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**Multibeam Cellular Communication Systems with Dynamic Channel
Assignment across Multiple Sectors**

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Abstract

In cellular communication systems, directional multibeam antennas at cell sites can be used to reduce co-channel interference, increase frequency reuse and improve system capacity. When combined with dynamic channel assignment (DCA), additional improvement is possible. We propose a multibeam scheme using dynamic channel assignment across multiple sectors. A cell is divided into several sectors, each of which is covered by several directional beams. Specific channels are allocated to each sector as in fixed channel assignment (FCA). A channel of a sector is dynamically assigned to a wireless user who communicates through one of the several beams of the sector. The assignment is made so that constraints on the allowable co-channel interference are satisfied. Limitations due to co-channel interference are analyzed. A tractable analytical model for the proposed scheme is developed using multidimensional birth-death processes. Theoretical traffic performance characteristics such as call blocking probability, forced termination probability, hand-off activity, carried traffic and channel rearrangement rate are determined. With the proposed scheme, call blocking probability can be reduced significantly for a fixed offered traffic. Alternatively, system capacity can be increased while blocking probability is maintained below the required level. Smaller forced termination probability is obtainable in comparison with corresponding FCA schemes.

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1. Introduction

As the number of wireless users grows rapidly, increased system capacity is sought. Sectorization and cell splitting are used to allow increased system capacity while interference is limited to maintain signal quality. Space Division Multiple Access (SDMA) is a technique which can increase system capacity by using directional multibeam antennas to reduce co-channel interference [1], [2]. This allows reduced frequency reuse distances for the same link quality of communication links and results in higher system capacity.

One approach in SDMA uses switched multibeams. Multiple beams are used to cover the service area of a base station and the beam with the strongest signal power is selected to serve the user [3]. Recent work on switched multibeam schemes includes the investigation of the gain improvement achieved with a multibeam antenna compared to the traditional sector configuration [4]. The tradeoffs between hysteresis level, switching time and gain for a multibeam antenna system are considered in [4]. The effects of incorrect beam selection on average signal-to-noise (SNR) power ratio and signal-to-interference (SIR) power ratio with a switched multibeam antenna system are examined in [5]. The frequency reuse efficiency of multibeam antenna systems is investigated in [6], [7]. The possibilities of channel reuse within beams of the same cell are analyzed in [8], [9]. The multibeam antenna systems combined with dynamic channel assignment (DCA) scheme is investigated in [9]. In [9], we have considered a multibeam antenna system in which channels that are assigned to a sector can be reused in *different beams of the same sector* provided the required angular separation between beams is met. DCA within a single sector is used. System capacity can be increased significantly using this approach. Even if channels are not reused in different beams of the same sector, improvement in system capacity is still possible. In this paper, we consider such a case and propose a scheme which combines the advantages of switched multibeam antennas and dynamic channel assignment, in which *DCA is used across multiple sectors*.

DCA is a technique which can improve channel reuse in cellular communications [10], [11], [12], [13]. In DCA, all channels are dynamically assigned to wireless users, subject to the constraints on the allowable co-channel interference. Channel rearrangement can be used to avoid unnecessary blocking of calls and allows the channel resources to be used efficiently. In channel rearrangement, the channel used to serve a particular call is not fixed. Depending on the channel occupancy and interference conditions, a call may switch between several different channels during its lifetime.

We consider a large population of mobile wireless users in a large geographical region covered by cells. A cell is divided into several sectors, each of which is covered by several directional beams. Specific channels are allocated to each sector as in FCA, and DCA is used across multiple sectors. Channels of a sector are dynamically assigned to wireless users in the sector as long as the co-channel interference constraints are satisfied. A wireless user can access any of the channels of a sector without regard to the beam through which it communicates. Models to compute fundamental traffic performance measures for the proposed scheme are devised. These include call blocking probability, forced termination probability, hand-off activity, carried traffic and channel rearrangement rate. Multidimensional birth-death processes are discussed in [16], [17], [20]. The global balance equations are determined and solved for the state probabilities, using the framework developed in earlier work [16], [17]. Performance characteristics are found from these state probabilities.

System traffic performance can be improved by using a multibeam scheme and DCA technique. For a fixed offered traffic, the blocking probability of calls can be reduced significantly. Alternatively, more new call traffic can be supported while the blocking probability is maintained. From a wireless subscriber's point of view, forced termination probability is a major concern. Smaller forced termination probability means a wireless user's call will be less likely to experience interruption during its lifetime. The proposed multibeam scheme has smaller forced termination probability than traditional sectorized FCA schemes.

2. Model Description

We consider a large geographical region covered by cells and traversed by large numbers of wireless platforms, such as vehicles and pedestrians. Each platform can support at most one call. A cell is divided into several sectors, each of which is covered by several directional beams. Specific channels are allocated to each sector as in FCA. There are C channels assigned to each sector. An example model that we consider is a 120° -sectored multibeam antenna system with two beams in each sector and a cluster size of four. This provides a total of six beams per cell as shown in Fig. 1. The beam in the counterclockwise direction is called the left beam and the other is called the right beam. Sectors which have the same angular orientation are allocated the same set of channels. These are co-channel sectors. For example, as shown in Fig. 2a, sectors 0, 1, 2, 3, and so on are co-channel sectors. *We define the interfering co-channel beams of a given sector as the beams in the neighboring co-channel sectors that point to the given sector. Here only the first tier co-channel sectors of a given sector are considered.* There are two interfering co-channel beams in the first tier co-channel sectors for a given sector. We can see from Fig. 2a, *the right beam of sector 0 and the left beam of sector 1 point to the same sector - - sector 7. These are interfering co-channel beams of sector 7. Similarly, the right beam of sector 1 and the left beam of sector 2 are the interfering co-channel beams of sector 8, and so on.* Since the simultaneous use of the same channel in the right beam of sector 0 and the left beam of sector 1 will cause excessive co-channel interference in sector 7, such use must be precluded. Similarly, the simultaneous use of the same channel in the right beam of sector 1 and the left beam of sector 2 will cause excessive co-channel interference in sector 8, and so on. Because there are C channels assigned to a sector, the total number of channels in interfering co-channel beams cannot exceed C .

A cut-off priority scheme is used to favor *hand-off calls* with respect to *new calls*. This approach reserves a certain number of channels for use by hand-off calls. Specific channels are not reserved, just the number. Referring to Fig. 2a, consider sector 1, which

has two beams (left and right). The left beam of sector 1 is an interfering co-channel beam of sector 7. We see from Fig. 2a that the right beam of sector 0 is also an interfering co-channel beam of sector 7. Since the simultaneous use of the same channel in the left beam of sector 1 and the right beam of sector 0 will cause excessive co-channel interference in sector 7, the use and assignment of channels to calls in sector 1 is coupled to the use of channels in sector 0. Succinctly, because the left beam of sector 1 and the right beam of sector 0 are interfering co-channel beams of the same sector (sector 7), the simultaneous use of channels in sectors 1 and 0 must be constrained. Similarly, the right beam of sector 1 and the left beam of sector 2 are interfering co-channel beams of the same sector (sector 8), the simultaneous use of channels in sectors 1 and 2 must be constrained. In order to provide cut-off priority for hand-off calls, therefore we will use the following channel assignment constraints. For clarity we discuss this assignment for sector 1, but the rules are similar for all sectors.

1. The total number of channels in use in sector 1 cannot exceed C . (The channel limit is C).
2. Sector 1 reserves C_{h1} channels of the C channels allocated to the sector for use by hand-off calls. Thus new calls that arrive in sector 1 will be blocked if the number of channels in use in sector 1 is $C - C_{h1}$ or greater.
3. The total number of channels in use in two interfering co-channel beams (the *left beam of sector 1* and the *right beam of sector 0*) cannot exceed C . Similarly, the number of channels in use in the *right beam of sector 1* and the *left beam of sector 2* cannot exceed C (and so forth, in like manner).
4. C_{h2} channels of the C channels are reserved for use by hand-off calls in two interfering co-channel beams. Thus new calls that arrive in the *left beam of sector 1* will be blocked if the number of channels in use in the *left beam of sector 1* and the *right beam of sector 0* is $C - C_{h2}$ or greater. Similarly, new calls that arrive in the *right beam of sector 1* will be blocked if the number of channels in use in the *right beam of sector 1* and the *left beam of sector 2* is $C - C_{h2}$ or greater. (These are interfering co-channel beams of sector 8).

5. Hand-off attempts (to sector 1) will fail if the number of channels in use in sector 1 is C . Hand-off attempts (to the left beam of sector 1) will also fail if the number of channels in use in the left beam of sector 1 and the right beam of sector 0 is C , and hand-off attempts (to the right beam of sector 1) will also fail if the number of channels in use in the right beam of sector 1 and the left beam of sector 2 is C .

In this way, hand-off calls have access to more channels than new calls do, and increasing C_{h1} or C_{h2} provides increasing priority for hand-offs at the expense of blocking new call originations. Thus forced termination and blocking performance can be exchanged.

Dynamic channel assignment improves channel reuse. Subject to the allowable co-channel interference, the channels of a *sector* are dynamically assigned to wireless users in that *sector* without regard to the *beam* through which they communicate. Channel rearrangement is used to avoid unnecessary blocking of calls and allow the channel resources to be used more efficiently.

In earlier work [15], [16], the concept of dwell time was used to characterize the mobility of platforms. This is the amount of time that a wireless platform is within communicating range of a given gateway. The dwell time depends on many factors such as propagation conditions, the path that a wireless platform follows, its velocity profile along the path, and especially the definition of communicating range. The dwell time is modeled by a random variable with a negative exponential distribution (n.e.d). The unencumbered call session duration of a call is assumed to be random with a negative exponential distribution (n.e.d). The new call originations are assumed to follow a Poisson point process.

3. Example Problem Statement

A large population of wireless platforms is considered. No platform can support more than one call at any given time. The new call origination rate from a non-communicating platform is denoted Λ . The number of non-communicating platforms in any sector is denoted ν . Therefore, the total call origination rate in a sector, Λ_n , is

$\Lambda_n = \Lambda \cdot v$. It is assumed that the number of non-communicating platforms is much larger than the number of channels in a sector so that the call generation rate does not depend on the number of calls in progress. This infinite population model is reasonable for commercial cellular systems.

When DCA is combined with channel arrangement and rearrangement, channel resources can be exploited even more efficiently and unnecessary blocking of calls can be avoided. The channels of a sector are numbered by integers from 1 to C. A channel is assigned to a new call or a hand-off call according to the following strategy. At the time of a new call or a hand-off call arrival in the left beam of a sector, an available channel with the lowest number is assigned to serve the call. If a new call or a hand-off call arrives in the right beam of a sector, an available channel with the highest number is assigned to serve the call. Examples of channel arrangement are shown in the top of Fig. 2b. Channels #1 to #3 are in use in the left beam and channels #C-2 to #C are in use in the right beam. When a new call or a hand-off call arrives in the left beam, channel #4 is the available channel with the lowest number and thus is assigned to serve the call. When a new call or a hand-off call arrives in the right beam, channel #C-3 is the available channel with the highest number and thus is assigned to serve the call. Channel rearrangement is used at a call completion and a hand-off departure. Channels in use are always rearranged to maintain a *compact pattern*. That is, channels in use in the left beam are always those channels with the lowest numbers and channels in use in the right beam are always those channels with the highest numbers. At the time of a call completion or a hand-off departure, channels in use are rearranged if the call completion or hand-off departure creates a pattern that is not compact. This fosters a high level of channel reuse. Examples of channel rearrangement are shown in the bottom of Fig. 2b. As an example for the left beam, the call using channel #2 in the left beam completes and the call using channel #5 switches to use channel #2 to maintain a *compact pattern*. As an example for the right beam, the call using channel #C completes and the call using channel #C-4 switches to use channel #C.

The unencumbered call session duration of a call is modeled as a n.e.d. random variable having a mean $T_c = 1/\mu_c$. The dwell time is determined by the duration of time that a two-way link between a wireless platform and its serving gateway can be maintained, for whatever reasons. The dwell time of a wireless platform in a beam is modeled as a n.e.d. random variable having a mean $T_D = 1/\mu_D$. Wireless users are distributed uniformly over the region of interest. The new call origination in a beam is a Poisson point process having a mean arrival rate $\Lambda_{nb} = \Lambda_n/2$.

4. Analysis of Co-channel Interference

We consider the usual hexagonal geometry and a 120° -sectorized multibeam scheme with two beams in each sector. Let R denote the radius of a cell. The reuse distance D is defined as the distance between the base stations of two nearest co-channel cells. Let N denote the cluster size, which is related to the reuse shift parameters (i, j) by $N = i^2 + ij + j^2$. The integers i and j determine the reuse pattern and identify co-channel cells. The co-channel reuse factor, Q , is defined as the ratio of D to R . This ratio is $\frac{D}{R} = \sqrt{3N}$.

We consider the carrier-to-interference ratio (CIR) of multibeam cellular communication systems in the worst case. Normal cellular practice specifies CIR to be 18 dB or higher. This is based on subjective tests and the criterion that 75 percent of the users say voice quality is "good" or "excellent" in 90 percent of the total covered area on a flat terrain [21]. Let I denote the normalized co-channel interference, which is normalized by the desired power. The CIR of multibeam cellular communication systems can be calculated as

$$CIR = 10 \log_{10} \left(\frac{1}{I} \right) \text{ dB.} \quad (1)$$

Consider a cellular system with cluster size $N=4$. As shown in Fig. 2a, sectors 0, 1, 2, 3, and so on are co-channel sectors. There are two co-channel beams in the first tier co-

channel sectors for a sector. The right beam of sector 4 and left beam of sector 5 point to the same sector - - sector 1, and are the co-channel beams of sector 1. Similarly, the right beam of sector 5 and left beam of sector 6 are co-channel beams of sector 2, etc. In the following discussion we use “up-link” to denote wireless user to base station and “down-link” to denote base station to wireless user.

In the worst case of up-link, the desired wireless user is at the vertex of a given sector and the interfering wireless user is at the position that is nearest to the gateway of the given sector in the co-channel sector. In the worst case of down-link, the desired wireless user is at the vertex of a given sector which is closest to the interfering gateway of its co-channel sector. For example, suppose that the desired wireless user is served by the right beam of the sector 1 as shown in Fig. 2. Consider the up-link case. In the worst case, the distance between the desired wireless user and its serving gateway is R , and the distance between the interfering wireless user in co-channel sector 8 and desired gateway of sector 1 is d_1 . From the geometry of Fig. 2, for a cluster of 4, d_1 is calculated as $2R\sqrt{3}$. Let I_u denote the value of I for the up-link in the worst case. The normalized co-channel interference on the up-link, I_u , can be calculated as

$$I_u = \left(\frac{R}{d_1}\right)^\gamma \quad (2)$$

in which, $d_1 = 2R\sqrt{3}$ and γ is a propagation constant that is heavily influenced by the actual terrain environment. The value of γ usually lies between 3 and 5.

Consider the down-link case. In the worst case, the distance between the desired wireless user and its serving gateway is R , and the distance between the desired wireless user and its nearest interfering gateway of sector 5 is d_2 . From geometry of Fig 2, d_2 is calculated as $\sqrt{13}R$. Let I_d denote the value of I for the down-link in the worst case. The normalized co-channel interference on the down-link, I_d , can be calculated as

$$I_d = \left(\frac{R}{d_2}\right)^\gamma \quad (3)$$

in which, $d_2 = \sqrt{13}R$.

The comparisons of *CIR* between traditional 6-sector scheme and the proposed 6-beam multibeam scheme are shown in Table 1 for both up-link and down-link in the worst case. It can be seen that the *CIR* on the up-link is worse than the *CIR* on the down-link for both schemes. Therefore up-link is the dominant one that limits quality of communication links. *CIR* on the up-link has the same value for both schemes.

<i>CIR</i>	Up-link	Down-link
Traditional 6-sector scheme	21.58 dB	25.58 dB
Proposed 6-beam multibeam scheme	21.58 dB	22.28 dB

Table 1. Comparisons of *CIR* between traditional scheme and proposed scheme in the worst case.

Parameters: Cluster size $N=4$, propagation constant $\gamma = 4$.

5. State Description

For the problem under consideration in this paper (infinite population model, single platform type and single call type), two state variables are needed to describe the status of each sector: one is the number of channels in use in the left beam and the other is the number of channels in use in the right beam. A complete state representation for the whole system will be a string of sector states with two state variables for each sector. However, the huge number of system states precludes pursuing this approach for most cases of interest. A simplified approach is to decouple a sector from others (using average new call arrival rates and average hand-off arrival rates from neighbors) and to model the statistical behavior of a given sector independently from the behavior of other sectors. Therefore, even though we calculate the theoretical performance characteristics from a given sector independently from others, we do consider the effect from its neighbors. This is similar to the approach used in [16], [17], [19]. Because of the homogeneous property of the system, the behavior of any sector in statistical equilibrium is the same as any other. However, due to the co-channel interference constraints, no two interfering co-channel beams of a given sector can use the same channel simultaneously. These two

interfering co-channel beams belong to two adjacent co-channel sectors. Thus the activities of any two adjacent co-channel sectors located in the same line perpendicular to the orientation of the co-channel sectors are not statistically independent. To account for this dependency, we consider two such adjacent co-channel sectors at the same time. That is, we consider two such adjacent co-channel sectors as a basic element. We can define the state of the basic element by a sequence of nonnegative integers, l_1, r_1, l_2, r_2 . In this sequence, the state variables, $l_i, i = 1, 2$, is the number of calls served by the left beam of sector i and, $r_i, i = 1, 2$, is the number of calls served by the right beam of sector i . Then, for convenience, we order the states using an index $s=0, 1, 2, \dots, S_{\max}$. Thereafter, l_i and $r_i, i = 1, 2$, can be shown as explicitly dependent on the state. That is, $l_i = l(s, i)$ and $r_i = r(s, i), i = 1, 2$, in which $l(s, i)$ is the number of calls served by the left beam of sector i when the basic element is in state, s , and $r(s, i)$ is the number of calls served by the right beam of sector i when the basic element is in state, s .

If C denotes the number of channels in each sector, we can specify the constraints on permissible states as

$$l(s, i) + r(s, i) \leq C, \quad \text{for all } i. \quad (4)$$

$$r(s, i) + l(s, i + 1) \leq C, \quad \text{for all } i. \quad (5)$$

The inequality, (4), means that the number of channels in use in any sector when the basic element is in state, s , cannot be larger than C . The inequality, (5), means that the number of channels in use in two interfering co-channel beams of any given sector when the basic element is in state, s , cannot be larger than C .

6. *Driving Processes and State Transition Flow*

There are four relevant driving processes. These are: {n} the generation of new calls in the sectors of interest; {c} the completion of calls in the sectors of interest; {h} the arrival of communicating wireless users at the sectors of interest; {d} the departure of communicating wireless users from the sectors of interest. We use Markovian

assumptions for the driving processes to render the problem amenable to solution using multidimensional birth-death processes.

6.1 New Call Arrivals

6.1.1 New call arrivals in the left beam of sector 1

A transition into state s due to a new call arrival in the left beam of sector 1 when the basic element is in state x_n , will cause the state variable $l(x_n,1)$ to be incremented by 1. We note that, because cut-off priority is used, C_{h1} channels are held for hand-off arrivals in a sector. A new call can be served in the left beam of sector 1 only if the number of channels in use in sector 1 does not exceed $C - C_{h1}$. Thus a permissible state x_n is a predecessor state of s for new call arrivals in the left beam of sector 1, if $l(x_n,1) + r(x_n,1) < C - C_{h1}$, and the state variables are related by

$$\begin{aligned} l(x_n,1) &= l(s,1) - 1, \\ l(x_n,2) &= l(s,2), \\ r(x_n,i) &= r(s,i), \quad i=1,2. \end{aligned} \tag{6}$$

Let Λ_{nb} denote the average arrival rate of new calls in a beam. We note that, only a fraction of the new call arrivals in the left beam of sector 1 contributes to the transition flow because the success of new call arrivals also depends on the states of sector 0. When the basic element is in state x_n , the value of $l(x_n,1)$ is known, however the number of calls in service in the right beam of sector 0, which is denoted $r(x_n,0)$, is unknown. The value of $r(x_n,0)$ may vary from 0 to $C - l(x_n,1)$. Because of the homogeneous property of the system, the statistical behavior of all basic elements in equilibrium is identical. Thus we can calculate the probability that there is no channel available for a new call in the left beam of sector 1 and right beam of sector 0 when the basic element is in state x_n , which is denoted $P_{bl}(x_n)$. Since the behavior of any two adjacent co-channel sectors is not independent, $P_{bl}(x_n)$ is actually a conditional probability and defined as follows.

$P_{bl}(x_n) = \text{Prob}\{ r(x_n,0) \geq C - C_{h2} - l(x_n,1) \mid \text{the numbers of calls in service in the left and right beams of sector 1 are } l(x_n,1) \text{ and } r(x_n,1) \text{ respectively when the basic element}$

is in state x_n } (7)

Thus the flow into state s from x_n due to new call arrivals in the left beam of sector 1 is

$$\gamma_{n1}(s, x_n) = \Lambda_{nb} \cdot (1 - P_{bl}(x_n)), \quad \text{if } l(x_n, 1) + r(x_n, 1) < C - C_{h1}. \quad (8)$$

6.1.2 New call arrivals in the right beam of sector 1

A transition into state s due to a new call arrival in the right beam of sector 1 when the basic element is in state x_n , will cause the state variable $r(x_n, 1)$ to be incremented by 1. We note that, because cut-off priority is used, C_{h1} channels are held for hand-off arrivals in a sector and C_{h2} channels are held for hand-off arrivals in two interfering co-channel beams of any given sector. A new call can be served in the right beam of sector 1 only if the number of channels in use in sector 1 does not exceed $C - C_{h1}$ and the number of channels in use all together in the right beam of sector 1 and left beam of sector 2 does not exceed $C - C_{h2}$. Thus a permissible state x_n is a predecessor state of s for new call arrivals in the right beam of sector 1, if $l(x_n, 1) + r(x_n, 1) < C - C_{h1}$ and $r(x_n, 1) + l(x_n, 2) < C - C_{h2}$, and the state variables are related by

$$\begin{aligned} l(x_n, i) &= l(s, i), & i &= 1, 2 \\ r(x_n, 1) &= r(s, 1) - 1, \\ r(x_n, 2) &= r(s, 2), \end{aligned} \quad (9)$$

The flow into state s from x_n due to new call arrivals in the right beam of sector 1 is

$$\gamma_{n2}(s, x_n) = \Lambda_{nb}, \quad \text{if } l(x_n, 1) + r(x_n, 1) < C - C_{h1} \text{ and } r(x_n, 1) + l(x_n, 2) < C - C_{h2}. \quad (10)$$

6.1.3 New call arrivals in the left beam of sector 2

A transition into state s due to a new call arrival in the left beam of sector 2 when the basic element is in state x_n , will cause the state variable $l(x_n, 2)$ to be incremented by 1. A new call can be served in the left beam of sector 2 only if the number of channels in use in sector 2 does not exceed $C - C_{h1}$ and the number of channels in use all together in the right beam of sector 1 and left beam of sector 2 does not exceed $C - C_{h2}$. Thus a permissible state x_n is a predecessor state of s for new call arrivals in the left beam of

sector 2, if $l(x_n,2) + r(x_n,2) < C - C_{h1}$ and $r(x_n,1) + l(x_n,2) < C - C_{h2}$, and the state variables are related by

$$\begin{aligned} l(x_n,1) &= l(s,1), \\ l(x_n,2) &= l(s,2) - 1, \\ r(x_n,i) &= r(s,i), \quad i=1,2. \end{aligned} \quad (11)$$

The flow into state s from x_n due to new call arrivals in the left beam of sector 2 is

$$\gamma_{n3}(s, x_n) = \Lambda_{nb}, \quad \text{if } l(x_n,2) + r(x_n,2) < C - C_{h1} \text{ and } r(x_n,1) + l(x_n,2) < C - C_{h2}. \quad (12)$$

6.1.4 New call arrivals in the right beam of sector 2

A transition into state s due to a new call arrival in the right beam of sector 2 when the basic element is in state x_n , will cause the state variable $r(x_n,2)$ to be incremented by 1. A new call can be served in the right beam of sector 2 only if the number of channels in use in sector 2 does not exceed $C - C_{h1}$. Thus a permissible state x_n is a predecessor state of s for new call arrivals in the right beam of sector 2, if $l(x_n,2) + r(x_n,2) < C - C_{h1}$, and the state variables are related by

$$\begin{aligned} l(x_n,i) &= l(s,i), \quad i=1,2. \\ r(x_n,1) &= r(s,1), \\ r(x_n,2) &= r(s,2) - 1. \end{aligned} \quad (13)$$

We note that, only a fraction of the new call arrivals in the right beam of sector 2 contributes to the transition flow because the success of new call arrivals also depends on the states of sector 3. When the basic element is in state x_n , the value of $r(x_n,2)$ is known, however the number of calls in service in the left beam of sector 3, which is denoted $l(x_n,3)$, is unknown. The value of $l(x_n,3)$ may vary from 0 to $C - r(x_n,2)$. Because of the homogeneous property of the system, the statistical behavior of all basic elements in equilibrium is identical. Thus we can calculate the probability that there is no channel available for a new call in the right beam of sector 2 and left beam of sector 3 when the basic element is in state x_n , which is denoted $P_{br}(x_n)$. Since the behavior of any two adjacent co-channel sectors is not independent, $P_{br}(x_n)$ is actually a conditional probability and defined as follows.

$P_{br}(x_n) = \text{Prob}\{ l(x_n,3) \geq C - C_{h2} - r(x_n,2) \mid \text{the numbers of calls in service in the left and right beams of sector 2 are } l(x_n,2) \text{ and } r(x_n,2) \text{ respectively when the basic element is in state } x_n \}$ (14)

Thus the flow into state s from x_n due to new call arrivals in the right beam of sector 2 is

$$\gamma_{n4}(s, x_n) = \Lambda_{nb} \cdot (1 - P_{br}(x_n)), \quad \text{if } l(x_n,2) + r(x_n,2) < C - C_{h1}. \quad (15)$$

6.2 Call Completion

6.2.1 Call completion in the left beam of sector 1

A transition into state s due to a call completion in the left beam of sector 1 when the basic element is in state x_c , will cause the state variable $l(x_c,1)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for call completion in the left beam of sector 1, if the state variables are related by

$$\begin{aligned} l(x_c,1) &= l(s,1) + 1, \\ l(x_c,2) &= l(s,2), \\ r(x_c,i) &= r(s,i), \quad i = 1,2. \end{aligned} \quad (16)$$

Let μ_c denote the average completion rate of a call. The flow into state s from x_c due to call completion in the left beam of sector 1 is

$$\gamma_{c1}(s, x_c) = \mu_c \cdot l(x_c,1) \quad (17)$$

6.2.2 Call completion in the right beam of sector 1

A transition into state s due to a call completion in the right beam of sector 1 when the basic element is in state x_c , will cause the state variable $r(x_c,1)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for call completion in the right beam of sector 1, if the state variables are related by

$$\begin{aligned} l(x_c,i) &= l(s,i), \quad i = 1,2. \\ r(x_c,1) &= r(s,1) + 1, \end{aligned}$$

$$r(x_c,2) = r(s,2), \quad (18)$$

The flow into state s from x_c due to call completion in the right beam of sector 1 is

$$\gamma_{c2}(s, x_c) = \mu_c \cdot r(x_c,1) \quad (19)$$

6.2.3 Call completion in the left beam of sector 2

A transition into state s due to a call completion in the left beam of sector 2 when the basic element is in state x_c will cause the state variable $l(x_c,2)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for call completion in the left beam of sector 2, if the state variables are related by

$$\begin{aligned} l(x_c,1) &= l(s,1), \\ l(x_c,2) &= l(s,2) + 1, \\ r(x_c,i) &= r(s,i), \quad i = 1,2. \end{aligned} \quad (20)$$

The flow into state s from x_c due to call completion in the left beam of sector 2 is

$$\gamma_{c3}(s, x_c) = \mu_c \cdot l(x_c,2) \quad (21)$$

6.2.4 Call completion in the right beam of sector 2

A transition into state s due to a call completion in the right beam of sector 2 when the basic element is in state x_c , will cause the state variable $r(x_c,2)$ to be decreased by 1. Thus a permissible state x_c is a predecessor state of s for call completion in the right beam of sector 2, if the state variables are related by

$$\begin{aligned} l(x_c,i) &= l(s,i), \quad i = 1,2. \\ r(x_c,1) &= r(s,1), \\ r(x_c,2) &= r(s,2) + 1. \end{aligned} \quad (22)$$

The flow into state s from x_c due to call completion in the right beam of sector 2 is

$$\gamma_{c4}(s, x_c) = \mu_c \cdot r(x_c,2). \quad (23)$$

6.3 Hand-off Arrivals (From Other Sectors)

6.3.1 Hand-off arrivals in the left beam of sector 1 from other sectors

Let Λ_h be the average rate at which hand-off arrivals (from other sectors) impinge on the beam of a sector. Initially, these parameters are assumed to be known, but ultimately their values are computed as part of the solution algorithm. A transition into state s due to a hand-off arrival in the left beam of sector 1 from other sectors when the basic element is in state x_h , will cause the state variable $l(x_h,1)$ to be incremented by 1. We also note that hand-off arrivals have access to all C channels in a sector. Thus a permissible state x_h is a predecessor state of s for hand-off arrivals in the left beam of sector 1 from other sectors, if $l(x_h,1) + r(x_h,1) < C$, and the state variables are related by

$$\begin{aligned} l(x_h,1) &= l(s,1) - 1, \\ l(x_h,2) &= l(s,2), \\ r(x_h,i) &= r(s,i), \quad i = 1,2. \end{aligned} \tag{24}$$

We note that, only a fraction of the hand-off arrivals in the left beam of sector 1 from other sectors contributes to the transition flow because the success of hand-off arrivals also depends on the states of sector 0. When the basic element is in state x_h , the value of $l(x_h,1)$ is known, however the number of calls in service in the right beam of sector 0, which is denoted $r(x_h,0)$, is unknown. The value of $r(x_h,0)$ may vary from 0 to $C - l(x_h,1)$. Because of the homogeneous property of the system, the statistical behavior of all basic elements in equilibrium is identical. Thus we can calculate the probability that there is no channel available for a hand-off call in the left beam of sector 1 and right beam of sector 0 when the basic element is in state x_h , which is denoted $P_{fl}(x_h)$. Since the behavior of any two adjacent co-channel sectors is not independent, $P_{fl}(x_h)$ is actually a conditional probability and defined as follows.

$$P_{fl}(x_h) = \text{Prob}\{ r(x_h,0) \geq C - l(x_h,1) \mid \text{the numbers of calls in service in the left and right beams of sector 1 are } l(x_h,1) \text{ and } r(x_h,1) \text{ respectively when the basic element is in state } x_h \} \tag{25}$$

The flow into state s from x_h due to hand-off arrivals in the left beam of sector 1 from other sectors is

$$\gamma_{h1}(s, x_h) = \Lambda_h \cdot (1 - P_{fl}(x_h)), \quad \text{if } l(x_h, 1) + r(x_h, 1) < C. \quad (26)$$

6.3.2 Hand-off arrivals in the right beam of sector 1 from other sectors

A transition into state s due to a hand-off arrival in the right beam of sector 1 from other sectors when the basic element is in state x_h , will cause the state variable $r(x_h, 1)$ to be incremented by 1. We also note that hand-off arrivals have access to all C channels in a sector and in two interfering co-channel beams of a sector. Thus a permissible state x_h is a predecessor state of s for hand-off arrivals in the right beam of sector 1 from other sectors if $l(x_h, 1) + r(x_h, 1) < C$, $r(x_h, 1) + l(x_h, 2) < C$, and the state variables are related by

$$\begin{aligned} l(x_h, i) &= l(s, i), & i &= 1, 2. \\ r(x_h, 1) &= r(s, 1) - 1, \\ r(x_h, 2) &= r(s, 2), \end{aligned} \quad (27)$$

The flow into state s from x_h due to hand-off arrivals in the right beam of sector 1 from other sectors is

$$\gamma_{h2}(s, x_h) = \Lambda_h, \quad \text{if } l(x_h, 1) + r(x_h, 1) < C \text{ and } r(x_h, 1) + l(x_h, 2) < C \quad (28)$$

6.3.3 Hand-off arrivals in the left beam of sector 2 from other sectors

A transition into state s due to a hand-off arrival in the left beam of sector 2 from other sectors when the basic element is in state x_h , will cause the state variable $l(x_h, 2)$ to be incremented by 1. Thus a permissible state x_h is a predecessor state of s for hand-off arrivals in the left beam of sector 2 from other sectors, if $l(x_h, 2) + r(x_h, 2) < C$, $r(x_h, 1) + l(x_h, 2) < C$, and the state variables are related by

$$\begin{aligned} l(x_h, 1) &= l(s, 1), \\ l(x_h, 2) &= l(s, 2) - 1, \\ r(x_h, i) &= r(s, i), & i &= 1, 2. \end{aligned} \quad (29)$$

The flow into state s from x_h due to hand-off arrivals in the right beam of sector 2 from other sectors is

$$\gamma_{h3}(s, x_h) = \Lambda_h, \quad \text{if } l(x_h, 2) + r(x_h, 2) < C \text{ and } r(x_h, 1) + l(x_h, 2) < C. \quad (30)$$

6.3.4 Hand-off arrivals in the right beam of sector 2 from other sectors

A transition into state s due to a hand-off arrival in the right beam of sector 2 from other sectors when the basic element is in state x_h will cause the state variable $r(x_h, 2)$ to be incremented by 1. Thus a permissible state x_h is a predecessor state of s for hand-off arrivals in the right beam of sector 2 from other sectors, if $l(x_h, 2) + r(x_h, 2) < C$, and the state variables are related by

$$\begin{aligned} l(x_h, i) &= l(s, i), & i &= 1, 2. \\ r(x_h, 1) &= r(s, 1), \\ r(x_h, 2) &= r(s, 2) - 1, \end{aligned} \quad (31)$$

We note that, only a fraction of the hand-off arrivals in the right beam of sector 2 from other sectors contributes to the transition flow because the success of hand-off arrivals also depends on the states of sector 3. When the basic element is in state x_h , the value of $r(x_h, 2)$ is known, however the number of calls in service in the left beam of sector 3, which is denoted $l(x_h, 3)$, is unknown. The value of $l(x_h, 3)$ may vary from 0 to $C - r(x_h, 2)$. Because of the homogeneous property of the system, the statistical behavior of all basic elements in equilibrium is identical. Thus we can calculate the probability that there is no channel available for a hand-off call in the right beam of sector 2 and left beam of sector 3 when the basic element is in state x_h , which is denoted $P_{fr}(x_h)$. Since the behavior of any two adjacent co-channel sectors is not independent, $P_{fr}(x_h)$ is actually a conditional probability and defined as follows.

$$P_{fr}(x_h) = \text{Prob}\{ l(x_h, 3) \geq C - r(x_h, 2) \mid \text{the numbers of calls in service in the left and right beams of sector 2 are } l(x_h, 2) \text{ and } r(x_h, 2) \text{ respectively when the basic element is in state } x_h \} \quad (32)$$

Thus the flow into state s from x_h due to hand-off arrivals in the right beam of sector 2 from other sectors is

$$\gamma_{h4}(s, x_h) = \Lambda_h \cdot (1 - P_{fr}(x_h)), \quad \text{if } l(x_h, 2) + r(x_h, 2) < C. \quad (33)$$

6.4 Hand-off Departures (To Other Sectors)

6.4.1 Hand-off departures from the left beam of sector 1 to other sectors

A transition due to a hand-off departure of a wireless platform from the left beam of sector 1 to other sectors when the basic element is in state x_d , will cause the state variable $l(x_d, 1)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a platform from the left beam of sector 1 to other sectors, if the state variables are related by

$$\begin{aligned} l(x_d, 1) &= l(s, 1) + 1, \\ l(x_d, 2) &= l(s, 2), \\ r(x_d, i) &= r(s, i), \quad i = 1, 2. \end{aligned} \quad (34)$$

Let F_1 denote the fraction of hand-off departures from a beam of a sector to other sectors. Thus the corresponding transition flow is given by

$$\gamma_{d1}(s, x_d) = \mu_D \cdot F_1 \cdot l(x_d, 1) \quad (35)$$

6.4.2 Hand-off departures from the right beam of sector 1 to other sectors

A transition due to a hand-off departure of a wireless platform from the right beam of sector 1 to other sectors when the basic element is in state x_d , will cause the state variable $r(x_d, 1)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departures of a platform from the right beam of sector 1 to other sectors, if the state variables are related by

$$\begin{aligned} l(x_d, i) &= l(s, i), \quad i = 1, 2. \\ r(x_d, 1) &= r(s, 1) + 1, \\ r(x_d, 2) &= r(s, 2). \end{aligned} \quad (36)$$

The corresponding transition flow is given by

$$\gamma_{d2}(s, x_d) = \mu_D \cdot F_1 \cdot r(x_d, 1) \quad (37)$$

6.4.3 Hand-off departures from the left beam of sector 2 to other sectors

A transition due to a hand-off departure of a wireless platform from the left beam of sector 2 to other sectors when the basic element is in state x_d , will cause the state variable $l(x_d,2)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a platform from the left beam of sector 2 to other sectors, if the state variables are related by

$$\begin{aligned} l(x_d,1) &= l(s,1), \\ l(x_d,2) &= l(s,2) + 1, \\ r(x_d,i) &= r(s,i), \quad i = 1,2. \end{aligned} \quad (38)$$

The corresponding transition flow is given by

$$\gamma_{d3}(s, x_d) = \mu_D \cdot F_1 \cdot l(x_d,2) \quad (39)$$

6.4.4 Hand-off departures from the right beam of sector 2 to other sectors

A transition due to a hand-off departure of a wireless platform from the right beam of sector 2 to other sectors when the basic element is in state x_d , will cause the state variable $r(x_d,2)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departure of a platform from the right beam of sector 2 to other sectors, if the state variables are related by

$$\begin{aligned} l(x_d,i) &= l(s,i), \quad i = 1,2. \\ r(x_d,1) &= r(s,1), \\ r(x_d,2) &= r(s,2) + 1. \end{aligned} \quad (40)$$

The corresponding transition flow is given by

$$\gamma_{d4}(s, x_d) = \mu_D \cdot F_1 \cdot r(x_d,2) \quad (41)$$

6.5 Call Hand-offs (Between Beams of the Same Sector)

6.5.1 Call hand-offs in the left beam from the right beam of sector 1 that succeed

A transition into state s due to a successful call hand-off in the left beam from the right beam of sector 1 when the basic element is in state x_{hd} , will cause the state variable $l(x_{hd},1)$ to be incremented by 1 and $r(x_{hd},1)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for successful call hand-off arrivals in the left beam

due to hand-off departures from the right beam of sector 1, if the state variables are related by

$$\begin{aligned}
l(x_{hd},1) &= l(s,1) - 1, \\
l(x_{hd},2) &= l(s,2), \\
r(x_{hd},1) &= r(s,1) + 1, \\
r(x_{hd},2) &= r(s,2).
\end{aligned} \tag{42}$$

Let F_2 denote the fraction of hand-off departures from a beam to other beam of the same sector. The flow into state s from x_{hd} due to successful call hand-offs in the left beam from the right beam of sector 1 is

$$\gamma_{hd1}(s, x_{hd}) = \mu_D \cdot r(x_{hd},1) \cdot F_2 \cdot (1 - P_{fl}(x_{hd})) \tag{43}$$

6.5.2 Call hand-offs in the left beam from the right beam of sector 1 that fail

A transition into state s due to a call hand-off failure in the left beam from the right beam of sector 1 when the basic element is in state x_{hd} , will cause the state variable $l(x_{hd},1)$ to be remained unchanged and $r(x_{hd},1)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for call hand-off failures in the left beam from the right beam of sector 1, if the state variables are related by

$$\begin{aligned}
l(x_{hd},1) &= l(s,1), \\
l(x_{hd},2) &= l(s,2), \\
r(x_{hd},1) &= r(s,1) + 1, \\
r(x_{hd},2) &= r(s,2).
\end{aligned} \tag{44}$$

The flow into state s from x_{hd} due to call hand-off failures in the left beam from the right beam of sector 1 is

$$\gamma_{hd2}(s, x_{hd}) = \mu_D \cdot r(x_{hd},1) \cdot F_2 \cdot P_{fl}(x_{hd}) \tag{45}$$

6.5.3 Call hand-offs in the right beam from the left beam of sector 1

A transition into state s due to a successful call hand-off in the right beam from the left beam of sector 1 when the basic element is in state x_{hd} , will cause the state variable

$r(x_{hd},1)$ to be incremented by 1 and $l(x_{hd},1)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for successful hand-off arrivals in the right beam due to hand-off departures from the left beam of sector 1, if $r(x_{hd},1) + l(x_{hd},2) < C$, and the state variables are related by

$$\begin{aligned}
l(x_{hd},1) &= l(s,1) + 1, \\
l(x_{hd},2) &= l(s,2), \\
r(x_{hd},1) &= r(s,1) - 1, \\
r(x_{hd},2) &= r(s,2).
\end{aligned} \tag{46}$$

A transition into state s due to a call hand-off failure in the right beam from the left beam of sector 1 when the basic element is in state x_{hd} , will cause the state variable $r(x_{hd},1)$ to be remained unchanged and $l(x_{hd},1)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for hand-off failures in the right beam from the left beam of sector 1, if $r(x_{hd},1) + l(x_{hd},2) = C$, and the state variables are related by

$$\begin{aligned}
l(x_{hd},1) &= l(s,1) + 1, \\
l(x_{hd},2) &= l(s,2), \\
r(x_{hd},1) &= r(s,1), \\
r(x_{hd},2) &= r(s,2).
\end{aligned} \tag{47}$$

The flow into state s from x_{hd} due to call hand-offs in the right beam from the left beam of sector 1 is

$$\gamma_{hd2}(s, x_{hd}) = \mu_D \cdot l(x_{hd},1) \cdot F_2. \tag{48}$$

6.5.4 Call hand-off in the left beam from the right beam of sector 2

A transition into state s due to a successful hand-off arrival in the left beam from the right beam of sector 2 when the basic element is in state x_{hd} , will cause the state variable $l(x_{hd},2)$ to be incremented by 1 and $r(x_{hd},2)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for successful hand-off arrivals in the left beam due to

hand-off departures from the right beam of sector 2, if $r(x_{hd},1) + l(x_{hd},2) < C$, and the state variables are related by

$$\begin{aligned}
 l(x_{hd},1) &= l(s,1), \\
 l(x_{hd},2) &= l(s,2) - 1, \\
 r(x_{hd},1) &= r(s,1), \\
 r(x_{hd},2) &= r(s,2) + 1.
 \end{aligned} \tag{49}$$

A transition into state s due to a hand-off failure in the left beam from the right beam of sector 2 when the basic element is in state x_{hd} , will cause the state variable $l(x_{hd},2)$ to be remained unchanged and $r(x_{hd},2)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for call hand-off failures in the left beam from the right beam of sector 2, if $r(x_{hd},1) + l(x_{hd},2) = C$, and the state variables are related by

$$\begin{aligned}
 l(x_{hd},1) &= l(s,1), \\
 l(x_{hd},2) &= l(s,2), \\
 r(x_{hd},1) &= r(s,1), \\
 r(x_{hd},2) &= r(s,2) + 1.
 \end{aligned} \tag{50}$$

The flow into state s from x_{hd} due to call hand-off in the left beam from the right beam of sector 2 is

$$\gamma_{hd4}(s, x_{hd}) = \mu_D \cdot r(x_{hd},2) \cdot F_2, \tag{51}$$

6.5.5 Call hand-off in the right beam from the left beam of sector 2 that succeed

A transition into state s due to a successful call hand-off in the right beam from the left beam of sector 2 when the basic element is in state x_{hd} , will cause the state variable $r(x_{hd},2)$ to be incremented by 1 and $l(x_{hd},2)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for successful hand-off arrivals in the right beam due to hand-off departures from the left beam of sector 2, if the state variables are related by

$$\begin{aligned}
 l(x_{hd},1) &= l(s,1), \\
 l(x_{hd},2) &= l(s,2) + 1,
 \end{aligned}$$

$$\begin{aligned}
r(x_{hd},1) &= r(s,1), \\
r(x_{hd},2) &= r(s,2) - 1.
\end{aligned} \tag{52}$$

The flow into state s from x_{hd} due to successful call hand-offs in the right beam from the left beam of sector 2 is

$$\gamma_{hd5}(s, x_{hd}) = \mu_D \cdot l(x_{hd},2) \cdot F_2 \cdot (1 - P_{fr}(x_{hd})) \tag{53}$$

6.5.6 Call hand-off in the right beam from the left beam of sector 2 that fail

A transition into state s due to a call hand-off failure in the right beam from the left beam of sector 2 when the basic element is in state x_{hd} , will cause the state variable $r(x_{hd},2)$ to be remained unchanged and $l(x_{hd},2)$ to be decreased by 1. Thus a permissible state x_{hd} is a predecessor state of s for call hand-off failures in the right beam from the left beam of sector 2, if the state variables are related by

$$\begin{aligned}
l(x_{hd},1) &= l(s,1), \\
l(x_{hd},2) &= l(s,2) + 1, \\
r(x_{hd},1) &= r(s,1), \\
r(x_{hd},2) &= r(s,2).
\end{aligned} \tag{54}$$

The flow into state s from x_{hd} due to call hand-off failures in the right beam from the left beam of sector 2 is

$$\gamma_{hd5}(s, x_{hd}) = \mu_D \cdot l(x_{hd},2) \cdot F_2 \cdot P_{fr}(x_{hd}) \tag{55}$$

7. Flow Balance Equations

From the equations given above, the total transition flow into s from any permissible predecessor state x can be found using

$$q(s, x) = \sum_{i=1}^4 [\gamma_{ni}(s, x) + \gamma_{ci}(s, x) + \gamma_{hi}(s, x) + \gamma_{di}(s, x)] + \sum_{i=1}^6 \gamma_{hdi}(s, x) \tag{56}$$

in which $s \neq x$, and flow into a state has been taken as a positive quantity. The total flow out of state s is denoted as $q(s, s)$, and is given by

$$q(s, s) = - \sum_{\substack{k=0 \\ k \neq s}}^{S_{\max}} q(k, s) \quad (57)$$

To find the statistical equilibrium state probabilities for a sector, we write the flow balance equations for the states. These are a set of $S_{\max} + 1$ simultaneous equations for the unknown state probabilities $p(s)$. They are of the form

$$\sum_{j=0}^{S_{\max}} q(i, j) p(j) = 0, \quad i = 0, 1, 2, \dots, S_{\max} - 1.$$

$$\sum_{j=0}^{S_{\max}} p(j) = 1. \quad (58)$$

In which, for $i \neq j$, $q(i, j)$ represents the net transition flow into state i from state j , and $q(i, i)$ is the total transition flow out of state i . These equations express that in statistical equilibrium, the net probability flow into any state is zero, and the sum of the probabilities is unity.

8. Performance Measures

There are six performance measures of interest: 1) call blocking probability, 2) hand-off failure probability, 3) forced termination probability, 4) hand-off activity, 5) carried traffic, and 6) channel rearrangement rate. Once the statistical equilibrium state probabilities and transition flows are found, the required performance measures can be calculated.

8.1 Blocking probability

The blocking probability for a call is the average fraction of new calls that are denied access to a channel. Blocking of new calls occurs if there are no channels to serve the call. We define the following set of states

$$B = \{ s : l(s,1) + r(s,1) \geq C - C_{h1} \cup r(s,1) + l(s,2) \geq C - C_{h2} \} \quad (59)$$

Then the blocking probability is

$$P_B = \sum_{s \in B} p(s) \quad (60)$$

8.2 Hand-off failure probability

The hand-off failure probability for calls is the average fraction of hand-off attempts that are denied a channel. We note that hand-off attempts have potential access to all channels of a sector and the co-channel beams of any given site without regard to C_{h1} or C_{h2} . We define the following sets of states, in which hand-off attempts will fail.

$$H_1 = \{s: l(s,1) + r(s,1) = C \cup r(s,1) + l(s,2) = C\} \quad (61)$$

$$H_2 = \{s: l(s,1) + r(s,1) = C\} \quad (62)$$

Then the hand-off failure probability due to call hand-offs across sectors can be written as

$$P_{H_1} = \sum_{s \in H_1} p(s) \quad (63)$$

and the hand-off failure probability due to call hand-offs within beams of the same sector can be written as

$$P_{H_2} = \sum_{s \in H_2} p(s) \quad (64)$$

If F_1 denote the fraction of hand-off departures from a beam of a sector to other sectors and F_2 denote the fraction of hand-off departures from a beam to other beam of the same sector. Then the average hand-off failure probability is

$$P_H = F_1 \cdot P_{H_1} + F_2 \cdot P_{H_2} \quad (65)$$

8.3 Forced termination probability

The forced termination probability is defined as the probability that a call that is not blocked is interrupted due to hand-off failure during its lifetime. Let p be the probability that a non-blocked call satisfactorily completes before the hand-off attempt occurs, and q be the probability that hand-off attempt occurs first. Because of the negative exponential assumption, we have

$$p = \mu_C / (\mu_C + \mu_D) \quad (66)$$

$$q = \mu_D / (\mu_C + \mu_D) \quad (67)$$

The probability that a non-blocked call is forced to terminate on its k^{th} hand-off attempt is

$$Y(k) = P_H \cdot q^k \cdot (1 - P_H)^{k-1} \quad (68)$$

The forced termination probability is therefore

$$P_{FT} = \sum_{k=1}^{\infty} Y(k) \quad (69)$$

This can be compactly written in closed form as

$$P_{FT} = \frac{q \cdot P_H}{1 - q \cdot (1 - P_H)} \quad (70)$$

8.4 Hand-off activity

Hand-off activity, H_A , is the expected number of hand-off attempts that a non-blocked call will experience during its lifetime. There will be exactly k hand-off attempts if 1) the call fails at the k^{th} hand-off attempt or 2) the call succeeds at the k^{th} hand-off attempt but successfully completes before the $(k + 1)^{\text{th}}$ hand-off attempt.

The probability that the call fails at the k^{th} hand-off attempt is

$$Z_1(k) = q^k \cdot (1 - P_H)^{k-1} \cdot P_H \quad (71)$$

The probability that the call succeeds at the k^{th} hand-off attempt but successfully completes before the $(k + 1)^{\text{th}}$ hand-off attempt is

$$Z_2(k) = q^k \cdot (1 - P_H)^k \cdot p \quad (72)$$

Consequently, the hand-off activity is

$$H_A = \sum_{k=1}^{\infty} k \cdot [Z_1(k) + Z_2(k)] \quad (73)$$

This can be simplified to

$$H_A = q \cdot [P_H + (1 - P_H) \cdot p] / [1 - q \cdot (1 - P_H)]^2 \quad (74)$$

8.5 Carried traffic

The carried traffic in a beam is the average number of channels occupied by the calls in that beam. The carried traffic in a beam is

$$A_c = \sum_{s=0}^{S_{\max}} r(s,1) \cdot p(s) \quad (75)$$

8.6 Channel rearrangement rate

Channel rearrangement rate is the average rate of channels that have to be rearranged. The channel rearrangement rate in a beam is

$$R = \mu_c \cdot \sum_{s=0}^{S_{\max}} [r(s,1) - 1] \cdot p(s), \quad \text{if } r(s,1) > 0. \quad (76)$$

9. Numerical Results and Discussion

Numerical results were generated using the approach described in this paper. For all figures, an unencumbered call duration of 100 seconds was assumed. The fraction of hand-off departures from a beam of a sector to *other sectors* was assumed to be 2/3 and the fraction of hand-off departures from a beam to *another beam of the same sector* was assumed to be 1/3. Sectors have $C=20$ channels each, which corresponds to 10 channels per beam. The total number of platforms per beam is 200. For Fig. 3 to 5, the new call origination rate per platform was varied from 1.7×10^{-4} to 3.3×10^{-4} calls/sec. For Fig. 6 and 7, the dwell time of platforms was varied from 25 to 150 seconds.

Fig. 3 shows the dependence of call blocking probability on demand. The proposed 6-beam multibeam DCA scheme performs better than the traditional 6-sector FCA scheme. System capacity can be improved by using a multibeam scheme and DCA technique because of more channel reuse. For a fixed offered traffic, the blocking probability of calls can be reduced significantly. Alternatively, more new call traffic can be accommodated while the blocking probability is maintained below the required threshold.

Fig. 4 shows the dependence of forced termination probability on demand. The proposed 6-beam multibeam scheme has smaller forced termination probability than traditional 6-sector scheme for a fixed offered traffic. Alternatively, more new call traffic can be supported while the forced termination probability is maintained. Increasing the value of C_h (or C_{h1} , C_{h2}) reduces the forced termination probability at the cost of increasing blocking probability.

Fig. 5 shows how carried traffic depends on demand. The proposed 6-beam multibeam scheme carries more traffic than traditional 6-sector scheme. For low demand, the carried traffic increases linearly with increasing demand. For higher demand, the increase in traffic is less than the proportional increase in demand. This is especially true for larger C_h (or C_{h1} , C_{h2}) since blocking performance is sacrificed to accommodate hand-offs.

Fig. 6 shows how forced termination probability depends on platform mobility. For a fixed dwell time, the proposed 6-beam multibeam scheme has better forced termination performance than the traditional 6-sector scheme under the same offered traffic. Alternatively, higher mobility of wireless platforms can be supported while the forced termination probability is maintained.

Fig.7 shows the dependence of hand-off activity on platform mobility. When the blocking and hand-off failure probabilities are small, the average number of hand-off attempts per non-blocked call is approximately equal to the ratio of unencumbered call session duration of a call to its dwell time - - that is, T_c/T_D . This ratio is plotted as a solid curve in Fig. 7. The average number of hand-off attempts per non-blocked call calculated using the multidimensional birth-death model is also shown. These are indicated by points using symbols "O" and "+" for two different new call origination rates per platform (1 call per hour per platform and 1 call every one and half an hour respectively). It can be seen that these values are quite close to each other. This is because the range of interest for blocking and hand-off failure probabilities are small values,

usually around 0.01 or less. Hand-off activity is insensitive to the offered traffic, but is sensitive to the mobility. Wireless platforms with higher mobility cross cell boundaries more often. The average number of hand-off attempts per non-blocked call decreases as the dwell time increases.

10. Conclusions

We proposed a multibeam scheme using dynamic channel assignment across multiple sectors. The combined approach enhances channel reuse. The framework using a state description and multidimensional birth-death processes was used to compute theoretical traffic performance characteristics for the scheme. In comparison with traditional schemes using FCA, blocking probability of calls can be reduced significantly for a fixed offered traffic. Alternatively, system capacity can be increased while the blocking probability is maintained below the required threshold. The smaller forced termination probability means wireless users will less likely experience interruption during the life time of calls. The proposed multibeam scheme has smaller forced termination probability than the traditional scheme.

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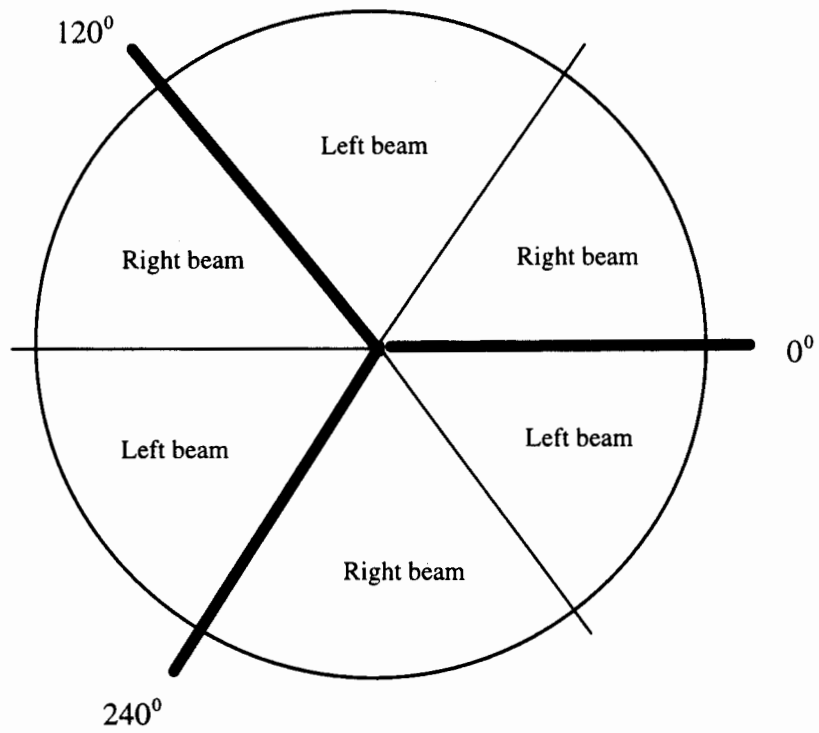


Fig. 1. The beam layout of 120° -sectorized multibeam cellular communication systems with 2 beams in each sector.

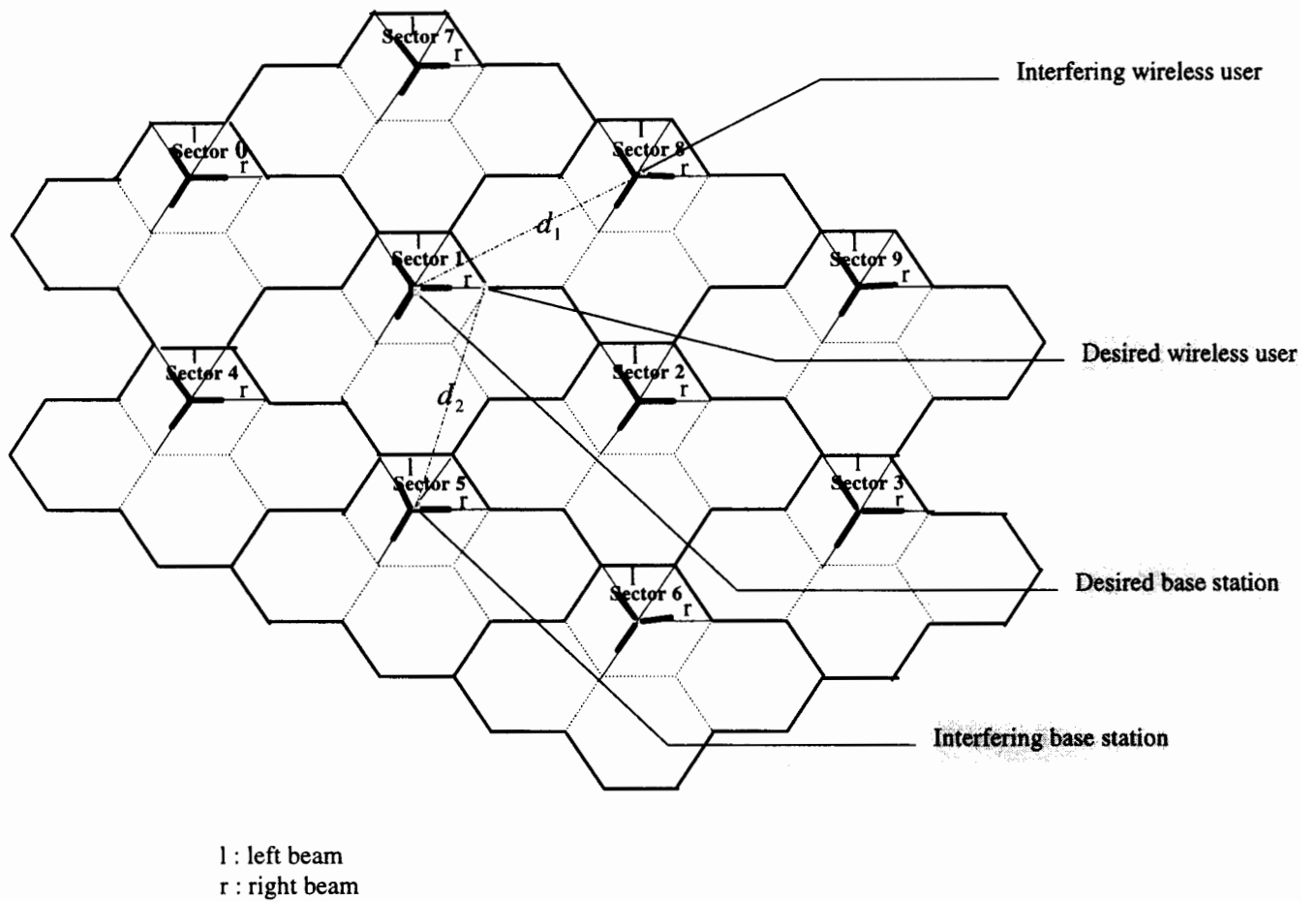
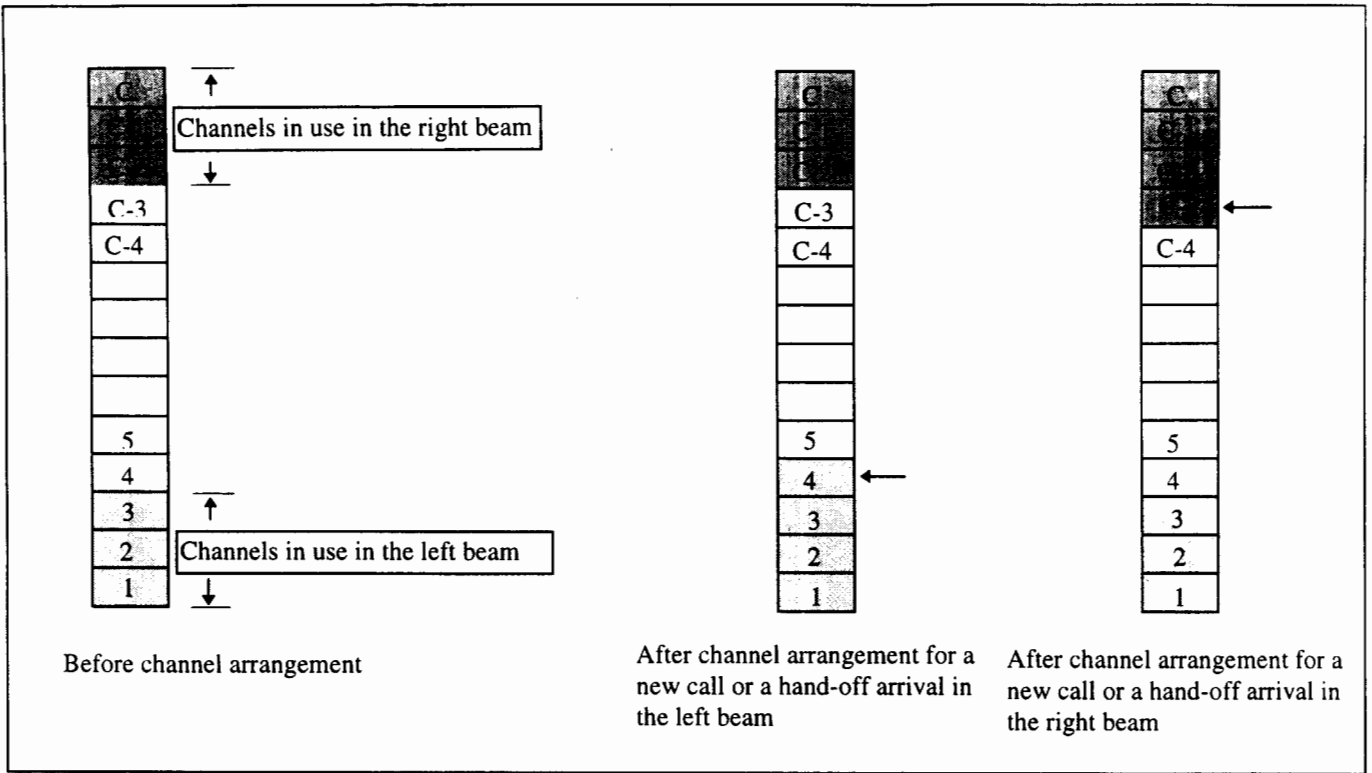
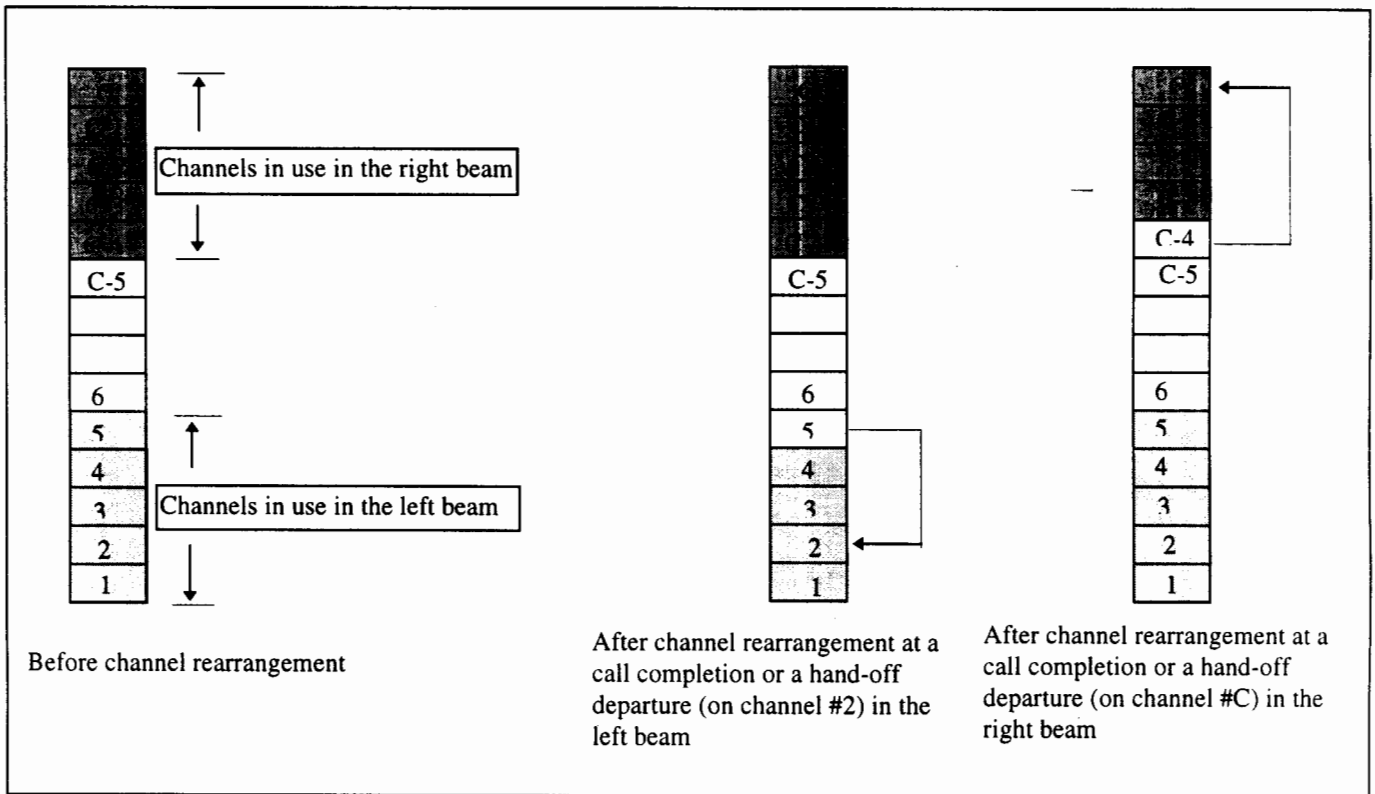


Fig. 2a. Analysis of co-channel interference for both up-link and down-link case in 120° -sectored multibeam system with 2 beams in each sector. The cluster size is 4 ($N=4$).



Channel arrangement



Channel rearrangement

Fig. 2b. Explanations of dynamic channel assignment with arrangement and rearrangement.

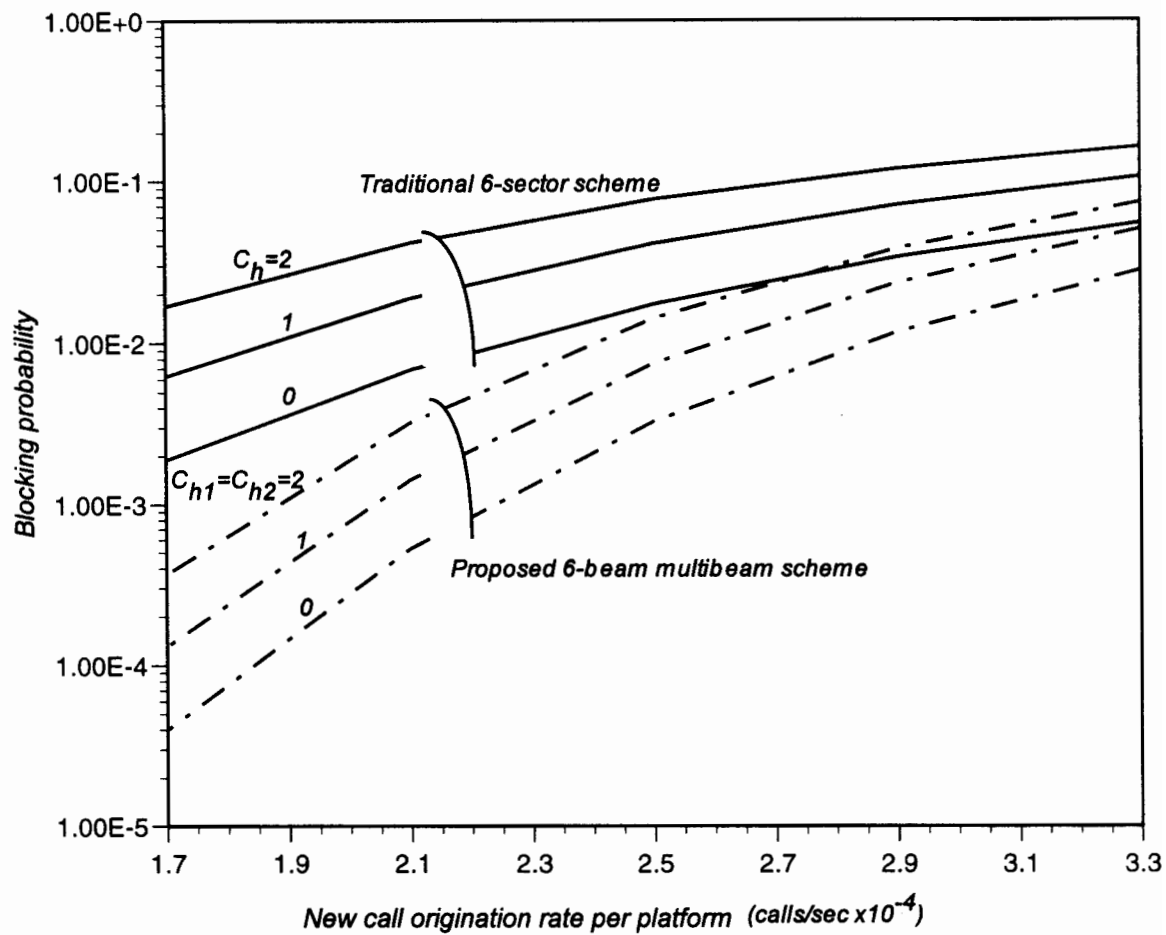


Fig. 3 Blocking probability depends on demand

Parameters: Total number of platforms per beam=200, number of channels per beam=10,
 $T_C=100$ seconds, $T_D=150$ seconds, $F_1=2/3$, $F_2=1/3$.

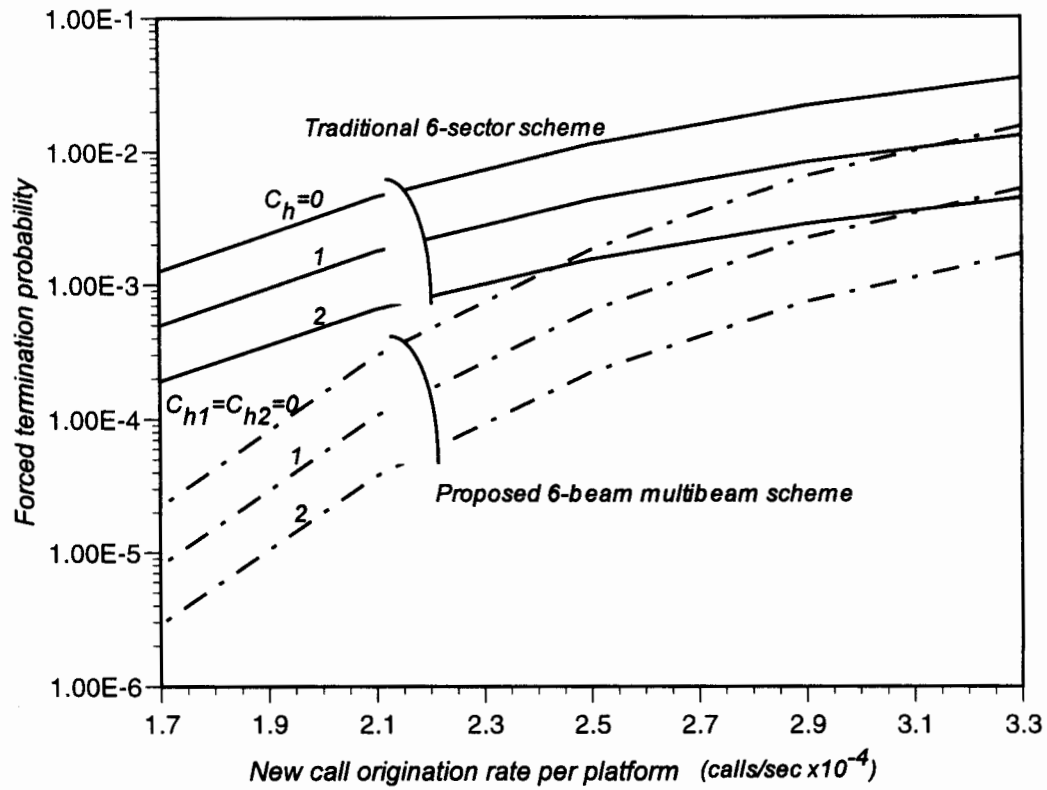


Fig.4. Forced termination probability depends on demand

Parameters: total number of platforms per beam=200, number of channels per beam=10,

$T_C=100$ seconds, $T_D=150$ seconds, $F_1=2/3$, $F_2=1/3$.

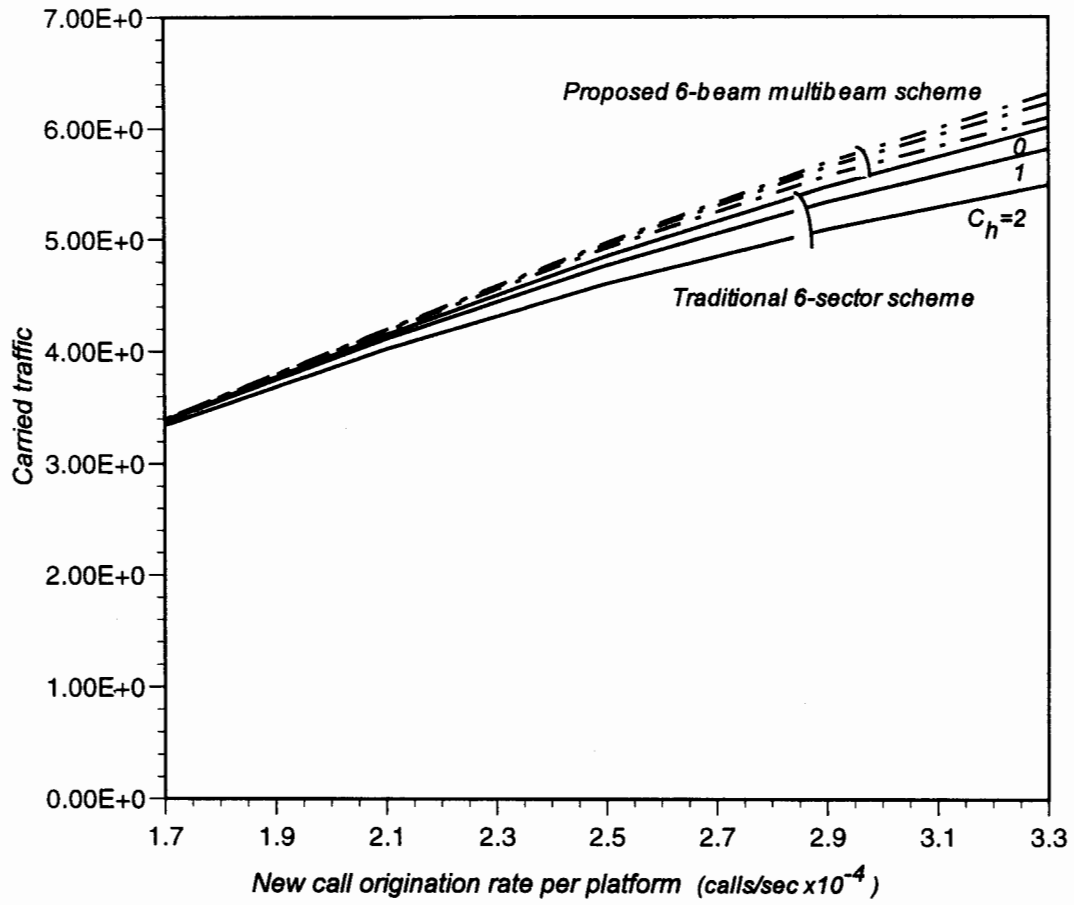


Fig. 5. Carried traffic depends on demand

parameters: total number of platforms per beam=200, number of channels per beam=10,
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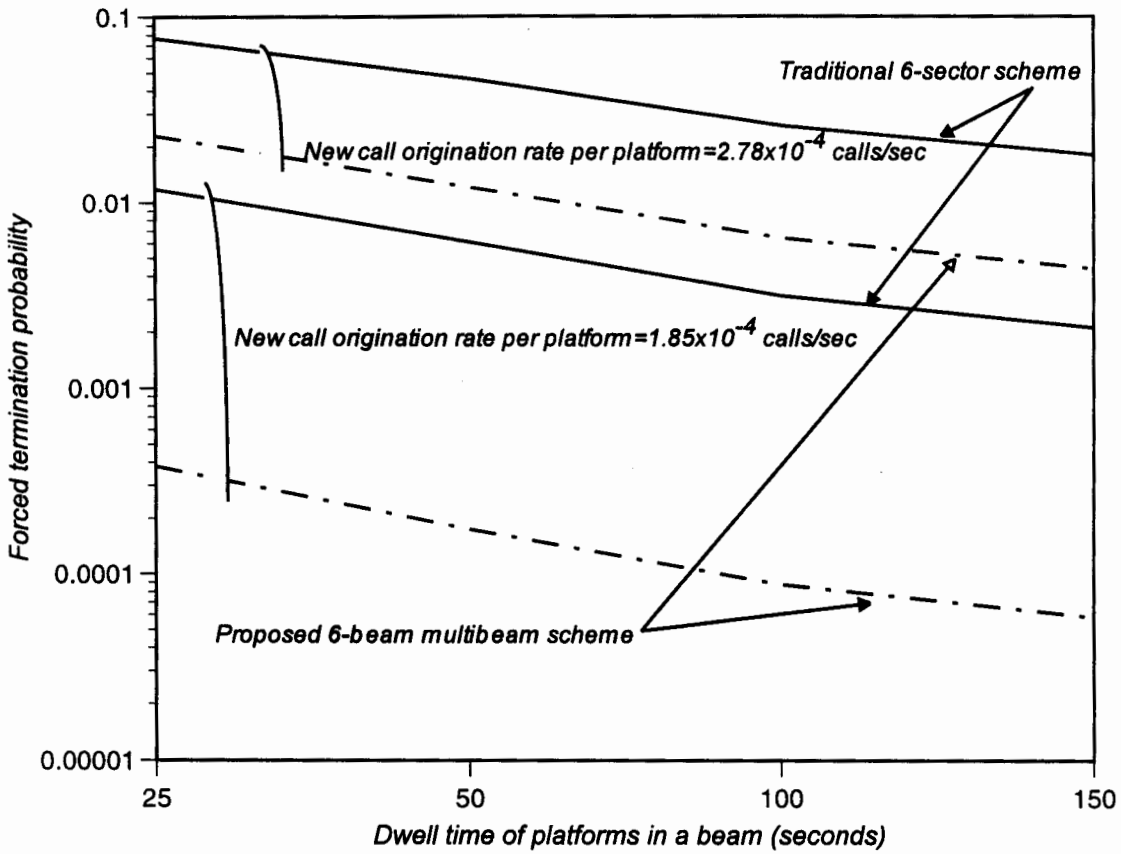


Fig. 6. Forced termination probability depends on mobility

Parameters: total number of platforms per beam=200, number of channels per beam=10,
 $T_C=100$ seconds, $C_{h1}=C_{h2}=0$, $F_1=2/3$, $F_2=1/3$.

- forced termination probability for traditional 6-sector scheme
- - - forced termination probability for proposed 6-beam multibeam scheme

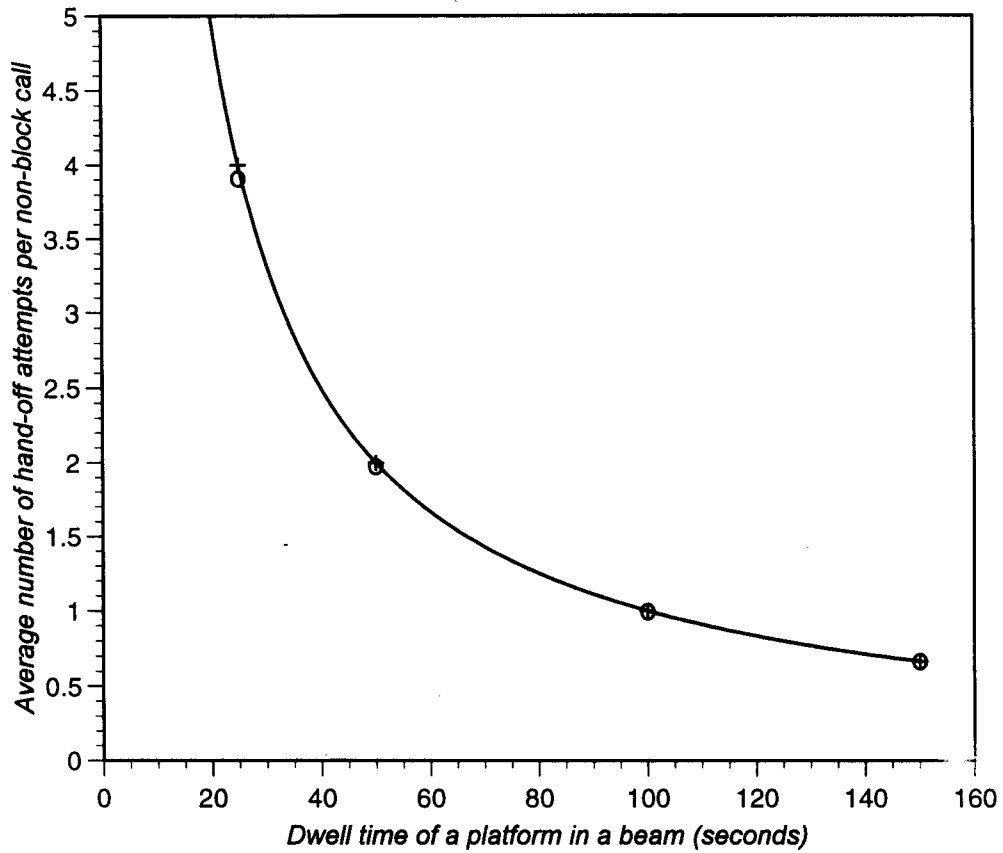


Fig. 7. Hand-off activity depends on mobility.

Parameters: total number of platforms per beam=200, number of channels per beam=10,

$T_C=100$ seconds, $C_{h1}=C_{h2}=0$, $F_1=2/3$, $F_2=1/3$.

O: hand-off activity for new call origination rate per platform is 1 call per hour (or 2.78×10^{-4} calls/sec),

+: hand-off activity for new call origination rate per platform is 1 call per one and half an hour (or 1.85×10^{-4} calls/sec).