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ON ELECTRODYNAMIC MODELING OF PLANETARY ATMOSPHERE

by

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Introduction

Recent geophysical modeling experiments have been quantitatively successful. It is impressive that this is true for the extremely large scale phenomena involved because modeling scales as small as 10^{-7} and smaller are necessary. The attainment of success can be closely correlated with the increasing attention paid to similarity requirements between prototype and model phenomena[1]. For continuum flows the application of the governing equations to the modeled phenomenon facilitates the definition of similarity variables and permits the selection of suitable measureable flow characteristics.

As might be suspected, complete similarity between model and prototype at such scales is not possible. Success with modeling techniques so far has seemed to depend on the existence in the prototype of one or more dominant force ratios which can be modeled. Examples for tropospheric flow are the Rossby number (the ratio of inertia to coriolis forces) and the Richardson number (the ratio of free convection energy transport to vertical momentum transport by horizontal viscous shear).

The techniques developed to date have their applicability mainly to the troposphere, the oceans, and to some large scale electromagnetic and hydromagnetic phenomena. Attempts to model the stratosphere are unknown to this author. However, it would appear that sufficient prototype information has accumulated[2] to encourage undertaking the design of such a modeling experiment. Incomplete knowledge of ionospheric flow seems, at this time, to place it rather far from the possibility of realistic total modeling. Nevertheless, considerable work has been done on the laboratory reproduction of isolated ionospheric phenomena (for example, references 5 and 12).

The present work concerns itself with the possibility of modeling planetary atmospheres (including planets other than Earth) by trapping an electrically charged gas in a sphere centered electric field (Fig. 1). Mechanical energy may be imparted to the gas by spinning the sphere about a polar axis. Thermal energy may be added by electromagnetic radiation of the gas as well as by thermal conduction between the bounding surface and the gas. It may be that the use of a disk, a cylinder, or a sphere segment rather than a sphere will simplify the experimentation. The ideas set down here apply qualitatively to all four configurations.

Field Modeling

A first consideration is modeling the gravitational field by an electrostatic field. A comparison of the field equations in the two cases will shed light on this situation.

The calculation of the conservative force field due to a continuous distribution of charge or mass leads to the well known result that at a point in space

$$\varphi(p) = k\int_V \frac{\delta\, dv}{r} \tag{1}$$

where δ ~ mass or charge density;

v ~ volume;

r ~ radius vector from origin of coordinates;

φ ~ potential function;

k ~ constant of proportionality

From this follow field equations for gravitational and for electrostatic fields.

For the gravitational field

$$\nabla\cdot\vec{G} = 4\pi\gamma\rho \tag{2}$$

$$\vec{G} = -\nabla U \tag{3}$$

$$\nabla^2 U = -4\pi\gamma\rho \tag{4}$$

and for the electrostatic field

$$\nabla\cdot\vec{E} = \rho_e/\epsilon \tag{5}$$

$$\vec{E} = -\nabla V \tag{6}$$

$$\nabla^2 V = -\rho_e/\epsilon \tag{7}$$

where $\gamma = 6.7 \times 10^{-11}\ \frac{\text{newton-m}^2}{\text{kg}^2}$; $\epsilon_v = \frac{1}{36\pi \times 10^9}\ ^{f}/_{m}$

ρ ~ mass density, kg/m^3 ; ρ_e ~ charge density, $coul/m^3$

G ~ $newt/kg$; E ~ $newt/coul$

in the MKS system.

For a homogeneous planet of mass density ρ_p surrounded by an atmosphere of mass density ρ_a (also assumed homogeneous), it is easy to show that

$$\frac{aG(r)}{aG(r_o)} = \left(\frac{r_o}{r}\right)^2 \left[1 + \overset{1}{\frac{\rho_a}{\rho_p}\left(\frac{r^3}{r_o^3} - 1\right)}\right] ; \quad (1 \leq r/r_o) \tag{8}$$

where aG(r) is the field outside the planet sphere (i.e., in the "atmosphere") and $aG(r_o)$ is the field at the planet surface where $r = r_o$. Normally, $\rho_a/\rho_p << 1$ and term 1 above can be neglected.

Similarly, for a homogeneous sphere of charge of density ρ_{em} ("m" for model) surrounded by a homogeneous atmosphere of charges of density ρ_{ea},

$$\frac{aE(r)}{aE(r_o)} = \left(\frac{r_o}{r}\right)^2 \left[1 + \frac{\rho_{ea}}{\rho_{em}}\left(\frac{r^3}{r_o^3} - 1\right)\right] ; \quad (1 \leq r/r_o) \tag{9}$$

for the electric field. Comparison of equations (8) and (9) shows the variations in field to be qualitatively the same and that the fields are identical if it is possible to establish identical density ratios at each point of the dimensionless space. Alternatively, since normally $\rho_a/\rho_p << 1$ and neglected; if $\rho_{ea}/\rho_{em} << 1$, the field distributions will be identical.

An extension of this reasoning to the comparison of potential functions in the two cases can be made. If the specification of the additional boundary condition can be made physically compatible for the two cases, congruence of the two distributions in dimensionless representation can be established just as for equations (8) and (9).

In summary, similarity of field and potential distributions exists for homogeneous atmospheres, identical charge and mass ratios, and physically compatible specification of an outer boundary condition. It is unlikely that these conditions will be completely met in the experiment. For example, it is probable that not $\rho_{ea}/\rho_{em} << 1$ because of the practical limitations on the voltage field due to the nature of the electrostatic force field in this application. Also, the assumption of a homogeneous atmosphere is justified in neither case. Nevertheless, it is probable that a near enough approach to these conditions may be made to ensure a practical degree of similarity in the fields. A more sophisticated analysis together with design calculations and experiments will be required to settle this question.

State of the Gas in Relation to Field

Given a conservative field of force, there exists a potential function such that $\vec{F} = -\nabla\varphi$. We consider the case of a gaseous atmosphere in static equilibrium under the influence of such a force field. It can be shown that isopotential surfaces correspond to constant pressure in such a static atmosphere. Thus

$$p = f(\varphi) \tag{10}$$

pressure is a function only of potential.

Furthermore, considerations of static equilibrium yield

$$\frac{dp}{dr} = -\rho(r)Fr(r) \tag{11}$$

for such a field. Fr(r) is the r-component of the field force. Here we permit variations of density to occur. Since,

$$Fr(r) = -\frac{d\varphi}{dr} \tag{12}$$

we obtain that

$$v(r) = \frac{1}{\rho(r)} = \frac{1}{f'(\varphi)} \frac{d\varphi}{dr} \tag{13}$$

or, the specific volume is, likewise, a function only of the force potential. Therefore, if the state equation for the atmosphere is of form

$$f(p,v,T) = 0 \tag{14}$$

specification of the field potential determines the thermodynamic state of the gas. The reasoning obviously applies equally well to electric and gravitational potentials.

In summary, the variation with altitude of the static state of the gas depends solely on the potential field of the sphere under the conditions

i) a conservative field of force;

ii) surface normal forces;

iii) any state equation of form equation (14).

Such an equation of state may be assumed to describe most neutral gas atmospheres. However, gas in the presence of external fields may present a more complicated physical situation.

Using the methods of macroscopic thermostatics, Chu[3] obtains

$$p = \overset{(1)}{\rho RT} + \frac{1}{2}\left[\vec{B}\cdot\vec{H} + \vec{E}\cdot\vec{D}\right] - \frac{1}{2}\rho\left[H^2\frac{\partial\mu}{\partial\rho} + E^2\frac{\partial\epsilon}{\partial\rho}\right] \tag{15}$$

the equation of state for an ideal gas, where

(1) ~ mechanical contribution for ideal gas;

(2) ~ "field" pressure;

(3) ~ electro-magnetostriction effect.

For the present case, in the absence of externally imposed magnetic fields and neglecting magnetic induction,

$$p = \rho RT + \tfrac{1}{2}\vec{E}\cdot\vec{D} - \tfrac{1}{2}\rho E^2\frac{\partial\epsilon}{\partial\rho} \tag{16}$$

where

$\vec{D}$ ~ displacement density, coul/m^2

ρ ~ mass density, kg/$_{m}3$

ϵ ~ gas dielectric coefficient, farads/$_m$

As the electric field approaches zero, equation (16) reduces to the equation of state for an ideal gas. Thus, direct application of hydrostatic analyses to the electrostatically modeled atmosphere implies neglect of the electrostrictive and field contributions to the pressure. For ϵ = constant and for isotropic media,

$$p = \rho RT + \frac{\epsilon}{2}E^2 \tag{17}$$

The former assumption is usually made in MHD applications[4], and justification of neglect of the field pressure will depend on

$$\frac{\epsilon}{2}\,\frac{E^2}{\rho RT} << 1$$

The practical circumstances under which this neglect is justified will have to receive further consideration.

Dynamic Modeling and Similarity

Dynamic modeling of momentum flow in a fluid field implies that the ratio of forces acting on a fluid element have the same value in the prototype and the model at corresponding positions and times in the dimensionless field. Energy flow modeling requires that the ratio of fluxes to or from a fluid element correspond in the same way. If, when dimensionlessly presented, the model field and the prototype field are congruent, dynamic similarity has been achieved.

Rarely can complete similarity be achieved by modeling of any complex fluid mechanical situation. However, in any given phenomenon two or three forces and/or fluxes will usually dominate. Experience has shown that it will generally suffice to match only these dominant ratios in model and prototype. Furthermore, it may prove unnecessary to model even these exactly if the theory is sufficiently well developed to permit extrapolation of experimental results. A common example is found in external aerodynamics where exact modeling of Reynolds number is rarely achieved. It is necessary, however, to model the airplane flow regime (laminar or turbulent) by approximating the Reynolds number range. Theoretical extrapolations may then be used to obtain predictions of prototype fluid forces.

Thus, the real power of modeling lies in its exercise as a "scientific art" wherein observation and analysis are combined to bring about advances in physical understanding even in the absence of exact modeling or sophisticated theories. Atmospheric modeling at its present stage of development is such an art.

The question which interests us is whether dynamic modeling of macroscopic motions of a planetary atmosphere by an ion gas held in a central force field on a rotating sphere is: 1) conceptually sound; 2) a practical possibility in the sense described above. We approach the answers to these questions by considering physical aspects of the equations of motion in general form.

For a single specie gas

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \overline{V} = 0 \tag{18}$$

mass conservation, and

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \rho_e \overline{V} = 0 \tag{19}$$

charge conservation; where

$$\rho = nm; \; \rho_e = nz_i e$$

; and, for a singly ionized gas,

$$\frac{\rho}{\rho_e} = \frac{m}{e} \tag{20}$$

the inverse of the specific charge. The momentum equation is written

$$\rho\frac{D\overline{V}}{Dt} = -\nabla p + \nabla\cdot\overline{\tau} + \underset{3}{\overline{J} \times \overline{B}} + \rho_e\overline{E} + \underset{5}{\rho\overline{G}} \tag{21}$$

where the pressure includes mechanical and electromagnetic contributions (equation 15). $\overline{\tau}$ is a fluid stress tensor excluding pressure; J is current density coul/m^2-sec; $\overline{B}$ is magnetic flux density, newt-sec/coul-m^2. We consider terms (3) and (5) to be negligible. In the absence of an externally imposed magnetic field (3) = 0 if magnetic induction is negligible. Term (5) can be neglected if the electrostatic force is sufficiently large relative to the gravitational force. This can easily be checked for the practical cases. As implied by equation (17), electrostrictive effects are neglected. Assuming these approximations for the moment, we proceed to the determination of dynamic similarity parameters.

In thin fluid layers of the character of atmospheres and oceans, application of the so-called boundary layer approximations would appear reasonable. The determination of similarity parameters from the boundary layer equations is straight forward and details of the method may be found in many textbooks.

Continuity, for this case, may be expressed

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \tag{22}$$

where u, v, w, x, y, z are the velocities and displacements respectively in a tangent plane coordinate system, and ρ is the mass density.

The x-component of the momentum equation becomes

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} \qquad (23)$$

$f \sim$ coriolis parameter, and

$$f = 2\Omega \sin \varphi$$

where Ω is the angular velocity and φ is the latitude.

Equations (22) and (23) may be made dimensionless by transforming in terms of the variables:

$$t^* = \Omega t; \quad u_i^* = \frac{u_i}{U}; \quad x_i^* = \frac{x}{a}; \quad p^* = \frac{p}{\rho_o U^2}; \quad \rho^* = \frac{\rho}{\rho_o}$$

U is a representative velocity relative to the rotating coordinate frame; a, the characteristic length, is taken to be the sphere radius. It results that

$$R_e = \frac{Ua}{\nu} \sim \frac{\text{inertia force}}{\text{viscous force}} \qquad \text{and} \qquad R_o = \frac{U}{a\Omega} \sim \frac{\text{inertia}}{\text{coriolis}}$$

are the modeling parameters for surface parallel motions in the thin boundary layer model.

For hydrostatic modeling, the surface normal component of momentum is

$$0 = -G(z) - \frac{1}{\rho}\frac{\partial p}{\partial z};$$ and, dimensionlessly,

$$\left[\frac{aG(o)}{U^2}\right] G^*(z) = -\frac{1}{\rho^*}\frac{\partial p^*}{\partial z^*} \qquad (24)$$

the electric body force for a non-conducting gas. For the case of a homogeneous non-neutral gas (as previously postulated) the last two terms are of no importance and buoyancy results from local perturbations of charge density, or field, or both. For the case of a neutral gaseous dielectric of non-zero susceptibility, the first term on the right vanishes and buoyancy depends on non-uniformity of field and properties. These electrostrictive forces have already been shown to be of significant practical importance in natural thermal convection[6].

It was previously indicated (equations (24) and (25)) that hydrostatic modeling of thin fluid layers required hydrodynamic and electrostatic Froude number matching. When vertical motions are admitted, however, the dominant forces become buoyancy and shear. This force ratio is expressed by the Richardson number

$$R_i = \frac{\beta\theta_s g\delta}{U^2} \tag{29}$$

commonly used in meteorological applications. This can be rewritten as

$$R_i = \frac{\beta\theta_s g\delta^3}{\nu^2} \cdot \left[\frac{\nu}{U\delta}\right]^2 = Gr \cdot R_e^{-2} \tag{30}$$

in terms of the Grashoff and Reynolds numbers commonly encountered in heat transfer literature. For these equations

$\beta = \frac{1}{T}$, the thermal coefficient of expansion;

$\theta_s = T_s - T$, the temperature difference locally between an element of fluid and the heated surface

$\delta \sim$ fluid layer thickness

Where the last two terms of equation (28) dominate, it has been shown[6] that the appropriate natural convection modeling parameter is

$$S_e = \frac{\beta \theta_s \gamma E_s^2 \delta^2}{v^2} \tag{31}$$

where $\gamma \sim$ expresses the temperature dependence of ϵ; $E_s \sim$ is the electric field at the model surface. For the case of a charged homogeneous gas, it can be shown that if gravitational effects are not negligible relative to electrostatic effects

$$F = F_G + F_e = \rho\left[\beta\theta_s g + \beta_e'(\Delta E)E\right] + \rho_e \Delta E \tag{32}$$

where

$$\beta_e' = \frac{1}{E}\left[\frac{2}{(2p/\epsilon E^2) - 1}\right]$$

and that, therefore, the additional dimensionless parameters should be of form $\frac{\rho\beta_e(\Delta E)\delta^3}{v^2}$ and $\left(\frac{e}{m}\right)\frac{E\delta^2}{v^2}$, where $\beta_e \equiv E\beta_e'$ for convenience.

The foregoing considerations indicate that where the thin fluid layer model is acceptable, Reynolds number, Rossby number and Froude number (gravitational and electrostatic) are the appropriate modeling parameters. Experience with hydrodynamic atmospheric modeling has indicated that Rossby number modeling is probably of greater importance than Reynolds and Froude number modeling for surface parallel motions. The importance of the electrostatic field in the proposed gas dynamical modeling technique is simply to hold the gas atmosphere to the model surface.

If a "thick" atmosphere model is chosen, the indication is that the modeling parameters are those commonly used in natural convection heat transfer; namely, Grashoff number (gravitational and electrostatic), Reynolds number, Rossby number, and Nusselts number. The first two of these parameters are combined to form the Richardson number commonly used in meteorological applications. It can be anticipated that Grashoff and Rossby number modeling will be of dominant importance. For this model the electric field technique offers advantages in increased versatility of the experimental approach.

The consideration of these two separate approaches exemplifies the necessity for compromising the demands for complete similarity. Such compromises may be based on possible practical limitations of size and physical equipment or on instrumentation, or other factors. The intent also is to indicate that with some ingenuity, practical difficulties can be overcome or, in some cases, even turned to advantage through placing emphasis on modeling one or another aspect of the prototype flows. It seems clear that modeling of planetary atmospheres by this approach is conceptually sound. Whether it is or not a practical possibility depends on numbers required and materials and techniques available. If practical, it has the advantage over other techniques to date of permitting greater freedom in body force modeling where vertical motions dominate. In addition, the tilt of the vorticity vector may be faithfully modeled where surface motions are of dominant importance.

What we have discussed is a representative first assessment of the problem. It should not be expected that the results presented are in any sense final, since several different experiments appear possible and the choice of modeling techniques and configurations will inevitably interact with the modeling analysis.

Laboratory Model of the Planetary Atmosphere

In considering possible techniques, the laboratory problem of producing and trapping a gaseous atmosphere in an electrostatic field must receive first consideration. At least four different approaches present themselves. They are the use of:

1) an electrically susceptible neutral gas on a slender rotating cylinder;
2) a low density plasma between counter rotating cylinders with continuous radio frequency excitation;
3) a test chamber with sphere, sphere segment, or disk model and an ion gas;
4) a thermionic emitting model surface with a mass enhanced charged atmosphere;
5) combinations and permutations of 1) through 4).

The first two of these techniques would rely on electrostrictive forces to alter gas body forces due to gravity or centrifugal mechanical fields. With the latter two, it should be possible to produce body forces of such magnitude that gravity and other mechanical effects can be ignored altogether. Since large fields and field gradients are entailed in these latter instances, it may prove of practical importance to investigate combinations of all these techniques.

Influence of the electrostrictive body force on natural thermal convection was first investigated by Senftleben and Braun[7]. They succeeded in showing experimentally that applying an electric field to a paraelectric gas surrounding a heated horizontal wire can produce an increase in heat rate up to 50%. This effect is due to the change in thermal circulation produced by the electrostrictive forces. Lykoudis and Yu formulated the dimensionless Senftleben number (equation (31)) and an empirical correlation which implies dominance of electrostrictive forces over gravitational forces under conditions beyond the range of presently existing experiments. Whether these can in fact be attained will depend on practical limitations such as breakdown potential of the gases. Nevertheless, this phenomenon offers an approach to increasing the range of body force and flux ratios regardless of the ultimate goal of attaining dominant non-gravitational body forces. This latter, after all, may or not be necessary depending on developments of the investigation.

A second possibility is the radio frequency excited, low density, low temperature plasma between counter rotating cylinders. The techniques for producing and probing such plasmas have been developed[11]. The application to thermally driven flows with radial force fields has not been attempted to the writer's knowledge. As a matter of fact, the neutrality of this atmosphere and the necessity for continuous excitation would appear to be strong deterrents against its successful application to the present problem. Nevertheless, it should receive study if only as a preliminary to more promising approaches since the experimental techniques will probably prove to be of value for the charged gas as well.

The possibility of production and trapping of a charged gas on a dielectric (or thermionic*) surface in an evacuated chamber is conceptually very attractive as was pointed out previously. In practice, an approach not unlike that used by Birkeland[5] in his Terella Experiments appears logical. The geometry would change and the magnitudes of applied fields and vacua would differ, but, in essence, the ground that he covered could be recovered if the emphasis were placed on magnetic rather than electric phenomena.

A first approach to the design configuration of such a chamber is shown schematically as Figure 2. The ionized gas is discharged tangentially into the chamber and charge separation is effected as the positively charged particles spiral into the dielectric coated surface of the model and the electrons are induced to the wall and conducted away. It is expected that establishing the atmosphere of charges about the sphere model could proceed at low rates and under continuous evacuation of the chamber. Thus, an order of magnitude difference in mass density at the surface and at the chamber boundary would exist. It would be expected to establish the field such that $h/R \ll 1$ (see Figure 1).

Item (4) above represents a different approach to the production of the charged atmosphere of item (3). A thermionic surface would be used to produce an electron atmosphere and a strong negative bias would hold the charge cloud at the cathode

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* A configuration suggested by R. W. Glasheen.

surface. Since both equation (26) and fluid mechanical common sense indicate the desirability of increasing the mass to charge ratio, a neutral dust of micron size would be added to the charge cloud. The technique has already been developed in a different application[10]. This configuration, if successful, would have the advantage of eliminating the necessity for plasma input and charge separation. Obvious disadvantages are the necessity for cathode heating and its possible attendant complications so far as rotation of the sphere is concerned as well as possible additional difficulty with probing the atmosphere.

Experimental Techniques

It is hoped that the foregoing comments provide a clear enough picture of proposed practical approaches to producing an atmospheric model. In what follows, a similarly abbreviated and preliminary approach to the experimental techniques is outlined. The object will be to communicate the concept of the experimental approach without devoting space and time to details of technique many of which remain to be worked out.

Three main aims of the experimental design program are: providing for mechanical and thermal energy addition; for addition of "pollutants"; and for instrumentation.

Mechanical energy addition would be by rotation of the "sting" which supports the model at the chamber center. Thermal energy addition could be by conduction from the model surface, by selective irradiation through metalized glass ports in the chamber walls, or by both. Simulation of thermal effects by local charge density perturbation is possible through neutralization by electron beam, charge dilution by addition of dust, or by local variations of the external field (for example, through dielectric layer inhomogeneities). Externally induced field oscillations will produce mhd waves in the atmosphere.

It will be of interest to study the dispersal of clouds of pollutant in the model atmosphere. Possibilities are dust clouds of neutral particles (charge diluent) or dust clouds of radioactive particles.

The consideration of similarity indicates that local measurements of velocity, pressure, temperature, and field are desirable for detailed flow studies. Experience shows that practical considerations will probably limit some of the measurements to the boundaries but that adequate data for modeling studies can be obtained despite such limitations. For detection of fluid velocities and for charge density mapping, it would appear that applications of methods associated with particle physics research are in order. Modifications of conventional fluid mechanical pressure and temperature sensors for use in strong fields should provide for those measurements.

As suggested, many of the techniques needed for energy addition to and probing of an atmospheric model already exist and can be applied either directly or with a reasonable degree of modification. Similarly, a reasonable amount of experience with electrostrictive gases and low temperature plasmas has accumulated. However, the author is unaware of previous attempts to model with ion gases as is proposed here. Therefore, a few preliminary calculations were made to explore the gross limitations on such a venture.

The Homogeneous Planetary Atmosphere Model

Referring to Figure 1. and 2., it is assumed that a perfectly conducting model sphere, 20 cm in diameter and with a perfect dielectric surface, is placed at the center of such a chamber. An atmosphere of charges such that $h/R \ll 1$ is established as described in the preceding section and the following questions are asked:

1) can an atmosphere of continuum flow density be established by application of a field of reasonable magnitude;
2) will the gravitational effect be negligible in this range of applied fields;
3) will one-to-one modeling of the gravitational field occur;
4) will consideration of the field effect on the thermodynamic state be required?

For continuum flow, a mean free path of roughly 1/100 the sphere diameter is assumed. For the idealized spherical configuration, Poisson's equation becomes

$$\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = -\frac{\rho e}{\epsilon}\, r^2 \tag{33}$$

where $V \sim$ volts. We assume that the gas is singly ionized argon. We assume, further, that the charge atmosphere is homogeneous. Although this latter assumption is physically unrealistic, it is probable that the calculation of the quantities of interest here will not be qualitatively affected. The boundary conditions are: at $r = r_o$, $V = 0$ and at $r = h, \frac{dV}{dr} = 0$ the latter being set for optimum field strength. A charge atmosphere thickness of 1 cm will be assumed. With this simplification, it results that

$$V(r) = \frac{\rho_e}{3\varepsilon} \cdot \left[h^3 \left[\frac{1}{r_o} - \frac{1}{r} \right] + \frac{1}{2} \left[r_o^2 - r^2 \right] \right] \qquad (34)$$

As examples of limiting cases, it results that for a charge density of one Earth's atmosphere,

$$V(h) = 10^{12} \text{ volts} \qquad (35)$$

which is obviously out of the question; but for a representative imospheric charge density ($n = 10^{10}$ charges/cm^3)

$$V(h) = 3.6 \times 10^4 \text{ volts} \qquad (36)$$

a moderate potential.

However, such an atmospheric density corresponds to mean free paths of 30 meters ... clearly a free molecule flow situation unless a neutral susceptible gas can be mixed with the charges as previously suggested. A mixture ratio of 10,000 neutrals per charge would be required to reduce the mfp to continuum flow magnitudes. This mixture yields an atmosphere of density 10^{20} particles/m^3 with a corresponding mean pressure of about 0.01 mm. Assuming practical working voltages up to 10^6 volts, a relatively wide selection of neutral-to-charge ratios exists. Thus, assuming these field calculations to be conservative as seems reasonable, there exists a range of practical dimensions, voltages, and densities in which modeling is possible if neutrals can be successfully mixed with the charges and held in the field.

The second question deals with the magnitude of gravitational forces relative to electrostatic body forces. From equation (34) the field strength at the sphere surface can be expressed

$$E_o = \frac{\rho e}{3\epsilon}\left[r_o - \frac{h^3}{r_o^2}\right] \tag{37}$$

The electrostatic and gravity body forces are expressed in equation (21). For the case of a 40000 volt applied potential, the charge to mass ratio for equal gravity and coulomb forces is

$$\frac{e}{m} = \frac{1}{7.5 \times 10^6} \quad \frac{\text{coul}}{\text{kg}}$$

For neglect of gravity forces, obviously,

$$\frac{e}{m} >> \frac{1}{7.5 \times 10^6} \quad \frac{\text{coul}}{\text{kg}} \tag{38}$$

For argon, singly ionized,

$$\left.\frac{e}{m}\right]_{\text{argon}} = 2.5 \times 10^7 \quad \frac{\text{coul}}{\text{kg}} \tag{39}$$

for ionospheric density. Therefore, gravity forces are negligible.

Modeling of the gravitational field depends on establishing congruence of equations (8) and (9). For similarity either

$$\frac{\rho_a}{\rho_p} = \frac{\rho_{ea}}{\rho_{em}}$$

or both should be negligible. For earth,

$$\frac{\rho_a}{\rho_b} = 2.56 \times 10^{-5} \approx 0$$

But for $r_o = 10$ cm and at ionospheric density

$$\frac{\rho_{ea}}{\rho_{em}} = 3$$

for a singly ionized gas. Hence, modeling of the gravitational field should not be expected. However, this is not surprising and may prove to be of little importance in view of the nature of the electrostatic body forces.

Finally, it was pointed out that unless $\left(\epsilon E^2/\rho RT \ll\right)1$, the electrical equation of state (equation (17)) will be required to describe the thermodynamic state of the gas. For the present case of ionospheric charge density $\left(\epsilon E^2/\rho RT\right) \approx 2$ and field effects are important as would be expected.

Summary and Conclusion

The electrodynamic approach to planetary atmospheric modeling shows promise of providing four advantages over present techniques: 1) more realistic body force vector modeling; 2) a sufficiently wider range of surface normal similarity parameters to permit model studies of planets other than earth; 3) modeling of the outer boundary conditions $\rho \to 0$, $\frac{\partial \rho}{\partial r} \to 0$, as $r/r_o \gg 1$; and 4) more realistic vorticity vector modeling than with disk or liquid sphere models.

The preliminary calculations indicate that experimental difficulties due to the large field strengths required may be expected. However, it may be possible to overcome these by using an ion gas diluted with a polar neutral gas. In any event, Earth's gravitation effects can easily be made negligible; one-to-one gravitational field modeling will not occur; and an equation of state equivalent to that of equation (17) will be required.

The next steps in a planetary atmospheric modeling program are clearly indicated by the results of the present investigation. They are: 1) a boundary layer study of a gas trapped in a central force field on a rotating heated sphere; 2) a continuing study of the practicability of using a diluted charged gas (or equivalent technique) for reducing the required applied voltages. A laminar boundary layer study at this time should prove of general interest in the area of macroscopic planetary atmospheric motions. Positive results from the second of these two investigations should justify design and construction of a facility for experimental investigations.

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Acknowledgment

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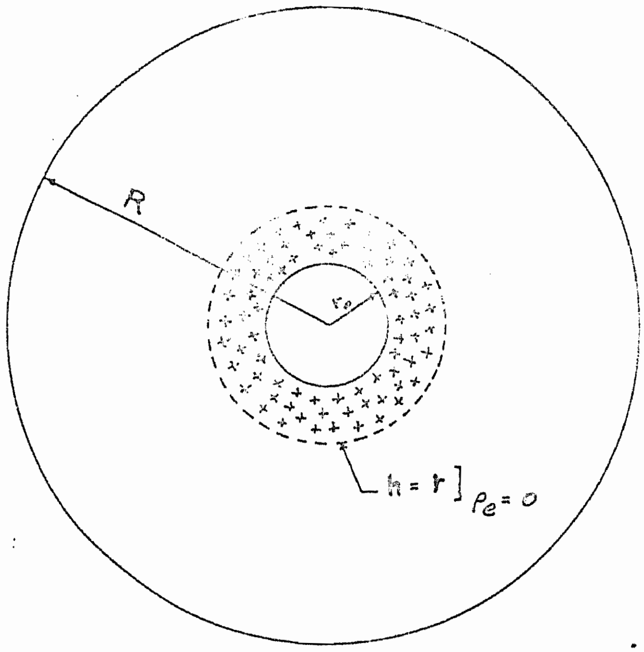
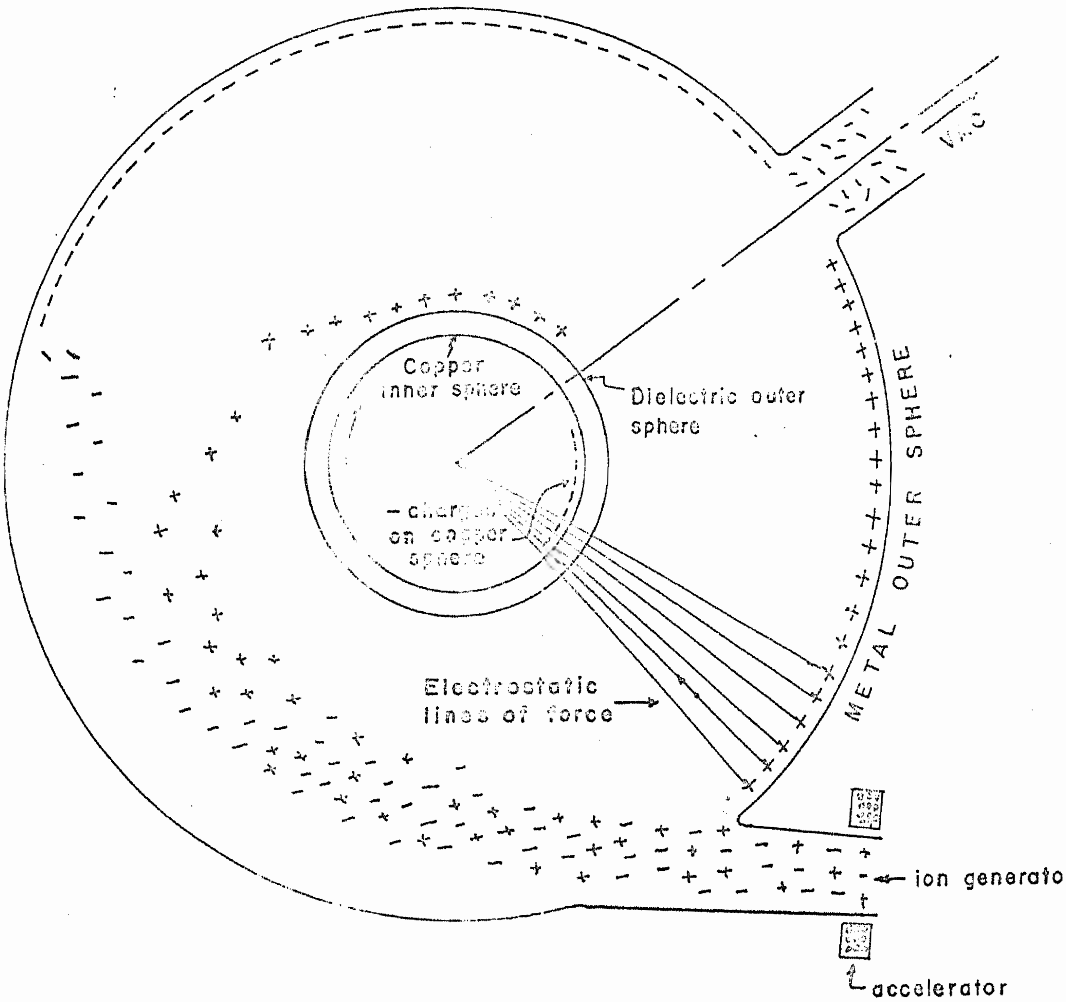


The Homogeneous Charged Gas Atmosphere

Figure 1



Schematic Diagram of Ion Gas Test Chamber

Figure 2