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## SYNOPSIS

In applying the moiré method for stress analysis, it is sometimes difficult to attribute proper signs to fringes. In the conventional way the sign of direct derivatives ( $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$) are determined from the physical consideration of boundary conditions, and the sign of cross derivatives ($\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$) are then deduced from the slope of fringes and the proper interpretation of singular points. Presented herein are methods for the determination of signs of moiré fringes based on their intrinsic properties. These methods can be applied to any portion of a moiré pattern without a priori knowledge of the boundary conditions. As a result the moiré method can be effectively used as a means for stress separation in three-dimensional photoelasticity. Heretofore the moiré pattern obtained from a stress-frozen photoelastic slice upon annealing is most difficult to analyze, because the boundary conditions are either not well defined or sometimes even "non-existing" due to the destructive nature of the stress-frozen technique.

Also presented is a method for fringe ordering whereby the signs of both direct derivative and cross derivatives are obtained automatically once the fringes are properly ordered.

# I. INTRODUCTION

In applying the moiré method for strain analysis [1],* the determination of signs is usually the most confusing part. The moiré fringes represent displacements in the direction perpendicular to the lines of the grating. Strains are obtained by graphical (or numerical) differentiation of the displacement curves plotted from these fringes. However, from the appearance of the fringes, it is impossible to attribute proper signs to the strains. While the relative signs among regions can be obtained by a careful study of the moiré pattern, the absolute signs have to be derived from known boundary conditions and it is no easy matter. Usually, one starts from a region containing a portion of the boundary with a known boundary condition to obtain the signs of the direct derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ (u and v are the displacements in the directions of x and y, respectively). The signs of cross derivatives $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$ are then derived from the slopes of the fringes and the signs of the direct derivatives. Once the sign of one region is known, the signs of other regions are deduced from this region through the knowledge of singular points [2]. There appears to be no standard procedure to follow and, as a result, each moiré pattern presents itself as a challenge to the interpreter.

Recently, moiré method has been applied to three dimensional photoelasticity as a supplementary technique for stress separation [3]. In this application, gratings are printed on stress-frozen slices and then annealed. The deformation of grating caused by the annealing (i.e. the releasing of stresses) produced a moiré pattern upon superposition with a master grating. Moiré patterns of this type are most difficult to interpret because of the

*Numbers in brackets denote references at the end of the paper.

fact that a photoelastic slice is only a section or a sub-section of the entire model. As a result, the boundary conditions may not be well defined to render a proper interpretation of the pattern or, in cases of a sub-slice, there may not be any known boundary conditions at all. It will then be impossible to attribute proper signs to the fringes. For example, as in the case shown in Fig. 1, there are at least two ways of plotting displacement curves depending upon which of the two moiré patterns these fringes are a portion of. And there is no boundary condition to help because the neighboring portions of the slice have been destroyed due to the destructive nature of the three dimensional photoelastic technique. Here the conventional method for the determination of signs breaks down completely.

Therefore, in order to simplify the interpretation of moiré patterns and in order to extend the usefulness of moiré method to the photoelasticians, new methods for the determination of signs are called for. Presented herein are several methods developed for this need. They are based on the intrinsic properties of moiré fringes and hence can be applied to any type of moiré fringe patterns without the need to have a priori knowledge of the boundary conditions.

Also presented at the end of the paper is a proposed convention for fringe ordering, which is closely related to the methods of determination of signs. The essence of the convention is such that once the fringe orders are properly attributed, the signs of all the necessary derivatives of displacements for strain analysis (i.e. $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$) follow automatically.

## II. DIFFERENT METHODS FOR THE DETERMINATION OF SIGNS

The essence of the determination of signs is to find out whether a set of moiré fringes in a small region is caused by the local contraction, elongation, rotation or a combination of either contraction and rotation or elongation and rotation, of the model grating. The case of pure rotation is easily recognized (fringe perpendicular to the line bisecting the angle of rotation) and hence will be excluded from the analysis.

As mentioned previously it is impossible to deduce the signs from the appearance of moiré fringes (e.g. a homogeneous tensile strain gives identical fringes to that of a homogeneous compressive strain of equal magnitude) any method that will reveal the signs will have to come from something else. The approach used here is to change the appearance of the moiré fringes by superimposing another set of fringes of known characteristics and it is hoped that the resulting fringe pattern, when compared with the original, will reveal the necessary information. Since the model grating is attached to the model under loading, it would be desirable not to disturb it while the changing of the moiré pattern is made. Three methods are developed to this effect: 1) the change is accomplished by superimposing on the model another master grating of different line density (or the equivalent) --- the linear mismatch method, 2) the change is accomplished by rotating the master grating --- the rotational mismatch method, and 3) the change is accomplished by varying the distance between the camera and the object if there is a gap between the model grating plane and the master grating plane --- the gap effect method. Details of the three methods are presented in the following.

### 1) Determination of Signs by Linear Mismatch

Linear mismatch is defined to be the difference in pitch (distance between two neighboring lines) between the master and model gratings.

In the following it will be shown that if a linear mismatch of known sign is superimposed on an existing moiré pattern, the resulting pattern, when compared with the original one, can be used as a means to determine the signs of the original pattern. Indeed, it is recalled that the fringe spacing $s$ for a homogeneous moiré pattern is given by [4]

$$s = \frac{pp'}{p-p'} \tag{1}$$

in which $p$ and $p'$ are the pitches of master and model gratings, respectively. The fringes are, of course, parallel to the lines of the gratings. Now if the pitch of the master grating is changed from $p$ to $p + dp$, the spacing between the fringes will also be changed and it is given by

$$s + ds = \frac{(p+dp)p'}{(p+dp)-p} = \frac{pp'}{(p-p')+dp} \tag{2}$$

with $p'dp$ neglected when compared with $pp'$. It is seen that if a positive $dp$ is imposed on the master, the fringe spacing will decrease if the local strain is compressive (i.e. $p>p'$), whereas the fringe spacing will increase if the local strain is tensile (i.e. $p<p'$). The exact opposite will be true if a negative $dp$ is imposed. In other words, the fringe density (fringes per unit length) increases if the local strain is of the same sign as that of the imposed (fictitious) strain, decreases if it is of opposite sign. The principle still holds if the local deformation contains shear as well as normal strains. The only caution is that the fringe density has to be taken along the direction normal to the lines of the master grating.

Whenever local deformation contains rotation the moiré fringes will no longer be parallel to the lines of the master grating. Whether the rotation is combined with a positive or negative normal strain can also be determined by the following method. Instead of observing the change of fringe densities along the direction normal to the lines of master grating, it is also possible to reveal the signs by observing the rotations of the fringes when a linear mismatch is superimposed. As shown in Fig. 2, when a positive dp is imposed on the master grating, the fringes will rotate away from the normal to the grating lines if the local strain is compressive (i.e. $p>p'$) and rotate toward the normal if the local strain is tensile (i.e. $p<p'$). A negative dp, with opposite effects as that of the positive dp, can also be used for the determination of signs. The following statement covers both cases: if the superimposing (fictitious) strain and the actual local strain are of the same sign, the fringes will turn away from the normal to the lines of the master grating; and if they are of opposite signs the fringes will turn toward the normal. The phenomenon can be easily visualized if it is recalled that when there is no normal strain but rotation alone, the fringes are nearly normal to the grating lines.

In applying the techniques of linear mismatch, it should be cautioned that the magnitude of the imposed (fictitious) strain should not exceed that of the local strain, if they are of opposite signs. Otherwise the fringes will behave differently from what have been previously described. The reason is easy to see from Eq. (1) in that the imposed dp should not be so large as to reverse the sign of $p-p'$. Therefore, the restriction for the linear mismatch method is such that

$$|dp| \leq |p-p'| \tag{3}$$

if dp and $(p-p')$ are of opposite signs. When dp approaches $(p-p')$ with opposite signs, the fringe spacing approaches infinite; and the field should have no fringe at all. Hence if dp can be applied in a continuous fashion (or discrete but with small increment), there is no danger to exceed the limitation imposed by Eq. (3). A useful dp is the one below the limit set by Eq. (3) but gives a detectable change of fringe density (or fringe rotation).

There are three methods of imposing linear mismatches to the master grating: the first and the obvious one is to have a set of master gratings with slightly different pitches. When there is a need to determine the signs of a moire pattern formed by a grating of pitch $p$ the pattern is compared with the one formed by a grating of pitch $p_1$ where $p_1 = p+dp$, starting with smallest dp so as not to violate the restriction set by Eq. (3), and gradually changing the master until the change of moire pattern is large enough for easy comparison. However, this method is less practical because it requires an almost "continuous" set of masters which is usually not available in most laboratories.

The second method of imposing linear mismatch is to form the moiré pattern at the back of the camera by imaging the model grating on the ground glass of the camera against which a master grating is erected. Under one to one magnification, the moiré pattern is the same as the one formed by model and master gratings in direct contact. Since the size of the model can be varied by changing the magnification, equivalent dp of both signs can be easily imposed onto the master grating. Most cameras are provided with a lens slideable along the optical axis, an equivalent positive dp is imposed on the master by moving the lens away from the model and a negative dp is obtained by moving the lens toward the model. The movement is continuous, hence the change of dp is also continuous. Not only is there no danger in exceeding the condition of Eq. (3) but it is easy to see when the condition $|dp| = |p-p'|$ is approaching. If there is no rotation of the model grating, the fringe spacing will gradually widen and finally disappear when $|dp| = |p-p'|$. The fringes will reappear with gradually decreasing fringe spacing if the movement of the lens is continued along the same direction. If there is rotation of the model grating the fringes will rotate toward the normal to the master grating (with decreasing fringe spacing) and then finally will swiftly shift to the other side of the normal and start to move away with increasing fringe density as the point where $|dp| = |p-p'|$ is reached and passed.

The third method of imposing dp to a master grating is by introducing a constant gap in between the model and master gratings. This method will be discussed in detail in section II (3)-(A).

2) Determination of Signs by Rotational Mismatch

Rotational mismatch is defined as the angular difference between the master and model gratings and it will be shown that the imposition of a rotational mismatch to the master grating is also a means for the determination of signs.

As shown in Fig. 2, if $\theta$ denoted the acute angle between the two gratings and $\phi$ denotes the angle between the tangent to a fringe and the lines of master grating, both measured from the master grating in a counter clockwise direction, it can be shown that [4]

$$\tan \phi = \frac{p \sin\theta}{p \cos\theta - p'} \quad . \tag{4}$$

For small $\theta$ Eq (4) can be approximated by

$$\tan \phi = \frac{p\theta}{p-p'} \tag{5}$$

It is seen from Eq. (5) that for a given $\theta$, the orientation of moiré fringes is determined by the signs of the local strain (i.e. $p-p'$). For example, for positive $\theta$, the fringes (or the tangents to them) are in the first and third quardrants when p-p' is positive, and in the second and fourth quardrants when p-p' is negative, as shown in Fig. 2.

Eq. (5) can be used for the determination of signs in the following two ways. First, if the moiré fringes are parallel to the master grating, i.e. when the deformation of the local model grating is such that there is no rotation involved, the signs of the normal strains can be determined by imposing a rotation $\theta$ to the master grating and the resulting orientation of the fringes is then an indication of the signs*. Second, if the moiré fringes are originally not parallel to the master grating, the signs of local strain are then indicated by the orientation of the fringes. This, of course, requires the a priori knowledge of the rotation of the model grating.

If Eq. (5) is differentiated, with p and p' being held as constants, the resulting equation

$$\sec^2\phi \; d\phi = \frac{p}{p-p'}\, d\theta, \tag{6}$$

is a more useful equation for the determination of signs. Eq. (6) states that if an additional rotation $d\theta$ is imposed on the master grating, the resulting rotation of the fringes $d\phi$ will have the same sign as that of $d\theta$ if $p>p'$, or the opposite sign if $p<p'$. In other words, upon the imposition of a rotational mismatch to the master grating, the moiré fringes will rotate the same way as the master grating if the local strain is tensile and the opposite way if the local strain is compressive. This is true for any moiré pattern. A graphical illustration of Eq. (6) is shown in Fig. 3.

The method of rotational mismatch is easy to use if moiré patterns are

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* This method was briefly described in an earlier paper of the author [5] for the determination of the signs of linear mismatch.

formed by direct contact between model and master gratings. If moiré is formed on the back of a camera, it is necessary to have a "revolving camera back" (commercially available) to facilitate the rotation of master grating.

3) Determination of Signs by Gap Effect

When two gratings are placed together with a small gap in between, a set of moiré fringes will be observed by eye or by a camera even if they are completely parallel in orientation. This effect is due to the fact that the two gratings have different object distances from the camera lens (or the eyes) and, as a result, they form images of different sizes at the film plane (or the retina). If the depth of field is such that both gratings are in focus, the two images of the gratings will interfere to form a moiré pattern. If the grating closer to the camera is called master grating, the other one model grating, the moiré pattern formed by the gap effect is always equivalent to a linear compressive mismatch, because the image of the master grating is always larger than that of the model grating due to magnification difference. The relation between the gap $z$ and the fictitious strain thus caused is given by [6]

$$\varepsilon = \frac{\Delta z}{z} \tag{7}$$

in which $z$ is the distance between the master grating and camera lens and $\varepsilon$ is the normal strain in the direction perpendicular to the lines of master grating.

Eq. (7) can be used for the determination of signs of a moiré pattern in either one of the following two ways.

(A) If the master grating is displaced from the deformed model grating with a constant gap, the gap effect is equivalent to having imposed a linear compressive mismatch to the moire pattern (i.e. a positive dp). Therefore, the technique described in section II-(1) can be applied for the determination of signs. It is not difficult to devise an apparatus so as to vary the gap continuously. The simplest way to impose a gap, however, is the following: usually the master grating is printed on a piece of photographic glass plate with a thickness of about 1/16 inch; a constant gap is easily introduced by turning the plate over with the non-emulsion side in contact with the model grating. The technique is not suitable to use when the moiré pattern is formed on the back of a camera.

(B) There are cases where the master grating has to be mounted at a fixed distance away from the model because of the loading device (e.g. the loading apparatus used in the dynamical moiré study in reference [7]); it is then impossible either to rotate the master to impose a rotational mismatch or to change the gap to give a linear mismatch to the moiré pattern. The gap Eq. (7) can then be used in the following fashion for the determination of signs of the fringes: if the distance z is changed, it is seen from the equation, the magnitude of the fictitious strain $\varepsilon$ is also changed. Therefore, if the set up is such that a gap is already presented between the master and model gratings, the moiré pattern can be changed by photographing the two gratings at different distances. For example, if two pictures are taken at two different distances $z_1$ and $z_2$ (with $z_2 > z_1$), the picture taken at $z_2$ is equivalent to having had a negative dp (since $\varepsilon_2 < \varepsilon_1$) imposed on the master grating, when compared with the picture taken at $z_1$. Effectively, it is then an imposition of linear (tensile) mismatch, The techniques described in section II-(1) can again be used here for the determination of signs.

## III. EXAMPLES FOR THE DETERMINATION OF SIGNS BY VARIOUS TECHNIQUES

A disk of 4 inches in diameter under three-point loading as shown in Fig. 4 is chosen as an example to demonstrate the various techniques presented in the preceding section. The moiré pattern is formed by a grating of 300 lines per inch for both the model and master. The moiré is of the v-field, i.e. the grating lines are horizontal. While it is possible to figure out the signs of the pattern from physical considerations, it is chosen to ignore the boundary conditions. The problem is, say, to find out the signs of strains along the vertical axis of the model from these fringes.

<u>The linear mismatch method</u>: A compressive linear mismatch is first applied to the pattern, the left half of the resulting pattern is then compared with the right half of the original pattern as shown in Fig. 6. According to the method presented in section II (1), the change of fringe density should indicate the sign of the local strain. It is easy to see from Fig. 5 that along the vertical axis, the fringes at the upper 1/4 (approximately) of the pattern have a higher density than before, whereas the lower 3/4 (approximately) has a lower density than the original. Since the mismatch is compressive (i.e. positive dp), according to Eq. (2) the place where fringe density increases (the upper 1/4 of the pattern) should have compressive strain, and the place where the fringe density decreases (the lower 3/4 of the pattern) should have tensile strain. This, of course, is compatible to the physical consideration of the boundary conditions. The compressive linear mismatch is obtained by introducing a constant gap in between the two gratings. The point where the strain changes from compressive to tensile is located somewhere at the mid-point of the upper half of the pattern. It is a singular point.

Rotational mismatch method: A small counter clockwise rotation is imposed on the master grating and the resulting moiré pattern is compared with the original as shown in Fig. 6 in which a little more than one half of the original pattern is shown at right so as to show better the curvatures of the fringes along the vertical axis. According to section II (2), if the fringes rotate the same way as the master grating, the local strain is tensile and if the fringes rotate the opposite way, the local strain is compressive. A comparison of the curvatures of the fringes along the vertical axis before and after the rotation of the master grating reveal that the upper 1/4 of the fringes have rotated clockwise, the strains then are compressive, and the lower 3/4 of the fringes have rotated counter-clockwise, the strains are then tensile. Results agree, of course, with that of linear mismatch method.

Gap-effect method: An initial gap is first introduced to the pattern so that it facilitates the demonstration of the method presented in section II (3) -(B). Two pictures are then taken at two different object-to-camera distances and are compared as shown in Fig. 7. The right half of the composite moiré pattern is taken at a smaller object-to-camera distance whereas the left half a larger object-to-camera distance. It is seen that the fringe density at the upper 1/4 portion of the fringes along the vertical axis is decreased whereas the lower 3/4 increased. According to Eq. (7), this indicates that the strains are compressive at the upper portion and tensile at the lower portion. It may be pointed out that the gap equation as it is used here is equivalent to an imposition of linear mismatch. Since a larger $z$, when compared to the pattern at a smaller $z$, is equivalent to a tensile mismatch, the effect is the opposite as that shown in Fig. 6, where a compressive mismatch is imposed.

As a last example, it may be interesting to see what effect it would have on a moiré pattern when too large a mismatch is introduced to the master grating. In Fig. 8 the left half of the picture is the original pattern and the right half is the pattern superimposed with a large compressive mismatch by using a different master grating with a smaller line density. The behavior of the resulting fringes along the vertical axis can be roughly grouped into three regions: upper, middle and lower. The state of strain at the upper region is compressive, hence the fringe density increases as a fictitious compressive strain is added to it. The middle region has a tensile state of strain and with approximately the same magnitude as that of the imposed compressive mismatch. As a result the fringes cancel out as demonstrated by the nearly blank region. The lower part of the model also has a tensile state of strain but with magnitude smaller than the middle region as evidenced by the original smaller fringe density. The imposed compressive mismatch has apparently exceeded in magnitude the local tensile strain. The fringe's density starts to increase again as predicted in section II-(1). Therefore, it is important that in order to use the techniques correctly, linear mismatch should be introduced starting from zero and then gradually increasing to the necessary amount. The same caution should be given to the rotational mismatch methods. If the imposed rotation is so large as to exceed in magnitude the local rotation of opposite sign, the fringe will change its direction of rotation.

## IV. SOME REMARKS ON THE ORDERING OF FRINGES

In the preceding sections methods are presented for the determination of signs of fringe so as to determine whether they correspond to local elongation or contraction. However, having found this does not guarantee a proper interpretation of the moiré pattern in the sense that both the direct and cross derivatives will be attributed with proper signs. In fact all these methods give are the signs of the direct derivatives of the displacement. Nothing is said about the signs of the cross derivatives. The conventional way to determine the signs of the cross derivatives, as mentioned earlier, is to deduce them from the signs of the direct derivatives and the slope of the fringes. Proper attention has to be directed to the properties of singular points if the signs in one region are to be deduced from that of the other. However, it will be shown in the following that if certain rules are followed in ordering the fringes with the help of the knowledge of the signs of direct derivatives, the signs of cross derivatives can be obtained automatically.

The rules of fringe-ordering, it seems, is not very explicitly explained in the literature except that fringe orders can be arbitrary but consecutive. While it is nice to associate a zero order to a fringe corresponding to a zero displacement, the use of it is not necessary, because moiré fringes represent relative displacements. The same can be said about negative orders. Therefore, in order to avoid possible confusions caused by the signs of fringe order, zero and negative orders will not be used in the following proposed rules for fringe ordering:

Rule One Assign a number to a fringe in a region where the sign of local strain has been determined by one of the previously described methods (or by boundary condition if more convenient). The number should be so large that there will be no "danger" of running into zero or negative orders (For example, one hundred is a proper number, because a moiré pattern seldom has more than one hundred fringes.)

Rule Two A x-y coordinate system is set up according to the grating lines. For u-field (or v-field fringes), consecutively increasing orders are assigned to the fringes in the direction of positive x-axis (or y-axis) if $\frac{\partial u}{\partial x}$ (or $\frac{\partial v}{\partial y}$) is positive, and consecutively decreasing orders are assigned if $\frac{\partial u}{\partial x}$ (or $\frac{\partial v}{\partial y}$) is negative. In other words, the fringe order will increase along the positive direction if local strain is tensile and decrease if compressive.

In following the above mentioned two rules it is not necessary to start from one region only. Sometimes it may be more convenient to start from two regions and match the difference when fringe orders from both sides meet. An example is given in Fig. 9 where it has been found by the methods shown in Fig. 5 through Fig. 7 that along the central vertical section the upper one quarter (approximately) portion of the fringes correspond to compressive strain, whereas the rest tensile strain. Therefore, decreasing orders starting with 100 are given to the upper fringe and increasing orders given to the rest of the fringes according to the x-y coordinate system shown. Fringe orders from 100 to 98 and from 98 to 103 are thus attributed. The rest of the fringes (97, 96 and 95) are easily obtained from the consecution consideration of the fringe orders as shown in the figure by the four arrows along which fringes orders have to be consecutively decreasing. It may be asked why the fringe orders along the central vertical axis change from 98 to 98 instead of 99 when it travels from the compressive side to the tensile. Indeed if it were 99 the consecution condition would have been violated at some place because there would be either two "closely" neighboring

98 fringes or 99 fringes. As it is seen that this technique requires no knowledge of singular points except, perhaps, their existence.

After the fringes are thus ordered, it can be seen that the signs of both direct and cross derivatives follow directly from the very nature in which the fringes are ordered. That is, along any positive direction, the derivative (either direct or cross) will be positive if fringe order increases (because either $\Delta u$ or $\Delta v$ is positive), and negative is fringe order decreases ($\Delta u$ or $\Delta u$ negative). The reason for this being true can be seen by representing the u (or v) displacement field as a function of x and y, and plotting the displacement surface with u (or v) as the third coordinate. The moire fringes are the projections of contour lines of equal displacements, obtained by intersecting the displacement surface with planes of u (or v) = np, where n and p are the fringe order and pitch of master grating, respectively [1].

## V. CONCLUSION

It may be concluded that with the methods presented above, the determination of signs in moiré method is much simplified and is no longer dependent on the physical consideration of boundary conditions. As a result the moiré method can be effectively extended into the domain of three-dimensional photoelasticity as a means for stress-separation. It is now always possible to determine the signs of fringes obtained from a stress-annealed slice of which the boundary condition is either not well defined or "non-existing" due to the destructive nature of the method of three-dimensional photoelasticity.

Furthermore, with the proposed fringe ordering technique, the interpretation of moiré pattern is greatly simplified in that once the sign of fringes in one region (or more) is known and the whole pattern ordered accordingly, the signs of both direct and cross derivatives of displacements follow automatically. This method does not require any understanding of the properties of singular points of a moiré pattern, which heretofore is absolutely necessary for a proper interpretation of moiré patterns.

## VI. ACKNOWLEDGMENT

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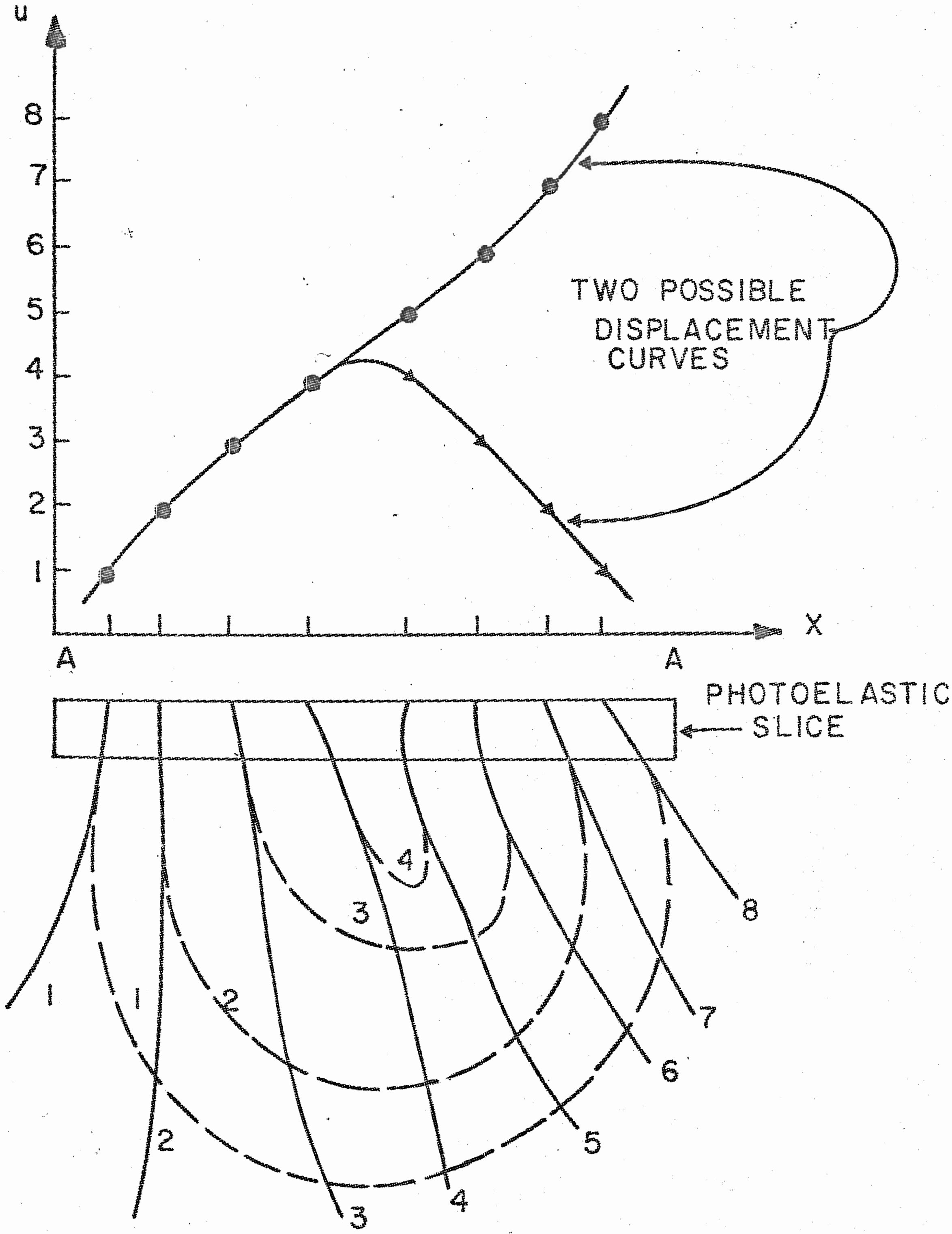


Fig. 1 Two Possibilities for Plotting Displacement Curves from a Moiré Pattern on Annealed Photoelastic Slice

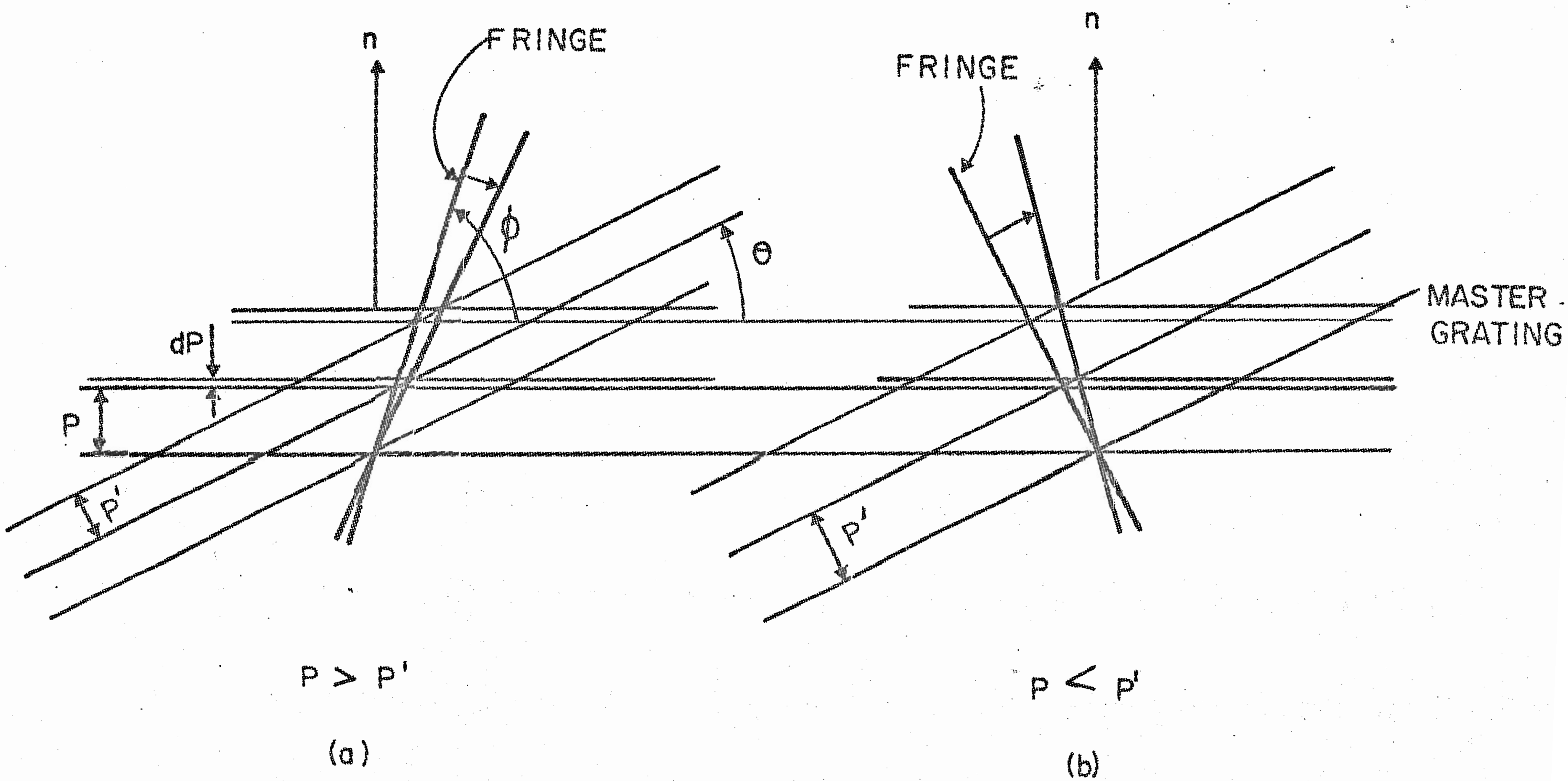


Fig. 2  Behavior of Fringe under the Imposition of a Positive dp to the Master Grating

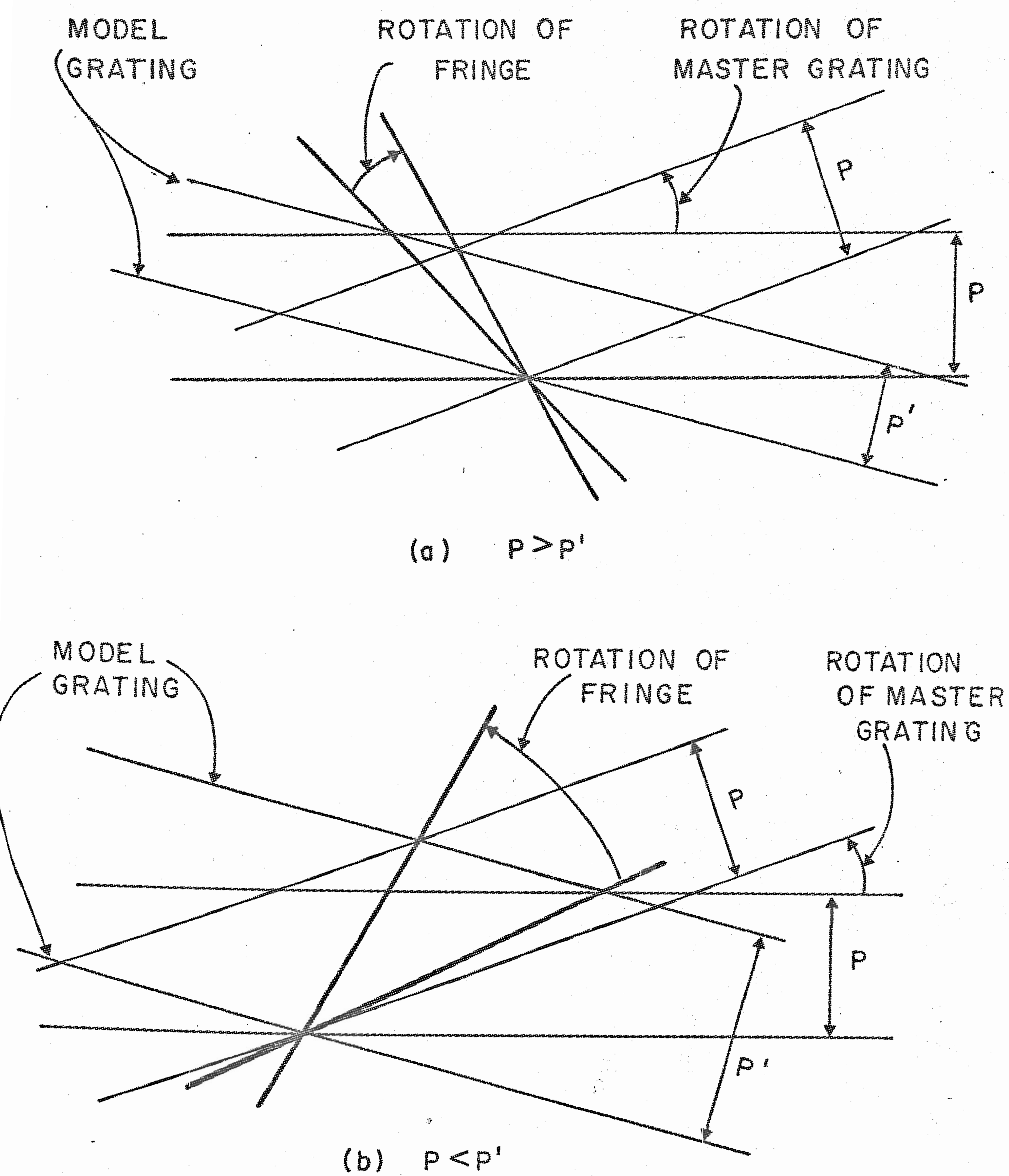


Fig. 3 Behavior of Fringe under the Imposition of a Clockwise Rotation to the Master Grating

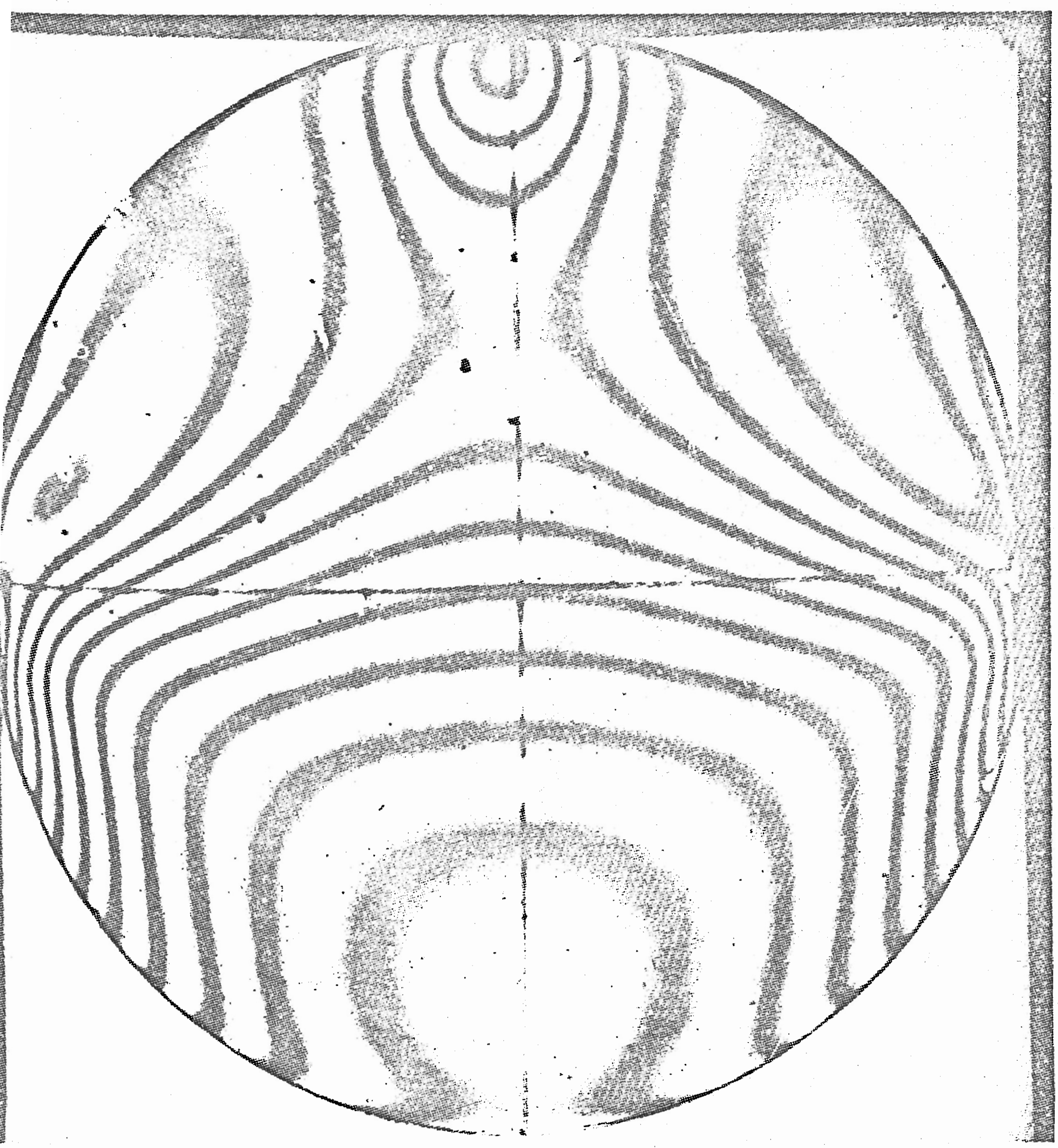

Fig. 4. Moiré Pattern of a Disk under Three-point Loading

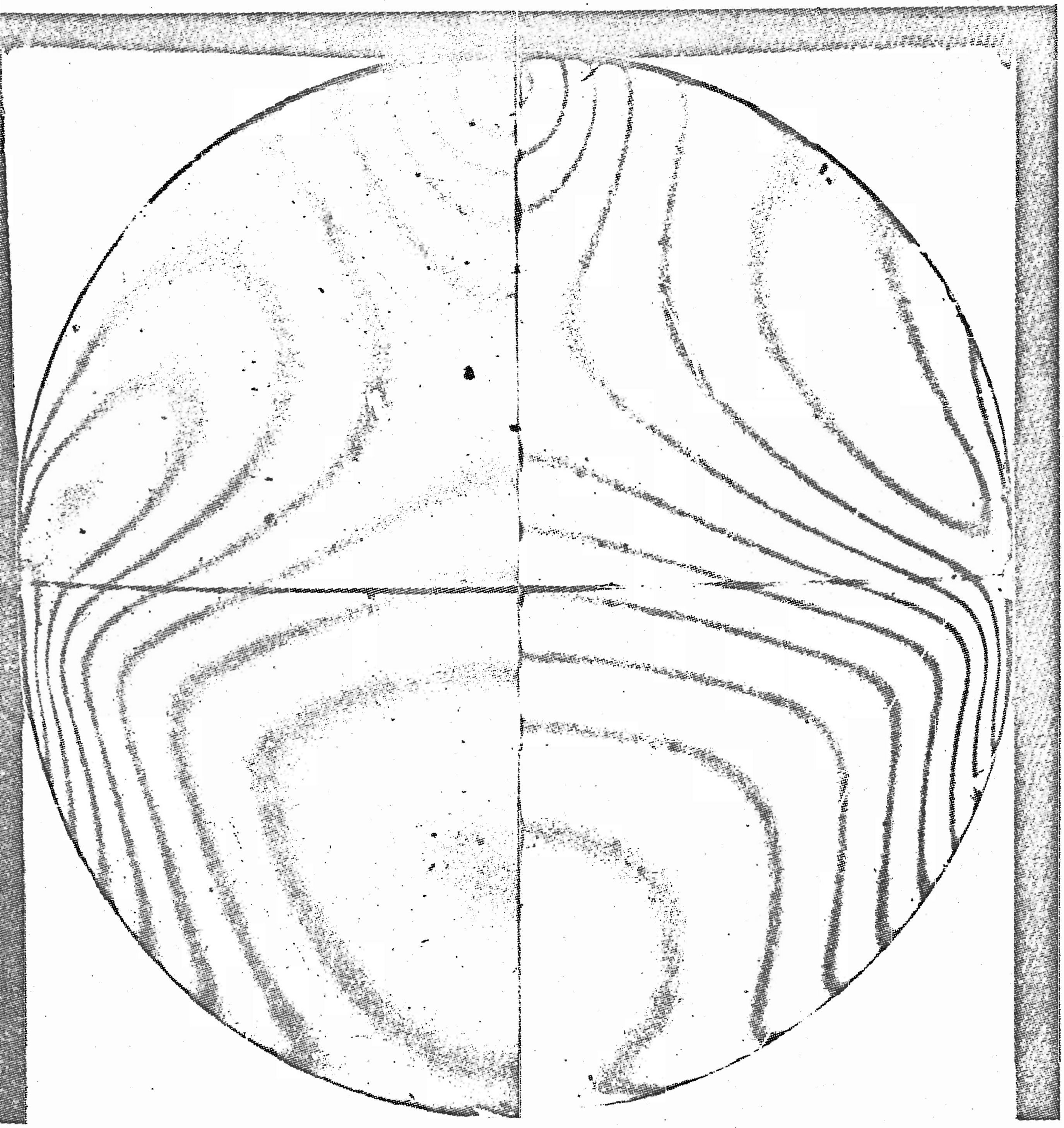

Fig. 5. Determination of Signs along Central Vertical Axis by Linear Mismatch

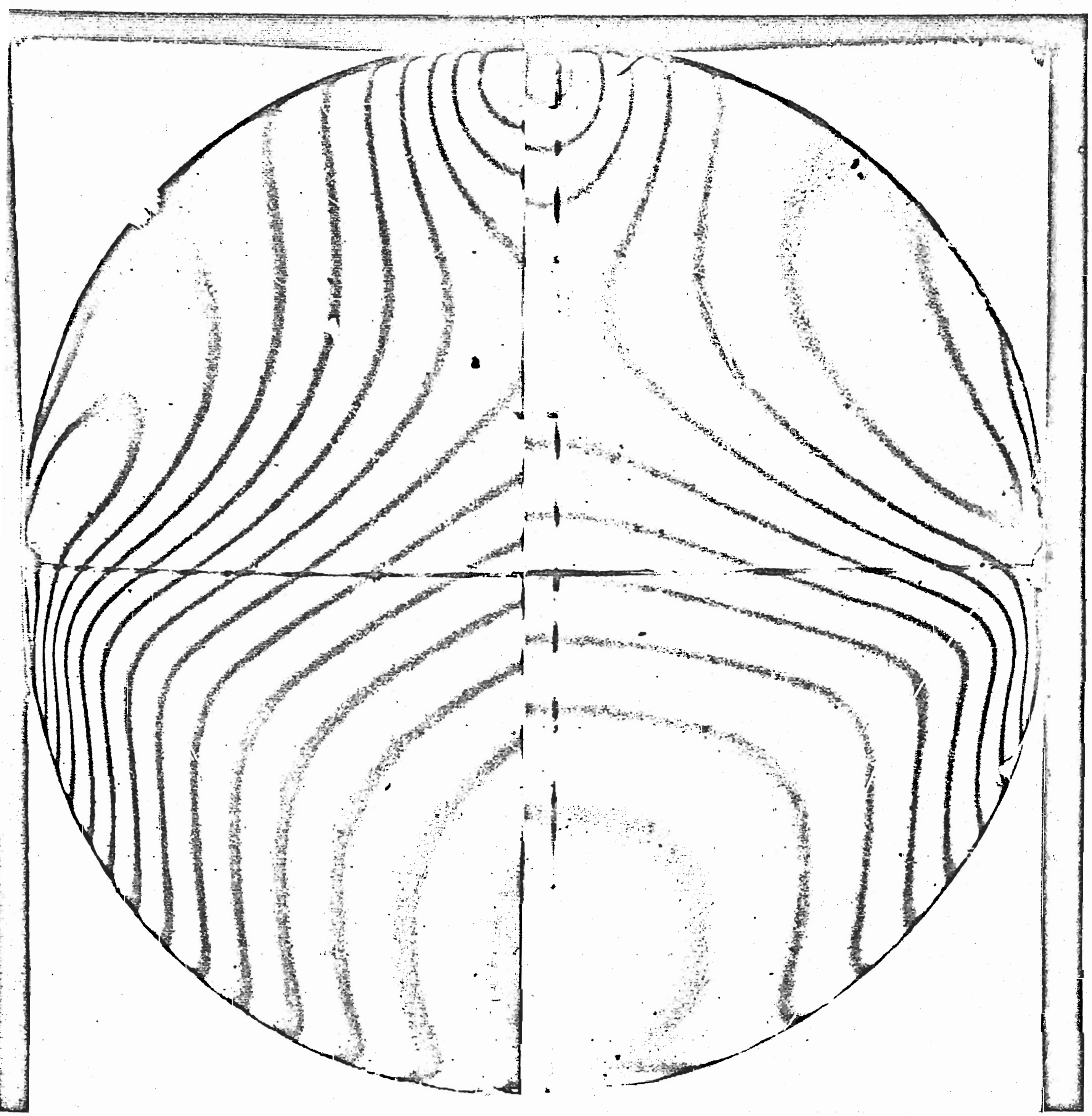

Fig. 6. Determination of Signs along Central Vertical Axis by Rotational Mismatch

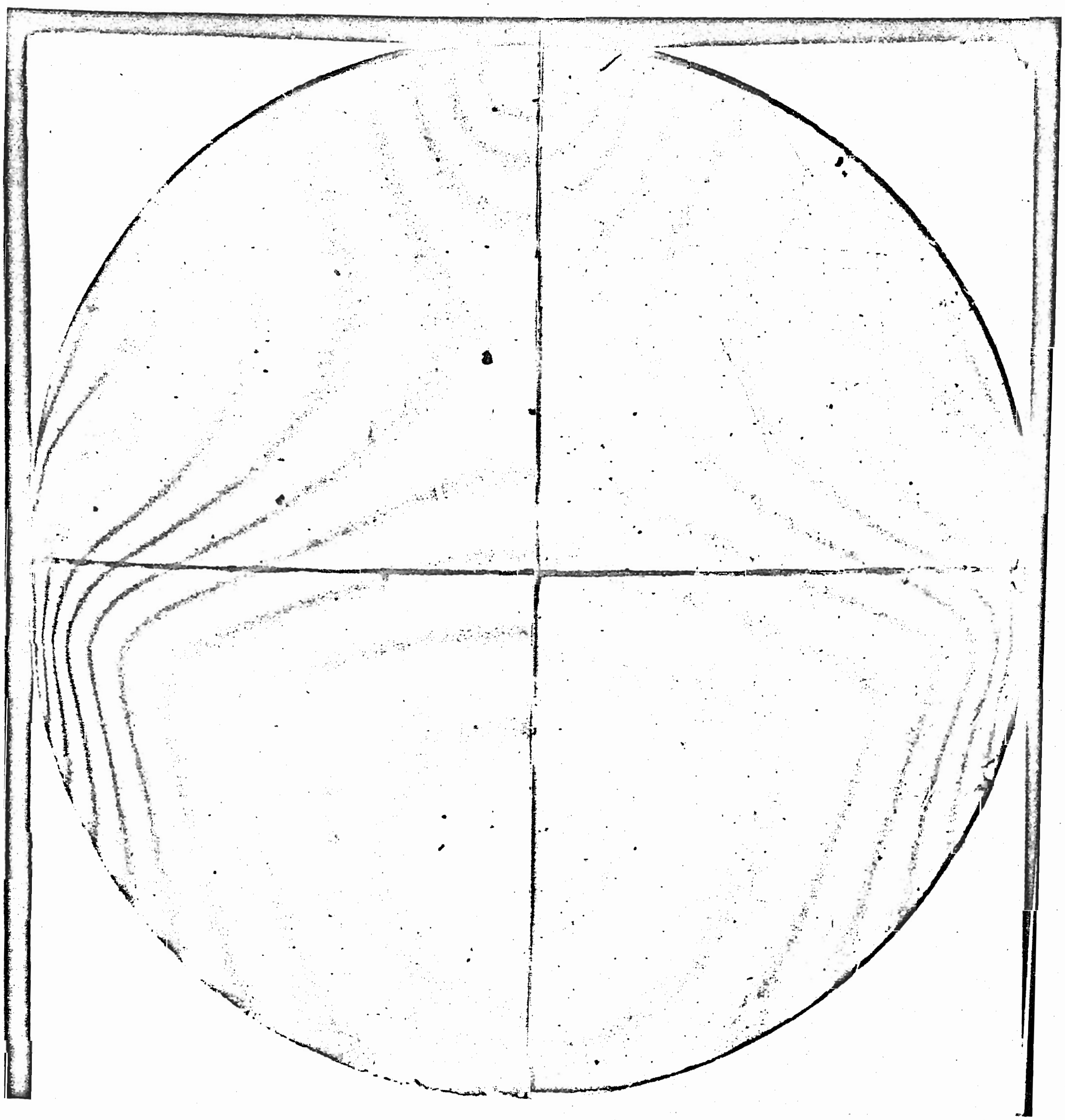

Fig. 7. Determination of Signs by Gap Effect

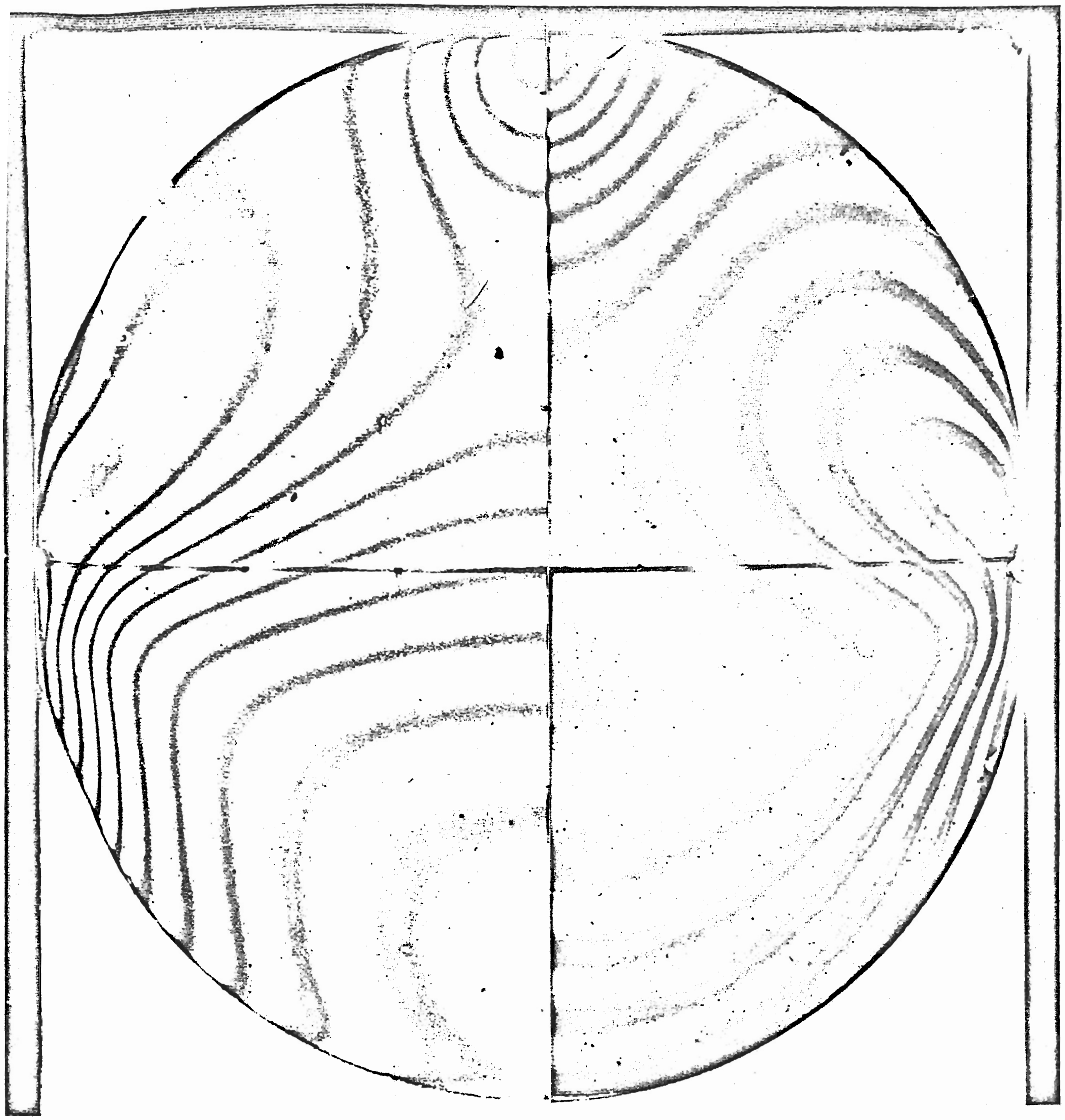

Fig. 8. Behavior of Fringes when the Imposed Linear Mismatch is too Large

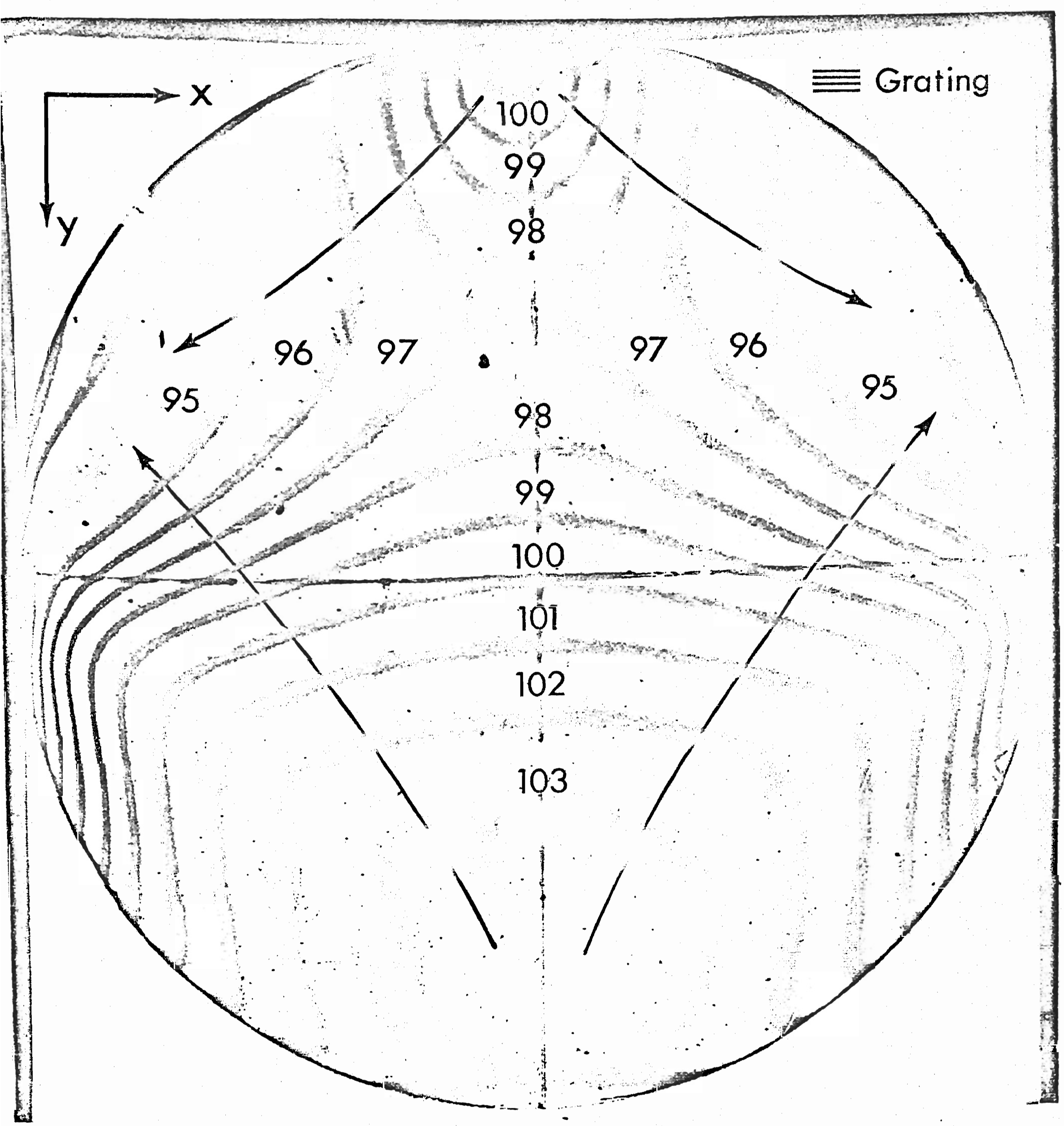


Fig. 9. Fringe Ordering