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DIFFUSION OF CLOUDS OF CONTAMINANT IN A TURBULENT BOUNDARY LAYER

by

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Abstract

Under the assumption that the detailed turbulence structure in the constant-stress region of a fluid flowing over a rough wall is in a universal state determined wholly by the friction velocity u_* a similarity analysis of a Lagrangian type can be applied to the relative dispersion of clouds of contaminant released in such a layer. It is found that for the case in which the initial dimension of the cloud, say L, and the initial mean height are comparable there are only two temporal regimes and they are characterized by times smaller than or greater than L/u_*.

For the small time regime all relative diffusivities are linearly dependent on time and their directional properties are related to Eulerian measurements. Of more interest are the predictions in the long time regime. It is found that in this region also relative diffusion is characterized by a diffusivity that is linearly dependent on time. Such a prediction differs from that of homogeneous turbulence and seems to have some support from experimental data in the earth's atmosphere.

Diffusion of Clouds of Contaminant in a Turbulent Boundary Layer

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Edward E. O'Brien[1]

<u>Introduction</u>: This paper is concerned with the problem of predicting the rate of growth of a small cloud of contaminant in a turbulent boundary layer and for that purpose employs an extension of Batchelor's (1959) Lagrangian similarity hypotheses which, in its original form, was concerned only with the statistics of the velocity of single particles. Batchelor (1964) and others, Ellison (1959), Gifford (1962) and Cermak (1963), have made use of the similarity concept to obtain satisfactory predictions of measurable consequences of continuous source emission and it seems worthwhile to pursue its extension to the behavior of clouds.

We consider the region of fluid near a rigid rough boundary in which the Reynolds stress is approximately constant and equal to the stress τ_o at the boundary. This stress is represented in the usual way by a friction velocity u_* $[= \sqrt{(\tau_o/\rho)}]$. Such a uniform stress region has been long recognized as being characterized by similarity of various single point Eulerian statistical functions of the turbulent velocity

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field and there are measurements in a circular pipe by Laufer (1955) supporting its extension to two point Eulerian functions such as velocity correlations. A recent paper by Lumley (1964) assumes such a similarity for correlation functions and a discussion of the subject can be found in Townsend's (1956) book on turbulent shear flows. Later, Townsend (1961) indicates reservations concerning the notion that the turbulence is in a universal state determined by u_* , but it seems to be effectively so for transfer mechanisms.

We propose to extend the similarity concept to two particle relative dispersion which is of course properly viewed as a Lagrangian problem, but first it is necessary to define the relationship between two particle relative dispersion and cloud relative dispersion for the inhomogeneous situation of a turbulent boundary layer. The lucid presentation of Batchelor (1952) for the homogeneous case is a natural starting point and the extension to a situation with homogeneity in horizontal planes only is so straight-forward that we suppress the detail here and present only the results. Let the positions of the particles at the instant of release be $\underset{\sim}{x}'(t_o)$ and $\underset{\sim}{x}''(t_o)$. By virtue of statistical homogeneity of the boundary layer in horizontal planes the statistics of the subsequent separation $\underset{\sim}{x}'(t) - \underset{\sim}{x}''(t)$, which we denote by $\underset{\sim}{y}(t)$, are dependent on the velocity field structure, $t - t_o$, $\underset{\sim}{y}(o)$ and a characteristic initial vertical coordinate which for convenience we take as the release height $Z(o)$ of either of the two particles.

Thus for a pair of particles in such a turbulent layer the relative dispersion tensor $\langle y_i(t) y_j(t) \rangle$ can be written, using Batchelor's (1952) notation,

$$\langle y_i(t) y_j(t) \rangle = \sigma_{ij}(t - t_o;\ Z(o),\ \underset{\sim}{y}(o)\ ).$$

Similarly we find that the cloud dispersion tensor $\Sigma_{ij}\,(t - t_o)$, again following Batchelor, is related to $\sigma_{ij}\,(t - t_o;\ Z(o),\ \underset{\sim}{y}(o)\ )$ in the following fashion:

$$\Sigma_{ij}(t - t_o) = V^{-2} \iint\ p(\underset{\sim}{x}',\ \underset{\sim}{y}',\ t_o)\ \sigma_{ij}(t - t_o;\ Z',\ \underset{\sim}{y}')\ d\underset{\sim}{x}'\ d\underset{\sim}{y}'$$

where V is the initial cloud volume and $P(\underset{\sim}{x}',\ \underset{\sim}{y}',\ t_o) = 1$ if both $\underset{\sim}{x}'$ and $\underset{\sim}{y}'$ are in the initial cloud or zero otherwise.

It is therefore sufficient, as in homogeneous turbulence, to study two particle relative dispersion.

Consequences of the similarity hypothesis: With the knowledge that Eulerian two point similarity has been observed in turbulent boundary layers of the type discussed above, we generalize the single point Lagrangian similarity concept to two points in the following way.

"The statistical properties of the relative displacement of two marked fluid particles at time t after simultaneous release near the ground depend only on u_*, t, the initial separation of the particles $\underset{\sim}{y}(o)$ and initial height of one of the particles $Z(o)$."

It has been pointed out by Ellison (1957) and others that the roughness height of the solid boundary determines only the horizontal velocity of the axes of reference without affecting the turbulence structure. It should have no influence on relative displacements.

An immediate conclusion of the similarity hypothesis is that the effective relative diffusivities take the form

$$\frac{1}{2}\frac{d}{dt}\sigma_{ij}(t;\ Z(o),\ \underset{\sim}{y}(o)) = u_*^2 t\, g_{ij}\left(\frac{Z(o)}{u_* t};\ \frac{\underset{\sim}{y}(o)}{u_* t}\right), \qquad (1)$$

where g_{ij} are universal functions.

The form presented as equation (1) gives rise to just two time scales (assuming the boundary layer is of infinite extent otherwise the time for particles to diffuse out of the layer would introduce a third time regime) $t_1 = \frac{Z(o)}{u_*}$, $t_2 = \frac{|\underset{\sim}{y}(o)|}{u_*}$.

Specific predictions are possible in only two cases $t << t_1$ or t_2 and $t >> t_1$ or t_2. For instantaneous cloud releases near the ground t_1 will often be of the order of t_2 and in such cases there will simply be two regimes, a small time and a large time, both of which are detailed below.

Small Time Behavior

For times much shorter than t_1 or t_2 it is well known from the work of Taylor (1921) and others that the diffusive action of turbulence is simply convection by the velocity field at the points of release.

We have

$$\langle y_i(t)y_j(t)\rangle = y_i(t_o)y_j(t_o) + \langle v_i(t_o)v_j(t_o)\rangle(t - t_o)^2 ,$$

where $v_i(t_o)$ is the difference velocity of the carrier fluid at the two points of release.

Eulerian two point similarity predicts

$$\langle u_i(\underset{\sim}{x})u_j(\underset{\sim}{x} + \underset{\sim}{y})\rangle = u_*^2 f_{ij}\left(\frac{\underset{\sim}{y}}{Z}\right) .$$

Whence we find

$$\frac{1}{2}\frac{d}{dt} \sigma_{ij}(t; Z(o), \underset{\sim}{y}(o)) = \frac{1}{2}\frac{d}{dt} \langle y_i(t)y_j(t)\rangle = u_*^2 t\left[2f_{ij}(o) - f_{ij}\left(\frac{\underset{\sim}{y}(o)}{Z(o)}\right) - f_{ji}\left(\frac{\underset{\sim}{y}(o)}{Z(o)}\right)\right] , \qquad \ldots.(2)$$

where $f_{ij}\left(\frac{\underset{\sim}{y}(o)}{Z(o)}\right)$ is the experimentally accessible Eulerian velocity correlation function.

Such a solution is consistent with the similarity hypothesis at small times when we demand, by kinematic arguments similar to the above, that $\frac{1}{2}\frac{d}{dt} \sigma_{ij}(t; Z(o), \underset{\sim}{y}(o))$ should be linear in t. That is, from (1), the appropriate small time predictions are

$$\frac{1}{2}\frac{d}{dt} \sigma_{ij}(t; Z(o), \underset{\sim}{y}(o)) = u_*^2 t g_{ij}\left(\frac{\underset{\sim}{y}(o)}{Z(o)}\right) . \qquad (3)$$

The result therefore is the usual linearly time dependent diffusivity but with directional sensitivity. Some measurements of $f_{11}\left(\frac{\underset{\sim}{y}(o)}{Z(o)}\right)$ in a circular pipe are reported by Laufer (1955).

Large Time Behavior

In the temporal regime, $t >> t_1$ or t_2, one expects loss of statistical dependence on the details of the position of the particles at release. That is, neither $\underset{\sim}{y}(o)$ nor $Z(o)$ is expected to enter significantly into the statistical description when the separation vector $|\underset{\sim}{y}(t)| >> |\underset{\sim}{y}(o)|$ and $Z(o)$. A similar kind of argument was used by Batchelor for relative diffusion in homogeneous turbulence except that an upper bound of t was necessary in his case so that $\underset{\sim}{y}(t)$ was still sensitive mainly to Fourier elements in the inertial subrange. Thus the equivalent regime was termed by him the intermediate time range. The existence of but two regimes in the boundary layer case is a consequence of the universal state of the turbulence so that u_* characterizes the entire structure and therefore the entire range of Fourier components.

In view of the lack of dependence on $y(o)$ and $Z(o)$ similarity theory in the large time regime predicts

$$\frac{1}{2} \frac{d}{dt} \sigma_{ij}(t) = a_{ij} u_*^2 t , \tag{4}$$

where a_{ij} are absolute constants.

The continued dependence of diffusivity on time in the large time limit is considerably different to the predictions associated with homogeneous turbulence where the diffusivity in the intermediate time stage is quadratic in t and in the large time asymptote is time independent, but as Lumley (1964) has shown the transfer mechanisms interpreted spectrally, are vastly more complicated than those of homogeneous turbulence and where equilibrium layers are in fact attained one cannot expect inertial subrange arguments to be applicable.

Equation (4) is really an asymptotic result and differs again from the classical homogeneous predictions by not yielding a constant diffusivity for large times unless the coefficients a_{ij} are zero. Since in the ideal equilibrium layer of infinite depth the turbulence structure grows with distance from the wall and since the individual particles have been shown by Batchelor (1964) to have a mean motion which is also outwards from the wall the arguments which lead to constant diffusivity in homogeneous flows seem not to be relevant. Saffman (1962) has reached the same conclusion in a more general context at least for the diffusivity in the horizontal direction.

Comparison with Experimental Data

Frenkiel and Katz (1956) report measurements of the increase in radius of a number of clouds released from within 100 meters of the ground and conclude that their data supports $\left|\overline{\underset{\sim}{y}^2}\right| \sim t^2$. This is precisely the prediction of the Lagrangian similarity theory for both small and large times. However the question of the existence of an adiabatic surface layer is not easily answered from their data and furthermore an analysis shows that the time of viewing is never more than twice $\left|\frac{\underset{\sim}{y}(o)}{u_*}\right|$. Thus the long time asymptote cannot be considered to be established. It is interesting however that a diffusivity linear in time seems to apply even around the time scale not associated with either small or large times.

Tank (1957) made dosage experiments with clouds released at 1.5 meters from the ground. Conversion of cloud growth rate to dosage involves assumptions about detailed concentration distributions in the cloud which are not implied by the similarity theory. However, Zimmerman (1965) has shown that in the conditions of high wind velocity the results are compatable with $\left|\overline{\underset{\sim}{y}^2}\right| \sim t^2$ and in this case it is fairly clear that the time scale extends well into the asymptotic time range. It is, however, not clear how nearly the atmospheric layer was an equilibrium layer or to what height it extended if it was. It is reasonable to expect in the absense of detailed measurement that dominance of wall friction over bouyancy effects is more likely at the highest wind speeds. In this sense then Tank's data is compatable with the conclusions of this analysis. Zimmerman ascribes the $\left|\overline{\underset{\sim}{y}^2}\right| \sim t^2$ behavior to a result due to Tchen (1954, 1959), who argued from Heisenberg's (1948) approximation to the same result in a flow where the mean shear is sufficiently large. The two explanations are not inconsistent although Tchen's should be merely a result for intermediate times and not valid for times asymptotically large.

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