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Photographic Realization of an Image-Deconvolution Filter for Holographic Fourier-Transform Division*

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The theory of the photographic realization of the Fourier-transform filter required for the authors' method (Phys. Letters, **25A**, (1967) 89) of *a posteriori* image-correcting 'deconvolution' by holographic Fourier-transform division is given. Subsequent to the first presentation of this work, the author's method of Phys. Letters **25A**, (1967) 89 has now been experimentally verified by A. W. Lohmann and H. W. Werlich [Phys. Letters **25A** (1967) 570] for two-dimensional "black-on-white" objects (point and letter), and for the deblurring of the photograph of a three-dimensional object by G. W. Stroke, G. Indebetouw and C. Puech [Phys. Letters, **26A** (1968) 443].

A considerable new interest appears to have recently arisen in methods of *a posteriori* image-correcting 'deconvolution' (also called 'convolution'), notably in view of the solution of astronomical (*e.g.* defocusing), radio-astronomical and spectroscopic problems among others. The 'deconvolution' problem may be solved in the 'apodisation' (*e.g.* lens-aperture) domain, or in the Fourier-transform *i.e.* "focused" field) domain. The spatial electric field-vector distributions of these two domains are related by two-dimensional spatial Fourier-transform relations.[1,2]

A holographic scheme for the solution of the image-correcting problem in the Fourier-transform domain was recently first proposed by us in a general theoretical form,[3] in application of the Maréchal and Croce spatial filtering concepts, first proposed in 1953[11]. Our method can be made to give a particularly straightforward solution of the deconvolution problem, provided that a scheme can be found for the purpose of realizing (*i.e.* materializing) the Fourier-transform division filter needed for the solution of the holographic Fourier-transform division problem in the Fourier-transform domain. In

our ref. 3, we presented the general scheme for the solution of the deconvolution problem.

In this letter, we give in summary theoretical form a scheme for the realization of the required Fourier-transform division filter, which we have now developed, in view of the practical solution of the image deconvolution problem. Details of the mathematical development, which we present, and experimental results will be given in a more complete publication, upon completion of the experimental verifications.

For the sake of clarity, it may be in order to briefly recall the image-deconvolution problem. Given $f(x, y)$, an intensity distribution of the electric-field vector distribution in the geometrical image of an object, and given $h(x, y)$, the "spread function" (*i.e.* the intensity distribution in the image of a point source in the image domain [where $h(x, y)$ may be the out-of-focus image of the object point, or the image of a point object as formed by an imperfect optical system, for example, then the intensity distribution recorded in the actual image is $g(x, y) = h(x, y) \otimes f(x, y)$, where $\otimes$ describes a spatial convolution.[1,2,3] In the Fourier-transform domain, we have $\bar{G}(u, v) = \bar{H}\bar{F}$, where $\bar{G}$, $\bar{H}$ and $\bar{F}$ are the spatial Fourier transforms of g, h and f, respectively.[1] We recall, for example, that $f(x, y) = \langle \bar{E}_f \bar{E}_f^* \rangle = |\bar{E}_f|^2$, where $\bar{E}_f$ is the distribution of the electric field vector in the *aerial* geometrical image of the objects (*i.e.* before recording). Similar relations hold for $g(x, y)$ and $h(x, y)$.

In order to 'retrieve' $\bar{F}$, from $\bar{G}$, and from $\bar{F}$, by spatial Fourier transformation, the function $f(x, y)$, as desired, we multiply $\bar{G}$

by $\bar{H}^{-1}$, *i.e.* by $\bar{H}^{-1}=\bar{H}^*|\bar{H}|^{-2}$. In what follows, we describe, as a model, one method of realization of the $\bar{H}^{-1}$ filter. Let us assume that we place a point source of light into the object domain (or carry out an equivalent operation), and that we record the corresponding intensity distribution, which will be $h(x, y)$, under the usual photographic assumptions,[1] in the image domain. We may call the first record a negative. Its *amplitude* transmittance T_{1-} (*i.e.* the spatial distribution of the electric field vector in the wavefront transmitted through the photographic transparancy upon illumination with a wave of unit amplitude) is the function $[h(x, y)]^{-\tau_N/2}$, in the linear part of the photographic H-D (Hurter and Driffield) curve, where the slope is equal to γ_N (see *e.g.* ref. 1, 10). A contact print of the negative (having the transmittance T_{1-}), which we may call the positive (which will have a transmittance T_{1+}) will record the *intensity* distribution $[h(x, y)]^{-\tau_N}$. The *amplitude* transmittance T_{1+} of the positive is then

$$T_{1+}=[h(x, y)]^{(-\tau_N)(-\tau_P/2)} \qquad (1)$$

where γ_P is the slope ("gamma") of the positive. It may be readily seen from eq. (1) that the amplitude transmittance T_{1+} of the positive will be $h(x, y)$, provided that we realize the condition

$$\gamma_N\gamma_P=2 . \qquad (2)$$

We may call the condition of eq. (2) a "coherent-optics" Goldberg linearity condition (in honour of Prof. E. Goldberg[4] and his work on sound recording, where the condition $\gamma_N\gamma_P=1$ is of importance).

It is the positive [with transmittance $T_{1+}=h(x, y)$] which serves for the generation of the two filter "components" $|\bar{H}|^{-2}$ and $\bar{H}^*$, the product of which is $\bar{H}^{-1}$, and which we first presented in our paper of ref. 3.

The H^* filter component may be realized from the positive [which has a transmittance $T_{1+}=h(x, y)$] simply in a holographic Fourier-transform arrangement,[1,5] as we previously mentioned,[3] except that the positive-to-negative step of our ref. 3, may be omitted.

The $|H|^{-2}$ filter component may also be quite simply realized from the same positive [of transmittance $T_{1+}=h(xy)$] by recording the *intensity* distribution $|H|^2$ in the Fourier transform of $h(x, y)$. The *amplitude* trans-

mittance of this third record (which we may call T_{3-}) is nothing but $|H|^{-2}$, provided that the photographic $\gamma_{N3}=2$ condition is respected for the "gamma" of this record, as we mentioned in our paper of ref. 3.

The "in-series" use of the two filter components H^* and $|H|^{-2}$ has the desired amplitude transmittance H^{-1}, as we stressed in our ref. 3.

It may be in order to note that we have verified the preceding analysis by a detailed mathematical development, which includes suitable proportionality constants, as well as the necessary assumptions of spatial invariance and linearity, together with appropriate "coherent" and "incoherent" optical imaging conditions, and suitable corresponding optical geometries for the spatial filtering arrangements, which are needed to justify the operational Fourier-transform treatment summarized above.[2] The effect of "noise" will be further discussed in our more extensive publication, following completion of the experimental verifications, now under way [see also ref. 12]. As one practical result, of interest, we may mention, according to preliminary experiments, that a "gamma" of 2 appears to be readily obtainable with some precautions with Kodak 649F emulsions (frequently used for high-resolution holography) with a 2.5 minute development in Kodak D19 developer and 68°F.

Relations of our work to the work on image-deconvolution by apodisation,[6] and by other non-holographic methods (*e.g.*[7,8,9]), including those using analytical and digital-computer processing, need to be stressed again.[3] The important relations of this work to the work of D. Gabor[10] should also again be noted.

References

1) For general background and for the notations

used, see, for example, G. W. Stroke, *An Introduction to Coherent Optics and Holography* (Academic Press, New York, 1967) [270 pages, including reprints of the three original "wavefront-reconstruction imaging" papers by D. Gabor]. [See also: *Selected Papers in Physics*, Vol. 155, (on Holography), issued by the Physical Society of Japan, March 20, 1967].

2) For an electromagnetic-theory justification and for a detailed derivation of the domains of applicability of the optical Fourier-transform relations used, and for the corresponding experimental arrangements, see *e.g.* G. W. Stroke, "Diffraction Gratings," pp. 426–754 in *Handbuch der Physik*, vol. 29, edited by S. Flügge (Springer Verlag, Berlin and Heidelberg, 1967).

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