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THERMAL ASPECTS OF MAGNETOHYDRODYNAMIC LUBRICATION

by

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Introduction

In recent years a considerable interest has been manifested in magnetohydrodynamic (MHD) lubrication [1-8][1]. Most of the work reported to date in the literature has been analytical although the author is aware of at least one experimental investigation currently being conducted in MHD lubrication [9]. The analytical studies have served to demonstrate the possibility of increasing the load capacity of liquid metal lubricants by means of an applied magnetic field.

All of the analyses to date have considered only the equation of motion and the continuity equation. If the assumption of constant thermophysical properties is made, these two equations suffice to determine the pressure distribution and therefore the load capacity of the bearing. In the present paper, the energy equation is utilized to determine the temperature rise of the lubricant in flowing through a slider bearing. In spite of the fact that liquid metals have large thermal conductivities which would result in rapid conduction of the energy arising from local dissipation, the presence of electric currents in the lubricant means that the strength of the local dissipation function may become quite large under certain operating conditions.

The present analysis must be viewed as a preliminary investigation of the lubricant temperature rise in MHD lubrication because a number of simplifying assumptions have been introduced

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1 Numbers in brackets designate References at end of paper.

to render the problem tractable analytically (without the use of a computer) and at the same time preserve the essential physical aspects of the problem. The analysis suggests that the influence of the applied electric and magnetic fields on the lubricant temperature rise will be significant in all cases for which significant increases in load capacity due to the applied fields are obtained. This in turn suggests that a more accurate analysis will require the inclusion of the effect of viscosity variation with temperature instead of a constant mean viscosity as used in the present analysis.

The Analysis

The linear slider bearing will be considered in the present analysis and the geometry is shown in Figure 1. The walls are assumed to be electrical insulators. The influence of finite electrical conductivity of the walls on MHD lubrication has been considered in reference [2]. The thermophysical properties of the fluid are assumed constant so that the equation of motion and the energy equation are uncoupled. This assumption implies that a mean value of viscosity is used based on a temperature between the inlet and exit temperatures. The appropriate temperature to use can be determined only after a more exact analysis is performed which considers variable viscosity. The bearing is considered to be infinite in the z-direction and a uniform magnetic field is applied in the y-direction.

The equation of motion has been discussed in reference [1] and may be written as

$$-\frac{dP}{dx} + \mu \frac{\partial^2 u}{\partial y^2} - \sigma B_y \left(E_z + u B_y \right) = 0 \qquad (1)$$

It is shown in reference [1] that the electric field E_z is constant for the present configuration. It is convenient to define the following dimensionless parameters.

$$\gamma = \frac{u}{V} \qquad \eta = \frac{w}{w_o} \qquad \zeta = \frac{y}{w_o} \qquad \alpha = \frac{x}{w_o} \qquad \pi = \frac{P}{\frac{\mu V}{w_o}}$$

In terms of the dimensionless parameters, the equation of motion is

$$\frac{\partial^2 \gamma}{\partial \xi^2} - M^2 \gamma = \frac{d\Pi}{d\alpha} + M^2 \Phi \tag{3}$$

where $M = W_0 B_Y \left(\frac{\sigma}{\mu}\right)^{1/2}$

$$\Phi = \frac{E_z}{V B_Y}$$

Equation (3) is integrated across the film, neglecting the pressure variation in the trans film direction, subject to the boundary conditions $\gamma(\xi = 0) = 1$ and $\gamma(\xi = \zeta) = 0$. The result may be written

$$\gamma = \left\{\Phi + \frac{1}{M^2}\frac{d\Pi}{d\alpha}\right\}\left\{\frac{\sinh M\zeta\,(\cosh M\xi - 1) - \sinh M\xi\,(\cosh M\zeta - 1)}{\sinh M\zeta}\right\} + \left\{\frac{\sinh M\zeta \cosh M\xi - \sinh M\xi \cosh M\zeta}{\sinh M\zeta}\right\} \tag{4}$$

The details of applying the continuity equation to determine the pressure distribution are contained in reference (2) and only the results are summarized here. The mass flow rate is given by

$$m = \rho \int_0^W u \, dY = \rho V W_0 \int_0^\zeta \gamma \, d\xi \tag{5}$$

Substituting for γ from Equation (4) into Equation (5) and integrating the pressure gradient with the boundary conditions

$\pi(\zeta = 1) = \pi_0$, $\pi(\zeta = \frac{W_1}{W_0} = \zeta_1) = \pi_0$ gives the following results which are required later in solving the energy equation.

$$\frac{1}{M^2}\frac{d\pi}{d\alpha} + \Phi = \frac{Mm^* - \dfrac{\cosh M\zeta - 1}{\sinh M\zeta}}{\dfrac{2(\cosh M\zeta - 1)}{\sinh M\zeta} - M\zeta} \tag{6}$$

$$Mm^* = \frac{I_2(\zeta_1) - \Phi(1-\zeta_1)}{I_1(\zeta_1)} \tag{7}$$

where $$m^* = \frac{m}{\rho V W_0}$$

$$\zeta_1 = \frac{W_1}{W_0}$$

$$I_1(\zeta) = \int_1^{\zeta} \frac{\operatorname{ctnh}\frac{M\xi}{2}}{2 - M\xi \operatorname{ctnh}\frac{M\xi}{2}}\, d\xi$$

$$I_2(\zeta) = \int_1^{\zeta} \frac{d\xi}{2 - M\xi \operatorname{ctnh}\frac{M\xi}{2}}$$

The integrals $I_1(\zeta)$ and $I_2(\zeta)$ must be evaluated numerically.

The energy equation including viscous dissipation and Joule heating is

$$\rho c u \frac{\partial T}{\partial X} = \sigma (E_z + u B_Y)^2 + \mu \left(\frac{\partial u}{\partial Y}\right)^2 + K\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) \tag{8}$$

In dimensionless form, Equation (8) may be written

$$Re\, Pr\, \gamma \frac{\partial \theta}{\partial \alpha} = M^2 (\Phi + \gamma)^2 + \left(\frac{\partial \gamma}{\partial \xi}\right)^2 + \frac{\partial^2 \theta}{\partial \xi^2} + \frac{\partial^2 \theta}{\partial \alpha^2} \tag{9}$$

where

$$\theta = \frac{T}{\frac{V^2}{c} Pr}$$

$$Pr = \frac{\mu c}{k}$$

$$Re = \frac{\rho V W_0}{\mu}$$

Equation (9) is uncoupled from the equation of motion through the assumption of constant thermophysical properties. The terms involving γ can be evaluated from Equations (4), (6), and (7) so that in principal, Equation (9) can be solved subject to specified boundary conditions. Even though the equation is linear, it is a complex equation and an exact solution would be extremely difficult to obtain without the aid of a computer. An approximate solution can be readily obtained if certain assumptions are made as follows.

A Rayleigh type of approximation will be used for the convective term. This means that the exact velocity expression is replaced by the average velocity across the film defined as

$$\bar{\gamma} = \frac{1}{\eta} \int_0^{\eta} \gamma \, d\xi = \frac{m^*}{\eta} \tag{10}$$

A Rayleigh approximation is more valid for an MHD flow than for a flow in the absence of a magnetic field because the effect of the magnetic field is to distort the otherwise parabolic profile into a flatter profile. The approximation becomes more exact as the magnetic field strength increases. The other assumption to be introduced is to replace the dissipation terms by the average dissipation across the film. The average dissipation is defined as

$$g(\zeta) = \frac{1}{\zeta} \int_0^{\zeta} \left[M^2 (\phi + \gamma)^2 + \left(\frac{\partial \sigma}{\partial \xi} \right)^2 \right] d\xi \tag{11}$$

and can be evaluated from Equations (4), (6), and (7). The approximate energy equation then becomes

$$Re\, Pr\, w' \frac{m^*}{\zeta} \frac{\partial \theta}{\partial \zeta} = g(\zeta) + \frac{\partial^2 \theta}{\partial \xi^2} + (w')^2 \frac{\partial^2 \theta}{\partial \zeta^2} \tag{12}$$

where the transformation $\frac{\partial}{\partial \alpha} = \frac{\partial}{\partial \zeta} \frac{\partial \zeta}{\partial \alpha} = w' \frac{\partial}{\partial \zeta}$ has been used with $w' = \frac{W_1 - W_0}{L}$.

If a solution to Equation (12) is assumed of the form

$$\theta(\xi, \zeta) = \theta^*(\xi, \zeta) + F(\zeta) \tag{13}$$

substitution of Equation (13) into (12) shows that the functions $\theta^*(\xi, \zeta)$ and $F(\zeta)$ satisfy the equations

$$F'' - \frac{Re\, Pr\, m^*}{w' \zeta} F' = -\frac{1}{(w')^2} g(\zeta) \tag{14}$$

$$\frac{RePr\, m^* w'}{\zeta}\frac{\partial \theta^*}{\partial \zeta} = \frac{\partial^2 \theta^*}{\partial \xi^2} + (w')^2 \frac{\partial^2 \theta^*}{\partial \zeta^2} \qquad (15)$$

A solution to Equation (14) can be obtained by means of an integrating factor with the result

$$F(\zeta) = -\frac{1}{(w')^2}\int_1^{\zeta}\left[\frac{1}{g}\int_1^{z} g\, g\, dx\right]dz + C_1 \int_1^{\zeta}\frac{dz}{g} + C_2 \qquad (16)$$

where $g = \zeta^{-\frac{RePr\, m^*}{w'}}$ is the integrating factor and C_1 and C_2 are constants to be determined by the boundary conditions.

A solution to Equation (15) may be obtained by assuming a product solution of the form $\theta^*(\xi,\zeta) = N(\zeta)\, Z(\xi)$ with the result

$$\theta^* = \sum_{n=0}^{\infty} N_n(\zeta)\left(C_n \sin\lambda_n \xi + D_n \cos\lambda_n \xi\right) \qquad (17)$$

where $N_n(\zeta)$ satisfies the equation

$$(w')^2 N_n'' - \frac{RePr\, m^* w'}{\zeta} N_n' - \lambda_n^2 N_n = 0 \qquad (18)$$

The constants C_n , D_n , and λ_n are to be determined from the boundary conditions.

A number of possible boundary conditions could be applied including various combinations of isothermal, adiabatic, and constant heat flux conditions at the two surfaces. The extreme case of maximum temperature rise of the lubricant would occur if both surfaces are adiabatic. The assumption of adiabatic surfaces will thus be made in the present analysis. The mathematical condition of adiabatic surfaces is

$$\left.\frac{\partial \theta}{\partial \xi}\right)_{\eta} = 0 \quad at \quad \xi = 0 \quad and \quad \xi = \eta \tag{19}$$

Applying Equation (19) to Equations (13) and (17) gives

$$C_n = 0 \qquad\qquad \lambda_n = \frac{n\pi}{\eta}$$

and thus the temperature distribution becomes

$$\theta(\xi, \eta) = \sum_{n=0}^{\infty} N_n(\eta)\, D_n \cos\frac{n\pi\xi}{\eta} + F(\eta) \tag{20}$$

The constants D_n would be determined from a specification of the temperature distribution at the inlet, .

The mean temperature across the film would be obtained by integrating Equation (20) across the film. If $\bar{\theta}(\eta)$ is the mean film temperature, we may write

$$\bar{\theta}(\eta) = \frac{1}{\eta}\int_0^{\eta} \theta(\xi, \eta)\, d\xi = F(\eta) \tag{21}$$

since the integration of the cosine terms between $\xi = 0$ and $\xi = \zeta$ vanishes. The change of mean film temperature thus depends only on $F(\zeta)$ which is given by Equation (16). There are still two undetermined constants in Equation (16), however. These constants are determined by the conditions

$$\frac{\partial \bar{\theta}}{\partial \zeta} = 0 \quad at \quad \zeta = 1 \qquad \bar{\theta}(\zeta) = \theta_0 \quad at \quad \zeta = 1 \tag{22}$$

The first condition gives $C_1 = 0$ and the second $C_2 = \theta_0$ with the final result

$$\bar{\theta}(\zeta) = \theta_0 - \frac{1}{(w')^2} \int_1^{\zeta} \left[\frac{1}{g} \int_1^{z} g\, g\, dx \right] dz \tag{23}$$

The overall mean temperature rise from inlet to outlet then becomes

$$\bar{\theta}(\zeta_1) - \theta_0 = - \frac{1}{(w')^2} \int_1^{\zeta_1} \left[\frac{1}{g} \int_1^{z} g\, g\, dx \right] dz \tag{24}$$

with

$$g = \zeta^{-\frac{Re Pr\, m^*}{w'}}$$

$$g = \frac{1}{\zeta} \int_0^{\zeta} \left[M^2 (\phi + \delta)^2 + \left(\frac{\partial \delta}{\partial \xi} \right)^2 \right] d\xi$$

It is clear that the integrals of Equation (24) would have to be evaluated numerically.

Solution Neglecting Axial Conduction

In reference [10], the effect of axial conduction on the heat transfer in the entrance regions of parallel plates and tubes is investigated. The ratio of the axial convection to axial conduction is given by the Peclet number, $Pe = Re\, Pr$. It is found in that reference that for $Pe \geq 100$, the effect of axial conduction on the temperature profile is completely negligible. There is only a slight effect of a few percent for Pe as low as 10 and the effect of axial conduction becomes significant only for $Pe < 10$. It is of interest to investigate a typical magnitude of Pe for a liquid metal lubricant. Considering mercury and the operating conditions $W_0 = 10^{-3}$ in., $V = 50$ ft/sec. gives $Pe = 42$. The effect of axial conduction should be negligible under these conditions. It should be emphasized, however, that realistic operating ranges of a MHD bearing are possible for which Pe could be much smaller than the above value and thus the axial conduction would be significant.

The mean film temperature is still given by Equation (21) with axial conduction neglected if the function $F(\eta)$ is understood to be the solution of Equation (14) with axial conduction neglected, i.e. the condition $F'' = 0$. The expression for overall film temperature rise analogous to

Equation (24) but with axial conduction neglected can be written as

$$\bar{\theta}(z_1) - \theta_0 = \frac{1}{Re\,Pr\,m^* w'} \int_1^{z_1} z\,g\,dz \tag{25}$$

with $g(z)$ defined as before.

A measure of the effect of the electric and magnetic fields on the temperature rise of the film can be obtained from the expression analogous to Equation (25) for the case of zero electric and magnetic fields. A closed form solution can be easily obtained in the absence of a magnetic field. Denoting this solution by the subscript $M=0$, the result is (neglecting axial conduction)

$$\left[\bar{\theta}(z_1) - \theta_0\right]_{M=0} = \frac{1}{Re\,Pr\,w'} \left[\frac{4(1+z_1)}{z_1} \ln z_1 + \frac{12(1-z_1)}{z_1} - \frac{6(1-z_1)}{(1+z_1)}\right] \tag{26}$$

The ratio of Equations (25) and (26) can be taken as a measure of the effect of the magnetic field on the mean temperature rise of the lubricant. Denoting this quantity by R_T gives

$$R_T = \frac{\bar{\theta}(z_1) - \theta_0}{\left[\bar{\theta}(z_1) - \theta_0\right]_{M=0}} = \frac{\frac{1}{m^*} \int_1^{z_1} z\,g\,dz}{\frac{4(1+z_1)}{z_1} \ln z_1 + \frac{12(1-z_1)}{z_1} - \frac{6(1-z_1)}{(1+z_1)}} \tag{27}$$

The quantity R_T is the ratio of the mean temperature rise of the lubricant in the presence of the magnetic field to the mean temperature rise in the absence of the magnetic field. Adiabatic surfaces are assumed in both cases and the values of Re and Pr are the same for both cases.

Discussion of Results

A plot of the ratio of temperature rise with the magnetic field to the corresponding quantity without the magnetic field, as given by Equation (27), is shown in Figure 2. The temperature rise ratio is plotted against Φ for three values of M. It is shown in references [5] and [2] that favorable conditions of MHD pressurization occur for negative Φ values and thus only negative values of Φ are used in Figure 2. It is seen that for large negative values of Φ and large values of M, the temperature rise with the magnetic field can be two orders of magnitude larger than the temperature rise in the absence of the magnetic field. The results also indicate that the increase in Joule heating in the film more than offsets the decrease in viscous heating in the central part of the film due to the flattening of the velocity profile by the magnetic field.

The above analysis suggests that the thermal aspects of MHD lubrication will always be significant in the range of electric and magnetic field strengths for which favorable MHD pressurization can occur. The actual temperature rise of the lubricant as given by Equation (25) would be reduced by allowing heat transfer at the two surfaces. Several possible areas for further investigation are suggested such as the influence of thermal boundary conditions on the film temperature distribution, the relative importance of viscous heating and Joule heating, the influence of variable thermophysical properties on the heat transfer, and exact analyses instead of the approximate treatment of the present paper. Some of these problems would undoubtedly require the use of a computer and further work along the lines suggested above is contemplated by the author.

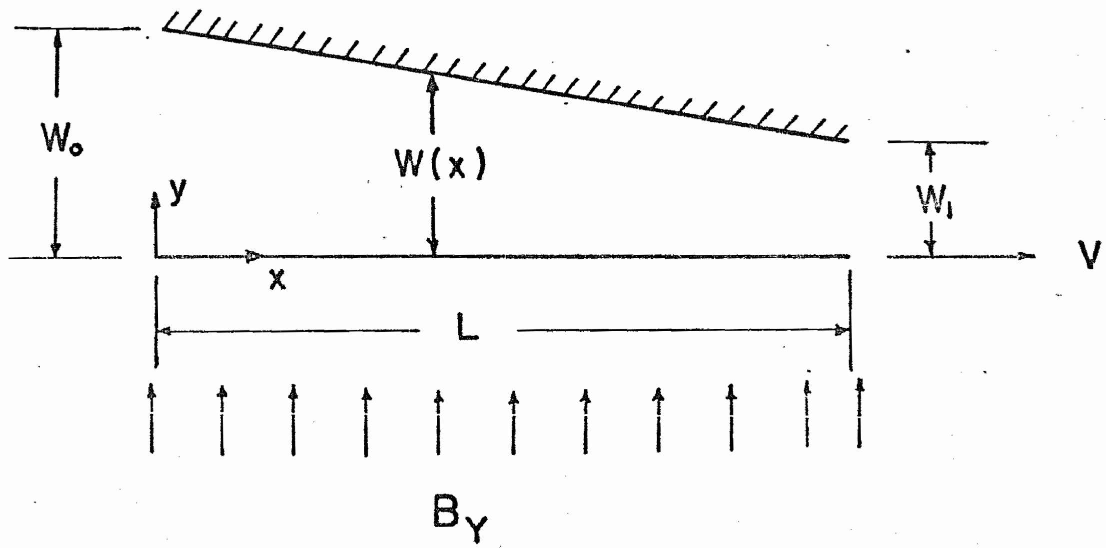

Figure 1. MHD Slider Bearing Geometry