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A NEW CHANNEL ASSIGNMENT AND SHARING METHOD FOR CELLULAR COMMUNICATION SYSTEMS

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Abstract

A new scheme that allows cell gateways (base station) to borrow channels from adjacent gateways in a cellular communication system is presented. Borrowed channels are used with reduced transmitted power to limit interference with co-channel cells. No channel locking is needed. The scheme, which can be used with various multiple access techniques, permits simple channel control management without requiring global information about channel usage throughout the system. It provides enhanced traffic performance in homogeneous environments and also can be used to relieve spatially localized traffic overloads (tele-traffic "hot spots"). Co-channel interference analysis show the scheme can maintain the same SIR as non-borrowing schemes. Analytical models using multi-dimensional birth-death processes and decomposition methods are devised to characterize the performance of the scheme. The results which are also validated by simulation indicate that significant traffic capacity can be achieved in comparison with non-borrowing schemes.

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1 INTRODUCTION

Because the demand for spectrum to serve mobile users is expected to grow rapidly in the foreseeable future, techniques that enhance capacity for a given grade of service are of strong interest. Additionally it is desirable for a mobile communication system to be able to accommodate spatially localized communications traffic overloads. Such overloads (or "hot spots") are often caused by events such as vehicular traffic jams, or the termination of a game at a major stadium. *Hot spots* can result in unacceptable degradation of system performance and can be particularly disturbing to users caught in highway traffic who are impatient to call their homes or offices.

One solution is to use channel borrowing schemes. In dynamic channel assignment (DCA) [1], [2], there is a central pool of all channels. A channel is borrowed from the central pool to a cell for use by a call in the cell. When the call is ended the channel is returned to the pool. In hybrid channel assignment (HCA) [3], [4], some channels are permanently assigned to each cell as in fixed channel assignment (FCA), and others are kept in a central pool for borrowing as in DCA. HCA and DCA can accommodate traffic hot spots and have better performance than FCA in light traffic. In HCA and DCA, *channel locking* is used to prevent an increase in co-channel interference [5]. That is, cells within the required minimum channel reuse distance from a cell that borrows a channel cannot use the same channel. Channel locking has some disadvantages. One is that the number of channels that are available for lending to a cell is limited, since a channel can be borrowed by a cell only when it is idle in *all of the cells within the required channel reuse distance* of the borrowing cell. Another disadvantage is the difficulty in maintaining co-channel reuse distance at the minimum required value everywhere in the system. Because of this difficulty, DCA and HCA generally perform less satisfactorily than FCA under high loads [1], [3]. Their other disadvantages relate to physical complexity. The transmitter of each base station (gateway) must be able to transmit not only on the channels allocated permanently to that cell, but also on any of the channels which belong to the central pool. Finally, to implement channel borrowing at a *given* cell, information about the channel usage at the cells within the channel reuse distance of the given cell must be known. This causes some complexity in management of system resources. In these schemes, a *borrowed channel is temporarily transferred* to a different gateway. We call these *type 1 schemes.*

Several modified DCA and HCA schemes have been suggested to mitigate the shortcomings. In [4], rearrangements are used when a regular channel becomes available. This minimizes the traffic carried on borrowed channels. In [5], *directional channel-locking* and locally optimized dynamic assignment are used to increase the number of channels available for borrowing and minimize the channel reuse distance of borrowed channels. This permits a reduction in blocking probability. A comparison of these strategies is given in [5]. Nevertheless, since all use channel locking, they cannot completely overcome the inherent disadvantages.

In a second type of channel borrowing, channels are assigned to each gateway as in FCA but a new call that finds all channels in its cell to be occupied, will not necessarily be blocked. Instead, if the user is also within range of a gateway of a neighboring cell, it will try to use a channel of that gateway. The link is established *through the gateway of the neighboring cell.* This type of channel sharing exploits the overlapping areas of cells. In this type of borrowing, *channels are not temporarily transferred* from the gateway of a cell to the gateway of another cell. Only the *right* to use a channel is transferred. Generalized FCA with ($k > 1$) in [6] and directed retry and load sharing in [7] and [8] are examples of this type of channel borrowing. We call these *type 2 schemes.*

In Generalized FCA *all* users in a cell can access the nearest k gateways, while in directed retry and load sharing only a fraction of the users in a cell can access the gateway of an adjacent cell. An advantage of this type of channel borrowing is that the gateway equipment must only accommodate the channels assigned to the gateway itself. The simplicity is accompanied by limitation. Users of borrowed channels must be in the coverage overlap among cells and therefore tend to be relatively far from the gateways through which they communicate. As a result, the quality of borrowed channels is lower than that of the regular channels. Co-channel interference is also increased, because the borrowed channels are used beyond the usual range. In order to limit co-channel interference, the overlap among cells cannot be too great. Thus overlap among two or three adjacent cells is usual and a given user can only access one or two additional gateways. This limits the number of channels that are potentially available to service a call.

Comparing existing schemes for the two types of channel borrowing, we find that type 1 schemes use channel locking to avoid co-channel interference while the type 2 schemes do not use channel locking but cause an increase in co-channel interference. Because channel locking limits system performance, we propose an alternative for systems that employ channel borrowing. We call the approach presented here *Channel Borrowing Without Locking*, and use the acronym, CBWL. *The scheme has most of the advantages of type 1 and type 2 schemes and overcomes their shortcomings.* In comparison with FCA, it allows improved channel utilization in (more or less) homogeneous environments and also can accommodate hot spots.

In Section 2, the operation of CBWL is described in detail. In Section 3, we consider co-channel interference for CBWL. We show, that by using a borrowed channel with restricted power and without channel locking, improved traffic performance can be achieved with no more co-channel interference than in FCA. In Section 4, we develop an analytical model for performance analysis of CBWL. Numerical results are discussed in Section 5.

2 ARCHITECTURE OF CBWL SCHEMES

Two aspects of CBWL are *organization* and *operation.* For convenience, we discuss each separately.

2.1 Organization of CBWL

The general organization of the underlying scheme is as follows. The system has a total of C_T channels, an omni-directional wireless gateway is located nominally at the center of each cell. With a cluster of size, N, the C_T channels are divided into N groups with about $C = C_T/N$ channels in each group. These channel groups are assigned to gateways in the service area as in FCA. If all channels of the gateway of a cell are occupied and a new call arrives, channel borrowing is employed according to certain rules. A channel can be borrowed only from an adjacent cell. For a hexagonal layout geometry there are six of these. A borrowed channel is temporarily transferred to the gateway that borrows the channel for the duration of time needed to accommodate the call. During this time, the borrowed channel can not be used in the original lending cell but it can still be used in any nearby co-channel cells (that also own the channel). Thus, there is no channel locking. To avoid the increase of co-channel interference caused by channel borrowing, borrowed channels are used with limited transmitted power. Therefore, they can be accessed only in part of borrowing cell. When a mobile station wants to make a call and finds all the channels in its current cell occupied, the mobile station may use a borrowed channel. To determine whether the mobile is in a region that can be served by a borrowed channel, the mobile station tunes its receiver to a predetermined signal which is transmitted by each gateway. The signal is called a borrowed channel sensing signal (BCSS). If the BCSS is not above some suitable threshold at the mobile, a borrowed channel cannot be used and the call is blocked; otherwise, its gateway attempts to borrow a channel from an adjacent gateway. If any of the neighboring gateways has an idle channel available for lending, the call uses one of them. If no such channel is available, the call is blocked.

Structured CBWL

Without further structuring, CBWL allows a gateway to borrow channels from its (six) adjacent gateways, and each gateway must be capable of transmitting and receiving on its own channels as well as on any of the channels of its six neighbors. If the number of cells in a cluster is greater than 7, these channels comprise just a part of those available to the system. However, if the number of cells in a cluster is less than or equal to 7, it might be necessary for any gateway to access all channels of a system. This can be avoided if the scheme is organized in a special way. For convenience of discussion, the neighboring cells of any cell are identified in the following manner. Beginning at the gateway of any given cell, draw a perpendicular line to one side of the cell. This establishes a reference direction. The cell adjacent to this side is labeled neighbor 1. The remaining five adjacent neighboring cells are numbered sequentially proceeding clockwise from the first. The given cell is labeled cell 0. To reduce complexity, the CBWL scheme can be implemented by dividing the C channels that are assigned to a gateway into seven distinct groups. The seven groups are numbered 0, 1, ..., 6. The channels of group 0 are reserved for exclusive use of the given cell. Channels in each of the other six groups can be lent to adjacent cells. The *ith* adjacent cell can only borrow channels from the *ith* group. The number of channels in the *ith* group is denoted C_i, $i = 0, 1, \ldots, 6$. For convenience we consider a symmetrical arrangement with $C_1 = C_2 = \ldots = C_6 = l$. However, C_0 can be any integer such that $0 \leq C_0 \leq C - 6$.

Figure 1 shows an example for which the cluster size is equal to 7.If the total number of channels

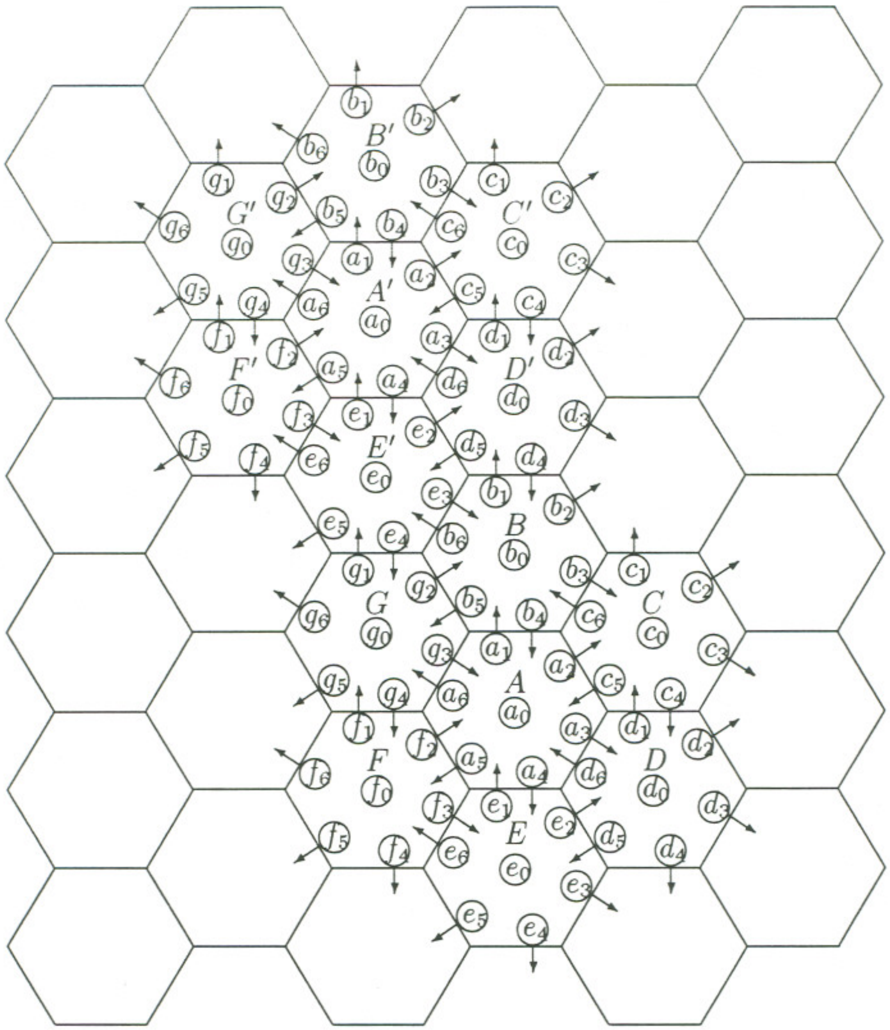


Figure 1: Channel structure of CBWL (cluster size =7).

is 210, thirty channels are nominally assigned to each gateway. Of these 30 channels, perhaps 6 specific channels will be used *only* in the cell served by the assigned gateway. The remaining $30 - 6 = 24$ channels can be divided into 6 groups of 4 each. Each group corresponds to one of the neighboring cells. *Any* of the 30 channels assigned to a gateway can be used in the cell. But if the given cell lends a channel to a given adjacent cell, only a channel from the corresponding group can be lent. In Figure 1, each cell is shown with seven circles, each representing a group of channels. The channels of group i (i=1, 2, ..., 6) are directed by an arrow to the *ith* adjacent cell (the cell to which channels of the group can be lent). Any given gateway can borrow up to 24 channels (4 from each of its 6 neighbors) and its equipment needs only the capability to transmit and receive on a specific set of 54 channels (30 of its own plus the 24 possible borrowed channels) rather than all 210 channels allocated to the system. The parameters for this example are: $C_T = 210$, $N = 7$, $l = 4$, $C_0 = 6$.

While the channel assignment and borrowing structure does not require a complicated channel management algorithm, it nevertheless prevents the occurrence of serious co-channel interference that can arise with channel borrowing. To emphasize this, consider channel borrowing from an adjacent gateway where the channels that can be borrowed are not assigned as described above. Without this lending channel structure, a gateway can borrow any of the idle channels available from an adjacent gateway. As shown in Figure 1, it may happen that cell B borrows a channel x from A and E' borrows the same channel x from A'. In the example, A and A' are closest co-channel cells. The cells, B and E' are adjacent to A and A' respectively, and B and E' are adjacent to one another. So if this borrowing pattern is allowed, the borrowed channel x would have to be used with a much reduced power to prevent excessive co-channel interference between cells B and E' on channel x. An alternative requirement is to allow only one of the cells, either B or E', borrow x at the same time. The requirement would make it necessary for any gateway to "know" the channel usage in nearby clusters. Thus, channel management would be significantly more complicated. In CBWL, the assignment of specific channels that can be lent to each adjacent cell does not permit the channel borrowing pattern just described. This is because for any pair of nearest co-channel cells, adjacent cells located between them can never borrow the same channels. In Figure 1, the channel group a_1 can be borrowed only by B and B', a_2 can be borrowed only by C or C', and so on. Therefore, in lending a channel, it is not necessary for a cell to know the channel usage of its co-channel cells – *the assignment scheme itself precludes borrowing conflicts* and excessive co-channel interference.

The reduction of complexity of structured CBWL in comparison with unstructured CBWL is at the cost of a decreased number of channels that can be accessed by call arrivals in a cell. In the example given earlier, a gateway can use 54 channels instead of all 210 channels. But from our analysis and simulation, if the traffic in each cell is not *very* different from that of the others, (the usual design assumption), *rarely* would a gateway *simultaneously* borrow many channels from *every* adjacent gateway, even though it is allowed to do so. Therefore, with appropriate system sizing, we would not expect any major difference in the performance between structured and unstructured CBWL schemes, while the former is considerably simpler to implement and manage.

A further advantage is that by appropriately organizing channel groups, it is possible to avoid using *adjacent channels* in the same cell, even with channel borrowing allowed. Thus, adjacent channel interference is not increased by channel borrowing. In current cellular systems, adjacent channels are not allowed to be used in a cell, because of relaxed filtering characteristics of most mobile units. By dividing channels for lending into different groups, it is possible to structure CBWL in a way which ensures that *adjacent* channels are not used in the same cell.

2.2 Operation of CBWL

Channel swapping to enhance borrowing for gateways

Without channel swapping, when a given gateway receives a request to borrow a channel from one of its adjacent gateways, the request would be denied if all channels which belong to the channel group that is associated with the requesting gateway are occupied. But it may happen that at least one of the channels in this group is occupied by a user in the given gateway's own cell. If then there is an idle channel in some other group, the user can be switched to *that* channel. This will free a channel in the fully occupied group, and the freed channel can be lent to the gateway making the current borrowing request. Not only does channel swapping allow more effective use of resources, but it considerably simplifies analysis.

Channel rearrangement to enhance borrowing for users

Because the transmitted power used on borrowed channels is smaller than that used on regular channels, only those users whose link path loss to the base does not exceed a certain maximum can use borrowed channels. The fraction of users that have access to borrowed channels is denoted as p. If p is small the number of users with access to borrowed channels is small and this tends to limit the potential improvement of CBWL. To mitigate this effect *channel rearrangements* can be used. With channel rearrangements, if a new call arrival finds all channels of its gateway occupied and the BCSS is not sufficiently strong, the call is *still not necessarily blocked.* If at the same time another call in the cell that is using a regular channel receives the BCSS above threshold, and at least one adjacent gateway can *lend channels to the given gateway,* the latter call will use a borrowed channel from an adjacent gateway and give its regular channel to the new call. In this way, calls that cannot use borrowed channels directly also benefit from the borrowing scheme. Therefore, the difference of blocking probabilities between two types of calls is lessened and the number of calls that can use borrowed channels (directly or indirectly) is increased. We use the acronym CBWL/CR to denote *channel borrowing without locking with channel rearrangement.* For convenience, we call CBWL without channel rearrangement as CBWL/NR.

Priority for calls arising in the cell

Since it may be desirable to limit the impact of borrowing on the calls that arise in the given cell, some priority can be given to them. This can be accomplished using cut-off priority (Other priority schemes are possible as well). So if a gateway receives a request to borrow from a neighboring gateway, the request will be denied if the number of occupied channels equals or exceeds some number, m. In addition, to give preference to its "own" users, a gateway will never lend more than n channels at any time. The constraint involving n is conceptually distinct from that involving m. The latter sets a limit on the number of occupied channels, beyond which, no new borrowing request is accepted. The former limits the total number of channels at a gateway that can be lent at any time.

At a gateway, the assignment of channels to calls proceeds roughly as follows. As demands for channels in a gateway arise from calls in the cell, channels from group zero are first allocated to these calls. Then the channels in group 1, 2, ..., 6 are allocated uniformly. If a gateway, A, receives a borrowing request from an adjacent gateway, and if all the constraints are met, the request is granted and a channel from the appropriate group is lent to the adjacent gateway (using channel swapping if necessary). A gateway always tries to make the distribution of occupied channels (including channels used by its own calls and those lent to adjacent cells) as uniform as possible in groups 1, ..., 6.

When all channels of a gateway are occupied when a new call arrives, the gateway will request to borrow a channel from its adjacent gateways. There are various possible ways in which the borrowing demands of a given gateway can be directed to the adjacent gateways. One possibility is that the gateway that wants to borrow a channel sends its demand to all of its adjacent gateways. Each adjacent gateway examines its own current state, and responds to the potential borrowing gateway with a message of YES or NO respectively, if it can or cannot lend a channel to the requesting gateway. If the number of gateways that answer YES is more than one, the borrowing gateway chooses one of them randomly. In this paper, this possibility is analyzed. Another possibility is that each adjacent gateways responds with more specific state information. The borrowing gateway can choose, for example, the gateway having the most idle channels.

In summary, if $C_k = l$ (k =1, ..., 6), a given gateway can borrow at most l channels from an adjacent gateway. It can borrow at most $6l$ channels from 6 adjacent gateways. A borrowing request from a gateway will be granted by the lending gateway, if

1. The total number of channels that the lending gateway has lent to the borrowing gateway is strictly less than l .
2. The total number of occupied channels at the lending gateway is strictly less than m.
3. The total number of channels that the lending gateway has lent to all of its adjacent gateways is strictly less than n.

Note that these constraints are in addition to those required by the channel assignment and allocation scheme.

2.3 Comparison of CBWL with Other Schemes

CBWL offers advantages in comparison with DCA and HCA. In CBWL, ordinarily only a fraction of the total channels of the system need to be accessible at each gateway. The cost of base stations is reduced. Without channel locking, channel reuse distance can always be kept at a desired minimum. Thus, the CBWL scheme exhibits better performance in light as well as heavy communications traffic loads. Furthermore, in CBWL, channel borrowing at a gateway does not require global information about channel usage in the system, so the control and management tasks are simplified. In CBWL, by appropriately organizing lending channel groups, it is possible to avoid using *adjacent channels* in the same cell, even with channel borrowing allowed.

In comparison with generalized FCA and directed retry, CBWL ensures good quality for borrowed and regular channels, yet co-channel interference is not increased. In CBWL, a user can borrow channels from any of the adjacent gateways. There are six of these in the standard hexagonal layout geometry. The greater number of channels that are potentially available to an arriving call results in superior performance.

A major advantage of CBWL is that it can be employed in current cellular systems without additional costly infrastructure. Unlike cell splitting, CBWL does not require new cell sites and additional antenna towers to increase system capacity.

3 Analysis of Co-Channel Interference in CBWL

In this section, we consider a cellular system with the usual hexagonal geometry and omni-directional antennas. Similar analysis for directional antennas is possible (although more complicated) and is deferred. Mobile users communicate via the gateways located at the centers of the cells. A group of channels is assigned to the gateway of each cell. The group of channels is reused in cells which are sufficiently distant so that the co-channel interference is acceptably low. Denote the radius of a cell as R. The reuse distance D is defined as the distance between the gateways of two nearest co-channel cell. N is the total number of channel groups needed (the cluster size), and is related to the reuse shift parameters (i,j) by

$$N = i^2 + ij + j^2 \quad . \tag{1}$$

The integers i and j determine the reuse pattern and identify co-channel cells [9]. The ratio of D to R is a measure of suppression of co-channel interference due to propagation loss. With flat, uniform propagation, this ratio is

$$\frac{D}{R} = \sqrt{3N} \; . \tag{2}$$

To gauge the potential of CBWL for performance improvement, we consider the signal-to-interference ratio (SIR) of CBWL in the worst case and compare it with the SIR of a system using only FCA.

For simplicity, the following assumptions are made.

1. All cells are the same size.

2. Flat uniform propagation conditions are in effect. Thus, (for a given propagation exponent) the received powers are determined by distances.

3. All gateway antennas have the same height, gain and emit the same maximum transmitted power.

4. All mobile stations use omni-directional antennas and have the same maximum transmitted power.

5. Fading is not included in the model presented here. We calculate SIR as the ratio of median signal power to the sum of median interference powers. This will not significantly affect our conclusions, because both CBWL and FCA are compared on the same basis. Protection against fading would require almost the same margin in SIR's in the two systems. Thus one can expect a similar SIR ordering when fading is considered.

6. Only the interference from the first ring of neighboring co-channel sectors is considered.

For FCA, in the worst case, the mobile unit is at the cell boundary, R, and the distance from its nearest co-channel interfering gateway is $D - R$ (see Figure 2). Then the SIR of a cellular

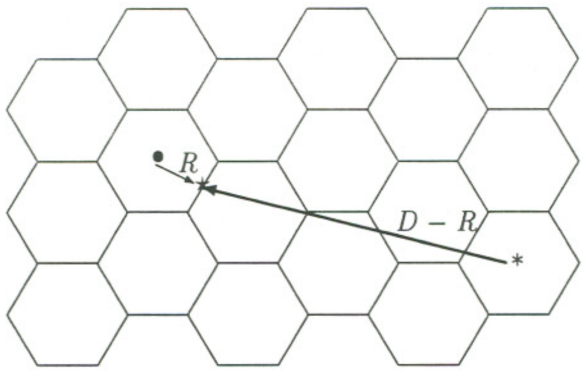


Figure 2: Determination of SIR in the worst case for FCA. △: transmitting gateway. •: receiving mobile station. ×: interfering gateway.

system using FCA is given by

$$SIR_0 = (\frac{D}{R} - 1)^\gamma = (\sqrt{3N} - 1)^\gamma \tag{3}$$

where the parameter, γ is a propagation path-loss slope that is heavily influenced by the actual terrain environment. The value of γ usually lies between 2 and 5. The value of SIR_0 is the same for mobile-to-gateway and gateway-to-mobile transmission.

For CBWL, in the worst case, a mobile station using a regular channel is at the boundary of a cell and a mobile station which is using a borrowed channel is at the boundary of the smaller region where use of borrowed channels is allowed (See Figure 4). To obtain pessimistic results, we assume that both regular and borrowed channels are used with their maximum allowable transmitted power. We denote (in the worst case), the ratio of power transmitted on a *borrowed channel* to the transmitted power on a *regular channel* for the forward link (gateway to mobile) as P_f, and the same ratio for the reverse link (gateway to mobile) as P_r. Because of the uniform propagation. Because of the uniform propagation assumptions, a borrowed channel can be used within a smaller circle with radius r ($r < R$). The principal interference arises from the co-channel cells in the first tier of the given cell as shown in Figure 3.

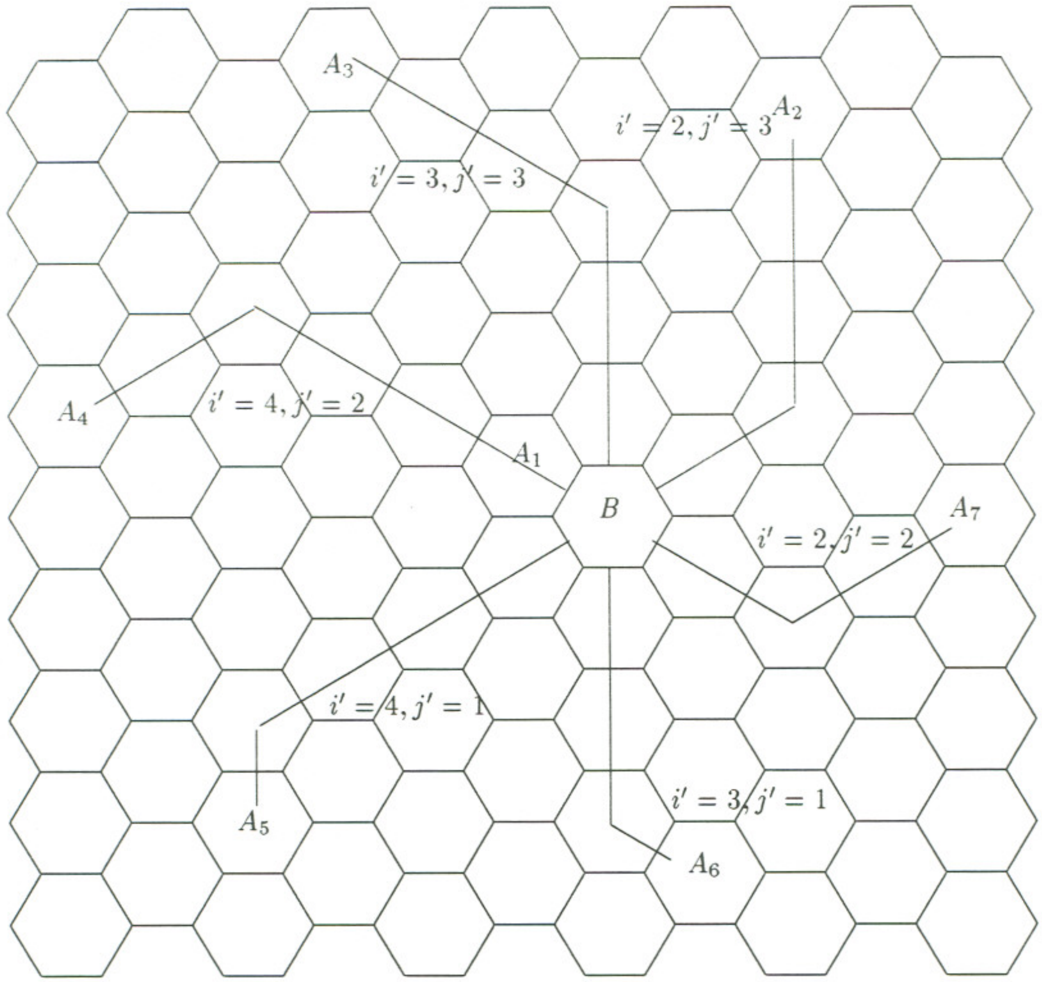


Figure 3: Change of channel-reuse pattern in CBWL ($i = 3, j = 2$). A_i: a group of co-channel cells. B: a borrowing cell.

For simplicity, we consider only a single interferer in each (CBWL and FCA) scheme. This will not significantly affect our final judgment, because including another interferer in each case would only cause the SIR to decrease at most by 3 dB. For a fair comparison, the same number of interferers should be considered for each scheme. Because additional interferers for the worst case scenario in each scheme will cause about the same degradation, we can base the calculation on the results for a single interferer.

Assume the original reuse pattern is defined by the shift parameters i and j, which determine the cluster size, N ($N = i^2 + ij + j^2$). The distance between the center of any cell and the center of a nearest co-channel cell is $R\sqrt{3N}$. In CBWL, if a channel x is lent to an adjacent cell, the original reuse pattern is disrupted. The distance from x's new cell to each of its nearby co-channel cells is decreased or increased depending on which neighbor of the original cell borrowed the channel (see Figure 3). Because of the symmetry, we can without any loss in generality, consider $i > j$ and furthermore the borrowing cell can be any neighbor of the original cell. In Figure 3, the original cell is A_1, whose nearest co-channel cells are A_i (i =2, 3, ..., 7). The borrowing cell is B. The figure is drawn for shift parameters $(i, j) = (3, 2)$ corresponding to $N = 19$. If the shift parameters from the borrowing cell is defined by (i', j'), then the values of (i', j') to the six nearest co-channel cells of B are $(i-1, j)$, $(i, j-1)$, $(i-1, j+1)$, $(i+1, j-1)$, $(i, j+1)$ and $(i+1, j)$ respectively. The corresponding distances are reflected in the values of N' ($N' = i'^2 + i'j' + j'^2$) that are determined by the shift parameters i' and j'. We are interested in the smallest N', since that corresponds to a worst case. The first two (i', j')correspond to $i' + j' = i + j - 1$ which indicates a decrease of reuse distance. The last two correspond to $i' + j' = i + j + 1$ which indicates an increased reuse distance in comparison with the original reuse distance. The middle two correspond to $i' + j' = i + j$, since

$$
\begin{aligned}
(i-1)^2 + (i-1)(j+1) + (j+1)^2 &= i^2 + ij + j^2 + i - j + 1 \\
(i+1)^2 + (i+1)(j-1) + (j-1)^2 &= i^2 + ij + j^2 - i + j + 1 \quad , \qquad (4)
\end{aligned}
$$

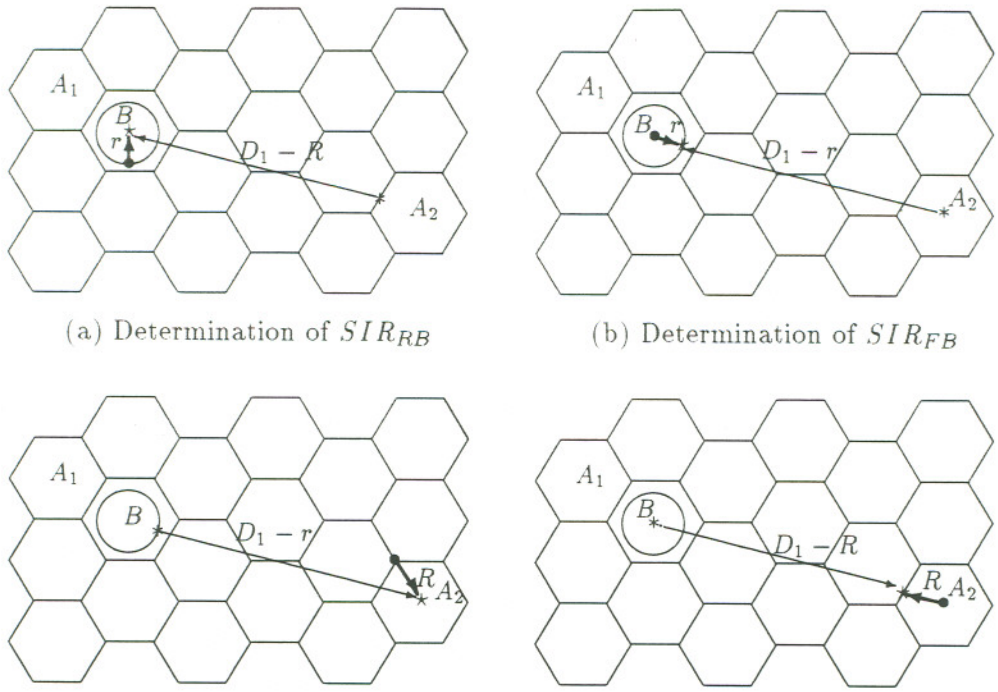
if $i = j$, the reuse distances of both cells increase. If $i = j + 1$, the reuse distance of one cell increases while that of another remains the same. If $i > j + 1$, the reuse distance of one cell increases while that of another cell decreases. In any case, the co-channel distance corresponding to equation (4) is greater than that from the first two (i', j'). Therefore, for given i and j, the corresponding smallest value of N' is given by

$$
N_1 \triangleq \min[i^2 + i(j-1) + (j-1)^2, (i-1)^2 + (i-1)j + j^2] \qquad (5)
$$

The minimum co-channel distance in the worst case D_1 is

$$
D_1 = \sqrt{3N_1}R \quad . \qquad (6)
$$

For CBWL, since channel borrowing disrupts the symmetry of the reuse pattern, we have to distinguish four types of signal-to-interference ratios (SIR's): (1) SIR of reverse-link on borrowed channel, SIR_{RB}. (2) SIR of forward-link on borrowed channels, SIR_{FB}. (3) SIR of reverse-link on regular channels, SIR_{RN}. (4) SIR of forward-link on regular channels, SIR_{FN}. The SIR's are calculated below. The reader is referred to Figure 4.



(a) Determination of SIR_{RB}

(b) Determination of SIR_{FB}

(c) Determination of SIR_{RN}

(d) Determination of SIR_{FN}

Figure 4: Determination of SIR in the worst case for CBWL. •: transmitter. ⋆: receiver. ∗: interferer. A_1, A_2: two co-channel cells. B: borrowing cell.

(a) Calculation of SIR_{RB}

In the worst case, the mobile unit is at the boundary of the smaller circle. Its distance to gateway is r. The interfering mobile unit is at the boundary of the nearest new co-channel cell, its distance to the gateway is $D_1 - R$ (See Figure 4(a)). The SIR of reverse-link on the borrowed channel is

$$SIR_{RB} = P_r \left(\frac{D_1 - R}{r}\right)^{\gamma} . \tag{7}$$

(b) Calculation of SIR_{FB}

Figure 4(b) shows that in the worst case, the distance between the mobile unit that borrows a channel and its gateway is r, and its distance to the closest interfering gateway is $D_1 - r$. The SIR of forward-link on the borrowed channel is

$$SIR_{FB} = P_f \left(\frac{D_1 - r}{r}\right)^{\gamma} . \tag{8}$$

(c) Calculation of SIR_{RN}

The worst case is shown in Figure 4(c). The distance between the signal transmitter (mobile unit) and the receiver (gateway) is R. The shortest distance from interferers is $D_1 - r$. The SIR of the reverse-link on a regular channel is

$$SIR_{RN} = \frac{1}{P_r}\left(\frac{D_1 - r}{R}\right)^{\gamma} . \tag{9}$$

(d) Calculation of SIR_{FN}

The worst case is shown in the Figure 4(d). The distance between signal transmitter (gateway) and receiver (mobile unit) is R. The closest interferer is at the distance of $D_1 - R$. Then the SIR of the forward-link on a regular channel is

$$SIR_{FN} = \frac{1}{P_f}\left(\frac{D_1 - R}{R}\right)^{\gamma} . \tag{10}$$

Recall that the the SIR of an FCA system without channel borrowing is given by (3). If CBWL is to be useful, the SIR's given by (7), (8), (9) and (10) must be greater than or equal to that given by (3). The reverse-link SIR requirements are $SIR_{RB} \geq SIR_0$ and $SIR_{RN} \geq SIR_0$. These lead to

$$\frac{\sqrt{3N_1} - \frac{r}{R}}{\sqrt{3N} - 1} \geq (P_r)^{\frac{1}{\gamma}} \geq \frac{\sqrt{3N} - 1}{\sqrt{3N_1} - 1}\frac{r}{R} . \tag{11}$$

The forward-link SIR requirements are $SIR_{FB} \geq SIR_0$ and $SIR_{FN} \geq SIR_0$. These lead to

$$\frac{\sqrt{3N_1} - 1}{\sqrt{3N} - 1} \geq (P_f)^{\frac{1}{\gamma}} \geq \frac{\sqrt{3N} - 1}{\sqrt{3N_1} - \frac{r}{R}}\frac{r}{R} . \tag{12}$$

The inequalities (11) and (12) define the ranges of power ratio for which the SIR of CBWL is greater than that of an FCA scheme if r is given. When equalities are taken in (11), the maximum allowable r/R is determined. This is

$$\frac{r}{R} = \frac{\sqrt{3N_1}(\sqrt{3N_1} - 1)}{(\sqrt{3N} - 1)^2 + \sqrt{3N_1} - 1} , \tag{13}$$

and

$$P_r = \left(\frac{\sqrt{3N_1} - \frac{r}{R}}{\sqrt{3N} - 1}\right)^{\gamma} . \tag{14}$$

Equalities in (12) yield the same value of r/R and

$$P_f = \left(\frac{\sqrt{3N_1} - 1}{\sqrt{3N} - 1}\right)^{\gamma} . \tag{15}$$

In Table 1, values of r/R that were determined using (13) are shown for different reuse sizes. When N is small, r/R is also small, since the closest co-channel cell is adjacent to the cell that borrows the channel. The value of p in the last column of the table is the fraction of region covered by borrowed channel. For the homogeneous case, p is determined by following formula:

$$p = \frac{\text{area of the micro-cell}}{\text{area of the regular cell}} = \frac{\pi r^2}{(3\sqrt{3}/2)R^2} \approx 1.1(\frac{r}{R})^2 . \tag{16}$$

Table 1: The fraction of area in a cell in which borrowed channels can be accessed.

N	i	j	N_1	r/R	P_f	P_r	p
3	1	1	1	.27	.02	.29	.08
4	2	0	1	.19	.01	.15	.04
7	2	1	3	.40	.10	.28	.18
9	3	0	4	.43	.12	.28	.20
12	2	2	7	.57	.26	.41	.36
13	3	1	7	.53	.22	.36	.31
16	4	0	9	.55	.25	.38	.33
19	3	2	12	.63	.34	.45	.44
21	4	1	13	.61	.33	.43	.41
25	5	0	16	.64	.36	.46	.45
27	3	3	19	.70	.45	.54	.54

about 18% of the service area is covered by borrowed channels. For $N > 12$, at least 28% of the area is covered by a region in which a borrowed channel can be used. Table 1 also lists the reduced power ratio of CBWL on the forward link and on the reverse link, P_f and P_r, respectively.

Based on this analysis, it appears that the CBWL scheme can maintain the same SIR as FCA while also allowing borrowed channels to be used in a significant portion of the system's coverage area. With greater access to channels than is possible with FCA, the blocking probability must be lower. In the subsequent sections, we quantify the blocking probability for the CBWL scheme.

4 Traffic Performance of CBWL

Since the analysis of CBWL when $m = C$ and $m < C$ are different, we will discuss them separately. In this paper, we limit our discussion in the case with $m = C$, which implies that no cut-off priority is given to the calls that arise in the given cell. The other case is with $0 \leq m < C$, which gives cut-off priority for calls that arise in the given cell. The last case will be presented in another paper.

For convenience of presentation, at first, we limit our analysis to a homogeneous system. That is, each gateway has the same number of channels, the same number of neighbors, the same offered traffic and the same organization of channels for CBWL. The non-homogeneous and hot spot cases are considered in a subsequent subsection. For the homogeneous case, we can discard the notation needed to distinguish different cells and just use an arbitrary cell to represent every cell.

We assume that: new calls in a cell arise at an average rate λ (new call arrivals per second per cell) according to a Poisson process, call holding times have a negative exponential probability distribution with mean $1/\mu$, and calls originate uniformly throughout the service area. In this case, the fraction of calls in a cell that can be served by borrowed channels is the same as the fraction of service area that is covered by borrowed channels, which is denoted as p and is given in Table. 1 for different cluster size. We note that channel borrowing requests to a given gateway from one of its adjacent gateways arise from an overflow process (at the adjacent gateway) and therefore do not conform to a Poisson process [10]. However at the adjacent gateway (i.e., the source of borrowing requests), borrowing requests are randomly split into six parts, only one of which is directed to the given gateway. The random splitting tends to smooth the peakedness of the overflow traffic directed to a given gateway. We model the overflow traffic directed to a given gateway by a Poisson process with intensity λ'. The parameter, λ' will be determined. Our simulation results indicate that this Poisson assumption is valid.

We will discuss the analysis of CBWL/NR in the first subsection. Based on the analysis in this subsection, the analysis of CBWL/CR is discussed in the second subsection.

4.1 Analysis of CBWL/NR

In CBWL/NR, at any given time a gateway is in one of a finite number of states characterized by the vector $\mathbf{I} = (i_0, i_1, i_2, i_3, i_4, i_5, i_6)$. The component i_0 is the number of channels occupied by calls that arise in the cell. The number of channels at the gateway that are (currently) lent to the kth adjacent gateway is i_k, (k = 1, 2, ..., 6). In state I, the total number of the gateway's channels that are occupied is given by

$$J(\mathbf{I}) \triangleq \sum_{k=0}^{6} i_k \ . \tag{17}$$

The total number of channels that are (currently) lent to all adjacent gateways is

$$L(\mathbf{I}) \triangleq \sum_{k=1}^{6} i_k \ . \tag{18}$$

In CBWL/NR, the maximum number of channels that a gateway can lend at any given time is

$$L_{max} = \min(n, \sum_{k=1}^{6} C_k) \ . \tag{19}$$

Permissible states correspond to those sequences of non-negative integers, i_0, i_1, i_2, ..., i_6, for which,

$$\begin{aligned} &0 \leq i_0 \leq C \\ &0 \leq i_k \leq l \qquad k = 1, 2, \ldots, 6 \\ &0 \leq J(\mathbf{I}) \leq C \\ &0 \leq L(\mathbf{I}) \leq L_{max} \end{aligned} \tag{20}$$

In order to characterize performance of the system, it is first necessary to determine the state probabilities.

4.1.1 Determination of State Probabilities

We define a function, $O(i_k)$, by

$$O(i_k) \triangleq \begin{cases} 1 & \text{if } 0 \leq i_k < l \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

According to the operation of the structured CBWL/NR scheme, the kth adjacent gateway can possibly borrow a channel from the given gateway only if $O(i_k) = 1$. Furthermore, we define a unit state vector $\delta_\mathbf{k}$ as a vector in which $i_k = 1$ and all other elements are 0. Also, let $p(\mathbf{I})$ denote the equilibrium probability of state $\mathbf{I}$. Denote Ω as the set of permissible states determined by (20). Define a function, $Z(\mathbf{I})$, such that

$$Z(\mathbf{I}) \triangleq \begin{cases} 1 & \text{if } \mathbf{I} \in \Omega \\ 0 & \text{if } \mathbf{I} \notin \Omega\ . \end{cases} \tag{22}$$

In statistical equilibrium, the probability flow out of each state $\mathbf{I}$ must equal the probability flow into that state. Application of this principle leads to a set equations which must be solved to find the state probabilities. In those permissible states for which the number of occupied channels of a gateway, $J(\mathbf{I})$, is less than C, the flow out of $\mathbf{I}$ consists of three parts: that due to new call arrivals, that due to channel borrowing demands from adjacent gateways, and that due to the completion of calls in state $\mathbf{I}$. The flow out of state $\mathbf{I}$ due to new call arrivals is given by

$$\{\text{flow out due to new call arrivals}\} = \lambda p(\mathbf{I})\ . \tag{23}$$

The flow out of state $\mathbf{I}$ due to channel borrowing demands (on resources of the given gateway) consists of six sub-flows, each from one adjacent gateway. The probability flow rate due to the kth adjacent gateway is λ' if $i_k < l$, otherwise, the flow rate is zero. Thus,

$$\{\text{flow out due to channel borrowing demands}\} = \lambda' \sum_{k=1}^{6} O(i_k) p(\mathbf{I})\ . \tag{24}$$

The flow out due to call completions is given by

$$\{\text{flow out due to call completions}\} = J(\mathbf{I}) \mu p(\mathbf{I})\ . \tag{25}$$

Now let us consider the probability flow components *into* state $\mathbf{I}$. The probability flows into $\mathbf{I}$ arise from other permissible states. It consists of up to three parts. That due to new call arrivals from state $\mathbf{I} - \delta_\mathbf{0}$, (if $\mathbf{I} - \delta_\mathbf{0} \in \Omega$); that due to channel borrowing demands from adjacent gateways occurring when the given gateway is in state $\mathbf{I} - \delta_\mathbf{k}$ ($k = 1, \ldots, 6$); and, that due to call completions in $\mathbf{I} + \delta_\mathbf{k}$,($k = 0, 1, \ldots, 6$) if $\mathbf{I} + \delta_\mathbf{k}$ is a permissible state. Otherwise flow-in components are zero. The first part is given by

$$\{\text{flow in due to new call arrivals}\} = \lambda p(\mathbf{I} - \delta_\mathbf{0}) Z(\mathbf{I} - \delta_\mathbf{0})\ . \tag{26}$$

The flow into **I** due to channel borrowing demands is given by

$$\{\text{flow in due to channel borrowing}\} = \lambda' \sum_{k=1}^{6} O(i_k - 1) p(\mathbf{I} - \delta_{\mathbf{k}}) Z(\mathbf{I} - \delta_{\mathbf{k}}) \ . \tag{27}$$

The last part is given by

$$\{\text{flow in due to call completions}\} = \sum_{k=0}^{6} (i_k + 1)\mu p(\mathbf{I} + \delta_{\mathbf{k}}) Z(\mathbf{I} + \delta_{\mathbf{k}}) \ . \tag{28}$$

In any state **I** with $J(\mathbf{I}) < C$, the flow balance equation is

$$\begin{aligned}
[\lambda + \lambda' \sum_{k=1}^{6} O(i_k) + J(\mathbf{I})\mu] p(\mathbf{I}) &= \lambda p(\mathbf{I} - \delta_{\mathbf{0}}) Z(\mathbf{I} - \delta_{\mathbf{0}}) \\
\lambda' \sum_{k=1}^{6} O(i_k - 1) p(\mathbf{I} - \delta_{\mathbf{k}}) Z(\mathbf{I} - \delta_{\mathbf{k}}) &+ \sum_{k=0}^{6} (i_k + 1)\mu p(\mathbf{I} + \delta_{\mathbf{k}}) Z(\mathbf{I} + \delta_{\mathbf{k}}) \\
&(\text{for any permissible } \mathbf{I} \text{ with } J(\mathbf{I}) < C) \ .
\end{aligned} \tag{29}$$

For any permissible state **I** with $J(\mathbf{I}) = C$,

$$\begin{aligned}
C\mu p(\mathbf{I}) = \lambda p(\mathbf{I} - \delta_{\mathbf{0}}) Z(\mathbf{I} - \delta_{\mathbf{0}}) &+ \lambda' \sum_{k=1}^{6} O(i_k - 1) p(\mathbf{I} - \delta_{\mathbf{k}}) Z(\mathbf{I} - \delta_{\mathbf{k}}) \\
&(\text{for any permissible } \mathbf{I} \text{ with } J(\mathbf{I}) = C) \ .
\end{aligned} \tag{30}$$

Since it can be cumbersome to numerically calculate the state probabilities when there are many states, it is reasonable to first seek a product form solution. When such a solution exists, calculation of the state probabilities can be expedited.

In this case, all C channels are shared by seven streams of calls arising from different cells. Six are streams of borrowing requests, and one stream is comprised of call arrivals in the given cell itself. The analytical structure of this problem is essentially the same as that in which several types of customers share a finite group of servers. It has been shown that the state probabilities can be expressed in product form [11]. A mathematically similar situation that arises in the context of satellite communications is described in [12].

In the present discussion, each of the six streams of borrowing requests has a parameter λ'. The stream of calls that arise in the given cell has a parameter λ. The service rate μ is the same for all streams. The probability that a given gateway is in state, **I**, can be expressed in product form by

$$p(\mathbf{I}) = \frac{1}{G(\Omega)} \frac{\left(\frac{\lambda}{\mu}\right)^{i_0}}{i_0!} \prod_{k=1}^{6} \frac{\left(\frac{\lambda'}{\mu}\right)^{i_k}}{i_k!} \ , \tag{31}$$

in which, the normalization constant $G(\Omega)$ is the sum of probabilities of all permissible states. Specifically, the constant is given by

$$G(\Omega) = \sum_{\mathbf{I} \in \Omega} p(\mathbf{I}) \tag{32}$$

The formula (32) is deceptive because many numerical operations are required, if there are a large number of states. Instead of summing over all states in some arbitrary order, Buzen's convolution algorithm can be used to calculate $G(\Omega)$ recursively, [13]. This algorithm has been used in the analysis of queueing networks [14].

We define a function

$$h(x) = \frac{\left(\frac{\lambda}{\mu}\right)^x}{x!} \qquad x = 0, 1, \ldots, C \ . \tag{33}$$

and another function

$$f(x) = \frac{\left(\frac{\lambda'}{\mu}\right)^x}{x!} \qquad x = 0, 1, \ldots, l \ . \tag{34}$$

Then, from (31), the probability of state $\mathbf{I}$ can be rewritten as

$$p(\mathbf{I}) = \frac{1}{G(\Omega)} h(i_0) \prod_{k=1}^{6} f(i_k) \ . \tag{35}$$

Since the sum of $p(\mathbf{I})$ over all states must add to unity, we have

$$G(\Omega) = \sum_{\mathbf{I}\in\Omega} h(i_0) \prod_{k=1}^{6} f(i_k) \ . \tag{36}$$

Let L_ν be the maximum number of channels that a gateway can lend to exactly ν adjacent gateways without regard to which gateways they are. Recall from section 2.2 that n is a limit on the number of channels that a gateway will lend. Thus

$$L_\nu \triangleq \min(n, \nu l) \ . \tag{37}$$

Because we are considering the case $m = C$, it follows from (19) and (37) that $L_6 = L_{max}$.

Define a function $g(j)$ such that $g(j)/G(\Omega)$ is the probability that exactly j channels of a gateway are occupied (including those being lent). Let Ω_j be a subset of Ω, which consists of all states for which $J(\mathbf{I}) = j$. That is

$$\Omega_j = \{\mathbf{I} : \mathbf{I} \in \Omega, J(\mathbf{I}) = j\} \ . \tag{38}$$

Thus,

$$\Pr\{\Omega_j\} = \frac{g(j)}{G(\Omega)} \ . \qquad j = 0, 1, \ldots, C. \tag{39}$$

Since Ω_j's are disjoint and the union of Ω_j's from $j = 0$ to C is Ω, the sum of $g(j)/G(\Omega)$ over $j = 0, 1, \ldots, C$ must add to unity. Thus,

$$G(\Omega) = \sum_{j=0}^{C} g(j) \ . \tag{40}$$

From (35) and (39),

$$g(j)/G(\Omega) = \frac{1}{G(\Omega)} \sum_{\mathbf{I}\in\Omega_j} h(i_0) \prod_{k=1}^{6} f(i_k) \ . \qquad j = 0, 1, \ldots, C. \tag{41}$$

One can cancel $G(\Omega)$ from the both sides of (41), to

$$g(j) = \sum_{\mathbf{I}\in\Omega_j} h(i_0) \prod_{k=1}^{6} f(i_k) \; . \qquad j = 0, 1, \ldots, C. \tag{42}$$

Given that j channel are occupied, if $j > L_{max}$, the minimum number of channels that can be used by the gateway's own calls is $j - L_{max}$. However, if $j \leq L_{max}$, the minimum number of channels that can be used by the gateway's own calls is 0. Thus, the gateway's own calls can use $i_0 = w$, $w+1, \ldots, j$ channels, in which,

$$w = \min(0, j - L_{max}) \; . \tag{43}$$

The remaining $j - i_0$ channels are lent to other gateways. Considering all possibilities of i_0 for given j, we can factor out the terms of $h(i_0)$ from (42). Thus,

$$g(j) = \sum_{i_0=w}^{j} h(i_0) \sum_{\substack{i_1, i_2 \ldots, i_6 \\ \mathbf{I}\in\Omega_j \\ \sum_{k=1}^{6} i_k = j - i_0}} \prod_{k=1}^{6} f(i_k) \; . \qquad j = 0, 1, \ldots, C. \tag{44}$$

Note that the expression in the second summation of (44) involves summing over only six state variables, i_1, i_2, $\ldots$,i_6. We define a six-vector

$$\mathbf{I_6} \triangleq (i_1, i_2, i_3, i_4, i_5, i_6) \; , \tag{45}$$

The six-vector represents the numbers of channels that are lent to each adjacent gateway. Note that the components of $\mathbf{I_6}$ are constrained $\mathbf{I} = (i_0, \mathbf{I_6})$ where $\mathbf{I} \in \Omega$. Then, define $S(x, 6)$ as the set of six-vectors whose components sum to x. That is,

$$S(x, 6) \triangleq \{\mathbf{I_6} = (i_1, i_2, i_3, i_4, i_5, i_6) : (i_0, \mathbf{I_6}) \in \Omega, \sum_{k=1}^{6} i_k = x\} \; . \tag{46}$$

From (46), we can see that $S(x, 6)$ includes all ways in which exactly x channels can be lent to the six adjacent gateways. If $x = j - i_0$, the set $S(x, 6)$ is coincides exactly with the region defined by the indices of the multiple summation that appears inside the brackets in (44). Equation (44) can be rewritten as

$$g(j) = \sum_{i_0=w}^{j} h(i_0) \left[\sum_{\mathbf{I_6}\in S(j-i_0,6)} \prod_{k=1}^{6} f(i_k) \right] \; , \qquad j = 0, 1, \ldots, C. \tag{47}$$

We define

$$g_6(x) \triangleq \sum_{\mathbf{I_6}\in S(x,6)} \prod_{k=1}^{6} f(i_k) \; , \qquad x = 0, 1, \ldots, L_{max}. \tag{48}$$

Then, (47) can be written as

$$g(j) = \sum_{i_0=w}^{j} h(i_0) g_6(j - i_0) \ , \qquad j = 0, 1, \ldots, C \tag{49}$$

which is a discrete convolution of functions $h(i_0)$ and $g_6(x)$ defined in (33) and (48) respectively. One needs to first calculate $g_6(x)$ to find $g(j)$ using (49).

As in (45) and (46), generally, we define a ν-vector, $\mathbf{I}_\nu$, by

$$\mathbf{I}_\nu \triangleq (i_1, \ldots, i_\nu) \ , \qquad \nu = 1, 2, \ldots, 6, \tag{50}$$

in which, the components of the ν-vector, $\mathbf{I}_\nu$, represent the numbers of channels that are lent individually to the first through the νth adjacent gateways. Thus, we must have

$$\mathbf{I} = (i_0, \mathbf{I}_\nu, i_{\nu+1}, \ldots, i_6) \in \Omega \ , \qquad (\nu = 1, 2, \ldots, 6). \tag{51}$$

We define $S(x, \nu)$ as the set of ν-vectors whose components sum to x. That is,

$$S(x, \nu) \triangleq \left\{ \mathbf{I}_\nu = (i_1, \ldots, i_\nu) : (i_0, \mathbf{I}_\nu, i_{\nu+1}, \ldots, i_6) \in \Omega, \sum_{k=1}^{\nu} i_k = x \right\} \ . \tag{52}$$

Given that x channels are lent, if $x < l$, the sixth adjacent gateway can borrow at most x channels, if $x \geq l$ channels, the sixth adjacent gateway can borrow at most l channels. Therefore, the sixth adjacent gateway may borrow $i_6 = 0,\ 1, \ldots, q$ channels, where q is given by

$$q = \min(x, l) \ . \tag{53}$$

The remaining $x - i_6$ channels are lent to the other five gateways. Considering all possibilities of i_6 for given x, we can factor out the terms of $f(i_6)$ from (48). Thus,

$$g_6(x) = \sum_{i_6=0}^{q} f(i_6) \sum_{\substack{i_1, i_2, \ldots, i_5 \\ (i_0, \mathbf{I_5}, i_6) \in \Omega \\ \sum_{k=1}^{5} i_k = x - i_6}} \prod_{k=1}^{5} f(i_k) \ , \qquad x = 0, 1, \ldots, L_{max}. \tag{54}$$

From (52), we can see that the indices of the second summation of (54) is the same as $S(x - i_6, 5)$. Thus, (54) can be rewritten as

$$g_6(x) = \sum_{i_6=0}^{q} f(i_6) \left[\sum_{\mathbf{I_5} \in S(x - i_6, 5)} \prod_{k=1}^{5} f(i_k) \right] \ , \qquad x = 0, 1, \ldots, L_{max}. \tag{55}$$

Now we define

$$g_5(x) \triangleq \sum_{\mathbf{I_5} \in S(x,5)} \prod_{k=1}^{5} f(i_k) \ , \qquad x = 0, 1, \ldots, L_5. \tag{56}$$

Then (55) can be rewritten as

$$g_6(x) = \sum_{i_6=0}^{q} f(i_6) g_5(x - i_6) \; , \qquad x = 0, 1, \ldots, L_{max}. \tag{57}$$

The convolution can be used to calculate $g_6(x)$ from $f(i_6)$ and $g_5(x)$.

In general, we define

$$g_\nu(x) \triangleq \sum_{\mathbf{I}_\nu \in S(x,\nu)} \prod_{k=1}^{\nu} f(i_k) \; , \qquad x = 0, 1, \ldots, L_\nu. \tag{58}$$

Factor out the terms of $f(i_\nu)$. Thus

$$g_\nu(x) = \sum_{i_\nu=0}^{q} f(i_\nu) \left[\sum_{\mathbf{I}_{\nu-1} \in S(x-i_\nu, \nu-1)} \prod_{k=1}^{\nu-1} f(i_k) \right] \; , \qquad x = 0, 1, \ldots, L_\nu. \tag{59}$$

The expression in the brackets can be identified as $g_{\nu-1}(x - i_\nu)$, so

$$g_\nu(x) = \sum_{i_\nu=0}^{q} f(i_\nu) g_{\nu-1}(x - i_\nu) \; , \qquad \nu = 2, \ldots, 6. \quad x = 0, 1, \ldots, L_\nu. \tag{60}$$

In this way, we can use the convolution form recursively to calculate $g_6(x)$. The initial conditions for the algorithm can be obtained directly from the definition of $g_\nu(x)$. From (58), we find,

$$\begin{aligned} g_1(x) &= f(x) \; , & x &= 0, 1, \ldots, L_1 \\ g_\nu(0) &= 1 \; , & \nu &= 1, 2, \ldots, 6. \end{aligned} \tag{61}$$

Beginning with (61) and (34), we can recursively compute $g_\nu(L_\nu)$, $g_\nu(L_\nu - 1)$, $\ldots$, $g_\nu(1)$ from $g_{\nu-1}(x)$ using (60). After five recursions, $g_6(x)$ ($x = 1, \ldots, L_{max}$) are obtained. Then from (49) and (33), $g(j)$ can be obtained. Finally, from (40), $G(\Omega)$ can be found. Once $G(\Omega)$ is calculated, any state probability can be obtained using (35).

4.1.2 Determination of Blocking Probabilities

Important performance measures can be expressed in terms of the state probabilities.

A borrowing request from a specific adjacent gateway will be denied by the given gateway if any of the following three events are true at the time that the borrowing request arises.

Event E_1: All channels of the given gateway are occupied.

Event E_2: The total number of channels that have been lent to all adjacent gateways is equal to the maximum possible number, L_{max}.

Event E_3: The adjacent gateway has already borrowed its maximum allowable channel quota, (l channels).

The probability that a borrowing request from a specific adjacent gateways denied by the given gateway is the probability of the union of the events. Thus

$$\begin{aligned}\Pr\{\text{borrowing request is denied}\} &= \Pr\{E_1 \bigcup E_2 \bigcup E_3\} \\ &= \Pr(E_1) + \Pr(E_2\bar{E}_1) + \Pr(E_3\bar{E}_2\bar{E}_1)\end{aligned} \tag{62}$$

in which an overbar denotes the complementary event.

We denote the probability of event E_1 by p_c, we find from (39) that

$$p_c \triangleq \Pr(E_1) = \frac{g(C)}{G(\Omega)} \; . \tag{63}$$

Since the event $E_2\bar{E}_1$ consists of all permissible states with $L(\mathbf{I}) = L_{max}$ and $J(\mathbf{I}) < C$, its probability is,

$$\Pr(E_2\bar{E}_1) = \sum_{\substack{\mathbf{I} \in \Omega \\ J(\mathbf{I}) < C \\ L(\mathbf{I}) = L_{max}}} \frac{1}{G(\Omega)} h(i_0) \prod_{k=1}^{6} f(i_k) \; . \tag{64}$$

With $J(\mathbf{I}) < C$ and $L(\mathbf{I}) = L_{max}$, the maximum of i_0 is $C - 1 - L_{max}$. The states for which $L(\mathbf{I}) = L_{max}$ are those which have $\mathbf{I_6}$ in $S(L_{max}, 6)$. Factoring out the terms of $h(i_0)$ in (64), we have

$$\Pr(E_2\bar{E}_1) = \frac{1}{G(\Omega)} \sum_{i_0=0}^{C-1-L_{max}} h(i_0) \sum_{\mathbf{I_6} \in S(L_{max},6)} \prod_{k=1}^{6} f(i_k) \; . \tag{65}$$

The second summation in (65) is identified as $g_6(L_{max})$. Thus

$$\Pr(E_2\bar{E}_1) = \frac{g_6(L_{max})}{G(\Omega)} \sum_{i_0=0}^{C-1-L_{max}} h(i_0) \; . \tag{66}$$

Let the specific adjacent gateway that is mentioned in defining E_3 be the sixth adjacent gateway of the given gateway. Since we are considering a homogeneous system, the result is the same regardless of which adjacent gateway is chosen. The event $E_3\bar{E}_2\bar{E}_1$ consists of all permissible states with $i_6 = l$, $L(\mathbf{I}) < L_{max}$ and $J(\mathbf{I}) < C$. From (35), we have

$$\Pr(E_3\bar{E}_2\bar{E}_1) = \sum_{\substack{\mathbf{I} \in \Omega \\ i_6 = l \\ J(\mathbf{I}) < C \\ L(\mathbf{I}) < L_{max}}} \frac{1}{G(\Omega)} h(i_0) f(l) \prod_{k=1}^{5} f(i_k) \; . \tag{67}$$

Given that $i_6 = l$ and $J(\mathbf{I}) < C$, the remaining adjacent gateways and the given gateway's own calls can use j channels, where $j = 0, 1, \ldots, C - 1 - l$. In the j channels, the given gateway's own calls can use $i_0 = w(j), w(j) + 1, \ldots, j$ channels, in which

$$w(j) = \min(0, j - L_{max} + 1) \; , \tag{68}$$

the remaining $j - i_0$ channels are lent to the remaining five adjacent gateways. Factoring out the terms of $h(i_0)$ from (67), we find

$$\Pr(E_3 \bar{E}_2 \bar{E}_1) = \frac{f(l)}{G(\Omega)} \sum_{j=0}^{C-1-l} \sum_{i_0=w(j)}^{j} h(i_0) \sum_{\mathbf{I_5} \in S(j-i_0,5)} \prod_{k=1}^{5} f(i_k) \; . \tag{69}$$

The last summation in (69) is identified as $g_5(j - i_0)$. Thus,

$$\Pr(E_3 \bar{E}_2 \bar{E}_1) = \frac{f(l)}{G(\Omega)} \sum_{j=0}^{C-1-l} \sum_{i_0=w(j)}^{j} h(i_0) g_5(j - i_0) \; . \tag{70}$$

From (62), (66) and (70), the probability that a borrowing request (from a specific adjacent gateway) is denied is given by

$$p_f = \frac{1}{G(\Omega)} \left[g(C) + g_6(L_{max}) \sum_{j=0}^{C-1-L_{max}} h(j) + f(l) \sum_{j=0}^{C-1-l} \sum_{i_0=w(j)}^{j} h(i_0) g_5(j - i_0) \right] \; . \tag{71}$$

Average Rate of Borrowing Requests from An Adjacent Gateway

In the CBWL/NR schemes, requests that cannot be served on a channel of a gateway may be served on a channel borrowed from an adjacent gateway. Clearly borrowing requests from a specific gateway will be more frequent when that adjacent gateway's *own* C channels are more heavily used. Thus the states of adjacent gateways are coupled. In principle one could define an overall *system* state as a concatenation of gateway states, for all gateways in the system. However this approach is not fruitful because the number of states that must be considered is analytically unmanageable. We retreat from this more rigorous approach and instead account for the coupling between gateways by considering the *average* rate of borrowing and lending between gateways. We found that this approach permits the construction of an analytically tractable model. Theoretical performance characteristics were calculated and were then compared with those obtained by Monte Carlo simulation. The results compared favorably and are discussed in section 5.

We let λ' denote the average rate of borrowing requests made at a given gateway by a specific adjacent gateway. When this rate is known, the state probabilities of the given gateway can be determined completely without the knowledge of the states of other gateways using the foregoing analysis. Actually this rate depends on the state probabilities themselves. The result is a set of implicit equations for the state probabilities and the rate, λ'.

The expression for the average rate of borrowing requests from an adjacent gateway can be discerned as follows. Because of the co-channel interference requirements considered in Section 3, only a fraction of calls that arise in a given cell can access borrowed channels — specifically, only those which can obtain adequate signal quality on the borrowed channels. The fraction is denoted as p. When all channels of a gateway are busy, the rate that new call arrivals make channel borrowing requests is λp. We suppose that the borrowing gateway initially directs a request to an adjacent gateway chosen at random. If the request is denied, the borrowing gateway will randomly select from the remaining adjacent gateways. The process is continued until the request is accepted or all adjacent gateways deny the request. A specific neighboring gateway may be selected by a given gateway (as target for borrowing request) on the first, second, ..., sixth try. Denote the average rate of the given gateway's borrowing requests that are directed into the specific gateway on the kth try as $\lambda'(k)$. The probability that the given gateway on the first try selects the specific adjacent gateway is 1/6. Thus

$$\lambda'(1) = \frac{\lambda p p_c}{6} . \tag{72}$$

if the given gateway on the first try select one of other five adjacent gateways and its request is denied, it selects the specific gateway on the second try with probability 1/5. thus

$$\lambda'(2) = 5\frac{\lambda p p_c}{6} p_f \frac{1}{5} = \frac{\lambda p p_c}{6} p_f . \tag{73}$$

The probability that the given gateway selects other four gateway on the second try is 4/5. If the second try fails, among the remaining four gateways, the specific gateway is selected on the third try with probability 1/4. Thus

$$\lambda'(3) = 4\frac{\lambda p p_c}{6}(p_f)^2 \frac{1}{4} = \frac{\lambda p p_c}{6}(p_f) . \tag{74}$$

In general, the average rate of the given gateway's borrowing requests that is directed to the specific gateway on the kth try is

$$\lambda'(k) = \frac{\lambda p p_c}{6}(p_f)^{k-1} . \tag{75}$$

From the viewpoint of the gateway at which demands are being made, the average rate of borrowing requests coming from a specific neighboring gateway is the sum of $\lambda'(k)$ from k=1 to 6. thus,

$$\lambda' = \sum_{k=1}^{6} \lambda'(k) = \frac{\lambda p p_c}{6} \sum_{k=0}^{5} p_f^k = \frac{\lambda p_c}{6} \frac{(1-p_f^6)}{(1-p_f)} . \tag{76}$$

Because p_c and p_f in (76) depend on λ, the equation is actually an implicit equation, which together with equation (71) (63) that can be used to obtain λ', p_c and p_f simultaneously. An iterative procedure was used as outlined below:

Step 1 The procedure starts with an arbitrary guess of λ'.

Step 2 Use the last updated λ' in convolution algorithm (40), (49) and (60) to calculate p_c (63) and p_f (71).

Step 3 The average rate of borrowing requests λ' from an adjacent gateway is updated using (76).

Step 4 Step 2 and 3 of the procedure is continued until the absolute value of the difference between λ''s from two consecutive iterations agree within the desired number of *significant figures*.

The probability of blocking is different for users in different positions of a cell. For a call that cannot access a borrowed channel, the blocking probability β_{NR} is equal to p_c. On the other hand, a call that can access a borrowed channel will be blocked if *all of the regular channel are occupied AND no channel can be borrowed from any adjacent gateway.* Its blocking probability, α_{NR}, is given by

$$\alpha_{NR} = p_c p_f^6 \ . \tag{77}$$

The blocking probability B_{NR} (averaged over all users) is

$$B_{NR} = p\alpha_{NR} + (1-p)\beta_{NR} = p_c[pp_f^6 + (1-p)] \ . \tag{78}$$

4.2 Analysis of CBWL/CR

4.2.1 Modeling of CBWL/CR

The seven-dimensional state vector $\mathbf{I} = (i_0, i_1, i_2, i_3, i_4, i_5, i_6)$, which we used to analyze the CBWL/NR scheme cannot completely describe the CBWL/CR scheme. In CBWL/CR, two types of calls must be distinguished. The first type of calls are those for which the BCSS is below threshold. These calls, denoted type B, cannot use borrowed channels directly, but they can benefit from channel rearrangement. Let i_a and i_b denote the numbers of a gateway's regular channels that are used by A type calls and B type calls respectively. For CBWL/CR, we use an eight-dimensional state vector,

$$\mathbf{I_r} = (i_a, i_b, i_1, i_2, i_3, i_4, i_6) \ , \tag{79}$$

to characterize the state of a gateway. With channel rearrangement, a new B call that finds all channels occupied is not necessary blocked. If an A call is in progress, it may be possible to borrow a channel to serve this call.

4.2.2 State Aggregation and Decomposition

Due to channel rearrangement, the equilibrium distribution of states $\mathbf{I_r}$ is not in product form. Using the states, $\mathbf{I_r}$, we can construct the probability flow balance equations and solve them to find the state probabilities. However, the number of states is prohibitively large (millions for a system with $C = 24$ and l=4). Instead, we will use a state aggregation and decomposition method and use the results from the analysis of CBWL/NR to expedite the calculation. The method is based on the method in [15].

State aggregation

Consider a gateway in state $\mathbf{I_r}$ $=(i_a, i_b, i_1, \ldots, i_6)$, in which all channels of the gateway are busy and $i_a > 0$. If a new B call arrives and an adjacent gateway can lend one channel to the given gateway, channel rearrangement is used. After channel rearrangement, the state of the given gateway is changed to $(i_a - 1, i_b + 1, i_1, \ldots, i_6)$. Therefore, channel rearrangement only causes the changes of i_a and i_b. Specifically, channel rearrangement reduces i_a by one and adds one to i_b but keeps $i_a + i_b$ unchanged. If we let $i_0 = i_a + i_b$, the set of states $\Phi(i_0) \triangleq \{\mathbf{I_r} : i_a + i_b = i_0\}$ are identical to state $\mathbf{I}$. That is, $\mathbf{I}$ is a macro-state correspondence to $\Phi(i_0)$. The effect of channel rearrangement is cancelled by the aggregation of states. By using these state aggregation, the eight-dimensional model is reduced to a seven-dimensional model. The aggregated space is the same as that of CBWL/NR. The problem of non-product form is reduced to the problem that has a product form solution. We can use the algorithm described in the last subsection to find the distributions of $\mathbf{I}$. From the distributions, we can find some important probabilities such as p_c (63) and p_f (71) (we will show that λ' used in these equations is different from that in CBWL/NR). However, because we have not distinguished i_a and i_b in $\mathbf{I}$, the distribution cannot give us the information about channel rearrangement. Further analysis is needed.

Probability that v channels are lent, $\Pr(v)$

The probability, $\Pr(v)$, will be needed in later analysis. It can be calculated from the results of convolution algorithm described in the last subsection. We note from (46) that $S(v,6)$ is the set of all states in which v channels are lent to neighbors. It follows that the probability that a given gateway has lent exactly v channels to neighbors and has i_0 channels used by the given gateway's own calls is given by

$$\sum \{\mathbf{I} \in \Omega : \mathbf{I_6} \in S(v,6)\} = \sum_{\mathbf{I_6} \in s(v,6)} p(i_0, \mathbf{I_6}) \ . \tag{80}$$

Then from (35) and (48), we find that $h(i_0)g_6(v)/G(\Omega)$ $(0 \le i_0 \le C - v)$ is the joint probability that the given gateway lends exactly v channels to the adjacent gateways and i_0 channels are used by the given gateway's own calls. Thus

$$\Pr(v) = \frac{g_6(v)}{G(\Omega)} \sum_{i_0=0}^{c-v} h(i_0) \ , \qquad v = 0, 1, \ldots, L_{max}. \tag{81}$$

Probability that channel rearrangement cannot be used

If all channels of a gateway are occupied and the number of channels that are occupied by A calls is zero, a new B call arrival cannot use channel rearrangement and is blocked. We denote the probability as p_a. Thus,

$$p_a = \Pr\{\mathbf{I_r} \in \Psi : i_a = 0, i_b + \sum_{k=1}^{6} i_k = C\} \tag{82}$$

where Ψ is the set of all permissible states, $\mathbf{I_r}$.

Because the distribution of $\mathbf{I_r}$ is unknown, we cannot use (82) to find p_a. We will use a decomposition method to calculate p_a.

Decomposition method

In following paragraphs, we will first show that because the distribution of $\mathbf{I}$ is in product form, we can use the decomposition method to calculate the marginal probability, $Pr(i_0)$. With this method, we decompose the system as a set of "independent" subsystems which can be analyzed separately from one another. Then we will show the method can be extended to find the marginal probability, $\Pr(i_a, i_b)$, from which, p_a can be obtained.

We can calculate $\Pr(i_0)$ by two ways. First, because $\Pr(\mathbf{I})$ is in product form, we can calculate it directly from (35). That is

$$\Pr(i_0) = \frac{h(i_0)}{G(\Omega)} \sum_{v=0}^{C-i_0} \sum_{\mathbf{I_6} \in S(v,6)} \prod_{k=1}^{6} f(i_k) = \frac{h(i_0)}{G(\Omega)} \sum_{v=0}^{C-i_0} g_6(v) \; . \tag{83}$$

On the other hand, we can use a decomposition method to find $\Pr(i_0)$. We decompose the system as a set of "independent" subsystems which can be analyzed separately from one another. Each subsystem corresponds to a fixed number of lending channels, v ($v = 0, \ldots, L_{max}$). For a fixed v, the corresponding subsystem is a $M/M/C-v/C-v$ queue with the given gateway's own calls as the only arrivals. Denote the conditional probability of i_0 given v as $p_v(i_0)$. From the Erlang B formula, we find

$$p_v(i_0) = \frac{h(i_0)}{\sum\limits_{x=0}^{C-v} h(x)} \; , \qquad i_0 = 0, \ldots, C-v, \tag{84}$$

in which $h(.)$ is defined in (33). The marginal probability of $\Pr(i_0)$ can be find from

$$\Pr(i_0) = \sum_{v=0}^{L_{max}} \Pr(v) p_v(i_0) \; . \tag{85}$$

Substituting (81) and (84) into (85), we find

$$\Pr(i_0) = \frac{h(i_0)}{G(\Omega)} \sum_{v=0}^{C-i_0} g_6(v) \; . \tag{86}$$

Note that (83) and (86) are exactly the same. Thus, we have shown that we can use the decomposition method to calculate the marginal probability of i_0 from (84) and (85). In the method, we consider in term that the value of v is held fixed for each possible value of v. Thus we can find the conditional probability $p_v(i_0)$ (for any given permitted value of v). Even though *the number of channels, i_0, which are used by the given gateway's own calls* and *the number of channels, v, that are lent to adjacent gateways* are dependent random variables, the dependence is reflected only in the maximum number of available channels to the gateway's own calls. That is, if v channels are lent, no more than $C - v$ channels can be used by the given gateway's own calls.

Since $i_0 = i_a + i_b$ and $p_v(i_0)$ does not depend on individual i_k's ($k \geq 1$), the interaction between i_a and i_b (when v is given) can also be studied without consideration of the individual i_k's ($k \geq 1$). We also note just as an additional comments, that, $\Pr(i_0)$can be directly (and most conveniently) obtained using (83), rather than using (84). However, computation of this probability is not needed to find the performance characteristics of the scheme. The probability is used to show that the decomposition method can be used to calculate marginal probability, $\Pr(i_a, i_b)$.

4.2.3 Conditional Probability of (i_a, i_b) Given v

Denote $p_v(i_a, i_b)$ as the conditional joint probability of (i_a, i_b) given that v channels are lent. With v channels being lent, the permissible states of i_a and i_b are constrained by following conditions:

$$\begin{aligned} i_a &\geq 0 \\ i_b &\geq 0 \\ i_a + i_b &\leq C - v \ . \end{aligned} \tag{87}$$

When a gateway is in any state (i_a, i_b) with $i_a + i_b < C - v$, if an A call arrives, the gateway's state is changed to $(i_a + 1, i_b)$. Denote λ_1 as the arrival rate of A calls. Thus,

$$\lambda_1 = p\lambda \ , \tag{88}$$

If a B call arrives, the gateway's state is changed to $(i_a, i_b + 1)$. Denote λ_2 as the arrival rate of B type of calls. Thus,

$$\lambda_2 = (1 - p)\lambda \ . \tag{89}$$

When a gateway is in a state (i_a, i_b) with $i_a + i_b = C - v$ and $i_a > 0$, if a B call arrives, and if its channel borrowing request is not denied by adjacent gateways, channel rearrangement will be used. As the result of channel rearrangement, an A call is switched to a borrowed channel and the released channel is given to the B call. Thus, the gateway's state is changed to $(i_a - 1, i_b + 1)$. Denote λ_3 as this transition rate. The probability that a borrowing request is accepted by an adjacent gateway is $1 - p_f^6$. Thus,

$$\lambda_3 = \lambda(1 - p)(1 - p_f^6) \ . \tag{90}$$

The rate at which regular channels are released by A calls or B calls at state, (i_a, i_b), with $i_a + i_b < C - v$ is $i_a\mu$ or $i_b\mu$, respectively. The state-transition diagram of the conditional probabilities for $C - v = 6$ is shown in Figure 5. From the figure, we can see the states in the upper diagonal line are not "doubly-connected". Therefore, the sufficient condition for a product form distribution is not satisfied [14]. We cannot use product form solution for distribution of i_a and i_b. That also can be used to explain why we cannot use product form solution in the distribution of $\mathbf{I_r}$. If we let $i_0 = i_a + i_b$, the states of Figure 5. are aggregated into some states concatenated in a straight line. The "doubly-connected" states are disappeared in the aggregation. The subsystem becomes a M/M/C-v queue. Because i_0 is a component of $\mathbf{I}$, the distribution of $\mathbf{I}$ is in product form.

The flow balance equations of $p_v(i_a, i_b)$ are as follows:

$$\begin{aligned} [\lambda + (i_a + i_b)\mu]p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\ +(i_a + 1)\mu p_v(i_a + 1, i_b) + (i_b + 1)\mu p_v(i_a, i_b + 1) \end{aligned}$$

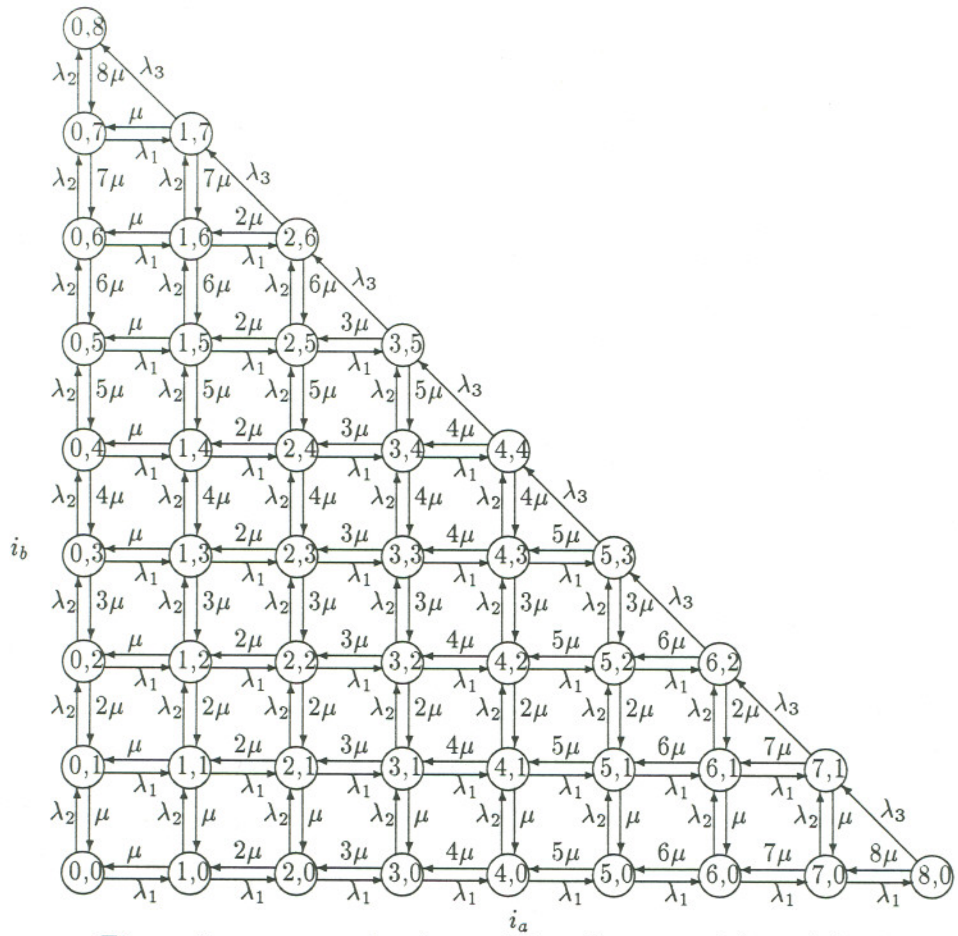


Figure 5: an example of transition diagram of (i_a, i_b) $(c - v = 8)$.

$$
\begin{gathered}
(0 \le i_a + i_b < C - v) \\
[\lambda_3 + (i_a + i_b)\mu] p_v(i_a, i_b) = \lambda_1 p_v(i_a - 1, i_b) + \lambda_2 p_v(i_a, i_b - 1) \\
+ \lambda_3 p_v(i_a + 1, i_b - 1) \\
(i_a + i_b = C - v, i_a > 0) \\
(C - v)\mu p_v(0, C - v) = \lambda_2 p_v(0, C - v - 1) + \lambda_3 p_v(1, C - v - 1) \\
(i_a = 0,\ i_b = C - v)
\end{gathered}
\tag{91}
$$

where $p_v(x, y) = 0$, if $x < 0$ or $y < 0$.

The balance equations can be solved by Gauss-Seidel Iteration. Since the dimensions have been reduced, the number of states have been decreased greatly and computation time is saved.

Among $p_v(i_a, i_b)$'s, $p_v(0, C-v)$ is the conditional probability that channel rearrangement cannot be used given v. We can calculate p_a from

$$
p_a = \sum_{v=0}^{L_{max}} p_v(0, C - v) \Pr(v) \ . \tag{92}
$$

In (92), $L_{max} + 1$ groups of equations of (91) for v from 0 to L_{max} must be solved. However, the number of groups of equations to be solved can be reduced greatly. Because the probability that a gateway borrows a lot of channels is very small and the contribution of these small $\Pr(v)$ to p_a can be omitted. In our algorithm, when $\Pr(v)$ from (81) is less than a desired precision, it is not necessary to solve the equations that correspond to that v.

4.2.4 Average Rate of Borrowing Requests, λ'

The average rate of borrowing request from a gateway to an adjacent gateway, λ', in CBWL/CR scheme is different from that in CBWL/NR scheme. In CBWL/NR, the borrowing requests arise only from the arrival of A calls. The request rate is given in (76). In CBWL/CR, some borrowing requests are due to B calls. If all channels of a gateway are busy, a new B call arrival in the cell will make a borrowing request with probability of $p_c - p_a$. Using the same analysis approach for (76), we can find

$$
\Pr\{\text{borrowing request to a gateway from } B \text{ calls}\} = \frac{(1-p)\lambda}{6} \frac{1 - p_f^6}{1 - p_f} (p_c - p_a) \ . \tag{93}
$$

Totally,

$$
\lambda' = \lambda \frac{1}{6} \frac{1 - p_f^6}{1 - p_f} \left[p p_c + (1 - p)(p_c - p_a) \right] \ . \tag{94}
$$

Because p_c, p_a and p_f in (94) depend on λ, the equation is actually an implicit equation that can be used to obtain p_f, p_c and λ' simultaneously. An iterative procedure was used as outlined below.

4.2.5 The Iterative Procedure

Step 1 The procedure starts with an arbitrary guess of λ'.

Step 2 Use the last updated λ' in convolution algorithm to calculate p_c, p_f and $\Pr(v)$ from (63), (71) and (81).

Step 3 For any v, with $\Pr(v)$ large enough, $v = 0, \ldots, L_{max}$, solve (91) by Gauss-Seidel iteration to get $p_a(v)$.

Step 4 Calculate p_a with equation (92).

Step 5 Update λ' using Equation (94).

Step 6 Step 2–6 of the procedure is continued until the absolute value of the difference between λ''s from the two consecutive iterations agree with the desired number of significant figures.

4.2.6 Blocking Probabilities

Once p_c, p_f and p_a are found, we can find blocking probabilities. First, the blocking probability experienced by an A call is just probability that all of channels of a gateway are occupied and the gateway cannot borrow any channel from its neighboring gateways,

$$\alpha_{CR} = p_c p_f^6 \ . \tag{95}$$

With channel rearrangement, B calls can use borrowed channel indirectly. A new B call arrival will be blocked if any of the following two events are true at the time the call arrives.

Event E_a: All channels of the given gateway are occupied and the gateway cannot make channel rearrangement.

Event E_b: All channels of the given gateway are occupied and the adjacent gateways cannot lend any channels to the given gateway.

Thus the probability that a B call is blocked is equal to the probability of the union of the two events. Thus

$$\begin{aligned} \Pr\{\text{a } B \text{ call is blocked}\} &= \Pr\{E_a \bigcup E_b\} \\ &= \Pr(E_a) + \Pr(E_a \bar{E}_b) \end{aligned} \tag{96}$$

The probability of event E_a is p_a. The second probability is equal to $(p_c - p_a)p_f^6$. Thus,

$$\beta_{CR} = p_a + (p_c - p_a)p_f^6 \ . \tag{97}$$

The overall blocking probability in a gateway is

$$B_{CR} = p\alpha_{CR} + (1-p)\beta_{CR} \ . \tag{98}$$

4.3 Nonhomogeneous Cellular System

In a nonhomogeneous cellular system, each cell may have different offered traffic and different number of channels. We cannot use an arbitrary cell to represent every cell. The average rate of borrowing requests between each pair of cells may be different. The rate cannot be calculated from (76) or (94). Let $\lambda(Y)$ be the offered traffic in cell Y, $p_c(Y)$ be the probability that all channels of cell Y are occupied, $p_f(Y, X)$ be the probability that a borrowing request from cell Y is rejected by cell X, $\lambda'(Y, X)$ be the average rate of borrowing requests to X from Y. Figure 6 is used to illustrated how to calculate $\lambda'(Y, X)$ for any pair of cells X and Y. In Figure 6, Y and A_i(i =1, 2,

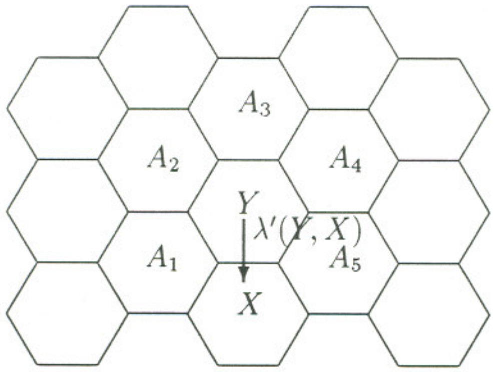


Figure 6: Illustration of overflow rate $\lambda'(Y, X)$

3, 4, 5) are adjacent cells of X. The average rate of borrowing requests that arise from gateway Y is denoted as $\lambda'(Y)$. The rate is dependent on the schemes.

$$\lambda'(Y) = \begin{cases} p\lambda(Y)p_c(Y) & \text{for CBWL/NR scheme} \\ \lambda(Y)[pp_c(Y) + (1-p)(p_c(Y) - p_a(Y))] & \text{for CBWL/CR scheme} \end{cases} \tag{99}$$

If no A_i can lend any channel to Y, the borrowing request of Y will definitely go to X. If there is only one A_i that can lend channels to Y, the borrowing request from Y will go to gateway X with probability of 1/2, and so on. Thus,

$$\begin{aligned} \lambda'(Y, X) = \lambda'(Y) \Bigg\{ & \prod_{i=1}^{5} p_f(Y, A_i) + \frac{1}{2}\sum_{i=1}^{5}[1 - p_f(Y, A_i)] \prod_{k=i} p_f(Y, A_k) \\ & + \frac{1}{3} \sum_{\substack{i,j=1,\ldots,5 \\ i \neq j}} [1 - p_f(Y, A_i)][1 - p_f(Y, A_j)] \prod_{k \neq i,j} p_f(Y, A_k) \\ & + \frac{1}{4} \sum_{\substack{i,j=1,\ldots,5 \\ i \neq j}} p_f(Y, A_i)p_f(Y, A_j) \prod_{k \neq i,j} [1 - p_f(Y, A_k)] \\ & + \frac{1}{5}\sum_{i=1}^{5} p_f(Y, A_i) \prod_{k \neq i}[1 - p_f(Y, A_k)] + \frac{1}{6}\prod_{i=1}^{5}[1 - p_f(Y, A_i)] \Bigg\} . \end{aligned} \tag{100}$$

An extended algorithm from homogeneous case can be used for nonhomogeneous case. A similar iterative procedure is employed. In one iteration, all $\lambda'(Y, X)$ between each pair of adjacent gateway Y and X and equilibrium state distribution of each gateway are calculated. The iterative procedure is stopped only when for each pair of adjacent gateways, obtained $\lambda'(Y, X)$'s from two consecutive iterations agree with the desired number of significant figures. Thus the computation time is much more than in the homogeneous case, but we found it to be much less than that needed for simulation.

5 Numerical Results And Discussion

As an example, we considered CBWL/NR and CBWL/CR for a cellular system with 24 channels in each gateway. For each scheme two cases were considered. One is homogeneous. The other has a "hot spot" in the central cell. Each of the other cells has less traffic, but the same as each other. For simplicity, we assume in the homogeneous case, that the system has a very large (essentially infinite) number of cells. Thus we do not need to distinguish the boundary cells and the internal cells. For the "hot spot" case, since the performance of each cell may be different, we consider a system of 37 cells.

The preceding analytical development was used to numerically calculate performance characteristics. The computed blocking probabilities, B_{NR} of CBWL/NR scheme and B_{CR} of CBWL/CR scheme with 95% confidence intervals of simulation results are shown in Figure 7 and Figure 8 respectively.

To simulate the system with an infinite number of cells, we used a 37 cell configuration with each cell having six adjacent neighboring cells. The boundary cells on one side are adjacent to the boundary cells on the other side. In each run, we generated *about* 2000 call arrivals in each cell and determined the fraction of blocking calls for each cell. Since the cells are statistically the same, in one simulation run, 37 blocking probabilities can be found, as well as the mean, variance and confidence intervals for the blocking probabilities. The simulation was written in the *Simscript* simulation language, and was executed on a *Sun* workstation. From the figures, we can see that the results of analysis are close to those obtained by simulation. The results displayed in these figures compare the performance of FCA (for which $p = 0$) to the CBWL schemes with $p = 0.1, 0.3$ and 0.5. It is seen that with $p = 0.5$ CBWL/NR can reduce the blocking probability about 50% and that CBWL/CR can reduce the blocking probability by orders of magnitude. Thus, CBWL/CR can improve the performance significantly.

Figure 9 depicts blocking probabilities (B_{NR}, α_{NR} and β_{NR}) of CBWL/NR against offered traffic per cell.

It is seen that $\beta_{NR} > B_{FCA} > B_{NR} > \alpha_{NR}$ for any $p > 0$ (α_{NR} is too small to show up in the figure). Thus channel borrowing without channel rearrangement causes very different blocking probabilities for the A-type calls and B-type calls. Because channel borrowing increases the

Table 2: The offered traffic per cell of CBWL/NR and CBWL/CR for 2% blocking probability, $B_{CR} = .02$.

p	CBWL/NR offered traffic (Erlang)	CBWL/NR percent increase	β_{NR}	α_{NR}	CBWL/CR offered traffic (Erlang)	CBWL/CR percent increase	β_{CR}	α_{CR}
0.0	16.63	0.0%	.0200	.0000	16.63	0.0%	.0200	.0000
0.1	16.83	1.3%	.0222	.0000	18.22	9.6%	.0222	.0000
0.2	17.05	2.5%	.0250	.0000	19.74	18.7%	.0254	.0000
0.3	17.31	4.1%	.0286	.0000	21.00	26.3%	.0292	.0000
0.4	17.63	6.0%	.0335	.0000	21.97	32.1%	.0335	.0001
0.5	18.00	8.2%	.0400	.0000	22.72	36.6%	.0395	.0011
0.6	18.53	11.4%	.0510	.0000	23.18	39.3%	.0396	.0067
0.7	19.14	15.1%	.0667	.0000	23.37	40.5%	.0301	.0160
0.8	20.18	21.3%	.0999	.0000	23.42	40.8%	.0214	.0200

number of channels that A-type calls can access, but channel lending reduces the number of channels that B-type calls can access. Therefore, α_{NR} is lower than B_{FCA} and β_{NR} is higher than B_{FCA}. In average, B_{NR} is still lower than B_{FCA}. Figure 10 is a similar plot for CBWL/CR. In Figure 10, $B_{FCA} > \beta_{CR} > B_{CR} > \alpha_{CR}$, and that *differences* between B_{CR}, α_{CR} and β_{CR} are much less than that of CBWL/NR.

Figure 11 compares the overall blocking probabilities of FCA, CBWL/NR and CBWL/CR schemes. The performance of CBWL/CR with $p = 0.1$ is even better than that of CBWL/NR with $p = 0.3$. Furthermore, it is better than that of CBWL/NR with $p = 0.5$ in light traffic. The figure indicates that for CBWL with channel rearrangement, the performance of the system is enhanced significantly.

Table 2 shows a comparison of offered traffic that can be accommodated at a 2% blocking probability for CBWL/NR and CBWL/CR cut-off priority.

Specifically, it tabulates the percentage increase (in offered traffic) in comparison with the corresponding FCA scheme. When the fraction of A-type calls, p is increased, the offered traffic of CBWL/NR and CBWL/CR is increased in p. For CBWL/NR, the increasing rate of offered traffic is small. For CBWL/CR, when $0 < p < 0.6$, the offered traffic increases from 16.63 Erlangs to 23.18 Erlangs (about 39.3% of increment). When p is greater than 0.6, the increase is slowed. Thus increasing p beyond 0.6 helps little to improve system performance for CBWL/CR. The effect is not a severe limitation for CBWL/CR scheme, because co-channel interference usually requires small p for CBWL schemes. Table 2 also shows the blocking probabilities of A-type and B-type calls for given $B_{NR} = B_{CR} = .02$. We notice that β_{CR} has a peak at about $p = 0.6$. The effect can be explained as follows. When p is increased from 0, for a fixed $B_{CR} = 0.02$, the offered traffic that a cell can accommodate is increased. The increasing of offered traffic causes p_c and p_f to increase, this causes increase of β_{CR}. But, when p is increased, the fraction of A-type calls is increased and the probability that a B-type call cannot use channel rearrangement, p_a (which is defined in (82), the probability that all channels of a gateway are occupied, and no channel is occupied by A-type of calls) becomes relatively small. The decrease of p_a causes the decrease of β_{CR}. When p is greater than 0.4, the rate of decrease is greater than the rate of increase, β_{CR} thus is decreased. When p continue to increase, β_{CR} approximates to B_{CR} and α_{CR}.

Figure 12. shows the blocking probabilities of CBWL/CR with different group size l. It is seen from the figure that group size has little influence to the performance. In this example, under light offered traffic, the performances are almost the same for any value of l between 1 and 4. In the heavy traffic, the blocking probability of $l = 1$ is slightly greater than blocking probabilities of other values of l. That can be explained as a gateway can borrow any available ones from $6l$ channels, even l is small, $6l$ is large enough for a gateway to borrow.

Figure 13 shows the performance of CBWL/CR under hot-spot traffic. An example system with a hot-spot was considered. The central cell has 1.5 times the offered traffic of any other cell. We use the analysis for nonhomogeneous case to calculate the blocking probability of each cell. The results of analysis agree with that from simulation. With CBWL/CR ($p > 0$), the blocking probability of the central hot spot cell and any other cell is significantly reduced in comparison with FCA ($p = 0$).

6 CONCLUSION

A new channel assignment and sharing method for cellular communication systems is presented in this paper. It allows cell gateways to borrow channels from adjacent gateways. To overcome co-channel interference caused by channel borrowing, the new scheme use reduced transmitted power on the borrowed channels rather than channel locking. We have shown that CBWL cannot cause more interference than FCA scheme. The analyses of this paper have shown that CBWL schemes have a better channel utilization than the conventional cellular system with FCA, DCA and HCA, and can provide a good performance in hot spot scenarios. Furthermore, CBWL is easier to implement. CBWL/NR scheme has a disadvantage that it causes the different blocking probabilities for the calls in different positions. The disadvantage is overcome by CBWL/CR scheme. CBWL/CR has a much better performance than CBWL/NR. Our analysis and simulation have shown that CBWL/CR can reduce blocking probability by factor of 10–1000 in comparison with FCA. It can increase the offered traffic about 30% for a given grade of service. It has a good performance under both light and heavy traffic, and in this respect is superior to DCA and HCA. Our analysis algorithm is simple and effective in computation. Its accuracy is proved by comparison with the results from simulation.

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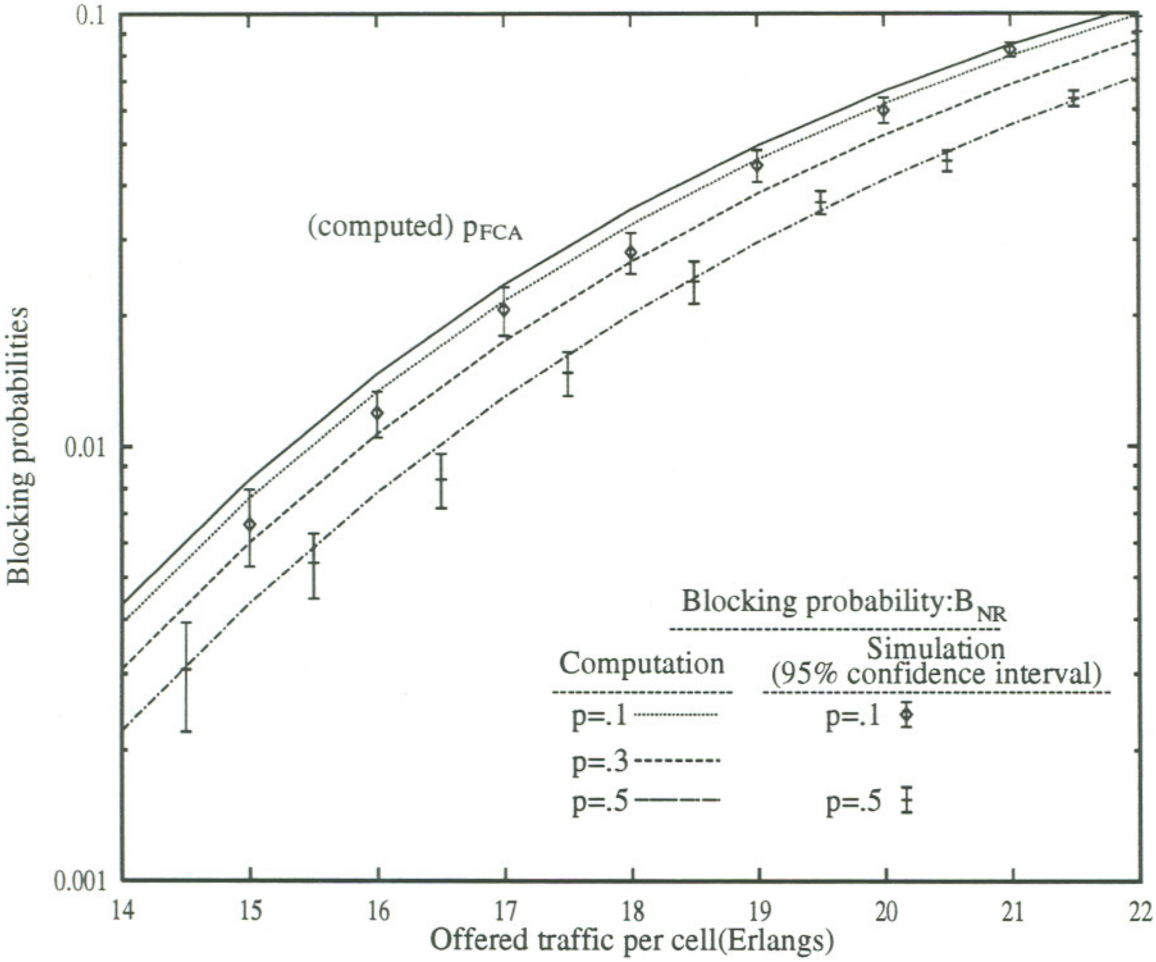

Figure 7: The performance of CBWL/NR scheme ($C = m = 24, l = 3$).

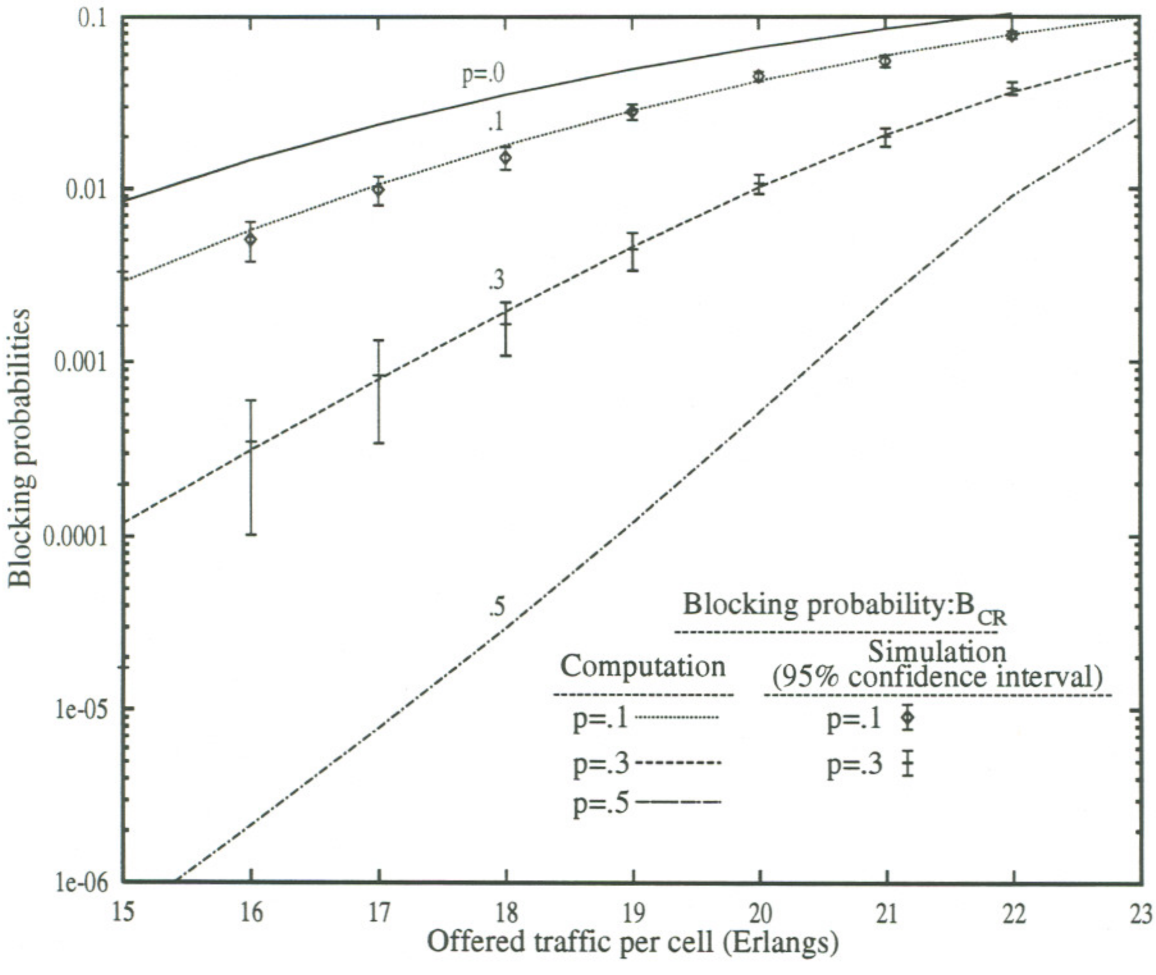

Figure 8: The performance of CBWL/CR scheme ($C = m = 24, l = 3$).

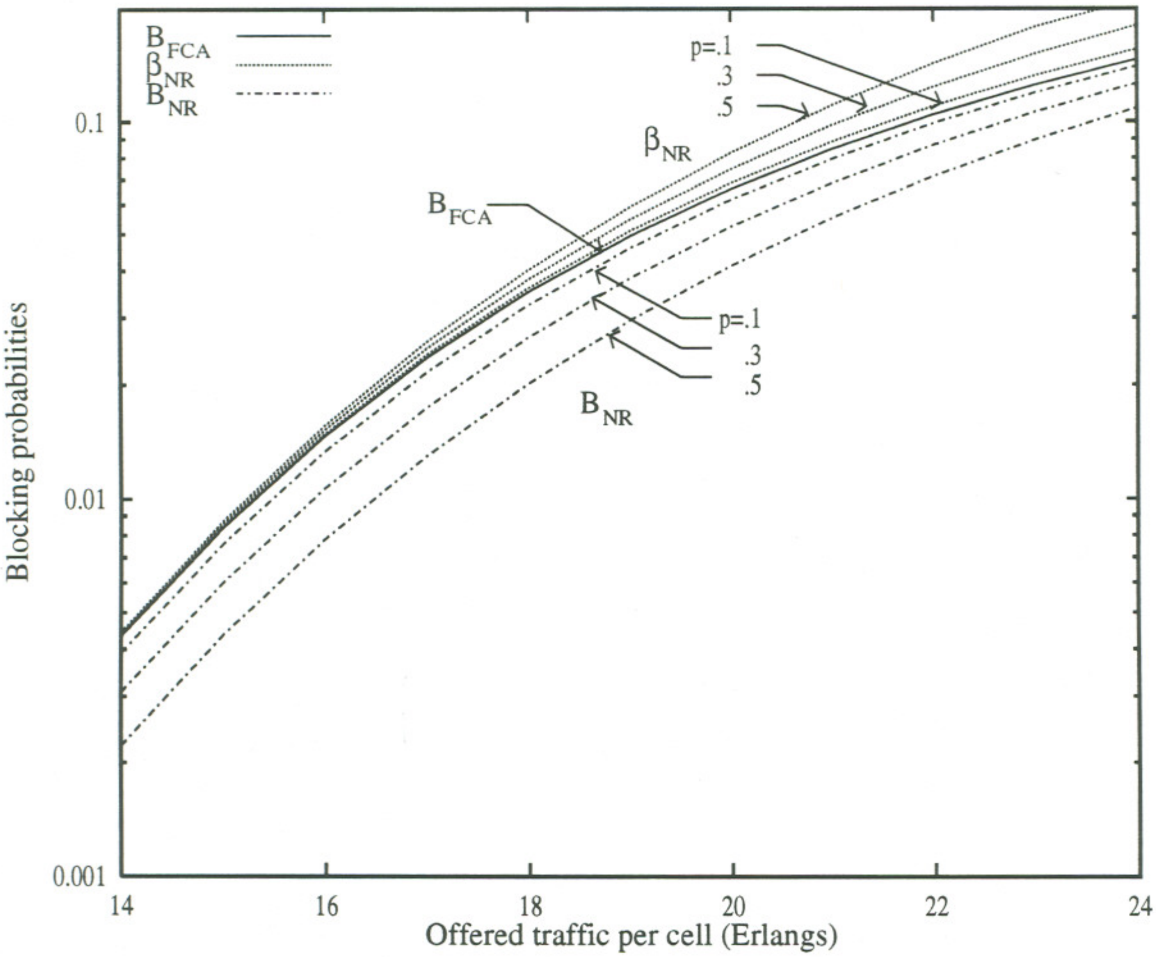

Figure 9: Blocking probabilities of CBWL/NR ($C = m = 24, l = 3$)

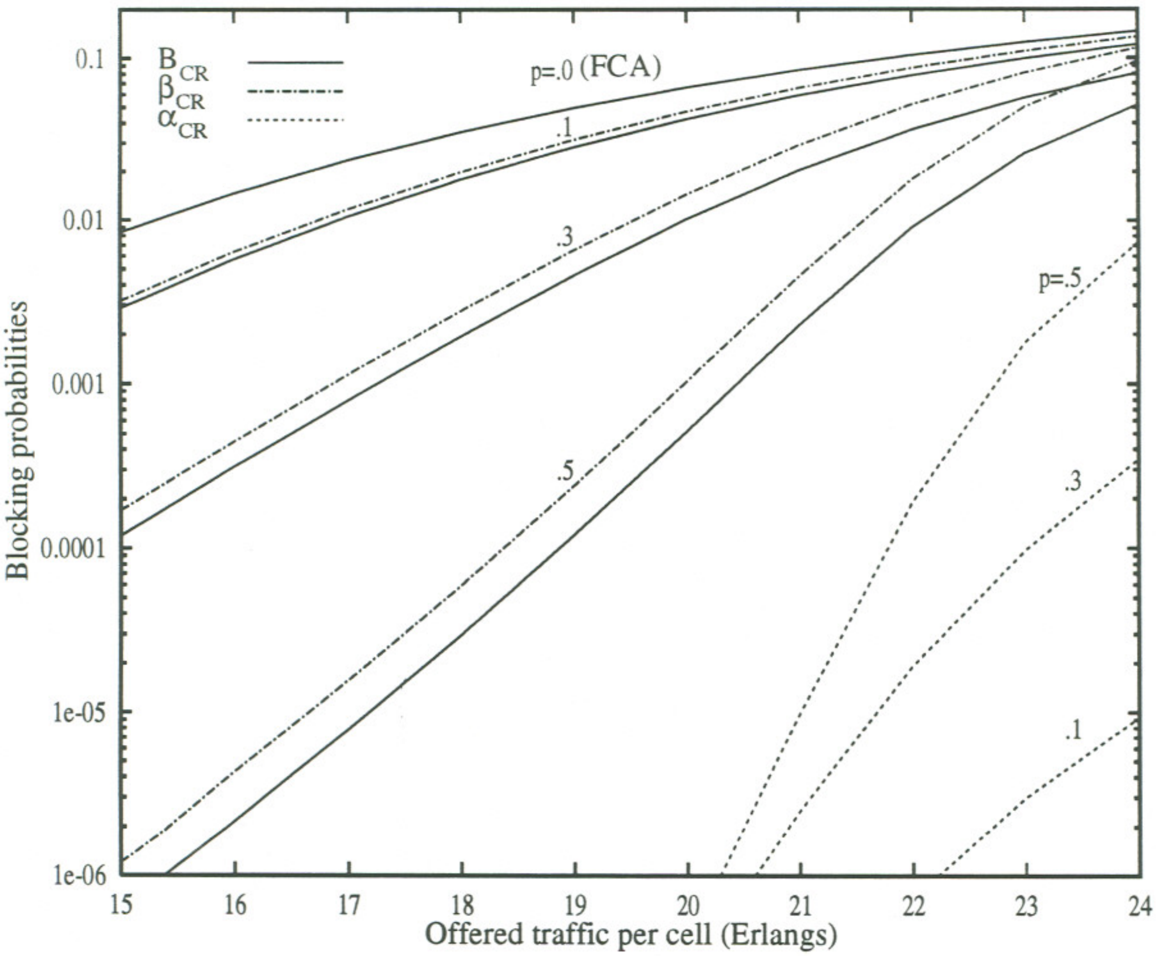

Figure 10: Blocking probabilities of CBWL/CR ($C = m = 24, l = 3$)

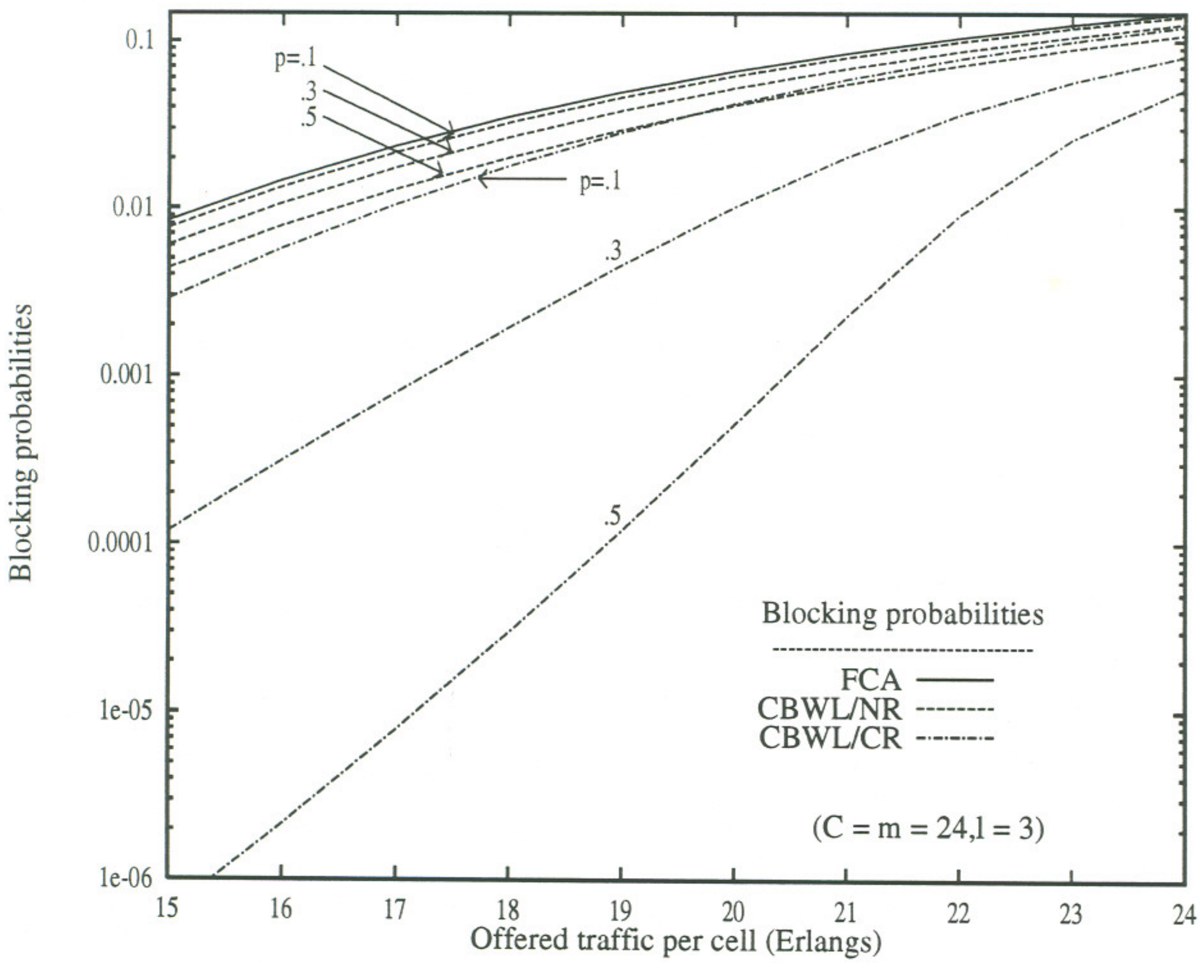

Figure 11: Blocking probabilities of CBWL/NR and CBWL/CR.

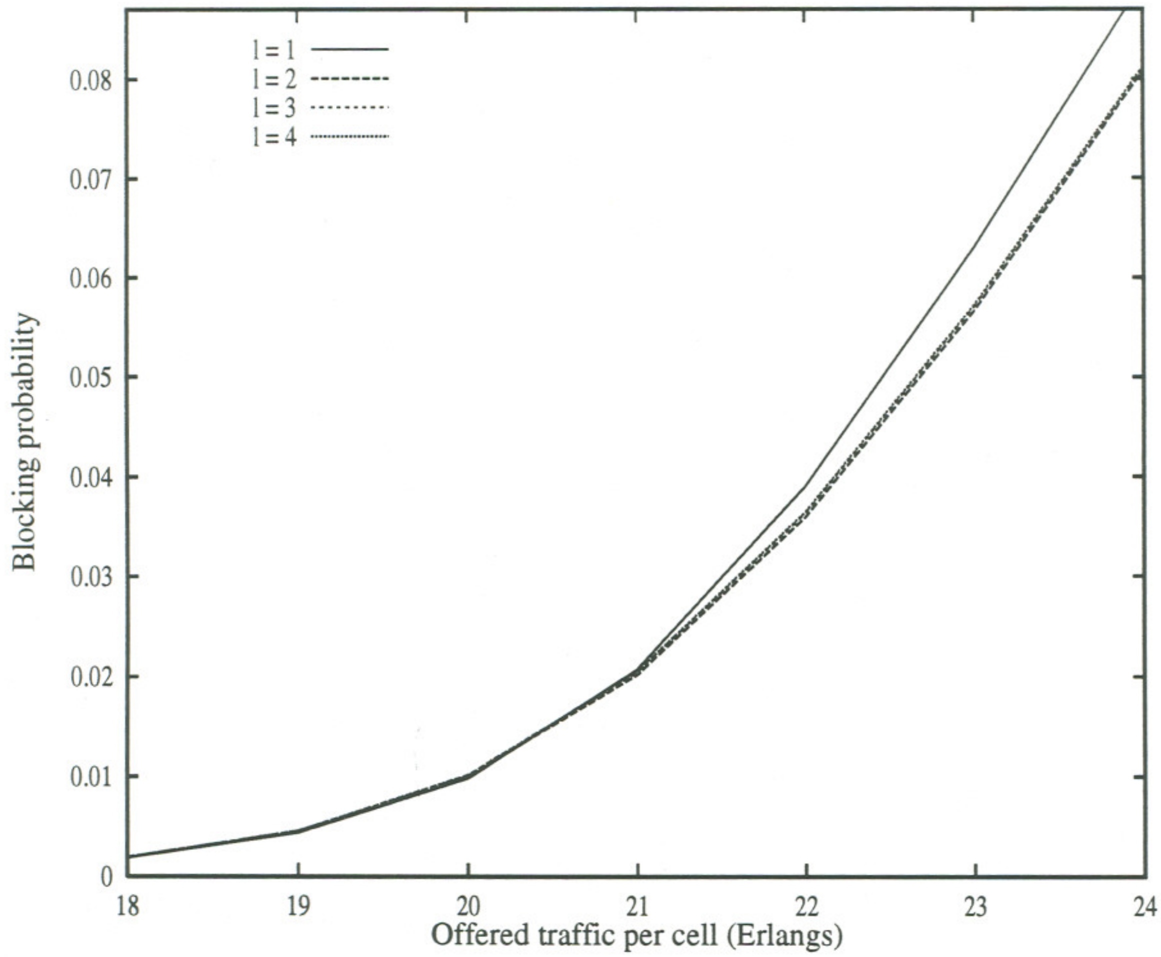

Figure 12: Performance of CBWL/CR under different group size ($C = m = 24, p = .3$).

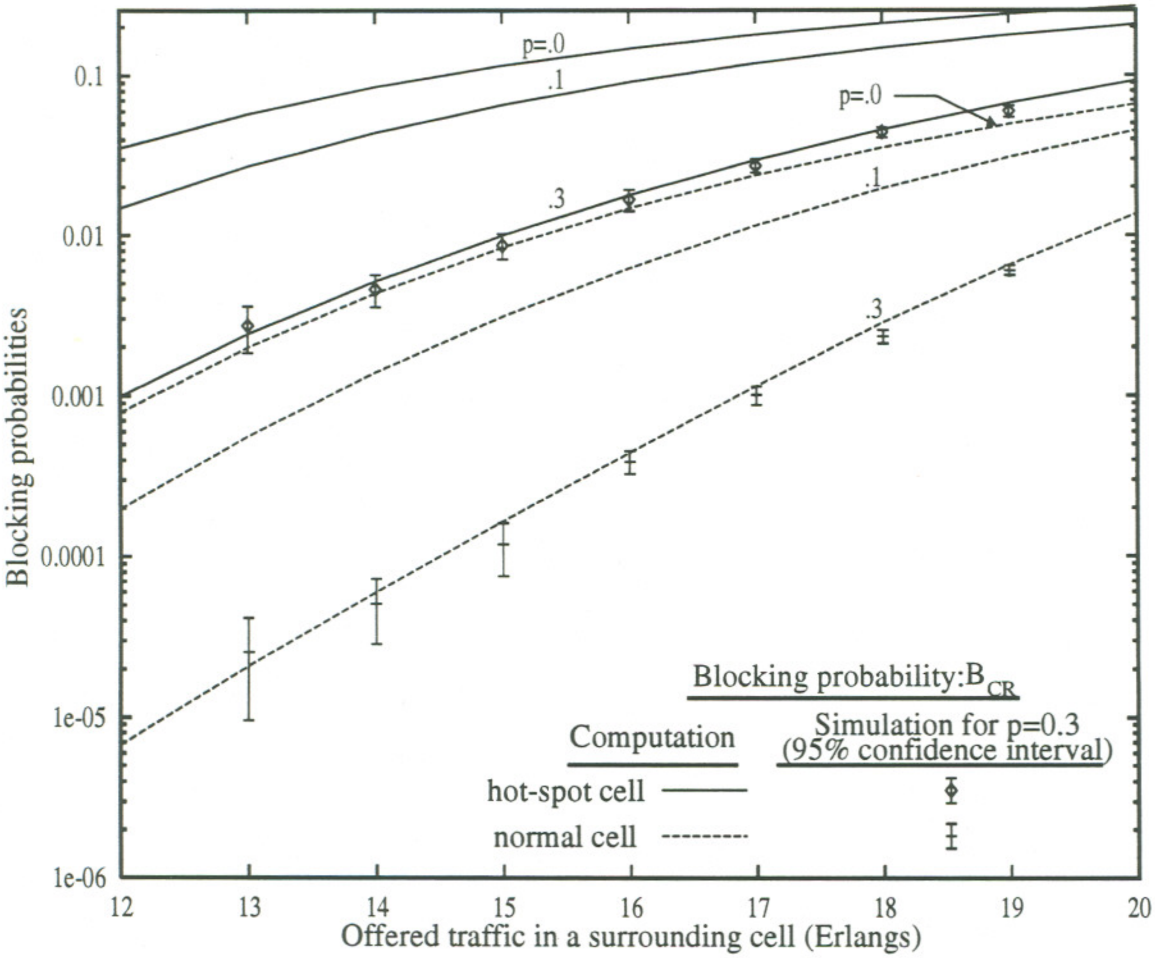

Figure 13: Blocking probabilities of CBWL/CR in hot spot case ($C = m = 24, l = 3$).