STATE UNIVERSITY OF NEW YORK

AT STONY BROOK

COLLEGE OF ENGINEERING

Report No. 91

DISCRETE SYSTEMS AND DIGITAL COMPUTER CONTROL

by

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May, 1967

APPLIED MECHANICS REVIEWS

VOL. 20, NO. 5 MAY 1967

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Introduction

Digital computers are finding more and more use in the control of large complex systems because of their versatility and accuracy. It is expected that this trend will continue, and the future development of control science will be centered around digital technology.

In general, a digital computer may serve any or all of three separate functions in a control system:

(1) Data processing and monitoring, e.g., computation of system which cannot be directly measured, data logging, scanning, and supervisory control, etc.;

(2) Direct control or compensation of system dynamics; and

(3) Implementation of adaptive or learning loop.

Of the three functions mentioned, (1) is simply using a computer as a computer. The present review is concerned essentially with (2) and (3), in which the computer is used for direct *on line* control.

System Considerations

There are a few unique system considerations in digital computer control, namely:

1. Data sampling and reconstruction

Since the computer takes its inputs in the form of a sequence of numbers, the input signal is sampled, normally at equal intervals. The output from computer is a sequence of numbers. It must be reconstructed to a continuous analog signal before it can be applied to the input of a continuous plant. Normally this is done by a *data hold* which holds the signal for one sampling period.

2. Time Sharing

Quite often a large number of independent systems are controlled by a single digital computer on a time sharing basis. It can be done in two ways:

(i) The computer is assigned to each controlled system at some fixed instants in turn. Sufficient time for computation must be allowed to each system, and the sampling time is unnecessarily large.

(ii) The computation is done continuously, and the computed results are stored and sampled at preassigned times. There is no waste of computer time, however, there is an extra delay in the sampled output from the computer.

(iii) The computer is assigned to each controlled system in turn. There is no waste of computer time, and only a minimum of computer delay. However, the sampling periods for each system becomes random and fluctuable.

3. Quantizing error in computations and in analog-to-digital conversion.

The quantizing error is significant when there is not a sufficient number of significant figures. To estimate its effect, one may regard each place where roundoff occurs as a element of unity gain with an additive noise of mean square value

$$\overline{n(t)^2} = \frac{q^2}{12}$$

where q is the value represented by a change of one in the lowest digit. The equation is valid if the system is sufficiently exercised so that the average change in successive samples is many times q. Otherwise, nonlinear oscillations may occur, and the amplitude of oscillation may reach a few times q [14], [155], [156].

From the given system considerations, it readily is seen that the quantization error is negligible when a general purpose computer is used, and the problem of time sharing is basically an engineering design problem once the basic problem of discrete control is solved. The major problem in digital computer control is that both the measured data and computer output are *sampled*. It is referred to in the literature as *sampled-data system*, or *discrete (time) control* system.

The development of discrete control theory can be divided into three stages. In the first stage, methods are developed which treat some aspects of a discrete system as that of a continuous system. In the second stage, discrete control technology is developed parallel with that of continuous control. In the third stage, computer control moves ahead of continuous control, not only in theory but also in doing many things which a continuous control has no possibility of doing.

Mathematical Representation of Sampling and Data Reconstruction

The first important development is the z-transform method developed independently by Barker, Ragazzini, and Zadeh, which makes it possible to analyze the effect produced by discrete or sampled signal on continuous systems using a transform method which is a slight modification of the Laplace transform method and can be used in conjunction with the Laplace transform [4], [122].

A sampled-signal $r^*(t)$ of the continuous signal $r(t)$ is represented as:

$$r^*(t) \equiv \sum_n r(nT)\delta(t - nT) = r(t) \sum_n \delta(t - nT). \tag{1}$$

It is noted that the sequence of numbers $r(nT)$ is not a signal because it cannot be applied to a continuous system to produce any meaningful result, but $r^*(t)$ is a signal in the given sense. For instance, if $r^*(t)$ is applied to a circuit with an impulse response $h(t)$:

$$\begin{aligned} h(t) &= 1 \qquad 0 < t \leq T \\ &= 0 \qquad t \leq 0,\ t > T. \end{aligned} \tag{2}$$

The output is a stepwise signal $m(t) = r(nT)$, $nT < t \leq (n + 1)T$.

The effect of sampling can be visualized by performing a Laplace transform on $r^*(t)$. Since

$$\sum_{n=\infty}^{n=\infty} \delta(t - nT) = \frac{1}{T} \sum_{n=-\infty}^{n=\infty} e^{jn\omega_s t} \tag{3}$$

where $\omega_s = 2\pi/T$.

$$\begin{aligned} R^*(s) \equiv L\{r^*(t)\} &= \frac{1}{T} \sum_{n=-\infty}^{n=\infty} L\{r(t)e^{-jn\omega_s t}\} \\ &= \frac{1}{T} \sum_{n=-\infty}^{n=\infty} R(s + jn\omega_s). \end{aligned} \tag{4}$$

In (4), $R(s)$ is the Laplace transform of $r(t)$. Equation (4) shows that one effect of sampling is to create many side bands which are exact duplicates of the spectrum of $r(t)$ but are shifted along the frequency axis by multiples of ω_s.

The result gives some insight on the required sampling rate for any given $r(t)$. In an overly simplified case, $r(t)$ is strictly band-limited: $R(j\omega) = 0$, for all $\omega \geq \omega_b$. If $\omega_s > 2\omega_b$ the sidebands are completely separated from the main spectrum, and $r(t)$ can be recovered from $r^*(t)$ by filtering out the sidebands. The condition $\omega_s > 2\omega_b$ is the same as

$$\frac{1}{T} > 2f_b \tag{5}$$

where $f_b = \omega_b/2\pi$. That (5) is the necessary and sufficient condition for errorless reconstruction of $r(t)$ from $r^*(t)$ is known as Shannon's sampling theorem [130].

In general, a signal $r(t)$ is not strictly band-limited. The sampling rate ω_s is usually selected so that only a negligible amount of power lies outside the frequency range $0 - \omega_s/2$ radians.

Linear Autonomous Systems

It is obvious from (4) that each pole s_1 in $R(s)$ corresponds to infinitely many poles in $R^*(s)$ at $s_1 + jn\omega_s$, $n = -\infty \ldots . -1,0,1,2 \ldots \infty$. The task of using Laplace transform to study sampled systems is formidable. However, if one substitutes the new variable $z = e^{sT}$ in $R^*(s)$, each pole s_1 in $R(s)$ becomes a single pole $z_1 \equiv e^{s_1 T}$ in the new function R^*. In z-transform literature, this new function is denoted as $R(z)$. It is worthy of note that $R(z)$ is not the same function $R(s)$ as is illustrated by the following example:

$$r(t) = e^{-at}$$

$$R(s) = \frac{1}{s + a}$$

$$r^*(t) = \sum_n e^{-anT}\,\delta(t - nT)$$

$$R^*(s) = \sum_n e^{-anT}e^{-nTs} = \frac{1}{1 - e^{-aT}e^{-sT}}$$

$$R(z) = \frac{1}{1 - e^{-aT}z^{-1}}$$

For a continuous system to be stable, all the poles of the system function $F(s)$ are located in the LHP (left half plane). In a sampled-system, the entire s-plane is mapped into a strip of width ω_s in the complex plane of $F^*(s)$, and the LHP is mapped into a half strip, which is, in turn, mapped into a unit circle in the z-plane.

The unique correspondence between the LHP of $F(s)$ and unit circle of $F(z)$ makes it possible to modify conventional techniques for analysis and design of continuous systems, such as root locus method, polar plot, Bode plot, Routh-Hurwitz criterion, and Nyquist criterion, to do the same for sampled systems [80], [81], [83], [84], [85], [96], [97], [108]. A further step is represented by development of techniques which apply only to sampled systems: studies of the basic limitations of closed-loop sampled-systems and design for a specified performance within these limitations, systems which are error free (at sampling instants), and ripple-free (errorless at all times) within a few sampling periods after the application of a deterministic input, and the introduction of a staleness factor to slow down the system response so that saturation will not become a serious problem [10], [26], [64], [65], [125].

Consider a feedback system with the sampled value $e^*(t)$ of the system error $e(t)$ as an input to the computer and the computer output $m^*(t)$ is fed to a data hold element. The output of the data hold element is the input of the controlled plant. A plant output controlled variable $c(t)$ is subtracted from the reference value $r(t)$ of the controlled variable to obtain $e(t)$. The system is assumed to be linear with plant transfer function $G_p(s)$ and hold circuit $H(s)$, and the computer is represented by the pulse transfer function $D(z)$. Let $G(z) \equiv HG_p(z)$. The system transfer function $K(z)$ is then

$$K(z) = \frac{C(z)}{R(z)} = \frac{D(z)\,G(z)}{1 + D(z)\,G(z)}. \qquad (6)$$

It is shown by Bergen and Ragazzini [107] that for $D(z)$ to be realizable, $F(z)$ must contain the same delay as $G(z)$, and all the zeros of $G(z)$ outside the unit circle as its zeros. To be more precise, let $Q(z)$ denote a function of which the reciprocal $1/Q(z)$ is analytic for all $|z| > 1$. If

$$G(z) = z^{-m} \prod_i (1 - \lambda_i z^{-1})\, Q(z) \qquad (7)$$

where $|\lambda_i| > 1$, and none of λ_i is a pole of $Q(z)$, then for a realizable stable closed-loop system $K(z)$ is necessarily of the form

$$K(z) = z^{-m} \prod_i (1 - \lambda_i z^{-1})\, F(z) \qquad (8)$$

where $F(z)$ is analytic for $|z| \geq 1$.

The basic ideas of error-free and ripple-free systems are as follows: Let $r(t)$ denote a deterministic input function. If $r(t)$ is a polynomial of t of the k-th degree, $R(z)$ has an $(k+1)$-th order pole at $z = 1$. Since

$$E(z) = [1 - K(z)]\, R(z) \qquad (9)$$

one can obtain an error-free system by selecting a $K(z)$ which satisfies both (8) and (10)

$$1 - K(z) = (1 - z^{-1})^{k+1}\, P(z) \qquad (10)$$

where $P(z)$ is a polynomial in z^{-1}. From (9) and (10), one sees that $E(z)$ is also a polynomial in z^{-1}, and consequently $e(nT) = 0$ for any n higher than the order of $E(z)$ in z^{-1}.

To obtain a ripple-free design, one needs only to modify (8) so that the set $\{\lambda_i\}$ include *all* zeros of $G(z)$ instead of only the zeros outside the unit circle. The modified (8) and (10) specify a ripple-free design.

If one chooses $P(z)$ to be a polynomial of minimum order among the ones which satisfy (8) and (10) simultaneously, the system is called a prototype system. Prototype systems have minimum settling time in which the error is reduced to zero. However, it is not always the most desirable because it usually has high overshoots and also large values for $m^*(t)$. To overcome these difficulties, a higher-order polynomial is selected as $P(z)$, and the extra coefficients are used to minimize the overshoot or $m^*(t)$ or both [26], [125].

There are various extensions of the basic theory: (1) multi-rate systems where the computer output samples are more frequent than its input, so that the hold circuit delay can be reduced [72], [73], [74]; (2) modified z-transform, which allows the ripple between sampling instants to be determined mathematically so that it can be taken into consideration in system design [86], [90], [94], [125], [139]; (3) the extension of Wiener's technique of optimum filtering to sampled systems [47], [48]; and (4) statistical design techniques for minimizing the mean square error for systems with random inputs, or minimizing the integral value of a quadratic cost function for systems with deterministic inputs [23], [24], [26], [141]. In both cases, the averaging can be over sampled errors only or over errors at all times, and the minimization is carried out under various constraints: plant input saturation, and the condition of Eq. (8), etc. A noteworthy point is that while the modified z-transform is very useful in computing error in between sampling, it is not the only method by which one can design a system with some control over the error at all times. The technique for analyzing multi-rate systems can be used to determine the error at a number of subintervals. A ripple-free design results in a system with no error at all after an initial period, and a least mean square error design results in a system with least mean square error averaging over all times.

Linear Time Varying Systems

One advantage of the z-transform method is its simplicity. Most problems can be solved analytically, and the resulting $D(z)$ is realizable as a recurrence relation which takes very few operations and very little computer memory. Its major use, however, is limited to linear, autonomous systems. For nonlinear or time varying systems, the method becomes cumbersome and difficult to work with. An age-old concept, which is at least as old as Newtonian mechanics, becomes the basic tool of analysis. The state of a dynamical system is represented completely by a state vector $x = (x_1, x_2 \ldots x_n)$ in the sense that:

1. x as a function of time describes the motion of the system;
2. The future motion of the system depends only on the present value of x and the input variable u from now on, and is independent of past history.

The dynamics of a continuous system is represented by

$$\frac{dx}{dt} = f_c(x, u, t). \qquad (11)$$

The dynamics of a discrete system is represented by

$$x(t+T) = f(x(t), u(t), t) \quad t = 0, T, 2T \ldots. \qquad (12)$$

If a continuous system is sampled, the sampled $x(t)$ obviously satisfies an equation of the form (12), and f is the solution of (11) at $t + T$ later with the initial value $x(t)$. As an example, let

$$f_c(x, u, t) = Fx + Bu \qquad (13)$$

where F and B are constant matrices, and $u(t)$ is a constant for $nT \leq t < (n+1)T$, then

$$f(x(t), u(t), t) = e^{FT} x(t) + F^{-1}(e^{FT} - 1) B u(t). \qquad (14)$$

For linear time-varying systems with a quadratic cost function, Kalman worked out what is now called Kalman Filter and a duality principle for optimum filtering and control [67], [68]. This principle was later extended by other investigators to various applications [22, 30, 57, 82, 140]. The random process to be estimated is described by

$$\frac{dx}{dt} = F(t)x + w. \qquad (15)$$

Measurements $y(t_i)$ are made at random instants t_i

$$y(t_i) = H(t_i)\, x(t_i) + v(t_i). \tag{16}$$

The best estimate of x is given as $\bar{x}$:

$$\frac{d\bar{x}}{dt} = F(t)\bar{x} + \sum_i K(t_i)\,[y(t_i) - H(t_i)\bar{x}(t_i-)]\,\delta(t-t_i). \tag{17}$$

Equation (17) has the following meaning:

In between measurements, $\bar{x}$ follows the same dynamics as x is (15) without the noise term. If the measured $y(t_i)$ differs from its expected value $\bar{y}(t_i) \equiv H(t_i)\bar{x}(t_i-)$ a step correction equal to $K(t_i)[y(t_i) - \bar{y}(t_i)]$ is made on the estimated $\bar{x}$ at t_i. The error matrix $\phi \equiv (x-\bar{x})(x-\bar{x})^T$ satisfies the differential equation

$$\phi = F\phi + \phi F + \phi_w - \sum_{t_i} \phi H^T\,[H\phi H^T + \phi_v]^{-1} H\phi\delta(t-t_i). \tag{18}$$

The matrix ϕ is discontinuous at t_i, and its value immediately *before* the discontinuity $(\phi(t_i-))$ is used in the last term, of (18). The solution of (18) is in the form of recurrence relations which can be readily programmed on a computer. The correction gain matrix $K(t_i)$ is given as

$$K(t_i) = \phi(t_i-)\, H^T(t_i)\,[H(t_i)\,\phi(t_i-)\,H^T(t_i) + \phi_v(t_i)]^{-1}. \tag{19}$$

For optimum control of a dynamic system described by

$$\frac{dx}{dt} = Fx + Lu + w \tag{20}$$

under the criterion

$$J = \frac{1}{2}\int_{t_1}^{t_2} (x^T p x + u^T Q u)\,dt + x^T(t_2)\,R(t_2)\,\dot{x}(t_2) = \text{minimum} \tag{21}$$

the co~~n~~t.ol law is

$$u(t) = -C(t)\,x(t) \tag{22}$$

where

$$C(t) = Q^{-1}(t)\,L(t)\,R(t) \tag{23}$$

$$\frac{dR}{dt} = -RF - F^T R - p + RLQ^{-1}L^T R. \tag{24}$$

Note that $R(t)$ can be obtained by solving (24) backwards with the final condition $R(t_2)$. If $x(t)$ is not measurable, and the best estimate $\bar{x}(t)$ is used in (22), it is proved to be still the optimum control law. Equations (18) and (24) are readily solved on a computer but quite impossible to solve analytically. Consequently, the gain matrices K and C must be obtained from a computer. Optimum filtering and control systems are readily realized on a digital computer and are very useful, and they cannot be realized by any other means.

If a continuously measured signal $z(t)$ also is available, it can be used to improve the system performance, as shown in reference [30].

Nonlinear Discrete Systems

The stability of a nonlinear discrete system can be readily studied by the second method of Lyapunov [56], [66], [100]. Consider the system described by (12). The system is assumed to be autonomous and there is a control law which gives u as a function of the state variable x:

$$u(t) - U(x(t)). \tag{25}$$

The motion of the closed loop system is described by

$$x(t+T) = f(x(t),\, U(x(t)) \equiv g(x(t)). \tag{26}$$

Equation (26) defines $g(x)$. The function $g(x)$ is assumed to be continuous in x. A point x_0 is said to be an equilibrium point if

$$x_0 = g(x_0). \tag{27}$$

Lyapunov's theorem can be stated in many ways. One of the simplest is as follows:

If there exists a function $V(x)$ such that

$$V(x) > 0 \quad \text{if } x \neq x_0 \qquad = 0 \quad \text{if } x = x_0 \tag{28}$$

$$V(\infty) = \infty \tag{29}$$

$$V(g(x)) \leq V(x) \tag{30}$$

the system is stable about x_0. If the equality sign in (29) is satisfied only by $x = x_0$, then the system is asymptotically stable about x_0.

Lyapunov's method can be used not only for stability analysis, but also for system design. The problem becomes the choice of a control law $U(x)$ such that (30) is satisfied for some $V(x)$ which satisfies (28) and (29).

Optimization of nonlinear discrete systems can be achieved either by Bellman's dynamic programming [7], [8], [140] or a discrete version of Pontryagin's maximum principle. As dynamic programming is a suitable topic for a separate review article, the present review will be limited to the discrete maximum principle [26], [27], [29], [69], [121]. A general formulation of the problem is as follows:

Consider the dynamical system as described in (12) with an additional component x_0, and f_0 satisfying an equation of the same form. The problem is to select a sequence $u(t)$ to transfer x from some initial value ξ_1 at t_1 to a final value ξ_2 at t_2 (or a final set) and to minimize $x_0(t_2)$ among all sequences which transfers from ξ_1 to ξ_2. A necessary condition for $u(t)$ to be optimal is the existence of an adjoint function ψ $(\psi_0, \psi_1 \ldots \psi_n)$ which satisfies (31) and some boundary conditions (or transversality conditions) at t_1 and t_2

$$\psi_i(t) = \frac{\partial H(\psi, x, u, t)}{\partial x_i} \qquad i = 0, 1 \ldots n \tag{31}$$

and that the Hamiltonian function

$$H(\psi, x, u, t) \equiv \sum_{i=0}^{i=n} \psi_i(t+1)\, f_i(x, u, t) \tag{32}$$

is either a local maximum or stationary point for sufficiently small variation of f due to allowed change in

If the range of the function f is convex, then the given condition can be strengthened to require that u maximizes $H(\psi, x, u, t)$ at every t [58]. If the state variable x is restricted to a certain region X, then for the part of the trajectory lying on the boundary of X, (31) is modified to

$$\dot{\psi}_i(t) = \frac{\partial H(\psi, x, u, t)}{\partial x_i} - \zeta(t)\,\eta_i(x), \ i = 0, 1, n \quad (33)$$

where $\zeta(t) \geq 0$, and $\eta(x)$ is the normal of X pointing outward from x [29]. Similar conditions also are obtained for more complex boundaries of X. For linear systems with convex allowed regions X and U, the necessary condition (32) is also sufficient for optimal control [31].

While Lyapunov's method and discrete maximum principle do not give a complete solution of the design problem of computer controlled nonlinear systems, they are very useful concepts in aiding the design processes.

Adaptive and Learning Systems

Adaptive and learning controllers are used when certain aspects of the controlled system are not known or are changing with time, and the control law is automatically modified to adapt to the unknown or changing situation.

The division line between an adaptive and a learning system is quite ambiguous. Perhaps one way to distinguish these two types of systems is that adaptive systems are the ones which have fixed simple adaptive strategy, and learning systems place greater emphasis on search, pattern recognition, storage, and inference [26], [28], [144].

Adaptive systems are usually of the following types: (1) measurement and estimation of the controlled plant characteristics and adjusting the controller accordingly [63]; (2) forcing the controlled plant to conform to a given model [26], [28]; (3) using a dimensionless control law and adjusting the scale factors automatically [2]; (4) adjusting for the improvement of certain performance index by hill-climbing technique [42], [25], [62]; and (5) adjusting for certain specified values for a number of performance parameters [3], [25], [62]. Theoretically, the first type should give best results but it is also the most difficult to implement. The second type gives tolerable performance but it is also the fastest in responding to a change in plant characteristics. The third type gives best performance wherever it can be applied, but its application is limited. The last two types are comparatively simple to implement.

The controller to an adaptive learning system may represent a threshold logic with a bang-bang output, and the learning logic may represent a "teacher" which generates a signal to be compared. If the two signals are identical, the threshold logic is left undisturbed. Otherwise, it is adjusted [15], [112]. Alternatively, the "teacher" may evaluate the result, and punishes the controller if it failed its mission [157].

In learning systems of a different type, the learning logic may represent hill-climbing search technique, either of the single peak [71] or a multiple peak type [77], [78]. It may include a categorizer and a storage element. The categorizer determines the control situation from a set of observable parameters, and the best prior setting for each control situation is memorized in the storage. Once the best setting for a certain situation is recalled for active use, it is also improved at the same time by the learning process [59], [60].

One possible way of modeling the learning process itself is to regard it as a probabilistic automata. A probabilistic automata is a Markov chain with transition probabilities depending on the input [126]. Using such models, the effectiveness of the learning processes can be studied [70]. Two excellent papers on learning control systems have recently been published by Sklansky [132] and Tou [144]. The reader is referred to these papers for a detailed account.

In summary, the digital computer greatly multiplies the adaptive and learning capabilities of control systems. With the rapid development of high speed computers, the day soon may come that any engineering procedure of measurement and design which can be written into an algorithm can be used as part of an on-line adaptive or learning system. In such a system, the control law is always the best within limits of engineering know-how and the limited on-line as well as prior-test knowledge about the system.

A basic limitation is that in writing the algorithm, the engineer has to foresee all possible developments, and specifies the immediate as well as learning response. Is there a way of constructing a learning system which programs itself by interacting with the environment and generates its own algorithm in a way which cannot be foreseen by the engineer? Lerner, Vapnik, and Chervonenkis proposed an evolutionary principle as the basis for the synthesis of such a learning system [104]. The design starts from a small initial device which is capable of developing its own structure through interaction with the environment. It is not clear how the authors propose to accomplish this. One thing seems certain; if a complex system is to grow out of elementary electronic cells, or building blocks, by itself, the most likely candidates for the building blocks are "and," "or," "not" logical circuits.

Acknowledgment

This work of organizing the material was partially sponsored by the Air Force Office of Scientific Research, Air Research and Development Command, Washington, DC, under Grant No. AF-AFOSR-542-67.

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