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Evaluation of Continuous-Time Priority Queueing Systems under Different Buffering Strategies with Applications to High Speed Switching

J.-W. Jeng and T.G. Robertazzi

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Jr-Wei Jeng, Student Member, IEEE
and
Thomas G. Robertazzi, Senior Member, IEEE

Department of Electrical Engineering
State University of New York at Stony Brook
Stony Brook, N.Y. 11794-2350

Telephone: 516-632-8412
FAX: 516-632-8494

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Abstract

In this paper, several different continuous time queueing system models applicable to high speed networks are proposed. They differ on the basis of various buffer access control disciplines and service disciplines. The average loss rate for each queueing system is studied in detail by analytical or simulation methods. Hardware implementation complexity for each queueing system is also discussed.

1. Introduction

In the future telecommunication environment, the demand for communication service may very well evolve towards high speed (broadband) integrated networks. These networks will carry a variety of traffic, such as voice, video, data and images. Planned access rates to the network would be in the range of several hundred million bits per second of information.

The transmitting speed of optical fibers is so fast that error detection and correction features in traditional packet switched networks will not be applicable to future high speed networks. Such networks will use simplified protocols and move most of the link-by-link layer protocols to higher edge-to-edge layers so that there will be no error protection on a link-by-link basis. The conventional dynamic flow control is also replaced by preventive control which actually means no flow control in the nodes. Due to this reason, loss of packets due to queue overflow is an important problem for high speed networks. The cell loss rate usually has to be set extremely low ($O(10^{-8})$). However, by carefully dimensioning the network, the loss rate of packets can be reduced to a very small value. For example, [1] studies a continuous tandem queueing system and [2] various dimensioning policies for a slotted queueing system.

While networks which can carry multiple classes of traffic have obvious benefits, their implementation raises difficult questions in terms of resource allocation and sharing. A particular problem examined in this paper is the choice of buffer allocation and service discipline policy to be placed in a generic high speed network. Among questions to be addressed are

- Is it better to use fixed buffer partitions for each class or to use a shared buffer?
- Can an optimal fixed partition be calculated?
- What are the trade-offs in terms of performance and implementation between simple and complex buffer management policies?

Using a variety of analysis, numerical techniques and simulation these questions will be answered. In this paper, several queueing models with different buffer allocation and server disciplines for a communication node in a multichannel network architecture are proposed. Continuous time models will be used because of their tractability and their ability to capture asynchronous effects. Two generic classes of traffic will be modeled which are treated identically in terms of buffering and service discipline. The reason for this is to emphasize buffer allocation issues rather than specific priority policies [2-10].

The paper is organized as follows. In section 2, different queueing systems are presented and the results are also given for each model. In section 3, a comparison of performance for each system is provided. A discussion of hardware implementation complexity on the basis of the server disciplines and buffer allocation policies is presented. Finally, a brief conclusion appears in section 4.

2. System Models and Results

In this paper, a continuous time queueing system with finite buffer capacity, N-2, and two servers (channels) is used for the system model. Hence, the total queue capacity is N-2+2=N. Since the characteristic of different traffic patterns on multi-class networks can vary a great deal, the network may provide different quality of service to satisfy the requirement for each traffic class. As mentioned in [11], there are two kinds of priorities, time priority and semantic priority. A time priority system allows some cells to remain longer in the network than others. In a semantic priority system, some cells are loss-sensitive and others are not. Priority classes will be naturally associated with these different types of traffics. In ATM, one bit of cell header has been reserved for the explicit indication of priority. In this paper only the case that the priorities are assigned on a per call basis will be considered. Assignment on the basis of VCI (virtual channel identifier) and VPI (virtual path identifier) will not be studied here.

Two priority classes of traffic will be used for the incoming packets to the queueing system, class 1 and class 2. In the queueing policies that follow a distinction will be made between the two generic classes of traffic though they will be treated identically. Both the arrival processes are assumed to be Poisson process and uncorrelated with each other. The service time is exponentially distributed with mean service time of $1/\mu$ for both servers, i.e. with the same service rate. This means that each class of packets will use the same bandwidth in the transmission capacity. For the case of different bandwidth utilization, adjusting the respective service rate will allow the queueing models to remain valid [6].

According to different resources arrangement and service discipline, five queueing systems are introduced and studied in this paper. They will be described as follows. First-come first-serve (FCFS) will be assumed for each class of packets.

2.1 Partitioning Fixed Buffer Assignment (PFBA) Queueing System:

Figure 1 shows a PFBA queueing system. The buffers and servers in a system will be partitioned into two independent queueing systems with the total number of buffers fixed. Each system will serve a specific class of traffic and that class of traffic will only go to that queueing system. It can be shown that for a queueing system with finite buffer length N its blocking probability will be [5]:

$$P_b(N) = \frac{1-\frac{\lambda}{\mu}}{1-(\frac{\lambda}{\mu})^{N+1}}(\frac{\lambda}{\mu})^N \tag{1}$$

Thus, for a PFBA queueing system, the average loss rate for two classes of packets will be:

$$\eta(N_1) = \frac{\lambda_1}{\lambda_1+\lambda_2}(\frac{1-\frac{\lambda_1}{\mu}}{1-(\frac{\lambda_1}{\mu})^{N_1+1}}(\frac{\lambda_1}{\mu})^{N_1}) + \frac{\lambda_2}{\lambda_1+\lambda_2}(\frac{1-\frac{\lambda_2}{\mu}}{1-(\frac{\lambda_2}{\mu})^{N_T-N_1+1}}(\frac{\lambda_2}{\mu})^{N_T-N_1}) \tag{2}$$

where N_1 : buffer length of the queueing system for class 1 traffic

N_T : total buffer length of both queueing systems

From equation (2), the average loss rate will change if one assigns different buffer lengths to each class of traffic. Since η is a function of only one variable, N_1, for a fixed total buffer length of N_T, the minimum value of η can be obtained when the variable N_1 satisfies the following derivative equation:

$$\frac{d\eta(N_1)}{dN_1} = 0 \tag{3}$$

Unfortunately the above equation does not yield a closed form solution for N_1. The average loss rate can be calculated for different value of N_1 to find the optimal value N_{1opt} which minimizes the average loss rate. The optimal value of N_{1opt} must be an integer. Figure 2 and 3 show average loss rate under different partitioning buffer sizes for a total buffer length of 25 and 50 in a PFBA environment. Table 1 gives the minimum average loss rate for different total buffer lengths using the PFBA policy.

2.2 Partitioning Fixed Buffer Assignment in Proportion to Arrival Rate (PFBA+PAR) Queueing System:

This model has the same scheme as the above queueing system except for the strategy for buffer assignment. In this model, each queue will get its assigned buffers in proportion to the arrival rate of its incoming traffic. For instance if the first queue has traffic with λ_1 arrival rate, it will receive the assignment of buffer length:

$$N_1 = Nearest\ integer\ to\ \left(\frac{\lambda_1}{\lambda_1 + \lambda_2} N_T\right) \tag{4}$$

A similar allocation holds for N_2.

Table 2 gives the average loss rate when using PFBA+PAR policy . A comparison of choosing N_1 and the ratio of average loss rate for PFBA and PFBA+PAR is illustrated in Table 3. It can be seen that this model has a quasi-optimal result most of the time. Intuitively, one will assign more buffers to a queue with heavier load so that it will decrease the blocking probability of that queue. From figure 3, we can see that the curve has a shallow minima if plotted linearly, and this will give flexibility in choosing the quasi-minimum value of N_1 and allow the system to have a quasi-optimal average loss rate.

2.3 Shared Buffer with Class Based Servers (SBCBS) Queueing:

In this model, both classes of traffic will share the same buffer and each server will serve only one preset class of traffic. When a server is available and there is one of its preset class of packets in the buffer, the packet of the preset class will immediately enter the server. The packets which are of another class in front of the preset class of packet will not block it. The schematic diagram of this system is shown in figure 4.

Since the arrival processes are Poisson processes and the service rate is exponential (which makes the system memoryless), there will be a Markovian transition state diagram. The state of this system is specified by the number of different classes of packets in the system. The transition state diagram is shown in figure 5 for a SBCBS queueing system with N=6. Here $\mu_1 = \mu_2 = \mu$. There are three different forms of global balance equations for the transition state diagram:

Form 1: Global balance equations for the state with two adjacent states

$$(\mu + \lambda_2)P(N-1,0) = \lambda_1 P(N-2,0) + \mu P(N-1,1) \quad (5)$$

$$(\mu + \lambda_1)P(0,N-1) = \lambda_2 P(0,N-2) + \mu P(1,N-1) \quad (6)$$

$$\mu P(n_1,n_2) + \mu P(n_1,n_2) = \lambda_1 P(n_1-1,n_2) + \lambda_2 P(n_1,n_2-1) \quad (7)$$

$$\text{where } (n_1,n_2) = (n_1, N-n_1) \;\; 1 \leq n_1 \leq N-1$$

$$(\lambda_1 + \lambda_2)P(0,0) = \mu P(1,0) + \mu P(0,1) \quad (8)$$

Form 2: Global balance equations for the state with three adjacent states

$$(\lambda_1 + \lambda_2 + \mu)P(n_1,0) = \mu P(n_1+1,0) + \mu P(n_1,1) + \lambda_1 P(n_1-1,0) \quad (9)$$

$$1 \leq n_1 \leq N-2$$

$$(\lambda_1 + \lambda_2 + \mu)P(0,n_2) = \mu P(0,n_2+1) + \mu P(1,n_2) + \lambda_2 P(0,n_2-1) \quad (10)$$

$$1 \leq n_2 \leq N-2$$

Form 3: Global balance equation for the state with four adjacent states

$$(\lambda_1 + \lambda_2 + 2\mu)P(n_1,n_2) = \mu P(n_1,n_2+1) + \mu P(n_1+1,n_2) +$$

$$\lambda_2 P(n_1, n_2 - 1) + \lambda_1 P(n_1 - 1, n_2) \quad (11)$$

$$\text{where } (n_1, n_2) \in \{(n_1, 1), (n_1, 2), \ldots, (n_1, N - n_1 - 2), (n_1, N - n_1 - 1)\}$$
$$1 \leq n_1 \leq N - 2$$

It can be verified that the following joint state pdf satisfies all the above global balance equations.

$$P(n_1, n_2) = \rho_1^{n_1} \rho_2^{n_2} f(\rho 1, \rho_2) \quad (12)$$

where

$$f(\rho_1, \rho_2) = \frac{(1-\rho_1)(1-\rho_2)(\rho_2-\rho_1)}{(1-\rho_1^{N+1})(\rho_2-\rho_1) - \rho_2(1-\rho_1)(\rho_2^{N+1}-\rho_1^{N+1}) - (1-\rho_1)(1-\rho_2)(\rho_2-\rho_1)(\rho_1^N+\rho_2^N)}$$
$$\rho_1 = \frac{\lambda_1}{\mu}$$
$$\rho_2 = \frac{\lambda_2}{\mu}$$

A little work can show that the blocking probability for class 1 packets in the SBCBS queueing system is:

$$P_{b_1} = \left(\frac{\rho_2^{N+1} - \rho_1^{N+1}}{\rho_2 - \rho_1} - \rho_2^N - \rho_1^N + \rho_1^{N-1}\right) f(\rho_1, \rho_2) \quad (13)$$

The blocking probability for class 2 packets is:

$$P_{b_2} = \left(\frac{\rho_2^{N+1} - \rho_1^{N+1}}{\rho_2 - \rho_1} - \rho_2^N - \rho_1^N + \rho_2^{N-1}\right) f(\rho_1, \rho_2) \quad (14)$$

From above two equations, the average loss rate for a SBCBS queueing system will be:

$$\eta_{loss} = \frac{\lambda_1}{\lambda_1 + \lambda_2} P_{b_1} + \frac{\lambda_2}{\lambda_1 + \lambda_2} P_{b_2}$$
$$= \frac{(\rho_2^{N+1} - \rho_1^{N+1})(\rho_1 + \rho_2) + (\rho_2 - \rho_1)(1 - \rho_1 - \rho_2)(\rho_1^N + \rho_2^N)}{\rho_2^2 - \rho_1^2} P(0,0) \quad (15)$$
$$\text{where } P(0,0) = f(\rho_1, \rho_2)$$

The results of average loss rate for different available buffers are given in Table 4. The average loss rate for SBCBS can be seen to be superior to that of the previous two system. This is because of the flexibility sharing the buffer gives in accommodating traffic flow.

2.4 Shared Buffer with Preemptive Priority (SBPP) Queueing System:

The scheme of the SBPP queueing system is the same as that shown in figure 4. The discipline for both servers is not the same as that in SBCBS. In this model, each server will mainly serve its preset class of packets. It can also serve another class of packets only if there is no packet which is of its preset class in the system. In the case that a server is serving a packet whose class is not the preset class for the server and if one of its preset class of packets arrives to the system, then the arriving preset class of packet will preempt the one already in service. This system is also memoryless.

There still exists a transition state diagram similar to figure 5 with some modifications. Figure 6 illustrates the transition state model for a SBPP queueing system with N=6. Nevertheless, there is no product-form solution for this diagram. One way to find the probability of each state is using the iterative method. Since each state has its own global balance equation, a program can iterate all these global balance equations until they converge and are within a range of error control . Another approach to solve these state probabilities is solving simultaneous global balance equations and normalization equation by using a Gauss-elimination linear equation solver. An example of a SBPP queueing system with N=3 under the matrix analytic method is:

$$AP = B \tag{16}$$

The entries of the matrix P are the state probability of each state and the entries of the matrix A are the coefficients of global balance equations except for the last row which are all 1 from the normalization equation. Thus:

$$A = \begin{bmatrix} \lambda_1+\lambda_2 & -\mu & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 \\ -\lambda_2 & \lambda_1+\lambda_2+\mu & -2\mu & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & -\lambda_2 & \lambda_1+\lambda_2+2\mu & -2\mu & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & -\lambda_2 & 2\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ -\lambda_1 & 0 & 0 & 0 & \lambda_1+\lambda_2+\mu & -\mu & 0 & -2\mu & 0 & 0 \\ 0 & -\lambda_1 & 0 & 0 & -\lambda_2 & \lambda_1+\lambda_2+2\mu & -\mu & 0 & -\mu & 0 \\ 0 & 0 & -\lambda_1 & 0 & 0 & -\lambda_2 & 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda_1 & 0 & 0 & \lambda_1+\lambda_2+2\mu & -\mu & -2\mu \\ 0 & 0 & 0 & 0 & 0 & -\lambda_1 & 0 & -\lambda_2 & 2\mu & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} P(0,0) \\ P(0,1) \\ P(0,2) \\ P(0,3) \\ P(1,0) \\ P(1,1) \\ P(1,2) \\ P(2,0) \\ P(2,1) \\ P(3,0) \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

For a queueing system with total buffer length of N, the iterative method will only need computation time of O(N^2) for one iteration compared to the matrix analytic method which needs O(N^6) [6] computation time to get the final steady state probability P of the equation $AP = B$ by the Gauss-elimination method. The iterative method is more efficient and less computer memory will be needed when the buffer length becomes larger. Although directly solving state probability equations can yield the exact value of each state probability, the error of the result from the iterative method can be controlled to be under 0.1% (based on the exact solution for SBCBS) in our study. The loss rate for this system is illustrated in table 5 for the iterative method.

2.5 Non-Preemptive Priority SBCBS (NP+SBCBS) Queueing:

This model has the same service discipline but a different buffer management policy as the model of SBCBS. Each packet entering the buffer will keep its position in order of arrival. When a server is free, its preset class of packet will not be able to enter the server if there is any one of the other class of packets in front of it in the queue. That is, packets of the other class in front of a packet will block it.

This queueing system model does not have a very tractable state transition diagram, nor does this queueing system model have an analytical solution. Figure 7 & 8 show the transition state diagrams for the queueing system having total queue capacity of 4 and 5. As the capacity increases, it will give rise to dauntingly complex state transition diagrams.

For this reason simulation was used to find the average loss rate. Table 6 shows the performance result of this queueing system. The reason that the table shows fewer buffers in the system than before is because when the buffer length becomes larger, its loss rate is so small as to cause simulation difficulty. For example, when the loss rate is approximated around the order of 10^{-12}, one needs to generate more than 10^{12} packets in the program to get a reliable result.

3. Comparison of Performance Results and Discussion

The above queueing models can be divided into two different groups. One is buffer sharing and the other one is fixed buffer assignment. PFBA and PFBA+PAR belong to the group of fixed buffer assignment. The group of buffer sharing consists of SBCBS , SBPP and NP+SBCBS.

The above results in Table 7 & 8 show that any one of the queueing models from the buffer sharing group has a better performance than that of PFBA or PFBA+PAR. The reason is that under the buffer sharing policy the queueing system will have a higher utilization of the buffers than that with a fixed buffer assignment policy. Put another way, buffer sharing inherently allows more flexibility in buffer allocation.

3.1 PFBA and PFBA+PAR queueing systems

The choice of N_1 in PFBA+PAR can be made simply once the system has information about the arrival rates of two lasses of traffic. This can be adjusted dynamically. In a PFBA queueing system, tables of optimal assignments could be stored. Then for each different combinations of arrival rates and available buffers the optimal assignment of N_1 can be retrieved. Apparently PFBA will need substantial memory to keep the presolved informations concerning optimal N_1.

The PFBA+PAR policy is thus more practical. Although the average loss rate when PFBA+PAR is used is not an optimal minimum value, the previous result shows that it yields a value very close to the optimal one most of the time.

3.2 Buffer sharing queueing systems

The constraint that packets in front of a packet will block it in a NP+SBCBS queueing system results in higher blocking probability when compared to that of the policy of SBCBS and SBPP. The average loss rate in a NP+SBCBS queueing system can be thought to be an upper bound for buffer sharing systems with the same length of buffers and two servers of the same service rate.

The service discipline of the server in the SBPP queueing system is less restrictive than that in the SBCBS queueing system. From the point of view of the utilization of the server, a SBPP system makes fuller use of the server. If the service rates of both servers are the same and only two classes of traffic will enter the system, the result of average loss rate in a SBPP system can be considered to be the lower bound for the buffer sharing system with the same length of buffer.

Both the SBPP and SBCBS queueing systems need a more complex buffer management system than that of NP+SBCBS. Though the SBPP queueing system has a more flexible service discipline, it will need a more sophisticated server than that in the SBCBS and the

NP+SBCBS systems. Finally, the preemptive feature in SBPP and SBCBS will necessitate more registers to temporarily store the packet which suffers an interruption of service.

4. Conclusion

On the basis of different queue control and service discipline, five different queueing systems are proposed and studied in this paper. Various methods which are analytical, numerical or simulative are also provided for analyzing the queueing systems. It was found that a buffer shared by two classes of traffic yields superior average loss probability compared to fixed buffer management. Results also show that the SBPP can achieve a better performance of average loss rate at the expense of more complicated hardware implementation.

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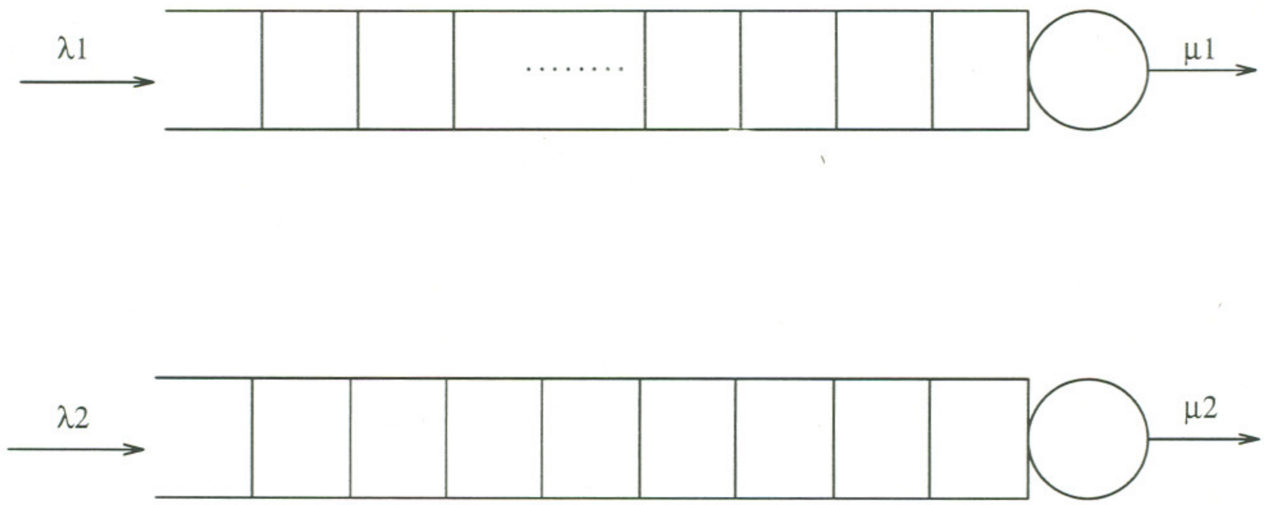


Figure 1: The queueing system scheme for PFBA and PFBA+PAR

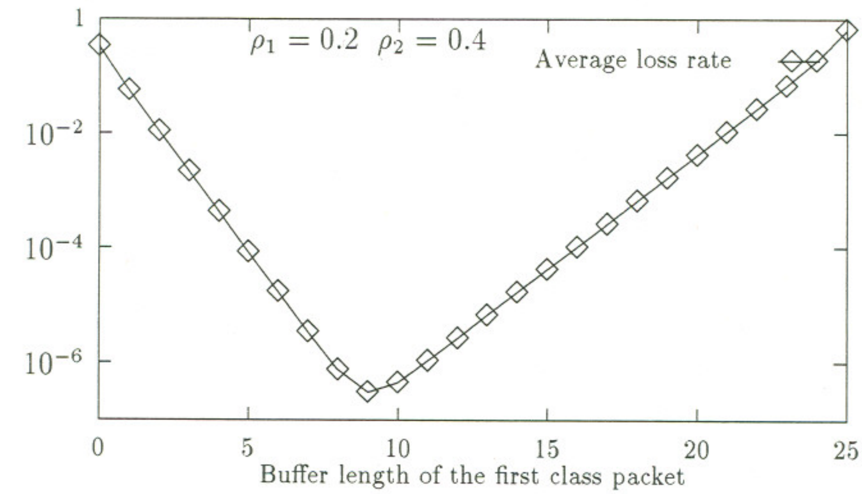


Figure 2: Average loss rate under different assignments of N_1 for 25 buffers in the system

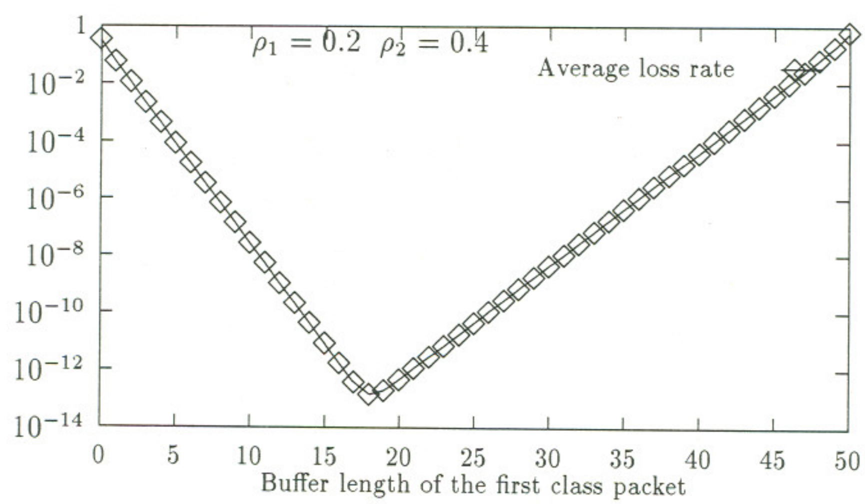


Figure 3: Average loss rate under different assignments of N_1 for 50 buffers in the system

Table 1.a: Average loss rate for PFBA

Arrival Rate		N_T(Total queue capacity)					
ρ_1	ρ_2	50	55	60	65	70	75
0.2	0.2	2.7×10^{-18}	6.4×10^{-20}	8.6×10^{-22}	2.1×10^{-23}	2.7×10^{-25}	6.6×10^{-27}
0.2	0.4	1.4×10^{-13}	7.5×10^{-15}	4.1×10^{-16}	2.4×10^{-17}	1.4×10^{-18}	6.7×10^{-20}
0.2	0.6	1.9×10^{-9}	2.7×10^{-10}	3.8×10^{-11}	5.4×10^{-12}	7.9×10^{-13}	1.2×10^{-13}
0.2	0.8	1.3×10^{-5}	4.9×10^{-6}	1.9×10^{-6}	6.8×10^{-7}	2.6×10^{-7}	9.7×10^{-8}

Table 1.b: Average loss rate for PFBA

Arrival Rate		N_T(Total queue capacity)				
ρ_1	ρ_2	80	85	90	95	100
0.2	0.2	8.8×10^{-29}	2.1×10^{-30}	2.8×10^{-32}	6.8×10^{-34}	9.0×10^{-36}
0.2	0.4	3.5×10^{-21}	1.9×10^{-22}	1.1×10^{-23}	6.2×10^{-25}	3.2×10^{-26}
0.2	0.6	1.7×10^{-14}	2.3×10^{-15}	3.3×10^{-16}	4.7×10^{-17}	7.5×10^{-18}
0.2	0.8	3.6×10^{-8}	1.4×10^{-8}	5.1×10^{-9}	1.9×10^{-9}	7.2×10^{-10}

Table 2.a: Average loss rate for PFBA+PAR

Arrival Rate		N_T(Total queue capacity)					
ρ_1	ρ_2	50	55	60	65	70	75
0.2	0.2	2.7×10^{-18}	6.4×10^{-20}	8.6×10^{-22}	2.1×10^{-23}	2.7×10^{-25}	6.6×10^{-27}
0.2	0.4	3.8×10^{-13}	7.1×10^{-14}	2.8×10^{-15}	1.1×10^{-16}	2.2×10^{-17}	9.0×10^{-19}
0.2	0.6	2.0×10^{-9}	2.7×10^{-10}	3.8×10^{-11}	5.4×10^{-12}	9.3×10^{-13}	1.2×10^{-13}
0.2	0.8	2.1×10^{-5}	8.7×10^{-6}	3.6×10^{-6}	1.5×10^{-6}	6.0×10^{-7}	2.5×10^{-7}

Table 2.b: Average loss rate for PFBA+PAR

Arrival Rate		N_T(Total queue capacity)				
ρ_1	ρ_2	80	85	90	95	100
0.2	0.2	8.8×10^{-29}	2.1×10^{-30}	2.8×10^{-32}	6.8×10^{-34}	9.0×10^{-36}
0.2	0.4	3.6×10^{-20}	7.2×10^{-21}	2.9×10^{-22}	1.1×10^{-23}	2.3×10^{-24}
0.2	0.6	1.7×10^{-14}	2.3×10^{-15}	4.3×10^{-16}	5.7×10^{-17}	7.6×10^{-18}
0.2	0.8	1.0×10^{-7}	4.1×10^{-8}	1.7×10^{-8}	6.9×10^{-9}	2.8×10^{-9}

Table 3.a: Comparison of the average loss rate between PFBA and PFBA+PAR

		$N_T = 50$			$N_T = 55$		
Arrival Rate		PFBA	PFBA+PAR	ratio	PFBA	PFBA+PAR	ratio
ρ_1	ρ_2	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$
0.2	0.2	25	25	1.0	28	28	1.0
0.2	0.4	18	17	2.64	20	18	9.40
0.2	0.6	12	13	1.05	14	14	1.0
0.2	0.8	7	10	1.65	8	11	1.79

Table 3.b: Comparison of the average loss rate between PFBA and PFBA+PAR

		$N_T = 60$			$N_T = 65$		
Arrival Rate		PFBA	PFBA+PAR	ratio	PFBA	PFBA+PAR	ratio
ρ_1	ρ_2	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$
0.2	0.2	30	30	1.0	33	33	1.0
0.2	0.4	22	20	6.87	24	22	4.83
0.2	0.6	15	15	1.0	16	16	1.0
0.2	0.8	8	12	1.91	9	13	2.15

Table 3.c: Comparison of the average loss rate between PFBA and PFBA+PAR

		$N_T = 70$			$N_T = 75$		
Arrival Rate		PFBA	PFBA+PAR	ratio	PFBA	PFBA+PAR	ratio
ρ_1	ρ_2	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$
0.2	0.2	35	35	1.0	38	38	1.0
0.2	0.4	25	23	16.15	27	25	13.34
0.2	0.6	17	18	1.18	18	19	1.03
0.2	0.8	10	14	2.29	10	15	2.54

Table 3.d: Comparison of the average loss rate between PFBA and PFBA+PAR

		$N_T = 80$			$N_T = 85$		
Arrival Rate		PFBA	PFBA+PAR	ratio	PFBA	PFBA+PAR	ratio
ρ_1	ρ_2	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$
0.2	0.2	40	40	1.0	42	43	1.0
0.2	0.4	29	27	10.44	31	28	30.31
0.2	0.6	20	20	1.0	21	21	1.0
0.2	0.8	11	16	2.78	11	17	2.93

Table 3.e: Comparison of the average loss rate between PFBA and PFBA+PAR

		$N_T = 90$			$N_T = 95$		
Arrival Rate		PFBA	PFBA+PAR	ratio	PFBA	PFBA+PAR	ratio
ρ_1	ρ_2	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$
0.2	0.2	45	45	1.0	48	48	1.0
0.2	0.4	33	30	27.07	35	32	18.43
0.2	0.6	22	23	1.29	23	24	1.16
0.2	0.8	12	18	3.32	13	19	3.56

Table 3.f: Comparison of the average loss rate between PFBA and PFBA+PAR

		$N_T = 100$		
Arrival Rate		PFBA	PFBA+PAR	ratio
ρ_1	ρ_2	N_{1opt}	N_1	$\frac{\eta_{PFBA+PAR}}{\eta_{PFBA}}$
0.2	0.2	50	50	1.0
0.2	0.4	36	33	71.75
0.2	0.6	24	25	1.01
0.2	0.8	13	20	3.91

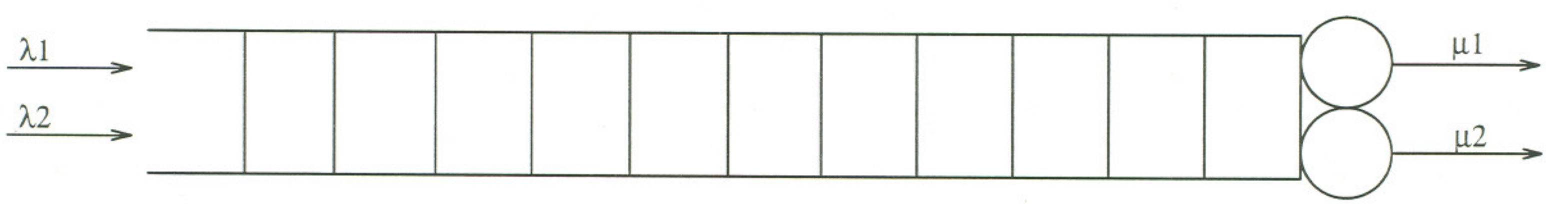


Figure 4: Buffer sharing queueing scheme for SBPP, SBCBS and NP+SBCBS

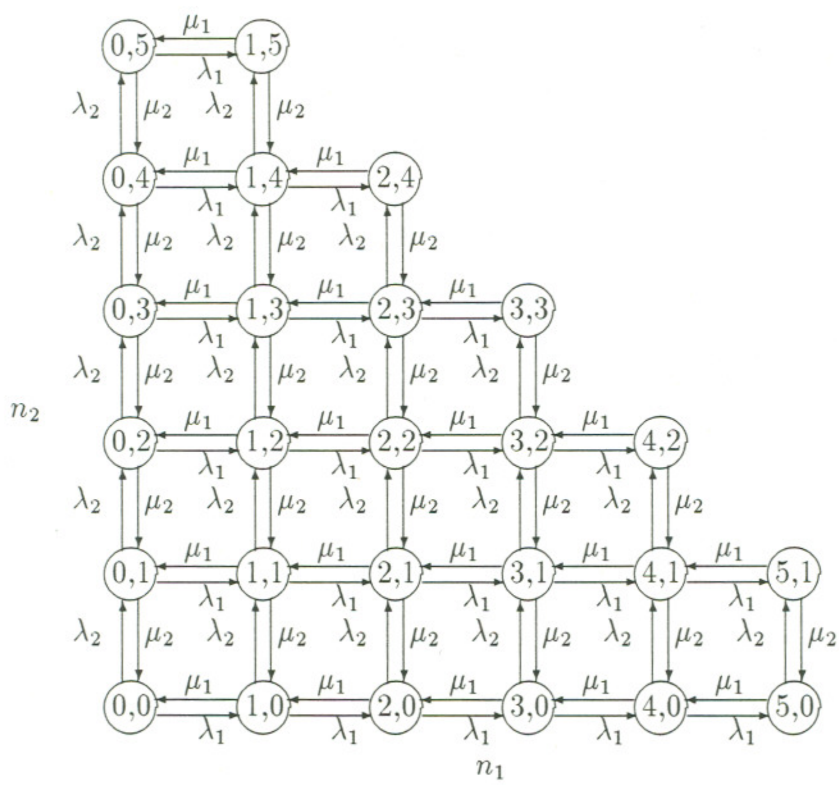


Figure.5 State Transition Diagram for SBCBS queueing system with N=6

Table 4.a: Average loss rate for SBCBS

Arrival Rate		N_T(Total queue capacity)					
ρ_1	ρ_2	50	55	60	65	70	75
0.2	0.2	3.9×10^{-32}	1.4×10^{-37}	4.7×10^{-41}	1.6×10^{-44}	5.6×10^{-48}	1.9×10^{-51}
0.2	0.4	1.6×10^{-20}	1.7×10^{-22}	1.7×10^{-24}	1.7×10^{-26}	1.8×10^{-28}	1.8×10^{-30}
0.2	0.6	4.5×10^{-12}	3.5×10^{-13}	2.7×10^{-14}	2.1×10^{-15}	1.7×10^{-16}	1.3×10^{-17}
0.2	0.8	3.0×10^{-6}	1.0×10^{-6}	3.3×10^{-7}	1.1×10^{-7}	3.5×10^{-8}	1.2×10^{-8}

Table 4.b: Average loss rate for SBCBS

Arrival Rate		N_T(Total queue capacity)				
ρ_1	ρ_2	80	85	90	95	100
0.2	0.2	6.5×10^{-55}	2.2×10^{-58}	7.4×10^{-62}	2.5×10^{-65}	8.4×10^{-69}
0.2	0.4	1.9×10^{-32}	1.9×10^{-34}	2.0×10^{-36}	2.0×10^{-38}	2.0×10^{-40}
0.2	0.6	1.0×10^{-18}	7.8×10^{-20}	6.1×10^{-21}	4.7×10^{-22}	3.7×10^{-23}
0.2	0.8	3.8×10^{-9}	1.2×10^{-9}	4.0×10^{-10}	1.3×10^{-10}	4.3×10^{-11}

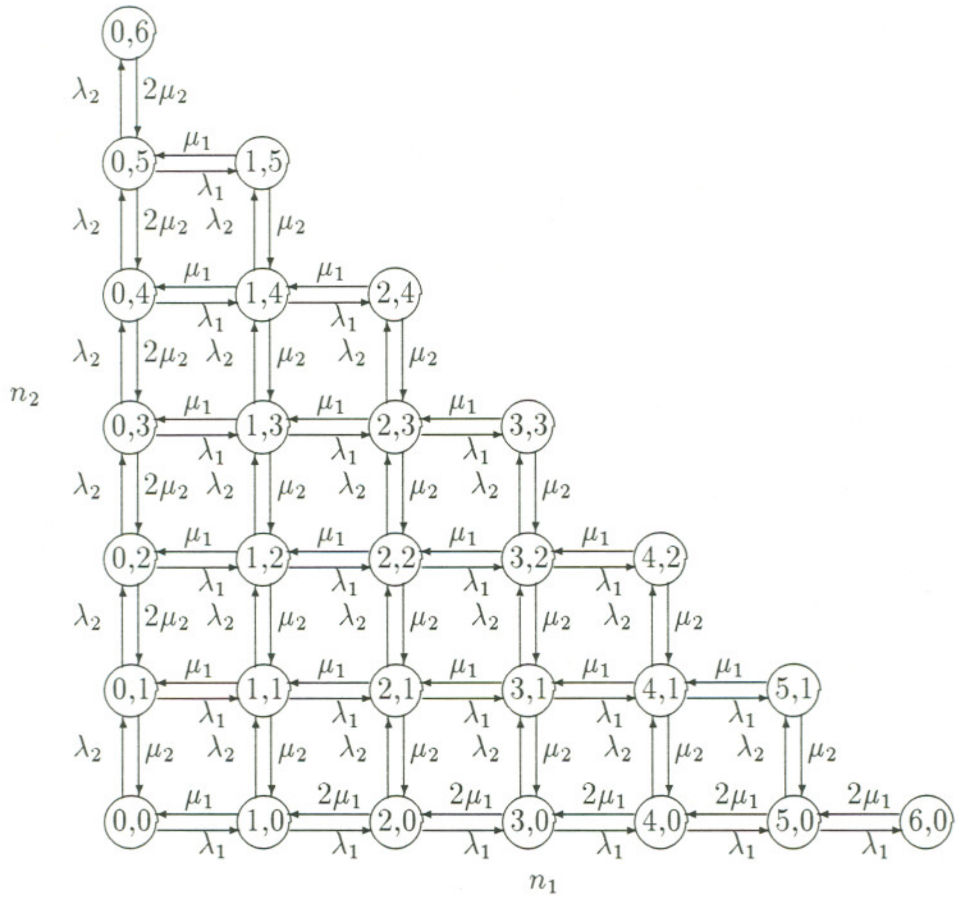
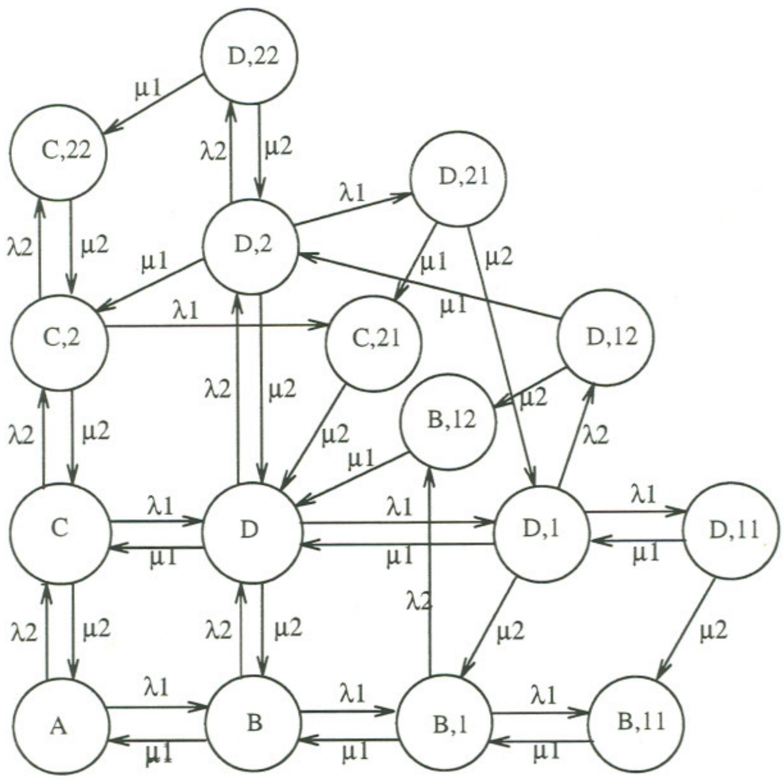


Figure.6 State Transition Diagram for SBPP queueing system with N=6



Legend of State:

A: no packet in the queueing system

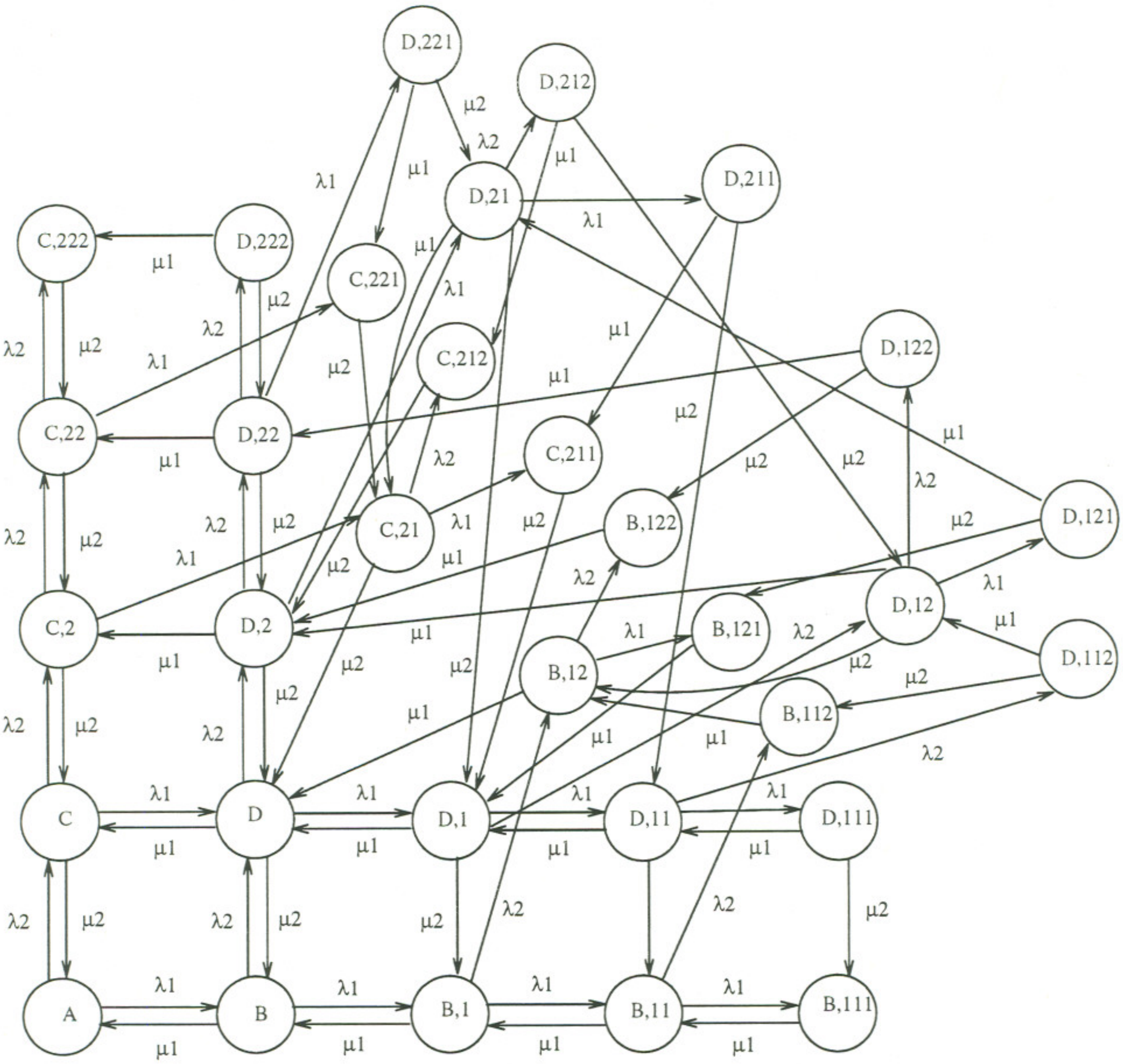
B: Only one class 1 packet in the server

C: Only one class 2 packet in the server

D: Both servers have packets

Sequential number after letter: Class information of packets in order of arrival to the buffer

Figure 7: State Transition Diagram for NP+SBCBS queueing system for N=4



Legend of State:

A: No packet in the queueing system

B: Only one class 1 packet in the server

C: Only one class 2 packet in the server

D: Both servers have packets

Sequential number after letter: Class information of packets in order of arrival to the buffer

Figure 8: State Transition Diagram for NP+SBCBS queueing system for N=5

Table 6: Average loss rate for NP+SBCBS

		N_T(Total queue capacity)				
ρ_1	ρ_2	5	10	15	20	25
0.2	0.2	7.0×10^{-3}	3.4×10^{-5}			
0.2	0.4	3.0×10^{-2}	1.0×10^{-3}	3.8×10^{-5}		
0.2	0.6	7.8×10^{-2}	1.0×10^{-2}	1.4×10^{-3}	2.5×10^{-4}	3.5×10^{-5}
0.2	0.8	1.5×10^{-1}	4.7×10^{-2}	1.9×10^{-2}	8.5×10^{-3}	3.6×10^{-3}

Table 7.a: Comparison of average loss rate

		N_T=50			
ρ_1	ρ_2	PFBA	PFBA+PAR	SBCBS	SBPP
0.2	0.2	2.7×10^{-18}	2.7×10^{-18}	3.9×10^{-32}	1.5×10^{-35}
0.2	0.4	1.4×10^{-13}	3.8×10^{-13}	1.6×10^{-20}	7.7×10^{-27}
0.2	0.6	1.9×10^{-9}	2.0×10^{-9}	4.5×10^{-12}	1.1×10^{-20}
0.2	0.8	1.3×10^{-5}	2.1×10^{-5}	3.0×10^{-6}	5.9×10^{-16}

Table 7.b: Comparison of average loss rate

		N_T=80			
ρ_1	ρ_2	PFBA	PFBA+PAR	SBCBS	SBPP
0.2	0.2	8.8×10^{-29}	8.8×10^{-29}	6.5×10^{-55}	1.6×10^{-56}
0.2	0.4	3.5×10^{-21}	3.6×10^{-20}	1.9×10^{-32}	1.6×10^{-42}
0.2	0.6	1.7×10^{-14}	1.7×10^{-14}	1.0×10^{-18}	1.3×10^{-32}
0.2	0.8	3.6×10^{-8}	1.0×10^{-7}	3.8×10^{-9}	5.5×10^{-25}

Table 8.a: Comparison of average loss rate for different queueing system

		N_T=5				
ρ_1	ρ_2	PFBA	PFBA+PAR	SBCBS	SBPP	NP+SBCBS
0.2	0.2	1.9×10^{-02}	1.9×10^{-02}	1.8×10^{-03}	4.3×10^{-04}	7.0×10^{-03}
0.2	0.4	3.7×10^{-02}	3.7×10^{-02}	1.3×10^{-02}	2.6×10^{-03}	3.0×10^{-02}
0.2	0.6	8.3×10^{-02}	8.4×10^{-02}	4.7×10^{-02}	8.8×10^{-03}	7.8×10^{-02}
0.2	0.8	1.3×10^{-01}	1.3×10^{-01}	1.0×10^{-01}	2.1×10^{-02}	1.5×10^{-01}

Table 8.b: Comparison of average loss rate for different queueing system

		N_T=10				
ρ_1	ρ_2	PFBA	PFBA+PAR	SBCBS	SBPP	NP+SBCBS
0.2	0.2	2.6×10^{-04}	2.6×10^{-04}	9.2×10^{-07}	1.4×10^{-07}	3.4×10^{-05}
0.2	0.4	2.1×10^{-03}	2.8×10^{-03}	1.3×10^{-04}	6.4×10^{-06}	1.0×10^{-03}
0.2	0.6	1.0×10^{-02}	1.0×10^{-02}	3.4×10^{-03}	9.0×10^{-05}	1.0×10^{-02}
0.2	0.8	3.7×10^{-02}	3.7×10^{-02}	2.6×10^{-02}	6.5×10^{-04}	4.7×10^{-02}

Table 8.c: Comparison of average loss rate for different queueing system

		N_T=15				
ρ_1	ρ_2	PFBA	PFBA+PAR	SBCBS	SBPP	NP+SBCBS
0.2	0.2	6.1×10^{-06}	6.1×10^{-06}	4.0×10^{-10}	4.4×10^{-11}	
0.2	0.4	1.2×10^{-04}	1.3×10^{-04}	1.4×10^{-06}	1.5×10^{-08}	3.8×10^{-05}
0.2	0.6	1.4×10^{-03}	1.4×10^{-03}	2.6×10^{-04}	9.2×10^{-07}	1.4×10^{-03}
0.2	0.8	1.3×10^{-02}	1.3×10^{-02}	7.8×10^{-03}	2.0×10^{-05}	1.9×10^{-02}