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DOES THE OPERATING POINT OF A SERIES-PARALLEL
NETWORK OF MONOTONE RESISTORS SATISFY ALL RESISTOR
RATINGS?

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DOES THE OPERATING POINT OF A SERIES-PARALLEL NETWORK OF MONOTONE RESISTORS SATISFY ALL RESISTOR RATINGS? *

A. H. Zemanian

Abstract — An important issue in circuit design is whether every element operates within its rated values. For the particular case of a series-parallel network of monotone resistors (i.e., one-ports) whose characteristic curves are continuous and monotonically increasing at least within the ranges between their rated values, we present a rapid procedure for resolving this question. If at least one resistor operates outside of its ratings, our procedure will reveal this fact without having to determine the operating point of the entire network or even of that resistor alone. On the other hand, if all elements operate within their ratings, our procedure determines that operating point easily without using the standard iterative numerical procedures, such as the Newton-Raphson, secant, or homotopy methods, which can be computationally prolonged.

Index Terms — Nonlinear networks, monotone resistors, operating points, rated values, series-parallel networks.

1 Introduction

There have been many papers about the determination of (DC) operating points for nonlinear resistive networks. The references [3], [4], [6], [7], [8], and [9] are only a sampling of some more recent works. Their bibliographies contain references to many earlier papers on that subject. An important question in this regard, which does not seem to have been addressed explicitly, is whether an operating point of a proposed network satisfies the ratings specifica-

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tions of all the elements in the network. If any rating is violated, that operating point must be avoided, necessitating a redesign of the network. Of course, once all the voltages and currents corresponding to an operating point are determined, the satisfaction of all element ratings can easily be checked. However, the standard methods for determining all those voltages and currents in a nonlinear network, such as Newton-Raphson, secant, or homotopy techniques, require iterative computations, which can be computationally onerous.

In a recent paper [9] a related question was completely resolved for any series-parallel network of resistors with two-way infinite characteristic curves. A fast procedure was presented that determined whether or not the network had an operating point without actually computing that operating point. The nonexistence of any operating point meant that the network was senseless. However, the existence of an operating point did not ensure that all elements operated within their ratings.

In this paper we again restrict our attention to series-parallel networks¹ of sources and continuous monotonically increasing resistors.² We present herein a fast method for determining whether or not a series-parallel network has an operating point under which no resistor rating is violated. Such an operating point will be unique if it exists, in which case our method will then yield that operating point with very little additional computation. All this is accomplished without using any of the conventional, robust, iterative, and computationally expensive methods for solving simultaneous nonlinear equations such as those mentioned above.

In our approach every resistor is represented by its "characteristic segment," namely, that finite portion of the characteristic curve lying between the ratings limits for the resistor. For example, a conventional p-n junction diode is rated by specifying a maximum forward current and a maximum reverse voltage. These determine the two endpoints for that diode's characteristic segment. Then, that diode's voltage-current pair must lie on its characteristic segment if it corresponds to an allowable operating point.

¹Duffin [2] characterized series-parallel graphs in several ways. Those ways that are of importance to us are pointed out in [9, Section 3]. That discussion will not be repeated here.

²The series-parallel structure with monotone resistors is a severe restriction if practical electronic circuits are to be considered in their entirety, but many subcircuits have this form. The determination of the (DC) operating point of a series-parallel subcircuit is the kind of problem addressed herein.

Our procedure consists of a sequence of series and parallel combinations of resistors which reduces the network to just two resistors forming a loop. At each step of the sequence a combined characteristic segment is computed. If at any step a void characteristic segment arises, our procedure stops and we conclude that the network does not have an allowable operating point. If, on the other hand, the final two-resistor loop is achieved, it is then easy to determine whether an allowable operating point exists. In that case a reversal of the sequence of combinations readily yields all the voltages and currents throughout the network. We get the same result no matter which sequence of series and parallel combinations we choose.

Let us now explicate some of our terminology and notations. A positive voltage drop and a positive current flow in any branch are measured in the same direction — the assigned orientation of the branch. We use “nonlinear” in an inclusive way to encompass “linear” as a special case. In this work all resistors and sources are one-ports with no coupling between them. Each branch contains a resistor and possibly a source. An *operating point* for a network of such branches is a set of all branch voltages and currents determined by Kirchhoff’s laws and the resistors’ characteristics. If no voltage or current violates a resistor’s ratings, we say that the operating point is a *within-ratings operating point*. To avoid repeating this phrase many times, we shall use instead its acronym “WROP.” Given any “characteristic segment” C (i.e., a finite portion) of a characteristic curve in the voltage-current plane (voltage v plotted along the horizontal axis and current i plotted along the vertical axis), the operators³ N , E , S , and W are defined by $N(C) = \sup\{i: (v, i) \in C\}$, $E(C) = \sup\{v: (v, i) \in C\}$, $S(C) = \inf\{i: (v, i) \in C\}$, and $W(C) = \inf\{v: (v, i) \in C\}$; this is illustrated in Fig. 1. Furthermore, for $(v, i) \in C$, we let $R: i \mapsto v$ and $G = R^{-1}: v \mapsto i$ be the resistance and conductance mappings specified by C .

2 Characteristic Segments

Throughout this paper the following is assumed.

³ N for “north,” E for “east,” S for “south,” and W for “west.”

Conditions 2.1: The network is connected, nonseparable, with at least three branches and at most finitely many branches. Those branches are in a series-parallel configuration. Each branch consists of a nonlinear resistor, which possibly incorporates a source, and its voltage-current pairs comprise a continuous monotonically increasing curve that terminates at two (finite) points of the voltage-current plane.

That curve will be called a “characteristic segment.” When dealing with a single resistor, it will always be understood that the two endpoints of its characteristic segment are the resistor’s rated values shifted in accordance with any incorporated source, as is illustrated in Fig. 2. A branch that is a pure source can be taken into account by shifting that source into resistive branches [5, pages 131-132]. More generally, we shall use the following definition of the characteristic segment for a single resistor or for any series-parallel combination of resistors.

Definition 2.2: A *characteristic segment* is a finite, continuous, monotonically increasing curve in the voltage-current plane having two endpoints. Those end point will be called the *ratings points* of the characteristic segment (whether or not the characteristic segment is for a single resistor or for a series-parallel combination of resistors). Furthermore, a single point is allowed as degenerate case of a characteristic segment.

At times, we may need to reverse the orientation of a branch. This will replace the branch’s voltage and current by their negative values. Thus, if $C = \{(v, i) : v = R(i), S(C) \leq i \leq N(C)\}$ is the characteristic segment before the reversal, then $\tilde{C} = \{(-v, -i) : (v, i) \in C\}$ is the characteristic segment after the reversal. This too is illustrated in Fig. 1.

3 Duffin’s Theorem and a Consequence

We will make use of a classical theorem due to R.J. Duffin [1]. For this theorem, it is assumed that each resistor is specified by a characteristic curve that is continuous, monotonically increasing, and two-way infinite with $v = R(i)$ relating voltage v and current i at each point on the characteristic curve, with $v \rightarrow \infty$ as $i \rightarrow \infty$, and with $v \rightarrow -\infty$ as $i \rightarrow -\infty$. Every independent voltage or current source is incorporated into a resistor, and thus $R(0) \neq 0$ is possible.

Theorem 3.1 (Duffin): A finite network of nonlinear resistors having the properties just stated has a unique operating point.

A consequence of this theorem is the following result.

Theorem 3.2: A network satisfying Conditions 2.1 either has a unique WROP or none at all.

Note: We are not asserting here that the network has no other operating point if the actual characteristic curves extend beyond the characteristic segments. It may indeed if its entire characteristic curves are nonmonotone with perhaps multivalued resistance or conductance operators. What we are asserting is that either exactly one of those operating points or none of them is a WROP.

Proof: We can extend each characteristic segment into a new characteristic curve that satisfies Conditions 2.1 (thus, outside the characteristic segment this new characteristic curve may be different from the actual one.) The new resulting network will have a unique operating point by virtue of Theorem 3.1, which means that no other combination of points on those new characteristic curves (one point for each curve) will satisfy Kirchhoff's laws throughout the network. This implies that there can be at most one WROP. \square

4 The Series Sum of Characteristic Segments

Consider two branches b_1 and b_2 connected in series. If they are not confluent oriented, reverse the orientation of one of them to obtain the circuit shown in Fig. 3. Their currents are the same: $i = i_1 = i_2$, and their voltages add to give the voltage $v = v_1 + v_2$ across the combination — with again the same orientation. Our definition of the series sum $C_1 \diamond C_2$ of their characteristic segments C_1 and C_2 requires this confluent orientation of b_1 and b_2 .

Set $N(C_1 \diamond C_2) = \min(N(C_1), N(C_2))$ and $S(C_1 \diamond C_2) = \max(S(C_1), S(C_2))$. If

$$S(C_1 \diamond C_2) \leq N(C_1 \diamond C_2), \quad (1)$$

we set

$$v = R_1(i) + R_2(i) \text{ for } S(C_1 \diamond C_2) \leq i \leq N(C_1 \diamond C_2). \quad (2)$$

The set of all such points (v, i) is by definition the characteristic segment $C_1 \diamond C_2$ for the

series combination of the confluent oriented b_1 and b_2 . This is illustrated in Fig. 4. If on the other hand (1) does not hold, we take it that $C_1 \diamond C_2$ does not exist.

The following is obvious.

Lemma 4.1. An existent series sum $C_1 \diamond C_2$ of two characteristic segments is also a characteristic segment. Moreover, each endpoint of $C_1 \diamond C_2$ is the series sum of a point of C_1 and a point of C_2 , at least one of which is an endpoint of C_1 or of C_2 .

Thus, if the series combination of two resistors is operating at one of its ratings points, then at least one of those resistors is also operating at one of its ratings points.

If C_1 and C_2 are given by formulas, then $C_1 \diamond C_2$ is given as a formula directly by (2). If however at least one of C_1 and C_2 is given graphically or by a table of points, our next step is to compute a table of points (perhaps 30 of them) for the characteristic segment of $C_1 \diamond C_2$. In each case (1) must hold of course. When only a table is available, interpolation may be used to obtain intermediate points. We want the points computed for $C_1 \diamond C_2$ to be more or less uniformly spaced. To this end, we proceed as follows.

Set $i_{min} = S(C_1 \diamond C_2)$ and $v_{min} = R_1(i_{min}) + R_2(i_{min})$, and set $i_{max} = N(C_1 \diamond C_2)$ and $v_{max} = R_1(i_{max}) + R_2(i_{max})$. We can do this because all the needed values are obtainable from the lowest and highest values in the tables or graphs for C_1 and C_2 . Next, set $i_{mid} = (i_{min} + i_{max})/2$ and $v_{mid} = R_1(i_{mid}) + R_2(i_{mid})$, where $R_1(i_{mid})$ and $R_2(i_{mid})$ are obtained by interpolation in general.

Now, let us assume that the points (v_k, i_k) for $C_1 \diamond C_2$ have been computed, where $k = 1, \dots, K$ and $K \geq 3$. In effect, we have partitioned $C_1 \diamond C_2$ into $K - 1$ segments S_k ($k = 1, \dots, K - 1$) whose endpoints may be denoted by $(v_{k,min}, i_{k,min})$ and $(v_{k,max}, i_{k,max})$. Define the *size* of S_k to be $v_{k,max} - v_{k,min} + i_{k,max} - i_{k,min}$. Choose a segment S_l having the largest size. Set $i_{l,mid} = (i_{l,min} + i_{l,max})/2$, and get $v_{l,mid} = R_1(i_{l,mid}) + R_2(i_{l,mid})$, interpolating if need be. This adds one more point $(v_{l,mid}, i_{l,mid})$ to our accruing table for $C_1 \diamond C_2$. Continue in this way until enough (30 or so?) points for $C_1 \diamond C_2$ are obtained. Since it is always a largest segment that is being partitioned into two segments, the final set of points will be approximately uniformly distributed. All this can be easily programmed for a computer.

Of course, if C_1 or C_2 is degenerate, there is only one point in $C_1 \diamond C_2$ whenever (2) holds.

5 The Parallel Sum of Characteristic Segments

We now examine how the characteristic segments of two parallel resistors combine. We start with two confluent oriented parallel branches b_1 and b_2 , as shown in Fig. 5; if need be, reverse the orientation of one of the branches to achieve this confluence, which is required for the following definitions. In this case, the voltages are the same: $v = v_1 = v_2$, and the currents add to give $i = i_1 + i_2$. Set $E(C_1 \square C_2) = \min(E(C_1), E(C_2))$ and $W(C_1 \square C_2) = \max(W(C_1), W(C_2))$. If

$$W(C_1 \square C_2) \leq E(C_1 \square C_2), \quad (3)$$

we set

$$i = G_1(v) + G_2(v) \text{ for } W(C_1 \square C_2) \leq v \leq E(C_1 \square C_2). \quad (4)$$

The set of all such points (v, i) is by definition the characteristic segment $C_1 \square C_2$ for the parallel combination of the confluent oriented branches b_1 and b_2 . This is illustrated in Fig. 6. If, on the other hand, (3) does not hold, we take it that $C_1 \square C_2$ does not exist.

As before, the following result is an immediate consequence of this definition.

Lemma 5.1. An existent parallel sum $C_1 \square C_2$ of two characteristic segments is also a characteristic segment. Moreover, each endpoint of $C_1 \square C_2$ is the parallel sum of a point of C_1 and a point of C_2 , at least one of which is an endpoint of C_1 or of C_2 .

Thus, if a parallel combination of two resistors is operating at one of its ratings points, then at least one of those resistors is also operating at one of its ratings points.

As before, if C_1 and C_2 are given by formulas, then $C_1 \square C_2$ is given as a formula by (4). So, consider the case where either or both of C_1 and C_2 are given graphically or by a table of points and where (3) holds. We wish to compute a table of points for $C_1 \square C_2$. Interpolation will yield intermediate points. Our procedure is the same as that for a series sum except that voltages and currents exchange their roles, but let us be specific. To obtain a fairly uniform set of points on $C_1 \square C_2$, we compute as follows.

Set $v_{min} = W(C_1 \square C_2)$ and $i_{min} = G_1(v_{min}) + G_2(v_{min})$, and set $v_{max} = E(C_1 \square C_2)$ and $i_{max} = G_1(v_{max}) + G_2(v_{max})$. Then, set $v_{mid} = (v_{min} + v_{max})/2$ and $i_{mid} = G_1(v_{mid}) + G_2(v_{mid})$, where $G_1(v_{mid})$ and $G_2(v_{mid})$ are obtained by interpolation in general.

Now, assume that the points (v_k, i_k) for $C_1 \square C_2$ have been computed, where $k = 1, \dots, K$, $K \geq 3$. This partitions $C_1 \square C_2$ into $K - 1$ segments S_k . Let us denote their endpoints by $(v_{k,min}, i_{k,min})$ and $(v_{k,max}, i_{k,max})$. Choose a segment S_l with the largest size $v_{l,max} - v_{l,min} + i_{l,max} - i_{l,min}$. Set $v_{l,mid} = (v_{l,min} + v_{l,max})/2$, and compute $i_{l,mid} = G_1(v_{l,mid}) + G_2(v_{l,mid})$, interpolating if need be. Thus, $(v_{l,mid}, i_{l,mid})$ can be added as a new point to our accruing table of points for $C_1 \square C_2$. Our procedure will yield a more or less uniformly distributed set of points on $C_1 \square C_2$.

6 The Final Two-Branch Loop

Our procedure for reducing the network will consist of a sequence of series and parallel combinations of resistors ending in a loop⁴ consisting of two branches b_0 and b_1 , as shown in Fig. 7 if at every step of the sequence the combined characteristic segment exists. Assuming that this is so, let us orient the final two-branch loop in accordance with Fig. 7, reversing a branch orientation if need be. For a WROP to exist in the original network, we must now have $v_1 = -v_0$ and $i_1 = i_0$.

To state this another way, let \hat{C}_0 be the mirror image through the i -axis of the characteristic segment for b_0 :

$$\hat{C}_0 = \{(v, i) : (-v, i) \in C_0\}. \quad (5)$$

Thus, \hat{C}_0 is a continuous monotonically decreasing curve in the (v, i) -plane with two endpoints (except in the degenerate case of a single point for \hat{C}_0). Then, a WROP will exist in the original network if \hat{C}_0 and C_1 intersect; in this case, the intersection point will be unique by virtue of the monotonicities. To ascertain the existence of such an intersection, it is helpful to think in terms of the *characteristic rectangles* \hat{D}_0 and D_1 for \hat{C}_0 and C_1 . These are the smallest rectangles with sides parallel to the v -axis and i -axis such that the

⁴That procedure will not end in a two-branch series circuit with two distinct end nodes (like that of Fig 3) because the original network is not separable.

endpoints of \hat{C}_0 and C_1 coincide with two corners of \hat{D}_0 and D_1 respectively. This is illustrated in Fig. 8. \hat{D}_0 contains all continuously monotonically decreasing curves \hat{C}_0 with the fixed endpoints⁵ $NW(\hat{C}_0)$ and $SE(\hat{C}_0)$. Except for their endpoints, those curves lie in the interior of \hat{D}_0 . In the same way, D_1 contains all continuous monotonically increasing curves C_1 with the fixed endpoints⁶ $NE(C_1)$ and $SW(C_1)$. However, in the degenerate case, the characteristic rectangle is also a single point.

In order for \hat{C}_0 and C_1 to intersect as stated, it is necessary that \hat{D}_0 and D_1 overlap, but the converse is not true. Nonetheless, by virtue of the intermediate-value theorem, a sufficient condition for the intersection of \hat{C}_0 and C_1 to exist is the following set of four inequalities:

$$N(\hat{C}_0) > N(C_1), S(\hat{C}_0) < S(C_1), E(\hat{C}_0) < E(C_1), W(\hat{C}_0) > E(C_1), \quad (6)$$

See Fig. 9. Another such sufficient set of inequalities for that intersection occurs when \hat{C}_0 and C_1 interchange their positions in (6):

$$N(C_1) > N(\hat{C}_0), S(C_1) < S(\hat{C}_0), E(C_1) < E(\hat{C}_0), W(C_1) > W(\hat{C}_0) \quad (7)$$

See Fig. 10.

On the other hand, each of the following inequalities is a sufficient condition by itself for the nonexistence of a intersection between \hat{C}_0 and C_1 :

$$N(\hat{C}_0) < S(C_1) \quad (8)$$

$$S(\hat{C}_0) > N(C_1) \quad (9)$$

$$E(\hat{C}_0) < W(C_1) \quad (10)$$

$$W(\hat{C}_0) > E(C_1) \quad (11)$$

In each case, the characteristic rectangles fail to overlap.

There are cases however where neither (6) nor (7) holds but where an intersection point between \hat{C}_0 and C_1 is still possible. Such for example, is the case illustrated in Fig. 8.

⁵NW for "northwest" and SE for "southeast."

⁶NE for "northeast" and SW for "southwest."

\hat{C}_0 and C_1 may intersect as shown, or C_1 may pass below all of \hat{C}_0 , thereby avoiding an intersection. In short, if each of the conditions (6) through (11) fail, we must then compute \hat{C}_0 and C_1 to ascertain whether or not they intersect. This is easily done. C_0 and C_1 can be computed as stated in the preceding two sections, and then \hat{C}_0 can be gotten from (5). With \hat{C}_0 and C_1 given by tables,⁷ their successive points are scanned for identical points or more generally for two pairs of successive points where currents reverse their larger and smaller relative values as voltage increases (or alternatively where voltages do so as current increases). If no such occurrence is found, no WROP exists. If such is found, there will be only one such occurrence, and a unique WROP will exist. In the latter case, interpolation will yield the intersection point between \hat{C}_0 and C_1 .

7 All Reduction Sequences Yield the Same Result

So far, we can draw the following conclusions for any chosen sequence of series and parallel reductions. If any such reduction fails to produce a characteristic segment or if they all do produce characteristic segments but then \hat{C}_0 and C_1 fail to intersect, then there is no WROP. On the other hand, if the reduction sequence produces a \hat{C}_0 and a C_1 and they intersect, then a unique WROP exists. The question that must now be addressed is whether or not these conclusions depend upon the choice of the sequence of series and parallel combinations.

Lemma 7.1: It is impossible for one sequence of series and parallel combinations to lead to the existence of a WROP and for another such sequence to lead to the nonexistence of a WROP.

Proof: We shall invoke Theorem 3.1. To this end, we can extend each characteristic segment into a two-way infinite characteristic curve satisfying the conditions hypothesized in that theorem. After doing this, let N_x denote the resulting network. Theorem 3.1 holds for N_x . Thus, N_x has a unique operating point.

Suppose this lemma is not true; that is, let one sequence lead to the existence of a WROP and a second sequence to the nonexistence of a WROP. For the first sequence, every resistor operates within its ratings. For the second sequence, the operating point for

⁷In the unlikely event that \hat{C}_0 and C_1 can be obtained as formulas, it is computationally more convenient to convert them to tables.

N_x is such that at least one resistor operates beyond its ratings. Thus, N_x must have at least two operating points, in contradiction to Theorem 3.1. \square

8 A Test for the Existence of a WROP

At this point we have the following procedure for answering the question, "Does the series-parallel network have a WROP?" by either a "yes" or a "no." As always, it is assumed that Conditions 2.1 hold and that every branch has an arbitrarily assigned orientation.

Procedure 8.1:

- 1) Search the network for a series or parallel circuit of two branches. (There will be at least one such pair.)
- 2) If they are not confluent oriented, reverse the orientation of one of them and adjust its formula or table accordingly.
- 3a) If they are in series and if (1) is violated, state "no." Then, stop.
- 3b) If they are in series and if (1) holds, reduce the network by combining the two branches and computing the series sum of their characteristic segments (Section 4).
- 3c) If they are in parallel and if (3) is violated, state "no." Then, stop.
- 3d) If they are in parallel and if (3) holds, reduce the network by combining the two branches and computing the parallel sum of their characteristic segments (Section 5).
- 4) If the reduced network has at least three branches, return to Step 1.
- 5) If the reduced network has exactly two branches, reverse the orientation of one of them if need be to obtain the relative orientations of Fig. 7.
- 6a) If either (6) or (7) holds, state "yes."
- 6b) If any one of (8) through (11) holds, state "no."
- 6c) If none of the conditions (6) through (11) holds, compute \hat{C}_0 and C_1 and check for an intersection point (Section 6). If an intersection point exists, state "yes"; if not, state "no."

7) Stop.

All this is readily programmed for a computer.

9 Computation of the WROP

In the event that Step 7 is reached and a "yes" has been obtained in Procedure 8.1, it is a simple matter to compute all the branch voltages and currents throughout the network, thereby completely determining the WROP (if indeed such complete information is desired). This is because all characteristic segments have already been computed as tables or formulas except possibly for C_0 and C_1 ; the latter can be determined as in Section 4 or Section 5 again. Next, the intersection point between \hat{C}_0 and C_1 can be computed as in Section 6. Then, a reversal of the sequence of series and parallel combinations by which the network was reduced will yield all branch voltages and currents by solving series and parallel summations. Solving a series summation simply requires the insertion of the obtained common value of current into both characteristic segments, using possibly interpolation, to obtain both branch voltages. Similarly, a parallel summation is solved by inserting the common voltage value to get both branch currents. Of course, branch orientations must be obeyed by adjusting the signs of currents and voltages.

If, on the other hand, the voltage and current of one particular branch is all that is desired, just designate that branch as b_0 and choose a reduction sequence accordingly. Then, the intersection point between \hat{C}_0 and C_1 , computed as in Section 6, gives that voltage and current directly.

10 Conclusions

Our principle purpose is to ascertain quickly whether or not a WROP exists in a network satisfying Conditions 2.1. If a WROP does not exist, Procedure 8.1 may reveal that fact well before all resistors are examined. If a WROP does exist, Procedure 8.1 will show this too but only after all resistors are examined. In the latter case, all branch voltages and currents can then be determined without much additional computation. At no point does our method employ the robust but computationally prolonged standard iterative numerical

methods. Moreover, if our interest is on a particular branch, we can get its voltage and current without having to determine the other branch voltages and currents. All this is easily programmed.

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Figure Captions

Fig. 1. The effect on a characteristic segment C of reversing the orientation of a branch.

The symbols with the tildes denote entities after the reversal has been made.

Fig. 2. (a) A characteristic segment that does not incorporate any source. (v_{max}, i_{max}) and (v_{min}, i_{min}) are its ratings points.

(b) A characteristic segment that incorporates a voltage source e . The ratings points are now $(v_{max} - e, i_{min})$ and $(v_{min} - e, i_{min})$.

(c) A characteristic segment that incorporates a current source h . The ratings points are now $(v_{max}, i_{max} - h)$ and $(v_{min}, i_{min} - h)$.

Fig. 3. Two confluent oriented series-connected branches.

Fig. 4. The series sum $C_1 \diamond C_2$ of the characteristic segments of two confluent oriented series-connected resistors.

Fig. 5. Two confluent oriented parallel branches.

Fig. 6. The parallel sum $C_1 \square C_2$ of the characteristic segments of two confluent oriented parallel-connected resistors.

Fig. 7. The final two-branch loop of the reduction process.

Fig. 8. The characteristic rectangles \hat{D}_0 for \hat{C}_0 and D_1 for C_1 . This figure has been drawn with intersecting \hat{C}_0 and C_1 , but this need not be the case.

Fig. 9. Illustration of the sufficient conditions (5) for the existence of a WROP.

Fig. 10. Illustration of the sufficient conditions (6) for the existence of a WROP.

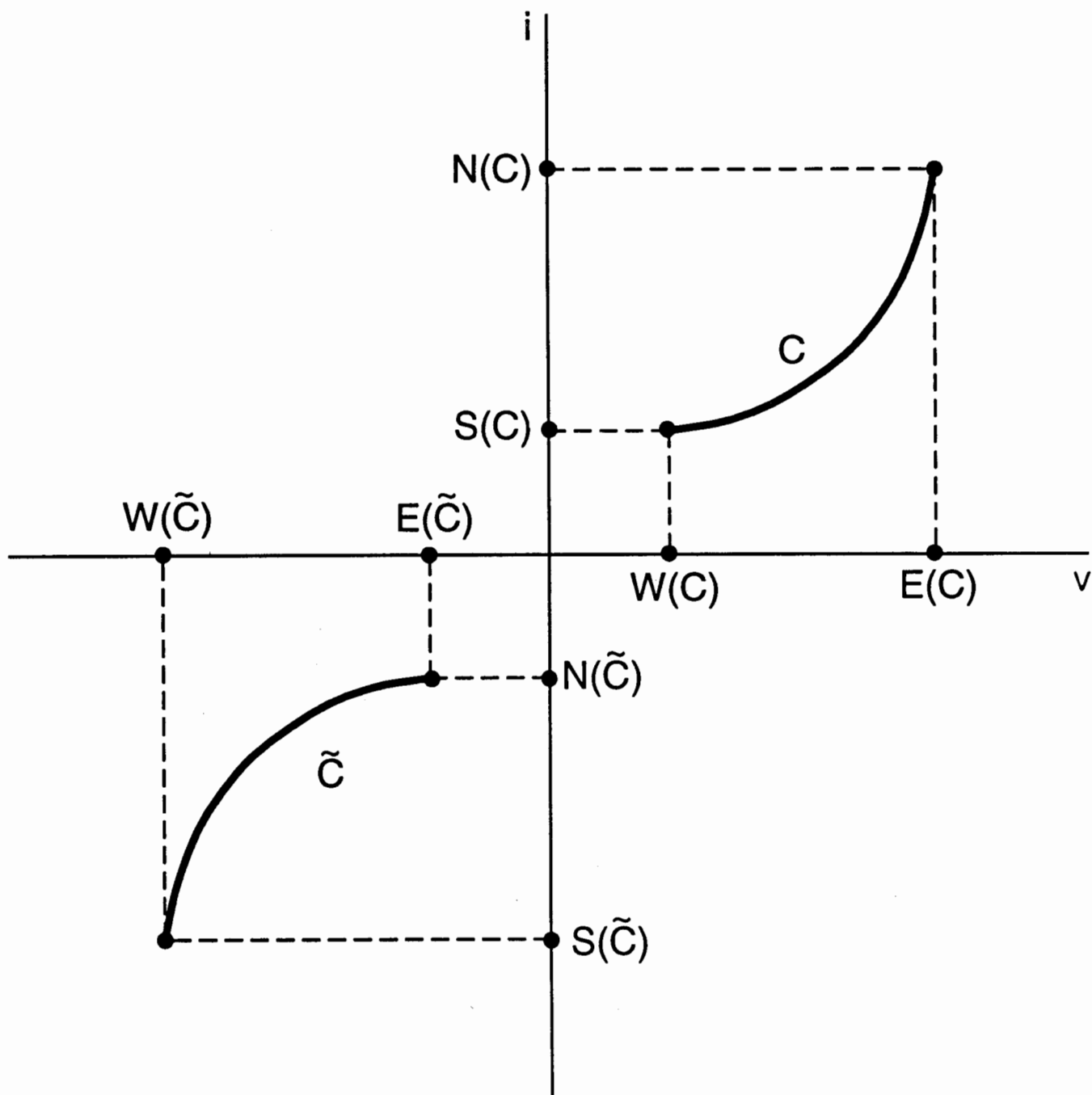
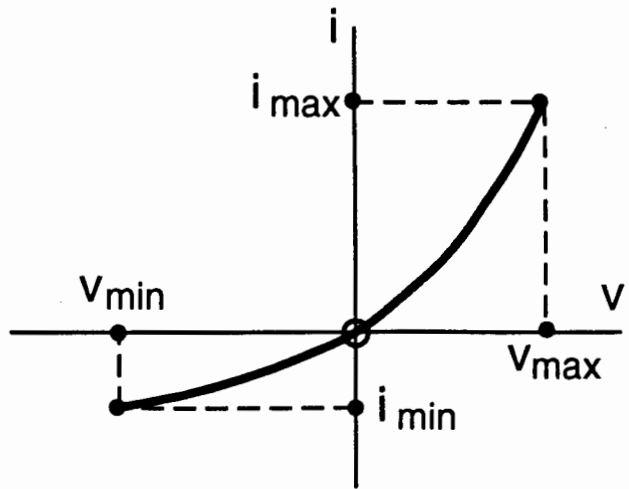
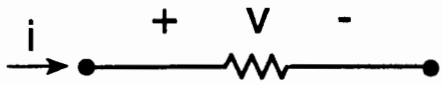
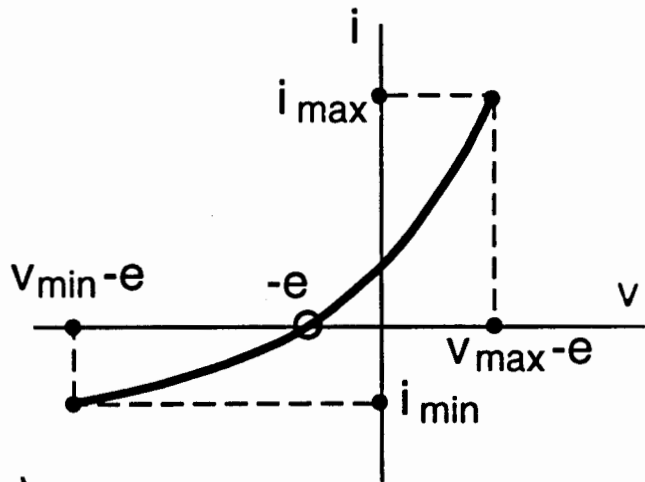
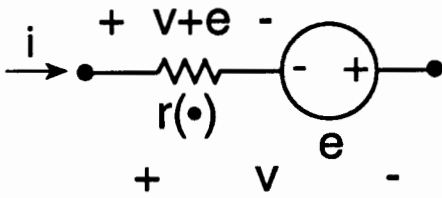


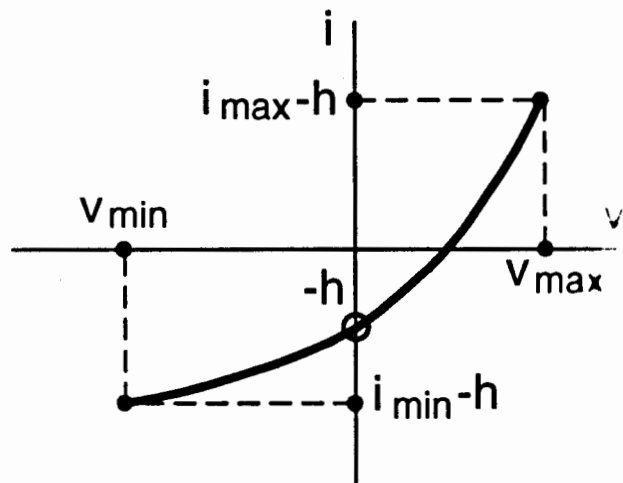
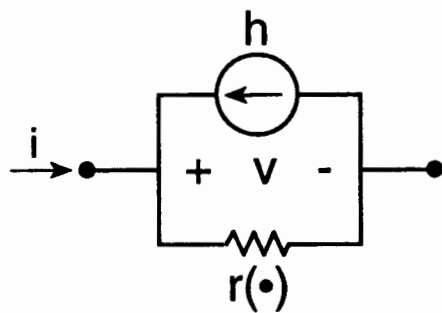
Fig. 1



(a)



(b)



(c)

Fig. 2 a/b/c

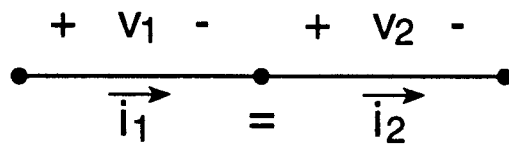


Fig. 3

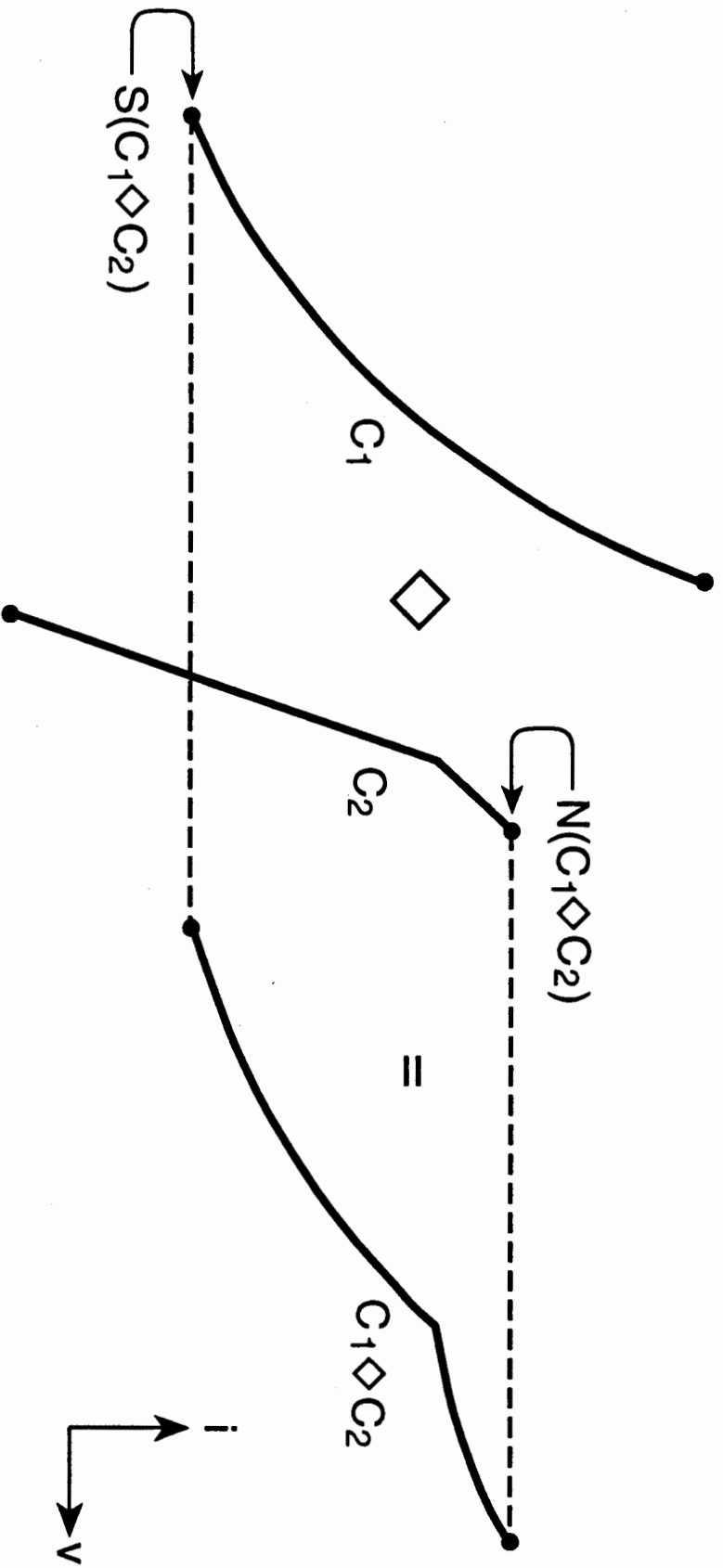


Fig. 4

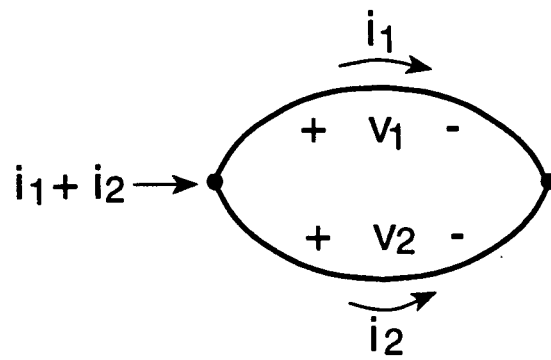


Fig. 5

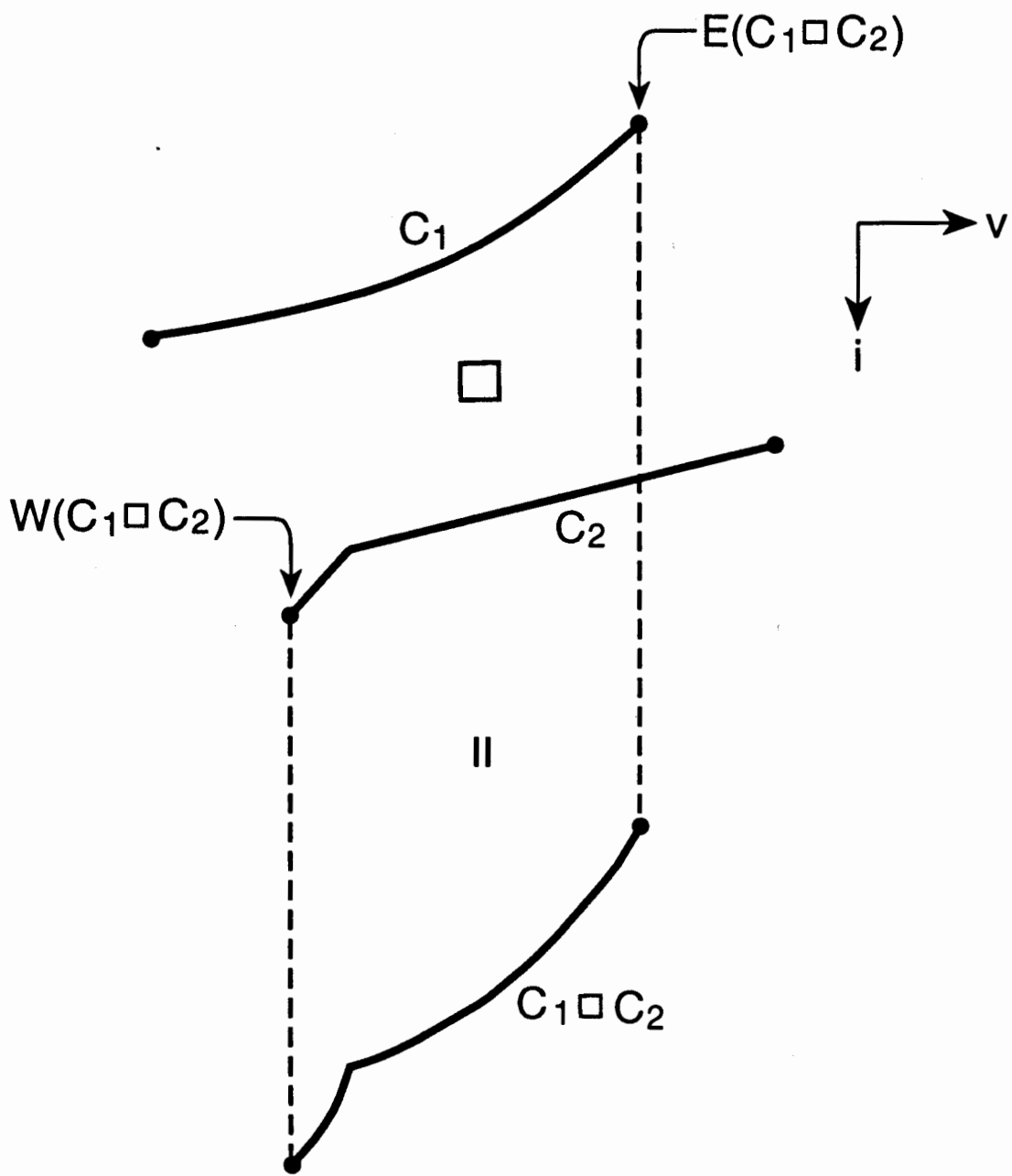


Fig. 6

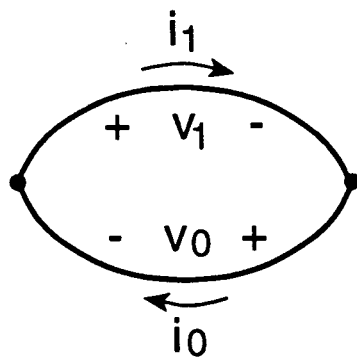


Fig. 7

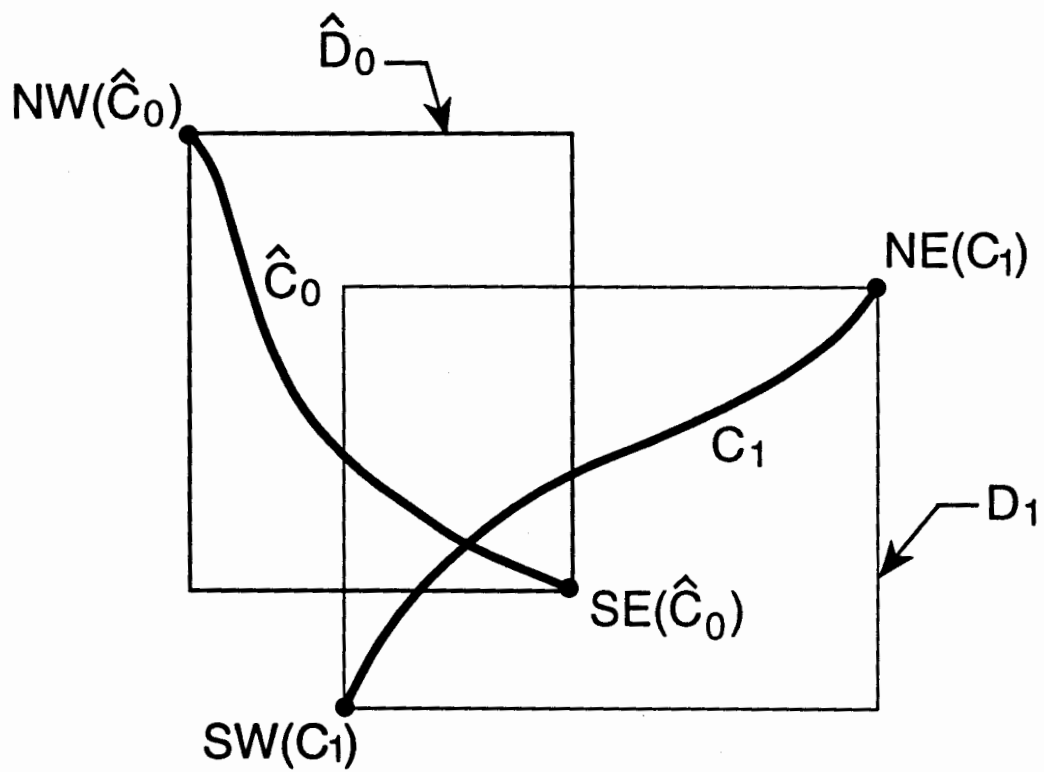


Fig. 8

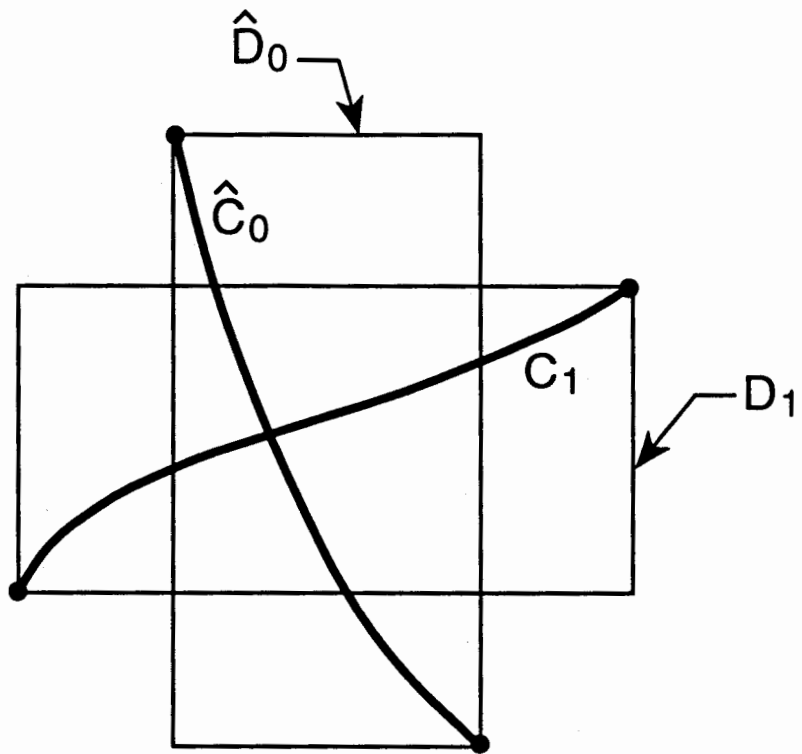


Fig. 9

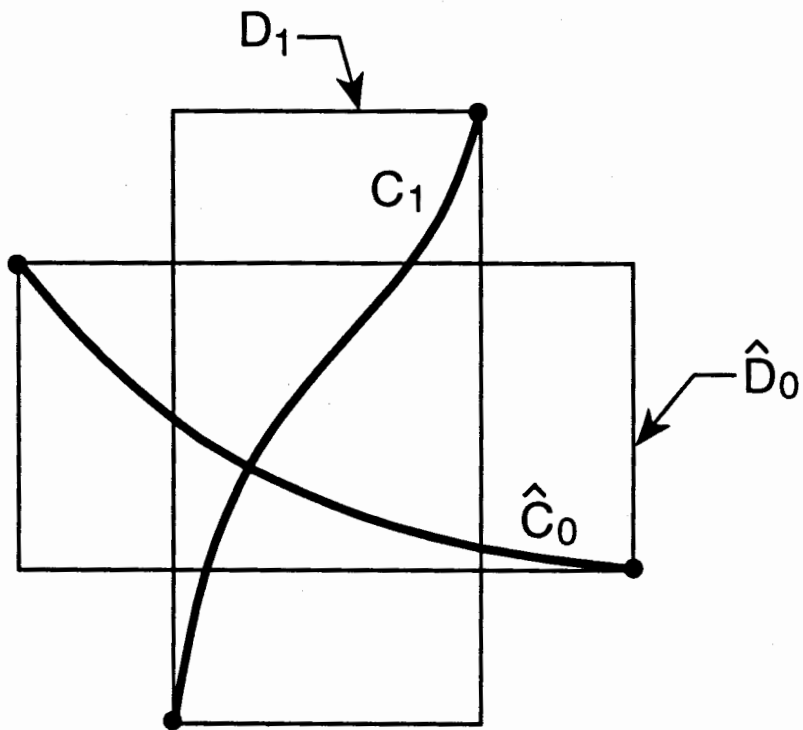


Fig. 10