CBWL WITH FAST RETURNING: A CHANNEL ASSIGNMENT AND USE STRATEGY FOR CELLULAR COMMUNICATION SYSTEMS

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Abstract

In recent work we suggested a new channel sharing method for cellular communications. The method, called Channel Borrowing Without Locking (CBWL) allows real-time borrowing of channels from adjacent cells without the need for channel locking in co-channel cells. CBWL provides enhanced traffic performance in homogeneous environments and also can be used to relieve spatially localized traffic overloads (tele-traffic "hot spots"). It can be applied in currently deployed as well as in next generation systems without additional costly infrastructure. CBWL permits simple channel control management and easy implementation.

This paper describes "fast channel returning", an enhancement of the CBWL scheme. With fast channel returning, a borrowed channel is returned as soon as a regular channel becomes available. CBWL with fast returning reduces unnecessary channel borrowing and improves the performance of CBWL.

System are modeled by multi-dimensional birth-death processes. An efficient method that uses macrostates, decomposition, combinatorial analysis and convolution algorithm is devised to find blocking probabilities. The results, which are also validated by simulation, indicate that channel fast returning can enhance system performance of CBWL scheme.

1 Introduction

A family of new channel assignment and sharing methods for cellular communication systems has been presented in [1] and [2]. The methods are called *channel borrowing without locking* (CBWL). CBWL can be used to enhance traffic capacity of cellular communication systems and to accommodate spatially localized communication traffic overloads (or *"hot spots"*). Variations of the schemes can be considered— but for convenience of presentation and explanation we consider a basic hexagonal layout with base stations (wireless gateways) using omni-directional antennas nominally located at cell centers. The system has a total of C_T channels. With a cluster of size, N, the C_T channels are divided into N groups with about $C = C_T/N$ channels in each group. As in fixed channel assignment (FCA), each gateway is assigned a group of channels which are reused at gateways of other cells that are sufficiently distant for the co-channel interference to be tolerable. However, in CBWL, if all channels of the gateway of a cell are occupied when a new call arrives, channel borrowing is employed according to certain rules.

Channel locking has been suggested in other channel assignment strategies such as dynamic channel assignment (DCA) and hybrid channel assignment (HCA) [3] to limit co-channel interference. That is, gateways within the required minimum reuse distance from a gateway that borrows a channel cannot use the same channel at the same time. Because of the difficulty in maintaining the reuse distance at the minimum value when channel locking is used, DCA and HCA generally perform less satisfactorily than FCA under high communication traffic loads [4], [5], [6].

In CBWL, a channel can be borrowed only from an adjacent gateway. The borrowed channels are temporarily transferred to the gateway that borrows the channel but are used with reduced transmitted power such that the co-channel interference caused by the channel borrowing is no worse than that of non-borrowing scheme. Therefore, *channel locking* is not necessary in CBWL schemes. The borrowed channels can be accessed only in part of the cell. To determine whether a mobile station is in the region that can be served by a borrowed channel, each gateway transmits a signal with the same reduced power as that on a borrowed channel. The signal is called borrowed channel sensing signal (BCSS). If the BCSS is not above some suitable threshold at a mobile station, a borrowed channel cannot be used by the mobile station; otherwise, the mobile station will use a borrowed channel if all its gateway's channels are occupied and any its neighboring gateways has a channel available for lending. Thus, there are two types of new call originations— those that arise in parts of a cell in which a borrowed channel *can* be used if one is available, and those that arise in parts of a cell where borrowed channels *cannot* be used. We denote these as A-type calls and B-type calls, respectively.

Neighboring gateways are identified in the following manner. With respect to the given gateway, choose the first adjacent gateway. The position of the reference adjacent gateway can be arbitrary, but once chosen for a given gateway, all other gateways label their neighbors in a corresponding manner. The remaining five adjacent gateways are numbered sequentially proceeding clockwise from the first. The given gateway is labeled gateway 0. The C channels of a gateway are divided into seven distinct groups. The seven groups are numbered 0, 1, ..., 6. The channels of group 0 are reserved for exclusive use of the given gateway. Channels in the other six groups can be lent to neighbors. The *ith* neighbor can only borrow channels in the *ith* group. The number

of channels in the ith group is denoted C_i, $i = 0, 1, \ldots, 6$. Thus $C = \sum_{k=0}^{6} C_k$. For convenience we consider a symmetrical arrangement with $C_1 = C_2 = \ldots = C_6 = l$. An example of the channel layout structure of CBWL is shown in Figure 1.
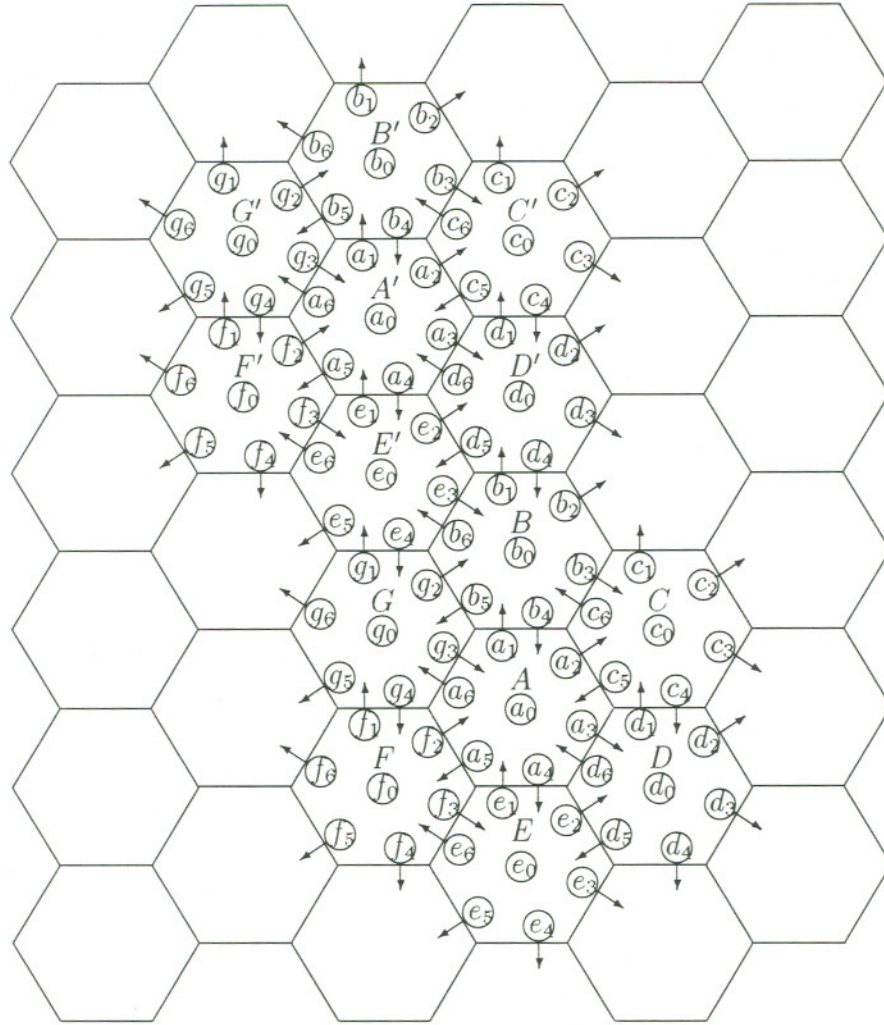

Figure 1: Channel structure of CBWL (cluster size =7).

CBWL with the structure described above has three advantages: 1) In the scheme, a gateway does not need to transmit and receive on all channels of its neighboring gateways. It only needs to access the channels that are assigned to it and the borrowed channels of six groups, one group from one neighbor. Therefore, the transmitter of a gateway only needs to access a total of $C + 6l$ channels instead of $7C$ channels. The cost and complexity of a gateway are reduced. 2) The scheme eliminates the possibility that two co-channel gateways lend the same channel simultaneously to a pair of closely located gateways (which would result in unacceptable co-channel interference). 3) With careful organization, the scheme can ensure that no *adjacent channels* are used in a given cell even though channel borrowing is employed.

As described in [1], *channel rearrangement* can be used in CBWL. With channel rearrangements, if a new B-type call arrival finds all channels of its gateway occupied, the call is *still not*

necessarily blocked. If at the same time an A-type call in the cell is using a regular channel, and at least one neighbor can *lend channels to the given gateway,* the A-type call will use a borrowed channel from a neighbor and give its regular channel to the B-type call. In this way, calls that cannot use borrowed channels directly also benefit from the borrowing scheme. Therefore, the difference of blocking probabilities between two types of calls is lessened and the number of calls that can use borrowed channels (directly or indirectly) is increased. For convenience, We call CBWL without channel rearrangement as CBWL/NR and CBWL with channel rearrangement as CBWL/CR.

Further discussion and comparison of the various channel assignment schemes including FCA, DCA, HCA, Generalized FCA, and Directed Retry is presented in [1]. The reader is referred to [4] –[9] for specific details of the schemes.

It may not be unusual in these schemes for any given gateway *simultaneously* to borrow from and to lend channels to its neighbors—even to the same neighbor from which it has borrowed. This is increasingly likely as traffic loading increases. Thus there can be unnecessary borrowing. Since borrowed channel can only be used by a fraction of users in a cell while regular channels can be accessed by all users in a cell, unnecessary borrowing limits the performance of the scheme at high traffic loading. To increase the number of potential users that can be served by a channel, the number of unnecessary borrowing and lending channels must be reduced. One way to alleviate this problem is to use a cut-off priority structure in which gateways that have more than $m(< C)$ channels occupied, will not lend. Thus at high loading some channels will be available only for calls that arise in the cell. CBWL/NR and CBWL/CR with cut-off priority structure were presented in [2].

Fast returning of borrowed channels is another way to reduce unnecessarily borrowed channels in CBWL. Without fast returning, a borrowed channel is returned only after the call that uses the borrowed channel is completed. With *fast returning,* a call that is using a borrowed channel will be transferred to a regular channel as soon as one is available to service it and the borrowed channel is returned to its owner. The returning process of borrowed channel is accelerated. Thus, no call is served on a borrowed channel if a regular channel (that can accommodate it) is idle. If a gateway has borrowed more than one channel, various strategies can be used to determine which borrowed channel to return. The simplest is to choose one at random. This is relatively easy to implement and to analyze. We use this method in our analysis. Other possibilities include: returning the borrowed channel which was first borrowed; and, priority oriented strategies that tend to return those channels that belong to adjacent gateways with the highest channel occupancy. A channel that becomes available can be one that is freed because of the completion of a call in the given cell, or, one that is returned by another borrowing gateway. Thus *fast returning* a channel can initiate a series of channel returns, bringing the system to a channel use pattern with few calls on borrowed channels.

We call the CBWL/NR that uses fast returning as CBWL/NR-FR; the CBWL/CR that uses fast returning as CBWL/CR-FR.

If a gateway X receives a channel borrowing request from a neighbor Y, the request *is* or *is not* granted depends on the current channel occupancy of X. The following rules are observed.

1. X will deny the request, if the total number of occupied channels of gateway X is more than

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m. Thus gateway X gives a cut-off priority of $C - m$ channels to the calls arising in its cell.

2. X will deny the request, if the number of channels that are lent from X to Y is equal to l.

3. X will deny the request, if the number of total channels of cell X that are lent to other gateways (including Y) is equal to n.

In Section 2, the two types of schemes are modeled and analyzed. Numerical results from analysis and simulation are given in Section 3.

2 Traffic Analysis of CBWL with Fast Returning

2.1 Assumption

1. For simplicity, we limit our analysis to a homogeneous system. That is, each gateway has the same number of channels and the same offered traffic. Our algorithm can be extended to non-homogeneous and hot spot cases.

2. New calls in a cell arise at an average rate λ (new call arrivals per second per cell) according to a Poisson process, and calls originate uniformly throughout the service area. Call holding times have a negative exponential probability distribution with mean $1/\mu$.

3. The fraction of new call arrivals that can use borrowed channels is p.

4. We note that borrowing requests to a given gateway from an adjacent gateway arise from an overflow process (at the adjacent gateway) and therefore do not conform to a Poisson process [10]. However at the adjacent gateway, borrowing requests are randomly split into six parts, only one of which is directed to the given gateway. The random splitting tends to smooth the peakedness of the overflow traffic directed to the given gateway. Thus the borrowing requests directed to a given gateway from an adjacent gateway can be approximated by a Poisson process with intensity λ'.

2.2 Traffic Analysis of CBWL/NR-FR

2.2.1 Equilibrium state distribution of CBWL/NR-FR

Consider a gateway in CBWL/NR-FR and denote the number of used channel of the gateway as x, the number of channels that it *borrows* from its neighbors is denoted as y. With fast returning, if a regular channels becomes available, the gateway will transfer a call that is using a borrowed channel (if any) to the regular channel. The released borrowed channel is returned to the owning gateway. Thus, no channels are borrowed if a regular channel is available. Mathematically, if $x < C$, then $y = 0$. If $x = C$ and $y > 0$, when a regular channel is released, a call that uses a borrowed channel is transferred to the regular channel and the borrowed channel is returned. After completion of fast returning, x is not changed and y is decreased by one. Therefore, unlike the model for CBWL/NR (without fast returning) [1], [2], state of y must be known.

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At any given time a gateway is in one of a finite number of states. A state is identified by a vector $\mathbf{I} = (i_0, i_1, i_2, i_3, i_4, i_5, i_6)$. The component i_0 is the total number of calls that are served through the given gateway, (either on channels that belong to the given gateway or on channels that are *borrowed* from adjacent gateways). The meaning of i_0 is different from that in CBWL/NR where i_0 denote the number of the given gateway's channels that are occupied by the calls originate in the given gateway's own cell. The number of channels at the gateway that are (currently) lent to the kth neighbor is i_k, ($k = 1, 2, \ldots, 6$), where $0 \leq i_k \leq C_k$. Thus, in state $\mathbf{I}$, the total number of calls that are *served through a given gateway* as well as on channels that are *lent to adjacent gateway* is given by

$$J(\mathbf{I}) = \sum_{k=0}^{6} i_k \quad . \tag{1}$$

The total number of channels that are (currently) lent to all adjacent gateways is

$$L(\mathbf{I}) = \sum_{k=1}^{6} i_k \quad . \tag{2}$$

From the last section, the maximum number of channels that a cell can lend at any given time is

$$L_{max} = \min(6l, m, n) \quad . \tag{3}$$

Permissible states of $\mathbf{I} = (i_0, i_1, i_2, \ldots, i_6)$ must satisfy the constraints given in below:

$$
\begin{aligned}
0 \leq \; & i_0 \; \leq C + 6l \\
0 \leq \; & i_k \; \leq l \qquad k = 1, 2, \ldots, 6 \\
0 \leq & J(\mathbf{I}) \leq C + 6l \\
0 \leq & L(\mathbf{I}) \leq L_{max}
\end{aligned}
\tag{4}
$$

The set of all permissible states can be partitioned into two sets: S_1, and S_2. The set S_1 includes all permissible states, $\mathbf{I}$, for which $J(\mathbf{I}) \leq C$. In any state in S_1, the gateway does not borrow any channel. The set S_2 includes all states $\mathbf{I}$, for which $J(\mathbf{I}) > C$. If a gateway is in a state that belongs to S_2, at least one channel is borrowed.

With seven dimension, the number of states can be very large. We will merge the seven dimensions into two dimension in order to reduce the number of state. The probability transition rate in the merged dimension must be able to find easily. As in [1] and [2], we find that the distribution of numbers of channels that are borrowed by each neighbor is in product form. If we merge the six-dimensional variables that represent the number of channels lent to each of six neighbors into a one-dimensional variable that represents the total number of lending channels, a convolution algorithm can be devised to calculate the transition rate in the merged variable.

Equilibrium balance equation of two-dimensional variables, (u, v)

We introduce a macro-state variable $v = \sum_{k=1}^{6} i_k$, which represents the number of channels (belonging to the given gateway) that are lent to all neighbors. Calls that are on these channels ARE NOT served *through* the given gateway. Another variable, $u = i_0$, represents the number of calls that ARE served through the given gateway. This includes calls that are served through the

given gateway on channels that the given gateway BORROWS from its neighbors. From (4), all permissible states of (u, v) are constrained by the following conditions:

$$
\begin{aligned}
0 \le\ &u\ \le C + 6l \\
0 \le\ &v\ \le L_{max} \\
0 \le &u + v \le C + 6l\ .
\end{aligned}
\tag{5}
$$

Let Ω be the set of permissible states (u, v). Define a function, $Z(u, v)$, such that

$$
Z(u, v) \triangleq \begin{cases} 1 & \text{if } (u, v) \in \Omega \\ 0 & \text{if } (u, v) \notin \Omega \end{cases}
\tag{6}
$$

Denote $p(u, v)$ as the equilibrium probability of state $\mathbf{I}$. In statistical equilibrium, the probability flow out of each state (u, v) must equal to the probability flow into that state. Application of this principle leads to a set equations which must be solved to find the state probabilities.

In those permissible states for which $0 \le u + v < m$, the transitions out of (u, v) consist of four parts: that due to new call arrivals; that due to channel lending to neighbors; that due to the completion of the calls of the given cell in state (u, v); that due to the returning of channels that were lent to neighbors. The transition out of state (u, v) due to new call arrivals is given by

$$\{\text{transition out due to new call arrivals}\} = \lambda\ .$$

If the state of a gateway permits to lend channels $(0 \le L(\mathbf{I}) \le L_{max})$, the transition out of state (u, v) due to channel lending to neighbors is the sum of channel borrowing rate from six neighbors given that v channels are lent. Note that if $v \ge l$, it is possible that a specific neighbor borrows l channels from the given gateway, according to our rule, the neighbor cannot borrow any channel from the given gateway. For a given v, there can be many different combinations of i_k's ($k = 1$, ..., 6) such that $\sum_{k=1}^{6} i_k = v$. Each combination can have different channel lending rate (if $i_k < l$, the borrowing rate from the kth gateway is λ', if $i_k = l$, the borrowing rate from the kth gateway is 0). Therefore, an *average* rate of borrowing requests from neighbors given that v channels are lent, $\rho(v)$, is used as the rate of transition out due to channel borrowing demands. The rate is not given but can be determined.

$$\{\text{transition out due to channel borrowing demands}\} = \rho(v)\ .$$

The transition out due to completion of a call served *through* the given gateway is given by

$$\{\text{transition out due to completion of a call served through the gateway}\} = u\mu\ .$$

The transition out of state (u, v) due to the returning of channels that had been lent to neighbors is the sum of channel return rate from six neighbors given that v channels are lent. Without fast returning, returning rate from the kth adjacent gateway is $i_k \mu$. The sum of channel returning rate from six adjacent gateways can be uniquely determined by v. That is $v\mu$. With fast returning, if $i_k > 0$ ($k = 1, \ldots, 6$), when one of C *regular* channels of *the kth adjacent gateway* becomes available, a borrowed channel is returned by the kth adjacent gateway. The returned channel may

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belong to the given gateway or to other gateways. Therefore, with fast returning the channel returning rate from the kth adjacent gateway is a complicated function of i_k. Denote the channel returning rate from kth adjacent gateway as $\mu(i_k)$. The channel returning rate from all neighbors is the sum of $\mu(i_k)$'s. For a given v, there can be a lot of different combinations of i_k's ($k = 1$, ..., 6) such that $\sum_{k=1}^{6} i_k = v$. Each combination can have different channel returning rate. Given that v channels are lent, we can find an *average* channel returning rate. We denote the rate as $\beta(v)$. Thus

$$\{\text{transition out due to returning of channel}\} = \beta(v) \ .$$

Now let us consider the probability transition components *into* state (u, v) with $0 \leq u + v < m$. The probability transitions into (u, v) consist of four parts: that due to new call arrivals from a permissible state $(u - 1, v)$; that due to channel lending to neighbors; that due to channel returned from neighbor; and, that due to completions of calls that are served by the given gateway. The first part is given by

$$\{\text{transition in due to new call arrivals}\} = \lambda \ .$$

The transition into (u, v) due to channel lending is given by

$$\{\text{transition in due to channel lending}\} = \rho(v - 1) \ .$$

The transition into (u, v) due to channel returned from adjacent gateways is given by

$$\{\text{transition in due to channel returning}\} = \beta(v + 1) \ .$$

The transition into (u, v) due to completions of calls that are served by the given gateway is

$$\{\text{transition in due to completions of calls that are served by the given gateway}\} = u\mu \ .$$

In any permissible state (u, v) with $0 \leq u + v < m$, the flow balance equation is

$$[\lambda + \rho(v) + u\mu + \beta(v)] \, p(u, v) = \lambda p(u - 1, v) Z(u - 1, v) + \rho(v - 1) p(u, v - 1) Z(u, v - 1)$$
$$+ (u + 1)\mu p(u + 1, v) + \beta(v + 1) p(u, v + 1) Z(u, v + 1)$$
$$(0 \leq u + v < m) \tag{7a}$$

In any permissible state (u, v) with $u + v = m$, no channel is lent, the flow balance equation is

$$[\lambda + u\mu + \beta(v)] \, p(u, v) = \lambda p(u - 1, v) Z(u - 1, v) + \rho(v - 1) p(u, v - 1) Z(u, v - 1)$$
$$+ (u + 1)\mu p(u + 1, v) + \beta(v + 1) p(u, v + 1) Z(u, v + 1)$$
$$(u + v = m) \tag{7b}$$

In any permissible state (u, v) with $m < u + v < C$, the transition in due to channel borrowing demands is zero. The flow balance equation is

$$[\lambda + u\mu + \beta(v)] \, p(u, v) = \lambda p(u - 1, v) Z(u - 1, v)$$
$$+ (u + 1)\mu p(u + 1, v) + \beta(v + 1) p(u, v + 1) Z(u, v + 1)$$
$$(m < u + v < C) \tag{7c}$$

In any permissible state (u, v) with $u + v \geq C$, all channels of the given gateway are occupied. A new call arrival will use a borrowed channel if the gateway can borrow one from neighbors. The more channels the gateway borrows, the smaller the probability that its next borrowing is successful is. Thus, channel borrowing rate is a function of the number of channels that have been borrowed by the given gateway. Denote $\alpha(y)$ as the average channel borrowing rate given that y channels have been borrowed. The transition out of (u, v) due to channel borrowing is

$$\{\text{transition out due to channel borrowing}\} = \alpha(u + v - C) \, .$$

In any permissible state (u, v) with $u + v = C$, The flow balance equation is

$$[\alpha(0) + u\mu + \beta(v)] p(u, v) = \lambda p(u - 1, v) Z(u - 1, v) + (u + 1)\mu p(u + 1, v)$$
$$+ \beta(v + 1) p(u, v + 1) Z(u, v + 1)$$
$$(u + v = C) \tag{7d}$$

In any permissible state (u, v) with $C < u + v \leq C + 6l$, The flow balance equation is

$$[\alpha(u + v - C) + u\mu + \beta(v)] p(u, v) = \alpha(u + v - C - 1) p(u - 1, v)$$
$$+ (u + 1)\mu p(u + 1, v) Z(u + 1, v) + \beta(v + 1) p(u, v + 1) Z(u, v + 1)$$
$$(C < u + v \leq C + 6l) \tag{7e}$$

If $m = C$ (no cut-off priority), the flow balance equations are only distinguished for three cases. For any state in $0 \leq u + v < C$, balance equation is given by (7a). For any state with $u + v = C$, the balance equation is given by

$$[\alpha(0) + u\mu + \beta(v)] p(u, v) = \lambda p(u - 1, v) Z(u - 1, v) + \rho(v - 1) p(u, v - 1) Z(u, v - 1)$$
$$+ (u + 1)\mu p(u + 1, v) + \beta(v + 1) p(u, v + 1) Z(u, v + 1) \tag{7f}$$
$$(u + v = C)$$

For any state in $C < u + v \leq C + 6l$, the balance equation is given by (7e).

Figure 2. shows the state-transition diagram of an example of CBWL/NR-FR with $C = 5$, $m = 4$. In the figure, for simplicity, the maximum number of channels that a gateway can borrow is 3. For homogeneous hexagonal cellular system, the maximum number of channels that a gateway can borrow is multiples of six.

Probability, $p(u, v)$, is not in product form. Gauss-Seidel iteration [11] is employed to find the solution of the equations. Since the number of states has been reduced greatly in the two-dimensional model, the computation work is reduced. To solve the equations, the value of $\rho(v)$, $\beta(v)$ and $\alpha(y)$ must be found.

Average lending rate, $\rho(v)$

For a given gateway, the average rate of its kth adjacent gateway's borrowing requests to it is λ' when $i_k < l$, and the channel returning rate to it from the kth neighbor is $\mu(i_k)$ (see Appendix A). Define $f(x)$ as

$$f(x) \triangleq (\lambda')^x / \prod_{t=1}^{x} \mu(t) \, . \tag{8}$$
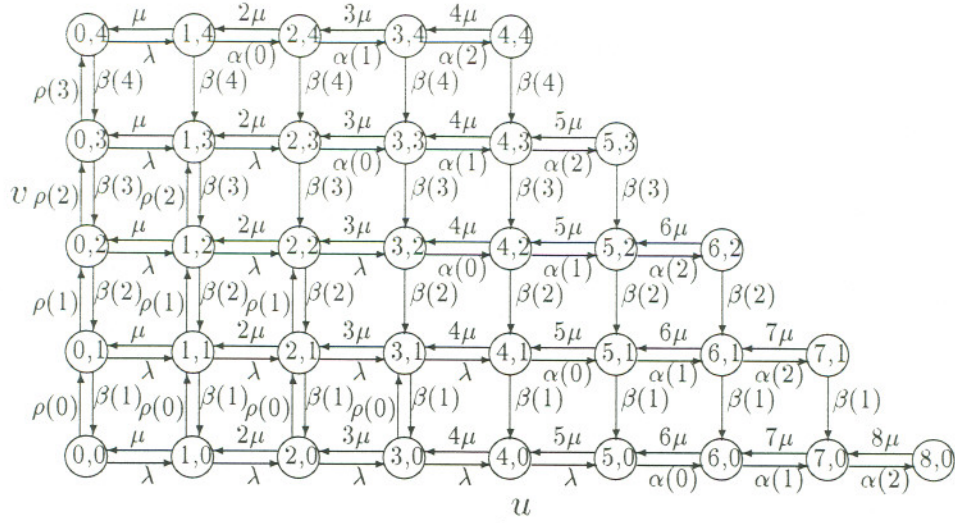
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Figure 2: State-transition diagram of two-dimensional macro-state (u, v) for a CBWL/NR-FR with $C = 5$ and $m = 4$ and $m = 4$ and $6l=3$.

Given that v channels of the given gateway are lent to total neighbors, the distribution of number of channels that are borrowed by each of neighboring gateways is in the following product form:

$$\Pr(i_1, i_2, i_3, i_4, i_5, i_6 | \sum_{k=1}^{6} i_k = v) = \frac{1}{b(v)} \prod_{k=1}^{6} f(i_k) \; .. \tag{9}$$

in which, $b(v)$ is normalization constant. It is equal to the sum of all probabilities that defined by (9).

Using the convolution algorithm that is described in [2], we can calculate the normalization constant $b(v)$, and the probability that exactly $6 - t$ adjacent gateways have borrowed exactly l channels from a given gateway given that total v channels of that gateway are lent. The probability is denoted as $b(t, v)/b(v)$ in [2]. If $6 - t$ gateway have borrowed l channels, they cannot borrow any more channels from the given gateway, the rate of borrowing requests from all adjacent gateways becomes $t\lambda'$. Thus,

$$\rho(v) = \frac{\lambda'}{b(v)} \sum_{t=1}^{6} tb(t, v) \; . \tag{10}$$

Average channel returning rate from all neighbors, $\beta(v)$

Define a six-vector

$$\mathbf{I_6} \triangleq (i_1, i_2, i_3, i_4, i_5, i_6).$$

Define $S(v, 6)$ as the set of six-vectors whose components sum to v. That is

$$S(v, 6) \triangleq \{\mathbf{I_6} = (i_1, i_2, i_3, i_4, i_5, i_6) : 0 \leq i_k \leq l, \sum_{k=1}^{6} i_k = v\} \; . \tag{11}$$

The returning rate from the first, second, ..., sixth gateways are $\mu(i_1)$, $\mu(i_2)$, ..., $\mu(i_6)$, respectively. The overall channel returning rate from all neighbors is the sum of $\mu(i_1)$, $\mu(i_2)$, ...,

$\mu(i_6)$. Consider all possible $\mathbf{I_6}$ that are in $S(6, v)$, we can find the average rate of channel returning from neighbors by (9). That is

$$\beta(v) = \frac{1}{b(v)} \sum_{\mathbf{I_6} \in S(v,6)} \left[\prod_{k=1}^{6} f(i_k) \sum_{t=1}^{6} \mu(i_t) \right] . \tag{12}$$

Although (12) can be used to calculate $\beta(v)$, we will not use it since many numerical operations are required. In Appendix B, we have shown that $\beta(v)$ can be easily computed from $b(t, v)/b(v)$, which can be found through the convolution algorithm. Specifically,

$$\beta(v) = \frac{\lambda'}{b(v)} \sum_{t=1}^{6} t b(t, v-1) \quad v = 1, \ldots, L_{max} \tag{13}$$

Channel borrowing rate, $\alpha(j)$

The quantity, $\alpha(j)$, is channel borrowing rate of a gateway when it has borrowed j channels. For CBWL/NR-FR, only a fraction of calls in a cell can use borrowed channels. The fraction is λp. The probability that a borrowing request is accepted by adjacent gateways given that j channels have been borrowed by the given gateway is denoted as $p_{bs}(j)$. The probability, $p_{bs}(j)$ will be determined in Appendix C. Thus,

$$\alpha(j) = \lambda p p_{bs}(j) \qquad j = 0, \ldots, 6l. \tag{14}$$

Once $\rho(v)$, $\beta(v)$ and $\alpha(j)$ are found, the equations (7) are determined. We can use usual numerical method to find $p(u, v)$.

2.2.2 Some Important Probabilities and Performance Measurement

From state probability $p(u, v)$, some important probabilities and blocking probabilities of each type of calls can be computed.

Probability of number of borrowed channels, $p_{br}(j)$

From $p(u, v)$, the distribution of borrowed channels can be calculated. The distribution is necessary for determination of $\mu(i)$ (in Appendix A). With fast returning, a gateway uses borrowed channels, only if $u + v > 0$. The number of borrowed channels is equal to $u + v - C$. Thus,

$$p_{br}(j) = \sum_{v=0}^{L_{max}} p(C + j - v, v), \qquad j = 0, 1, \ldots, 6l \tag{15}$$

in which $p_{br}(0)$ is the probability that no channel is borrowed *and* all channels of the gateway are occupied.

Probability that all channels of a gateway are occupied p_c

The probability that all channels of a gateway are occupied is the sum of state probabilities with $u + v \geq C$. From (15), we have

$$p_c = \sum_{j=0}^{6l} p_{br}(j) . \tag{16}$$

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Probability that a borrowing request of the given gateway is denied by a specific adjacent gateway: p_f

A borrowing request from a given gateway will be denied by a specific adjacent gateway if any of the following three events are true in the neighbor at the time that the borrowing request arises: 1) E_1, all channels of the neighbor are occupied; 2) E_2, the total lending channels of the neighbors equal to the maximum possible number, L_{max}; 3) E_3, the given gateway has already borrowed its maximum allowable channel quota, (l channels) from the neighbor. Thus, the probability, p_f, is the probability of the union of E_1, E_2 and E_3. That is,

$$p_f = \Pr\{E_1 \bigcup E_2 \bigcup E_3\} = \Pr(E_1) + \Pr(E_2 \bar{E}_1) + \Pr(E_3 \bar{E}_2 \bar{E}_1) \tag{17}$$

The first two terms of (17) are the probability that event E_1 or E_2 occurs at an adjacent gateway. We denote the probability as p_e, which will be used to find $p_{bs}(j)$ in Appendix C. We calculate it first.

The event E_1 includes all states with $u + v \geq m$. Its probability can be divided into two parts: one is p_c, other is the sum of $p(u,v)$'s with $m \leq u + v < C$. That is,

$$\Pr(E_1) = p_c + \sum_{v=0}^{L_{max}} \sum_{u=m-v}^{C-1-v} p(u,v) . \tag{18}$$

The event $\bar{E}_1 E_2$ consists of all permissible states with $v = L_{max}$ and $u + v < m$. That is,

$$\Pr(E_2 \bar{E}_1) = \sum_{u=0}^{m-1-L_{max}} p(u, L_{max}) . \tag{19}$$

Thus,

$$p_e = p_c + \sum_{u=0}^{m-1-L_{max}} p(u, L_{max}) + \sum_{v=0}^{L_{max}} \sum_{u=m-v}^{C-1-v} p(u,v) . \tag{20}$$

The third term in (17) is derived as follows. Since it is a homogeneous system, if s adjacent gateways of the considered gateway have borrowed l channels from the gateway, the probability that the given gateway is among those s gateways is $s/6$. Thus,

$$\Pr(E_3 \bar{E}_2 \bar{E}_1) = \sum_{v=l}^{L_{max}-1} \sum_{s=1}^{6} \frac{s}{6} \Pr(s \text{ gateways borrow } l \text{ channels} | v \text{ channels are lent})$$
$$\cdot \Pr(0 \leq u + v < m | v) \Pr(v \text{ channels are lent}) . \tag{21}$$

From [2], we know that the probability that s adjacent gateways have borrowed l channels given that v channels are lent is $b(6 - s, v)/b(v)$. And from the definition of conditional probability, we can get

$$\Pr(0 \leq u + v < m | v) = \sum_{u=0}^{m-v-1} p(u,v)/\Pr(v \text{ channels are lent}) . \tag{22}$$

Thus,

$$\Pr(E_3 \bar{E}_2 \bar{E}_1) = \sum_{v=l}^{L_{max}-1} \sum_{s=1}^{6} \frac{b(6 - s, v)}{b(v)} \frac{s}{6} \sum_{u=0}^{m-v-1} p(u,v) . \tag{23}$$

From (17), we have

$$p_f = p_e + \sum_{v=l}^{L_{max}-1} \sum_{s=1}^{6} \frac{b(6-s,v)}{b(v)} \frac{s}{6} \sum_{u=0}^{m-v-1} p(u,v) .$$ (24)

The average rate borrowing requests to a neighbor, λ'

First we consider the average channel borrowing rate of the given gateway from a specific neighbor. Denote the rate as λ''. The channel borrowing rate of the given gateway from the neighbor given j channels are borrowed is $\alpha(j)$. Thus, the average channel borrowing rate is

$$\lambda'' = \frac{1}{6} \sum_{j=0}^{6l} p_{br}(j)\alpha(j) .$$ (25)

Denote λ' as the average borrowing *request* rate of the given gateway to the specific neighbor. The probability that those requests are accepted by the neighbor is $1 - p_f$. That is

$$\lambda'(1 - p_f) = \lambda'' .$$ (26)

From (26), we find

$$\lambda' = \frac{1}{6(1-p_f)} \sum_{j=0}^{6l} p_{br}(j)\alpha(j) .$$ (27)

Blocking probability

In CBWL/NR-FR, the blocking probability of calls that cannot use borrowed channels is denoted as $\beta_{NR\text{-}FR}$. Those calls are blocked if all channels of a gateway are occupied. So, $\beta_{NR\text{-}FR} = p_c$. The blocking probability of calls that can use borrowed channels is denoted as $\alpha_{NR\text{-}FR}$. Those calls are blocked if all channels of a gateway are occupied and their borrowing requests are rejected by all neighbors. If the given gateway has borrowed j channels, the probability that the borrowing requests are denied by neighbors is $1 - p_{bs}(j)$ [see Appendix C: (C.3)]. Thus,

$$\alpha_{NR\text{-}FR} = \sum_{j=0}^{6l} p_{br}(j)[1 - p_{bs}(j)] = \sum_{j=0}^{6l} p_{br}(j) \sum_{s=s_1}^{s_2} \frac{6!}{s!(6-s)!} \frac{a(j-sl,6-s,l-1)}{a(j,6,l)} [p_e]^{6-s} .$$ (28)

The overall blocking probability $B_{NR\text{-}FR}$ is

$$B_{NR\text{-}FR} = p\alpha_{NR\text{-}FR} + (1-p)\beta_{NR\text{-}FR} .$$ (29)

2.2.3 Iterative procedure

As in [1] and [2], an iterative procedure is used to obtain p_f, p_c, p_e and λ'. The iterative procedure is described as follows:

Step 1: The procedure starts with an arbitrary guess of λ', p_e and a group of $\mu(i)$'s, $i = 1, \ldots, l$. Our experiences show that $\mu(i)$'s are close to C and they do not vary significantly with i.

Step 2: Use the last updated λ' and $\mu(i)$'s in convolution algorithm that is describe in [2] to calculate $b(t,v)/b(v)$. Then, $\rho(v)$ and $\beta(v)$ can be found from (10) and (13).

Step 3: Use the last updated p_e in (C.3) to find $p_{bs}(j)$, which is used in (14) to find $\alpha(y)$.

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Step 4: Use updated $\rho(v)$, $\beta(v)$ and $\alpha(y)$ in (7a)–(7f). Solve the equations by Gauss-Seidel iteration to get $p(u,v)$'s.

Step 5: From $p(u,v)$'s get $p_{br}(j)$, p_c, p_e and p_s. Update $\mu(i)$ by (A.3). Update the overflow traffic λ' by (27).

Step 6: Compare old λ', p_e and $\mu(i)$ with updated ones. If the differences between old values and new values are greater than desired quantities, go back to step 2 and start a new iteration. Otherwise, stop.

2.3 Analysis of CBWL/CR-FR

If the number of the gateway's channels that are used by A-type calls is zero, channel rearrangement cannot be used for a new B-type call. Therefore, in CBWL/CR-FR, to know the number of a gateway's channels that are used by A-type and B-type calls, we need two state variable: i_a and i_b. Variable, i_a, is the number of channels occupied by A-type calls and i_b is the number of channels occupied by B-type calls. We denote p_a as the probability that B-type calls cannot make channel rearrangement. To find p_a, a two-step *decomposition* procedure is used. The method divide all state space into $L_{max}+1$ subspaces, each of which corresponds to a fixed value of $v = \sum_{k=1}^{6} i_k$ ($v = 0, 1, \ldots, L_{max}$). The conditional distribution, $\Pr(i_a, i_b|v)$ can be calculated separately for each fixed v. However, because i_a and i_b cannot be completely separated from the other variables, the decomposition method is an approximation. If $\lambda \gg \lambda'$, the interactions between i_a and i_b are much stronger than the interactions between i_0 and other i_k's ($k \geq 1$). We can calculate $\Pr(i_a, i_b|v)$ separately for each fixed v and *neglect* the interactions between (i_a, i_b) and other i_k's ($k \geq 1$) as if those interactions do not exist [14]. The agreement between results of simulation and analysis validates this approximation.

2.3.1 First Step

In the first step, the purpose is to find the probability that a gateway lends v channels to neighbors, $p_l(v)$. In this step, we use macro-states (u,v) in CBWL/NR-FR in which i_a and i_b are not distinguished. The equilibrium state distributions of the states (u,v) can be calculated from (7). However, to use (7) in CBWL/CR-FR, channel borrowing rate of a gateway, $\alpha(j)$, is not defined by equation (14). Because with channel rearrangement, not only A-type calls can use borrowed channels directly, but some B-type calls can also use borrowed channels indirectly. The probability that a B-type call can use channel rearrangement is $1 - p_a$, so the borrowing rate that raised by B-type calls is $(1-p)(1-p_a)\lambda$. Thus, the average channel borrowing rate given that j channels are lent to adjacent gateways is

$$\alpha(j) = \lambda[p + (1-p)(1-p_a)]p_{bs}(j) . \tag{30}$$

From $p(u,v)$, the probability $p_l(v)$ can be found by

$$p_l(v) = \sum_{u=0}^{C+6l-v} p(u,v) \qquad v = 0, 1, \ldots, L_{max} . \tag{31}$$

The probabilities, p_c, p_f, and p_e can be determined from $p(u,v)$ as in CBWL/NR-FR. A probability that will be used in the second step is the probability that no channel is borrowed given that v channels are lent *and* all channels are occupied. The conditional probability is given by

$$p_{nb}(v) = p(C - v, v) / \sum_{u=C-v}^{C+6l-v} p(u,v) \qquad v = 0, \ldots, L_{max} .$$

(32)

2.3.2 Second Step

In the second step we find p_a. Given v channels are lent, we construct balance equations of i_a and i_b. The two-dimensional equations can be solved for every possible v to find conditional distribution, $p_v(i_a, i_b)$. From the distribution, we can find the probability that all remaining channels of the given gateway are occupied by B-type calls given that v channels are lent, $p_v(0, C - v)$. It is the probability that B-type calls cannot use channel rearrangement given that v channels are lent. Thus, p_a can be found from

$$p_a = \sum_{v=0}^{L_{max}} p_v(0, C - v) p_l(v) .$$

(33)

Denote $p_v(i_a, i_b)$ as the equilibrium distribution of (i_a, i_b) given that v channels are lent. If v channels have been lent, all permissible states of i_a and i_b are constrained by following conditions:

$$\begin{aligned}
0 \leq \quad i_a \quad &\leq C - v \\
0 \leq \quad i_b \quad &\leq C - v \\
0 \leq i_a + i_b &\leq C - v .
\end{aligned}$$

(34)

Denote λ_1 and λ_2 as the arrival rate of A-type and B-type calls, respectively. That is,

$$\lambda_1 = p\lambda ,$$

(35)

and

$$\lambda_2 = (1 - p)\lambda .$$

(36)

If a gateway is in a state (i_a, i_b) with $i_a + i_b = C - v$ and $i_a > 0$, a channel will be borrowed through channel rearrangement when a B-type call arrives. If the channel borrowing request is accepted by an adjacent gateway, an A-type call is transferred to the borrowed channel and the released regular channel is given to the new B-type call. Thus, the gateway's state is changed to $(i_a - 1, i_b + 1)$. Denote this transition rate as λ_3. If the gateway has borrowed j channels, the probability that its borrowing request is accepted by its neighbors is $p_{bs}(j)$. Thus,

$$\lambda_3 = \lambda(1 - p) \sum_{j=0}^{6l} \frac{p_{br}(j)}{p_c} p_{bs}(j) .$$

(37)

The channel releasing rate by A-type calls or B-type calls at state, (i_a, i_b), with $i_a + i_b < C - v$ is $i_a \mu$ or $i_b \mu$, respectively.

We consider the channel releasing rate in a state with $i_a + i_b = C - v$, In those states, the gateway may use some borrowed channels with probability of $1 - p_{nb}(v)$, or it may not borrow any channels with probability of $p_{nb}(v)$ (32). State transitions are different for the two cases. When a channel is released by an A-type call, if the gateway does not borrow any channel, the state is changed to $(i_a - 1, i_b)$ with the rate of $p_{nb}(v)i_a\mu$. When a channel is released by a B-type call and no channel is borrowed, the state is changed to $(i_a, i_b - 1)$ with the rate of $p_{nb}(v)i_b\mu$. If the gateway borrows at least one channel and a channel is released, the released channel is at once given to a call that uses a borrowed channel, and the borrowed channel is returned to its owner. Because the call that is transferred from the borrowed channel must be a call of type A, the size of i_a is increased by one. If the channel is released by an A-type call, the size of i_a must be decreased by one. The overall effect is that the (i_a, i_b) is not changed. If the channel is released by a B-type call, the size of i_b is decreased by one. Thus, the state is changed to $(i_a + 1, i_b - 1)$ with the rate of $(1 - p_{nb}(v))i_b\mu$.

The state-transition diagram of the conditional probabilities for $C - v = 6$ is shown in Figure 3.
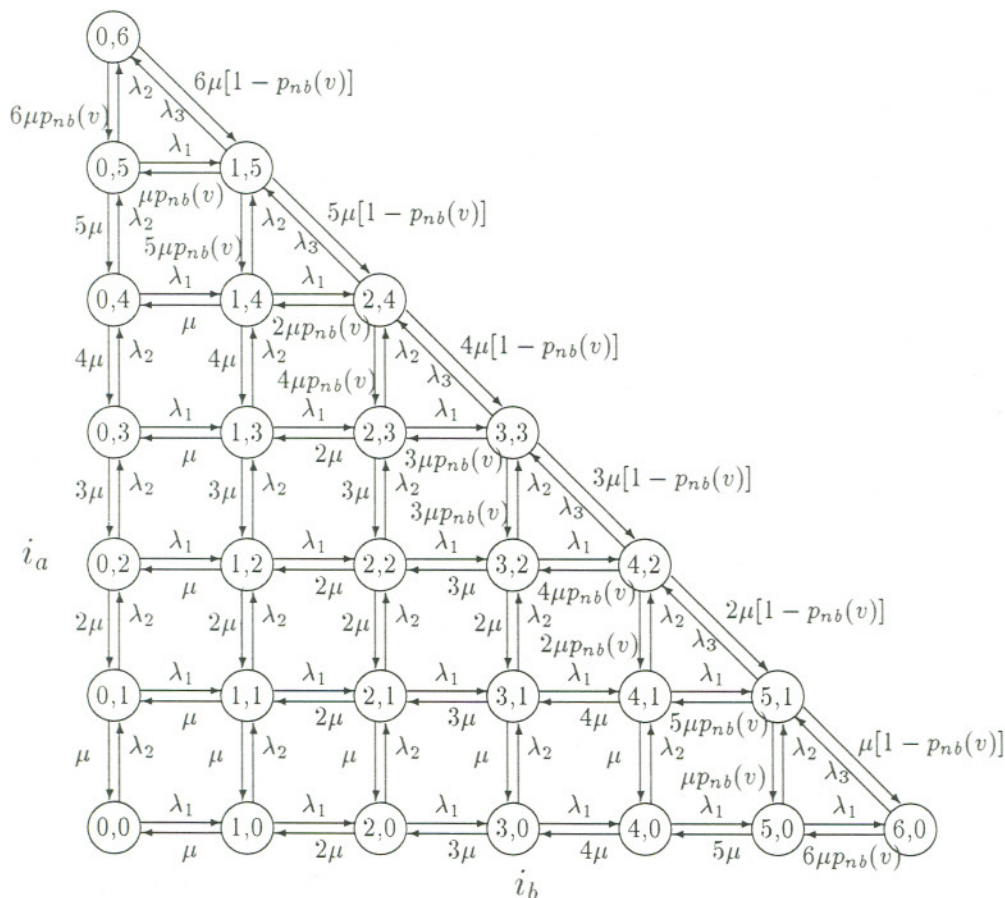

Figure 3: An example of transition diagram for macro-state (i_a, i_b) of CBWL/CR-FR, $(C - v = 6)$.

The flow balance equations of $p_v(i_a, i_b)$ are as follows:

$$[\lambda + (i_a + i_b)\mu]p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1)$$

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$$+(i_a+1)\mu p_v(i_a+1,i_b)+(i_b+1)\mu p_v(i_a,i_b+1)$$
$$(0 \le i_a+i_b < C-v-1)$$
$$[\lambda+(C-v-1)\mu]\,p_v(i_a,i_b) = \lambda_1 p_v(i_a-1,i_b)+\lambda_2 p_v(i_a,i_b-1)$$
$$+p_{nb}(v)(i_a+1)\mu p_v(i_a+1,i_b)+p_{nb}(v)(i_b+1)\mu p_v(i_a,i_b+1)$$
$$(i_a+i_b=C-v-1) \tag{38}$$
$$[\lambda_3+p_{nb}(v)i_a\mu+i_b\mu]p_v(i_a,i_b) = \lambda_1 p_v(i_a-1,i_b)+\lambda_2 p_v(i_a,i_b-1)$$
$$+\lambda_3 p_v(i_a+1,i_b-1)+(1-p_{nb}(v))(i_b+1)\mu p_v(i_a-1,i_b+1)$$
$$(i_a+i_b=C-v,i_a>0)$$
$$(C-v)\mu p_v(0,C-v) = \lambda_2 p_v(0,C-v-1)+\lambda_3 p_v(1,C-v-1)$$
$$(i_a=0,\ i_b=C-v)$$

where $p_v(x,y)=0$, if $x<0$ or $y<0$.

The balance equations are solved by Gauss-Seidel Iteration. Substitute $p_v(0,C-v)$ in (33) as $\Pr(0,C-v|v)$ and p_a is computed. In (33), $L_{max}+1$ groups of equations of (38) with v from 0 to L_{max} must be solved. However, the number of equation groups to be solved can be reduced greatly. Since fast returning reduces the usage of borrowed channels, the probability that a gateway lents a lot of channels is quite low. In our algorithm, when $p_l(v)$ is less than a desired precision, the contribution of the corresponding term in (33) to p_a is very small. It is not necessary to solve the equations that corresponds to that v.

2.3.3 Blocking probability and average rate of borrowing requests

Blocking probability

Besides p_a, in the two-step procedure, we also find probabilities, p_c, p_f and p_e. From them, blocking probability can be determined. First, the blocking probability of A-type calls, $\alpha_{CR\text{-}FR}$, is the same as $\alpha_{NR\text{-}FR}$ (28). If a B-type call can use channel rearrangement, it has the same blocking probability as an A-type call. If a new B-type call find all channels occupied and it cannot use channel rearrangement, it will be blocked. Thus

$$\beta_{CR\text{-}FR} = (1-\frac{p_a}{p_c})\alpha_{CR\text{-}FR}+p_a. \tag{39}$$

The overall blocking probability in a gateway is

$$B_{CR\text{-}FR} = p\alpha_{CR\text{-}FR}+(1-p)\beta_{CR\text{-}FR}. \tag{40}$$

Average rate of borrowing requests from one gateway to another gateway λ'

In CBWL/CR-FR, the average rate of borrowing requests from one gateway to another gateway, λ', is the same as equation (27). However, now $\alpha(j)$ in this equation is defined by (30).

2.3.4 Iterative Procedure

An iterative procedure is used to obtain λ' and p_c, p_f, p_e and p_a simultaneously.

step 1 Guess reasonable values for p_c, p_f, p_e, p_a, $\mu(i)$'s, $i = 1, \ldots, l$. Use p_c, p_f, p_a in (27) to calculate λ'.

step 2 Use last updated λ' and $\mu(i)$'s in convolution algorithm to calculate $b(t, v)/b(v)$. Then, $\rho(v)$ and $\beta(v)$ can be found from (10) and (13).

step 3 Use last updated p_e in (C.3) to find $p_{bs}(j)$, which is used in (30) to find $\alpha(y)$.

step 4 Use newly updated $\rho(v)$, $\beta(v)$ and $\alpha(y)$ in (7). Solve the equations by Gauss-Seidel iteration to get $p(u, v)$'s.

step 5 From $p(u, v)$'s get $p_{br}(j)$, $p_l(v)$, p_c, p_e and p_f. Update $\mu(i)$ by (A.3).

step 6 For any v, with $p_l(v)$ large enough, $(v = 0, \ldots, L_{max})$, solve (38) by Gauss-Seidel iteration to get $\Pr(0, C - v|v)$.

step 7 Calculate p_a with equation (33).

step 8 Update λ' using Equation (27).

step 9 Compare old p_c, p_k, p_f, p_e with updated ones. If all of them agree within the desired number of significant figures, the iteration is stopped. Otherwise, go back to step 2.

The reasonable initial values of p_c, p_f, p_e, p_a, $\mu(i)$'s is that $0 < p_c < 1$, $0 < p_f < 1$, $0 < p_a < 1$, $0 < p_e < 1$, and $p_a < p_c < p_e < p_f$, $p_e \approx p_f$, $\mu(i)$'s, $i = 1, \ldots, l$, are about C and do not vary much with i. If we choose initial values in those ranges, we found that the algorithm converged.

3   Numerical Results And Discussion

In our numerical examples, we consider CBWL/NR-FR and CBWL/CR-FR schemes for a mobile system with 24 channels in each gateway. For simplicity, we assume in the homogeneous case, that the system has a very large (essentially infinite) number of cells. Thus we do not need to distinguish the boundary cells and the internal cells.

Figure 4 shows overall blocking probability of CBWL/NR-FR obtained by numerical computation plotted against offered traffic in a cell. Simulation confidence intervals of 95% are also shown. Figure 5 is a similar plot for CBWL/CR-FR.

From the figures, we can see that the results of analysis are close to those by simulation. In Figure 5, a slight difference of results from analysis and simulation occurs when $p = .3$ and heavy offered traffic (more than 22 Erlangs). In that condition, a lot of channel borrowing is involved and λ' is increased significantly. The assumption of $\lambda' \ll \lambda$ that is used in the decomposition method is void. Thus, the results of analysis underestimate blocking probability in this range. The results displayed in Figure 4 and 5 compare the performance of FCA (for which $p = 0$) to the CBWL scheme with $p = 0.1$, 0.2 and 0.3. It is seen that CBWL/NR-FR cannot improve system performance significantly. While CBWL/CR-FR can reduce the blocking probability by order of magnitudes.
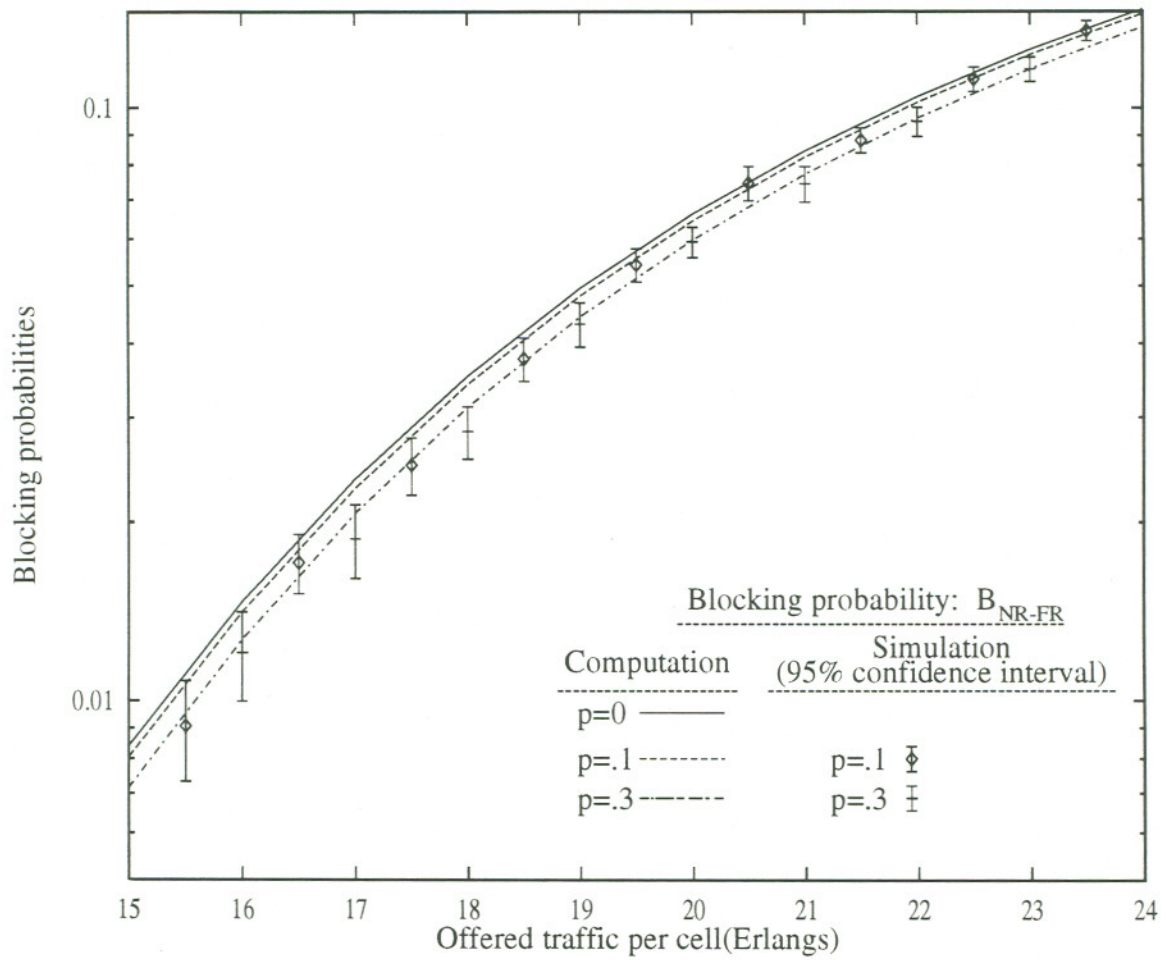
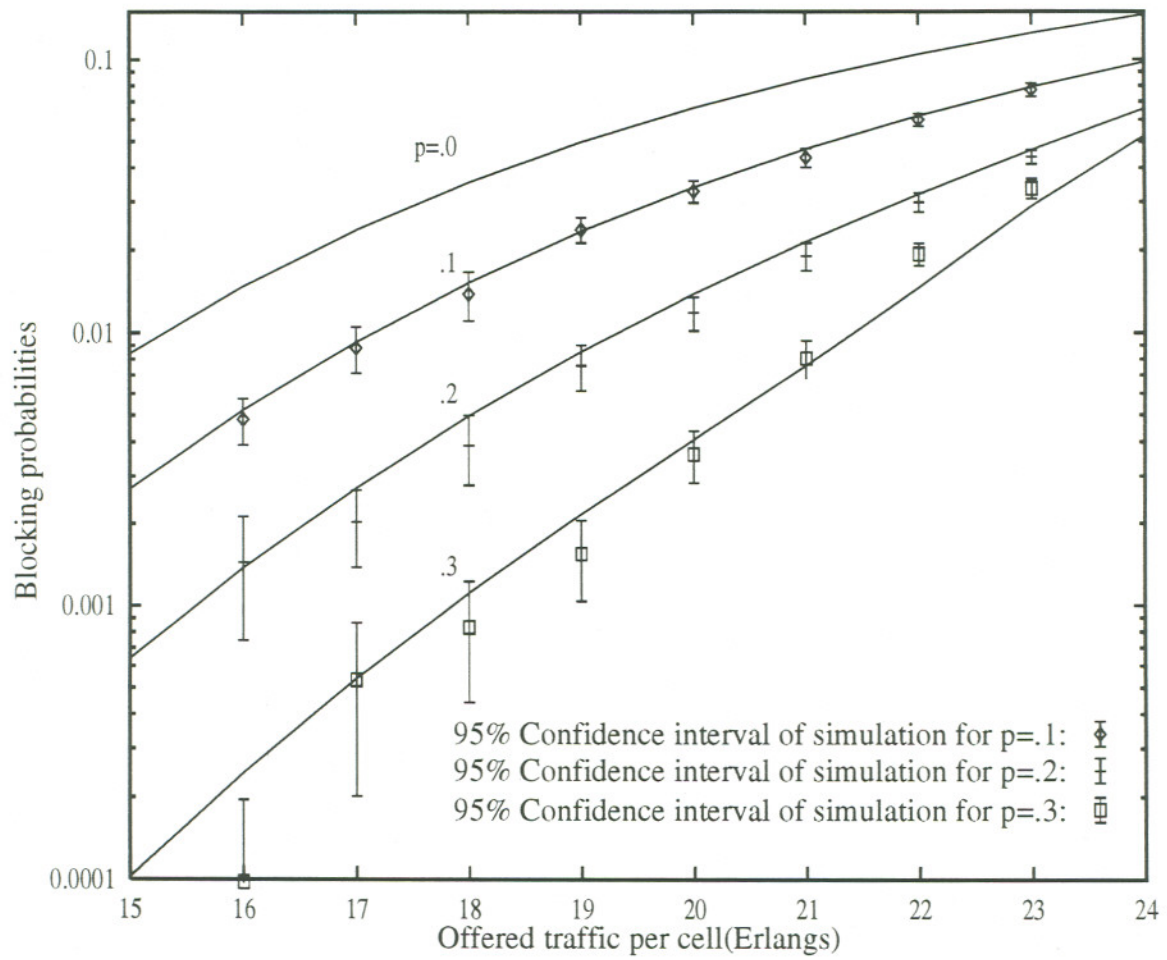Figure 4: Blocking probabilities of CBWL/NR-FR ($C = 24, m = 22, l = 4$).

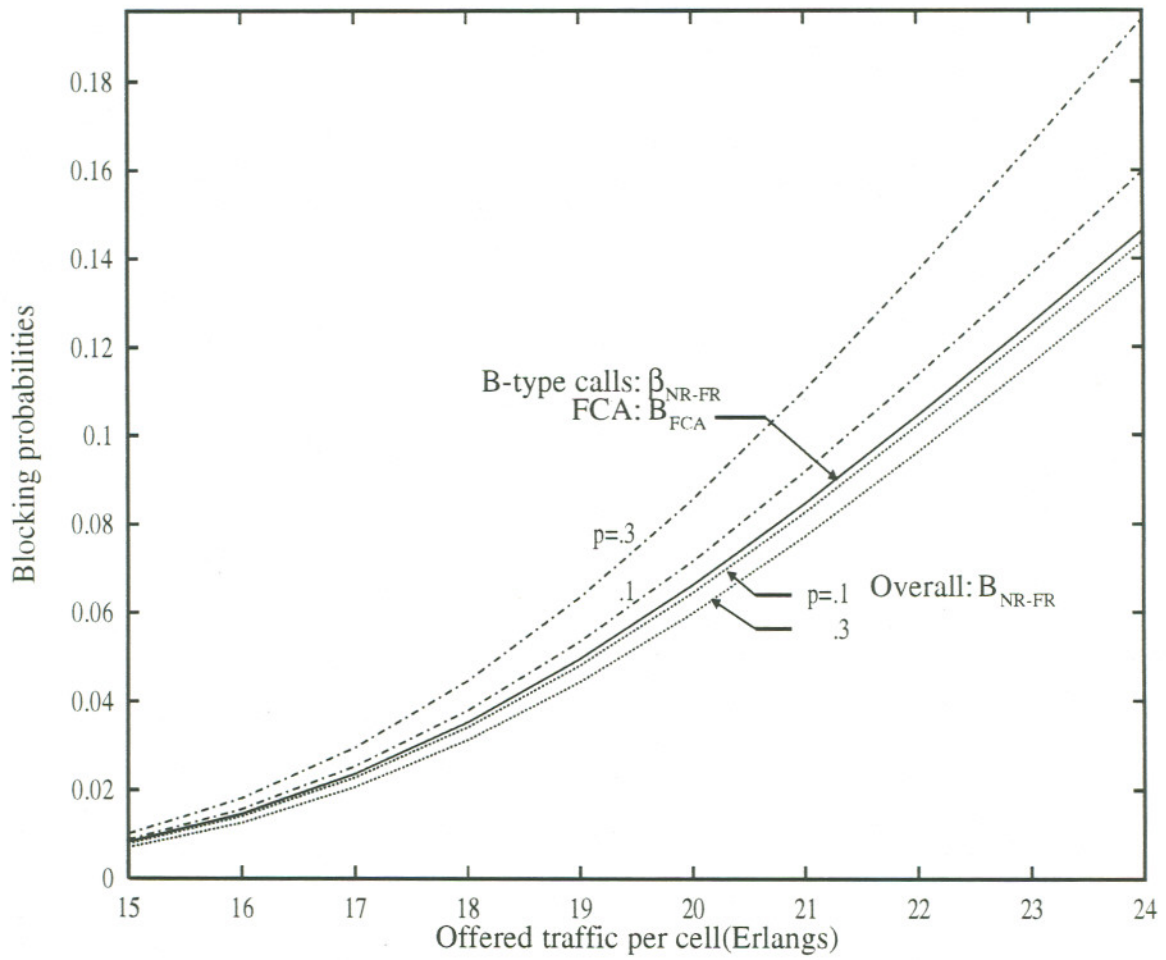Figure 5: Blocking probabilities of CBWL/CR-FR ($C = m = 24, l = 3$).

Figure 6: Blocking probabilities of different calls in CBWL/NR-FR ($C = m = 24, l = 3$).

p	CBWL/CR-FR				CBWL/CR	
	offered traffic (Erlang)	percent increase	$\beta_{CR\text{-}FR}$	$\alpha_{CR\text{-}FR}$	offered traffic (Erlang)	percent increase
0.0	16.63	0.0%	.0200	.0000	16.63	0.0%
0.1	18.63	12.0%	.0222	.0000	18.22	9.6%
0.2	20.84	25.3%	.0248	.0004	19.74	18.7%
0.3	22.28	33.9%	.0251	.0080	21.00	26.3%
0.4	22.66	36.3%	.0223	.0165	21.97	32.1%
0.5	22.75	36.8%	.0206	.0195	22.72	36.6%
0.6	22.76	36.9%	.0201	.0200	23.18	39.3%
0.7	22.76	36.9%	.0200	.0200	23.37	40.5%
0.8	22.76	36.9%	.0200	.0200	23.42	40.8%

Table 1: The offered traffic per cell of CBWL/CR-FR for 2% blocking probability, $B_{CR/FR} = .02, (C = m = 24, l = 3)$.

Figure 6 depicts blocking probability of B-type calls, $\beta_{NR\text{-}FR}$ and overall blocking probabilities, $B_{NR\text{-}FR}$ for a CBWL/NR-FR scheme (blocking probability of A-type calls, $\alpha_{NR\text{-}FR}$ is too small to show in the figure). Also shown is the blocking probability B_{FCA} for the fixed channel assignment. Figure 7 is a similar plot for CBWL/CR-FR. Like CBWL/NR and CBWL/CR, it is seen that for CBWL/NR-FR resulting blocking probabilities are ordered by $\beta_{NR\text{-}FR} > B_{FCA} > B_{NR\text{-}FR} > \alpha_{NR\text{-}FR}$. Thus channel borrowing without channel rearrangement causes very different blocking probabilities for the A-type calls and B-type calls. From Figure 7, it is seen that for CBWL/CR-FR, blocking probabilities are ordered by $B_{FCA} > \beta_{CR\text{-}FR} > B_{CR\text{-}FR} > \alpha_{CR\text{-}FR}$. The *differences* between $B_{CR\text{-}FR}$, $\alpha_{CR\text{-}FR}$ and $\beta_{CR\text{-}FR}$ are much less that of CBWL/NR-FR. The heavier offered traffic is or the larger p is, the smaller the difference is.

Figure 8 shows the comparison between the blocking probabilities of CBWL/NR, CBWL/CR, CBWL/NR-FR and CBWL/CR-FR. It is seen that the blocking probabilities are ordered by $B_{FCA} > B_{NR\text{-}FR} > B_{NR} > B_{CR} > B_{CR\text{-}FR}$. We note that the performance of CBWL/NR-FR is poorer than that of CBWL/NR. That can be explained as follows. With channel fast returning, the borrowed channels are used as less as possible, the probability that a gateway's all channels are occupied is increased, thus the probability that B-type calls are blocked is increased. Hence, the overall blocking probability is also increased. However, with channel rearrangement, fast returning can significantly enhance system performance of CBWL/CR-FR.

Table 1 shows a comparison of offered traffic that can be accommodated at a 2% blocking probability for CBWL/CR and CBWL/CR-FR. Specifically, it tabulates the percentage increase (in offered traffic) in comparison with the corresponding FCA scheme. When the fraction of A-type calls, p, is increased, the offered traffic is increased. For $p = 0.3$, traffic increases about 34%. The offered traffic that can be accommodated increases very fast with increase of p for $p < 0.3$. When p is greater than 0.3, the increase is slowed. It means that increasing p beyond $p > 0.3$ helps little to improve system performance for CBWL/CR-FR. The effect is not a severe limitation
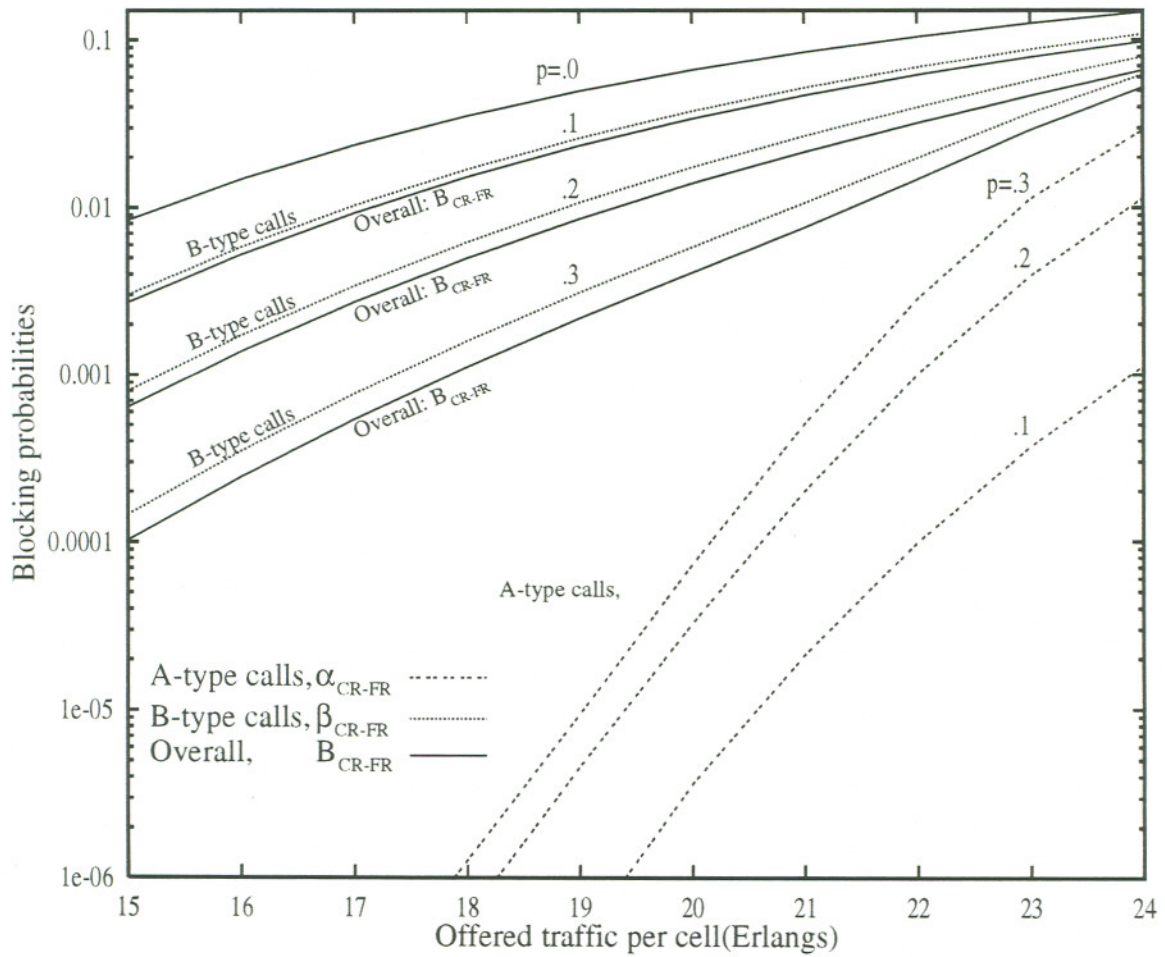
Figure 7: Blocking probabilities of different calls in CBWL/CR-FR ($C = m = 24, l = 3$).
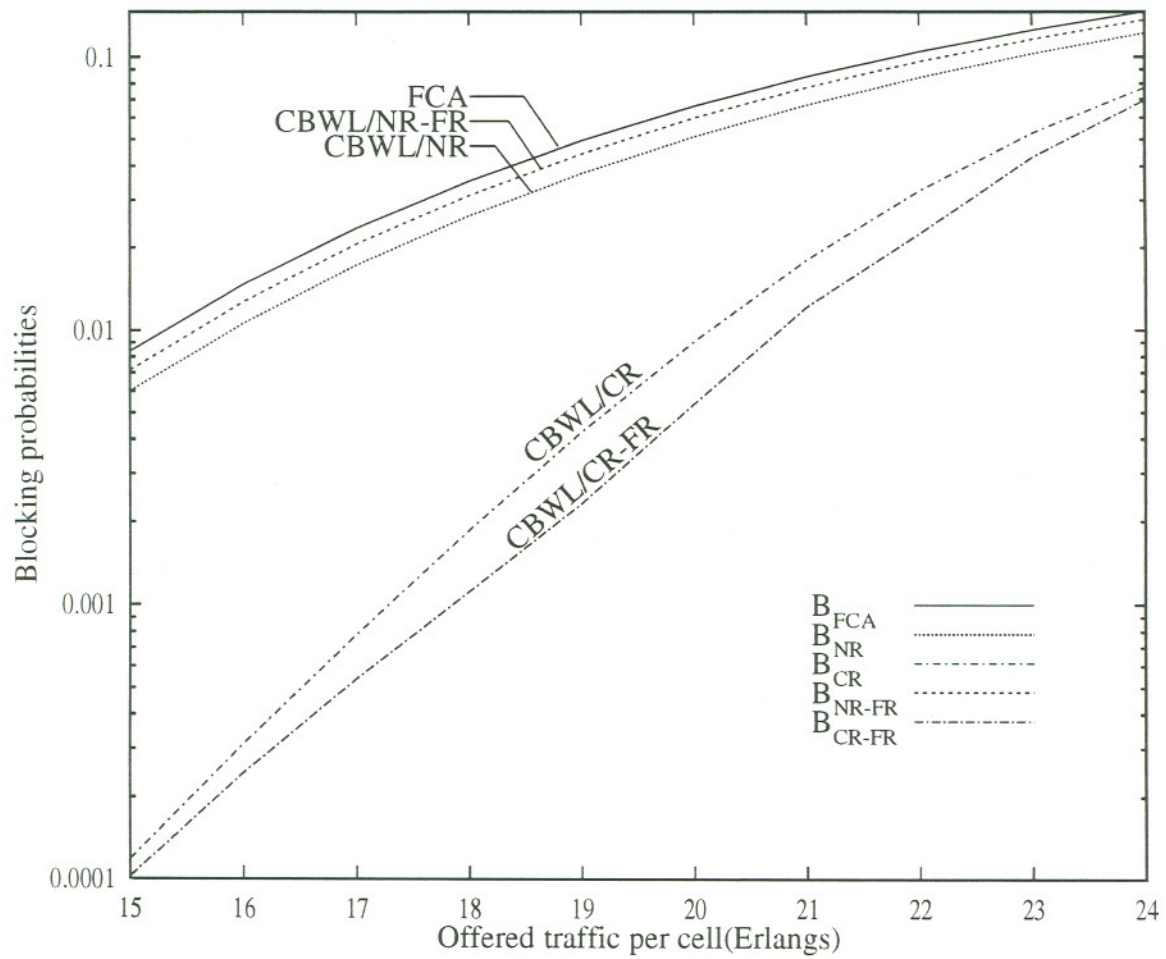
Figure 8: Blocking probabilities of CBWL/NR, CBWL/NR-FR, CBWL/CR and CBWL/CR-FR ($C = 24, m = 22, l = 4, p = .3$).

for CBWL/CR-FR scheme, because co-channel interference usually requires small p for CBWL schemes. In comparison with CBWL/CR (without fast returning), the maximum value of p that permits a fast increase of offered traffic is reduced from 0.5 to 0.3. Thus, CBWL/CR-FR can be used in system with smaller p. Table 1 also shows β_{CR-FR} and α_{CR-FR} given $B_{CR-FR} = .02$. We notice that β_{CR-FR} has a peak at about $p = 0.3$. The effect can be explained as follows. When p is increased from 0, for a fixed $B_{CR-FR} = 0.02$, the offered traffic that a cell can accommodate is increased. The increasing of offered traffic causes p_c and p_f to increase, this causes increase of β_{CR-FR}. But, when p is increased, the fraction of A-type calls is increased and the probability that a B-type call cannot use channel rearrangement, p_a (which is defined in (33), the probability that all channels of a gateway are occupied, and no channel is occupied by A-type calls) becomes relatively small. The decrease of p_a causes the decrease of β_{CR-FR}. When p is greater than 0.3, the rate of decrease is greater than the rate of increase, β_{CR-FR} thus is decreased. When p continue to increase, β_{CR-FR} approximates to B_{CR-FR} and α_{CR-FR}.

4   Conclusion

Our analysis and simulation have shown that fast returning with channel rearrangement (CBWL/CR-FR) can enhance the performance of CBWL schemes. CBWL/CR-FR exhibits excellent performance. It is especially useful in cellular systems with the small channel reuse factors, since they require small p. The difference between blocking probabilities of the users in different locations is reduced.

Appendix

A   Channel Returning Rate from An Adjacent Gateway, $\mu(i)$

Channel returning rate from a neighbor

In CBWL/NR-FR, the channel returning rate is accelerated. Assume all C channels of gateway, Y, are busy and Y borrows j channels from its six neighbors, i of them are borrowed from gateway, X. We consider the channel returning rate from Y to X. If one of i channels that are borrowed from X is released, the channel is definitely returned to X. If one of C channels of Y is released, Y randomly chooses one of j calls that uses a borrowed channels and transfers the call to the channel that is just released. The borrowed channel is returned to its owner. The chance that the owner is X is i/j. Thus, the rate of channel returning from Y to X is

$$R(i,j) = (i + \frac{i}{j}C)\mu \quad i = 0,\ldots,l, \quad i \le j \le L_{max} . \tag{A.1}$$

Denote $p_{br}(j)$ as the probability that a gateway borrows j channels from its neighbors. Denote $\Pr(i|j)$ as the conditional probability that given that the gateway borrows j channels, it borrows *exactly* i channels from a specific adjacent gateway. Denote $\Pr(j|i)$ as the conditional probability that given that a gateway borrows i channels from the specific adjacent gateway, the gateway

borrows j channels from all neighbors. From Bayes' Theorem,

$$\Pr(j|i) = \frac{\Pr(i|j)p_{br}(j)}{\sum_{j=i}^{5l+i} \Pr(i|j)p_{br}(j)} \quad i = 0,\ldots,l. \quad j = i,\ldots,5l+i. \tag{A.2}$$

Define $\mu(i)$ as the average channel returning rate from the given gateway to the specific adjacent gateway if the specific adjacent gateway borrows i channels from the given gateway. Thus,

$$\mu(i) = \sum_{j=i}^{5l+i} \Pr(j|i)R(i,j) = \frac{\sum_{j=i}^{5l+i} \Pr(i|j)p_{br}(j)R(i,j)}{\sum_{j=i}^{5l+i} \Pr(i|j)p_{br}(j)} \quad i = 0,\ldots,l. \tag{A.3}$$

The probability $p_{br}(j)$ is calculated in (15). The conditional probability $\Pr(i|j)$ is determined in following paragraphs.

Conditional probability, $\Pr(i|j)$

In our analysis, we assume a homogeneous system and use the strategy that when a gateway needs to borrow a channel, it randomly chooses a neighbor that has free channels for lending and directs a borrowing request to that gateway. Therefore, given that a gateway borrows j channels, the probability that one of j channels is borrowed from each neighbor is equally distributed. Thus, the distribution of number of channels borrowed from each neighbor given that j channels are borrowed is the same as the distribution of randomly distributing j identical balls into six distinct boxes if each box can have no more than l balls. The probability can be obtained by using combinatorial analysis.

Denote $a(j,k,l)$ as the number of ways to distribute j identical balls in k distinct boxes if each box can have no more than l balls. From Appendix D , we have

$$a(j,k,l) = \sum_{s=0}^{S}(-1)^s \binom{k}{s}\binom{j+k-1-s(l+1)}{j-s(l+1)} \tag{A.4}$$

where S is the largest integer that is less than or equal to $j/(l+1)$.

The probability $\Pr(i|j)$ is the fraction of number of ways that we first place i balls into a given box, then distribute $j-i$ balls into remaining 5 boxes. Thus,

$$\Pr(i|j) = \frac{a(j-i,5,l)}{a(j,6,l)} \quad j = 1,\ldots,6l, \; i = 0,\ldots,l . \tag{A.5}$$

B Average Channel Returning Rate from Adjacent Gateways

From (8), we have following equation,

$$f(i_k)\mu(i_k) = \lambda' f(i_k - 1) \tag{B.1}$$

Exchange the order of summation and production in the brackets of (12), and using (B.1) to reduce the size of i_s. The terms in the brackets in (12) can be expressed as

$$\prod_{k=1}^{6} f(i_k) \sum_{s=1}^{6} \mu(i_s) = \lambda' \sum_{s=1}^{6} u(i_s - 1) \left[f(i_s - 1) \prod_{\substack{k=1,\ldots,6 \\ k \neq s}} f(i_k) \right] \qquad (B.2)$$

where the $u(x)$ is defined as

$$u(x) = \begin{cases} 1 & 0 \leq x \leq l \\ 0 & \text{otherwise.} \end{cases}$$

Substituting (B.2) into (12, we have

$$\beta(v) = \frac{\lambda'}{b(v)} \sum_{\mathbf{I_6} \in S(v,6)} \sum_{s=1}^{6} u(i_s - 1) \left[f(i_s - 1) \prod_{\substack{k=1,\ldots,6 \\ k \neq s}} f(i_k) \right] . \qquad (B.3)$$

We recall the definition of (11 and since $\sum_{k=1}^{6} i_k = v$, we find the term in the brackets of (B.3) is the product of components of a $\mathbf{I_6} \in S(v-1,6)$. We introduce a transform in which for a $\mathbf{I_6} \in S(v,6)$, if its component, i_s, is greater than zero, we construct a new vector $\mathbf{I_6'} \in S(v-1,6)$. The five components of $\mathbf{I_6'}$ are the same as those of $\mathbf{I_6}$, except its sth component, $i_s' = i_s - 1$. Thus each $\mathbf{I_6}$ can transform into up to six $\mathbf{I_6'}$'s. With this transform in (B.3), each $u(i_s - 1)$ is changed into $u(i_s' + 1)$. Thus,

$$\beta(v) = \frac{\lambda'}{b(v)} \sum_{\mathbf{I_6'} \in S(v-1,6)} \sum_{s=1}^{6} u(i_s' + 1) \prod_{k=1}^{6} f(i_k') . \qquad (B.4)$$

For all possible value of $i_s'(0,\ldots,l)$, only when $i_s' = l$, $u(i_s' + 1) = 0$. Thus, if t components of an $\mathbf{I_6'}$ that are less than l, the second summation in (B.4) is just t. Recall in [2] that we define $b(t, v-1)$ as the sum of all possible $\sum_{k=1}^{6} f(i_k)$ that exactly 6-t components (i_k) are equal to l and $\sum_{k=1}^{6} i_k = v - 1$. Thus,

$$\beta(v) = \frac{\lambda'}{b(v)} \sum_{t=1}^{6} t b(t, v - 1) . \qquad (B.5)$$

C Probability that Borrowing Requests Success Given that j Channels Are Borrowed, $p_{bs}(j)$

If a gateway ,X, has borrowed j channels, the j channels may borrow from different adjacent gateways. If $j \geq sl$ ($s = 1, \ldots, 6$), it is possible that sl of j channels are borrowed from s neighbors. Thus, those s neighbors will not lend any channels to X. The remaining $6 - s$ neighbors *may or may not* lend channels to X depending on their states. In (20) of section 2.2.2 we have found

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the probability that one adjacent gateway deny the borrowing request of X even though X has not borrowed l channels from the neighbor is p_e. By considering the coupling between adjacent gateways as the average rate of borrowing requests, λ, and including the rate into the model for state analysis of a gateway, the state probabilities of a gateway can be determined completely without the knowledge of the states of other gateways [1]. Thus, the states of adjacent gateways can be considered as "independent". Therefore, the probability that $6-s$ neighbors deny borrowing requests of X even though X has not borrowed l channels from the neighbors is p_e^{6-s}.

Denote $p_g(s|j)$ as the probability that each of s neighbors lends l channels to X given that X has borrowed j channels. Thus,

$$p_{bs}(j) = \sum_{s=s_1}^{s_2} p_g(s|j)(1 - p_e^{6-s}) \qquad j = 0, \ldots, 6l \tag{C.1}$$

where s_1 and s_2 are the minimum and maximum number of neighbors that lend exactly l channels to X if j channels are borrowed by X, respectively. Specifically, $s_1 = \max(0, j - 6(l-1))$ and s_2 is equal to the maximum integer that less than or equal to j/l.

Since we use the strategy that a gateway send borrowing demand randomly to one of neighbors that have available channels for lending, and in homogeneous system, every gateway is considered to be the same, every way to distribute j channels among six neighbors is equally likely. Thus, we have a similar problem of distributing randomly j balls into 6 boxes with each box having at most l balls. The total number of ways to distribute exactly j balls into six boxes is denoted as $a(j, 6, l)$ (D.9). The number of ways to distribute j balls into six boxes so that s boxes have l balls can be obtained by following ways: The number of ways to choose randomly s boxes from the six boxes is $6!/[(6-s)!s!]$. We assign l balls to each of the s boxes. Then we distribute the remaining $j - sl$ balls into the remaining $6 - s$ boxes with each box getting at most $l - 1$ balls. The number of ways is $a(j - sl, 6 - s, l - 1)$. Thus,

$$p_g(s|j) = \frac{6!}{s!(6-s)!} \frac{a(j - sl, 6 - s, l - 1)}{a(j, 6, l)} . \qquad j = 0, \ldots, \lfloor j/sl \rfloor \tag{C.2}$$

From (C.1) and (C.2),

$$p_{bs}(j) = \sum_{s=s_1}^{s_2} \frac{6!}{s!(6-s)!} \frac{a(j - sl, 6 - s, l - 1)}{a(j, 6, l)} \{1 - p_e^{6-s}\} \qquad j = 0, \ldots, 6l . \tag{C.3}$$

D   Number of Ways to Distribute Balls into Boxes

We want to find number of ways to distribute j *identical* balls into k *different* boxes if each box can have at most l balls. This problem can be solved by *generating functions* [15]. Function $g(x)$ is a generating function of a combinatorial problem if $g(x)$ has the polynomial expansion

$$g(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_j x^j + \cdots$$

and c_j is the number of ways to distribute j objects in the method that the problem requires. Therefore, if we can model our problem into a generating function and find c_j, our problem will be solved.

Let $f(x) = 1 + x + x^2 + \cdots + x^l$. The power of x in $f(x)$ corresponds to the number of balls that are distributed into a specific box. Because the way to distribute s identical balls into a specific box is unique, all coefficients of $f(x)$ are 1. Consider the coefficient of x^j in the expansion of $[f(x)]^k$. It is the number of different formal product of $[f(x)]^k$ whose sum of exponents is j. Each of k $f(x)$'s represents the number of ways to distribute balls in each of k boxes. Thus, the coefficient of x^j in the expansion of $[f(x)]^k$ is equal to the number of ways to distribute j identical balls into k distinct boxes. Thus, our problem has a generating function

$$g(x) = [f(x)]^k . \tag{D.1}$$

The next step is to find the coefficient. Rewrite this generating function as

$$g(x) = (1 + x + x^2 + \cdots + x^l)^k = \left(\frac{1 - x^{l+1}}{1 - x}\right)^k = (1 - x)^{-k}(1 - x^{l+1})^k. \tag{D.2}$$

The first factor can be expanded as

$$(1 - x)^{-k} = 1 + \binom{k}{1} x + \binom{k+1}{2} x^2 + \cdots + \binom{s+k-1}{s} x^s + \cdots \tag{D.3}$$

and the second factor can be expanded as

$$(1 - x^{l+1})^k = 1 - \binom{k}{1} x^{l+1} + \binom{k}{2} x^{2(l+1)} - \cdots + (-1)^s \binom{k}{s} x^{s(l+1)} + \cdots$$

$$+ (-1)^k \binom{k}{k} x^{k(l+1)} . \tag{D.4}$$

Denote the first factor as

$$(1 - x)^{-k} = a_0 + a_1 x + a_2 x^2 + \cdots + a_s x^s + \cdots \tag{D.5}$$

and the second factor as

$$(1 - x^{l+1})^k = b_0 + b_{l+1} x^{l+1} + b_{2(l+1)} x^{2(l+1)} + \cdots + b_{s(l+1)} x^{s(l+1)} + \cdots + b_{k(l+1)} x^{k(l+1)} . \tag{D.6}$$

From the rules of multiplication of polynomials, we can know the coefficient of x^j in the expansion of $g(x)$ is given by

$$c_j = \sum_{i=0}^{S} a_{j-i(l+1)} b_{i(l+1)} . \tag{D.7}$$

where S is is the maximum integer that is less or equal to $j/(l+1)$. From (D.3) and (D.4), we have

$$c_j = \sum_{i=0}^{S} (-1)^i \binom{k}{i} \binom{j - i(l+1) + k - 1}{j - i(l+1)} . \tag{D.8}$$

Define $a(j, k, l)$ as the number of ways to distribute j identical balls into k distinct boxes with at most l balls in each box. Thus,

$$a(j, k, l) = \sum_{i=0}^{S} (-1)^i \binom{k}{i} \binom{j - i(l+1) + k - 1}{j - i(l+1)} . \tag{D.9}$$

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