transition into state s due to a new call arrival in the beam 2 of the sector when the sector is in state x_n, will cause the state variable $d(x_n,2)$ to be incremented by 1. A new data call can be served in the beam 2 of the sector only if the number of channels in use in beam 2 does not exceed C_H. Thus a permissible state x_n is a predecessor state of s for new call arrivals in the beam 2 of the sector, if $v_H(x_n,2) + d(x_n,2) < C_H$ and the state variables are related by

$$\begin{aligned} &v_L(x_n,i) = v_L(s,i), && i = 1,2 \\ &v_H(x_n,i) = v_H(s,i), && i = 1,2, \\ &d(x_n,1) = d(s,1), && \\ &d(x_n,2) = d(s,2) - 1. && (14) \end{aligned}$$

The flow into state s from x_n due to new call arrivals in the beam 1 of the sector is

$$\gamma_{n4}(s,x_n) = \Lambda_{nd}, \quad \text{if } v_H(x_n,2) + d(x_n,2) < C_H. \qquad (15)$$

2. Call Completion

2.1 Voice call completion on L-channels in the beam 1 of the sector

A transition into state s due to a voice call completion on L-channels in the beam 1 of the sector when the sector is in state x_c, will cause the state variable $v_L(x_c,1)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for voice call completion in the beam 1 of the sector, if the state variables are related by

$$\begin{aligned} &v_L(x_n,1) = v_L(s,1)+1, \\ &v_L(x_n,2) = v_L(s,2), \\ &v_H(x_n,i) = v_H(s,i), \qquad i=1,2 \\ &d(x_n,i) = d(s,i), \qquad i=1,2. \end{aligned} \tag{16}$$

Let μ_{Cv} denote the average completion rate of a voice call. The flow into state s from x_c due to voice call completion in the beam 1 of the sector is

$$\gamma_{c1}(s,x_c) = \mu_{Cv} \cdot v_L(x_c,1) \tag{17}$$

2.2 Voice call completion on L-channels in the beam 2 of the sector

A transition into state s due to a voice call completion on the L-channels in the beam 2 of the sector when the sector is in state x_c, will cause the state variable $v_L(x_c,2)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for voice call completion in the beam 1 of the sector, if the state variables are related by

$$\begin{aligned} &v_L(x_n,1) = v_L(s,1), \\ &v_L(x_n,2) = v_L(s,2)+1, \\ &v_H(x_n,i) = v_H(s,i), \qquad i=1,2 \\ &d(x_n,i) = d(s,i), \qquad i=1,2. \end{aligned} \tag{18}$$

The flow into state s from x_c due to call completion in the beam 2 of the sector is

$$\gamma_{c2}(s,x_c) = \mu_{Cv} \cdot v_L(x_c,2) \tag{19}$$

2.3 Voice call completion on H-channels in the beam 1 of the sector

A transition into state s due to a voice call completion on H-channels in the beam 1 of the sector when the sector is in state x_c, will cause the state variable $v_H(x_c,1)$ to be decreased by 1.

Thus a permissible state x_c is a predecessor state of s for voice call completion on H-channels in the beam 1 of the sector, if the state variables are related by

$$\begin{aligned}
&v_L(x_n, i) = v_L(s, i), && i = 1{,}2 \\
&v_H(x_n, 1) = v_L(s, 1) + 1, && \\
&v_H(x_n, 2) = v_H(s, 2), && \\
&d(x_n, i) = d(s, i), && i = 1{,}2.
\end{aligned} \tag{20}$$

Let μ_{Cv} denote the average completion rate of a voice call. The flow into state s from x_c due to voice call completion on H-channels in the beam 1 of the sector is

$$\gamma_{c3}(s, x_c) = \mu_{Cv} \cdot v_H(x_c, 1) \tag{21}$$

2.4 Voice call completion on H-channels in the beam 2 of the sector

A transition into state s due to a voice call completion on the H-channels in the beam 2 of the sector when the sector is in state x_c, will cause the state variable $v_H(x_c, 2)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for voice call completion on H-channels in the beam 2 of the sector, if the state variables are related by

$$\begin{aligned}
&v_L(x_n, i) = v_L(s, i), && i = 1{,}2 \\
&v_H(x_n, 1) = v_H(s, 1), && \\
&v_H(x_n, 2) = v_H(s, 2) + 1, && \\
&d(x_n, i) = d(s, i), && i = 1{,}2.
\end{aligned} \tag{22}$$

The flow into state s from x_c due to voice call completion on H-channels in the beam 2 of the sector is

$$\gamma_{c4}(s, x_c) = \mu_{Cv} \cdot v_H(x_c, 2) \tag{23}$$

2.5 Data call completion (on H-channels) in the beam 1 of the sector

A transition into state s due to a data call completion in the beam 1 of the sector when the sector is in state x_c, will cause the state variable $d(x_c, 1)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for data call completion in the beam 1 of the sector, if the state variables are related by

$$\begin{aligned}
&v_L(x_n, i) = v_L(s, i), && i = 1{,}2 \\
&v_H(x_n, i) = v_H(s, i), && i = 1{,}2 \\
&d(x_n, 1) = d(s, 1) + 1, &&
\end{aligned}$$

$$d(x_n,2) = d(s,2),. \quad (24)$$

Let μ_{Cd} denote the average completion rate of a data call. The flow into state s from x_c due to data call completion on H-channels in the beam 1 of the sector is

$$\gamma_{c5}(s,x_c) = \mu_{Cd} \cdot d(x_c,1) \quad (25)$$

2.6 Data call completion (on H-channels) in the beam 2 of the sector

A transition into state s due to a data call completion in the beam 2 of the sector when the sector is in state x_c, will cause the state variable $d(x_c,2)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for data call completion in the beam 2 of the sector, if the state variables are related by

$$v_L(x_n,i) = v_L(s,i), \qquad i = 1,2$$

$$v_H(x_n,i) = v_H(s,i), \qquad i = 1,2$$

$$d(x_n,1) = d(s,1),$$

$$d(x_n,2) = d(s,2) + 1. \quad (26)$$

The flow into state s from x_c due to data call completion on H-channels in the beam 2 of the sector is

$$\gamma_{c6}(s,x_c) = \mu_{Cd} \cdot d(x_c,2) \quad (27)$$

3. Hand-off Arrivals (From Other Sectors)

3.1 Voice call hand-off arrivals in the beam 1 of the sector from other sectors

Let Λ_h be the average rate at which hand-off arrivals (from other sectors) impinge on the beam of a sector. Initially, these parameters are assumed to be known, but ultimately their values are computed as part of the solution algorithm. A transition into state s due to a voice call hand-off arrival in the beam 1 of the sector from other sectors when the sector is in state x_h, will cause the state variable $v_L(x_h,1)$ to be incremented by 1. Thus a permissible state x_h is a predecessor state of s for voice call hand-off arrivals in the beam 1 of the sector from other sectors, if $v_L(x_h,1) + v_L(x_h,2) < 2C_L$, and the state variables are related by

$$v_L(x_n,1) = v_L(s,1) - 1,$$

$$v_L(x_n,2) = v_L(s,2),$$

$$v_H(x_n,i) = v_H(s,i), \qquad i = 1,2$$

$$d(x_n,i) = d(s,i), \qquad i = 1,2. \quad (28)$$

The flow into state s from x_n due to voice call hand-off arrivals in the beam 1 of the sector from other sectors is

$$\gamma_{h1}(s,x_n)=\Lambda_h, \qquad \text{if } v_L(x_n,1)+v_L(x_n,2)<2C_L. \tag{29}$$

We also note that voice calls can access H-channels if L-channels are not available. In this case, a transition into state s due to a voice call hand-off arrival in beam 1 of the sector when the sector is in state x_n, will cause the state variable $v_H(x_n,2)$ to be incremented by 1. A voice call hand-off arrival can be served in beam 1 of the sector only if the number of H-channels in use in the sector does not exceed C_H. Thus a permissible state x_n is a predecessor state of s for voice call hand-off arrivals in beam 1 of the sector, if $v_L(x_n,1)+v_L(x_n,2)=2C_L$ and $v_{H1}+d_1<C_H$, and the state variables are related by

$$\begin{aligned}
&v_L(x_n,i)=v_L(s,i), && i=1,2\\
&v_H(x_n,1)=v_H(s,1)-1, &&\\
&v_H(x_n,2)=v_H(s,2), &&\\
&d(x_n,i)=d(s,i), && i=1,2.
\end{aligned} \tag{30}$$

The flow into state s from x_n due to hand-off arrivals of voice calls in the beam 1 of the sector is

$$\gamma_{h1}(s,x_n)=\Lambda_h, \qquad \text{if } v_L(x_n,1)+v_L(x_n,2)=2C_L \text{ and } v_{H1}+d_1<C_H. \tag{31}$$

3.2 Voice call hand-off arrivals in the beam 2 of the sector from other sectors

A transition into state s due to a voice call hand-off arrival in the beam 2 of the sector from other sectors when the sector is in state x_h, will cause the state variable $v_L(x_h,2)$ to be incremented by 1. Thus a permissible state x_h is a predecessor state of s for voice call hand-off arrivals in the beam 1 of the sector from other sectors, if $v_L(x_h,1)+v_L(x_h,2)<2C_L$, and the state variables are related by

$$\begin{aligned}
&v_L(x_n,1)=v_L(s,1), &&\\
&v_L(x_n,2)=v_L(s,2)-1, &&\\
&v_H(x_n,i)=v_H(s,i), && i=1,2\\
&d(x_n,i)=d(s,i), && i=1,2.
\end{aligned} \tag{32}$$

The flow into state s from x_n due to voice call hand-off arrivals in the beam 2 of the sector from other sectors is

$$\gamma_{h2}(s,x_n)=\Lambda_h, \qquad \text{if } v_L(x_n,1)+v_L(x_n,2)<2C_L. \tag{33}$$

We also note that voice calls can access H-channels if L-channels are not available. In this case, a transition into state s due to a voice call hand-off arrival in beam 2 of the sector when the sector is in state x_h, will cause the state variable $v_H(x_h,2)$ to be incremented by 1. A voice call hand-off arrival can be served in beam 2 of the sector only if the number of H-channels in use in the sector does not exceed C_H. Thus a permissible state x_n is a predecessor state of s for voice call hand-off arrivals in beam 2 of the sector, if $v_L(x_n,1)+v_L(x_n,2)=2C_L$ and $v_{H1}+d_1<C_H$, and the state variables are related by

$$\begin{aligned}
&v_L(x_n,i)=v_L(s,i), && i=1,2\\
&v_H(x_n,1)=v_H(s,1), &&\\
&v_H(x_n,2)=v_H(s,2)-1, &&\\
&d(x_n,i)=d(s,i), && i=1,2. \qquad (34)
\end{aligned}$$

The flow into state s from x_n due to hand-off arrivals of voice calls in the beam 2 of the sector is

$$\gamma_{h2}(s,x_n)=\Lambda_h, \qquad \text{if } v_L(x_n,1)+v_L(x_n,2)=2C_L \text{ and } v_{H2}+d_2<C_H. \qquad (35)$$

3.3 Hand-off arrivals of data calls in the beam 1 of the sector from other sectors

A transition into state s due to a hand-off arrival of data call in the beam 1 of the sector when the sector is in state x_h, will cause the state variable $d(x_h,1)$ to be incremented by 1. A hand-off arrival of data call can be served in the beam 1 of the sector only if the number of H-channels in use in beam 1 does not exceed C_H. Thus a permissible state x_h is a predecessor state of s for hand-off arrivals of data calls in the beam 1 of the sector, if $v_H(x_h,1)+d(x_h,1)<C_H$ and the state variables are related by

$$\begin{aligned}
&v_L(x_h,i)=v_L(s,i), && i=1,2\\
&v_H(x_h,i)=v_H(s,i), && i=1,2,\\
&d(x_h,1)=d(s,1)-1, &&\\
&d(x_h,2)=d(s,2). && \qquad (36)
\end{aligned}$$

The flow into state s from x_h due to hand-off arrivals of data calls in the beam 1 of the sector from other sectors is

$$\gamma_{h3}(s,x_h)=\Lambda_h, \quad \text{if } v_H(x_h,1)+d(x_h,1)<C_H. \qquad (37)$$

3.4 Hand-off arrivals of data calls in the beam 2 of the sector from other sectors

A transition into state s due to a hand-off arrival of data call in the beam 2 of the sector when the sector is in state x_h, will cause the state variable $d(x_h,2)$ to be incremented by 1. A hand-off arrival of data call can be served in the beam 2 of the sector only if the number of H-channels in use in beam 2 does not exceed C_H. Thus a permissible state x_h is a predecessor state of s for hand-off arrivals of data calls in the beam 2 of the sector, if $v_H(x_h,2)+d(x_h,2)<C_H$ and the state variables are related by

$$\begin{aligned}
&v_L(x_h,i)=v_L(s,i), && i=1,2\\
&v_H(x_h,i)=v_H(s,i), && i=1,2,\\
&d(x_h,1)=d(s,1),\\
&d(x_h,2)=d(s,2)-1. && \quad (38)
\end{aligned}$$

The flow into state s from x_h due to hand-off arrivals of data calls in the beam 2 of the sector from other sectors is

$$\gamma_{h4}(s,x_h)=\Lambda_h, \ \text{ if } v_H(x_h,2)+d(x_h,2)<C_H. \quad (39)$$

4. Hand-off Departures (To Other Sectors)

4.1 Hand-off departures of voice calls on L-channels from the beam 1 of the sector to other sectors

A transition due to a hand-off departure of a voice call on L-channels from the beam 1 of the sector to other sectors when the sector is in state x_d, will cause the state variable $v_L(x_d,1)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a voice call on L-channels from the beam 1 of the sector to other sectors, if the state variables are related by

$$\begin{aligned}
&v_L(x_d,1)=v_L(s,1)+1,\\
&v_L(x_d,2)=v_L(s,2)\\
&v_H(x_d,i)=v_H(s,i), && i=1,2\\
&d(x_d,i)=d(s,i), && i=1,2. \quad (40)
\end{aligned}$$

Let F_1 denote the fraction of hand-off departures from a beam of a sector to other sectors. Thus the corresponding transition flow is given by

$$\gamma_{d1}(s,x_d)=\mu_D\cdot F_1\cdot v_L(x_d,1) \quad (41)$$

4.2 Hand-off departures of voice calls on L-channels from the beam 2 of the sector to other sectors

A transition due to a hand-off departure of a voice call on L-channels from the beam21 of the sector to other sectors when the sector is in state x_d, will cause the state variable $v_L(x_d,2)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a voice call on L-channels from the beam 2 of the sector to other sectors, if the state variables are related by

$$\begin{aligned}
&v_L(x_d,1) = v_L(s,1), \\
&v_L(x_d,2) = v_L(s,2) + 1 \\
&v_H(x_d,i) = v_H(s,i), && i = 1,2 \\
&d(x_d,i) = d(s,i), && i = 1,2.
\end{aligned} \tag{42}$$

The corresponding transition flow is given by

$$\gamma_{d2}(s,x_d) = \mu_D \cdot F_1 \cdot v_L(x_d,2) \tag{43}$$

4.3 Hand-off departures of voice calls on H-channels from the beam 1 of the sector to other sectors

A transition due to a hand-off departure of a voice call on H-channels from the beam 1 of the sector to other sectors when the sector is in state x_d, will cause the state variable $v_H(x_d,1)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a voice call on H-channels from the beam 1 of the sector to other sectors, if the state variables are related by

$$\begin{aligned}
&v_L(x_d,i) = v_L(s,i), && i = 1,2 \\
&v_H(x_d,1) = v_H(s,1) + 1 \\
&v_H(x_d,2) = v_H(s,2), \\
&d(x_d,i) = d(s,i), && i = 1,2.
\end{aligned} \tag{43}$$

The corresponding transition flow is given by

$$\gamma_{d3}(s,x_d) = \mu_D \cdot F_1 \cdot v_H(x_d,1) \tag{44}$$

4.4 Hand-off departures of voice calls on H-channels from the beam 2 of the sector to other sectors

A transition due to a hand-off departure of a voice call on H-channels from the beam 2 of the sector to other sectors when the sector is in state x_d, will cause the state variable $v_H(x_d,2)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a voice call on H-channels from the beam 2 of the sector to other sectors, if the state variables are related by

$$\begin{aligned}
&v_L(x_d,i)=v_L(s,i), && i=1,2\\
&v_H(x_d,1)=v_H(s,1)\\
&v_H(x_d,2)=v_H(s,2)+1,\\
&d(x_d,i)=d(s,i), && i=1,2.
\end{aligned} \tag{45}$$

The corresponding transition flow is given by

$$\gamma_{d4}(s,x_d)=\mu_D\cdot F_1\cdot v_H(x_d,2) \tag{46}$$

4.5 Hand-off departures of data calls from the beam 1 of the sector to other sectors

A transition due to a hand-off departure of a data call (on H-channels) from the beam 1 of the sector to other sectors when the sector is in state x_d, will cause the state variable $d(x_d,1)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a data call from the beam 1 of the sector to other sectors, if the state variables are related by

$$\begin{aligned}
&v_L(x_d,i)=v_L(s,i), && i=1,2\\
&v_H(x_d,i)=v_H(s,i), && i=1,2\\
&d(x_d,1)=d(s,1)+1,\\
&d(x_d,2)=d(s,2),.
\end{aligned} \tag{47}$$

The corresponding transition flow is given by

$$\gamma_{d5}(s,x_d)=\mu_D\cdot F_1\cdot d(x_d,1) \tag{48}$$

4.6 Hand-off departures of data calls from the beam 2 of the sector to other sectors

A transition due to a hand-off departure of a data call (on H-channels) from the beam 2 of the sector to other sectors when the sector is in state x_d, will cause the state variable $d(x_d,2)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a data call from the beam 2 of the sector to other sectors, if the state variables are related by

$$v_L(x_d, i) = v_L(s, i), \qquad i = 1,2$$

$$v_H(x_d, i) = v_H(s, i), \qquad i = 1,2$$

$$d(x_d, 1) = d(s,1),$$

$$d(x_d, 2) = d(s,2) + 1,. \tag{49}$$

The corresponding transition flow is given by

$$\gamma_{d6}(s, x_d) = \mu_D \cdot F_1 \cdot d(x_d, 2) \tag{50}$$

5. Call Hand-offs (Between Beams of the Same Sector)

5.1 Voice call hand-offs on L-channels in the beam 1 from the beam 2 of the same sector

A transition into state s due to a voice call hand-off on L-channels in the beam 1 from the beam 2 of the same sector when the sector is in state x_{hd}, will cause the state variable $v_L(x_{hd}, 1)$ to be incremented by 1, and $v_L(x_{hd}, 2)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for voice call hand-off arrivals on L-channels in the beam 1 due to hand-off departures from the beam 2 of the same sector, if the state variables are related by

$$v_L(x_{hd}, 1) = v_L(s,1) - 1,$$

$$v_L(x_{hd}, 2) = v_L(s,2) + 1,$$

$$v_H(x_{hd}, i) = v_H(s, i), \qquad i = 1,2$$

$$d(x_{hd}, i) = d(s, i), \qquad i = 1,2 \tag{51}$$

Let F_2 denote the fraction of hand-off departures from a beam to the other beam of the same sector. The flow into state s from x_{hd} due to successful voice call hand-offs in the beam 1 from the beam 2 of the sector is

$$\gamma_{hd1}(s, x_{hd}) = \mu_D \cdot v_L(x_{hd}, 2) \cdot F_2 \tag{52}$$

5.2 Voice call hand-offs on L-channels in the beam 2 from the beam 1 of the same sector

A transition into state s due to a voice call hand-off on L-channels in the beam 2 from the beam 1 of the same sector when the sector is in state x_{hd}, will cause the state variable $v_L(x_{hd}, 2)$ to be incremented by 1, and $v_L(x_{hd}, 1)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for voice call hand-off arrivals on L-channels in the beam 2 due to hand-off departures from the beam 1 of the same sector, if the state variables are related by

$$v_L(x_{hd}, 1) = v_L(s,1) + 1,$$

$$v_L(x_{hd}, 2) = v_L(s,2) - 1,$$

$$\begin{aligned} &v_H(x_{hd}, i) = v_H(s, i), && i = 1,2 \\ &d(x_{hd}, i) = d(s, i), && i = 1,2 \end{aligned} \tag{53}$$

The flow into state s from x_{hd} due to successful voice call hand-offs in the beam 2 from the beam 1 of the sector is

$$\gamma_{hd2}(s, x_{hd}) = \mu_D \cdot v_L(x_{hd}, 1) \cdot F_2 \tag{54}$$

5.3 Voice call hand-offs on H-channels in the beam 1 from the beam 2 of the same sector

At the time of a voice call hand-off arrival on H-channels in beam 1, there are three possibilities. 1) If there are available L-channels, then the voice call will switch to use L-channels from H-channels. 2) If there is no available L-channels, the voice call will continue using H-channels. 3) If there is no channel available for both L and H-channels, the hand-off of voice call will fail. A transition into state s due to a successful voice call hand-off on H-channels in the beam 1 from the beam 2 of the same sector when the sector is in state x_{hd}, will cause the state variable $v_H(x_{hd}, 1)$ to be incremented by 1, and $v_H(x_{hd}, 2)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for successful voice call hand-off arrivals in the beam 1 due to hand-off departures from the beam 2 of the same sector, if the state variables are related by

$$\begin{aligned} &v_L(x_{hd}, i) = v_L(s, i), && i = 1,2 \\ &v_H(x_{hd}, 1) = v_H(s, 1) - 1, && \\ &v_H(x_{hd}, 2) = v_H(s, 2) + 1, && \\ &d(x_{hd}, i) = d(s, i), && i = 1,2 \end{aligned} \tag{55}$$

The flow into state s from x_{hd} due to successful voice call hand-offs in the beam 1 from the beam 2 of the sector is

$$\gamma_{hd3}(s, x_{hd}) = \mu_D \cdot v_H(x_{hd}, 2) \cdot F_2 \tag{56}$$

5.4 Voice call hand-offs on H-channels in the beam 2 from the beam 1 of the same sector

A transition into state s due to a voice call hand-off on H-channels in the beam 2 from the beam 1 of the same sector when the sector is in state x_{hd}, will cause the state variable $v_H(x_{hd}, 2)$ to be incremented by 1, and $v_H(x_{hd}, 1)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for voice call hand-off arrivals on H-channels in the beam 2 due to hand-off departures from the beam 1 of the same sector, if the state variables are related by

$$
\begin{aligned}
&v_L(x_{hd},i) = v_L(s,i)\,, && i=1,2\\
&v_H(x_{hd},1) = v_H(s,1)+1\,,\\
&v_H(x_{hd},2) = v_H(s,2)-1\,,\\
&d(x_{hd},i) = d(s,i)\,, && i=1,2
\end{aligned}
\tag{57}
$$

The flow into state s from x_{hd} due to successful voice call hand-offs in the beam 2 from the beam 1 of the sector is

$$\gamma_{hd4}(s,x_{hd}) = \mu_D \cdot v_H(x_{hd},1)\cdot F_2 \tag{58}$$

5.5 Data call hand-offs on H-channels in the beam 1 from the beam 2 of the same sector

A transition into state s due to a data call hand-off on H-channels in the beam 1 from the beam 2 of the same sector when the sector is in state x_{hd}, will cause the state variable $d(x_{hd},1)$ to be incremented by 1, and $d(x_{hd},2)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for data call hand-off arrivals on H-channels in the beam 1 due to hand-off departures from the beam 2 of the same sector, if the state variables are related by

$$
\begin{aligned}
&v_L(x_{hd},i) = v_L(s,i)\,, && i=1,2\\
&v_H(x_{hd},i) = v_H(s,i)\,, && i=1,2\\
&d(x_{hd},1) = d(s,1)-1\,,\\
&d(x_{hd},2) = d(s,2)+1\,,
\end{aligned}
\tag{59}
$$

The flow into state s from x_{hd} due to data call hand-offs in the beam 1 from the beam 2 of the sector is

$$\gamma_{hd5}(s,x_{hd}) = \mu_D \cdot d(x_{hd},2)\cdot F_2 \tag{60}$$

5.6 Data call hand-offs on H-channels in the beam 2 from the beam 1 of the same sector

A transition into state s due to a data call hand-off on H-channels in the beam 2 from the beam 1 of the same sector when the sector is in state x_{hd}, will cause the state variable $d(x_{hd},2)$ to be incremented by 1, and $d(x_{hd},1)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for data call hand-off arrivals on H-channels in the beam 2 due to hand-off departures from the beam 1 of the same sector, if the state variables are related by

$$
\begin{aligned}
&v_L(x_{hd},i) = v_L(s,i)\,, && i=1,2\\
&v_H(x_{hd},i) = v_H(s,i)\,, && i=1,2\\
&d(x_{hd},1) = d(s,1)+1\,,
\end{aligned}
$$

$$d(x_{hd},2) = d(s,2) - 1, \tag{61}$$

The flow into state s from x_{hd} due to data call hand-offs in the beam 2 from the beam 1 of the sector is

$$\gamma_{hd6}(s, x_{hd}) = \mu_D \cdot d(x_{hd},1) \cdot F_2 \tag{62}$$

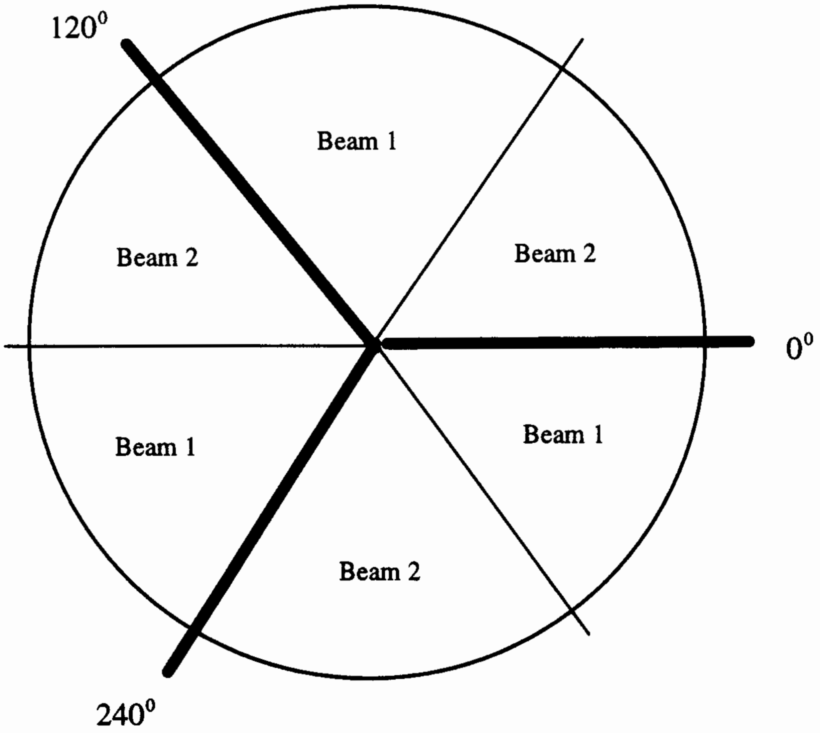

Fig. 1. The beam layout of 120^0-sectorized multibeam cellular communication systems with 2 beams in each sector.

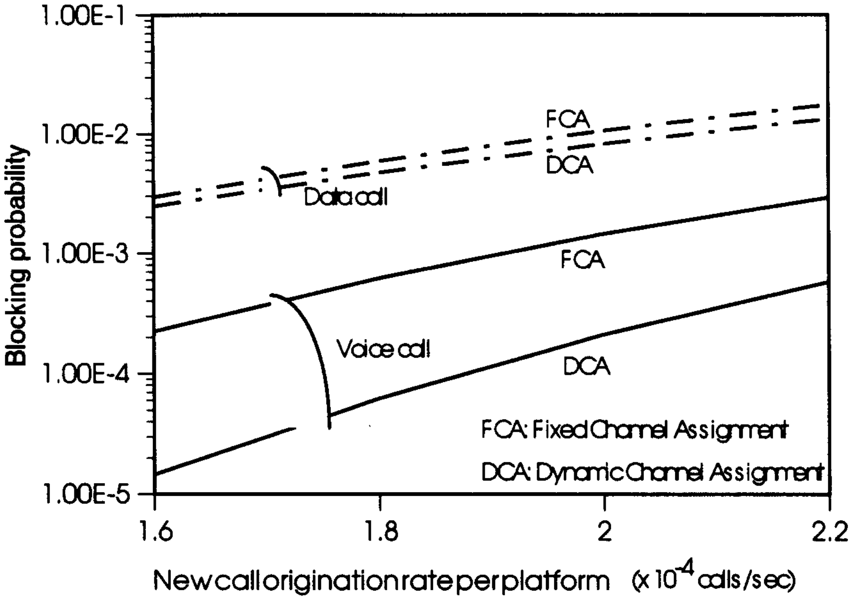

Fig 2. Blocking probability depends on demand

Parameters: Channel limit per beam C=14, C_H=8, T_C(voice)=100 s, T_C(data)=100 s, T_D=100 s, number of platforms per beam = 300, α=1/1 (voice/data new call origination ratio).

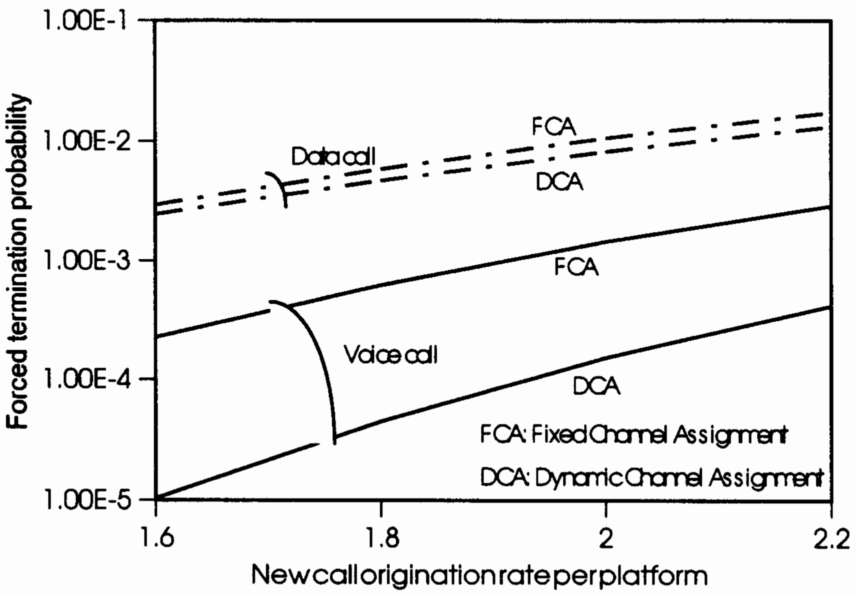

Fig 3. Forced termination probability depends on demand

Parameters: Channel limit per beam C=14, C_H=8, T_C(voice)=100 s, T_C(data)=100 s, T_D=100 s, number of platforms per beam = 300, α=1/1 (voice/data new call origination ratio).

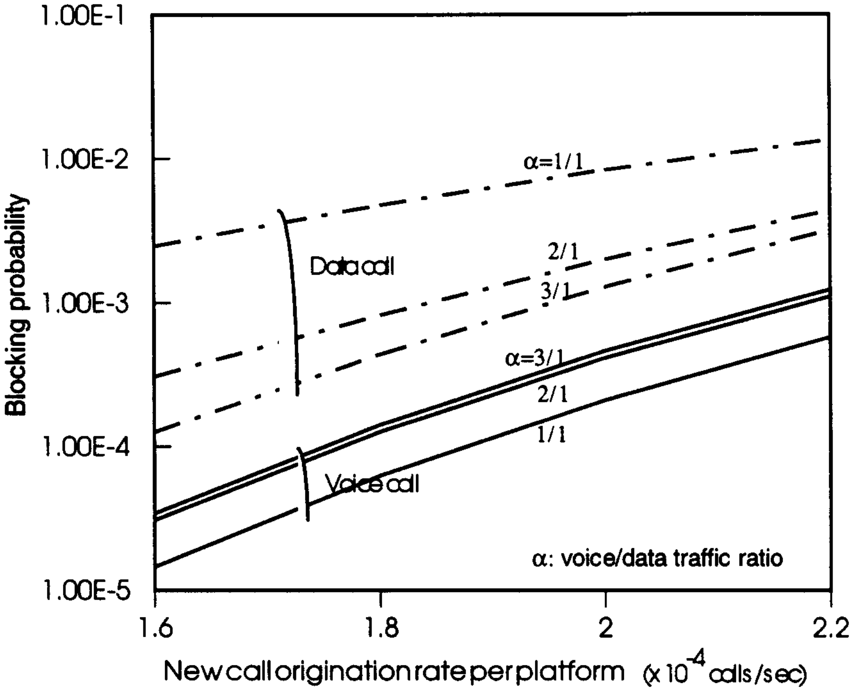

Fig 4. Blocking probability depends on demand

Parameters: channel limit per beam C=14, C_H=8, T_C(voice)=100 s, T_C(data)=100 s, T_D=100 s, number of platforms per beam = 300

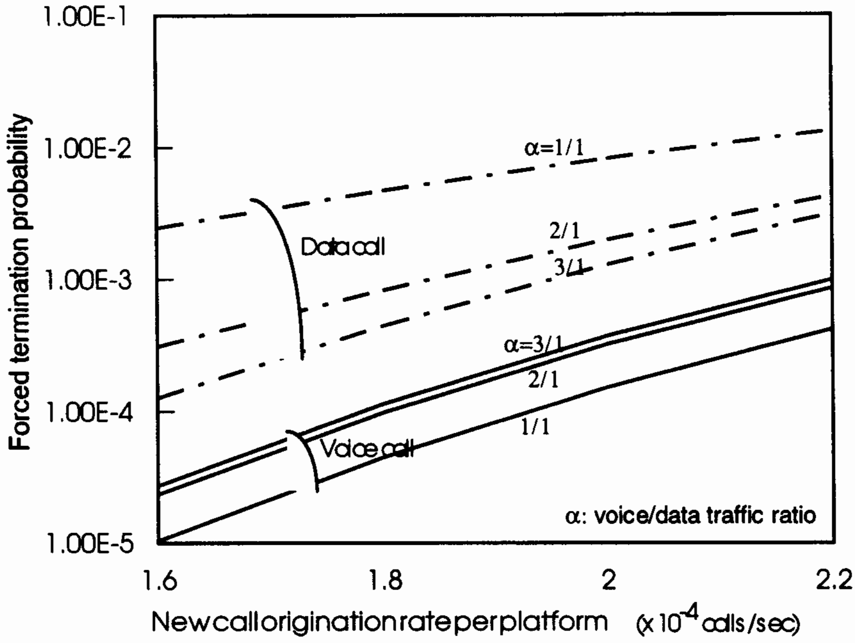

Fig 5. Forced termination probability depends on demand

Parameters: channel limit per beam C=14, C_H=8, T_C(voice)=100 s, T_C(data)=100 s, T_D=100 s, number of platforms per beam = 300

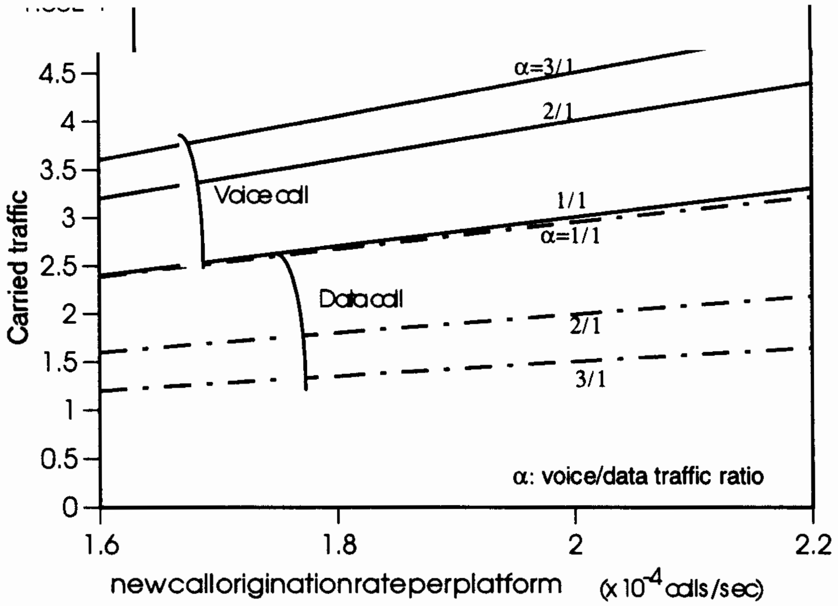

Fig 6. Carried traffic depends on demand

Parameters: channel limit per beam $C=14$, $C_H=8$, $T_C(\text{voice})=100$ s, $T_C(\text{data})=100$ s, $T_D=100$ s, number of platforms per beam = 300