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by

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Abstract

The use of real time channel borrowing in digital cellular TDMA systems is considered. Some systems (such as IS-54) are asynchronous in the sense that time slots in different cells are not aligned. CBWL (channel borrowing without locking) techniques can be applied but (without cell-to-cell synchronization) borrowing *individual* time slots from adjacent cells would violate co-channel interference constraints. Instead, frequency carriers can be borrowed. But (for example) in IS-54, a carrier supports three TDMA channels. So if only one TDMA channel is needed in the borrowing cell, two TDMA channels are unnecessarily transferred. We devised an appropriate carrier borrowing scheme and an analytical model to determine the traffic performance of TDMA/CBWL. *Fast carrier returning* is used to increase channel utilization by returning borrowed carriers as soon as possible. An efficient computational method that uses macro-states, decomposition, combinatorial analysis and the convolution algorithm is devised to find blocking probabilities. The results show that in comparison with FCA, the new CBWL scheme can significantly improve system performance of asynchronous TDMA cellular systems that use FDMA/TDMA multiplexes.

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1 Introduction

A family of new channel assignment and sharing schemes for cellular communication systems has been presented in [1]–[4]. The schemes, called *channel borrowing without locking* (CBWL), can be used to enhance traffic capacity of cellular communication systems and to accommodate spatially localized communication traffic overloads (or *"hot spots"*). Variations of the schemes can be considered — but for convenience of presentation and explanation we consider a basic hexagonal layout with base stations (wireless gateways) using omni-directional antennas nominally located at cell centers. In CBWL, each gateway is assigned a group of channels which is reused at gateways of other cells that are sufficiently distant for the co-channel interference to be tolerable. If all channels of the gateway of a cell are occupied when a new call arrives, channel borrowing is employed according to certain rules.

1.1 Background

Channel locking has been suggested to limit co-channel interference in other channel assignment strategies such as dynamic channel assignment (DCA) and hybrid channel assignment (HCA) [5]. That is, gateways within the required minimum reuse distance from a gateway that borrows a channel cannot use the same channel at the same time. Because of the difficulty in maintaining the reuse distance at the minimum value when channel locking is used, DCA and HCA generally perform less satisfactorily than FCA under high communication traffic loads [6], [7], [8].

In CBWL, a channel can be borrowed only from an adjacent gateway. Borrowed channels are used with reduced transmitted power such that co-channel interference caused by channel borrowing is no worse than that of a non-borrowing scheme. Therefore, *channel locking* is not necessary in CBWL schemes. The borrowed channels can be accessed only in part of the cell. To determine whether a mobile station is in the region that can be served by a borrowed channel, each gateway transmits a signal with the same reduced power as that on a borrowed channel. The signal is called borrowed channel sensing signal (BCSS). If the BCSS is not above some suitable threshold at a mobile station, a borrowed channel cannot be used by the mobile station; otherwise, the mobile station will use a borrowed channel if all its gateway's channels are occupied and any of its neighboring gateways has a channel available for lending. Thus, there are two types of new call originations—those that arise in parts of a cell in which a borrowed channel *can* be used, and those that arise in parts of a cell where borrowed channels *cannot* be used. We denote these as A-type and B-type calls, respectively.

Neighboring gateways are identified in the following manner. With respect to the given gateway, choose the first adjacent gateway. The position of the reference adjacent gateway can be arbitrary, but once chosen for a given gateway, all other gateways label their neighbors in a corresponding manner. The remaining five adjacent gateways are numbered sequentially proceeding clockwise from the first. The given gateway is labeled gateway 0. The C channels of a gateway are divided into seven distinct groups. The seven groups are numbered 0, 1, ..., 6. The channels of group 0 are reserved for exclusive use of the given gateway. Channels in the other six groups can be lent to neighbors. The ith neighbor can only borrow channels in the ith group. The number of channels in the ith group is denoted C_i, $i = 0, 1, \ldots, 6$. Thus $C = \sum_{k=0}^{6} C_k$. For convenience we consider a symmetrical arrangement with $C_1 = C_2 = \ldots = C_6 = l$. An example of the channel layout structure of CBWL is shown in Figure 1.

CBWL with the structure described above has three advantages: 1) In the scheme, a gateway does not need to transmit and receive on all channels of its neighboring gateways. It only needs to access the channels that

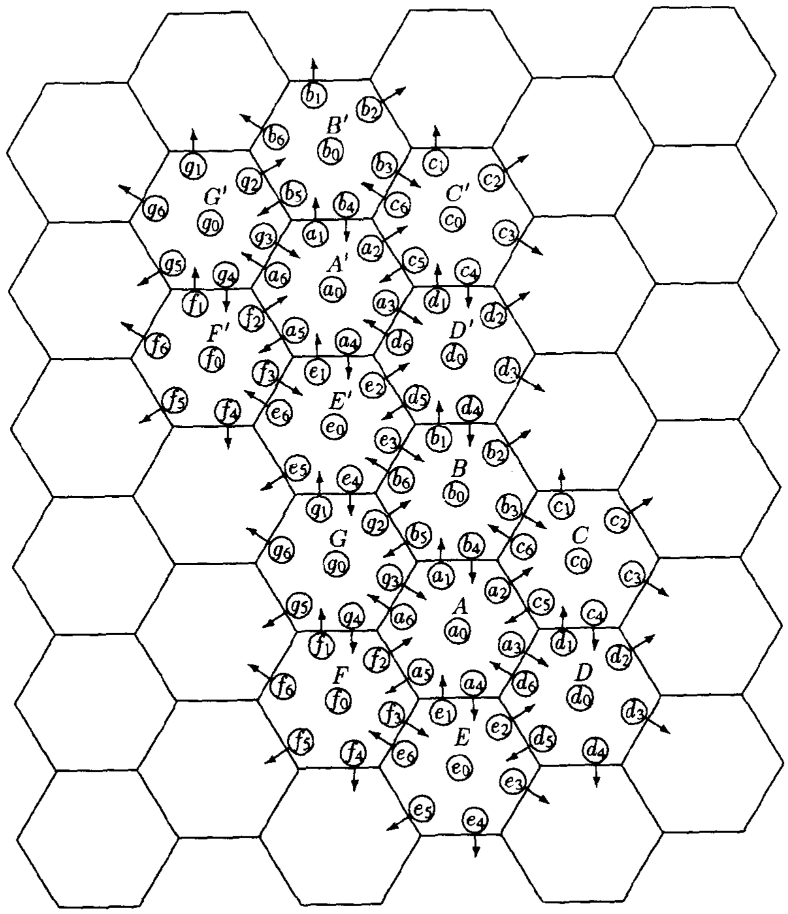

Figure 1: Channel structure of CBWL (cluster size =7).

are assigned to it and the channels of six groups, one group from each neighbor. Therefore, the transmitter of a gateway only needs to access a total of $C + 6l$ channels instead of $7C$ channels. The cost and complexity of a gateway are reduced. 2) The scheme eliminates the possibility that two co-channel gateways lend the same channel simultaneously to a pair of closely located gateways (the event would result in unacceptable co-channel interference). 3) With careful organization, the scheme can ensure that no *adjacent channels* are used in a given cell even though channel borrowing is employed.

Channel rearrangements can be used in CBWL [1]. With channel rearrangements, if a new B-type call arrival finds all channels of its gateway occupied, the call is *still not necessarily blocked.* If at the same time an A-type call in the cell is using a regular channel, and at least one neighbor can *lend channels to the given gateway,* the A-type call will use a borrowed channel from a neighbor and give its regular channel to the B-type call. In this way, calls that cannot use borrowed channels directly also benefit from the borrowing scheme. Therefore, the difference of blocking probabilities between two types of calls is lessened and the number of calls that can use borrowed channels (directly or indirectly) is increased.

A discussion and comparison of various channel assignment schemes including FCA, DCA, HCA, Generalized FCA, and Directed Retry is presented in [1]. Specific details of the schemes appear in [6] –[11].

In this paper, we consider the use of CBWL in TDMA systems such as IS-54. In IS-54, each carrier provides

3 TDMA slots and can accommodate 3 calls. If CBWL is used with TDMA, there can be two different kinds of channel borrowing. First, a cell can borrow a single TDMA channel (a time slot of a carrier) from a neighbor. Second, a cell can only borrow channels by carriers (3 TDMA channels together) from a neighbor. The first way is efficient, but it requires cell-to-cell time slot synchronization if co-channel interference is to be avoided. The second way does not require such synchronization and it can be easily implemented. But, with carrier borrowing, if the borrowing cell only needs one TDMA channel, two TDMA channels are unnecessarily transferred. To increase channel utilization, we can use "fast carrier returning"[3]. With fast carrier returning, a call that is using a borrowed TDMA channel will be transferred to a regular TDMA channel as soon as one is available to service it. After all the calls on the borrowed carrier are completed or transferred, the borrowed carrier is returned to its owner cell. Thus, borrowed carriers are returned as soon as possible and no call is served on a borrowed carrier if a regular TDMA channel (that can accommodate it) is idle. Without fast carrier returning, a borrowed carrier is returned only after all the calls (up to three) that uses the borrowed carrier are completed.

1.2 Carrier borrowing

We assume that each cell has C carriers. A gateway Y will not request to borrow if it can accommodate the new call on carriers that it already has. If Y sends a carrier borrowing request to a neighbor X, the request *is* or *is not* granted depending on the current channel occupancy of X. The rules for carrier borrowing are as follows:

1. X will deny the request, if the number of carriers that are lent from X to Y is equal to l.
2. X will deny the request, if the number of total carriers of cell X that are lent to other gateways (including Y) is equal to n.
3. A carrier can be borrowed from gateway X only if all slots on the carrier are unoccupied (at gateway X).

Fast carrier returning can be accomplished as follows: If gateway X has borrowed more than one carrier from neighbor, Y, one of the borrowed carriers from Y is chosen as *the primary borrowed carrier from* Y (usually this is the borrowed carrier which has the least number of calls). The primary borrowed carrier is given priority to be returned first. The priority is established by the following operating rules:

- If a regular channel (at gateway, X), say c_r, becomes free,
  1. X chooses a neighbor from which it has borrowed a carrier. We assume the chosen neighbor is Y. The choice can be done randomly. Alternatively the neighbor with the highest channel occupancy can be chosen. In this paper, the first method is assumed;
  2. One of the calls on the primary borrowed carrier from Y is transferred to c_r. One channel on the primary carrier of Y is freed.
- If a channel on a *nonprimary* borrowed carrier from a neighbor becomes available, one of calls on *the primary borrowed carrier of that neighbor* is transferred to that channel. One channel on the primary borrowed carrier of that neighbor is freed.
- If one channel on the primary borrowed carrier from a neighbor is freed, no transfer is needed.

Once all calls on the primary borrowed carrier from a neighbor is completed or transferred. The primary borrowed carrier is returned to the neighbor, and a new primary borrowed carrier is chosen randomly from all remaining borrowed channels from that neighbor until all borrowed carriers from that neighbor are returned. In this way, all nonprimary borrowed carriers and regular carriers are packed with the maximum number of (three) calls and primary borrowed carriers are returned as soon as possible. This strategy is fair to all neighbors.

If channels are borrowed by TDMA slots, the analysis models in [1]–[4] can be used since a channel is a TDMA slot. However, if channels are borrowed by carriers (3 slots), those models are not directly applicable. In this paper, we develop a new model for TDMA/CBWL schemes with *carrier* borrowing.

In Section 2, the scheme is modeled and analyzed. Numerical results from analysis are given in Section 3.

2 Traffic Analysis of TDMA/CBWL

We consider for the purpose of discussion, a homogeneous system. That is, each gateway has the same number of channels and the same offered traffic. New calls in a cell arise at an average rate λ (new call arrivals per second per cell) according to a Poisson process, and calls originate uniformly throughout the service area. Call holding times have a negative exponential probability distribution with mean $1/\mu$. Users are assumed to be essentially stationary so that hand-off issues are not considered here. This is a reasonable model for a system whose user population consists primarily of stationary computer terminal sources. The hand-off issues of the high mobility users can be analyzed using the approach in [13].

The fraction of new call arrivals that can use borrowed channels is taken as a parameter, p. The fraction can be approximated by the fraction of cell area where borrowed channels can be used [1]. Calls in progress in the part of a cell where borrowed channels *can be used* are denoted A-type calls even if they are not using a borrowed channel. Calls in progress outside this region are B-type calls.

2.1 Equilibrium state distribution of TDMA/CBWL

At any given time a gateway is in one of a finite number of states. A state is identified by a vector $\mathbf{I} = (i_a, i_b, i_1, i_2, i_3, i_4, i_5, i_6, n_b)$, in which

i_a is the number of regular TDMA *slots* that are used by the A-type calls in the given cell;

i_b is the number of regular TDMA *slots* that are used by the B-type calls in the given cell;

i_k $(k = 1, \ldots, 6)$ is the number of *carriers* that are lent to the kth adjacent cell;

n_b is the number of borrowed TDMA *slots* that are *used* by the calls in the given cell.

In state $\mathbf{I}$, the total number of TDMA channels that are used, including those that are *lent to adjacent gateways*, is given by

$$J(\mathbf{I}) = i_a + i_b + 3\sum_{k=0}^{6} i_k \,. \qquad (1)$$

The total number of carriers that are (currently) lent to all adjacent gateways is

$$L(\mathbf{I}) = \sum_{k=1}^{6} i_k \,. \qquad (2)$$

From the last section, the maximum number of carriers that a cell can lend at any given time is

$$L_{max} = \min(6l, n) \,. \qquad (3)$$

The total number of TDMA channels of a cell is $3C$. With fast carrier returning, if $J(\mathbf{I}) < 3C$, n_b is always equal to zero. That is, only if all TDMA channels of a gateway are busy (occupied or lent), is the gateway allowed to use a borrowed carrier. Therefore, permissible states of $\mathbf{I} = (i_a, i_b, i_1, \ldots, i_6, n_b)$ are constrained by the following conditions

$$
\begin{aligned}
0 \leq\ & i_a \leq 3C\,, \\
0 \leq\ & i_b \leq 3C\,, \\
0 \leq\ & i_k \leq l \qquad k = 1,2,\ldots,6\,, \\
0 \leq\ & J(\mathbf{I}) \leq 3C\,, \\
0 \leq\ & L(\mathbf{I}) \leq L_{max}\,, \\
n_b = 0, \qquad & \text{if} J(\mathbf{I}) < 3C\,, \\
0 \leq n_b \leq 18l, \qquad & \text{if} J(\mathbf{I}) = 3C\,.
\end{aligned} \tag{4}
$$

Since the state vector, **I**, has nine dimensions, the number of states can be very large. Macro-states are introduced to reduce the number of states. To calculate the probability of the macro-states, the probability transition rates of the macro-states must be determined. As in [1]–[3], we find that *for given carrier lending and returning rates and given the number of carriers that are lent to all six neighbors*, the joint distribution of the number of carriers that are lent to neighboring cells is in product form (That is, the conditional joint distribution of $i_1, i_2, \ldots, i_6$). The product-form solution can be used to calculate the transition rates of macro-variables obtained from merging variables i_k $(k = 1, \ldots, 6)$ into a single variable v that represents the total number of carriers that are lent by the gateway, that is.

$$
v = \sum_{k=1}^{6} i_k. \tag{5}
$$

After replacing six variables i_k $(k = 1, \ldots, 6)$ by v, the number of state dimensions becomes four. To further reduce the number of states, a second macro-variable, u, is introduced. The physical meaning of u is the number of calls that are served through the given gateway. This includes calls that are served through the given gateway on carriers that the given gateway BORROWS from its neighbors. Thus, the variable, u, can replace i_a, i_b, and n_b. That is,

$$
u = i_a + i_b + n_b\,. \tag{6}
$$

Note $n_b = 0$, when $J(\mathbf{I}) < 3C$.

Now, we have a two-dimensional macro-state variable, (u, v). From (4), all permissible states of (u, v) are constrained by the following conditions:

$$
\begin{aligned}
0 \leq\ & u \leq 3(C + 6l) \\
0 \leq\ & v \leq L_{max} \\
0 \leq\ & u + 3v \leq C + 6l\,.
\end{aligned} \tag{7}
$$

Equilibrium balance equation of two-dimensional variables, (u, v)

To find the equilibrium distribution of macro-state (u, v), we must first determine the state transition rates. For a state (u, v), the transition rates are: (1) the new call arrival rate, λ; (2) the *average carrier* lending rate given that v channels have been lent, $\alpha(v)$; (3) the call completion rate, $u\mu$; (4) the *average rate* of *carrier* returning from neighbors, $\rho(v)$; (5) the rate at which *borrowed channels* are used. Note that only some needs to borrow channels result in *carrier borrowing* requests. This is because a gateway will always use idle slots on a *carrier* that it has already borrowed before *borrowing* another *carrier*. Since no carriers are borrowed when the gateway

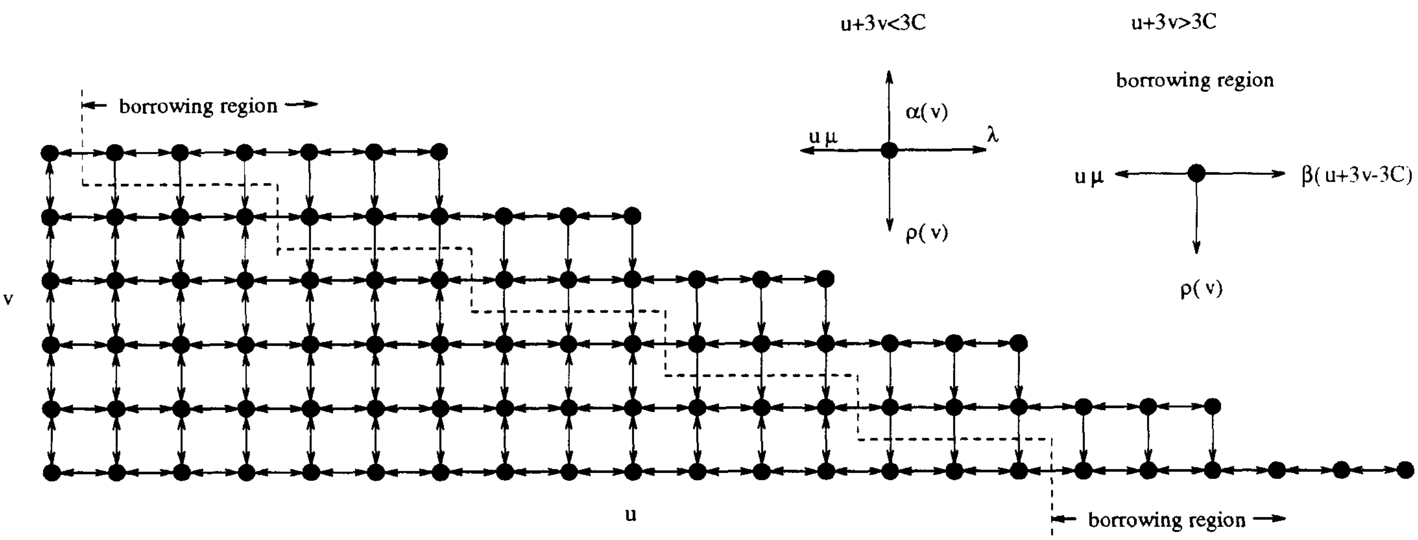

Figure 2: State-transition diagram of two-dimensional macro-state (u, v) for a TDMA/CBWL with $C = 5$ and a gateway can borrowed up to two carriers.

has idle carriers of its own, the rate at which borrowed channels are used is state dependent. Specifically this rate depends on the number of borrowed slots already in use at the gateway. Details are given in Appendix B. The rate is denoted $\beta(j)$ where j is the number of borrowed slots already in use at the gateway.

The rates of (1) and (3) are given system parameters. The rates of (2), (4) are state dependent. For a given v, there can be many different sequences of $i_i, i_2, \ldots, i_6$ with $\sum_{k=1}^{6} i_k = v$. For different sequences, their channel lending rates and channel returning rates from neighbors may be different. Therefore, for any given v, the channel lending rate and the returning rate must be determined by averaging over all sequences of $i_i, i_2, \ldots, i_6$ with $\sum_{k=1}^{6} i_k = v$. Since the variables that comprise v have product form solutions, a convolution algorithm is devised to find these averages efficiently [4]. These rates are calculated in Appendix B. The rate of (5) is also state dependent. The rate is determined in Appendix B.

Figure 2 shows the state-transition diagram of a TDMA/CBWL scheme.

Denote $p_{uv}(u, v)$ as the equilibrium probability of state (u, v). In statistical equilibrium, the probability flow out of each state must equal the probability flow into that state. Application of this principle leads to a set equations which must be solved to find the state probabilities.

$$
\begin{aligned}
[\lambda + \alpha(v) + u\mu + \rho(v)]\, p_{uv}(u, v) = {} & \lambda p_{uv}(u-1, v) + \alpha(v-1) p_{uv}(u, v-1) \\
& + (u+1)\mu p_{uv}(u+1, v) + \rho(v+1) p_{uv}(u, v+1)\,, \\
& \text{for } 0 \le u + v < C
\end{aligned} \tag{8a}
$$

$$
\begin{aligned}
[\beta(0) + u\mu + \rho(v)]\, p_{uv}(u, v) = {} & \lambda p_{uv}(u-1, v) + (u+1)\mu p_{uv}(u+1, v) + \rho(v+1) p_{uv}(u, v+1)\,, \\
& \text{for } u + v = C
\end{aligned} \tag{8b}
$$

$$[\beta(u+3v-3C)+u\mu+\rho(v)]\,p_{uv}(u,v)=\beta(u+3v-3C-1)p_{uv}(u-1,v)$$
$$+(u+1)\mu p_{uv}(u+1,v)+\rho(v+1)p_{uv}(u,v+1)\;,$$
$$\text{for } C<u+v\le C+6l \tag{8c}$$

where $p_{uv}(u,v)$=0, if $u<0, v<0$ and $v\le L_{\max}$.

If $\beta(j)$, $\rho(v)$ and $\alpha(v)$ are known, Gauss-Seidel iteration [15] can be used to solve the set of equations above. However, since these are not known and in fact are dependent on the state probabilities, the set of equations are in fact nonlinear. We use an iterative method, in which, we use an initial guess of some probabilities (they are specified later). From them we can calculate $\alpha(v)$, $\rho(v)$ and $\beta(j)$, then we can solve the set of equations. Because the dimensions of states has been reduced from eight to two, the computational effort is reduced.

2.2 The distribution of i_a and i_b: The decomposition method

Most of the probabilities for performance measurement can be found from $p_{uv}(u,v)$. However, we cannot find the distribution of (i_a,i_b) from $p_{uv}(u,v)$. Thus, from $p_{uv}(u,v)$, we cannot determine the channel rearrangement failure probability, p_a, which is needed to calculate $\beta(j)$ and other quantities.

To find p_a, a *decomposition* method is used. The method divides all state space into $L_{max}+1$ subspaces, each of which corresponds to a fixed value of v ($v=0,1,\ldots,L_{max}$). A subspace corresponding v is comprised all states in which the given gateway has lent exactly v carriers. If the distribution of u,v is product form, we can use decomposition method to calculate the conditional distribution, $p_v(i_a,i_b)$, separately for each subspace as if these subspaces were "independent" from one another. But, product form solution cannot be found for this case. Thus, the subspaces cannot be separated completely. Nevertheless, we can still use decomposition method as a good approximation. Since the calls that arise in the given cell usually occur much often than borrowed requests ($\lambda\gg\lambda'$), the interactions between i_a and i_b are much stronger than the interactions between u and v. We can calculate $p_v(i_a,i_b)$ separately for each fixed v and *omit* the interactions between u and v as if those interactions do not exist [17]. The agreement between results of simulation and analysis validates this approximation.

For the subspace corresponding v, if all $C-v$ carriers are occupied by B-type calls and no channel is used by A-type call, no channel rearrangement can be made. Thus, $p_v(0,3C-3v)$ is the probability of channel rearrangement failure given v carriers lent. Then, the probability of channel rearrangement failure, p_a, can be found from

$$p_a=\sum_{v=0}^{L_{max}}p_v(0,3C-3v)p_l(v)\;. \tag{9}$$

where $p_l(v)$ is the probability that a gateway lent v channels to its neighbors. It is found from $p_{uv}(u,v)$ in Appendix C.

If v carriers have been lent, all permissible states of i_a and i_b are constrained by following conditions:

$$\begin{aligned}0\le\ & i_a \le 3(C-v)\\ 0\le\ & i_b \le 3(C-v)\\ 0\le\ & i_a+i_b\le 3(C-v)\;.\end{aligned} \tag{10}$$

Denote λ_1 and λ_2 as the arrival rate of A-type and B-type calls, respectively. That is,

$$\lambda_1=p\lambda\;, \tag{11}$$

and

$$\lambda_2 = (1-p)\lambda\,. \tag{12}$$

If $i_a + i_b = 3(C-v)$ and $i_a > 0$, a borrowed channel is used for a new B-type call arrival through channel rearrangement. After channel rearrangement, the gateway's state is changed to $(i_a - 1, i_b + 1)$. Denote this transition rate as λ_3. If the gateway has borrowed j carriers and all $3j$ borrowed channels are occupied, the gateway will borrow one more carrier for a new call arrival. The probability that the borrowing successes is $p_{bs}(j)$ (Appendix C). If at least one borrowed slot is free, no carrier is borrowed. The probability that some borrowed slots are free given that j carriers are borrowed is $p_{br}(j) - p_{bf}(j)$ [See Appendix C (C.1) and (C.2)]. Thus

$$\lambda_3 = \frac{\lambda(1-p)}{p_c}\sum_{j=0}^{6l}[p_{bf}(j)p_{bs}(j) + p_{br}(j) - p_{bf}(j)]\,. \tag{13}$$

The channel releasing rate by A-type calls or B-type calls at state, (i_a, i_b), with $i_a + i_b < 3(C-v)$ is $i_a\mu$ or $i_b\mu$, respectively.

We consider the channel releasing rate in a state with $i_a + i_b = 3(C-v)$. In those states, the gateway may use some borrowed carriers with probability of $1 - p_{nb}(v)$, or it may not use any borrow channel with probability of $p_{nb}(v)$ (C.4). State transitions are different for the two cases. When a channel is released by an A-type call, if the gateway does not borrow any carrier, the state is changed to $(i_a - 1, i_b)$ with the rate of $p_{nb}(v)i_a\mu$. When a channel is released by a B-type call and no carrier is borrowed, the state is changed to $(i_a, i_b - 1)$ with the rate of $p_{nb}(v)i_b\mu$. If the gateway borrows at least one carrier and a channel is released, the released channel is at once reassigned to a call that uses a borrowed channel. Because the call that uses borrowed channel must be an A-type, the size of i_a is increased by one. If the channel is released by an A-type call, the size of i_a must be decreased by one. Totally, the state of (i_a, i_b) is not changed. If the channel is released by a B-type call, the size of i_b is decreased by one. Thus, the state is changed to $(i_a + 1, i_b - 1)$ with the rate of $[1 - p_{nb}(v)]i_b\mu$.

The state transition diagram of the conditional probabilities for $3(C-v) = 6$ is shown in Figure 3.

The flow balance equations of $p_v(i_a, i_b)$ are as follows:

$$\begin{aligned}
&[\lambda + (i_a + i_b)\mu]p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1)\\
&\qquad +(i_a+1)\mu p_v(i_a+1, i_b) + (i_b+1)\mu p_v(i_a, i_b+1)\,,\\
&\qquad \text{for } 0 \le i_a + i_b < 3(C-v-1)\\
&[\lambda + (C-v-1)\mu]\, p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1)\\
&\qquad +p_{nb}(v)(i_a+1)\mu p_v(i_a+1, i_b) + p_{nb}(v)(i_b+1)\mu p_v(i_a, i_b+1)\,,\\
&\qquad \text{for } i_a + i_b = 3(C-v-1)\\
&[\lambda_3 + p_{nb}(v)i_a\mu + i_b\mu]p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1)\\
&\qquad +\lambda_3 p_v(i_a+1, i_b-1) + [1 - p_{nb}(v)](i_b+1)\mu p_v(i_a-1, i_b+1)\,,\\
&\qquad \text{for } i_a + i_b = 3(C-v),\ i_a > 0\\
&(C-v)\mu p_v(0, C-v) = \lambda_2 p_v(0, C-v-1) + \lambda_3 p_v(1, C-v-1)\,,\\
&\qquad \text{for } i_a = 0,\ i_b = 3(C-v)
\end{aligned} \tag{14}$$

where $p_v(i_a, i_b) = 0$, if $i_a < 0$ or $i_b < 0$.

The balance equations are solved by Gauss-Seidel Iteration. where $p_l(v)$ is the probability that the given gateway lends v carriers to its neighbors [see Appendix C (C.3)]. In (9), $L_{max} + 1$ groups of equations of (14) with v from 0 to L_{max} must be solved. However, the number of equation groups to be solved can be reduced significantly. Since fast returning reduces the usage of borrowed carriers, the probability that a gateway lents a lot of carriers is quite small. When $p_l(v)$ is less than a desired precision, their contribution to p_a can be neglected and we do not need to solve the equations for corresponding v.

2.3 Blocking probabilities

To determine blocking probabilities, we define following probabilities. They are calculated in Appendix C.
p_c: probability that all channels of a gateway are occupied [see (C.5)].
p_{nf}: probability of no free channel (in both regular and borrowed carriers), a carrier needs to be borrowed from neighbors [see (C.6)].
p_e: probability that a borrowing request from a neighbor is denied due to carrier availability [see (C.8)].
p_f: probability that a borrowing request from a neighbor is denied [see (C.10)].
$p_{bs}(j)$: probability that a gateway's borrowing request successes given that the gateway has borrowed j carriers [see (C.13)].
$p_{bf}(j)$: probability that all borrowed channels are busy given that j carriers are borrowed [see (C.2)].

2.3.1 The average rate of borrowing requests to a neighbor

The rate is denoted as λ' and is needed for the convolution algorithm in Appendix B. First, we consider the average carrier borrowing rate of the given gateway from a specific neighbor. Denote the rate as λ''. The carrier borrowing rate given j carriers borrowed is $\beta(j)$. Thus, the average carrier borrowing rate is

$$\lambda'' = \frac{1}{6} \sum_{j=0}^{6l} p_{bf}(j)\beta(j) \,. \tag{15}$$

Denote λ' as the average borrowing *request* rate of the given gateway to the specific neighbor. The probability that those requests are accepted by the neighbor is $1 - p_f$ (C.10). That is

$$\lambda'(1 - p_f) = \lambda'' \,. \tag{16}$$

From (16), we find

$$\lambda' = \frac{1}{6(1 - p_f)} \sum_{j=0}^{6l} p_{bf}(j)\beta(j) \,. \tag{17}$$

Equations (B.3), (C.13), (8), (A.4), (14) and (9) form a set of simultaneous nonlinear equations which can be solved for system variables when parameters are given. Beginning with an initial guess of λ' and other parameters, the equations were solved numerically using the method of successive substitution.

2.3.2 Blocking probability

The blocking probability of A-type calls, α_{TC}, is the probability that all regular and borrowed channels of a gateway are occupied and their carrier borrowing requests are rejected by all neighbors. If the given gateway has borrowed j carriers, the probability that the borrowing requests are denied by neighbors is $1 - p_{bs}(j)$. Thus,

$$\alpha_{TC} = \sum_{j=0}^{6l} p_{bf}(j)[1 - p_{bs}(j)] = \sum_{j=0}^{6l} p_{bf}(j) \sum_{s=s_1}^{s_2} \frac{6!}{s!(6-s)!} \frac{a(j - sl, 6 - s, l - 1)}{a(j, 6, l)} p_e^{6-s} \,. \tag{18}$$

If a B-type call can use channel rearrangement, it has the same blocking probability as an A-type call. If a new B-type call find all channels occupied and it cannot use channel rearrangement, it will be blocked. Thus

$$\beta_{TC} = (1 - \frac{p_a}{p_{nf}})\alpha_{TC} + p_a. \tag{19}$$

where p_{nf} is the probability that a carrier must be borrowed to accommodate a new call arrival (C.6).

The overall blocking probability in a gateway is

$$B_{TC} = p\alpha_{TC} + (1-p)\beta_{TC} \, . \tag{20}$$

3 Numerical Results And Discussion

In our numerical examples, we consider TDMA/CBWL schemes for a mobile system with 24 carriers in each gateway. For simplicity, we assume in the homogeneous case, that the system has a very large (essentially infinite) number of cells. Thus we do not need to distinguish the boundary cells and the internal cells.

To simulate the system with a large number of cells, we used a 37 cell configuration with each cell having six adjacent neighboring cells. The simulation programs made boundary cells on one side of the region adjacent to the boundary cells on the other side. In each run, we generated about 2500 call arrivals *in each cell* and determined the fraction of calls for that were blocked in each cell. Since the cells are statistically the same, in one simulation run, 37 blocking probabilities can be found, as well as the mean, variance and confidence intervals for the blocking probabilities. The simulation was written in the *Simscript* simulation language, and was executed on a *Sun* workstation. About one hour of execution time was required for each simulation point plotted on Figure 5 and 7.

Figure 4 shows overall blocking probability of TDMA/CBWL obtained by numerical computation plotted against offered traffic in a cell. Simulation confidence intervals of 95% are also shown.

From the figures, we can see that the results of analysis are close to those by simulation. In light load, CBWL can improve blocking probability significantly in comparison with FCA. While in heavy load, the blocking probability is close to that of FCA (saturation). When $p = .3$, for a light traffic load, CBWL improves the system significantly. But when the traffic load increases, the performance improvement is reduced. Because it is difficult to borrow a carrier (three channels) in the much heavy traffic load.

In TDMA that uses FCA scheme, a cell with 24 carriers can accommodate 58 Erlangs at 1% blocking probability. At this level of blocking probability, in TDMA/CBWL, a cell with 24 carriers can accommodate about 65 Erlangs with $p = .1$ and 70 Erlangs with $p = .2$. The channel capacity is increased by 12% and 21% respectively. Figure 5 shows the probabilities of the different types of calls in TDMA/CBWL scheme. It is seen that for TDMA/CBWL, The *differences* between different calls are small for $p = .3$ and heavy traffic load.

Figure 6 compares the TDMA/CBWL with and without carrier fast returning. It is seen that fast carrier returning can increase channel efficiency and system capacity, especially under heavy traffic load.

4 Conclusion

Our analysis and simulation have shown that CBWL can enhance the performance of TDMA cellular systems. CBWL does not require complex channel management. Fast carrier returning is suggested for TDMA/CBWL to increase channel efficiency. For a 24 carrier/cell TDMA/CBWL systems, the traffic capacity is increased about 21% in comparison with FCA scheme when the fraction of calls that can use borrowed channels is just 0.2.

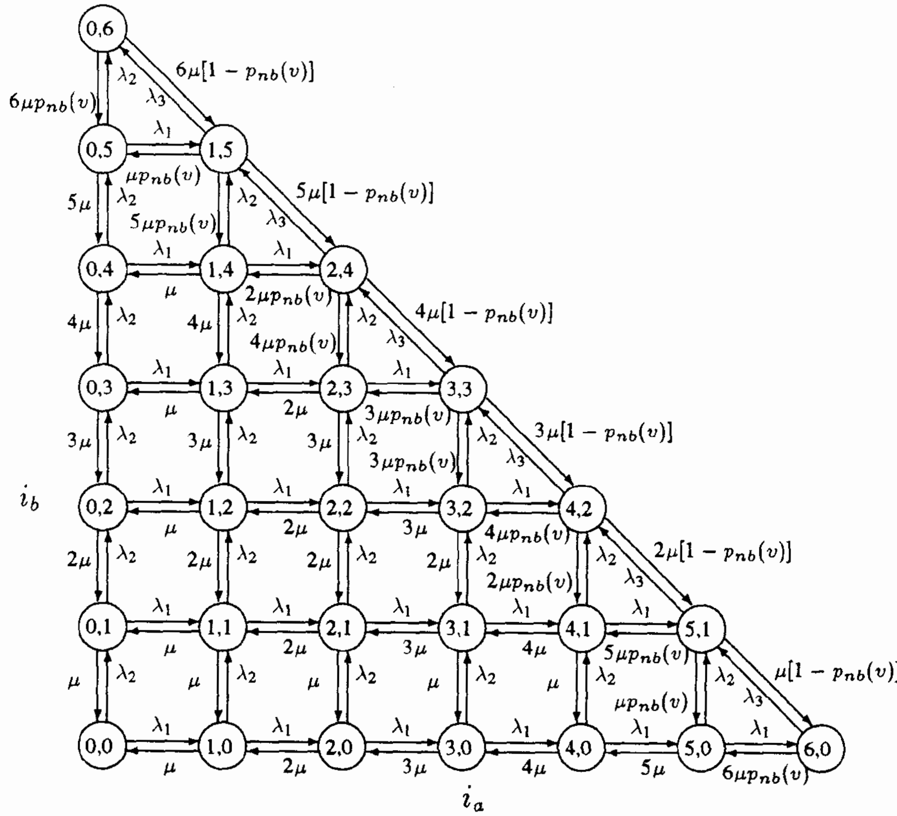

Figure 3: An example of transition diagram for macro-state (i_a, i_b) of CBWL/CR-FR, $(3(C - v) = 6)$.

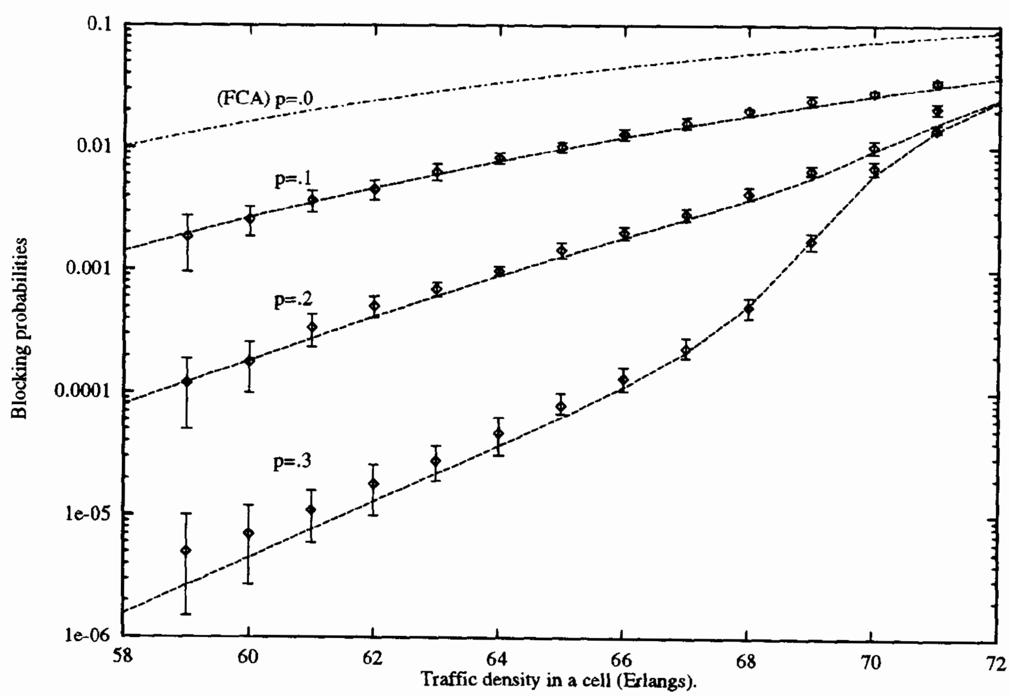

Figure 4: Blocking probabilities of TDMA/CBWL $(C = 24, l = 4)$.

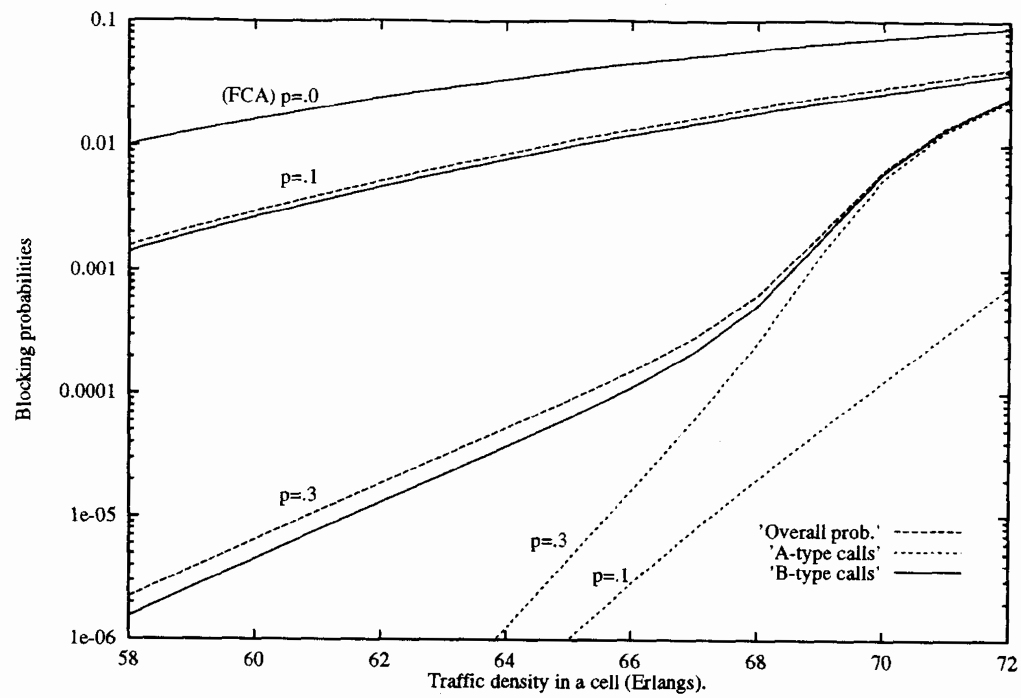

Figure 5: Blocking probabilities of different calls in TDMA/CBWL ($C = m = 24, l = 4$).

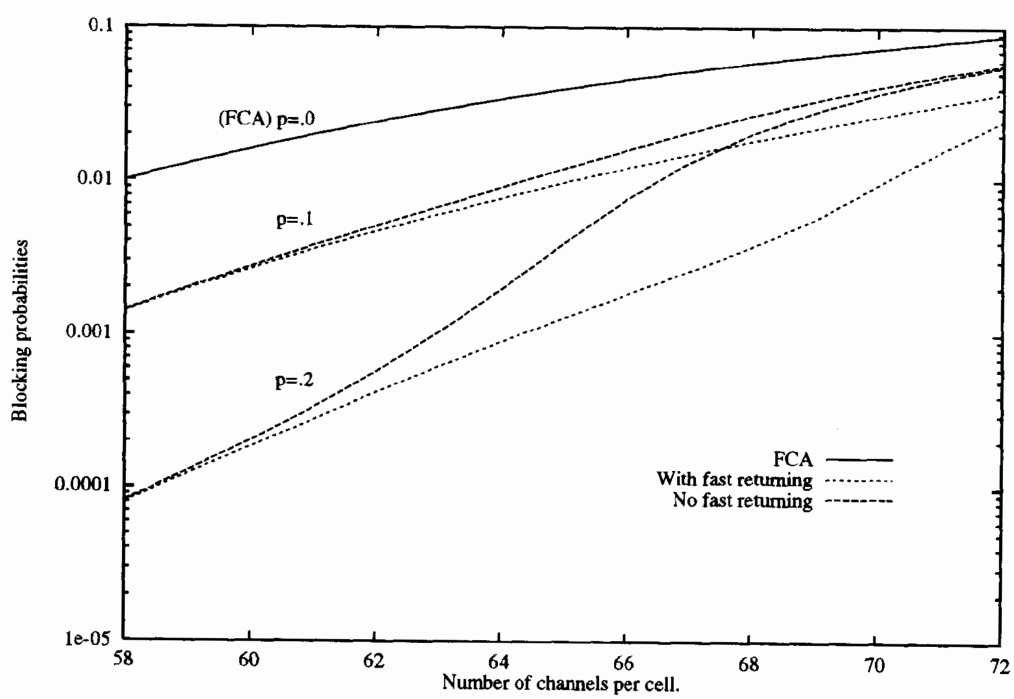

Figure 6: Comparison of TDMA/CBWL with or without fast carrier returning ($C = m = 24, l = 4$).

Appendices

Appendix A: Channel Returning Rate from An Adjacent Gateway, $\mu(i)$

Fast carrier returning accelerates channel returning rate of borrowed carriers. Assume a gateway, Y borrows j carriers from its k ($k \leq 6$) neighbors, i of them are borrowed from neighbor, X. We consider the channel returning rate from Y to X. If one of $3C$ regular channels of Y is released, Y randomly chooses one of the k neighbors and transfers one of calls on the primary borrowed carrier from the neighbor to the channel that is just released. The chance that X is chosen is $1/k$. If one of $3(i-1)$ channels on the $i-1$ nonprimary borrowed carriers from X becomes free, one of calls on the primary borrowed carrier is transferred to the freed channel. Thus, the call transfer rate on a primary carrier is

$$R(i,k) = 3(i-1+C/k)\mu \quad i = 0,\ldots,l, \;\; i \leq j \leq L_{max}\,. \tag{A.1}$$

Denote $p_{br}(j)$ as the probability that Y borrows j carriers from its neighbors. Denote $\Pr(k|i,j)$ as the conditional probability that Y borrows from k neighbors given that Y borrows i carriers from X and j carriers from all neighbors. The average call transfer rate of X's primary carrier in gateway Y is

$$\alpha(i) = \frac{\sum_{j=i}^{5l+i} \sum_{k=1}^{6} \Pr(k|i,j) p_{br}(j) R(i,k)}{\sum_{j=i}^{5l+i} \Pr(i|j) p_{br}(j)}\,. \tag{A.2}$$

where $\Pr(i|j)$ is the conditional probability that given that Y borrows j carriers, Y borrows i carriers from X [see equation (C.1)]. From Bayes' Theorem, The denominator is just $\Pr(i)$.

With call transfer rate on primary carrier, we can determine, $\mu(i)$, the average carrier return rate from Y to X if X lends i carriers to Y. Because a carrier has three channels, we use a three-state queuing model to describe a primary carrier. The model is show in Figure 7. The new call arrival rate on a primary borrowed carrier is

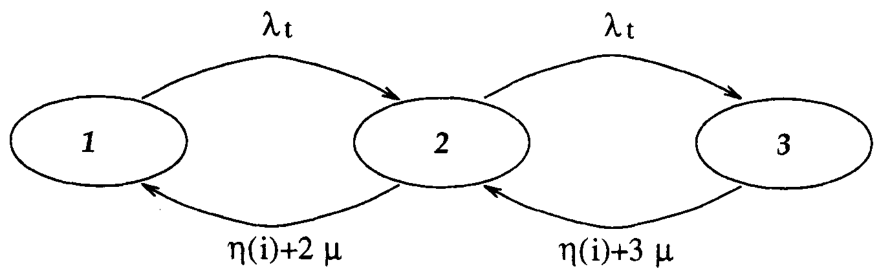

Figure 7: State-transition diagram in the primary borrowed carrier

$\lambda_b = \lambda[1-(1-p)p_a/p_c]$. The channel release rate on the primary carrier is the call transfer rate plus call completion rate of the primary carrier.

the return of the carrier can only occur in the first state. The probability is

$$r_p = \frac{1}{1+\lambda_b/[\alpha(i)+2\mu]+\lambda_b^2/[\alpha(i)+3\mu]}\,. \tag{A.3}$$

Thus, the carrier returning rate is

$$\mu(i) = r_p[\alpha(i) + \mu] \,. \tag{A.4}$$

To use (A.2), we must know $\Pr(i|j)$ and $\Pr(k|i,j)$. Since the system is homogeneous and a gateway randomly chooses a neighbor to borrow a carrier, every neighbor has the same chance to be borrowed. The problem is identical to that one randomly distributes j identical balls into six distinct boxes if each box can have no more than l balls. In Appendix D, we find $a(j,k,l)$ is the number of ways to distribute j identical balls in k distinct boxes if each box can have no more than l balls [see equation (D.9)]

The probability $\Pr(k|i,j)$ is the fraction of number of ways that we first place i balls into a given box and $k-1$ balls into each of chosen $k-1$ boxes, then distribute $j-i-k+1$ balls into the $k-1$ boxes. Thus,

$$\Pr(k|i,j) = \binom{5}{k} \frac{a(j-i-k+1, k-1, l-1)}{a(j,6,l)} \qquad j = 1,\ldots,6l,\ i = 0,\ldots,l \,. \tag{A.5}$$

The probability $\Pr(i|j)$ is the fraction of number of ways that we first place i balls into a given box, then distribute $j-i$ balls into the remaining 5 boxes. Thus,

$$\Pr(i|j) = \frac{a(j-i,5,l)}{a(j,6,l)} \qquad j = 1,\ldots,6l,\ i = 0,\ldots,l \,. \tag{A.6}$$

Appendix B: Transition Rate of Macro-variable (u, v)

Average lending rate, $\alpha(v)$

We note that borrowing requests to a given gateway from an adjacent gateway arise from an overflow process (at the adjacent gateway) and therefore do not conform to a Poisson process [14]. However at the adjacent gateway, borrowing requests are randomly split into six parts, only one of which is directed to the given gateway. The random splitting tends to smooth the peakedness of the overflow traffic directed to the given gateway. Thus the borrowing requests directed to a given gateway from an adjacent gateway can be approximated by a Poisson process with intensity λ'. The quantity is calculated from (17).

For a given gateway, its kth adjacent gateway's borrowing request rate is λ' when $i_k < l$, and the carrier returning rate to it from the kth neighbor is $\mu(i_k)$ (see Appendix A). Define $f(x)$ as

$$f(x) \triangleq (\lambda')^x / \prod_{j=1}^{x} \mu(j) \,. \tag{B.1}$$

Given that v carriers of the given gateway are lent to all neighbors, each neighbor borrows channels from different subset carriers of the given gateway. The number of carriers that lent to each neighbor is independent each other provided the sum is less than or equal to v. Thus, the distribution of lending carriers to each neighbor is of product form. Thus,

$$\Pr(i_1, i_2, i_3, i_4, i_5, i_6 | \sum_{k=1}^{6} i_k = v) = \frac{1}{b(v)} \prod_{k=1}^{6} f(i_k) \,. \tag{B.2}$$

in which, $b(v)$ is normalization constant. It is equal to the sum of all probabilities that defined by (B.2) and can be found by the convolution algorithm in [4]. From the algorithm we can find simultaneously the probability that exactly $6-j$ adjacent gateways have borrowed exactly l carriers from a given gateway with v carriers lent

The probability is denoted as $B(j, v)$. Since the $6 - j$ neighbors cannot borrow any more carriers from the given gateway, the borrowing request rate from all neighbors becomes $j\lambda'$. Thus,

$$\alpha(v) = \sum_{j=1}^{6} jB(j, v) . \tag{B.3}$$

Average carrier returning rate from all neighbors, $\rho(v)$

Define a six-vector

$$\mathbf{I_6} \triangleq (i_1, i_2, i_3, i_4, i_5, i_6) .$$

Define $S(v, 6)$ as the set of six-vectors whose components sum to v. That is

$$S(v, 6) \triangleq \{\mathbf{I_6} = (i_1, i_2, i_3, i_4, i_5, i_6) : 0 \le i_k \le l, \sum_{k=1}^{6} i_k = v\} . \tag{B.4}$$

The returning rate from the kth neighbor is $\mu(i_k)$. The overall carrier returning rate from all neighbors is the sum of $\sum_{k=1}^{6} \mu(i_k)$. Consider all possible $\mathbf{I_6}$ that are in $S(6, v)$, we find the average rate of carrier returning from neighbors from (B.2). That is

$$\rho(v) = \frac{1}{b(v)} \sum_{\mathbf{I_6} \in S(v,6)} \left[\prod_{k=1}^{6} f(i_k) \sum_{j=1}^{6} \mu(i_j) \right] . \tag{B.5}$$

Equation (B.5) requires a lot of numerical operations. A simpler form is derived as follows. From (B.1), we have following equation,

$$f(i_k)\mu(i_k) = \lambda' f(i_k - 1) \tag{B.6}$$

Exchange the order of summation and production in the brackets of (B.5), and using (B.6) to reduce the size of i_s. The terms in the brackets in (B.5) can be expressed as

$$\prod_{k=1}^{6} f(i_k) \sum_{s=1}^{6} \mu(i_s) = \lambda' \sum_{s=1}^{6} u(i_s - 1) f(i_s - 1) \prod_{\substack{k = 1, \ldots, 6 \\ k \ne s}} f(i_k) \tag{B.7}$$

where the $u(x)$ is defined as

$$u(x) = \begin{cases} 1 & 0 \le x < l \\ 0 & \text{otherwise.} \end{cases}$$

Substituting (B.7) into (B.5), we have

$$\rho(v) = \frac{\lambda'}{b(v)} \sum_{\mathbf{I_6} \in S(v,6)} \sum_{s=1}^{6} u(i_s - 1) \left[f(i_s - 1) \prod_{\substack{k = 1, \ldots, 6 \\ k \ne s}} f(i_k) \right] . \tag{B.8}$$

From (B.4), we find the term in the brackets of (B.8) is $f(i_k)$'s product of an $\mathbf{I_6}$ that is in $S(v - 1, 6)$. We introduce a transform in which for an $\mathbf{I_6} \in S(v, 6)$, if its any component, i_s, is greater than zero, we construct a new vector $\mathbf{I}'_6 \in S(v - 1, 6)$. The five components of $\mathbf{I}'_6$ are the same as those of $\mathbf{I_6}$, except its sth component, $i'_s = i_s - 1$.

Thus each $\mathbf{I}_6$ can be transformed into up to six $\mathbf{I}'_6$'s. Using this transform in (B.8), each $u(i_s - 1)$ is changed into $u(i'_s)$. Thus,

$$\rho(v) = \lambda' \left[\frac{1}{b(v)} \sum_{\mathbf{I}'_6 \in S(v-1,6)} \sum_{s=1}^{6} u(i'_s) \prod_{k=1}^{6} f(i'_k) \right] . \qquad \text{(B.9)}$$

For all possible value of i'_s $(0, \ldots, l)$, only when $i'_s = l$, $u(i'_s) = 0$. Thus, if j components of an $\mathbf{I}'_6$ that are less than l, the second summation in (B.9) is just j. Recall that $B(j, v-1)$ is the probability that exactly $6-j$ adjacent gateways have borrowed exactly l carriers from a given gateway with $v-1$ carriers lent. Thus the terms in the brackets of (B.9) are equal to $jB(j, v-1)$. We find

$$\rho(v) = \lambda' \sum_{j=1}^{6} jB(j, v-1) . \qquad \text{(B.10)}$$

Rate of borrowed channel being used, $\beta(j)$

The quantity, $\beta(j)$, is the rate at which borrowed channels are used at a gateway given that j borrowed channels are occupied. If a gateway has idle channels on its own carriers, no carriers are borrowed and the rate is zero. This occurs when a gateway is at states at which $j = u + 3v - 3C < 0$. If all channels are busy on the regular and borrowed carriers, more carrier is borrowed for the needs of channels at the gateway. Usually a carrier (3 channels) is borrowed for the demand of a single channels. When a carrier is borrowed, usually some idle slots can be found on the borrowed carrier. A gateway always uses idle slots on a *carrier* that it has already borrowed before *borrowing* another *carrier*. Specifically, if $(u + 3v - 3C)/3$ is an integer, a carrier must be borrowed from neighbors for a channel demand. If $(u + 3v - 3C)/3$ is not an integer, there are at least one idle channels on borrowed carriers. The idle slots can be used for the new call arrival and no carrier needs to be borrowed. Therefore, the rate at which borrowed channels are used is different for the two states. That is

$$\beta(j) = \begin{cases} 0 & j = u + 3v - 3C < 0 \\ \lambda[p + (1-p)p_a/p_c]p_{bs}(j) & \text{if } j = u + 3v - 3C \text{ is multiples of three} \\ \lambda[p + (1-p)p_a/p_c] & \text{otherwise} \end{cases} \qquad \text{(B.11)}$$

Appendix C: Some Important Probabilities

Probability of number of borrowed carriers

From $p_{uv}(u, v)$, the distribution of number of borrowed carriers can be calculated. Denote the distribution as $p_{br}(j)$. Thus,

$$p_{br}(j) = \sum_{v=0}^{L_{max}} \sum_{k=0}^{2} p_{uv}(3(C - v + j) - k, v) , \qquad j = 1, \ldots, 6l \qquad \text{(C.1)}$$

Since one carrier has three channels and channels are borrowed by carriers, some slots of borrowed carrier may not be idle. If j carriers are borrowed, the probability that all $3j$ borrowed channels are busy is

$$p_{bf}(j) = \sum_{v=0}^{L_{max}} p_{uv}(3(C - v + j), v) , \qquad j = 1, \ldots, 6l \qquad \text{(C.2)}$$

Distribution of lending carriers

Denote the distribution as $p_l(v)$. The probability $p_l(v)$ is determined as follows:

$$p_l(v) = \sum_{u=0}^{3(C+6l-v)} p_{uv}(u, v) \quad v = 0, 1, \ldots, L_{max} . \qquad \text{(C.3)}$$

Probability that no carrier is borrowed with v carriers lent

From $p_{br}(j)$, we can determine the probability that no carrier is borrowed given that v carrier are lent when all channels are occupied. The probability is denoted as $p_{nb}(v)$.

$$p_{nb}(v) = p_{uv}(3(C-v), v) / \sum_{u=3(C-v)}^{3(C+6l-v)} p_{uv}(u, v) \qquad v = 0, \ldots, L_{max} \,. \tag{C.4}$$

Probability that all channels of a gateway are occupied

The probability that all channels of a gateway are occupied is the sum of state probabilities with $u + 3v \geq 3C$. From (C.1), we have

$$p_c = \sum_{j=0}^{6l} p_{br}(j) \,. \tag{C.5}$$

Probability that all borrowed channel are busy

If a gateway is in the states with $u = 3(C - v + j), (j = 0, 1, \ldots, 6l - 1)$ all channels of regular and borrowed carriers are occupied and one more carrier should be borrowed for a new call arrival. We denote the probability of all these states as p_{nf}. That is,

$$p_{nf} = \sum_{v=0}^{L_{max}} \sum_{j=1}^{6l-1} Pr\{3(C - v + j), v\} \tag{C.6}$$

Probability that a borrowing request from a neighbor is denied

The probability is denoted as p_f. A borrowing request from a specific adjacent gateway will be denied by the given gateway if any of the following three events are true at the time that the borrowing request arises.

Event E_1: More than m carriers of the given gateway are occupied.

Event E_2: The total number of carriers that have been lent to all adjacent gateways is equal to the maximum possible number, L_{max}.

Event E_3: The adjacent gateway has already borrowed its maximum allowable carrier quota, (l carriers) from the given gateway.

The probability, p_f is the probability of the union of the events. Thus

$$p_f = \Pr\{E_1 \bigcup E_2 \bigcup E_3\} = \Pr(E_1) + \Pr(E_2\bar{E}_1) + \Pr(E_3\bar{E}_2\bar{E}_1) \tag{C.7}$$

The sum of the first and second term is denoted as p_e. That is

$$p_e = p_c + \sum_{v=0}^{L_{max}} \sum_{u=3(m-v)}^{3(C-1-v)} p_{uv}(u, v) + \sum_{u=0}^{3(m-1-L_{max})} p_{uv}(u, L_{max}) \,. \tag{C.8}$$

Recall that $B(6 - j, v)$ is the probability that exactly j neighbors have borrowed exactly l carriers from a given gateway with v carriers lent. The chance that a specific neighbor is among the j neighbors is $j/6$. Thus,

$$\Pr(E_3\bar{E}_2\bar{E}_1) = \sum_{v=l}^{L_{max}-1} \sum_{j=1}^{6} \frac{jB(6-j, v)}{6} \sum_{u=0}^{3(m-v-1)} p_{uv}(u, v) \,. \tag{C.9}$$

From (C.7), we have

$$p_f = p_e + \sum_{v=l}^{L_{max}-1} \sum_{j=1}^{6} \frac{jB(6-j, v)}{6} \sum_{u=0}^{3(m-v-1)} p_{uv}(u, v) \,. \tag{C.10}$$

Probability that a borrowing request success

Since the coupling between adjacent gateways is carrier borrowing requests and the borrowing rate, λ', has been included into the queueing model of X, the state probabilities of X can be determined completely without the knowledge of the neighbor states[1]. Thus, the states of adjacent gateways can be considered as "independent". Therefore, the probability that $6-s$ neighbors deny borrowing requests of X even though X has borrowed less than l carriers from those neighbors is p_e^{6-s}.

Denote $p_g(s|j)$ as the probability that each of s neighbors lends l carriers to X given that X has borrowed j carriers. Thus,

$$p_{bs}(j) = \sum_{s=s_1}^{s_2} p_g(s|j)(1-p_e^{6-s}) \qquad j = 0, \ldots, 6l \tag{C.11}$$

where, $s_1 = \max\{0, j-6(l-1)\}$ and s_2 is equal to the maximum integer that less than or equal to j/l.

Since a gateway randomly chooses a neighbor to borrow a carrier and the system is homogeneous, every way to distribute j carriers among six neighbors is equally likely. Thus, the problem is identical to that one distributes randomly j balls into 6 boxes with each box having at most l balls. The total number of ways is $a(j,6,l)$ (D.9). The number of ways to distribute j balls into six boxes so that s boxes have l balls can be obtained by following ways: The number of ways to choose s boxes from the six boxes is $6!/[(6-s)!s!]$. We assign l balls to each of the s boxes. Then we distribute the remaining $j-sl$ balls into the remaining $6-s$ boxes with each box getting at most $l-1$ balls. The number of ways is $a(j-sl, 6-s, l-1)$. Thus,

$$p_g(s|j) = \frac{6!}{s!(6-s)!}\frac{a(j-sl, 6-s, l-1)}{a(j,6,l)} . \qquad j = 0, \ldots, \lfloor j/sl \rfloor \tag{C.12}$$

From (C.11) and (C.12),

$$p_{bs}(j) = \sum_{s=s_1}^{s_2} \frac{6!}{s!(6-s)!}\frac{a(j-sl, 6-s, l-1)}{a(j,6,l)}(1-p_e^{6-s}) \qquad j = 0, \ldots, 6l . \tag{C.13}$$

Appendix D: Number of Ways to Distribute Balls into Boxes

We want to find number of ways to distribute j *identical* balls into k *different* boxes if each box can have at most l balls. This problem can be solved by *generating functions* [18]. Function $g(x)$ is a generating function of a combinatorial problem if $g(x)$ has the polynomial expansion

$$g(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_j x^j + \cdots$$

and c_j is the number of ways to distribute j objects. Therefore, if we can determine the generating function and find c_j, our problem will be solved.

Let $f(x) = 1 + x + x^2 + \cdots + x^l$. The power of x in $f(x)$ corresponds to the number of balls that are distributed into a specific box. Because the way to distribute s identical balls into a specific box is unique, all coefficients of $f(x)$ are 1. Consider the coefficient of x^j in the expansion of $[f(x)]^k$. It is the number of different products whose sum of exponents is j. Each of the k $f(x)$'s represents the number of ways to place balls in the k boxes. Thus, the coefficient of x^j in the expansion of $[f(x)]^k$ is equal to the number of ways to distribute j identical balls into k distinct boxes. Thus, our problem has a generating function

$$g(x) = [f(x)]^k . \tag{D.1}$$

The next step is to find the coefficient. Rewrite this generating function as

$$g(x) = (1 + x + x^2 + \cdots + x^l)^k = \left(\frac{1 - x^{l+1}}{1 - x}\right)^k = (1 - x)^{-k}(1 - x^{l+1})^k. \tag{D.2}$$

The first factor can be expanded as

$$(1 - x)^{-k} = 1 + \binom{k}{1} x + \binom{k+1}{2} x^2 + \cdots + \binom{s+k-1}{s} x^s + \cdots \tag{D.3}$$

and the second factor can be expanded as

$$\begin{aligned}(1 - x^{l+1})^k = 1 - \binom{k}{1} x^{l+1} + \binom{k}{2} x^{2(l+1)} - \cdots + (-1)^s \binom{k}{s} x^{s(l+1)} + \cdots \\ +(-1)^k \binom{k}{k} x^{k(l+1)}.\end{aligned} \tag{D.4}$$

Denote the first factor as

$$(1 - x)^{-k} = a_0 + a_1 x + a_2 x^2 + \cdots + a_s x^s + \cdots \tag{D.5}$$

and the second factor as

$$(1 - x^{l+1})^k = b_0 + b_{l+1} x^{l+1} + b_{2(l+1)} x^{2(l+1)} + \cdots + b_{s(l+1)} x^{s(l+1)} + \cdots + b_{k(l+1)} x^{k(l+1)}. \tag{D.6}$$

From the rules of multiplication of polynomials, we can know the coefficient of x^j in the expansion of $g(x)$ is given by

$$c_j = \sum_{i=0}^{S} a_{j-i(l+1)} b_{i(l+1)}. \tag{D.7}$$

where S is is the maximum integer that is less or equal to $j/(l+1)$. From (D.3) and (D.4), we have

$$c_j = \sum_{i=0}^{S} (-1)^i \binom{k}{i} \binom{j - i(l+1) + k - 1}{j - i(l+1)}. \tag{D.8}$$

Define $a(j, k, l)$ as the number of ways to distribute j identical balls into k distinct boxes with at most l balls in each box. Thus,

$$a(j, k, l) = \sum_{i=0}^{S} (-1)^i \binom{k}{i} \binom{j - i(l+1) + k - 1}{j - i(l+1)}. \tag{D.9}$$

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