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Initialization Technique in Variable and Multiple Target Tracking Systems

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Abstract

We present initialization of targets in multiple and variable number of target-tracking systems (where targets maneuver dynamically with random accelerations) in this paper where especially, received signal strength (RSS) model sensors are applied in wireless sensor networks. RSS measurement model does not allow to form one to one mapping between states and observations (or measurement). Based on *least squares method* (along with modified version), any newly appeared single target is detected and initialized by the particle filtering. We introduce "residue cancelation lateration (RCL)" method for initializing a newly appeared target besides existing target. Initialization of the targets in variable number of target tracking system is very important because under RSS measurement model, it is not very efficient to detect or track multiple and variable number of targets because it does not give the one to one mapping between measurement and true states. This fast and efficient initialization technique can be contributed to the multiple target tracking system where the number of targets varies.

CONTENTS

I. INTRODUCTION

Not only in signal processing area, but also in many other areas, the target tracking problems always have obtained researchers' attention persistently. The development of target tracking system solutions are also constantly achieved by researchers; from the classical joint probabilistic data association filter (JPDAF) [1], [2] or multiple hypothesis tracker (MHT) [3], [4] to recent probability hypothesis density (PHD) filter based on finite set statistics (FISST) [5], [6]. Regardless of algorithms , filters (e.g., Kalman filter [7], [8], particle filter [9], Monte Carlo Markov Chain [10], etc.), or processors (which produce the measurement of observations) that are adopted for the solutions for target tracking systems (even if the measurement is very efficient to be estimated for the states), once it is based on joint probabilistic data association (JPDA) [11], it is very difficult to avoid complex joint probability density function of the states. That is because the size of target states grows up exponentially even if the environmental scenario is not hostile (e.g., few number of targets, known number of targets tracking, the number of targets does not vary, etc). In some problems, depending on the measurement (observation) models, which we employ for the purpose of estimating states, JPDA is not relevant nor possible to be applied to the problem because the dimensional complexity of state will even huger, and it is impossible to estimate the states of targets under some measurement models. In that case, measurement models do not supply one to one mapping between measurement and state that will cause much wider range of probability density function of the target states, and uncertainty of the states will highly increase. For instance, in the field of sensor network where many sensors are used to take the observations from the data and process them to the measurement in order to estimate the states of, e.g., targets, there are many kinds of sensors that can be used depending on the problems. Usually expensive and complicated sensors will produce more efficient information after processing the raw data collected from the targets. Depending on the sensors, they will produce different measurement; position itself, range or/and bearing, time of arrival difference, frequency of narrow band signal emitted by targets, frequency difference due to Doppler shift sensed by two sensors, and signal strength emitted from the targets which we employ as the measurement in this paper. Among them, tracking targets by either bearing only tracking or received signal strength (RSS) is very challenging problem [12], [13], [14], [15], [16], [17]; especially if the unknown multiple number of targets varies, and targets maneuver with random accelerations. There are two ways of interpreting the term "tracking" in the literature. One is separately used from the "detection" of targets which means, "tracking" is performed after any detections of targets are executed. The other meaning of "tracking" does not specifically include detections of targets in the procedure of the whole tracking systems because estimating the states of targets with the time itself includes detection and “tracking” (in the first meaning of ‘tracking’) all together. People use it confusingly, but sometimes, we need to separate “detection” part from the whole “tracking” systems. In JPDA based target tracking systems, “detection” part does not have to be separated from the whole tracking system [18], [19], [20], [21], [22], [23] because detections of targets are part of estimating joint probability density function of the states in tracking systems. On the other hand, in RSS sensor model, we may not have to separate detection step from the whole tracking system, however, at least we have to initialize the locations of targets when we have belief of newly appeared targets, and it is very challenging problem, especially when unknown and multiple number of targets varies with the time [24]. In most literature where RSS sensor model is applied, the number of targets are known and fixed number [17]. In most JPDA based target tracking system (where the number of targets varies), “birth/death” move model [25] is adopted for newly appearing or disappearing targets after algorithm detects those moves while we generate all possible particles (birth, death and update) and compute the weight, find the best particle in RSS model sensor networked tracking system where “least squares method (residue cancelation)” and “particle filter” combined tracking system is used. Since we do not have any designated step for any moves (birth, death, update, merge, split, etc.), particle filter will smoothly take any move (Particle filter will detect a new target if the heaviest weight particle, by the MAP rule, has a new target.) and proceed with the stream of proper particles (Particle filter considers particle and the weight as the probability measure). Nonetheless, we need a initializing algorithm (see Section III-B and IV) of newly appearing targets unless we assume that it is known prior [26]. According to RSS sensor model [27], we can only estimate the distance between the source target and the sensor that receives the power of the signal by a single measurement. However, if we use 3 sensors or more, theoretically we are able to locate the source target by the *triangulation* technique which is sometimes used in cellular communications to pinpoint the geographic position of a user [28]. *Lateration* uses this triangulation technique and apply *least squares method* [29] to pinpoint the source target using the noise added received signal power [30], [31]. We take advantage of lateration technique to initialize newly appeared target in target tracking system where we apply particle filtering for the whole tracking system; particle filtering is now dominantly used in estimating parameters especially in state space and non-linear model, and most literature tries to show the superiority of particle filtering over traditional estimating solutions. We evolve this technique and introduce the “residue cancelation lateration (RCL)” method to initialize a newly appeared target besides existing targets (see Section V). Sensors are uniformly deployed in our wireless sensor network model. We only use 3 sensors that receive the strongest power among all sensors when we apply lateration for initialization. Most of the time, just a few sensors receive useful data information in RSS sensor model because as the strength of the signal decreases, the distance between the signal source and the sensor increases relatively quickly according to the RSS model [32].

The initialization method with particle filtering can be applied some other important area such as, localization of sensors in ad-hoc sensor networks, localization of advanced moving, smart sensors, and so on. We also introduce initialization of more than one target at the same time using iterative cancelation method (see Section VI). We present the Cramer-Rao bound for the estimator of a single target with different number of sensors used for the measurement (see Section VII).

II. Model of the tracking system

A. State space model and measurement

We have a 2 dimensional cartesian coordinate, rectangular field of interest with uniformly distributed sensors that follow RSS sensor model. Received signal strength from the source target at the sensor is described in non-linear model as follows according to [32]:

$$y_{n,t} = 10 \log_{10} \left(\sum_{k=1}^{K} \frac{\Psi_k d_0^{\alpha}}{|\mathbf{r}_n - \mathbf{l}_{k,t}|^{\alpha}} \right) + v_{n,t}, \quad n = 1, 2, \ldots, N \tag{1}$$

where $\mathbf{l}$ is the location of a source target, n is the sensor index, t is the time instant, K is the number of targets, Ψ_k is the received power from the source at the reference distance d_0, $\mathbf{r}$ is the sensor location, α is the attenuation factor ($\alpha \geq 1$), v is background zero-mean Gaussian noise, and N is the total number of sensors used in the field. Therefore, the received signal strength depends on the distance between the source targets and the sensor. The only information we can be provided from this type of sensor is the distance between the source target and the data receiving sensor, and we do not know where, or which direction from the source is. Sensors that are located closely to the source target receive strong signals while the strength decreases very quickly as the distance increases according to (1). Because of that, we do not use information from the sensors that are located relatively far from the target and does not receive very good source information because it takes big perturbation even by the small noise. We use 3 best sensors, which means 3 strongest received power when we use regular lateration and just 2 or 3 best measurement when we use modified lateration for initialization of newly appeared target. In tracking system, we have to be able to estimate the location of the targets in addition to the number of targets at every time instant using the sensors that are not very intelligent.

We can model the moving target systems by linear state space model as follows:

$$\mathbf{x}_t = \mathbf{f}_t(\mathbf{x}_{t-1}, \mathbf{u}_t) = \mathbf{G}_x \mathbf{x}_{t-1} + \mathbf{G}_u \mathbf{u}_t \tag{2}$$

where $\mathbf{x}_t = [\ddot{x}_{1,t} \;\; \ddot{x}_{2,t} \;\; \dot{x}_{1,t} \;\; \dot{x}_{2,t} \;\; x_{1,t} \;\; x_{2,t}]^{\top}$ is a state vector which indicates the acceleration , velocity, and position of the target respectively in a two-dimensional Cartesian coordinate system, $\mathbf{G}_x$ and $\mathbf{G}_u$ are known matrices defined by

$$\mathbf{G}_x = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ T_s & 0 & 1 & 0 & 0 & 0 \\ 0 & T_s & 0 & 1 & 0 & 0 \\ \frac{T_s^2}{2} & 0 & T_s & 0 & 1 & 0 \\ 0 & \frac{T_s^2}{2} & 0 & T_s & 0 & 1 \end{pmatrix}, \ \mathbf{G}_u \mathbf{u}_t = \mathbf{u}_t = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \text{and}$$

T_s is sampling time (s), σ_i is uniformly distributed in $[-W \ W]$ (m/s^2). Targets maneuver with random acceleration based on classical dynamics (discrete time sampled version). In this paper, acceleration is modeled by Markov Chain and part of the hidden state rather than random noise as in the most literature [1], [20]. Only the part of the state forms the measurement in the model because the state comprises location, velocity, and acceleration while only location of target contributes to the observation. Nonetheless, all states are related by classical mechanics, once we can estimate anyone of them, we can relate them and find the rest of states.

B. Varying pattern of the number of targets

As in the traditional or/and modern target tracking systems (especially multiple and varying number of targets), it is very difficult to avoid complexity of states, and problem itself which results in, so called, “dimensionality curse”. Especially, if we consider about all situations, e.g., clutters, false alarms, detection probabilities, and etc., it is not very easy to track all states of targets when it is multiple and varying. At this point, we can make the scenario a little bit comfortable with very realistic assumption. We can assume that the number of targets varies between two consecutive time steps according to 3 patterns as in [26] as follows:

1) The number of targets remains the same as previous time step with the same identities.
2) The number of targets increases by a newly appeared target.
3) The number of targets decreases by one that is in the tracked targets in the previous time step.

This assumption is realistic because if the sampling time for the discretization is fast enough, the assumption will be satisfied. Nonetheless we will introduce the technique how to initialize two newly appeared targets at the same time (see Section VI). This assumption may look like “birth/deatth” model which is adopted in many literature in multiple target tracking systems. However, according to our model, any particle (see Section III) that has the information of the state will have the same probability of move, e.g., “birth”, “death”, or “propagating” while birth/death rate is random variable in most literature.

III. Particle filtering as the solution of tracking system

We apply *lateration* (see Section IV) to the part of tracking system that we perform using particle filtering [33], [34], [9]. When we apply particle filtering, lateration is responsible for the detection part of tracking system. Any particles that are generated by *particle filtering* produce offspring of new target [26] even if it can be removed after resampling [9] due to the low weight. Therefore "lateration" is applied to every single particle at every time step of the algorithm.

A. Sampling importance resampling (SIR) filter

There are many versions of particle filtering [34] and our choice for tracking system is "sampling importance resampling (SIR)". SIR is relatively easier to apply and generally applicable to any models. If we denote state function, observation function, state, observation, and the weight by $\mathbf{f}_t$, $\mathbf{h}_t$, $\mathbf{x}_t$, $\mathbf{y}_t$, and w_t^i (where t is the time index and i is the particle index) respectively, the importance density, $q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_{1:t})$ will turn out to be the prior density, $p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)$, and because resampling is executed at every time step, the weight can be calculated as

$$w_t^i \propto p(\mathbf{y}_t|\mathbf{x}_t^i). \tag{3}$$

B. SIR particle filter combined with lateration

In multiple and varying number of target-tracking system, the state space equation must include the state of the number of targets; we denote it by $\mathbf{K}_t$ at time step t. $\mathbf{K}_t$ has 3 patterns to propagate as mentioned previously, e.g., $\mathbf{K}_t = \mathbf{K}_{t-1} + 1$, $\mathbf{K}_t = \mathbf{K}_{t-1}$, and $\mathbf{K}_t = \mathbf{K}_{t-1} - 1$ [26]. Because $\mathbf{K}_t$ is not a random variable in our model, every single particle will produce all possible descendants. The number of descendants depends on $\mathbf{K}_{t-1}$. If $\mathbf{K}_{t-1} = 0$, then offsprings will be 2 kinds; 0 and 1. If $\mathbf{K}_{t-1} = n > 0$, then the descendants will have the number of target, $n-1$, n, and $n+1$. However, when a particle produces a descendant which has the number of $n-1$, it will have C_n^{n-1} kinds of offsprings because all target has equal possibility of disappearance. When $\mathbf{K}_t = 0$, then the state space will be empty except for the number of targets. The posterior function of interest will be $p(\mathbf{K}_{1:t}, \mathbf{x}_{1:t}|\mathbf{y}_{1:t})$, and the distribution is approximated by the "random measure", $p(\{\boldsymbol{\chi}_{1:t}\}_{m=1}^M) = p(\{\mathbf{K}_{1:t}^m, \mathbf{x}_{1:t}^m\}_{m=1}^M) = \{w_{1:t}^m\}_{m=1}^M$ where M is the total number of the particles. We can compute the weight when we use SIR particle filter, i.e., $w_t^m \propto p(y_{1:N,t}|\mathbf{x}_t^m, \mathbf{K}_t^m)$. If we use only 3 best sensors and assuming that sensors are not correlated, then $w_t^m \propto \prod_{n=1}^3 p(y_{n,t}|\mathbf{x}_t^m, \mathbf{K}_t^m)$.

We need to apply lateration when we generate a particle that has newly appearing target; that is the case when $\mathbf{K}_t^{m'} = \mathbf{K}_{t-1}^m + 1$ (where $m' \neq m$ because we have to generate more than M particles before resampling step, nonetheless, particle m' is generated from particle m). It is shown how to apply and combine with particle filtering in the Table I. In Table I, we estimate the distance from the each sensor to the new target ($q_{s_n,t}$)by subtracting and canceling the predicted measurement part from the whole measurement. We may call this technique "*residue cancelation lateration* (RCL)". The reason why only the information of the best sensor and measurement is sent to lateration method is because we use the neighboring sensors of the best sensor to initialize a new target. Even though we use 3 best measurement for the estimation of the states, initialization algorithm has to use the sensors that are neighboring to each other. Sometimes 3 best sensors may not be neighbors to each other which may cause failure of the least squares method.

TABLE I

INITIALIZING A NEW TARGET BY LATERATION (RESIDUE CANCELATION LATERATION) ADOPTED IN PARTICLE FILTERING

At time t, from the all measurement $y_{s_{1:N}}$, find 3 best measurement ($y_{(s_1,s_2,s_3),t}$) and corresponding sensors' identities (s_1, s_2, s_3). Suppose $\mathbf{K}_{t-1}^m = 1$, and it can be easily generalized for any value of $\mathbf{K}_{t-1}^m$.

- $\eta = 0$,
- `For` m=1:M (M is the number of particles.)

  $*$ $\mathbf{K}_t^{M+1+\eta} = 0$ (particle $M+1+\eta$ is generated from particle m). The elements of the other states become empty.

  $*$ $\mathbf{K}_t^m = 1$ (particle m is generated from particle m). Generate a new particle $\boldsymbol{\chi}_t^m$ as follows:

  $\boldsymbol{\chi}_t^m = \{\mathbf{K}_t^m, \mathbf{x}_t^m\} = \{1, \mathbf{x}_t^m\}$, $\mathbf{x}_t^m \sim p(\mathbf{x}_{t-1}|\mathbf{x}_{t-1}^m)$ according to SIR particle filter.

  $*$ $\mathbf{K}_t^{M+2+\eta} = 2$ (particle $M+1+\eta$ is generated from particle m). Suppose

$$
\begin{aligned}
y_{s_n,t} &= 10\log_{10}\left(\sum_{k=1}^{\mathbf{K}_t^{M+2+\eta}} \frac{\Psi_k d_0^\alpha}{|\mathbf{s}_n - \mathbf{l}_{k,t}|^\alpha}\right) + v_{s_n,t} \triangleq 10\log_{10}\left[\left(\sum_{k=1}^{\mathbf{K}_{t-1}^m} \frac{\Psi_k d_0^\alpha}{|\mathbf{s}_n - \mathbf{l}_{k,t}^p|^\alpha}\right) + \frac{\Psi_{\text{new}} d_0^\alpha}{|\mathbf{s}_n - \mathbf{l}_{\text{new},t}|^\alpha}\right] \\
&\triangleq 10\log_{10}\left[\Gamma + \frac{\Psi_{\text{new}} d_0^\alpha}{{q_{s_n,t}}^\alpha}\right] \quad \text{then,} \quad q_{s_n,t} = \left(\frac{\Psi_{\text{new}} d_0^\alpha}{10^{(y_{s_n,t}/10)} - \Gamma}\right)^{1/\alpha} \qquad (4)
\end{aligned}
$$

  where $\Gamma = \sum_{k=1}^{\mathbf{K}_{t-1}^m} \frac{\Psi_k d_0^\alpha}{|\mathbf{s}_n - \mathbf{l}_{k,t}^p|^\alpha}$ which is predicted part of measurement by the continuing targets, $q_{s_n,t} = |\mathbf{s}_n - \mathbf{l}_{\text{new},t}|$ which is the estimated distance between the new target and each sensor. , Ψ_{new} is the reference power of the new target, and $\mathbf{l}_{k,t}^p$ is the predicted locations of targets propagating from the previous time step, which is propagated from previous particle.

  $*$ From $q_{s_{1:N},t}$, find the minimum of $q_{s_{min},t}$.

  $*$ Send the information of $\{s_{min}, q_{s_{min},t}\}$to *lateration (least squares method) algorithm*, and guess the initial location of newly appeared target using the neighboring sensors of the best sensor (s_{min}). Find the best two neighboring sensors that have shorter distances than the rest of the neighbors (make sure that these 3 sensors form "right triangle", but not straight line).

  $*$ $\eta = \eta + 2$

  `end`
- Select the best particle using the measurement, $y_{(s_1,s_2,s_3)}$ by the maximum a posteriori (MAP) rule.

IV. Lateration (Least squares method)

We compare the regular least squares method (also called lateration) and modified lateration regarding initializing a single newly appeared target in this section. When we apply regular lateration under the hostile situation, under very low SNR, often, imaginary solution is produced or estimate is far away from the real state while modified version is more robust under the hostile environment. However, regular version shows better performance under relatively non-hostile situation.

A. Regular Lateration

Suppose there is a target in a 2-dimensional Euclidean space and we know the distances from the target to the certain locations of sensors. Theoretically, we need exactly 3 distance information from the target to the sensors, and 3 circles that are found from the 3 distances are supposed to cross one another as in Fig 1. The distances from O to the points $A(a_1, a_2)$,

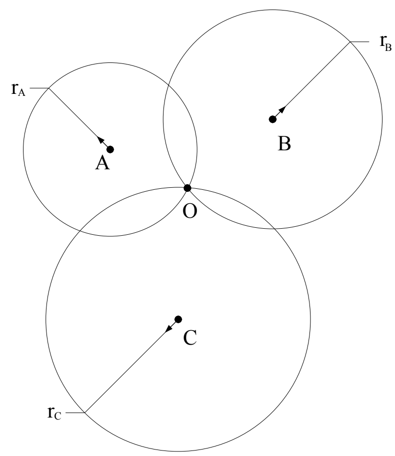


Fig. 1. Three circles cross at one point.

$B(b_1, b_2)$, and $C(c_1, c_2)$ are r_A, r_B, and r_C respectively. One way to find the point O by the 3 distances is that after finding the crossing straight line of two circles and plug in that line equation to the other circle equation. On the other hand, if we do not know the exact true radii of three circles but only noise added radii as in the Fig. 2, alternate solution can be the lateration by *least squares* approach where all circles does not have to cross one another, and that is performed as follows [30], [31]:
In X and Y, cartesian coordinate, three circles are expressed as,

$$
\begin{aligned}
(x-a_1)^2+(y-a_2)^2-{r_A}^2 &= x^2+y^2+2a_1x+2a_2y+{a_1}^2+{a_2}^2-{r_A}^2=0 \\
(x-b_1)^2+(y-b_2)^2-{r_B}^2 &= x^2+y^2+2b_1x+2a_2y+{b_1}^2+{b_2}^2-{r_B}^2=0 \\
(x-c_1)^2+(y-c_2)^2-{r_C}^2 &= x^2+y^2+2c_1x+2c_2y+{c_1}^2+{c_2}^2-{r_C}^2=0
\end{aligned}
$$

Then, according to the least squares method, we have two linear equation as,

$$\begin{aligned} {a_1}^2 - {c_1}^2 - 2(a_1 - c_1)x + {a_2}^2 - {c_2}^2 - 2(a_2 - c_2)y &= {r_A}^2 - {r_c}^2 \\ {b_1}^2 - {c_1}^2 - 2(b_1 - c_1)x + {b_2}^2 - {c_2}^2 - 2(b_2 - c_2)y &= {r_B}^2 - {r_c}^2 \end{aligned}$$

Least squares [29] solves these linear equations as follows:

$$H\mathbf{x} = d, \text{ then } \quad \hat{\mathbf{x}} = (H^\top H)^{-1} H^\top d. \tag{5}$$

where

$$H = \begin{bmatrix} 2(a_1 - c_1) & 2(a_2 - c_2) \\ 2(b_1 - c_1) & 2(b_2 - c_2) \end{bmatrix}, \; d = \begin{bmatrix} {a_1}^2 - {c_1}^2 + {a_2}^2 - {c_2}^2 + {r_C}^2 - {r_A}^2 \\ {b_1}^2 - {c_1}^2 + {b_2}^2 - {c_2}^2 + {r_C}^2 - {r_B}^2 \end{bmatrix}, \text{ and } \mathbf{x} = [x \;\; y]^\top.$$

When the distances (radii of the circles) are estimated from the measurement data, least squares find the point which gives the least sum of differences between the function of data and function of estimated point. When H is a singular matrix, there is not a solution or the solution will be imaginary. We may use more data measurement to solve more dimensional linear equations. However, in our RSS measurement model, the received power at the sensors that are very far from the target are not that good quality of measurement. Therefore we use only 3 best sensors, which means we use 3 strongest measurement received to estimate the location of initialized target. Table II summarize the steps of initializing a new target.

B. Modified Lateration

Regular lateration shows better result in the simulations which we will show later in this paper only under the relatively non-hostile situations. As the noise power increases, while "mean error" and "variances" of estimates increase too, modified lateration method shows less error and variance increment than the regular lateration in the simulations.

Modified lateration find two crossing points of the first best and second best circles; the smaller circle means the better measurement it is. Third best circle find better point out of two points that are found by 2 best circles. As shown in Fig. 2, there are 3 best estimated circles, A, B, and C. After comparing r_C with r_{CF} and r_{CE}, take the the estimated point which has closer distance from the C to r_C; in this case F is chosen, i.e., take E if $|r_{CE} - r_C| < |r_{CF} - r_C|$, take F otherwise. However, there is another way we can apply to choose the estimated point other than this algorithm. If we use particle filtering, we can take two points together with different weight after generating particles. But, this way can be more expensive because if we apply particle filtering to the first modified algorithm, we will still track back to the point closer eventually.

TABLE II

INITIALIZING A TARGET FROM THE EMPTY FIELD BY LATERATION ADOPTED IN PARTICLE FILTERING

At time t, from the all measurement, find 3 best measurement, ($y_{(s_1,s_2,s_3),t}$), and corresponding sensors' identities (s_1, s_2, s_3).

- `For` m=1:M (M is the number of particles.)

  $*$ Using the maximum of $y_{s_{max},t}$, compute $q_{s_{max},t}$ as follows:

$$
\begin{aligned}
y_{s_{max},t} &= 10\log_{10}\left(\sum_{k=1}^{1}\frac{\Psi_k d_0^\alpha}{|\mathbf{s}_{max}-\mathbf{l}_{k,t}|^\alpha}\right)+v_{s_{max},t} \\
&\triangleq 10\log_{10}\left(\frac{\Psi_{\text{new}} d_0^\alpha}{q_{s_{max},t}{}^\alpha}\right) \quad \text{, then } \; q_{s_{max},t}=\left(\frac{\Psi_{\text{new}} d_0^\alpha}{10^{(y_{s_{max},t}/10)}}\right)^{1/\alpha} \qquad (6)
\end{aligned}
$$

  where $q_{s_{max},t}=|\mathbf{s}_{max}-\mathbf{l}_{\text{new},t}|$ which is the estimated distance between the new target and the best sensor (s_{max}) , Ψ_{new} is the reference power of the new target.

  $*$ With the information of s_{max} and $q_{s_{max},t}$, send it to lateration (least squares method) algorithm and guess the initial location of newly appeared target using two more neighboring sensors of the best sensor (s_{max}). Make sure that these 3 sensors form "right triangle", but not straight line.

  `end`

- Select the particle according to the maximum a posteriori (MAP) rule using 3 best measurement, $y_{(s_1,s_2,s_3),t}$.

There has to be 2 crossing points to apply modified algorithm, but if first 2 best circles do not cross to each other, we take the middle point of two circles' gap as shown in Fig. 3. This can happen often in hostile situation. If SNR is low, then the third best estimated circle does not have very good information about the target's initial point and that is why regular lateration is not as good as modified lateration in the hostile situation.

C. Performance Comparison

In this section, we show the performance comparison between regular lateration and modified lateration regarding initializing a single target in wireless sensor networks using uniformly deployed sensors with RSS measurement model. The initialization problem is essential requirement to proceed any target tracking solutions, e.g., particle filtering for multiple and variable number of target tracking problems.

Simulation is executed with various different noise power or variances, from 0.001 (W) to 10 (W). 25 sensors are uniformly distributed in the 200×200 (m^2), 2 dimensional cartesian coordinate system. $d_0 = 1$ (m), $\Psi = 10,000$ (W), and true target initial point is (50, 120). SNR is computed only for the signals received at the sensors we used for estimation, usually 3 or 2 sensors in this paper. As the noise power increases, two methods' SNR gap increases too because if there is strong noise,

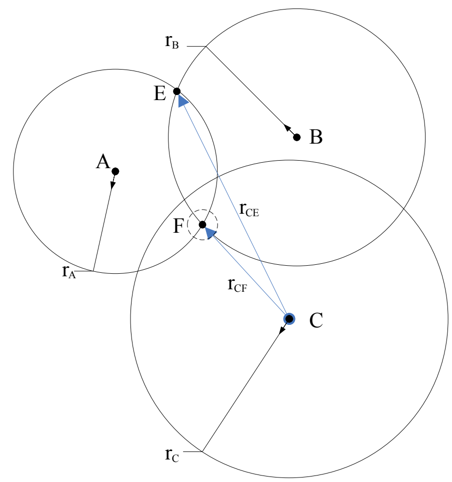


Fig. 2. Modified lateration using 3 best circles.

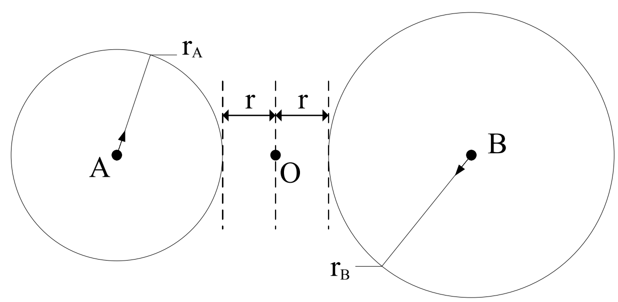
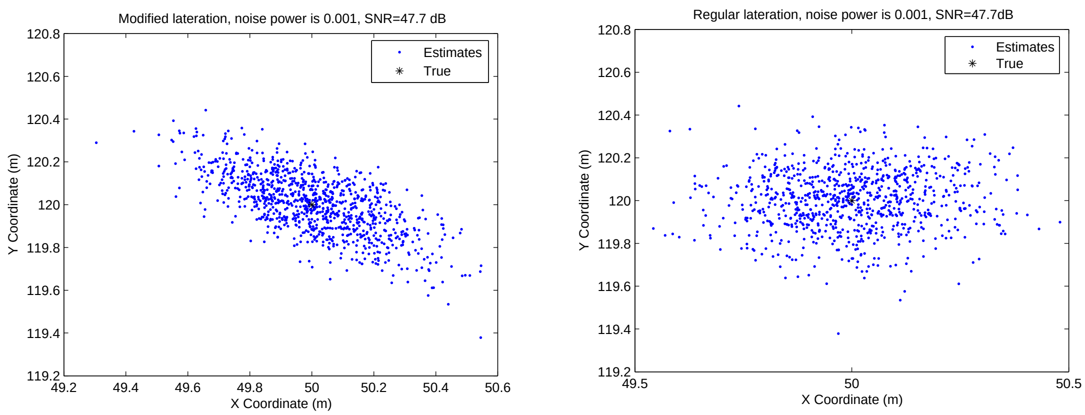


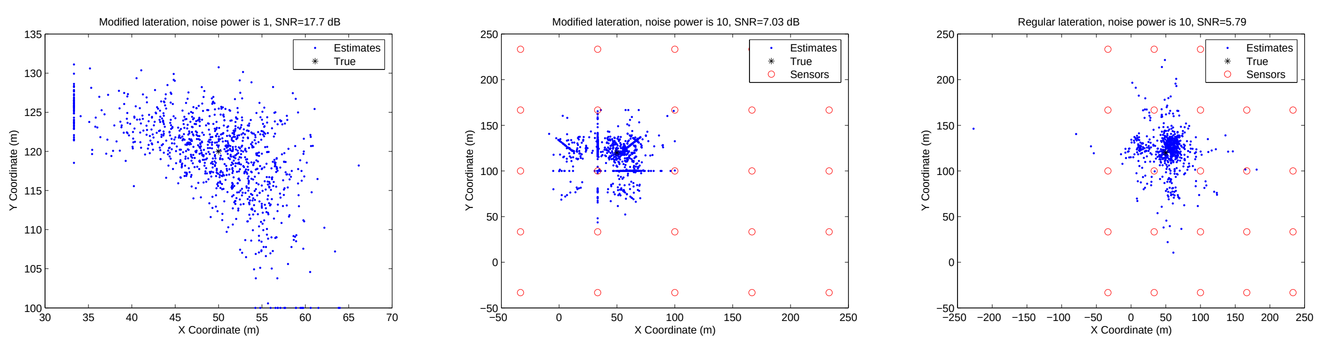
Fig. 3. Modified lateration when first 2 best circles do not cross.

modified lateration tends to use only two sensors while regular one always use 3 sensors, and 3rd sensor usually receives very weaker signal than the first or the second sensor. In summary, even though regular lateration started to fail to initialize a newly appeared target when the noise variance is equal to 1, it shows less “mean distance error” until the noise variance is larger than 1 as shown in Fig. 6(a), and also regular lateration shows less “variance of estimates” with 1000 runs till the noise variance is 1 as shown in Fig. 7(b). Note that from a certain point, modified lateration shows the pattern of straight line formed by initialized points (see Fig. 5(a) and (b)). That is because when only two sensors are used and many initial points are initialized just at the center of the gap of the two circles of which radii are the estimates of the distances from the target to the sensors. Even though modified lateration shows poorer performance than the regular lateration under a moderate environment in initializing a single target, it shows the best performance among 3 methods (one is regular, another one is modified, and the other is mixed lateration that is going to be explained in the Section VI-A.2) when initializing two targets simultaneously that are not located very closely. Fig. 4 shows the typical pattern of initialized points when the noise power is not so strong that the straight line pattern does not start to form yet in modified lateration, and Fig. 6 and Fig. 7 show the summary of the result regarding mean error distance between true value and initialized points, the variances of the initialized points, number of fails out of 1000 runs, and SNR respectively. Especially, in Fig 7, we compared Cramer-Rao bound (CRB) with the variance of the initialized points by regular lateration. We compared CRB with only regular lateration because, when we apply modified lateration, the number of sensors used is 2 or 3 that have different CRB respectively. When the target is located on the line between two sensors, it has very high pick CRB as shown in Fig. 27(a). Furthermore, it is compared with the noise only up to 0.1 because, if the noise is larger than that, even though the number of sensors used does not change, but identities of sensors changes. Depending on the geometrical locations of sensors, the CRB is different (see Fig. 27 (b)).



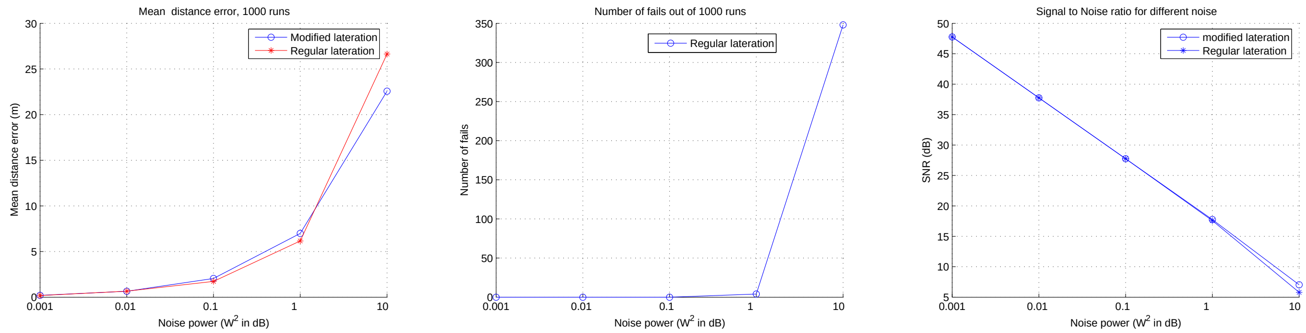
(a) Modified lateration. (b) Regular lateration.

Fig. 4. Comparison of regular and modified lateration when initializing a single target. The noise power is 0.001. True target location is (50, 120), 1000 runs.



(a) Initialized points by modified lateration. Noise power is 1. (b) Initialized points by modified lateration. Noise power is 10. (c) Initialized points by regular lateration. Noise power is 10.

Fig. 5. Initialized points by lateration. True location is (50, 120), 1000 runs. Note the pattern of straight line formed by many initialized points because only two sensors are used for many initial points in (a) and (b).

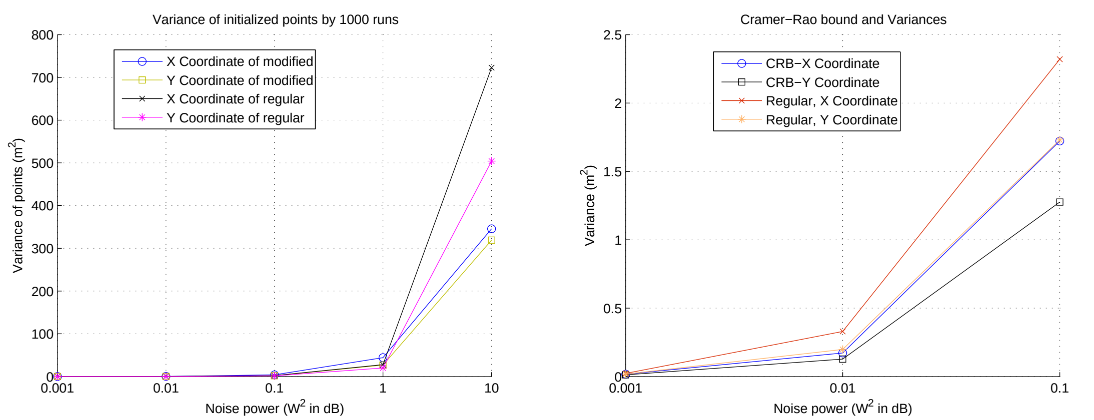


(a) Comparison of mean error distance between regular and modified lateration.

(b) The number of fails when using regular lateration, on the other hand, modified lateration never fails.

(c) Received signal to noise ratio.

Fig. 6. Initialized points by modified lateration with different noise power. True location is (50, 120), 1000 runs. Note the pattern of straight line formed by many initialized points because only two sensors are used for many initial points.



(a) Variance of the initialized points for two methods.

(b) Cramer-Rao bound and the variance of the initialized points by regular lateration .

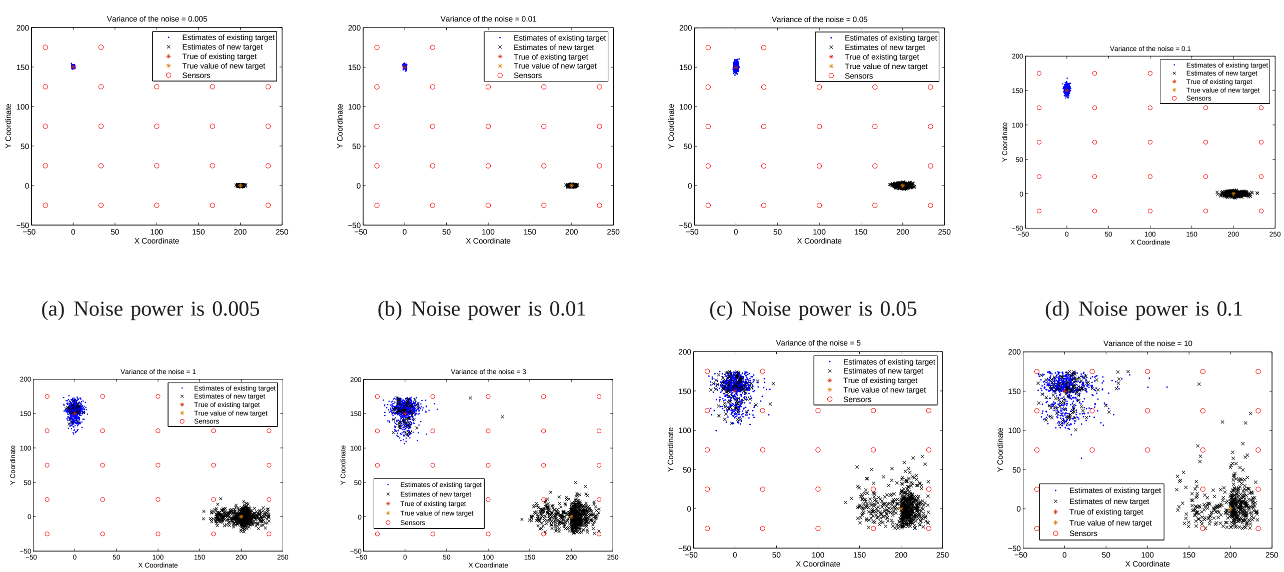
Fig. 7. Variances of the two methods and comparison of Cramer-Rao bound and the variance of the initialized points by regular lateration with 1000 runs.

V. Initialization of a New Target Besides Existing Targets

In this section, we present a new target initialization besides existing targets using the regular lateration. The particle filter detects any newly appeared target, and estimate the states of targets according to the weights of particles. A single particle has the information of the number of targets, identifications of targets, the locations (velocities and accelerations too) of all identified targets. Any single particle propagates producing multiple particles following the assumption in Section II-B. Suppose we have M particles that have the same identification of one target but different details of the states (locations, velocities, and accelerations). Each particle will produce 3 different kinds of particles; a particle which has no target (disappeared target), a particle which has same target as before and updated, and a particle which has additional new target and updated target. If there is a newly appeared target, the particle which has additional new target will have the heaviest weight, and will be selected when we use MAP estimation rule. Initializing a new target besides the existing targets using lateration by particle filtering is summarized in Table I. According to Table I, particle filter will select the particle which has the heaviest weight as the estimate of the state at time t, and does the *down-resampling* from $3M$ to M and keeps the prosperous particles. A simulation is following.

A. Simulation

The state space and measurement model follows as in Section II-A. Initially, there is a single target at the coordinate of $(0, 150)$, and another new target appears right next time step at the coordinate of $(200, 0)$. Fig. 8 shows the simulation result of initializing a new target following the steps in Table I. Each sub-figure shows the different result under the different noise variance. Generally the result shows similar pattern except untill the noise variance is 1 (W), but when the noise power is lager than 1 (W), it starts to have initial points around the existing target. Nevertheless continuing updated target never estimated around the new target because we assume that we know the state space equation and continuing target propagate according to the equation, but we have no information of the location nor velocity about newly appeared target. Fig. 9 shows the summary

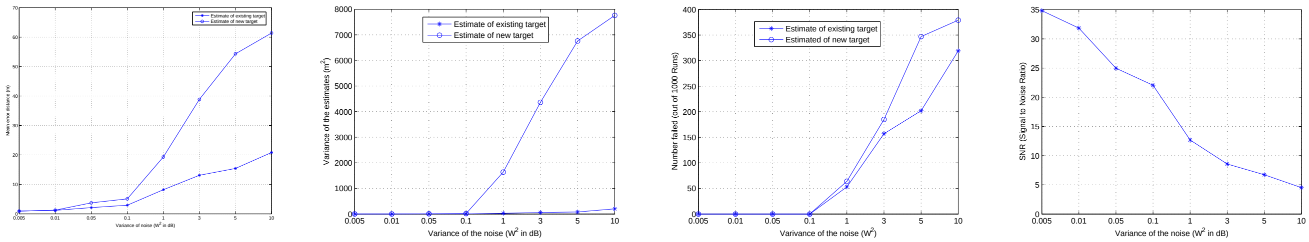


(a) Noise power is 0.005 (b) Noise power is 0.01 (c) Noise power is 0.05 (d) Noise power is 0.1

(e) Noise power is 1 (f) Noise power is 3 (g) Noise power is 5 (h) Noise power is 10

Fig. 8. Initializing a new target besides a continuing target by regular lateration, 1000 runs.

of the simulation result. The mean error distance of the new target increases more than the mean error distance of the updated target increases as the noise increase. Especially the variance gap far more larger as the noise increases. Overall, the lateration works well when initializing a new target with blind information about it.



(a) Mean error distance . (b) Variance of estimated points. (c) The number of fails. (d) The SNR (dB) of simulations.

Fig. 9. The summary of the result when initializing a new target besides existing target by regular lateration, 1000 runs.

VI. Initialization of two new target

In this section, we introduce the technique how to initialize two targets at the same time. This is not the case under the assumption in Section II-B. So, suppose we have to initialize two newly appeared targets at the same time step. We compare the performances of three different methods even though they are similar. Regular, modified, and mixed lateration are applied for initializing two newly appeared target at the same time. We have explained about first two methods in the previous sections, but not about the thrid one, *mixed lateration*. Mixed lateration is the combination of regular and modified lateration. The purpose of the mixed lateration is to take the advantages of each method. When we apply modified one, if two first best circles do not cross each other, then we take the middle point of the gap of the two circles. However, that can cause very ristricted initialization of the new targets, especially when limited number of sensors or hostile noise environment. Therefore, we apply regular lateration when we use only two best circles in applying modified lateration. Except for that, the rest is the same as modified lateration.

In the previous sections, we used 3 best measurement or sensors for the estimation of the states, but we have to use one best sensor and measurement with its neighboring two more sensors for the least squares method. The “3 best measurement” could be the same as “ one best measurement and two neighboring sensed measurement” or not. If there is an single target, usually 3 best sensors means also 3 neighboring sensors which may be used for the lateration (see Fig. 10). However, if there are multiple-targets to be tracked, most of the time the best 3 sensors does not mean that they are neighboring to each other as explained in detail in Fig. 11. Nonetheless, when we estimate the states of the targets, we still have to use the best 3 measurement of the sensors after initializing the new targets. According to the RSS sensor model (see (1)), the strength of the received signal drops quickly with respect to increasing distance between the sensor and the target. If we take the advantage of this property, we can initialize the first target of the two simultaneous target initializations. The best sensor is supposed to be very close to the one of two newly appeared targets; the further the second target is from the first target , the closer the first target is to the best sensor. We initialize a target as if there is only one newly appeared target at the first step, and then using *residue cancelation*, we initialize another target besides previously initialized target. These two steps can be repeated turn by turn to reduce the error, and all this procedure takes place at one time step. These steps can be summarized in the Table III. We present the simulation result comparing the performances of the three methods regarding different noise, and the number of iterations. Iterative method always works well with modified lateration up to certain number of iteration. Especially, from the simulation result, modified lateration performs better than the other two methods when two targets are not located very closely to each other (see Section VI-A.2) while modified lateration works poorer than the other two methods when two targets are located very close to each other (see Section VI-A.1). Iterative method is very effective especially for the firstly initialized target. Just one iteration makes the error of the first initialized target fall down dramatically(see Fig. 23).

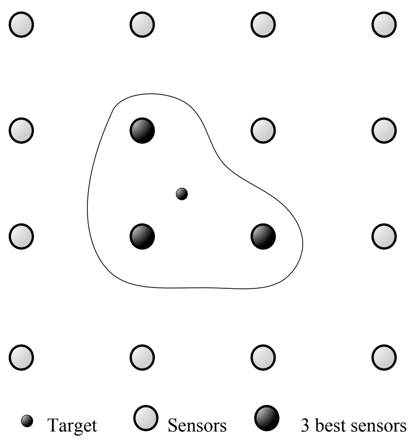


Fig. 10. 3 best neighboring sensors for initialization by lateration.

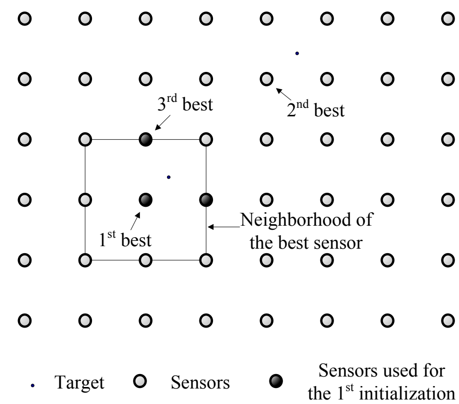


Fig. 11. Initialization of two targets by lateration.

A. Simulation

1) Two Close Targets: In this section, we show the simulation result of initializing two close targets simultaneously under the diverse conditions. We compare the performances of three kinds of laterations: regular, modified, and mixed lateration. We can make brief conclusion in advance here that regular and modified lateration performs almost the same,but modified one performs poorer when two targets are not very close. Fig. 12, 15, and 16 show the result of initializing two close targets regarding different noise and the number of iterations. As we can observe from the result, iterative method gives better result for modified lateration, especially for the firstly initialized target. Iteration makes almost no difference to regular and mixed lateration, and even worsen the performances as the number of iteration increases (see Fig. 17 and 18). If the number of iteration increases greater than 1, initial points start to diverge and finally overlap with the other initialized points as in Fig. 12, 15, and 16. This diverging pattern occurs for all methods as the number of iterations increases. It occurs, because as the iteration number increases, once initialized point becomes distant from the true point, the other initialized point reciprocally

TABLE III

INITIALIZATION OF TWO NEWLY APPEARED TARGETS AT THE SAME TIME.

---

1) At time t, from $y_{s_{1:N},t}$, find the 3 largest of $y_{(s_{m_1},s_{m_2},s_{m_3}),t}$.
2) Using $y_{s_{m_1},t}$, the best measurement, and corresponding s_{m_1}, compute $q_{s_{m_1},t}$, the estimated distance between the best sensor and the first target to be initialized, as follows:

$$y_{s_{m_1},t} = 10\log_{10}\left(\sum_{k=1}^{2}\frac{\Psi_k d_0^\alpha}{|\mathbf{s}_{m_1}-\mathbf{l}_{k,t}|^\alpha}\right)+v_{s_{m_1},t} \triangleq 10\log_{10}\left[\frac{\Psi_{\text{new1}} d_0^\alpha}{|\mathbf{s}_{m_1}-\mathbf{l}_{\text{new1},t}|^\alpha}\right], \text{ then} \quad (7)$$

$$q_{s_{m_1},t} \triangleq |\mathbf{s}_{m_1}-\mathbf{l}_{\text{new1},t}| = \left(\frac{\Psi_{\text{new1}} d_0^\alpha}{10^{\left(y_{s_{m_1},t}/10\right)}}\right)^{1/\alpha} \quad (8)$$

where Ψ_{new1} is the reference power of the first initialized target.

∗ Send the information of $\{s_{m_1}, q_{s_{m_1},t}\}$to *lateration (least squares method) algorithm*, and guess the initial location ($\mathbf{l}_{\text{new1},t}$) of newly appeared target using the neighboring sensors of the best sensor (s_{m_1}). Find the neighboring sensors and best two neighboring sensors that have stronger signal than the rest of the neighbors (make sure that these 3 sensors form “right triangle”, but not straight line).

3) Apply *residue cancelation* step as in Table I as follows:

$$y_{s_n,t} = 10\log_{10}\left[\frac{\Psi_{\text{new1}} d_0^\alpha}{|\mathbf{s}_n-\mathbf{l}_{\text{new1},t}|^\alpha}+\frac{\Psi_{\text{new2}} d_0^\alpha}{|\mathbf{s}_n-\mathbf{l}_{\text{new2},t}|^\alpha}\right]+v_{s_n,t} \triangleq 10\log_{10}\left[\boldsymbol{\beta}+\frac{\Psi_{\text{new2}} d_0^\alpha}{|\mathbf{s}_n-\mathbf{l}_{\text{new2},t}|^\alpha}\right], \text{ then} \quad (9)$$

$$q'_{s_n,t} \triangleq |\mathbf{s}_n-\mathbf{l}_{\text{new2},t}| = \left(\frac{\Psi_{\text{new2}} d_0^\alpha}{10^{\left(y_{s_n,t}/10\right)-\boldsymbol{\beta}}}\right)^{1/\alpha} \quad (10)$$

4) From $q'_{s_n,t}$, find the minimum of $q'_{s_{min},t}$.

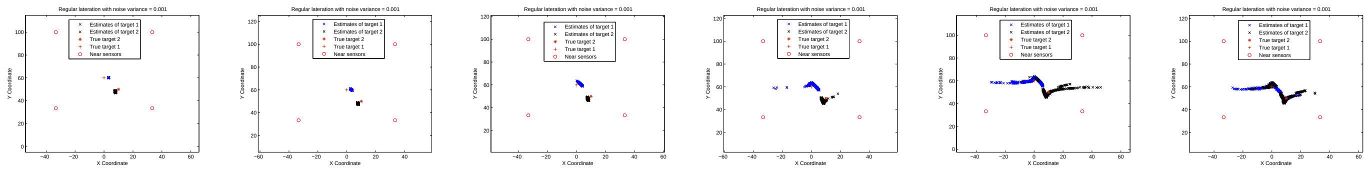
∗ Send the information of $\{s_{min}, q_{s_{min},t}\}$to *lateration (least squares method) algorithm*, and guess the initial location ($\mathbf{l}_{\text{new2},t}$) of second target using the neighboring sensors of the best sensor (s_{min}). Find the neighboring sensors and best two neighboring sensors that have shorter distances than the rest of the neighbors (make sure that these 3 sensors form “right triangle”, but not straight line).

5) Iteratively repeat from 2) to 4) to have reduced error (depending on the iteration number).

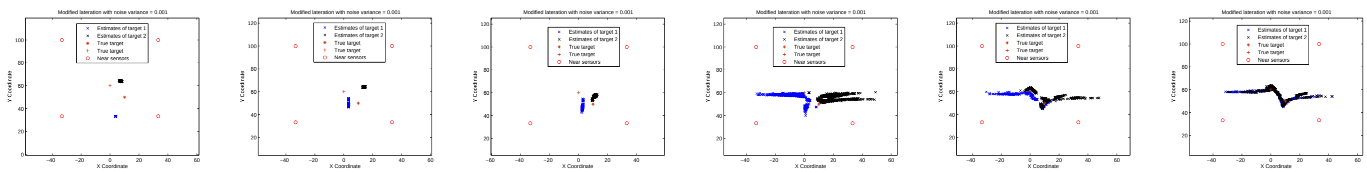
---

becomes distant too as shown in Fig. 13 and 14. Finally, when error reaches the limit point, these two initialized points overlap to each other because initialized points with large error must have limit with certain best sensors that already chosen (Compare Fig. 15(i) and Fig. 15(j)).

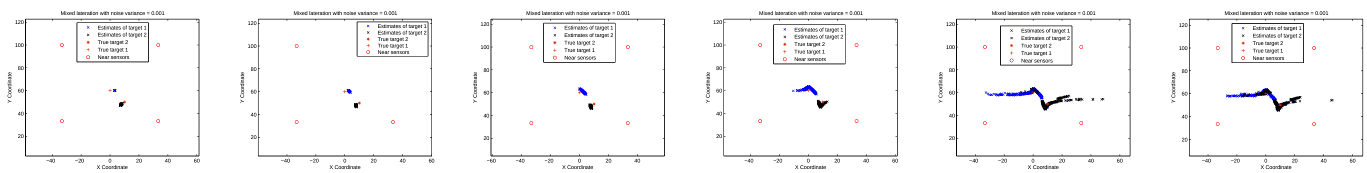
2) Two Far Targets: When two targets are located far to each other, all three methods perform very well and better than when two targets are very close to each other. Also, iterative methods performs well for all three methods, especially for the first initialized target. Iterative methods does not make difference for the initialization of second target, and after one iteration, the mean error distance for the first target is better than the second target while it was vise versa before the iteration as shown



(a) Regular, 0. (b) Regular, 1. (c) Regular, 2. (d) Regular, 5. (e) Regular, 10. (f) Regular, 100.



(g) Modified, 0. (h) Modified, 1. (i) Modified, 2. (j) Modified, 5. (k) Modified, 10. (l) Modified, 100.



(m) Mixed, 0 (n) Mixed, 1 (o) Mixed, 2 (p) Mixed, 5 (q) Mixed, 10 (r) Mixed, 100

Fig. 12. Comparison of 3 methods with different number of iterations on initializing two close targets when noise variance is 0.001. Each caption under the figures shows the lateration method and the number of iterations, 1000 runs.

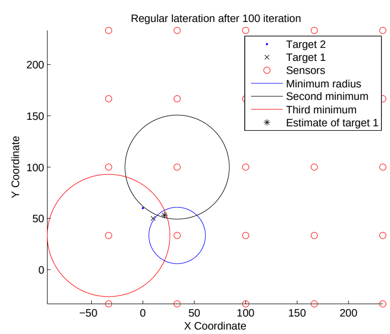


Fig. 13. Least squares method for the first target initialization with large error on initializing two close targets.

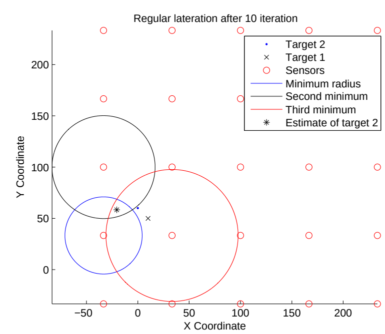
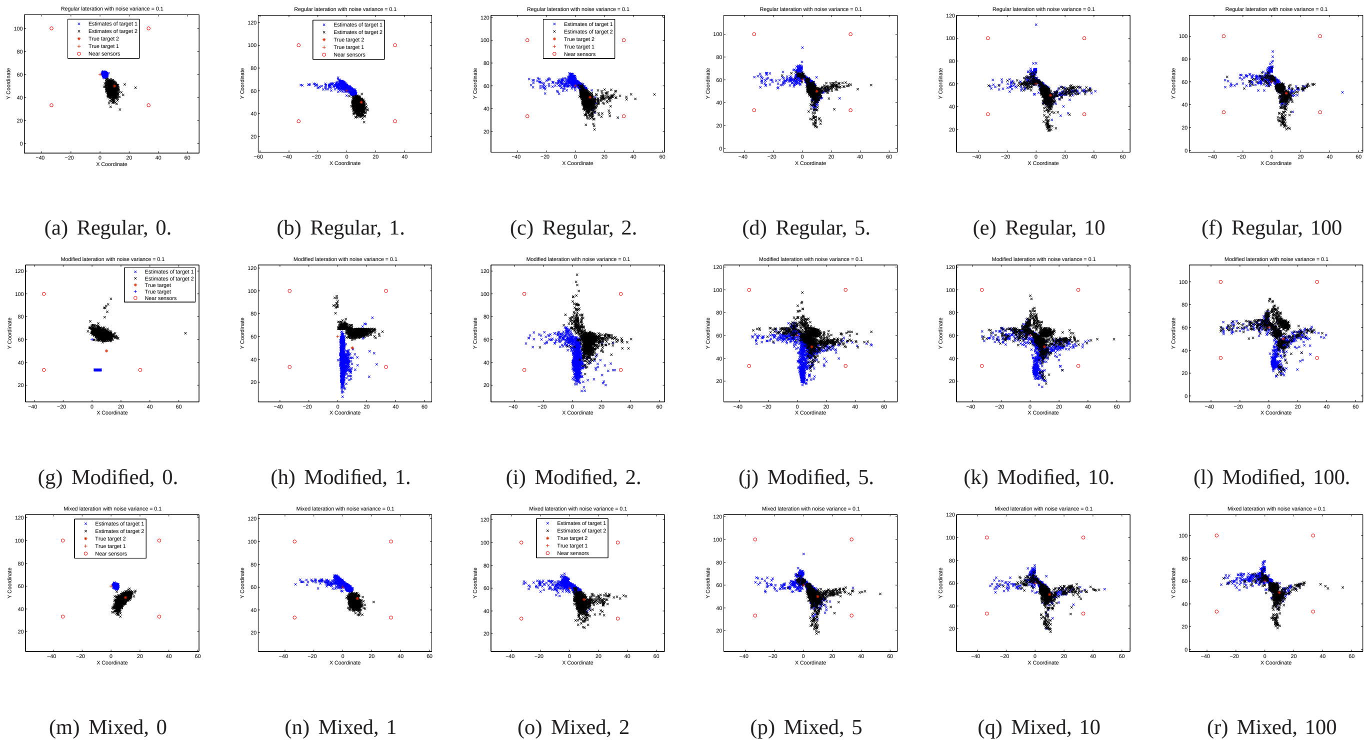


Fig. 14. Least squares method for the second target initialization with large error in initializing two close targets.

in Fig. 23(a). So, basically all three methods show the similar pattern of initialized points as shown in Fig. 20. When the noise variance if equal to 1 or greater than that, due to the large variance of the initialized points, it starts to have overlapping between two targets in the same area as shown in Fig. 21(c). That phenomenon occurs to all 3 methods. We can see the outstanding difference between “before the iteration” and “after the iteration” when applying modified lateration shown as in 21. Modified lateration uses only two best sensors sometimes which makes specific pattern of the initialized points as in Fig. 22(b). After iteration that specific pattern disappears as shown in Fig. 22(b) and gives far better performance for the initializing the first target.



(a) Regular, 0. (b) Regular, 1. (c) Regular, 2. (d) Regular, 5. (e) Regular, 10 (f) Regular, 100

(g) Modified, 0. (h) Modified, 1. (i) Modified, 2. (j) Modified, 5. (k) Modified, 10. (l) Modified, 100.

(m) Mixed, 0 (n) Mixed, 1 (o) Mixed, 2 (p) Mixed, 5 (q) Mixed, 10 (r) Mixed, 100

Fig. 15. Comparison of 3 methods with different number of iterations when noise variance is 0.1 on initializing two close targets. Each caption under the figures shows the lateration method and the number of iterations, 1000 runs.

VII. CRAMER-RAO LOWER BOUND (CRLB)

We show the CRLB of the estimator of the parameter, the location of a "single target" when we use only 3 sensors which forms right triangle in this section. The parameter is denoted by $\boldsymbol{\theta} = \mathbf{l} = [x\ y]^\top$. From (1), the likelihood function is

$$p(\mathbf{y};\boldsymbol{\theta}) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^3 \exp\left\{-\frac{1}{2\sigma^2}\sum_{n=1}^{3}[y_n - f_n(\boldsymbol{\theta})]\right\}$$

where

$$f_n(\boldsymbol{\theta}) = 10\log_{10}\left[\frac{\Psi}{g_n(\boldsymbol{\theta})}\right] \tag{11}$$

and

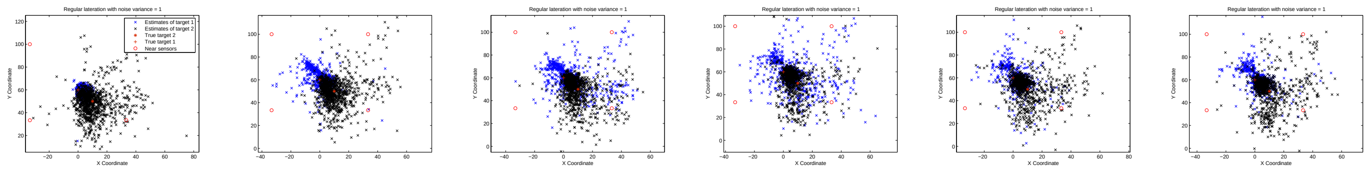
$$g_n(\boldsymbol{\theta}) = g_n(x,y) = |\mathbf{s}_n - \mathbf{l}|^\alpha = (s_{n_x} - x)^2 + (s_{n_y} - y)^2. \tag{12}$$

The log-likelihood function is,

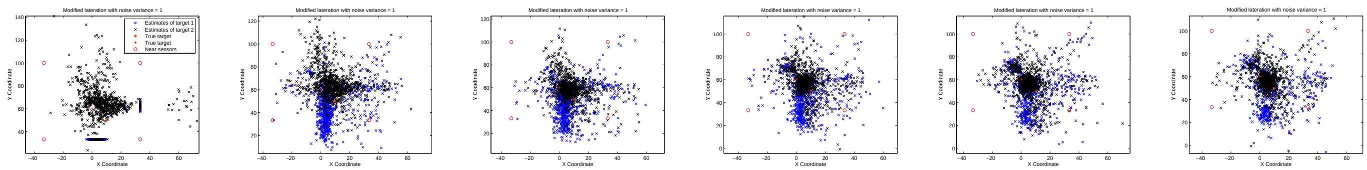
$$\ln p(\mathbf{y};\boldsymbol{\theta}) = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^3 + \left[-\frac{1}{2\sigma^2}\sum_{n=1}^{3}(y_n - f_n)\right] \tag{13}$$

from which the derivative of x coordinate follows as

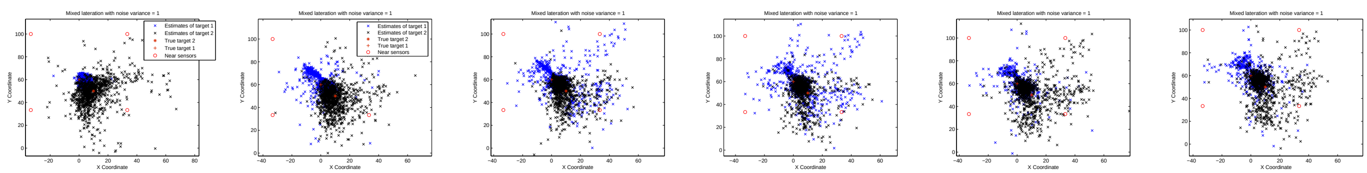
$$\frac{\partial \ln p}{\partial x} = \frac{\partial}{\partial x}\left[-\frac{1}{2\sigma^2}\sum_{n=1}^{3}(y_n - f_n)^2\right] = -\frac{1}{2\sigma^2}\sum_{n=1}^{3}\underbrace{\left\{\frac{\partial}{\partial x}\left[(y_n - f_n)^2\right]\right\}}_{\mathcal{A}}. \tag{14}$$



(a) Regular, 0. (b) Regular, 1. (c) Regular, 2. (d) Regular, 5. (e) Regular, 10. (f) Regular, 100.

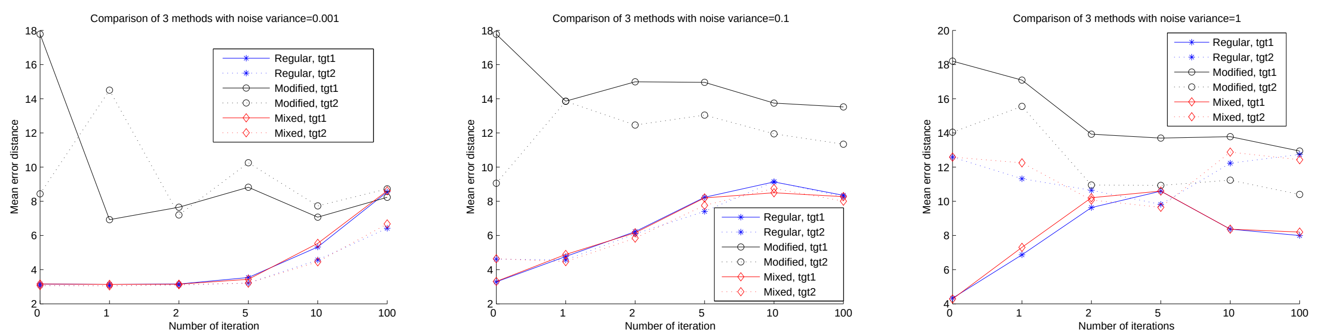


(g) Modified, 0. (h) Modified, 1. (i) Modified, 2. (j) Modified, 5. (k) Modified, 10. (l) Modified, 100.



(m) Mixed, 0. (n) Mixed, 1. (o) Mixed, 2. (p) Mixed, 5. (q) Mixed, 10. (r) Mixed, 100.

Fig. 16. Comparison of 3 methods with different number of iterations when noise variance is 1 on initializing two close targets. Each caption under the figures shows the lateration method and the number of iterations, 1000 runs.

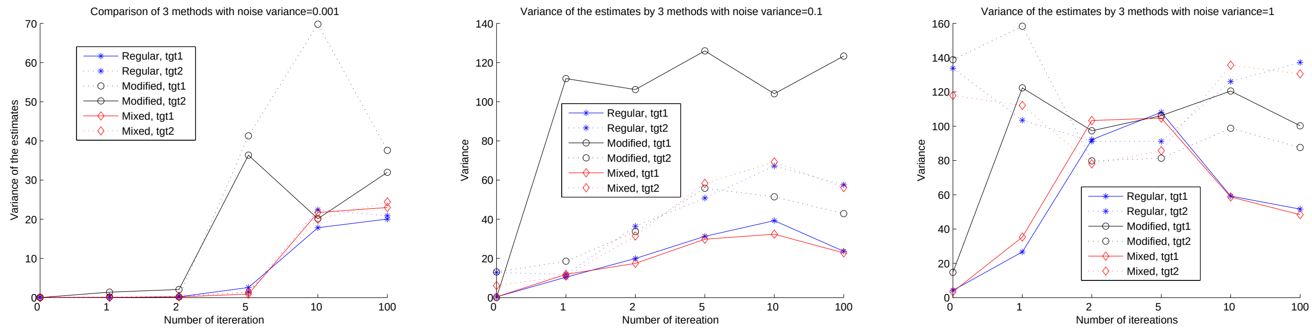


(a) Noise variance is 0.001. (b) Noise variance is 0.1. (c) Noise variance is 1.

Fig. 17. Mean error distances of 3 methods with different number of iterations on initializing two close targets, 1000 runs.

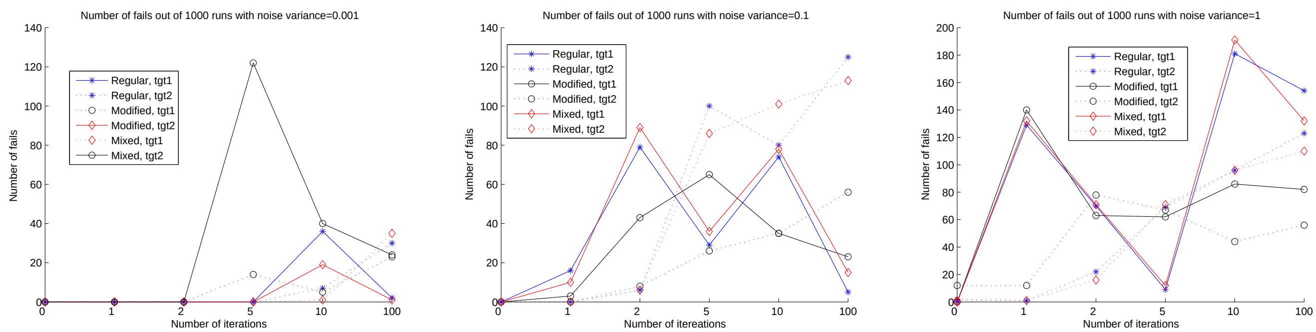
From $\mathcal{A}$,

$$\mathcal{A} = \frac{\partial}{\partial x}\left[(y_n - f_n)^2\right] = 2\left[y_n - f_n(\boldsymbol{\theta})\right]\underbrace{\left[-\frac{\partial f_n(x,y)}{\partial x}\right]}_{\mathcal{B}}. \quad (15)$$



(a) Noise variance is 0.001. (b) Noise variance is 0.1. (c) Noise variance is 1.

Fig. 18. Variances of the estimates of 3 methods with different number of iterations on initializing two close targets, 1000 runs.



(a) Noise variance is 0.001. (b) Noise variance is 0.1. (c) Noise variance is 1.

Fig. 19. The number of fails of 3 methods with different number of iterations on initializing two close targets, 1000 runs.

From $\mathcal{B}$,

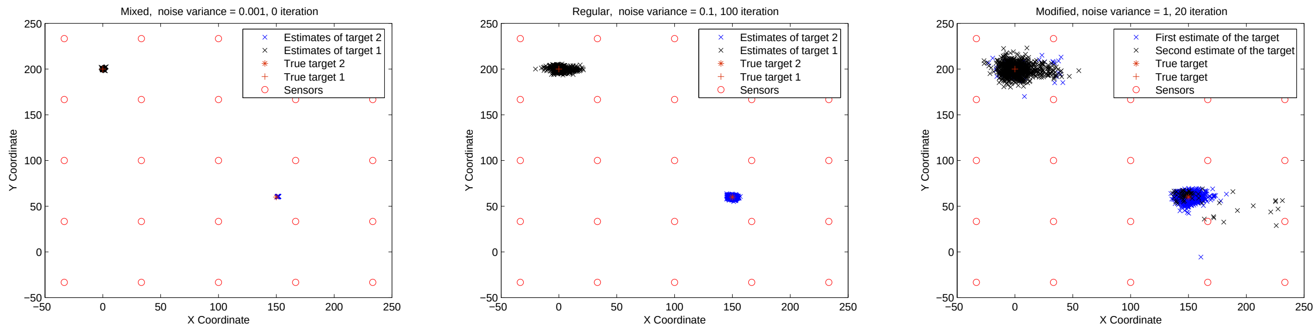
$$\begin{aligned}
\mathcal{B} &= \frac{\partial f_n(x,y)}{\partial x} = \frac{\partial}{\partial x}\left\{10\log_{10}\left[\frac{\Psi}{g_n(x,y)}\right]\right\} && (16)\\
&= \frac{\partial}{\partial x}\left[10\log_{10}\Psi - 10\log_{10} g_n(x,y)\right] = \frac{\partial}{\partial x}\left[10\log_{10} g_n(x,y)\right] && (17)\\
&= -10\frac{\partial}{\partial x}\left[\log_{10} g_n(x,y)\right] = -\frac{10}{\ln 10}\frac{[\partial g_n(x,y)/\partial x]}{g_n(x,y)} && (18)\\
&= \frac{20}{\ln 10}\frac{(s_{n_x}-x)}{g_n}\ \left(\because \frac{\partial g_n}{\partial x} = -2(s_{n_x}-x)\right). && (19)
\end{aligned}$$

If we plug $\mathcal{B}$ into $\mathcal{A}$,

$$\mathcal{A} = 2[y_n - f_n(\boldsymbol{\theta})]\left[-\frac{20}{\ln 10}\frac{(s_{n_x}-x)}{g_n}\right]. \tag{20}$$

Plugging $\mathcal{A}$ into (14),

$$\begin{aligned}
\frac{\partial \ln p}{\partial x} &= -\frac{1}{2\sigma^2}\sum_{n=1}^{3}\left\{2[y_n - f_n(\boldsymbol{\theta})]\left[-\frac{20}{\ln 10}\frac{(s_{n_x}-x)}{g_n(x,y)}\right]\right\} && (21)\\
&= \frac{20}{\sigma^2\ln 10}\sum\left\{[y_n - f_n(\boldsymbol{\theta})]\left[\frac{(s_{n_x}-x)}{g_n(x,y)}\right]\right\} && (22)\\
&= \frac{20}{\sigma^2\ln 10}\sum\left\{\frac{[y_n - f_n(\boldsymbol{\theta})](s_{n_x}-x)}{g_n(x,y)}\right\}. && (23)
\end{aligned}$$

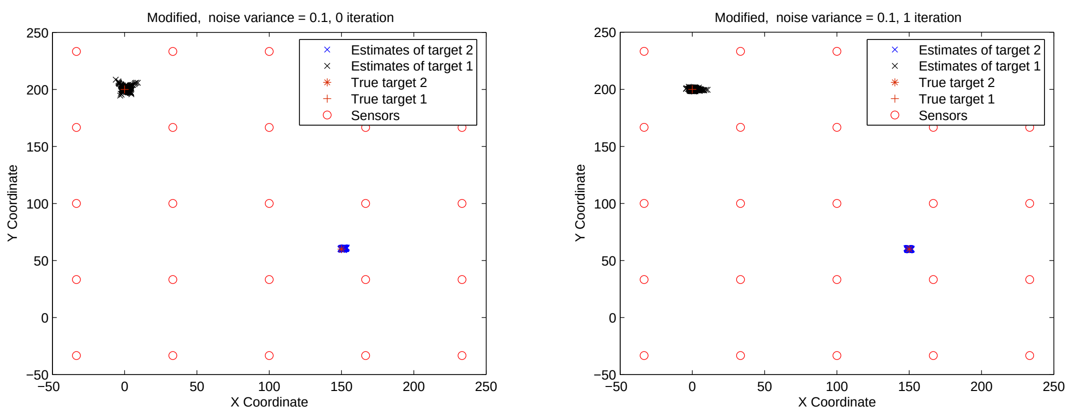


(a) Modified, 0.001, and 0.

(b) Regular, 0.1, and 100.

(c) Modified, 1, and 20.

Fig. 20. Initialized points by 3 different lateration methods on initializing two distant targets, captions under the figure show the lateration method, noise, and the number of iteration respectively, 1000 runs.



(a) Modified lateration, noise is 0.1, and 0 iteration.

(b) Modified lateration, noise is 0.1, and 1 iteration

Fig. 21. Modified lateration of two targets that are not very close, 1000 runs.

Similarly, we can derive derivative of y coordinate as

$$\frac{\partial \ln p}{\partial y} = \frac{20}{\sigma^2 \ln 10} \sum \left\{ \frac{[y_n - f_n(\boldsymbol{\theta})](s_{n_y} - y)}{g_n(x,y)} \right\}. \tag{24}$$

The second derivative of x coordinate follows as

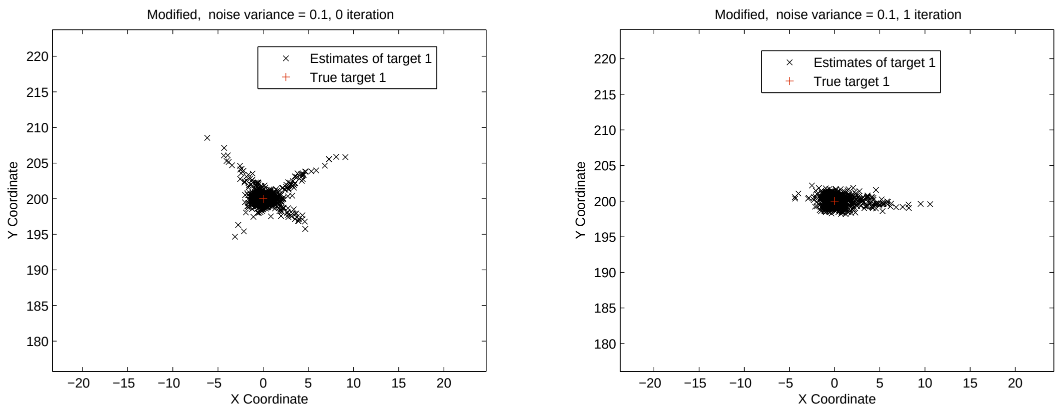
$$\begin{aligned} \frac{\partial^2 \ln p}{\partial x^2} &= \frac{\partial}{\partial x} \left\{ \frac{20}{\sigma^2 \ln 10} \sum \left[\frac{(y_n - f_n)(s_{n_x} - x)}{g_n(x,y)} \right] \right\} &(25)\\ &= \frac{20}{\sigma^2 \ln 10} \sum \frac{\partial}{\partial x} \left[\frac{(y_n - f_n)(s_{n_x} - x)}{g_n(x,y)} \right]. &(26) \end{aligned}$$

If we define

$$P_x(x,y) \triangleq (s_{n_x} - x)(y_n - f_n) \tag{27}$$

we have

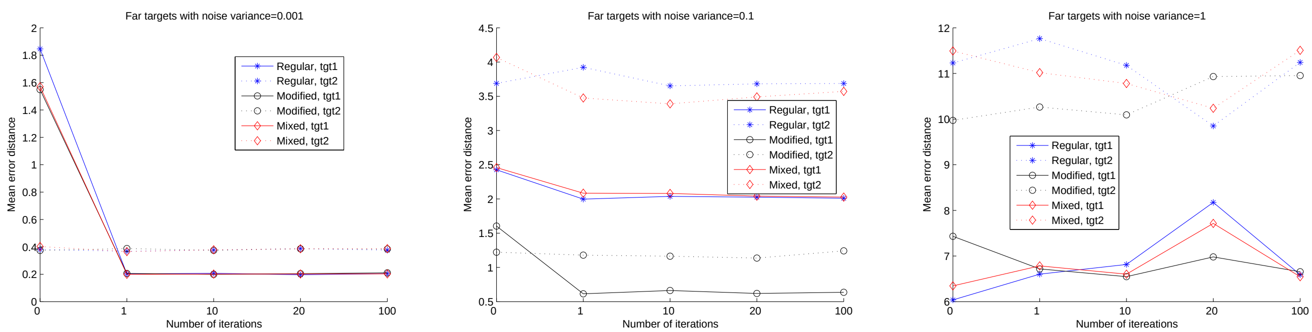
$$\frac{\partial^2 \ln p}{\partial x^2} = \frac{20}{\sigma^2 \ln 10} \sum \underbrace{\left[\frac{\partial}{\partial x} \left(\frac{P_x}{g_n} \right) \right]}_{\mathcal{G}}. \tag{28}$$



(a) Enlarged figure of Fig. 21(a)

(b) Enlarged figure of Fig. 21(b)

Fig. 22. Enlarged figures of Fig. 21.



(a) Noise variance is 0.001.

(b) Noise variance is 0.1

(c) Noise variance is 1.

Fig. 23. Mean error distances of 3 methods with different number of iterations, distant targets, and 1000 runs.

From $\mathcal{G}$,

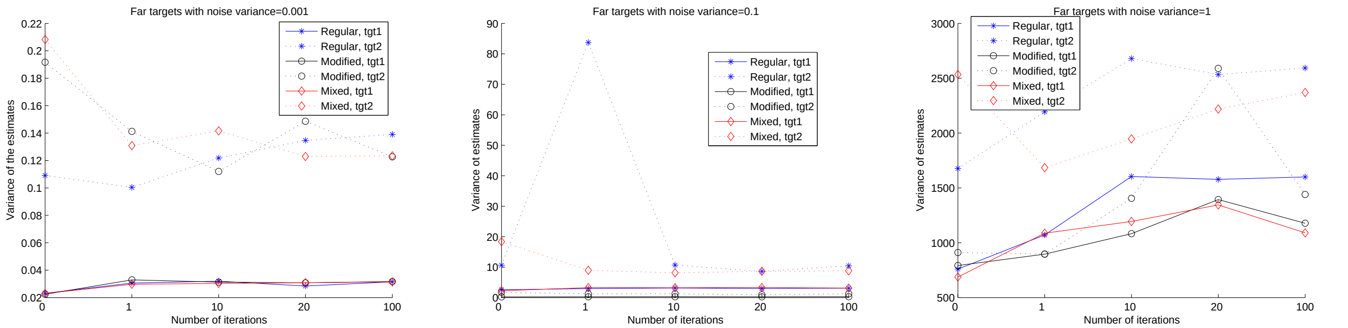
$$\mathcal{G} = \frac{\partial}{\partial x}\left(\frac{P_x}{g_n}\right) = \frac{{P_x}' g_n - P_x {g_n}'}{{g_n}^2} \tag{29}$$

where

$$\begin{aligned} {P_x}' &= -(y_n - f_n) - (s_{n_x} - x){f_n}' && (30)\\ {f_n}' &= \frac{20}{\ln 10}\,\frac{(s_{n_x} - x)}{g_n} \quad \text{from } \mathcal{B}, && (31)\\ {g_n}' &= -2(s_{n_x} - x) && (32) \end{aligned}$$

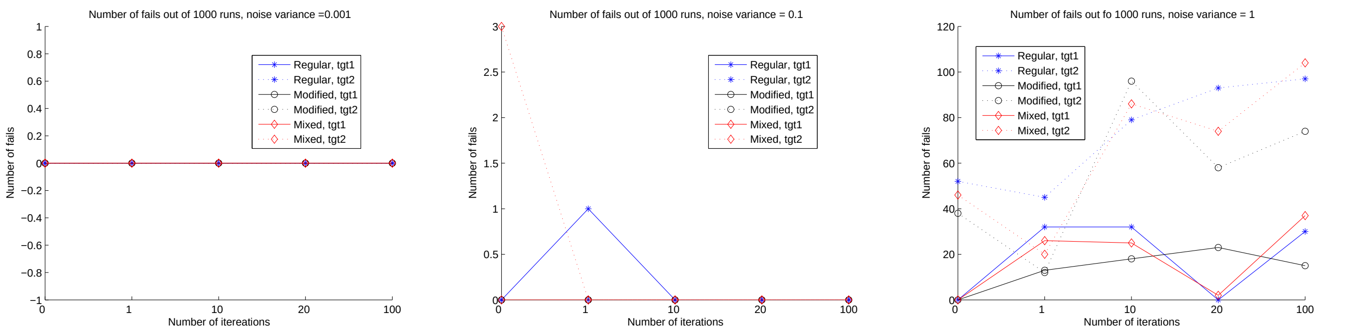
then

$$\begin{aligned} {P_x}' &= -(y_n - f_n) - (s_{n_x} - x)\left(\frac{20}{\ln 10}\cdot\frac{(s_{n_x} - x)}{g_n}\right) && (33)\\ &= -(y_n - f_n) - \frac{20}{\ln 10}\cdot\frac{(s_{n_x} - x)^2}{g_n}. && (34) \end{aligned}$$



(a) Noise variance is 0.001. (b) Noise variance is 0.1. (c) Noise variance is 1.

Fig. 24. Variances of the estimates of 3 methods with different number of iterations, distant targets, and 1000 runs.



(a) Noise variance is 0.001. (b) Noise variance is 0.1. (c) Noise variance is 1.

Fig. 25. The number of fails of 3 methods with different number of iterations, distant targets, and 1000 runs.

Plugging ${P_x}'$ into $\mathcal{G}$,

$$\mathcal{G} = \frac{-(y_n - f_n)\cdot g_n - \frac{20}{\ln 10}\cdot(s_{n_x} - x)^2 + 2(s_n - x)(y_n - f_n)(s_{n_x} - x)}{{g_n}^2} \tag{35}$$

$$= \frac{2(s_n - x)^2(y_n - f_n) - (y_n - f_n)\cdot g_n - \frac{20}{\ln 10}\cdot(s_{n_x} - x)^2}{{g_n}^2}. \tag{36}$$

Plugging $\mathcal{G}$ into (28),

$$\frac{\partial^2 \ln p}{\partial x^2} = \frac{20}{\sigma^2 \ln 10}\sum\left[\frac{2(s_n - x)^2(y_n - f_n) - (y_n - f_n)\cdot g_n - \frac{20}{\ln 10}\cdot(s_{n_x} - x)^2}{{g_n}^2}\right]. \tag{37}$$

Similarly, we can drive

$$\frac{\partial^2 \ln p}{\partial y^2} = \frac{20}{\sigma^2 \ln 10}\sum\left[\frac{2(s_n - y)^2(y_n - f_n) - (y_n - f_n)\cdot g_n - \frac{20}{\ln 10}\cdot(s_{n_y} - y)^2}{{g_n}^2}\right]. \tag{38}$$

To completely find the elements of the Fisher information matrix, we have to find

$$\frac{\partial^2 \ln p}{\partial y \partial x} = \frac{\partial}{\partial y}\left\{\frac{20}{\sigma^2 \ln 10}\sum\left[\frac{(y_n - f_n)(s_{n_x} - x)}{g_n}\right]\right\} \tag{39}$$

$$= \frac{20}{\sigma^2 \ln 10}\sum\left\{\frac{\partial}{\partial y}\left[\frac{(y_n - f_n)(s_{n_x} - x)}{g_n}\right]\right\} \tag{40}$$

$$= \frac{20}{\sigma^2 \ln 10}\sum\underbrace{\left[\frac{\partial}{\partial y}\left(\frac{P_x}{g_n}\right)\right]}_{Q_n}. \tag{41}$$

$$\frac{\partial Q_n}{\partial y} = \frac{{P_x}' g_n - P_x {g_n}'}{{g_n}^2} = \frac{2(s_{n_x} - x)(s_{n_y} - y)(y_n - f_n)}{{g_n}^2} - \frac{\frac{20}{\ln 10}(s_{n_x} - x)(s_{n_y} - y)}{{g_n}^2} \tag{42}$$

where $\frac{\partial g_n}{\partial y} = -2(s_{n_y} - y)$, $\frac{\partial f_n}{\partial y} = \frac{20}{\ln 10}\frac{(s_{n_x}-x)(s_{n_y}-y)}{{g_n}^2}$. Therefore

$$\frac{\partial^2 \ln p}{\partial y \partial x} = \frac{20}{\sigma^2 \ln 10}\sum\left[\frac{2(s_{n_x} - x)(s_{n_y} - y)(y_n - f_n) - \frac{20}{\ln 10}(s_{n_x} - x)(s_{ny} - y)}{{g_n}^2}\right]. \tag{43}$$

Similarly,

$$\frac{\partial^2 \ln p}{\partial x \partial y} = \frac{20}{\sigma^2 \ln 10}\sum\left[\frac{2(s_{n_x} - x)(s_{n_y} - y)(y_n - f_n) - \frac{20}{\ln 10}(s_{n_x} - x)(s_{ny} - y)}{{g_n}^2}\right]. \tag{44}$$

To find the Fisher information matrix,

$$\mathbf{I}(\boldsymbol{\theta}) = \begin{bmatrix} -E\left(\frac{\partial^2 \ln p}{\partial x^2}\right) & -E\left(\frac{\partial^2 \ln p}{\partial x \partial y}\right) \\ -E\left(\frac{\partial^2 \ln p}{\partial y \partial x}\right) & -E\left(\frac{\partial^2 \ln p}{\partial y^2}\right) \end{bmatrix} = \begin{bmatrix} -E\left(\frac{\partial^2 \ln p}{\partial x^2}\right) & 0 \\ 0 & -E\left(\frac{\partial^2 \ln p}{\partial y^2}\right) \end{bmatrix}, \tag{45}$$

note [expectation of f_n] $= y_n$, and from (37) and (43), we can compute

$$E\left(\frac{\partial^2 \ln p}{\partial y \partial x}\right) = \frac{20}{\sigma^2 \ln 10}\sum\left[\frac{-\frac{20}{\ln 10}\cdot(s_{n_x} - x)(s_{n_y} - y)}{{g_n}^2}\right]$$

$$= -\left(\frac{20}{\sigma \ln 10}\right)^2\sum\left[\frac{(s_{n_x} - x)(s_{n_y} - y)}{{g_n}^2}\right]. \tag{46}$$

Similarly,

$$E\left(\frac{\partial^2 \ln p}{\partial y \partial x}\right) = -\left(\frac{20}{\sigma \ln 10}\right)^2\sum\left[\frac{(s_{n_x} - x)(s_{n_y} - y)}{{g_n}^2}\right]. \tag{47}$$

$$E\left(\frac{\partial^2 \ln p}{\partial x^2}\right) = \frac{20}{\sigma^2 \ln 10}\sum\left[\frac{-\frac{20}{\ln 10}\cdot(s_{n_x} - x)^2}{{g_n}^2}\right] = -\left(\frac{20}{\sigma \ln 10}\right)^2\sum\left[\frac{(s_{n_x} - x)^2}{{g_n}^2}\right]. \tag{48}$$

Similarly,

$$E\left(\frac{\partial^2 \ln p}{\partial y^2}\right) = \frac{20}{\sigma^2 \ln 10}\sum\left[\frac{-\frac{20}{\ln 10}\cdot(s_{n_y} - y)^2}{{g_n}^2}\right] = -\left(\frac{20}{\sigma \ln 10}\right)^2\sum\left[\frac{(s_{n_y} - y)^2}{{g_n}^2}\right]. \tag{49}$$

Therefore,

$$\mathbf{I}(\boldsymbol{\theta}) = \left(\frac{20}{\sigma \ln 10}\right)^2 \cdot \begin{bmatrix} \sum\left[\frac{(s_{n_x}-x)^2}{{g_n}^2}\right] & \sum\left[\frac{(s_{n_x}-x)(s_{n_y}-y)}{{g_n}^2}\right] \\ \sum\left[\frac{(s_{n_x}-x)(s_{n_y}-y)}{{g_n}^2}\right] & \sum\left[\frac{(s_{n_y}-y)^2}{{g_n}^2}\right] \end{bmatrix} \triangleq \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \tag{50}$$

then

$$\mathbf{I}^{-1}(\boldsymbol{\theta}) = \frac{1}{ab}\begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix}. \tag{51}$$

Since

$$var\left(\hat{\theta}_i\right) \geq \left[\mathbf{I}^{-1}\left(\boldsymbol{\theta}\right)\right]_{ii} \tag{52}$$

$$var(\hat{x}) \geq \frac{1}{a} = \frac{\left(\frac{\sigma \ln 10}{20}\right)^2 \cdot \sum\left[\frac{(s_{n_y}-y)^2}{{g_n}^2}\right]}{\sum\left[\frac{(s_{n_x}-x)^2}{{g_n}^2}\right]\sum\left[\frac{(s_{n_y}-y)^2}{{g_n}^2}\right] - \left\{\sum\left[\frac{(s_{n_x}-x)(s_{n_y}-y)}{{g_n}^2}\right]\right\}^2} \tag{53}$$

$$var(\hat{y}) \geq \frac{1}{b} = \frac{\left(\frac{\sigma \ln 10}{20}\right)^2 \cdot \sum\left[\frac{(s_{n_x}-x)^2}{{g_n}^2}\right]}{\sum\left[\frac{(s_{n_x}-x)^2}{{g_n}^2}\right]\sum\left[\frac{(s_{n_y}-y)^2}{{g_n}^2}\right] - \left\{\sum\left[\frac{(s_{n_x}-x)(s_{n_y}-y)}{{g_n}^2}\right]\right\}^2} \qquad \square \tag{54}$$

Fig. 26 shows the Cramer-Rao (CRB) bound as the number of sensors increases. The sensors are added up from the close to distant one when the number of sensors increases. When there is only one single sensor, the Fishier information matrix becomes singular (see (50)). In that case we have to approach by other method to find CRB [35], [36]. We have to use more than 1 sensors to apply lateration in this paper, therefore, we do not discuss about that problem since it is beyond the scope of this paper.

Fig. 27 shows the surface CRB when 2, 3, and 4 sensors are used respectively. Fig. 28 shows the surface CRB of whole plane when 16 grid sensors are used. Note that when we use two sensors, according to (50), Fisher information matrix, CRB of the center point of the two sensors is infinity because denominator is 0. But, it is not shown in the figure because the surface plot is with respect to the grid point which does not include that center point in Fig. 27(a).

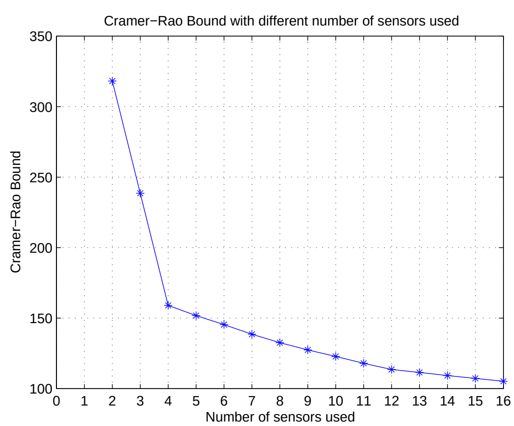
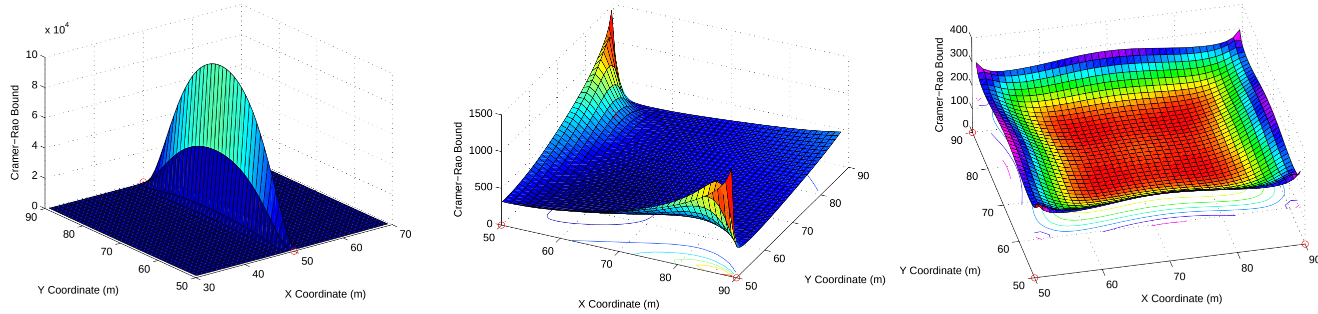


Fig. 26. Cramer-Rao bound as the number of sensors increases from 1 to 16.

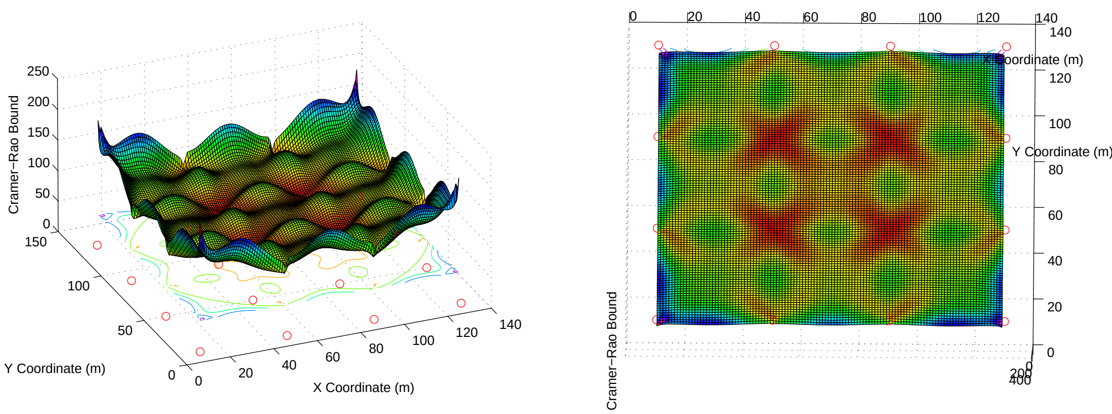


(a) Using 2 sensors .

(b) Using 3 sensors.

(c) Using 4 sensors.

Fig. 27. Surface plot of Cramer-Rao lower bound for different number of sensors used. Red circles show the sensors.



(a) Whole surface using 16 grid sensors .

(b) View from the top of (a).

Fig. 28. Surface plot of Cramer-Rao lower bound of whole plane with 16 grid sensors. Red circles show the sensors.

VIII. Conclusions

Initializations of the targets in variable, multiple target tracking system is a very important problem especially in non-one to one mapping model between “the target states” and “the measurement”, such as RSS sensor model (see (1)), in wireless sensor network system. We need cooperative combinational complex measurement to estimate a single state of the target. To overcome this difficulty, we adopted the principle of the “least squares” method and developed it to more methods which are more useful in certain situations depending upon the problems (those three methods is regular, modified, and mixed lateration), which are combined with “particle filtering”, the most recent statistical parameter estimation solution for non-linear and non Gaussian model. The regular lateration shows better performances than the modified lateration when initializing a single target and two very close targets at the same time while modified lateration shows better performances than regular or mixed lateration in initializing two distant targets simultaneously. We also applied iterative method for initializing more than one target simultaneously taking advantage of the residue cancelation lateration (RCL, see Section. III-B and Table. I). Iterative method takes very essential and critical role in adjusting and reducing the error of the firstly initialized target in initializing two targets simultaneously, especially with modified lateration. RCL is the key for the initialization of the multiple target tracking solution. The approach we present in the paper can be also applied to the localization of the sensors in ad-hoc wireless sensor network system.

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