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RADIATION HEAT TRANSFER IN AN
ABSORBING MEDIUM BOUNDED BY A
SPECULAR REFLECTOR

by

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RADIATION HEAT TRANSFER IN AN ABSORBING MEDIUM
BOUNDED BY A SPECULAR REFLECTOR¹⁾

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1. INTRODUCTION

In analyzing radiation heat transfer within a medium which absorbs and emits thermal radiation, a rather large number of assumptions are conventionally employed. One of these is that the bounding surfaces are diffuse reflectors, and the purpose of the present note is to investigate a simple situation involving a bounding surface which reflects in a specular rather than diffuse manner.

The physical model and coordinate system are illustrated in Figure 1, and this consists of two infinite parallel plates separated by an absorbing and emitting medium. The bottom plate is assumed to be isothermal, gray, a diffuse emitter and a specular reflector. For simplicity the upper plate is taken to be an isothermal black surface, and the following assumptions regarding the absorbing medium will be employed:

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1. The medium is gray with an index of refraction of unity and an absorption coefficient which is independent of temperature.
2. Scattering is negligible.
3. Radiation is the sole mode of energy transfer.

It should be noted that in the absence of an absorbing medium the net heat transfer between plates will be independent of whether the lower plate is a diffuse or specular reflector [1]. Thus, the present physical model is one which isolates the effect of surface reflection characteristics upon radiation transfer within the medium.

2. Results for Diffuse Reflection

Before proceeding with the analysis in which surface 1 is a specular reflector, it will be convenient to briefly summarize the equations which apply to the analogous situation where surface 1 reflects in a diffuse manner. Let e denote black-body emissive power ($e = \sigma T^4$ where σ is the Stefan-Boltzmann constant and T is absolute temperature), g_r denote the radiation heat flux within the medium, and R_1 denote the radiosity of the lower surface (i.e., the sum of emitted and reflected radiation from surface 1). Further, dimensionless quantities will be defined as

$$\phi = \frac{e - e_2}{R_1 - e_2} \quad , \quad \alpha = \frac{g_r}{R_1 - e_2}$$

The expression for the radiation heat flux q_r is given in a number of references, for example [2] and [3], and in dimensionless form is

$$Q = 2E_3(\tau) + 2 \int_0^{\tau} \varphi(t) E_2(\tau-t) dt - 2 \int_{\tau}^{\tau_0} \varphi(t) E_2(t-\tau) dt \quad (1)$$

where $\tau = ay$, $\tau_0 = aL$, a is the absorption coefficient of the medium, and $E_n(t)$ is the exponential integral

$$E_n(t) = \int_0^1 \exp\left(-\frac{t}{\mu}\right) \mu^{n-2} d\mu$$

The equations governing the emissive power profile $\varphi(\tau)$, the radiation heat flux Q , and the radiosity R_1 , are as follow:

Emissive power profile;

$$2\varphi(\tau) = E_2(\tau) + \int_0^{\tau_0} \varphi(t) E_1(|\tau-t|) dt \quad (2a)$$

Radiation heat flux

$$Q = 1 - 2 \int_0^{\tau_0} \varphi(\tau) E_2(\tau) d\tau \quad (2b)$$

Radiosity;

$$\frac{R_1 - e_2}{e_1 - e_2} = \frac{\epsilon}{\epsilon + (1-\epsilon)Q} \quad (2c)$$

where ϵ is the emissivity of surface 1.

Equation (2a), which is an integral equation describing $\varphi(\tau)$, is obtained by differentiating Equation (1) and noting that Q is a constant. Equation (2b) describes the net heat transfer between the two plates, and is simply Equation (1) with $\tau = 0$. The radiosity expression, Equation (2c), follows from combining the diffuse surface relation [3, 4]

$$R_1 = \epsilon E_1 + (2 - \epsilon) \left[e_2 E_3(\tau_0) + \int_0^{\tau_0} e(\tau) E_2(\tau) d\tau \right] \quad (3)$$

with Equation (2b), and this procedure is analogous to that previously employed by Perlmutter and Howell [5]. Results for $\varphi(\tau)$ and Q are given in a number of references, for example [2] and [6].

3. Analysis for Specular Reflection

Consider now the case in which surface 1 is a specular reflector, and let I^+ and I^- denote the intensities of radiation (i.e., the rate of energy transfer per unit solid angle per unit area normal to the pencil of rays) in the upper and lower directions as illustrated in Figure 1. With $\mu = \cos \Theta$, it follows from [2] that

$$I^+(\mu, \tau) = I^+(\mu, 0) \exp\left(-\frac{\tau}{\mu}\right) + \frac{1}{\pi} \int_0^{\tau} e(t) \exp\left(\frac{t-\tau}{\mu}\right) \frac{dt}{\mu} \quad (4a)$$

$$I^-(\mu, \tau) = \frac{e_2}{\pi} \exp\left(\frac{\tau_0 - \tau}{\mu}\right) - \frac{1}{\pi} \int_{\tau}^{\tau_0} e(t) \exp\left(\frac{t-\tau}{\mu}\right) \frac{dt}{\mu} \quad (4b)$$

These expressions are completely general as far as the directional emission or reflection characteristics of surface 1 are concerned. If surface 1 is a diffuse emitter and diffuse reflector, the quantity $I^+(\mu, 0)$ appearing in Equation (4a) is a constant, while for any other situation $I^+(\mu, 0)$ will be a function of μ .

For the present situation in which surface 1 is a diffuse emitter and specular reflector

$$I^+(\mu, 0) = \frac{\epsilon e_1}{\pi} + (1 - \epsilon) I^-(-\mu, 0)$$

where $(1 - \epsilon)$ is the reflectivity of a gray surface. Thus, from Equation (4b)

$$I^+(\mu, 0) = \frac{\epsilon e_1}{\pi} + (1 - \epsilon) \left[\frac{e_2}{\pi} \exp\left(-\frac{\tau_0}{\mu}\right) + \frac{1}{\pi} \int_0^{\tau_0} e(t) \exp\left(-\frac{t}{\mu}\right) \frac{dt}{\mu} \right] \quad (5)$$

The net radiation heat transfer, q_r , may now be obtained through integration over the solid angle, with the result

$$q_r = 2\pi \int_0^1 I^+(\mu, \tau) \mu d\mu - 2\pi \int_0^{-1} I^-(\mu, \tau) \mu d\mu$$

and from Equations (4) and (5)

$$q_r = 2\epsilon e_1 E_3(\tau) - 2e_2 E_3(\tau_0 - \tau) + 2 \int_0^{\tau} e(t) E_2(\tau - t) dt - 2 \int_{\tau}^{\tau_0} e(t) E_2(t - \tau) dt + 2(1 - \epsilon) \left[e_2 E_3(\tau + \tau_0) + \int_0^{\tau_0} e(t) E_2(t + \tau) dt \right] \quad (6)$$

For comparative purposes, it will be convenient to express Equation (6) in terms of the same dimensionless quantities previously considered; i.e., φ and Q . To this end, it is first necessary to obtain an expression for the radiosity R_1 . Since

$$R_1 = 2\pi \int_0^1 I^+(\mu, 0) \mu d\mu$$

then from Equation (5)

$$R_1 = \epsilon E_1 + 2(1-\epsilon) \left[e_2 E_3(\tau_0) + \int_0^{\tau_0} e(\tau) E_2(\tau) d\tau \right] \quad (7)$$

and this is identical with the result for diffuse reflection given by Equation (3). One may now express Equation (6) in the dimensionless form

$$Q = 2E_3(\tau) + 2 \int_0^{\tau} \varphi(t) E_2(\tau-t) dt - 2 \int_{\tau}^{\tau_0} \varphi(t) E_2(t-\tau) dt \\ + 2(1-\epsilon) \int_0^{\tau_0} \varphi(t) \left[E_2(t+\tau) - 2E_3(\tau) E_2(t) \right] dt \quad (8)$$

Referring to Equation (1) for diffuse reflection, it is seen that Equations (1) and (8) differ through the appearance of the last term in Equation (8).

The equations describing $\varphi(\tau)$, Q and R_1 may be obtained in exactly the same manner as previously described for diffuse reflection, and these equations are as follow:

Emissive power profile;

$$2\varphi(\tau) = E_2(\tau) + \int_0^{\tau_0} \varphi(t) E_1(1\tau-t) dt + (1-\epsilon) \int_0^{\tau_0} \varphi(t) [E_1(t+\tau) - 2E_2(\tau)E_2(t)] dt \quad (9a)$$

Radiation heat flux;

$$Q = 1 - 2 \int_0^{\tau_0} \varphi(\tau) E_2(\tau) d\tau \quad (9b)$$

Radiosity;

$$\frac{R_1 - \epsilon_2}{\epsilon_1 - \epsilon_2} = \frac{\epsilon}{\epsilon + (1-\epsilon)Q} \quad (9c)$$

Comparing the above equations with Equations (2) which apply for diffuse reflection, it is seen that only Equations (2a) and (9a) differ. Equation (9a) contains an additional term, and this term affects $\varphi(\tau)$ in two ways. First, it introduces the emissivity ϵ as a parameter; and second, $\varphi(\tau)$ is no longer antisymmetric as it is for diffuse reflection [7].

To obtain a first approximation for $\varphi(\tau)$, a particularly meaningful approach for the present problem is the exponential kernel approximation as employed by Lick [8], and this makes use of the expressions

$$E_2(t) \approx \frac{3}{4} e^{-3t/2}, \quad E_3(t) \approx \frac{1}{2} e^{-3t/2}$$

Upon substituting these into Equation (8) it is seen that the last term vanishes, and this is the term that makes Equation (8) differ from its diffuse reflection counterpart given by Equation (1). Thus, within the framework of the exponential kernel approximation diffuse reflection and specular reflection yield identical results, and from Lick [8] these are

$$\varphi(\tau) = \left(1 - \frac{Q}{2}\right) - \frac{3Q}{4}\tau \quad (10)$$

$$Q = \frac{1}{1 + 3\tau_0/4} \quad (11)$$

It is interesting to note that the above expressions have also been obtained by Deissler [9] through use of the Rosseland equation with jump boundary conditions.

A second approximation for $\varphi(\tau)$ can be obtained by substituting Equation (10) into the right hand side of Equation (9a); i.e., conventional use of the method of successive approximations. The resulting $\varphi(\tau)$ profiles are illustrated in Figures 2 and 3 for $\epsilon = 0.7$ and 0.3 respectively³⁾. Also illustrated are curves representing diffuse reflection⁴⁾,

3) For the sake of brevity, the rather lengthy expression for $\varphi(\tau)$ is omitted.

4) It should be recalled that for diffuse reflection $\varphi(\tau)$ is independent of ϵ .

and these were obtained in the same manner as the specular reflection results by substituting Equation (10) into the right hand side of Equation (2a). The present diffuse reflection curves agree almost precisely with those of References [2] and [6], and it was thus deemed unnecessary to consider higher approximations of $\varphi(\tau)$ for either diffuse or specular reflection.

It may be seen that the difference between the specular and diffuse results is quite small. It is slightly greater for $\epsilon = 0.3$ than for $\epsilon = 0.7$, since reflection plays a greater role at the lower emissivity value. One may also note that the difference decreases as τ_0 becomes large. The reason is that for large τ_0 radiation transfer within the medium approaches a diffusion process and thus becomes independent of the reflection characteristics of the bounding surfaces. A direct analogy to this involves molecular heat transfer in a gas, since the manner in which a surface reflects molecules plays no role in continuum heat conduction but is a factor only under rarefied conditions.

A comparison of net heat transfer is illustrated in Table 1, and these results were obtained through numerical integration of the expression

$$Q = 1 - 2 \int_0^{\tau_0} \varphi(\tau) E_2(\tau) d\tau$$

As would be expected from the close agreement of the emissive power profiles, there is no appreciable difference between the

specular and diffuse reflection results for net heat transfer.

Table 1. Radiation heat flux, $Q = q_r / (R_1 - e_2)$

τ_0	Diffuse	Specular	
		$\epsilon = 0.7$	$\epsilon = 0.3$
0.1	0.916	0.915	0.914
1	0.554	0.553	0.549
10	0.111	0.109	0.107

4. Concluding Remarks

In closing, it is worth mentioning the more general case in which both surfaces are specular reflectors. Letting ρ_1 and ρ_2 be the reflectivities of the two surfaces, the radiation flux equation will contain integrals of the type

$$\int_0^1 \frac{\exp(-t/\mu)}{1 - \rho_1 \rho_2 \exp(-2\tau_0/\mu)} \mu^{n-2} d\mu$$

This reduces to the exponential integral, $E_n(t)$, only for the case in which one of the two surfaces is black.

References

1. E. R. G. Eckert and R. M. Drake, Heat and Mass Transfer (McGraw-Hill Book Company, Inc., New York, 1959).
2. R. Viskanta and R. J. Grosh, Heat Transfer in a Thermal Radiation Absorbing and Scattering Medium, International Developments in Heat Transfer Part IV, 820-828 (ASME, New York, 1961).
3. R. D. Cess, The Interaction of Thermal Radiation with Conduction and Convection Heat Transfer, Advances in Heat Transfer, Vol. I (Academic Press, New York, 1964).
4. R. Viskanta and R. J. Grosh, Intern. J. Heat Mass Transfer 5, 729-734 (1962).
5. M. Perlmutter and J. R. Howell, Radiant Transfer Through a Gray Gas Between Concentric Cylinders Using Monte Carlo, ASME Paper No. 63-HT-39, (1963).
6. C. M. Usiskin and E. M. Sparrow, Intern. J. Heat Mass Transfer 1, 28 (1960).
7. R. V. Meghreblian, Intern. J. Heat Mass Transfer 5, 1051-1052 (1962).
8. W. Lick, Energy Transfer by Radiation and Conduction, Proceedings of 1963 Heat Transfer and Fluid Mechanics Institute, 14-26 (Stanford University Press, 1963).
9. R. G. Deissler, Diffusion Approximation for Thermal Radiation in Gases with Jump Boundary Conditions, ASME Paper No. 63-HT-13 (1963).

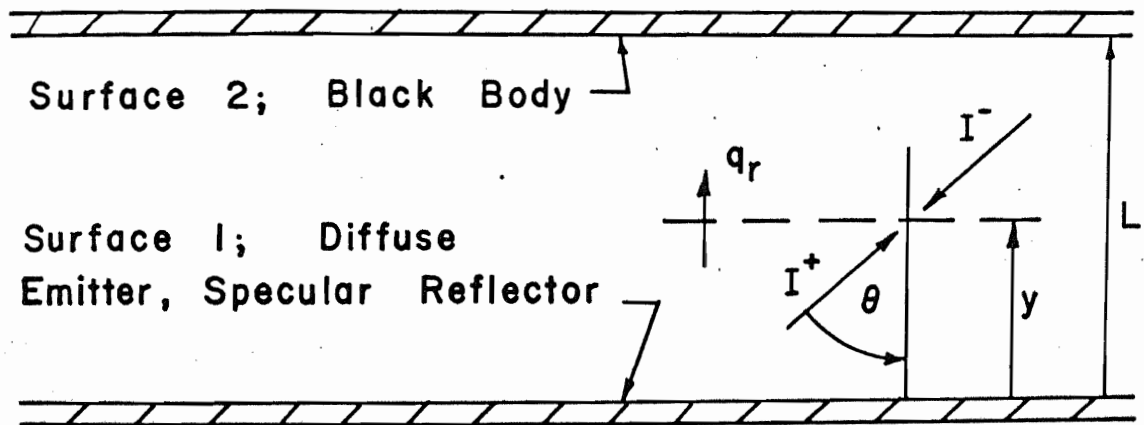


Fig. 1. Physical model and coordinate system

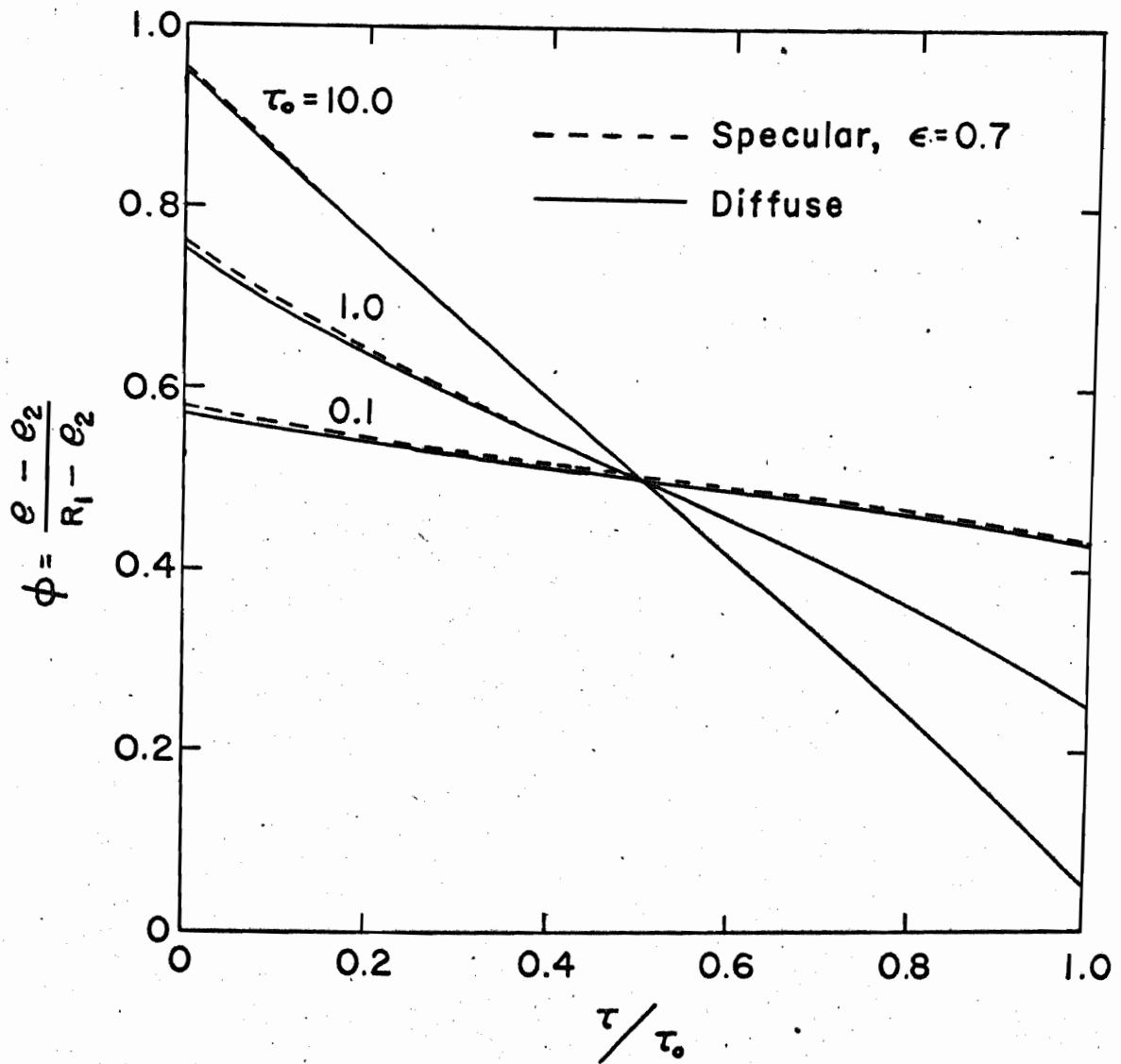


Fig. 2. Emissive power profiles for $\epsilon = 0.7$

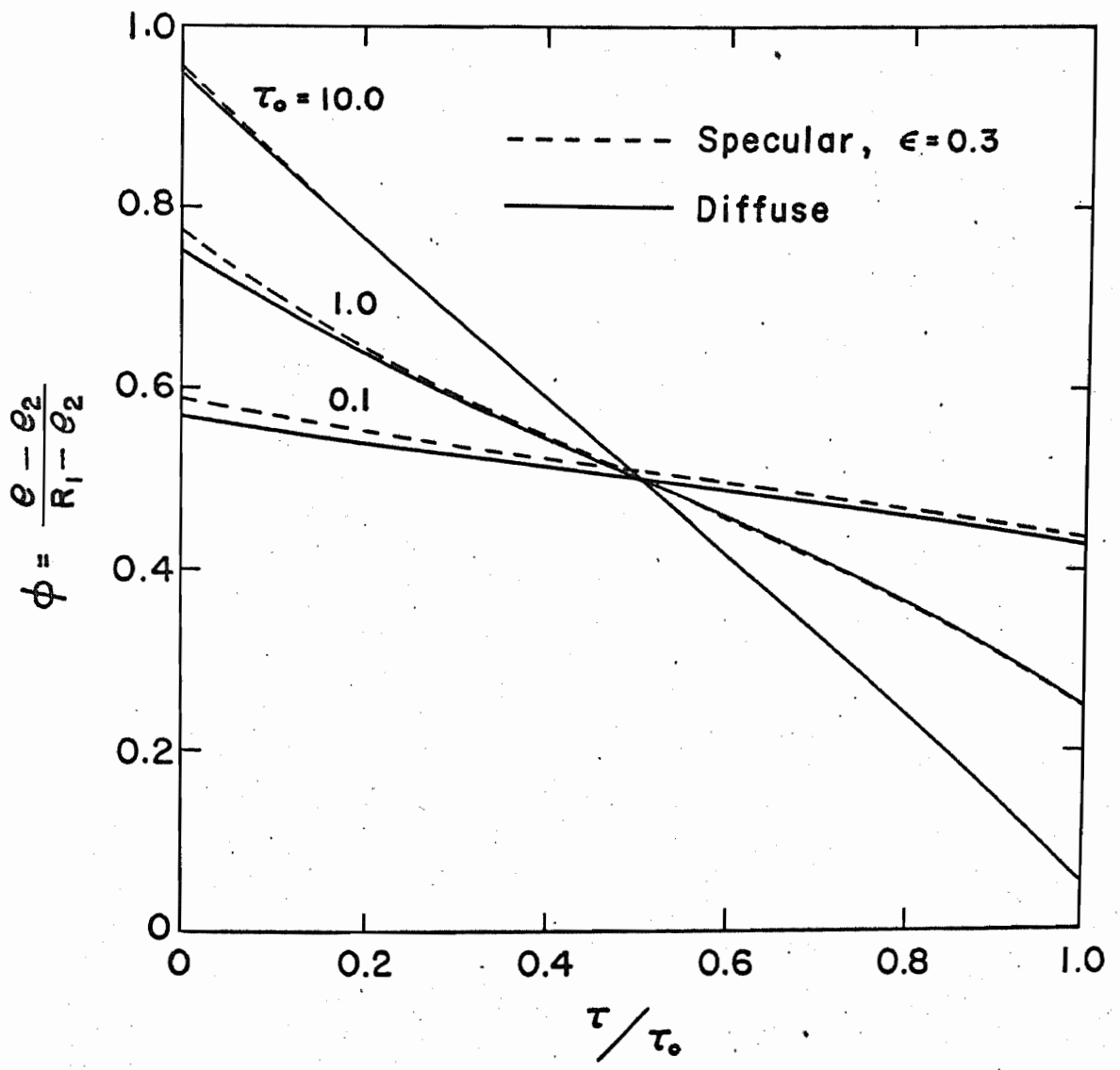


Fig. 3. Emissive power profiles for $\epsilon = 0.3$.