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# Multibeam Cellular Communication Systems with Dynamic Channel Assignment

**by**


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## ABSTRACT

Sectorized multibeam cellular communication systems with dynamic channel assignment to beams is considered. Limitations due to co-channel interference are analyzed. A model for traffic performance is developed using multidimensional birth-death processes. Theoretical traffic performance characteristics such as call blocking probability, channel rearrangement rates and overall carried traffic are determined.

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# 1. INTRODUCTION

As the number of mobile users grows, increased system capacity is sought. Sectorization and cell splitting are used to allow increased system capacity while interference is limited to maintain signal quality. Space Division Multiple Access (SDMA) is a technique to increase system capacity. With the use of adaptive directional antennas and additional hardware and software at the base stations, co-channel interference can be reduced, which indicates more efficient frequency reuse and results in improvement of system capacity [1],[2]. Users in different angular positions can be served on the same channel provided the required angular separation between them is met [3],[4],[5].

One approach in SDMA is the switched multibeam system, in which multiple beams are used to cover the entire coverage of base station. The beam with the strongest signal power for the desired user is selected to serve the user. Recent work on switched multibeam systems includes the investigation of the gain improvement achieved with a multibeam antenna compared to the traditional sector configuration [6]. The tradeoffs between hysterisis level, switching time and gain for a multibeam antenna system are considered in [6]. The effects of incorrect beam selection on average signal-to-noise (SNR) power ratio and signal-to-interference (SIR) power ratio with a switched multibeam antenna system are examined in [7]. The frequency reuse efficiency of multibeam systems is investigated in [3],[4]. Channels in different beams can be reused if the required angular separation between beams is met. System capacity can be increased significantly using this approach. We consider such a system in this paper. A cell is divided into several sectors, each of which is covered by several directional beams. Certain channels are allocated to each sector. Channels that are assigned to the sector can be reused in different beams of the sector provided that co-channel interference remains below the required level. Sectorized cellular communication systems with and without multibeam schemes are compared. System capacity can be improved by using a multibeam scheme because of channel reuse among beams. For a fixed offered traffic the blocking probability of calls can be reduced significantly. Alternatively, more new call traffic can be supported while the blocking probability is maintained.

Dynamic channel assignment can be used to provide even more efficient channel reuse. In dynamic channel assignment, the channel used to serve a particular call is not fixed. Depending on the channel occupation and interference conditions, a call may switch between several different channels during its lifetime. Sectorized multibeam cellular communications with and without channel rearrangements are compared. Channel rearrangements can reduce call blocking but system implementation entails increased complexity.

The trade-off between blocking probability and channel rearrangement rates is examined. We devise models to compute fundamental traffic performance measures including call blocking probability, channel rearrangement rates and overall carried traffic. Multidimensional birth-death processes are used [8], [9], [10]. The global balance equations are determined and solved for the state probabilities, using the framework developed in earlier work [8], [9]. Performance characteristics are found from these state probabilities.

In this paper, Section 2 presents the analytical model for the multibeam cellular communication systems with dynamic channel assignment. Section 3 analyzes the co-channel interference. The state representation is described in Section 4. Section 5 explains the driving processes and state transition flow. Flow balance equations are established in Section 6. Performance measures are discussed in Section 7. Sections 8 and 9 discuss numerical results and conclusions.

## 2. MODEL DESCRIPTION

We consider $120^0$-sectored multibeam cellular communication systems with three beams deployed in each sector. This provides a total of nine beams per cell. Each sector consists of one center beam (beam 2) and two side beams. The center beam has beamwidth $\omega_2$ degrees with the peak of its radiation pattern being located at angular position of $\phi_2$ degrees. The two side beams (beams 1 and 3) have beamwidths $\omega_1$ and $\omega_3$ degrees with the peaks of their radiation patterns being located at angular positions of $\phi_1$ and $\phi_3$ degrees respectively. Beams are numbered in a counterclockwise direction as shown in Figure 1. Each sector has a limit of C channels. A large population of wireless users is considered. New call originations are assumed to follow a Poisson point process. Co-channel interference in a given beam at a site arises from the use of the same channel in other beams at the same site as well as from the use of the same channel at other sites. Channels can be reused in the two side beams provided the minimum angular separation between them is met such that overall co-channel interference is below the required level.

Dynamic channel assignment to beams within sectors is used in order to utilize channel resources of the system efficiently. At the time of a new call arrival in a side beam, if there is an available channel in that side beam which is already in use in the other side beam, then this channel is assigned to serve the new call. If there is no such channel in the side beam, then an available channel is randomly selected to serve the new call. Thus channels that are allocated to the sector can be reused in the side beams.

At the time of a new call arrival in the center beam, if all the channels in the sector are occupied, then the call will be blocked if channel rearrangement is not possible. Channel rearrangements can be performed only if there are at least two channels, one of which is in use in beam 1 but not in use in beam 3, or vice versa. A rearrangement is made so that one of the two channels is reused to serve the two existing calls in beams 1 and 3, while the other channel is used to serve the new call in the center beam. For example, suppose that user A is assigned to channel $f_1$ in beam 1 but $f_1$ is not in use beam 3. Suppose further that user C is assigned to channel $f_3$ in beam 3 but $f_3$ is not used in beam 1. When a new call made by user B originates in the center beam and there is no channel that can serve it, $f_3$ can be reassigned to serve user A. Channel $f_1$ will then be available for use and can be assigned to serve user B. Thus blocking of user B's call is avoided.

## 3. ANALYSIS OF CO-CHANNEL INTERFERENCE

We consider the usual hexagonal geometry and a $120^0$- sectorized multibeam scheme with three beams in each sector. Let R denote the radius of a cell. The reuse distance D is defined as the distance between the base stations of two nearest co-channel cells. Let N denote the cluster size, which is related to the reuse shift parameters $(i, j)$ by $N = i^2 + ij + j^2$. The integers $i$ and $j$ determine the reuse pattern and identify co-channel cells. The co-channel reuse factor, Q, is defined as the ratio of D to R. This ratio is $\frac{D}{R} = \sqrt{3N}$

In multibeam cellular communication systems, the co-channel interference of a desired wireless user comes from other co-channel cells as well as other beam of the same cell. Let $I_1$ and $I_2$ denote the normalized co-channel interference from other cells and normalized co-channel interference from other beams of the same cell correspondingly. The co-channel interference $I_1$ and $I_2$ are normalized by the desired signal power. We consider the carrier-to-interference ratio (*CIR*) of multibeam cellular communication systems in the worst case. Normal cellular practice specifies *CIR* to be 18 dB or higher. This is based on subjective tests and the criterion that 75 percent of the users say voice quality is "good" or "excellent" in 90 percent of the total covered area on a flat terrain [11]. The *CIR* of multibeam cellular communication systems can be calculated as

$$CIR = 10\log_{10}(\frac{1}{I_1 + I_2}) \quad \text{dB}. \qquad (1)$$

Consider cellular systems with cluster size N=7. As shown in figure 2a, we use A, B, C, D, E, F and G to distinguish seven cells in each cluster. Each sector of the system is labeled by one of seven letters with two subscripts. The first subscript denotes the cluster to which the sector belongs. The second subscript denotes the angular orientation of the sector. Thus, sectors with the same second subscript have the same orientation. Sectors whose labels have the same letter and the same second subscript are co-channel sectors. For example $A_{01}$ represents the sector 1 of cell A in cluster 0, $A_{11}$ represents the sector 1 of cell A in cluster 1 and the sectors $A_{01}$ and $A_{11}$ are co-channel sectors. Consider the up-link case. Suppose that the desired wireless user is served by beam 1 of the sector $A_{01}$. The worst case of $I_1$ occurs when the two wireless users served by the beams 1 and 3 of the sector $A_{11}$ use the same channel and both contribute co-channel interference to the desired wireless user. This is shown in figure 2a. Assume that the desired received signal power at the base station site from each wireless user served by this base station is kept the same. This can be done by means of power control mechanism. The distances between the base station of the sector $A_{11}$ and the two co-channel interfering wireless users of the sector $A_{11}$ are R, and the distances between the base station of the sector $A_{01}$ (desired base station) and the two co-channel interfering wireless users of the sector $A_{11}$ are $d_1$ and $d_2$ respectively. From the geometry of figure 2a, $d_1$ and $d_2$ are calculated as $\sqrt{19}R$ and $\sqrt{31}R$ respectively. Let $I_{1u}$ denote the $I_1$ in the up-link case. So the co-channel interference from other cells on the up-link, $I_{1u}$, can be calculated as

$$I_{1u} = (\frac{R}{d_1})^{\gamma} + (\frac{R}{d_2})^{\gamma} \tag{2}$$

in which, $d_1 = \sqrt{19}R$, $d_2 = \sqrt{31}R$ and $\gamma$ is a propagation constant that is heavily influenced by the actual terrain environment. The value of $\gamma$ usually lies between 3 and 5.

Consider the down-link case. Suppose that the desired wireless user is served by beam 3 of the sector $A_{01}$. The worst case of $I_1$ occurs when the co-channel sectors $A_{31}$ and $A_{41}$ contribute co-channel interference to the desired wireless user. This is shown in figure 2b. The distance between the desired wireless user and the base station of $A_{01}$ is R, and the distances between the desired wireless user and the two co-channel interfering base stations of the sectors $A_{31}$ and $A_{41}$ are $d_3$ and $d_4$ respectively. From geometry of figure 2b, $d_3$ and $d_4$ are calculated as $2\sqrt{7}R$ and $\sqrt{19}R$ correspondingly. Let $I_{1d}$ denote the $I_1$ in the down-link case. So the co-channel interference from other cells on the down-link, $I_{1d}$, can be calculated as

$$I_{1d} = (\frac{R}{d_3})^{\gamma} + (\frac{R}{d_4})^{\gamma} \tag{3}$$

in which, $d_3 = 2\sqrt{7}R$ and $d_4 = \sqrt{19}R$.

The far-field radiation pattern of antenna arrays, $E(\theta)$, is the product of a space factor, $S(\theta)$, and a normalized array factor, $A(\theta)$ [12].

$$E(\theta) = S(\theta)A(\theta) \tag{4}$$

The individual radiating elements are assumed to be monopole antennas. The space factor for a single radiating element is [12]:

$$S(\theta) = \frac{\cos(\frac{\pi}{2}\sin\theta)}{\cos\theta}. \quad (5)$$

For a linear antenna array having M active radiating elements with a spacing of half wavelength spacing between elements, the normalized array factor is [13]:

$$A(\theta) = \frac{\sin(\frac{M}{2}\pi\sin\theta)}{M\sin(\frac{\pi\sin\theta}{2})}. \quad (6)$$

The normalized co-channel interference from other beam of the same cell, $I_2$, can be calculated from the radiation pattern of antennas. The desired received signal power at base station site from wireless users are assumed to be kept the same by means of power control mechanism. For the up-link, the worst case of $I_2$ occurs when the receiver of desired gateway receives the strongest interfering power from the interfering wireless user. For the down-link, the worst case of $I_2$ occurs when the desired wireless user receives the strongest interfering power from interfering gateway. Let $I_{2u}$ denote the $I$ in the up-link case. If the desired wireless user is served by beam 1, then $I_{2u}$ can be calculated using

$$I_{2u} = \frac{\max_{0\le\theta_d\le\omega_1} |E_3(\theta_d)|^2}{\min_{120-\omega_3\le\theta_i\le120} |E_3(\theta_i)|^2} \quad (7)$$

where $E_3(.)$ is the radiation pattern of beam 3, $\theta_d$ is the angular position of desired wireless user and $\theta_i$ is the angular position of interfering wireless user.

Due to symmetry, the value of $I_{2u}$ is the same when the desired wireless user is served by beam 3. Let $I_{2d}$ denote the $I_2$ in the down-link case. If the desired wireless user is served by beam 1, then $I_{2d}$ can be calculated using

$$I_{2d} = \max_{0\le\theta_d\le\omega_1} \frac{|E_3(\theta_d)|^2}{|E_1(\theta_d)|^2} \quad (8)$$

where $E_1(.)$ is the radiation pattern of beam 1 ( the desired antenna array ), $E_3(.)$ is the radiation pattern of beam 3 ( interfering antenna array ) and $\theta_d$ is the angular position of desired wireless user. Due to symmetry, the value of $I_{2d}$ is the same when the desired wireless user is served by beam 3.

The *CIR* of sectorized multibeam cellular systems with cluster size N=7 is found using equation (1) . Usually the co-channel interference from another beam of the same cell is reduced if the angular separation between the beams is increased. In order to see how the angular separation between two side beams influences the *CIR* of multibeam systems, two different beam layouts are considered. One beam layout has an angular separation of $40^0$ between two side beams, which corresponds to $\omega_1 = \omega_2 = \omega_3 = 40^0$ (or $\phi_1 = 20^0$, $\phi_2 = 60^0$ and $\phi_3 = 100^0$). The other beam layout has an angular separation of $50^0$ between two side beams, which corresponds to $\omega_2 = 50^0$, $\omega_1 = \omega_3 = 35^0$ (or $\phi_1 = 10^0$, $\phi_2 = 60^0$ and $\phi_3 = 110^0$). Here we consider an antenna array with two active radiation elements for beams, that is, M=2. Three beams in each sector provide $120^0$ coverage of the sector. Users are each served by a beam corresponding to the strongest received signal power. Both the up-link and the down-link are considered for each beam layout. The propagation constant $\gamma$ is chosen to be four. The resultant *CIR* is calculated in Table 1. It is seen that the co-channel interference is usually improved if the angular separation between the beams is increased.

| *CIR* | $\omega_1 = \omega_2 = \omega_3 = 40^0$ | $\omega_2 = 50^0, \omega_1 = \omega_3 = 35^0$ |
|---|---|---|
| Up-link | 17.88 dB | 23.90 dB |
| Down-link | 17.85 dB | 23.66 dB |

Table 1. The calculated *CIR* of sectorized multibeam cellular systems.

## 4. STATE DESCRIPTION

When we consider a single sector, we can define the state of that sector by a sequence of nonnegative integers, $v_1, v_2, v_3, \alpha$. In this sequence, the state variable, $v_i$, $i = 1,2,3$, is the number of calls served by the *i-th* beam and $\alpha$ is the number of channels in use in beam 1 and beam 3 that are the same. Then, for convenience, we order the states using an index s=0,1,2,......., $S_{max}$. Thereafter, $v_i$, $i = 1,2,3$, and $\alpha$ can be shown as explicitly dependent on the state. That is, $v_i = v(s,i), i = 1,2,3$ and $\alpha = \alpha(s)$, in which $v(s,i)$ is the number of calls served by the *i-th* beam when the sector is in state, *s*, and $\alpha(s)$ is the number of channels in use in beam 1 and beam 3 that are the same when the sector is in state, *s*.

If C denotes the number of channels in each sector, we can specify the constraints on permissible states as

$$v(s,i) \le C, \qquad i = 1,2,3 \tag{9}$$

$$\sum_{i=1}^{2} v(s,i) \le C, \tag{10}$$

$$\sum_{i=2}^{3} v(s,i) \le C, \tag{11}$$

$$\sum_{i=1}^{3} v(s,i) - \alpha(s) \le C, \tag{12}$$

and,

$$\alpha(s) \le \min\{v(s,1), v(s,3)\}. \tag{13}$$

The inequality, (9), means that the number of channels in use in any beam when the sector is in state, *s*, cannot be larger than C. The constraints, (10) and (11) mean that the number of channels in use in any two adjacent beams of a sector when the sector is in state, *s*, cannot be larger than C. Constraint inequality (12) indicates that the total number of channels in use in a sector when the sector is in state *s* can not be more than C. The constraint, (13), means that the number of channels in use in beam 1 and beam 3 that are the same when the sector is in state *s* cannot exceed the smaller of the numbers of channels in use in either side beam.

The number of calls served in each sector can be larger than the number of channels in the sector because of the channel reuse. The maximum number of calls that each sector can support is 2C. A new call in the center beam can be served when the sector is in state *s* if there is an available channel in the sector, which requires $\sum_{i=1}^{3} v(s,i) - \alpha(s) < C$. If there is no available channel in the sector, a new call can be served only if channel rearrangement is possible. This requires $v(s,2) + \max\{v(s,1), v(s,3)\} < C$. Otherwise, the new call will be blocked. That is, if $\sum_{i=1}^{3} v(s,i) - \alpha(s) < C$, then new calls in center beam can be served. If $\sum_{i=1}^{3} v(s,i) - \alpha(s) = C$, but $v(s,2) + \max\{v(s,1), v(s,3)\} < C$, then new calls still can be served after channel rearrangement is performed. If $v(s,2) + \max\{v(s,1), v(s,3)\} = C$, then new calls will be blocked.

## 5. DRIVING PROCESSES AND STATE TRANSITION FLOW

There are two relevant driving processes. These are: {n} the generation of new calls in the sector of interest; {c} the completion of calls in the sector of interest. The new call arrival processes in any state are assumed to follow the Poisson point process. We use Markovian assumptions for the driving processes to render the problem amenable to solution using multidimensional birth-death processes.

### 5.1 New call arrivals

#### 5.1.1 New call arrivals in beam 1

A transition into state s due to a new call arrival in beam 1 when the sector is in state $x_n$ will cause the state variable $v(x_n,1)$ to be incremented by one. A new call can be served in beam 1 only if the number of channels in use in both beam 1 and center beam (beam 2) does not exceed C. When a new call arrives in beam 1, if there is a channel available in beam 1 which is already in use in beam 3, then this channel is assigned to serve the new call. This causes the state variables $\alpha(x_n)$ and $v(x_n,1)$ to be incremented by one. Thus a permissible state $x_n$ is a predecessor state of $s$ for new call arrivals in beam 1, if $v(x_n,1)+v(x_n,2)<C$ and $v(x_n,3)>\alpha(x_n)$, and the state variables are related by

$$\begin{aligned} &v(x_n,1)=v(s,1)-1, \\ &v(x_n,j)=v(s,j), \qquad j\neq 1 \\ &\alpha(x_n)=\alpha(s)-1 \end{aligned} \tag{14}$$

If there is no such which is available in beam 1 and already in use in beam 3, then another available channel is selected randomly to serve the new call. This causes the state variable $\alpha(x_n)$ to remain unchanged and state variable $v(x_n,1)$ to be incremented by one Thus a permissible state $x_n$ is a predecessor state of $s$ for new call arrivals in beam 1, if $v(x_n,1)+v(x_n,2)<C$ and $v(x_n,3)=\alpha(x_n)$, and the state variables are related by

$$\begin{aligned} &v(x_n,1)=v(s,1)-1, \\ &v(x_n,j)=v(s,j), \qquad j\neq 1 \\ &\alpha(x_n)=\alpha(s) \end{aligned} \tag{15}$$

Let $\Lambda_n(i), i=1,2,3$, denote the average arrival rate of new calls in beam i. The flow into state $s$ from $x_n$ due to new call arrivals in beam 1 is

$$\gamma_{n1}(s,x_n)=\Lambda_n(1), \qquad \text{if } v(x_n,1)+v(x_n,2)<C. \tag{16}$$

### 5.1.2 New call arrivals in center beam

A transition into state $s$ due to a new call arrival in the center beam when the sector is in state $x_n$ will cause the state variable $v(x_n,2)$ to be incremented by one. A new call can be served in the center beam only if the number of channels in use in both the center and the side beams does not exceed $C$. When a new call arrives in the center beam and there is an available channel to serve the call, it causes the state variable $\alpha(x_n)$ to remain the same and the state variable $v(x_n,2)$ to be incremented by one. Thus a permissible state $x_n$ is predecessor state of $s$ for new call arrivals in center beam, if $\sum_{i=1}^{3} v(x_n,i) - \alpha(x_n) < C$, and the state variables are related by

$$\begin{aligned} &v(x_n,2) = v(s,2) - 1, \\ &v(x_n,j) = v(s,j), \qquad j \neq 2 \\ &\alpha(x_n) = \alpha(s) \end{aligned} \tag{17}$$

When a new call arrives in the center beam and there is no available channel to serve the call, channel rearrangement is carried out. This causes state variable $\alpha(x_n)$ and $v(x_n,2)$ to be incremented by one. Thus a permissible state $x_n$ is predecessor state of $s$ for new call arrivals in center beam, if $v(x_n,2) + \max\{v(x_n,1), v(x_n,3)\} < C$ and $\sum_{i=1}^{3} v(x_n,i) - \alpha(x_n) = C$, and the state variables are related by

$$\begin{aligned} &v(x_n,2) = v(s,2) - 1, \\ &v(x_n,j) = v(s,j), \qquad j \neq 2 \\ &\alpha(x_n) = \alpha(s) - 1 \end{aligned} \tag{18}$$

The flow into state $s$ from $x_n$ due to new call arrivals in the center beam is

$$\gamma_{n2}(s,x_n) = \Lambda_n(2), \qquad \text{if } v(x_n,2) + \max[v(x_n,1), v(x_n,3)] < C. \tag{19}$$

### 5.1.3 New call arrivals in beam 3

A transition into state s due to new call arrivals in beam 3 when the sector is in state $x_n$ will cause the state variable $v(x_n,3)$ to be incremented by one. A new call can be served in beam 3, only if the number of channels in use in both beam 3 and the center beam, (beam 2), is less than C. When a new call arrives in beam 3, if there is a channel available in beam 3, which is already in use in beam 1, this channel is assigned to serve the new call. This causes the state variables $\alpha(x_n)$ and $v(x_n,3)$ to be incremented by one.

Thus a permissible state $x_n$ is a predecessor state of $s$ for new call arrivals in beam 3, if $v(x_n,2)+v(x_n,3)<C$ and $v(x_n,1)>\alpha(x_n)$, and the state variables are related by

$$\begin{aligned} &v(x_n,3)=v(s,3)-1, \\ &v(x_n,j)=v(s,j), \qquad j\neq 3 \\ &\alpha(x_n)=\alpha(s)-1 \end{aligned} \tag{20}$$

If there is no such available channel in beam 3 that is already in use in beam 1, then another available channel is selected randomly to serve the new call. This causes the state variable $\alpha(x_n)$ to remain unchanged and state variable $v(x_n,3)$ to be incremented by one. Thus a permissible state $x_n$ is a predecessor state of $s$ for new call arrivals in beam 3, if $v(x_n,2)+v(x_n,3)<C$ and $v(x_n,1)=\alpha(x_n)$ , and the state variables are related by

$$\begin{aligned} &v(x_n,3)=v(s,3)-1, \\ &v(x_n,j)=v(s,j), \qquad j\neq 3 \\ &\alpha(x_n)=\alpha(s) \end{aligned} \tag{21}$$

The flow into state $s$ from $x_n$ due to new call arrivals in beam 3 is

$$\gamma_{n3}(s,x_n)=\Lambda_n(3), \qquad \text{if } v(x_n,2)+v(x_n,3)<C. \tag{22}$$

## 5.2 Call completions

### 5.2.1 Call completions in beam 1

A transition into state s due to a call completion in beam 1 when the sector is in state $x_c$ will cause the state variable $v(x_c,1)$ to be decreased by one. If the channel serving the call is also in use in beam 3, then this call completion will cause state variable $\alpha(x_c)$ to be decreased by one. Thus a permissible state $x_c$ is a predecessor state of $s$ for call completion in beam 1, if the state variables are related by

$$\begin{aligned} &v(x_c,1)=v(s,1)+1, \\ &v(x_c,j)=v(s,j), \qquad j\neq 1 \\ &\alpha(x_c)=\alpha(s)+1 \end{aligned} \tag{23}$$

Let $\mu_c$ denote the average completion rate of each call. The flow into state s from $x_c$ due to call completion in beam 1 is

$$\gamma_{c1}(s,x_c)=\alpha(x_c)\mu_c \tag{24}$$

If the channel serving the call is not in use in beam 3, then this call completion will cause state variable $\alpha(x_c)$ to remain unchanged and state variable $v(x_c,1)$ to be decreased by one. Thus a permissible state $x_c$ is a predecessor state of $s$ for call completion in beam 1, if the state variables are related by

$$
\begin{aligned}
&v(x_c,1) = v(s,1) + 1, \\
&v(x_c,j) = v(s,j), \qquad j \neq 1 \\
&\alpha(x_c) = \alpha(s)
\end{aligned}
\tag{25}
$$

The corresponding transition flow is given by

$$\gamma_{c1}(s,x_c) = (v(x_c,1) - \alpha(x_c))\mu_c \tag{26}$$

### 5.2.2 Call completions in center beam

A transition into state s due to a call completion in center beam when the sector is in state $x_c$ will cause the state variable $v(x_c,2)$ to be decreased by one and the state variable $\alpha(x_c)$ to remain unchanged. Thus a permissible state $x_c$ is a predecessor state of $s$ for call completion in center beam, if the state variables are related by

$$
\begin{aligned}
&v(x_c,2) = v(s,2) + 1, \\
&v(x_c,j) = v(s,j), \qquad j \neq 2 \\
&\alpha(x_c) = \alpha(s)
\end{aligned}
\tag{27}
$$

The flow into state s from $x_c$ due to call completion in center beam is

$$\gamma_{c2}(s,x_c) = v(x_c,2)\mu_c. \tag{28}$$

### 5.2.3 call completions in beam 3

A transition into state s due to a call completion in beam 3 when the sector is in state $x_c$ will cause the state variable $v(x_c,3)$ to be decreased by one. If the channel serving the call is also in use in beam 1, then this call completion will cause state variable $\alpha(x_c)$ to be decreased by one. Thus a permissible state $x_c$ is a predecessor state of $s$ for call completion in beam 3, if the state variables are related by

$$
\begin{aligned}
&v(x_c,3) = v(s,3) + 1, \\
&v(x_c,j) = v(s,j), \qquad j \neq 3 \\
&\alpha(x_c) = \alpha(s) + 1
\end{aligned}
\tag{29}
$$

The flow into state s from $x_c$ due to call completion in beam 3 is

$$\gamma_{c3}(s,x_c) = \alpha(x_c)\mu_c \tag{30}$$

If the channel serving the call is not in use in beam 1, then this call completion will cause state variable $\alpha(x_c)$ to remain unchanged and state variable $v(x_c,3)$ to be decreased by one. Thus a permissible state $x_c$ is a predecessor state of $s$ for call completion in beam 3, if the state variables are related by

$$\begin{aligned} v(x_c,3) &= v(s,3)+1, \\ v(x_c,j) &= v(s,j), \qquad j \neq 3 \\ \alpha(x_c) &= \alpha(s) \end{aligned} \tag{31}$$

The corresponding transition flow is given by

$$\gamma_{c3}(s,x_c) = (v(x_c,3) - \alpha(x_c))\mu_c \tag{32}$$

## 6. FLOW BALANCE EQUATIONS

From the equations given above, the total transition flow into $s$ from any permissible predecessor state $x$ can be found using

$$q(s,x) = \gamma_{n1}(s,x) + \gamma_{n2}(s,x) + \gamma_{n3}(s,x) + \gamma_{c1}(s,x) + \gamma_{c2}(s,x) + \gamma_{c3}(s,x) \tag{33}$$

in which $s \neq x$ , and flow into a state has been taken as a positive quantity. The total flow out of state $s$ is denoted as $q(s,s)$, and is given by

$$q(s,s) = -\sum_{\substack{k=0 \\ k \neq s}}^{S_{max}} q(k,s) \tag{34}$$

To find the statistical equilibrium state probabilities for a sector, we write the flow balance equations for the states. These are a set of $S_{max}+1$ simultaneous equations for the unknown state probabilities $p(s)$. They are of the form

$$\sum_{j=0}^{S_{max}} q(i,j)p(j) = 0, \qquad i = 0,1,2,\ldots\ldots,S_{max}-1.$$

$$\sum_{j=0}^{S_{max}} p(j) = 1. \tag{35}$$

In which, for $i \neq j$, $q(i,j)$ represents the net transition flow into state $i$ from state $j$, and $q(i,i)$ is the total transition flow out of state $i$. These equations express that in statistical equailibrium, the net probability flow into any state is zero, and the sum of the probabilities is unity.

## 7. PERFORMANCE MEASURES

There are three performance measures of interest: 1) individual beam and overall blocking probability, 2) channel rearrangement rates and 3) overall carried traffic. Once the statistic equilibrium state probabilities are found, the required performance measures can be calculated.

### 7.1 Blocking probability

The blocking probability for a call is the average fraction of new calls that are denied access to a channel. Blocking of new calls occurs if there are no channels to serve the call.

#### 7.1.1 Individual beam blocking probability

We define the following sets of states

$$B1 = \{ s : v(s,1) + v(s,2) = C \} \tag{36}$$
$$B2 = \{ s : v(s,2) + max[v(s,1), v(s,3)] = C \} \tag{37}$$
$$B3 = \{ s : v(s,2) + v(s,3) = C \} \tag{38}$$

Then the blocking probability in beam $i$, $P_b(i)$, can be calculated by

$$P_b(1) = \sum_{s \in B1} p(s), \quad P_b(2) = \sum_{s \in B2} p(s), \quad P_b(3) = \sum_{s \in B3} p(s) \tag{39}$$

#### 7.1.2 Overall blocking probability

The overall blocking probability can be calculated by

$$P_B = P_n(1) \times P_b(1) + P_n(2) \times P_b(2) + P_n(3) \times P_b(3) \tag{40}$$

where $P_n(i)$ is the average fraction of new calls that arrive in beam *i*. The details of calculation are described in the Appendix.

### 7.2 Channel rearrangement rate

Channel rearrangement rate is the average rate of channels that have to be rearranged. We define a set of states as

$$W = \{ s : v(s,2) + max[v(s,1), v(s,3)] < C, \sum_{i=1}^{3} v(s,i) - \alpha(s) = C \} \qquad (41)$$

The channel rearrangement rate can be calculated by

$$R = \Lambda_n(2) \times \sum_{s \in W} p(s) \qquad (42)$$

The details of calculation are described in the Appendix.

### 7.3 Overall carried traffic

The overall carried traffic per sector is the average number of channels occupied by the calls. The carried traffic in beam $i$, $A_c(i)$, can be calculated by

$$A_c(i) = \sum_{s=0}^{S_{max}} v(s,i) p(s), \qquad i = 1,2,3. \qquad (43)$$

Then the overall carried traffic, $A_C$, can be calculated as

$$A_C = \sum_{i=1}^{3} A_c(i) \qquad (44)$$

## 8. NUMERICAL RESULTS

Figure 3 and 4 are plotted for $C = 15$ channels and the peak of radiation pattern of center beam is located at angular position of $60^0$ with beamwidth of $40^0$. The peak of radiation pattern of beams 1 and 3 are located at angular position of $20^0$ and $100^0$ with beamwidth of $40^0$ for each. Figure 3 shows the blocking probability of sectorized multibeam cellular communication systems with and without channel rearrangement as well as the sectorized cellular communication systems without using multibeam schemes. It shows that multibeam systems have a better performance than the systems without using multibeam schemes in terms of blocking probability. In multibeam systems, the center beam usually has the worse blocking probability than side beams. Channel rearrangement is used to reduce the blocking of new calls in center beam and balance the blocking probabilities among center beam and side beams. Figure 4 shows the channel rearrangement rate. Channel rearrangement rate increases as new call origination rate increases but the relationship between them is not linear.

Figure 5 plots the five different example density functions of new call arrival rates. We label these curves by numbers. All these density functions except curve 1 have traffic concentrations on angular position of $60^0$ but with different deviations. Curve 1 represents the uniform distribution of traffic and curve 5 represents the most concentrated distribution of traffic among these curves. The new call arrival rate in a sector is fixed to be 0.1 calls/sec but has the different distribution. That is, the integrals (from $0^0$ to $120^0$) of different new call arrival rate density functions are all 0.1.

Figures 6 to 9 are plotted for $C = 15$ channels and $\Lambda_N = 0.1$ calls/sec. In figure 6 and 7, five different distributions of new call arrival rates which concentrate on the angular position of 60 degrees, as plotted in figure 5, are considered. Figure 6 shows the overall blocking probability. At low concentrated traffic conditions, like curve 1 to 3, the blocking probability achieves its minimum when center beam is centered at angular position of $60^0$. This is because the offered traffics in two side beams are balanced when center beam is centered at angular position of $60^0$. The system benefits from the better channel reuses when offered traffics in two side beams are balanced. So the best angular position of the center beam is centered where the offered traffic in two side beams are balanced. At highly concentrated traffic conditions, like curve 4 and 5, the offered traffic in center beam has increasingly significant impact on blocking probability so that the minimum of curve moves away from angular position of $60^0$ as the offered traffic is increasingly concentrated at the angular position of $60^0$. Figure 7 shows the channel rearrangement rate of systems.

In figure 8 and 9, another five different distributions of new call arrival rates which concentrate on angular position of $40^0$ instead of $60^0$ are considered. Curve 1 represents the uniform distribution of new call arrival rates. As the label number of curves increases, it means the traffic has more concentration on angular position of $40^0$. Figure 8 shows the overall blocking probability. Once again, the blocking probability achieves its minimum at angular position where the offered traffic in two side beams are balanced. So the minimum point moves to the direction of angular position of $40^0$ from angular position of $60^0$ as the offered traffic is increasingly concentrated at $40^0$. The best angular position of the center beam is centered where the offered traffic in two side beams are balanced. But at highly concentrated traffic conditions, like curve 4 and 5, the offered traffic in center beam has increasingly significant impact on blocking probability so that the minimum moves away from angular position of $40^0$ as the offered traffic is increasingly concentrated at $40^0$. Figure 9 shows the channel rearrangement rate of systems.

## 9. CONCLUSIONS

The framework that we are developing that employs a state description and multidimensional birth-death processes can be used to compute theoretical traffic performance characteristics for multibeam schemes in cellular communication systems. Sectorized multibeam cellular communication systems with three beams in each sector are considered. Multibeam cellular communication systems provide better blocking performance in comparison with systems that do not use multiple beams in sectors. Dynamic channel assignment is used in order to exploit channel resources efficiently. The blocking of calls in the center beam is usually worse than blocking of calls in side beams. Channel rearrangement can used to improve blocking conditions of calls in center beam and balance the blocking probability of calls between center and side beams. The co-channel interference is analyzed for sectorized multibeam cellular communication systems. Since in addition to other co-channel cells, other beam of the same cell also contributes co-channel interference to a desired wireless user, the carrier-to-interference ratio of sectorized multibeam cellular communication systems is slightly worse than the sectorized cellular communication systems without using multibeam schemes. The co-channel interference from other beam of the same cell can be limited provided the required angular separation between side beams is met.

System capacity can be improved by using a multibeam scheme because of channel reuse among beams. The blocking probability of new calls can be reduced significantly for fixed offered traffic, or more traffic can be accommodated while blocking probability of calls is maintained below the required level. Results indicate that multibeam cellular communication systems have a significant improvement on system capacity.

The best angular positions of beams change with the different offered traffic distributions. At uniform and low concentrated traffic conditions, the best angular position of the center beam is centered where the offered traffic in two side beams are balanced because balanced offered traffic in two side beams provides more efficient channel reuses. At highly concentrated traffic conditions, the offered traffic in center beam has increasingly significant impact on blocking probability of calls. The best angular position of center beam moves away from the angular position where the traffic is highly concentrated.

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# APPENDIX

## 1. Calculation of the overall blocking probability

The overall blocking probability, $P_B$, can be calculated as

$$
\begin{aligned}
P_B = {} & P_r\{\text{new call arrival in beam } 1\} \times P_r\{\text{call is blocked} \mid \text{new call arrival in beam } 1\} + \\
& P_r\{\text{new call arrival in beam } 2\} \times P_r\{\text{call is blocked} \mid \text{new call arrival in beam } 2\} + \\
& P_r\{\text{new call arrival in beam } 3\} \times P_r\{\text{call is blocked} \mid \text{new call arrival in beam } 3\}
\end{aligned}
\tag{A.1}
$$

Let $P_n(i)$ denote the probability of new call arrivals in beam $i$. Then the expressions can be rewritten as

$$P_B = P_n(1) \times P_b(1) + P_n(2) \times P_b(2) + P_n(3) \times P_b(3) \tag{A.2}$$

Assume that new call arrival rate density function in the sector of interest is $f_n(\theta)$, $0 \le \theta \le 120$. The beam $i$ has beamwidth $\omega_i$, and the peak of its radiation pattern is located at $\phi_i$ degree in angle where $\sum_{i=1}^{3} \omega_i = 120^0$. The new call arrival rate in each beam, $\Lambda_{ni}, i = 1,2,3$, can be calculated by

$$\Lambda_{n1} = \int_{\theta=0}^{\omega_1} f_n(\theta) d\theta \quad , \Lambda_{n2} = \int_{\theta=\omega_1}^{\omega_1+\omega_2} f_n(\theta) d\theta \quad \Lambda_{n3} = \int_{\theta=\omega_1+\omega_2}^{120} f_n(\theta) d\theta \tag{A.3}$$

and the total new call arrival rate in a sector, $\Lambda_N$, is given by

$$\Lambda_N = \int_{\theta=0}^{120} f_n(\theta) d\theta \tag{A.4}$$

So the probability of new call arrivals in each beam, $P_n(i), i = 1,2,3$, is calculated by

$$P_n(i) = \frac{\Lambda_{ni}}{\Lambda_N}, \quad i = 1,2,3 \tag{A.5}$$

## 2. Calculation of channel rearrangement rate

Channel rearrangement rate is the average rate of channels that require rearrangement. Channel rearrangement rate, $R$, is calculated by

$R = \Lambda_N \times P_r\{$new call arrivals in center beam $\} \times P_r\{$system is in state that would require channel rearrangement | new call arrivals in center beam $\}$

The first two terms of right side of above expression can be combined as new call arrival rate in center beam, $\Lambda_{n2}$, and the last term is the summation of all probabilities of channel rearrangement states, $\sum_{s \in W} p(s)$. So the channel rearrangement rate can be rewritten as

$$R = \Lambda_{n2} \times \sum_{s \in W} p(s) \qquad \text{(A.6)}$$

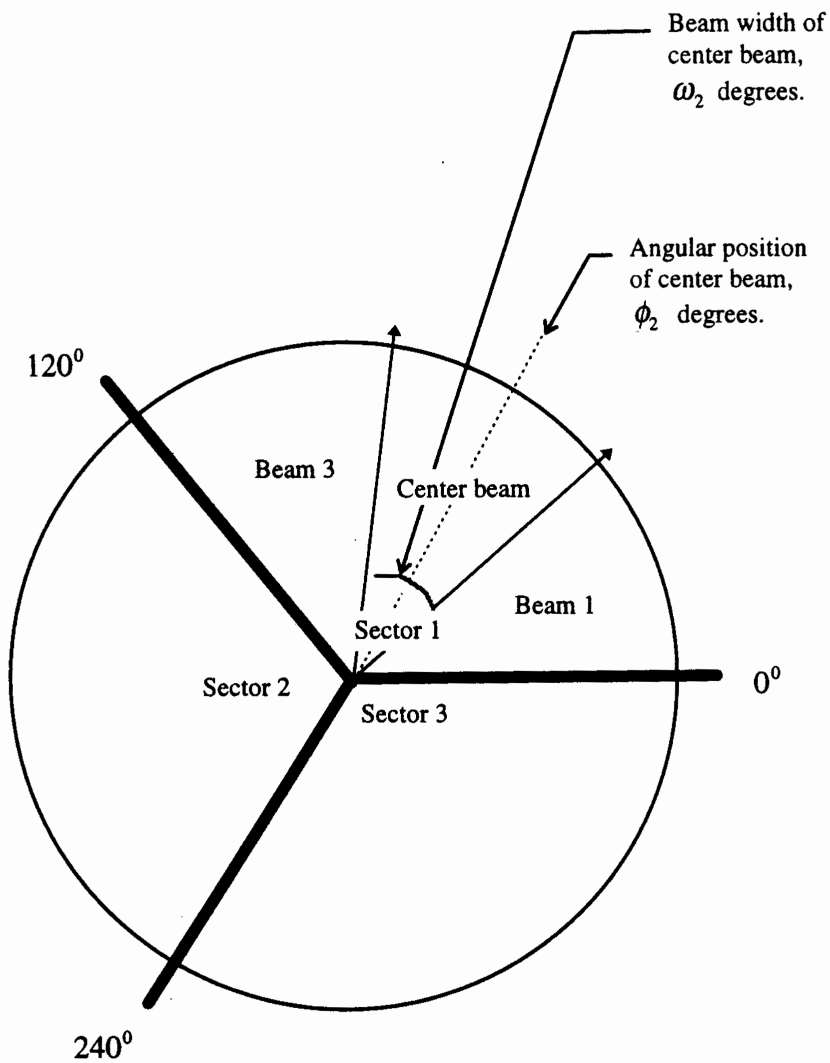



Figure 1. The system layout of sectorized multibeam cellular communication systems.

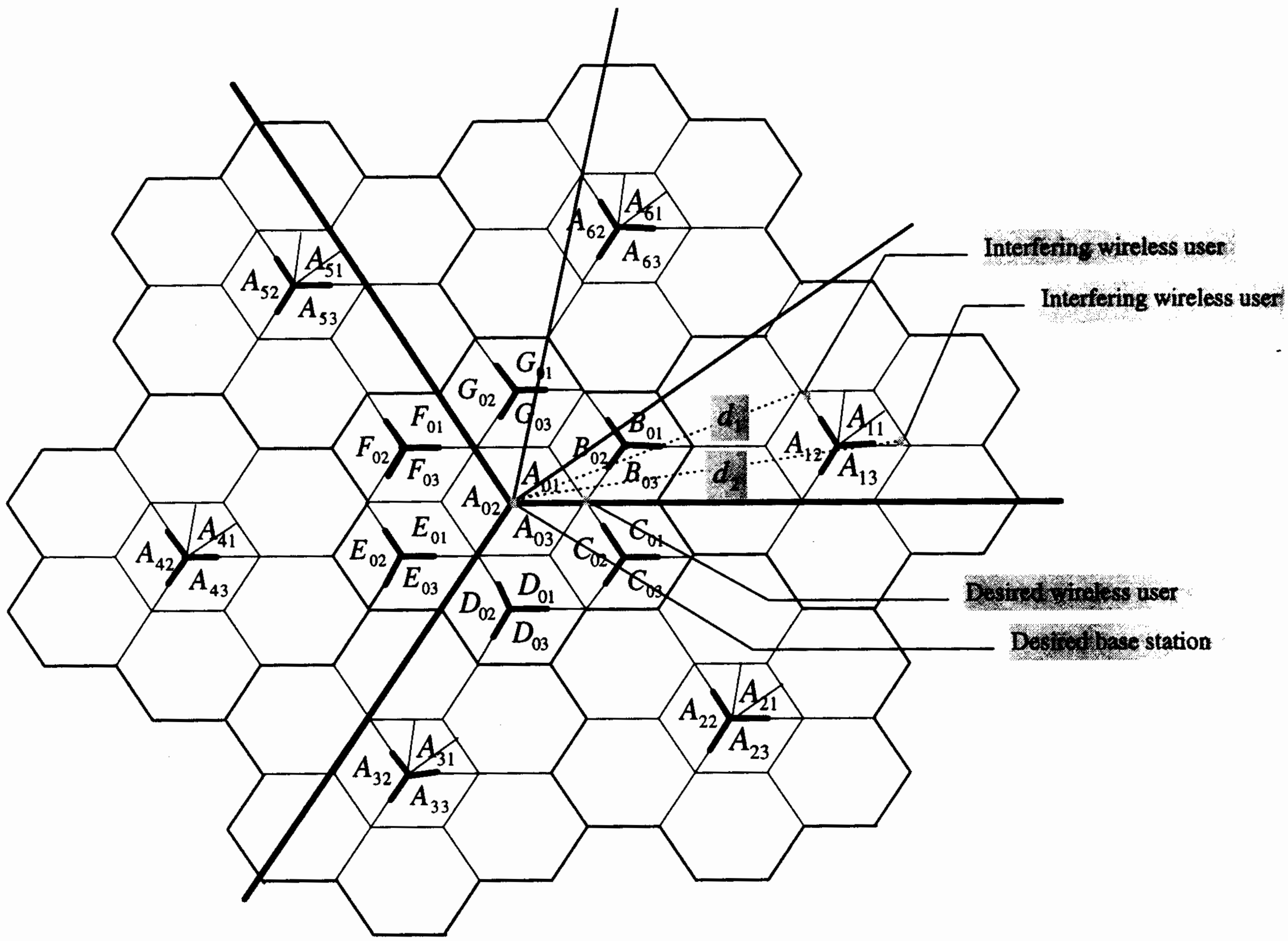



Fig 2 (a). $120^0$-sectored multibeam system with 3 beams in each sector for the up-link case. The cluster size is 7.

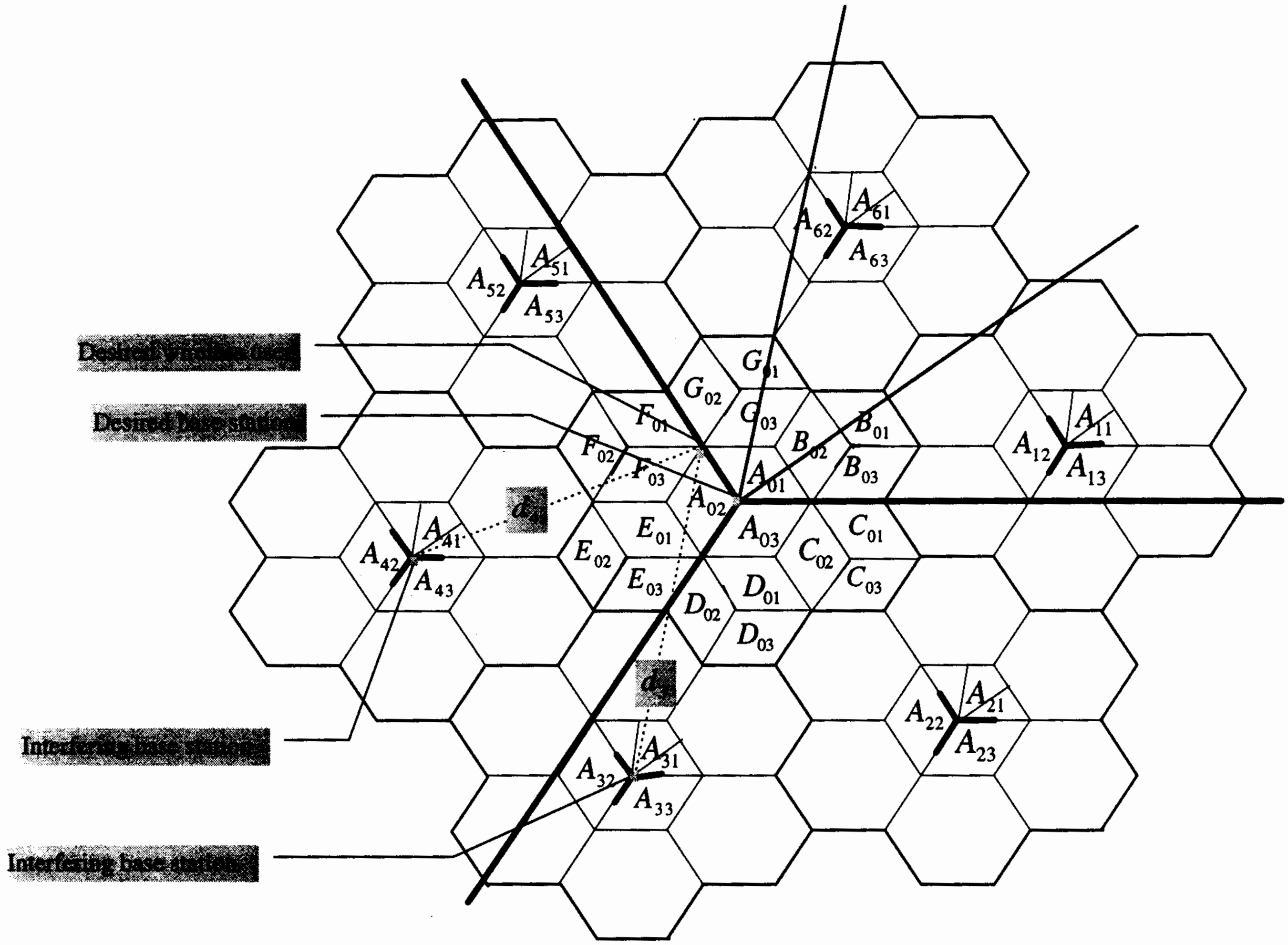



Fig 2 (b). $120^0$ -sectored multibeam system with 3 beams in each sector for the down-link case. The cluster size is 7.

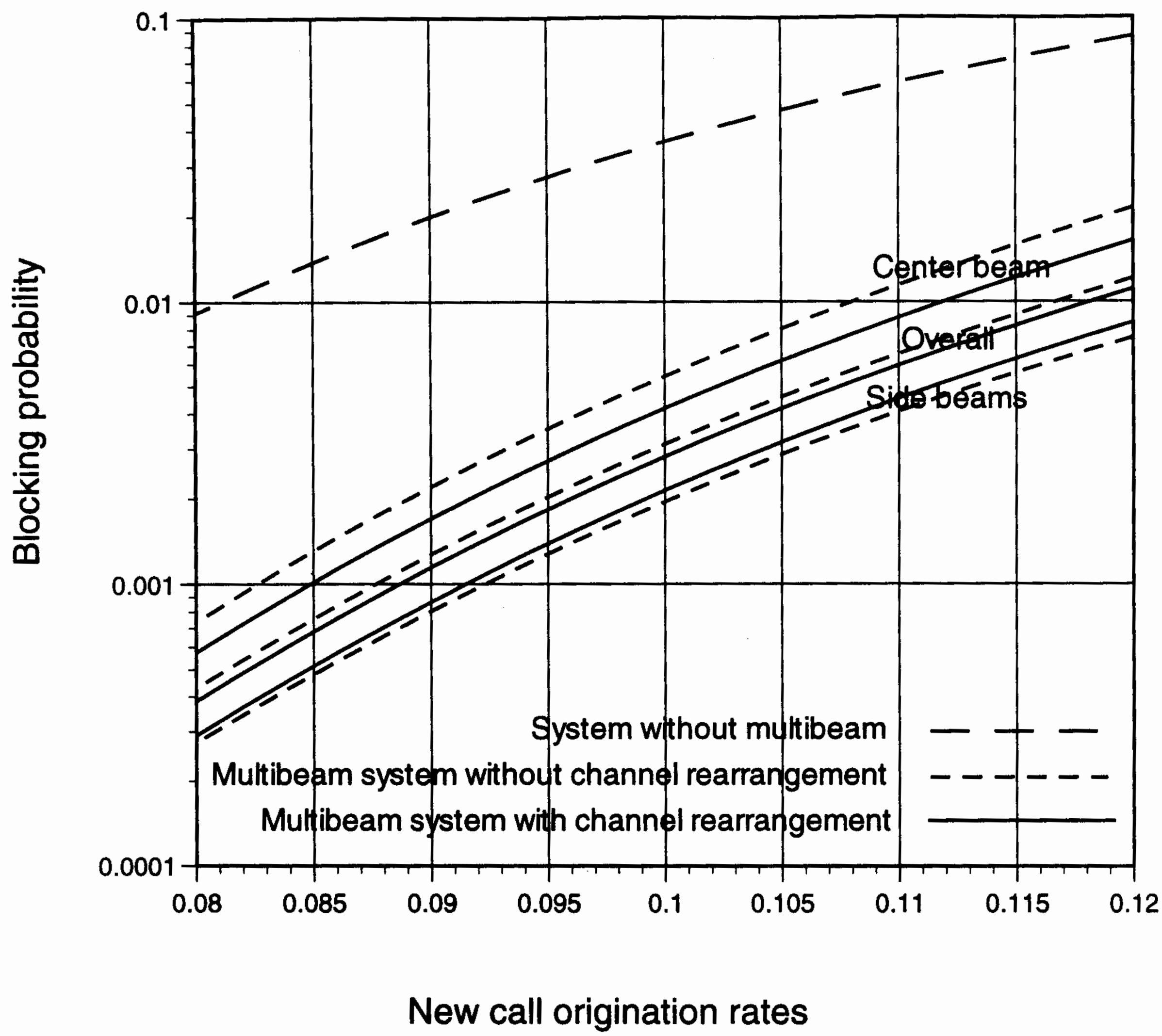



Figure 3. Blocking probability for C=15.

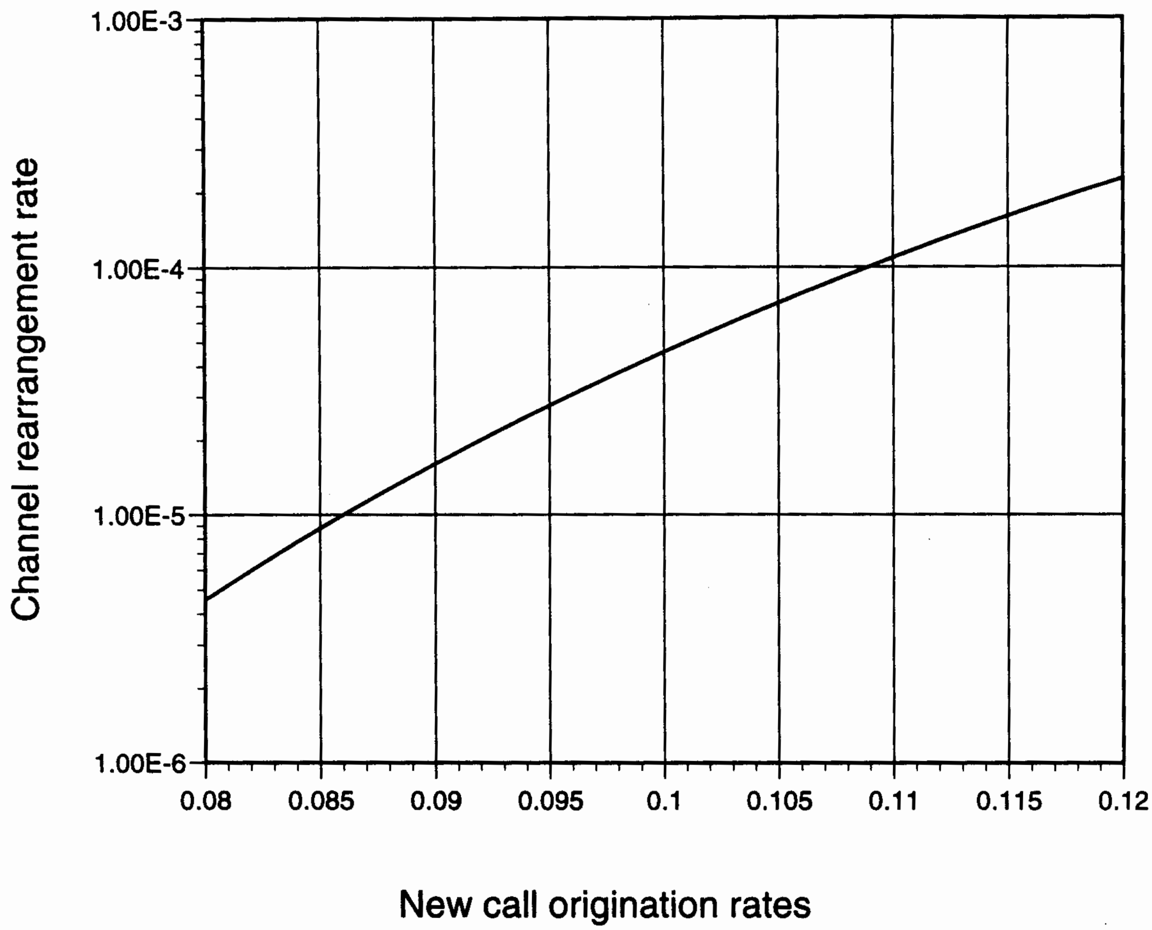



Figure 4. Channel rearrangement rate for C=15.

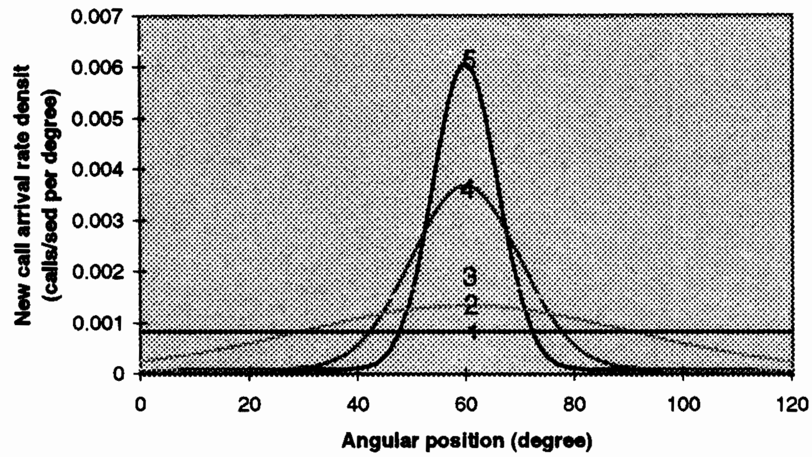



**Figure 5. New call arrival rate density functions**

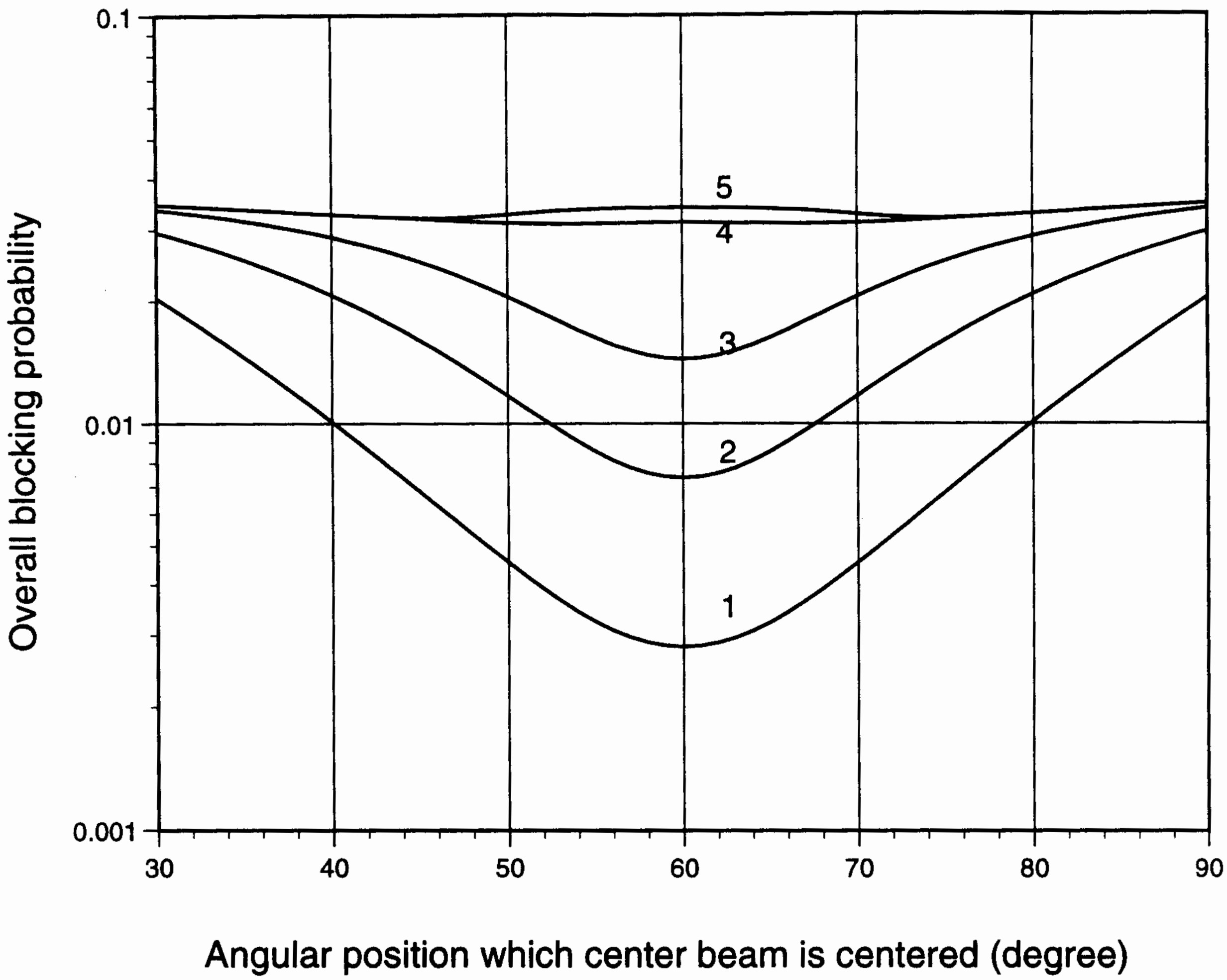



Figure 6. Overall blocking probability for different distributions of new call arrival rate concentrating at angular position of 60 degrees. It is plotted for C=15 and new call arrival rate/sector=0.1 calls/sed.

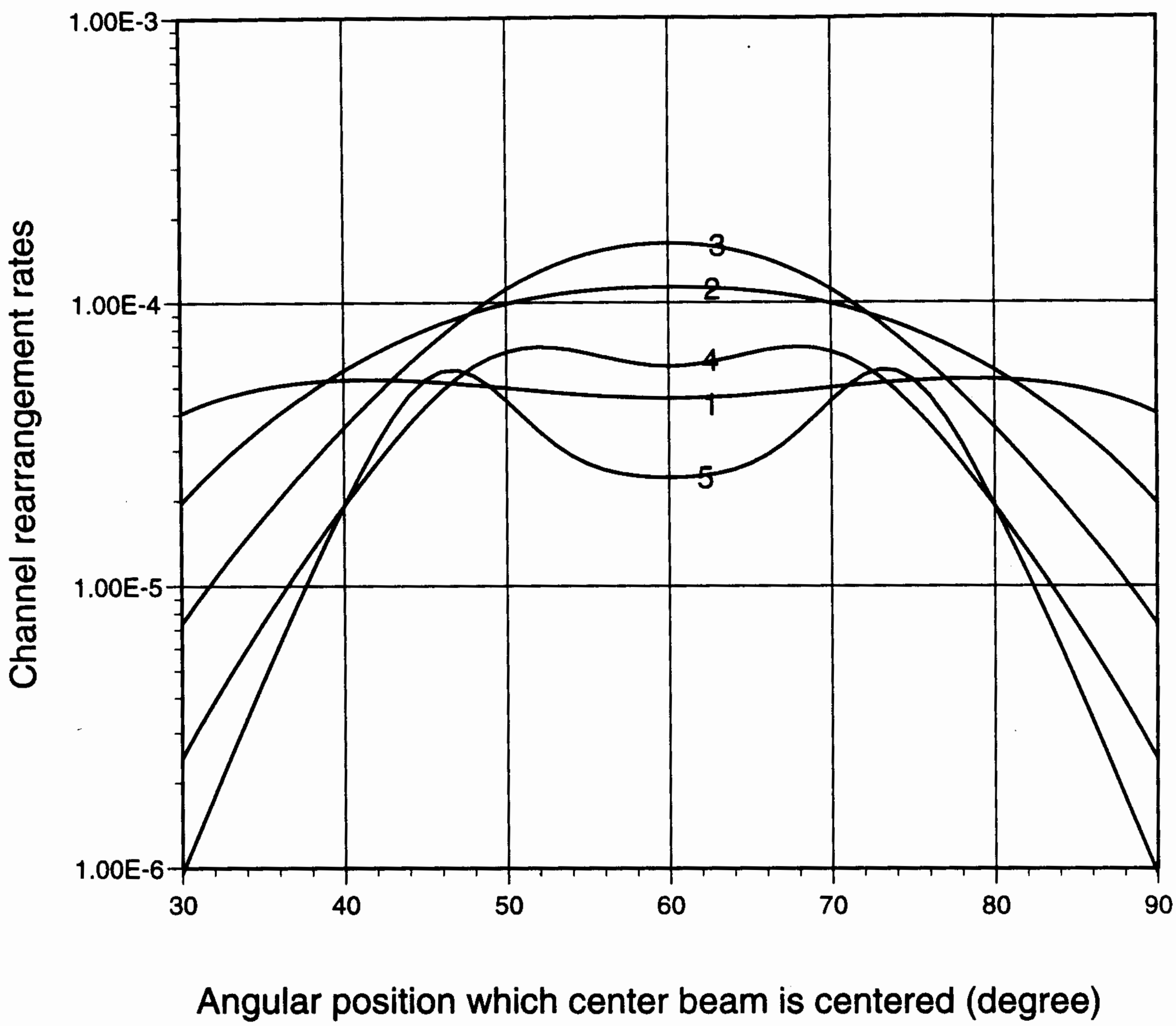
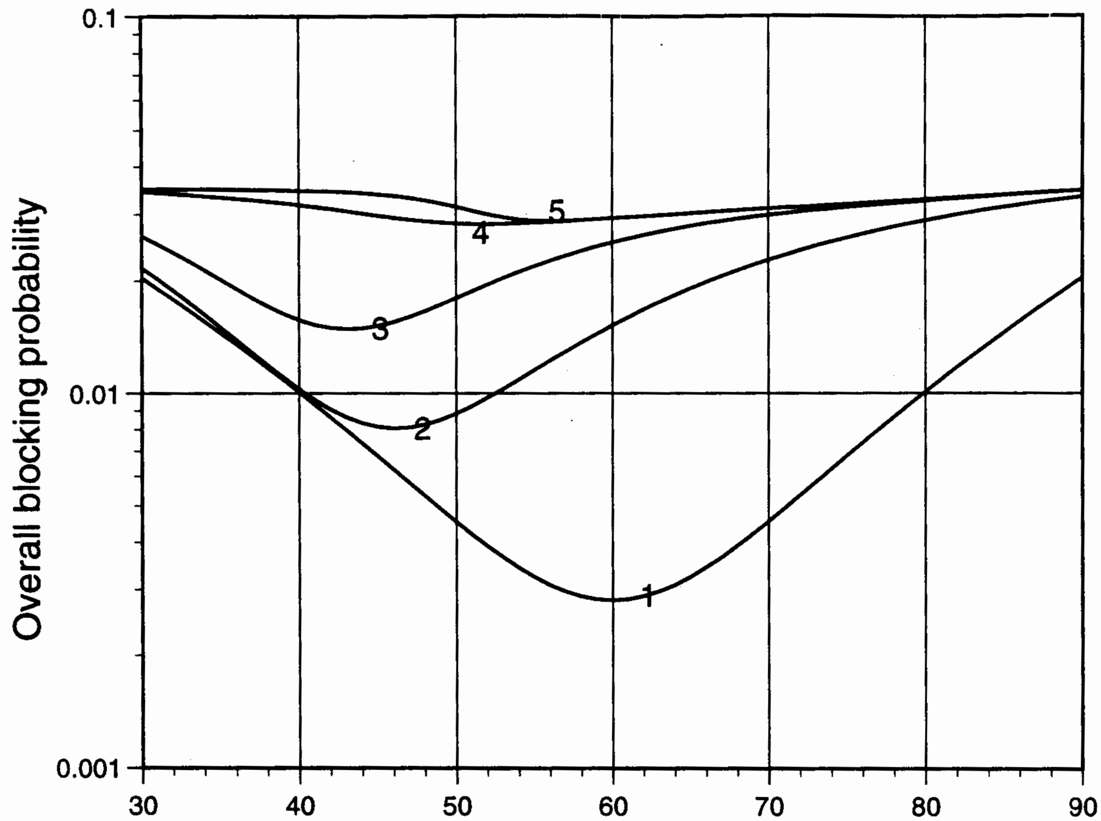



Figure 7. Channel rearrangement rates for different distributions of new call arrival rate concentrating at angular position of 60 degrees. It is plotted for C=15 and new call arrival/sector=0.1 calls/sec.




Angular position which center beam is centered (degree)

Figure 8. Overall blocking probability for different distributions of new call arrival rate concentrating at angular position of 40 degrees. it is plotted for C=15 and new call arrival rate/sector=0.1 calls/sec.

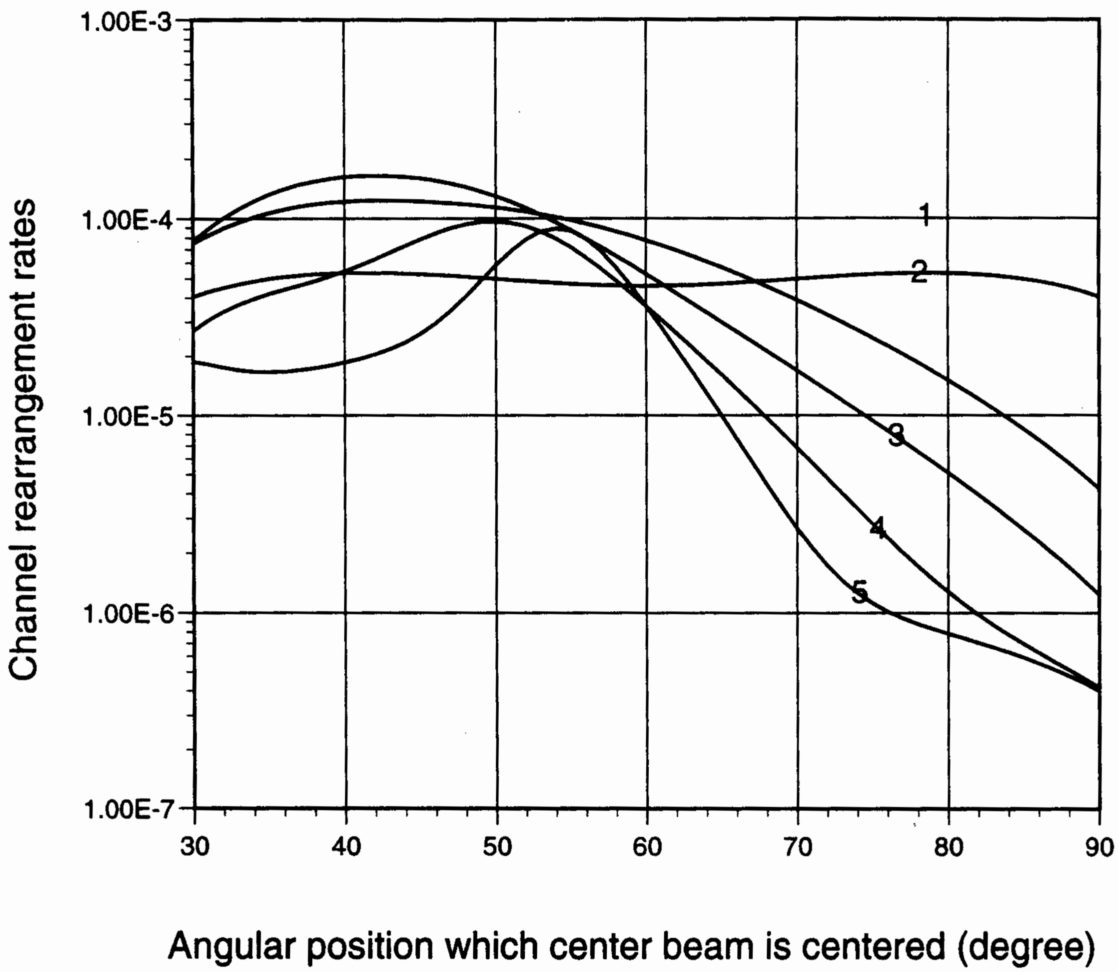



Figure 9. Channel rearrangement rates for different distributions of new call arrival rate concentrating at angular position of 40 degrees. it is plotted for C=15 and new call arrival rate/sector=0.1 calls/sec.