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A background data transmission scheme for cellular communications with fading

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ABSTRACT

Data transmission can be integrated into an existing voice only system by sending data as an overlay, transparent to existing services and utilizing resources that are temporarily idle. Aloha may be chosen as a media access scheme, owing to its protocol's simplicity. However, the traditional Aloha suffers from poor throughput-delay characteristics.

In the following we study methods intended to improve that performance by employing diversity Aloha. In addition we exploit capture effects. This combination is shown to be an effective countermeasure to combat deleterious effects of fading and background noise as well as self-interference. It successfully improves the throughput-delay characteristics in the entire range of offered data traffic.

A mathematical analysis is presented and performance results are calculated. These show the improvements attainable.

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1. INTRODUCTION

The goal of voice/data integration is to share network resources efficiently between classes of traffic, while preserving their unique performance requirements. Providing dedicated or switched circuit connections to data is quite inefficient for several reasons. Primarily, the need to assure low session blocking probability results in a significant portion of the system capacity being idle much of the time. Secondly, due to the bursty nature of most data transmissions, much of an individual channel's bandwidth is underutilized during a session. Finally, many data applications may not need a dedicated session to be established and therefore do not require complex signaling overhead necessary for session setup, maintenance and teardown.

There are several general ways to combine voice and data communications on shared facilities. One is to partition system resources for voice and data. For systems that are primarily intended for voice, this has a negative effect on the voice traffic, which must function with less resources.

Second approach is to provide the complete digital integration of voice and data allowing for sharing of communication resources by different classes of communication traffic. These methods, however, present a significant technical challenge in the areas of signaling and user synchronization. Moreover, they require system redesign from the ground up.

A third approach allows not only for relatively easy integration into the existing networks but also does not affect the performance of the voice connections. It takes advantage of the fact that the circuit switched cellular systems are designed to achieve low probability of new call blocking as well as low probability of hand-off failure. As a result, on average a significant number of channels are idle. This unused capacity can be utilized to transmit data in gaps between voice calls. This approach is taken by commercially available CDPD data networks.

In the following we analyze a system in which data is overlaid on circuit switched voice. The data subsystem employs a diversity Aloha scheme [1]. Whenever a data user generates a packet, he will transmit k copies of it. The packet is successfully transmitted if at least one of its copies is correctly received by the base station. This approach, while retaining the natural simplicity of Aloha protocol, improves the system's delay performance. We also show that exploiting the capture effect can benefit the system's delay-throughput characteristics in a diversity environment. This effect was previously studied in non-diversity setting[5]. Finally, diversity is shown to be an effective measure mitigating the effects of background noise.

2. MODEL DESCRIPTION

We consider a cellular system in which every cell (sector) is allocated C channels. We use the term channel to describe the smallest resource used to handle a voice transmission. This communication capacity is shared by two classes of traffic: voice and data. Voice services are provided in a circuit switched mode and retain absolute priority in channel assignment. This technique applies to analog (AMPS type) systems as well as to 2nd generation TDMA/FDMA configurations. In this approach, voice has exclusive use of a channel and no attempt is made to send data between talkspurts. Instead, the data subsystem operates in the background, sending data on channels that are temporarily unused by voice transmissions. No data session is established and data is sent on a packet by packet basis. Data transmissions use diversity Aloha to improve the system's packet loss and delay characteristics. We assume that copies of a packet sent by data sources are of duration τ. This approach is applicable to different physical system configurations. In an analog system, data would be sent on channels (frequencies) unused by voice, employing unslotted diversity ALOHA. In contrast, in TDMA systems data would utilize time slots temporarily not used by voice. Since the framing structure is already in place, slotted ALOHA would be the preferred mode of data transmission. For brevity and convenience we focus discussion on FDMA/TDMA type systems with slotted diversity Aloha data transmissions on idle channels. Applicability of both terminology and mathematical formulation to unslotted Aloha system is straightforward.

New voice call attempts originate from an infinite population of users. Voice calls arrive in a Poisson stream at an average rate λ, (calls/sec). A new voice call is admitted for a session whenever a number of active voice connections is less than C. Blocked calls are cleared from the system. The number of active voice calls is denoted by C_v. A newly admitted voice call will start using the channel assigned to it at the end of the current data packet transmission (if any) on that channel. The channel holding time is an exponentially distributed r.v. with mean $1/\mu$. The voice subsystem is modeled as an M/M/C/C queue.

A population of data sources is assumed to be infinite and generating packets in a Poisson stream of intensity Λ. Data sources are not admitted for a session. Instead, they transmit at will (constrained only by the channel access protocol), whenever data becomes available. Moreover, packets that were not successfully transmitted are not cleared from the system, but retransmitted. Therefore, the total packet arrival stream consists of both new and retransmitted packets. Specifically, the data user population generates packets at the rate $\Lambda=\Lambda_N + \Lambda_R$, where Λ_N denotes the average rate at which new packets are generated and Λ_R is the average rate of retransmissions. Hence, the total average traffic generated by the user population is $G= \Lambda\tau$ packets/sec. A data source that has a packet ready to be transmitted, will sense the downstream signaling channel (or scan all the channels) and find the number of channels available for data transmission. We assume that the channel sensing time is negligible. The observed number of channels available for data transmissions is denoted $C_D=C-C_V$. Similarly, the system makes an estimate of the traffic intensity G. Based on the two parameters, the data source will choose (or be informed of) the number of copies of a packet to be sent. We define $k(C_D,G)$ ($k(C_D,G)=0,1,\ldots C_D$) as the number of copies to be sent when the observed number of available channels is C_D and the average offered data traffic equals G. In the following for the sake of notational simplicity, we elect not to emphasize the dependence on G and denote the number by $k(C_D)$. Then, the data source sends that number of identical copies of the packet with probability $p_T(C_D)$ and defers the transmission with probability $(1-p_T(C_D))$. The block of $k(C_D,G)$ copies of a packet is called a transmission attempt. Copies can be lost due to collisions with transmission attempts of other users.

Collisions occur if more than one user transmit a packet copy in the same slot. In addition, copies may not be received by the base station due to propagation factors like fading. *A packet succeeds if any copy succeeds.*

A frame is defined as a collection of time slots, determined by the underlying TDMA scheme that supports isochronous communications. A transmission attempt is confined to a single frame. Throughput, S, is defined as a number of successful packets per frame. Note that if several copies of a given packet succeed in a frame, this amounts only to a single packet success.

3. INTEGRATED DATA/VOICE SCHEME IN ABSENCE OF FADING.

First, we consider an integrated voice/data scheme operating on the principles outlined above and utilizing a communication channel with no fading. Collisions among data packet copies are the only cause of packet and copy failures.

The primary measure of performance is the probability of a successful packet transmission in a single transmission attempt. Since the data subsystem is operating as an overlay on the voice subsystem, the number of channels available for data is determined by the activity in the voice subsystem. The probability of successful transmission can be determined by first calculating the conditional probability of packet success given the number of data channels C_D and the number of competing users and then calculating the marginal probability by averaging over the randomness of the conditions.

When a particular user U is ready in a certain frame, then the average probability of packet success in a transmission attempt can be determined as

$$P_S = \sum_{r=0}^{\infty} P_r \cdot \sum_{C_D=0}^{C} P_{s|r,C_D} \cdot P_{C_D} \tag{1}$$

In (1) $P_{s|r,C_D}$ denotes the probability of packet success for a transmission attempt in a frame when r other users are transmitting and there are C_D channels available for data. This probability is the same as the probability that at least one of the user U's $k(C_D)$ slot choices was not chosen by other users under the same conditions.

$$P_{s|r,C_D} = \sum_{n=1}^{k(C_D)} (-1)^{n+1} \binom{k(C_D)}{n} \left[\frac{\binom{C_D - n}{k(C_D)}}{\binom{C_D}{k(C_D)}} \right]^r \tag{2}$$

The above equation has been obtained by applying the inclusion/exclusion principle [4]. The factor in square parentheses is the probability that a particular set of n channels out of the $k(C_D)$ channels chosen by user U, was not chosen by any other user. The numerator is the number of ways that an interfering user may choose his $k(C_D)$

channels, without choosing any from a particular n-channel subset of channel choices made by the user U. The denominator is simply the number of ways one can choose $k(C_D)$ channels from C_D channels.

Since we assumed that the packets are generated according to a Poisson point process with rate Λ, the probability that r users have a packet to transmit is

$$P_r = \Pr(\text{r users have a packet to transmit}) = \frac{(\Lambda \cdot \tau)^r}{r!} \cdot e^{-\Lambda\tau} \tag{3}$$

However, due to transmission deferrals, the number of users actually attempting transmission will be smaller and is found to be

$$P_r = \Pr(\text{r users are transmitting in a slot} \mid t' \text{ users have a packet}) =$$
$$= \sum_{t'=r}^{\infty} \binom{t'}{r} \cdot p_T^{\,r} (1-p_T)^{t'-r} \frac{(\Lambda\tau)^{t'}}{t'!} \cdot e^{-\Lambda\tau} \overset{t=t'-r}{=}$$
$$p_T^{\,r} \cdot \frac{(\Lambda\tau)^r}{r!} \cdot e^{-\Lambda\tau} \sum_{t=0}^{\infty} (1-p_T)^t \cdot \frac{(\Lambda\tau)^t}{t!} = \frac{(p_T \Lambda\tau)^r}{r!} \cdot e^{-p_T \Lambda\tau} \tag{4}$$

In Appendix A we find the distribution of channels that are unused by voice calls. The probability that there are exactly C_D channels available for data transmission is

$$P_{C_D}(C_D) = P_{C_D}(0) \cdot \frac{C!}{(C-C_D)!} \cdot \left(\frac{\mu}{\lambda}\right)^{C_D} \tag{5}$$

Combining (2) and (4) we obtain $P_{s|C_D}$, the probability of successful packet transmission in a frame with C_D channels available for data transmission

$$P_{s|C_D} = \sum_{r=0}^{\infty} P_r \cdot P_{s|r,C_D} = \sum_{r=0}^{\infty} \frac{(p_T\Lambda\tau)^r}{r!} \cdot e^{-p_T\Lambda\tau} \cdot \sum_{n=1}^{k(C_D)} (-1)^{n+1} \binom{k(C_D)}{n} \left[\frac{\binom{C_D - n}{k(C_D)}}{\binom{C_D}{k(C_D)}} \right]^r =$$

$$= \sum_{n=1}^{k(C_D)} (-1)^{n+1} \binom{k(C_D)}{n} \cdot e^{-p_T\Lambda\tau} \sum_{r=0}^{\infty} \frac{1}{r!} \left[p_T \Lambda\tau \frac{\binom{C_D - n}{k(C_D)}}{\binom{C_D}{k(C_D)}} \right]^r =$$

$$= \sum_{n=1}^{k(C_D)} (-1)^{n+1} \binom{k(C_D)}{n} \cdot \exp\left[p_T \Lambda \tau \left(\frac{\binom{C_D - n}{k(C_D)}}{\binom{C_D}{k(C_D)}} - 1 \right) \right] \tag{6}$$

In order to maximize the packet success probability P_S, we will maximize the $P_{S|C_D}$ for each C_D and for every value of Λ. To this end $P_{S|C_D}$ is calculated for every value of $k(C_D,G) = 0 \ldots C_D$ as a function of G. Subsequently, the optimum $k(C_D,G)$ is found as a function of data traffic intensity G for each $C_D = 0 \ldots C$. Since each $P_{S|C_D}$ can be maximized independently of the others, this will yield the global maximum of P_S.

4. FADING MODELS.

Fading will have a detrimental effect on a delay/throughput performance of the data subsystem. In order to investigate the performance of the joint voice/diversity ALOHA system in presence of fading, we propose two simple fading channel models. For a different approach, concentrating on contiguous transmission of copies in a data only system see [3].

The first fading model, which we call *correlated fading*, is suitable to describe the effects of slow (as compared to other characteristic time constants in the system), frequency non-selective fading. This model is appropriate when fade duration is on the order of the time between retransmission attempts and when all copies in a transmission attempt are identically affected by the fading. We assume that fading impacts terminals independently. However, if fading does occur, all copies of the same packet in a given transmission attempt are lost. We circumvent the detailed modeling of a fading channel by using a probability of fading, p_{fdm}. This is the probability that the signal power drops below the level at which the receiver can successfully receive a packet. This probability is treated as a given parameter. Moreover, we assume that none of the packet copies in a faded transmission attempt can cause a collision with other users' packets. It should be emphesised that either or both types of fading can be present in the system, as factors such as rate of signal power variations and frequency selectivity characteristics do depend on mobile speed and the coherence bandwidth of the channel. Both of these factors will vary widly in diverse coverage areas.

The second model, termed *uncorrelated fading*, is applicable to fast and frequency selective fading. Individual copies of a packet are independently affected by fading, each with probability p_{fds}. Again, faded packet copies do not cause collisions.

5. CORRELATED FADING.

In order to describe the effects of correlated fading we proceed similarly to the development presented in equations (1) - (6). Although we start the analysis with an equation identical to (1), the definitions of quantities that appear are modified to include the effects of fading. One effect of correlated fading is that a smaller number of interfering users can cause collisions on the channel. Mathematically, it corresponds to thinning of the Poisson packet arrival stream. Therefore P_r is now defined as the probability that r users' transmission attempts did not fade.

$$\Pr(\text{r stations did not fade}) =$$

$$\sum_{m=r}^{\infty} \Pr(\text{r stations did not fade} \mid \text{m stations attempted transmission}) \cdot$$

$$\Pr(\text{m stations attempted transmission}) =$$

$$= \sum_{m=r}^{\infty} \binom{m}{r} \left(p_{fdm}\right)^{m-r} \left(1 - p_{fdm}\right)^r \cdot \frac{\left(p_T \Lambda \tau\right)^m}{m!} \cdot e^{-p_T \Lambda \tau} \overset{m'=m-r}{=}$$

$$= \frac{\left[\left(1 - p_{fdm}\right) p_T \Lambda \tau\right]^r}{r!} \cdot e^{-\left(1 - p_{fdm}\right) p_T \Lambda \tau} \tag{7}$$

Moreover, in the presence of fading a given user's attempt might be unsuccessful because of collisions with other users' transmissions or it may itself fade. Therefore, the conditional probability of success in a transmission attempt can now be calculated as

$$P_{S|r,C_D} = \left(1 - p_{fdm}\right) \sum_{n=1}^{k(C_D)} (-1)^{n+1} \binom{k(C_D)}{n} \left[\frac{\binom{C_D - n}{k(C_D)}}{\binom{C_D}{k(C_D)}} \right]^r \tag{8}$$

Combining (7) and (8) the conditional probability of successful packet transmission is found to be

$$P_{s|C_D} = \sum_{r=0}^{\infty} P_{s|r,C_D} \frac{\left[\left(1 - p_{fdm}\right) p_T \Lambda \tau\right]^r}{r!} \cdot e^{-\left(1 - p_{fdm}\right) p_T \Lambda \tau} =$$

$$= \sum_{r=0}^{\infty} \left(1 - p_{fdm}\right) \sum_{n=1}^{k(C_D)} (-1)^{n+1} \binom{k(C_D)}{n} \left[\frac{\binom{C_D - n}{k(C_D)}}{\binom{C_D}{k(C_D)}} \right]^r \cdot \frac{\left[\left(1 - p_{fdm}\right) p_T \Lambda \tau\right]^r}{r!} \cdot e^{-\left(1 - p_{fdm}\right) p_T \Lambda \tau} \tag{9}$$

The above equation can be simplified to

$$P_{s|C_D} = \left(1-p_{fdm}\right)\sum_{n=1}^{k(C_D)}(-1)^{n+1}\cdot\binom{k(C_D)}{n}\cdot\exp\left[\Lambda\tau\ p_T\left(1-p_{fdm}\right)\left(\left[\frac{\binom{C_D-n}{k(C_D)}}{\binom{C_D}{k(C_D)}}\right]-1\right)\right] \quad (10)$$

In comparison with (6) the equation shows that the probability of success in a transmission attempt is lower due to fading of a given terminal's transmissions, but the effect is mitigated by a "thinner" stream of (non-faded) packet copies from interferers.

6. UNCORRELATED FADING.

To consider uncorrelated fading we define an interference vector $\underline{\mathbf{J}} = \left(j_0, j_1, \ldots, j_{k(C_D)}\right)$. The component, j_i denotes the number of data users for which i copies in a transmission attempt did not fade. The probability that transmission attempts of m users in a frame result in interference vector $\underline{\mathbf{J}}$ can be determined by first calculating the probability that a user's transmission attempt results in i unfaded copies (it is the familiar Bernoulli trials formula in square parentheses) and applying it to all m users in a particular interference vector $\underline{\mathbf{J}}$.

$$P_{\underline{\mathbf{J}}|m} = \binom{m}{j_0 j_1 \cdots j_{k(C_D)}}\prod_{i=0}^{k(C_D)}\left[\binom{k(C_D)}{k(C_D)-i}\cdot\left(p_{fds}\right)^{k(C_D)-i}\left(1-p_{fds}\right)^{i}\right]^{j_i} \quad (11)$$

Conditional success probability can be found by adapting equation (2) to the situation in which only a fraction (specified by the interference vector) of copies sent by interfering users reaches the gateway and can cause collisions. In addition, individual fading of copies sent by a particular user, U, is taken into account.

$$P_{s|C_D,\underline{\mathbf{J}}} = \sum_{n=1}^{k(C_D)}(-1)^{n+1}\cdot\binom{k(C_D)}{n}\cdot\left(1-p_{fds}\right)^{n}\cdot\prod_{t=1}^{k(C_D)}\left[\frac{\binom{C_D-n}{t}}{\binom{C_D}{t}}\right]^{j_t} \quad (12)$$

Using (11) in (12) to remove conditioning on $\underline{\mathbf{J}}$, we obtain after some algebraic manipulation (see Appendix B for details)

$$P_{s|C_D} = \sum_{n=1}^{k(C_D)}(-1)^{n+1}\cdot\binom{k(C_D)}{n}\cdot\left(1-p_{fds}\right)^{n}\cdot\exp\left[(A-1)\Lambda\tau\right] \quad (13)$$

where

$$A = p_T \sum_{i=0}^{k(C_D)} \frac{\binom{C_D - n}{i}}{\binom{C_D}{i}} \binom{k(C_D)}{k(C_D)-i} \cdot \left(p_{fds}\right)^{k(C_D)-i} \left(1-p_{fds}\right)^i \tag{14}$$

7. PACKET CAPTURE AND COCHANNEL INTERFERENCE.

It has been recognized that exploiting packet capture in fading environments can be beneficial to the efficiency of the Aloha scheme. This is due to the fact that when received power levels are not equal (at least short term in the capture time), the mutual destruction of colliding packets is no longer assured. In [5] the analysis was carried out and the benefits of fading to throughput in a non-diversity Aloha environment were noted. In the following we show that similar beneficial effects can be observed in a diversity Aloha environment.

Let us concentrate on a single transmission attempt. Let there be r other users attempting transmission in the same frame and C_D slots available for data transmission. All users transmit $k(C_D)$ copies of a packet. Furthermore, let us focus our attention on a particular user U. Without any loss of generality, we can number $k(C_D)$ slot choices made by user U as $1,2,\dots,k(C_D)$. Moreover, let the vector $\underline{\mathbf{n}}$ denote the number of interfering users that chose (at least some) slots that were also chosen by user U. Specifically, if $\underline{\mathbf{n}}=(n_1,n_2,\dots,n_k)$, then n_i $i=1\dots k(C_D)$ denotes the number of users that chose the i-th slot chosen by user U i.e. there are n_i+1 simultaneous transmissions in slot i. If the power of user U's packet sufficiently exceeds the combined power of interfering packets in a slot, that packet might be successfully received (captured) despite the existence of other transmissions in that slot. The probability of capture in presence of n other transmissions in a slot is denoted $p_c(n)$. Then, the probability of packet success in a transmission attempt, when r other users transmit and C_D slots are available for data transmission can be expressed as

$$P_{s|r,C_D} = \sum_{\underline{\mathbf{n}}} p\left(n_1, n_2, \dots, n_{k(C_D)} \middle| r\right) \cdot \left(1 - \prod_{i=1}^{k(C_D)} \left(1 - p_c(n_i)\right)\right) \tag{15}$$

In order to determine the probability of capture in a slot and in presence of n interferers we assume that the system under consideration employs power control. As a result all packets are received with equal mean power, $\overline{R}$, and Rayleigh distributed envelope. Then the p.d.f. of the received power of the desired signal is an exponentially distributed R.V., R_d,

$$f_{R_d}(\xi) = \frac{1}{\overline{R}} \exp\left(-\frac{\xi}{\overline{R}}\right) \tag{16}$$

Depending on system characteristics and receiver design, the phasors of the received signals can add either coherently or incoherently. Reference [5] discusses the conditions under which one of the extreme cases of interference addition is applicable. Let us just emphasize here that the notion of coherency in this context relates to quasi-stability of all received signals' phases during the packet capture time. Assuming the coherent addition of phasors is applicable, the envelope of the sum of the n interfering signals will also be a Rayleigh R.V. and consequently the p.d.f. of the power of the interfering signals can be found again as an exponentially distributed R.V. with mean $n\bar{R}$

$$f_{R_i}(\eta) = \frac{1}{n\bar{R}} \exp\left(-\frac{\eta}{n\bar{R}}\right) \tag{17}$$

A packet can be captured successfully if the packet's power sufficiently exceeds background noise as well as the combined power of the interfering packets, i.e. $R_d > B\bar{R}$ and $R_d > zR_i$, where z denotes the capture ratio and B is based on background noise level. Then, the probability of packet capture in a slot in the presence of n interferers, denoted $p_c(n)$, can be determined as ([7])

$$p_c(n) = \Pr\left(R_d > B \cdot \bar{R} \text{ and } R_d > z \cdot R_i\right) =$$
$$= \int_{B\bar{R}}^{\infty} f_{R_d}(\xi) \int_0^{\xi/z} f_{R_i}(\eta)\, d\eta\, d\xi = \exp(-B)\left[1 - \frac{\exp\left(-B/_{n \cdot z}\right)}{1 + 1/_{n \cdot z}}\right] \tag{18}$$

If we limit our attention to the case for which background noise is ignored (by setting B=0), then we obtain (as in [5])

$$p_c(n) = (1 + n \cdot z)^{-1} \tag{19}$$

and the conditional probability of packet success in that case is shown to be

$$P_{slr,C_D} = \sum_{\underline{n}} p(n_1, n_2, \ldots, n_k | r) \cdot \left(1 - \prod_{i=1}^{k} \frac{n_i \cdot z}{1 + n_i \cdot z}\right) \tag{20}$$

Since the interference generated by all cochannel interferers behaves in a noiselike manner, it can be modeled as background noise. To that end an average (in respect to user's position) interference level originated by a single user is determined. Then the average (in respect to varying voice and data activities in cochannel cells) interference level from all cochannel cells is calculated. It, of course, depends on the geometry of the problem, in particular on the reuse distance. Finally the constant B can be calculated, such that it assures proper S/I ratio. Using the average value of the interference carries some risks for the reliability of the analysis. One can easily picture a situation, where all cochannel interferers are located as close as possible to the base station of interest, generating interference levels close to the worst case scenario (which incidentally is much simpler mathematically). However, since the number of interferers involved is relatively large, the typical behavior of the system will produce interference levels much closer to the average than to the worst case. The method of determining the average value of cochannel interference is presented in Appendix D. Then, constant B is determined for the nominal operating point of (heavily loaded) cochannel cells of 1% blocking for voice calls and 5% packet failure probability for the data packets. This procedure in effect establishes a bound on system perfomance which, although it does not constitute a true worst case scenario, is very unlikely to be violated while the system remains in the range of satisfatory performance. At the same time we avoid the mathematical complications involved in detailed modeling of cochannel interference and its statistics.

The number of interferers in each of user U's slot choices is the last variable to be determined so the calculations desribed in (15) can be performed. In its most general form, the determination of the probability distribution of that number presents a difficult combinatorial task. However, it can be determined relatively easily for some of the most interesting cases, when $k(C_D)=1,2,3$.

The simple case of non-diversity Aloha (k=1) was presented in [5] and in current notation it can be stated as

$$P_{s|C_D} = 1 - \sum_{n=0}^{\infty} \frac{\left(\Lambda\tau / C_D\right)^n}{n!} \exp\left(-\Lambda\tau / C_D\right) \cdot \left(1 - p_c(n)\right) \tag{21}$$

The case when two copies of a packet are sent (k=2) is developed in Appendix C and the probability of packet success is shown to be

$$\begin{aligned} P_{s|C_D} = 1 - \exp\left(\Lambda\tau\left(\frac{(C_D-2)(C_D-3)}{C_D(C_D-1)} - 1\right)\right) \cdot \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\left[\frac{2\Lambda\tau(C_D-2)}{(C_D-1)C_D}\right]^{n_1+n_2} \\ \sum_{j=0}^{\min(n_1,n_2)} \frac{1}{j!(n_1-j)!(n_1-j)!}\left[\frac{2\Lambda\tau(C_D-2)^2}{(C_D-1)C_D}\right]^{-j} \cdot \left(1-p_c(n_1)\right)\cdot\left(1-p_c(n_2)\right) \end{aligned} \tag{22}$$

The case k=3 can easily be obtained by substituting results of Appendix C to (15).

8. SYSTEM PERFORMANCE.

The importance of alternative measures of system performance depends on the characteristics of the data sources. For data that requires time constrained delivery but is tolerant of old packets being simply dropped and not retransmitted, probability of packet failure $P_F = 1 - P_S$ would be an important measure of the packet dropping probability. In contrast, for data requiring an absolutely reliable delivery, delay incurred in transmitting a packet is an appropriate measure of performance.

More formally, let us define the activity factor, R_a as the average number of transmission attempts needed to successfully transmit a packet. The activity factor can be found as the ratio of offered data traffic to system throughput.

$$R_a = G / S = 1 / P_s \tag{23}$$

A true measure of packet delay will also include the delay incurred by the user by deferring a transmission attempt. Then, the average number of frames between the time a packet originates and the time it is successfully received by the wireless gateway can be calculated as (assuming that p_T is a constant parameter, independent of C_D and G)

$$F = 1 / \left(P_S \cdot (1 - p_T)\right) = R_a / (1 - p_T) \tag{24}$$

9. NUMERICAL EXAMPLE AND DISCUSSION OF RESULTS.

In order to quantify the performance of an integrated voice/data system with diversity Aloha, we investigated the following example for a system with C=48 channels.

We considered four cases of channel fading characteristics:

1. no fading
2. correlated fading with fading probability p_{fdm}=0.1
3. uncorrelated fading with fading probability p_{fds}=0.1
4. uncorrelated fading with fading probability p_{fds}=0.3.

The system's performance was investigated at two different levels of voice traffic corresponding to 0.1% blocking and 1% blocking.

Figures1 through 4 show the performance in the absence of fading. Specifically, fig.1 displays the optimum number of copies of a packet to be transmitted in order to maximize the packet success probability. Although more than two copies is optimum, the improvement obtained by using more than two copies is not very significant. Therefore in our comparison of the traditional, single-copy Aloha and the optimal scheme, we have also included a third variant, in which users transmit at most two packets (i.e. two copies are transmitted at any time the optimum is two or more copies). As can be seen from the performance curves, such a scheme performs almost as well as the optimum scheme with much less complexity. Overall, in the absence of fading, diversity Aloha allows for an 3-4 fold improvement in packet failure probability, which in turn corresponds to an approximate 10% reduction in the average number of transmission attempts needed to successfully transmit a packet. The benefits of diversity diminish in heavy traffic, as in this range the optimum number of copies is one.

Figures 5 through 8 show the deleterious effects of correlated fading on system performance. Notice also the minuscule improvement offered by the diversity scheme in this case. However, in many instances, this type of fading can be aleviated by power control.

Independent fading also has a destructive effect on system performance (figs.9-16), but employing diversity Aloha significantly mitigates the impact. Note the huge (over an order of magnitude at low data traffic) decrease in packet failure probability and a corresponding 20% improvement in activity factor. This is coupled with an increase in throughput at low and moderate data traffic.

Fig.17 shows the optimum number of packet copies in different fading conditions. Correlated fading does not increase the optimal number of packet copies in comparison to a non-fading environment but rather extends the range of offered traffic where this number of copies is optimum. This is a direct result of "thinning" of the packet arrival stream. In contrast. uncorrelated fading calls for a larger number of copies to compensate for fading of individual packet copies.

Fig.18 contains an interesting comparison of two systems. One is equipped with 48 channels, the other with 16 channels. They are otherwise identical and operate under the same conditions. The figure shows a plot of normalized throughput to normalized traffic (normalization is in respect to the total number of channels). The higher trunking efficiency of the voice connections in the 48 channel system results in its achieving lower data throughput than the 16 channel system, at the same level of voice call blocking. At first this may seem counterintuitive, but it is in fact easily understood. Since the 48 channel system has higher voice trunking efficiency, generally there are (for a given voice call blocking probability) fewer idle channels available for data communications. The result is higher data throughput (for given data demand) in the system with fewer channels.

The performance of the system with capture capability compares favorably to systems without capture. Again, we investigate the system with C=48 channels at two different voice traffic levels, corresponding to 0.1% and 1% blocking. Figs.19-24 illustrate the performance gains due to both capture and diversity. Again, improvements attributable to diversity are the most pronounced when system with no diversity and system transmitting at most 2 copies are compared. Sending more copies of a packet produces only minute improvements, which are not shown in the figures. In low and moderate traffic most of the performance gains are clearly due to diversity, since in this range the collisions are rare and the opportunities to exploit capture are few. In heavy traffic, however, the opposite is true and impressive performance gains are realized by exploiting packet capture. At the same time, the diversity scheme adapts to very heavy traffic by sending only a single copy of a packet and offers virtually the same performance as non-diversity Aloha. It is worth noting that in heavy voice traffic (1% blocking) both the performance and performance gains due to capture and diversity are greatly diminished as we observe the effects of data traffic being "pushed out" of the system by the higher priority voice traffic.

Fig. 25 shows the effects of background noise on probability of packet failure. At the high offered data traffic, the noise has a small effect as collisions are the major impediment to a successful packet transmission. The presence of noise lowers the maximum attainable througput for a given failure probability. At the low offered traffic we observe significant benefits to employing the diversity scheme. It can be seen to be very beneficial in combating background noise and interference in addition to already mentioned benefits in fighting contention and fading. These improvements approach an order of magnitude when signal to interference ratio, S/I, of 10 dB is to be maintained.

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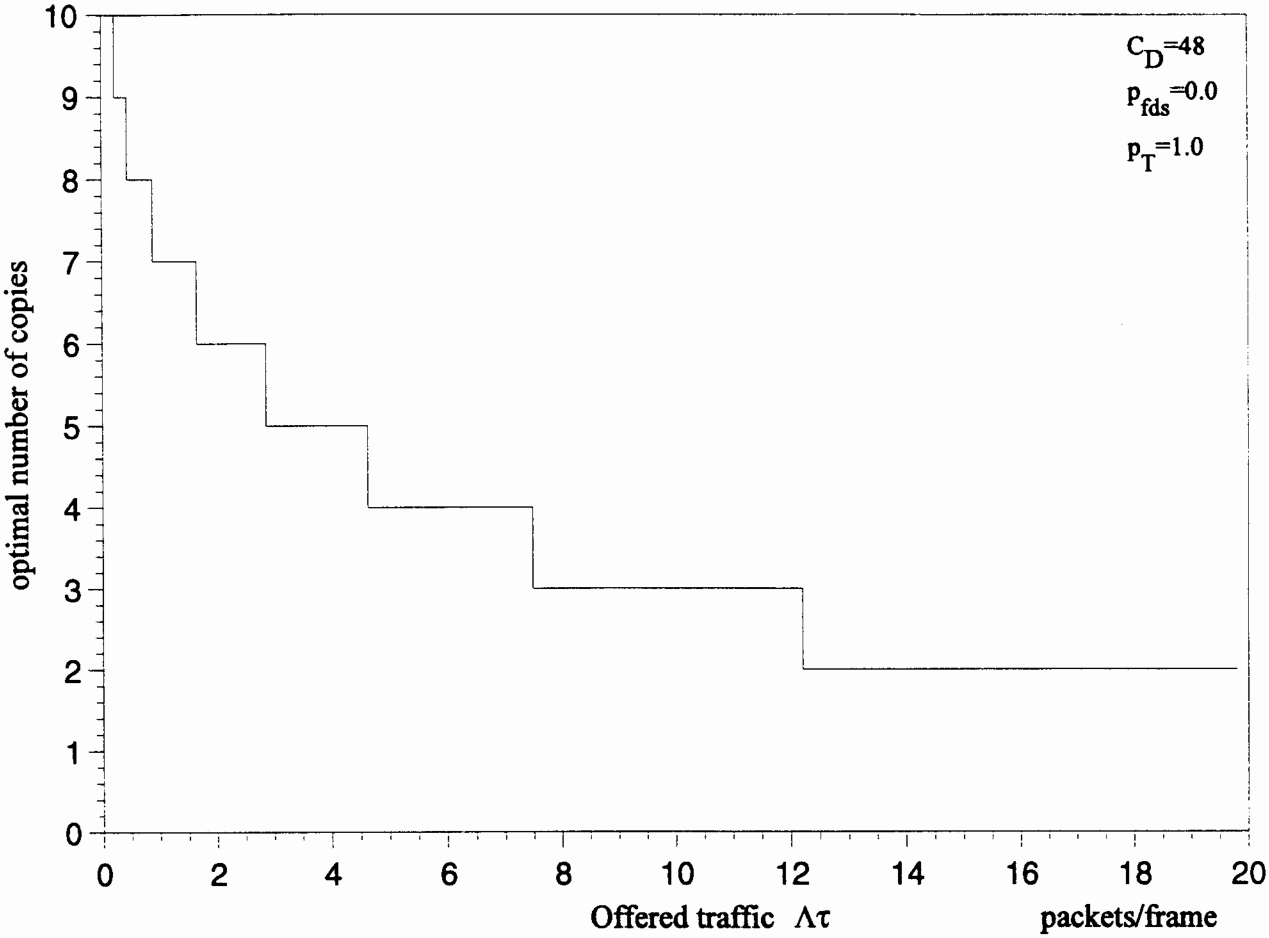

Fig.1 Optimal number of copies of a packet at different levels of data traffic and in absence of fading.

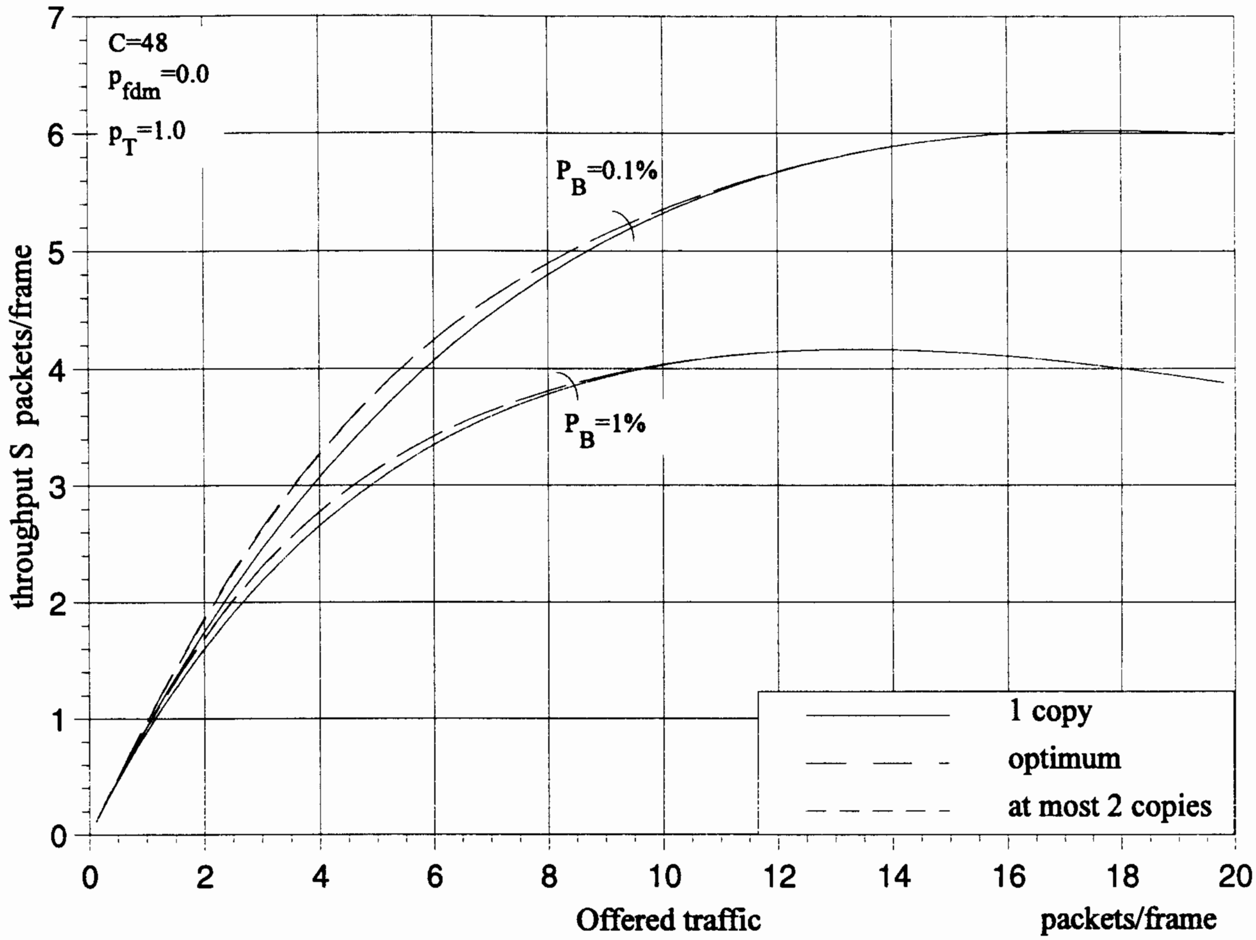

Fig.2 Data throughput for an integrated voice-data system in absence of fading at two different levels of voice traffic and for different diversity startegies.
C=48 p_{fdm}=0.0 p_T=1.0

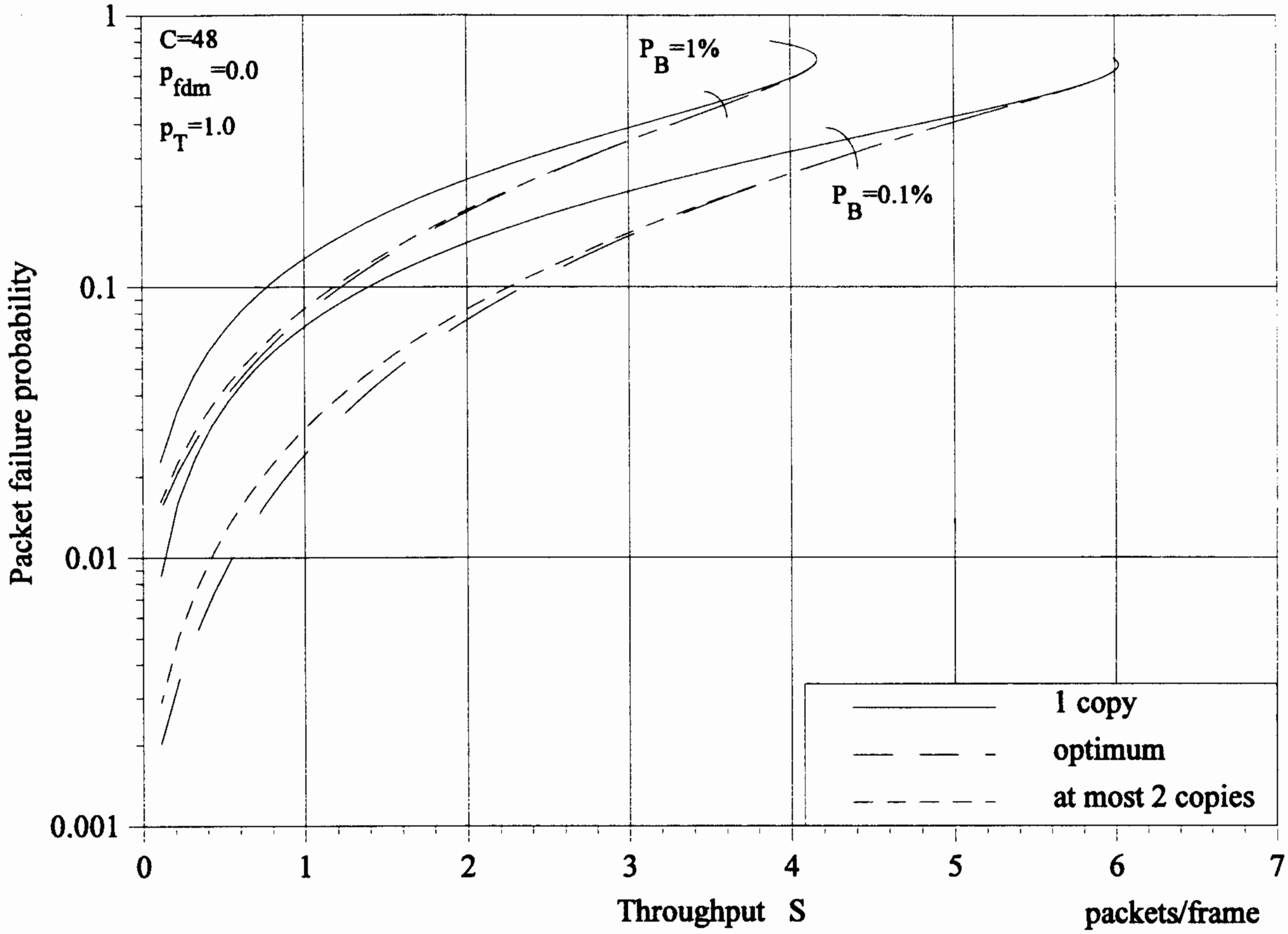

Fig.3 Packet failure probability for an integrated system in absence of fading at two different levels of voice traffic and for different diversity strategies.

C=48 p_{fdm}=0.0 p_T=1.0

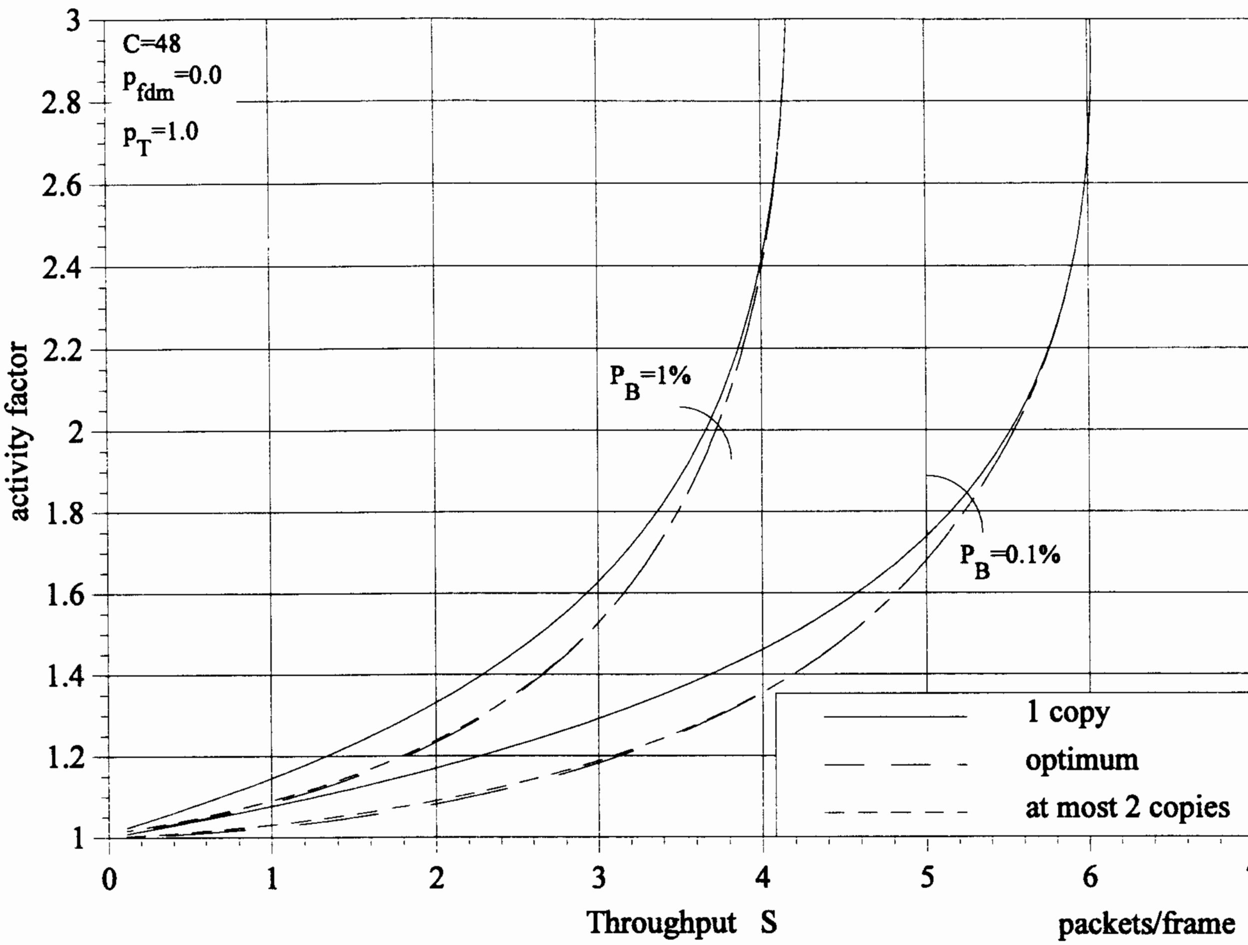

Fig.4 Activity factor for an integrated system in absence of fading at two different levels of voice traffic and for different diversity strategies.

C=48 p_{fdm}=0.0 p_T=1.0

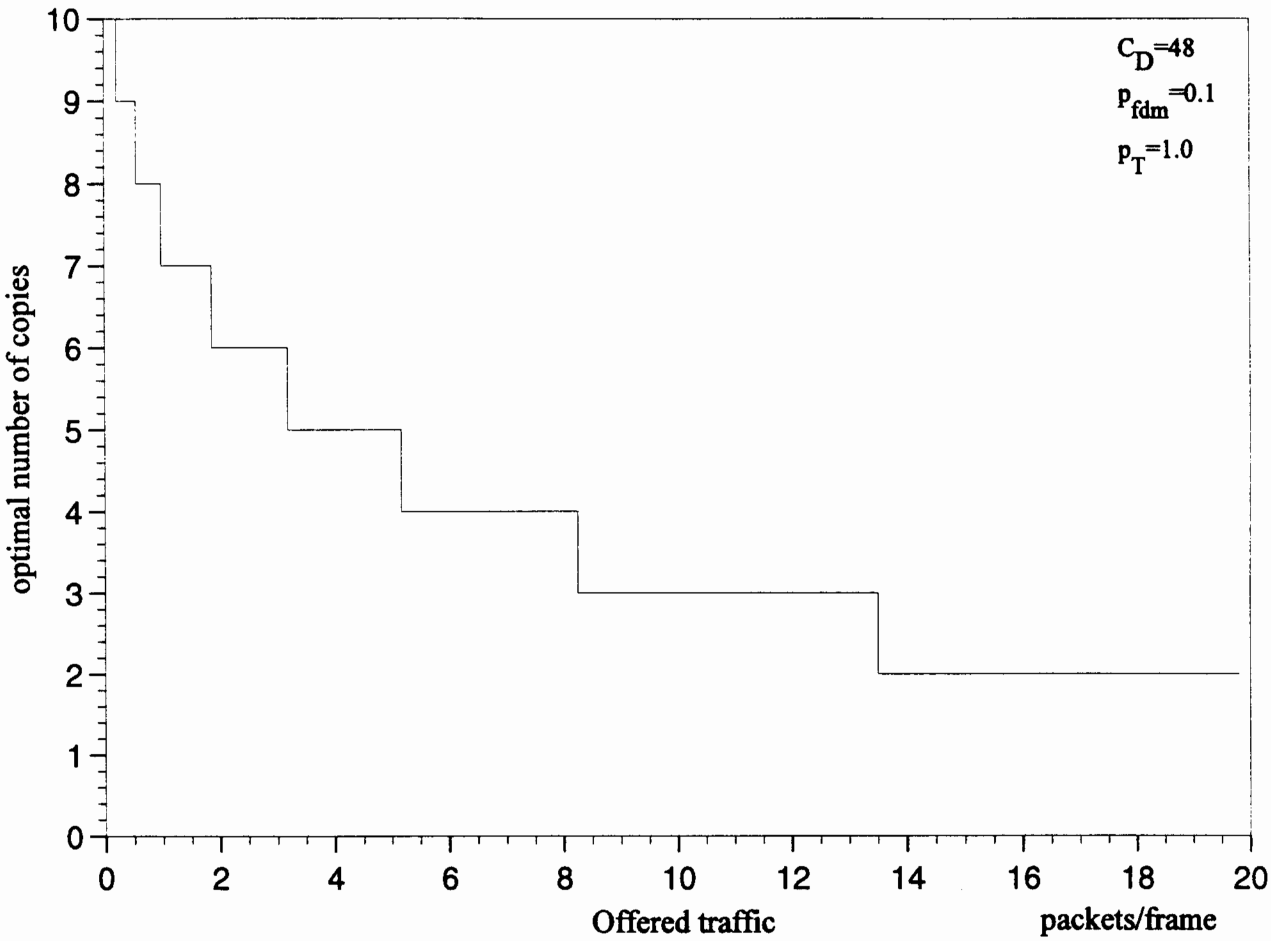

Fig.5 Optimal number of copies of a packet at different levels of data traffic and in presence of correlated fading.

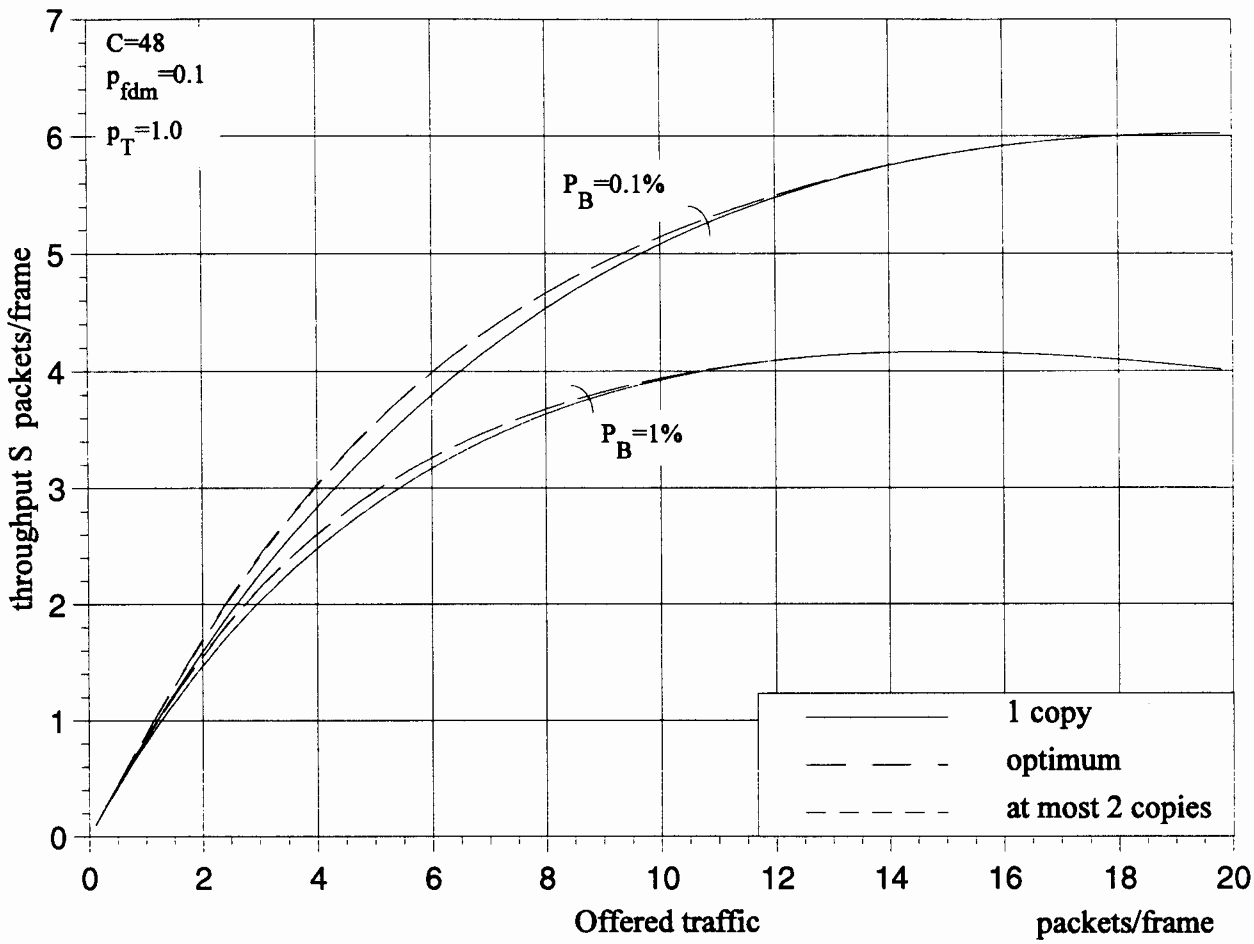

Fig.6 Data throughput for an integrated voice-data system in presence of correlated fading at two different levels of voice traffic and for different diversity startegies.

C=48 p_{fdm}=0.1 p_T=1.0

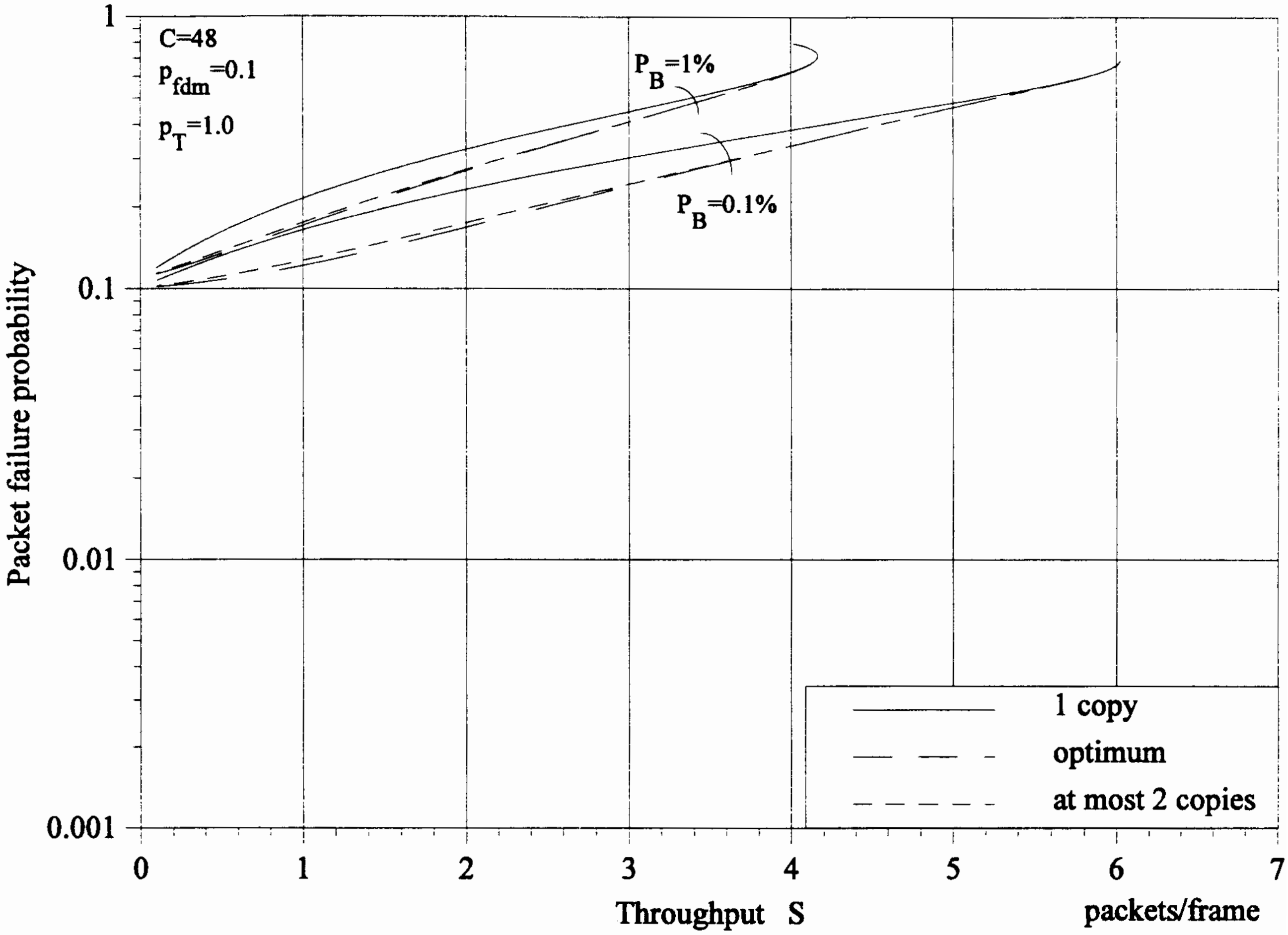

Fig.7 Packet failure probability for an integrated system in presence of correlated fading at two different levels of voice traffic and for different diversity strategies. $C=48$ $p_{fdm}=0.1$ $p_T=1.0$

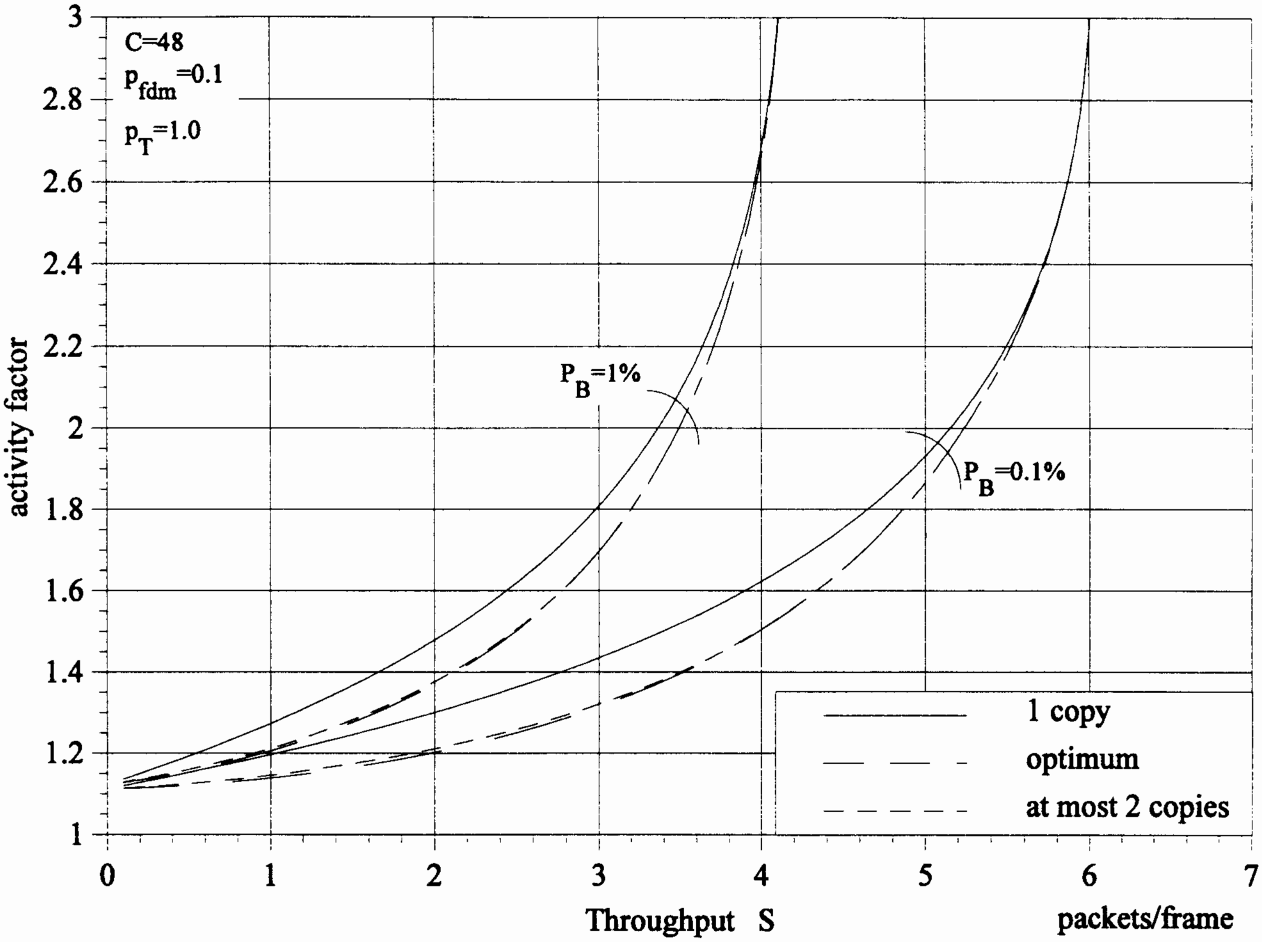

Fig.8 Activity factor for an integrated system in presence of correlated fading at two different levels of voice traffic and for different diversity strategies.

C=48 p_{fdm}=0.1 p_T=1.0

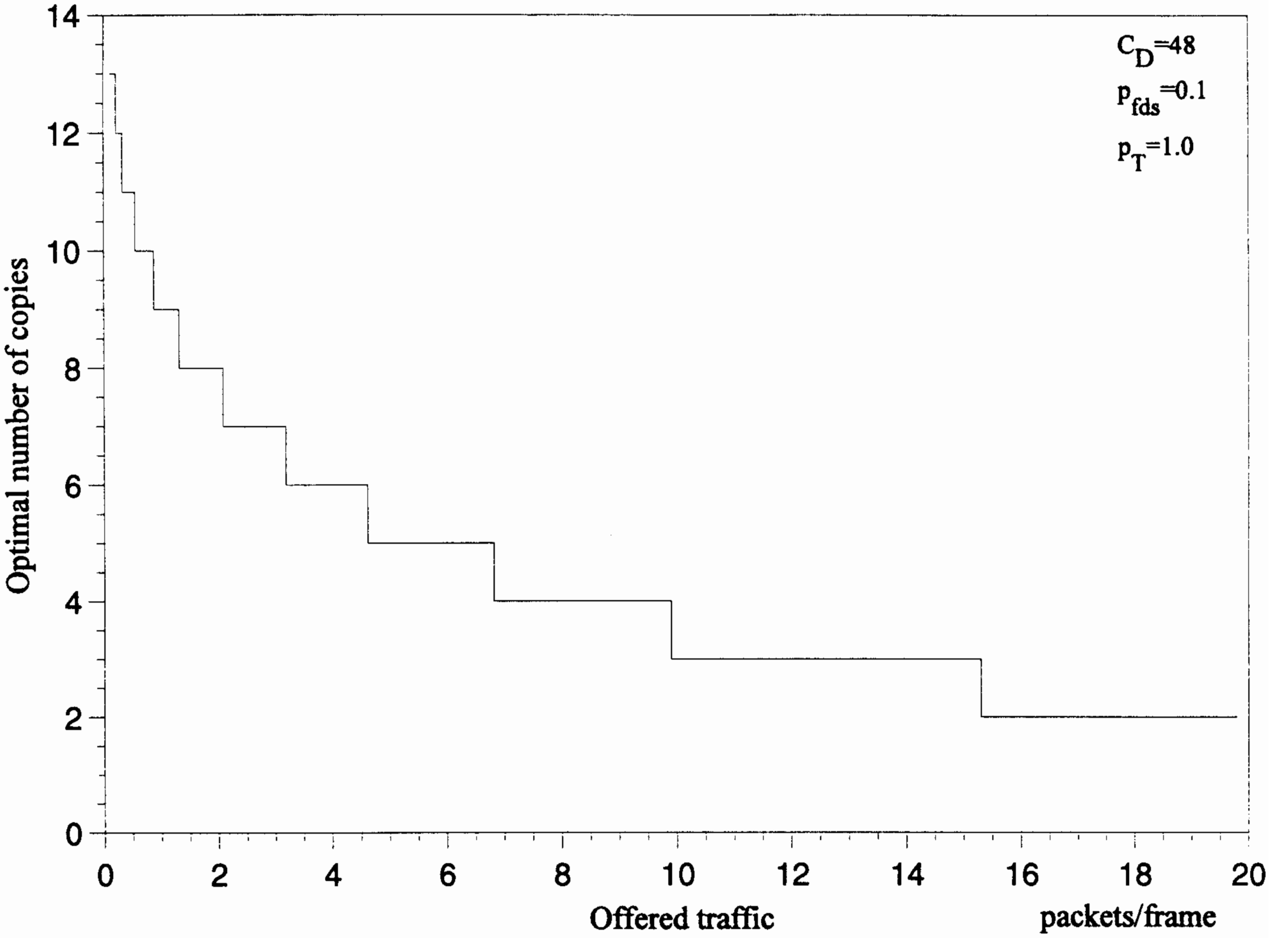

Fig.9 Optimal number of copies of a packet at different levels of data traffic and in presence of uncorrelated fading.

C=48 p_{fds}=0.1 p_T=1.0

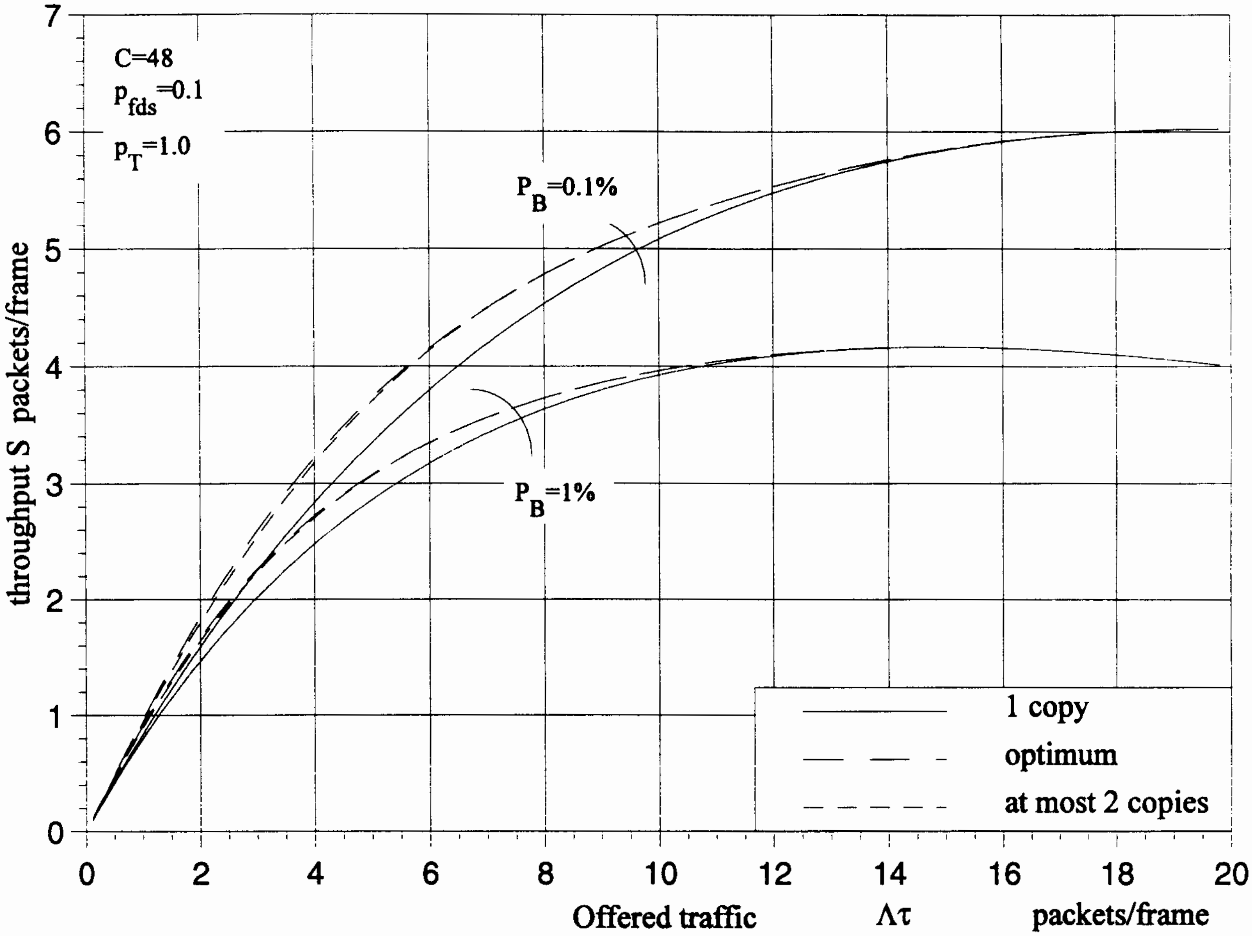

Fig.10 Data throughput for an integrated voice-data system in presence of uncorrelated fading at two different levels of voice traffic and for different diversity startegies.

C=48 p_{fds}=0.1 p_T=1.0

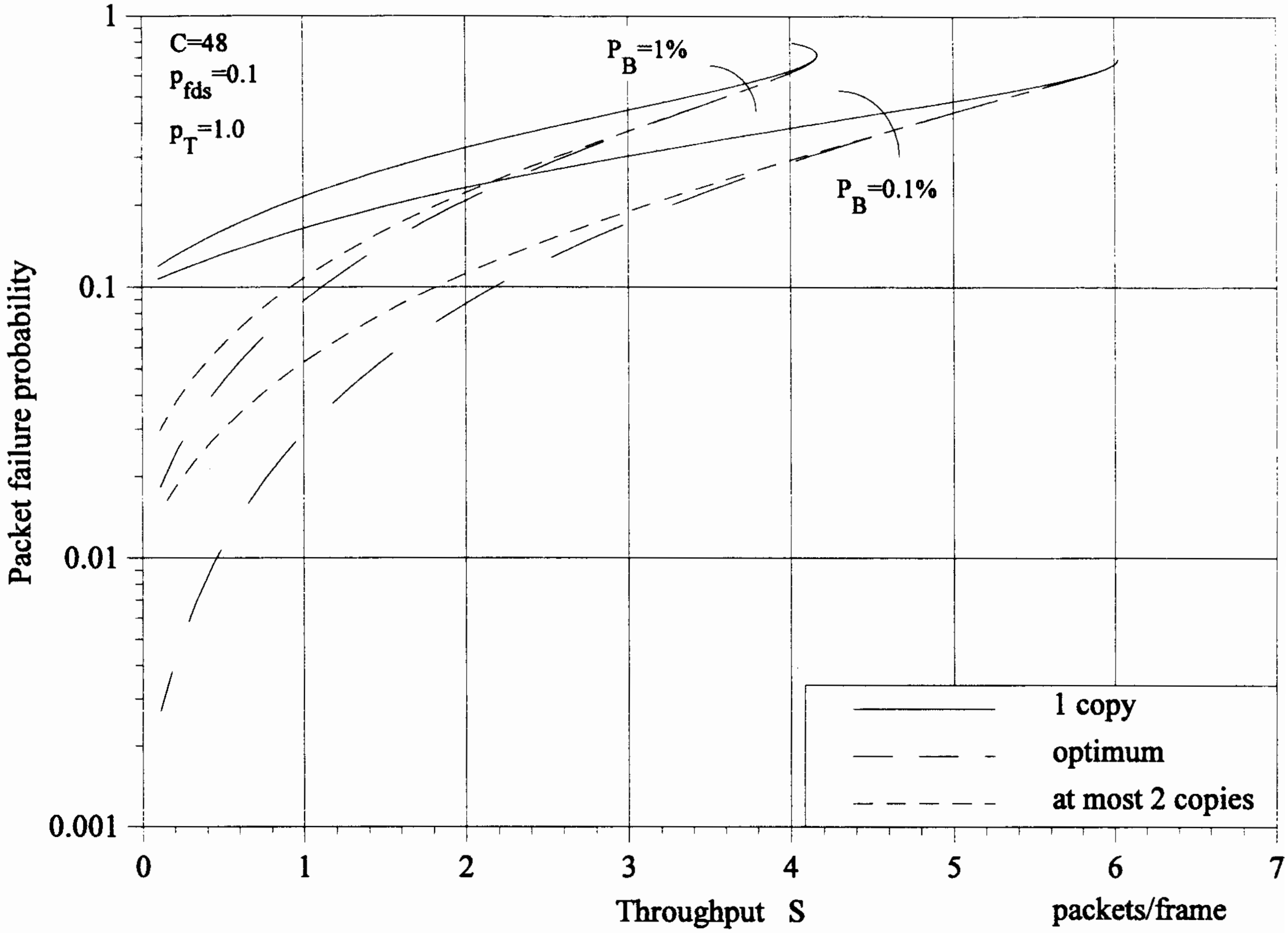

Fig.11 Packet failure probability for and integrated system in presence of uncorrelated fading at two different levels of voice traffic and for different diversity strategies.

C=48 p_{fds}=0.1 p_T=1.0

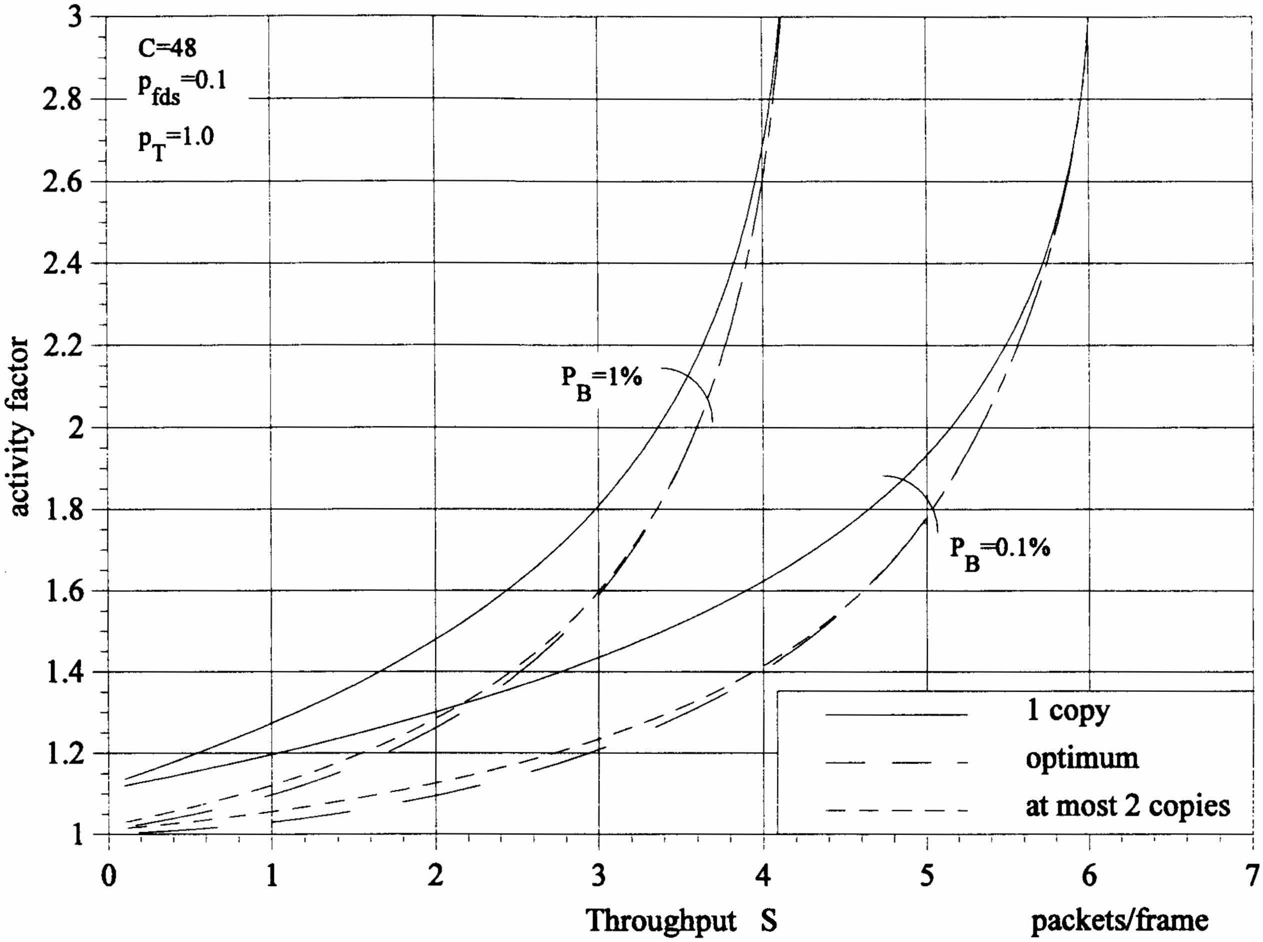

Fig.12 Activity factor for an integrated system in presence of uncorrelated fading at two different levels of voice traffic and for different diversity strategies.

C=48 p_{fds}=0.1 p_T=1.0

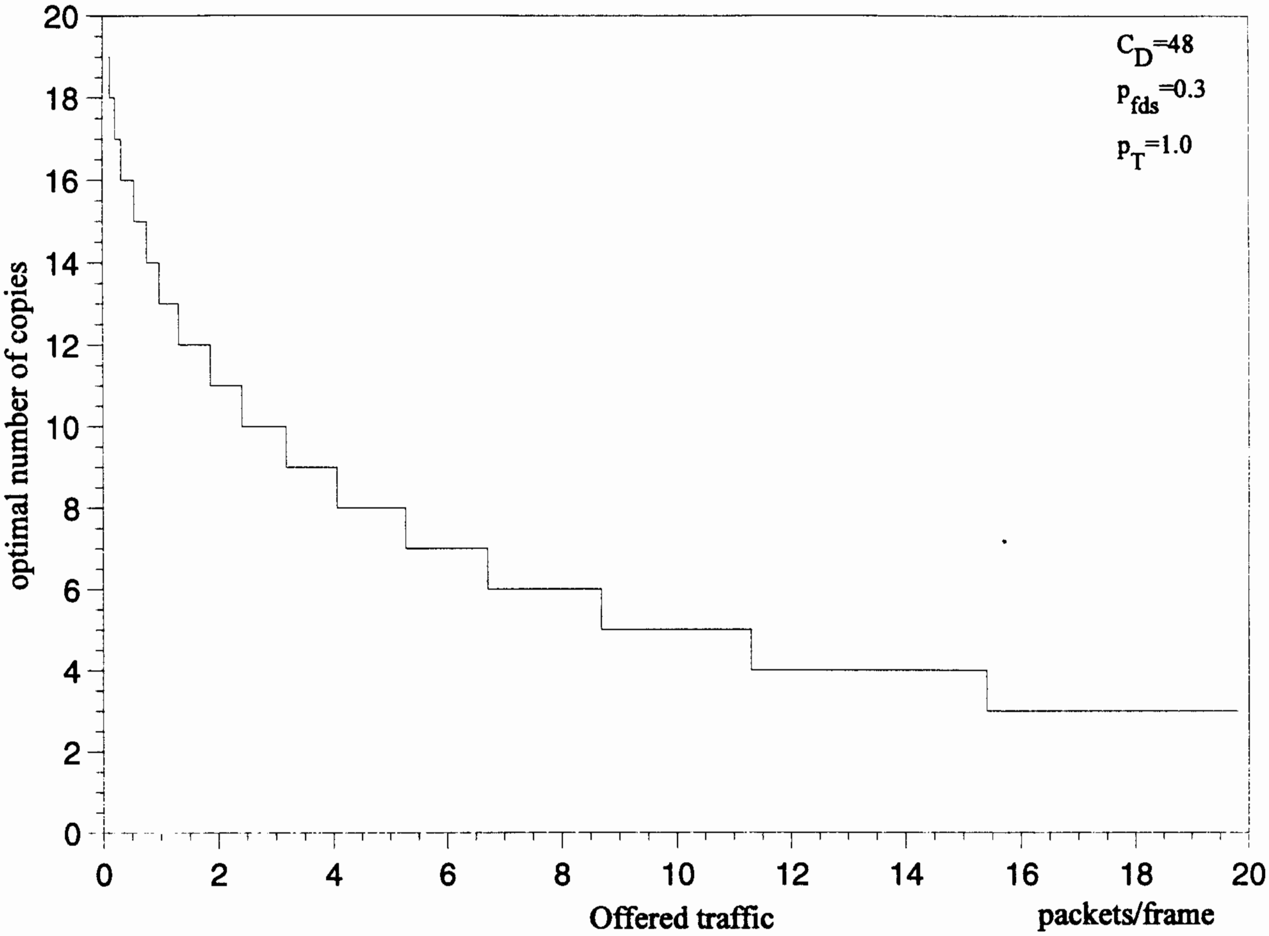

Fig.13 Optimal number of copies of a packets at different levels of data traffic and in presence of uncorrelated fading.

C=48 p_{fds}=0.3 p_T=1.0

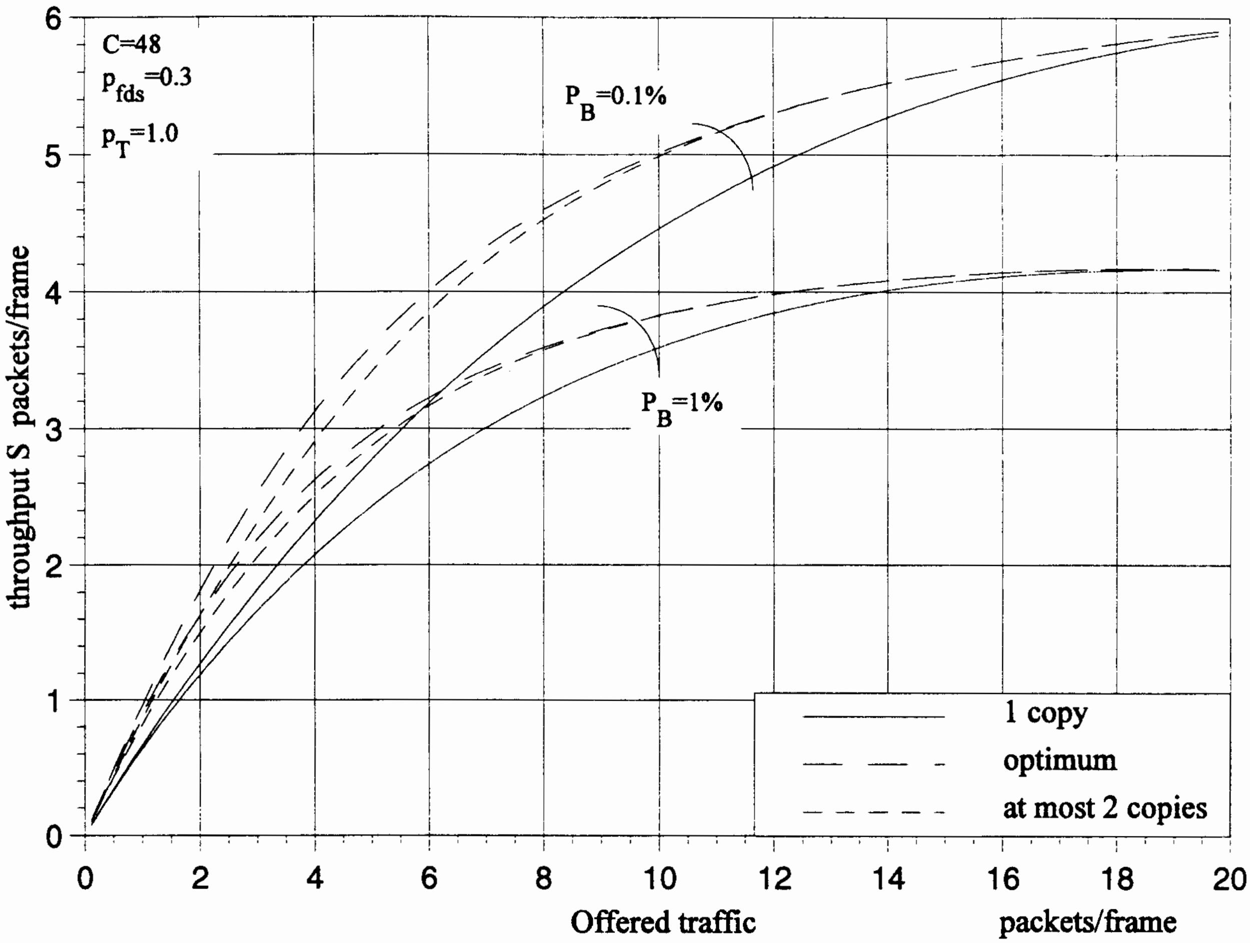

Fig.14 Data throughput for an integrated voice-data system in presence of uncorrelated fading at two different levels of voice traffic and for different diversity startegies.

C=48 p_{fds}=0.3 p_T=1.0

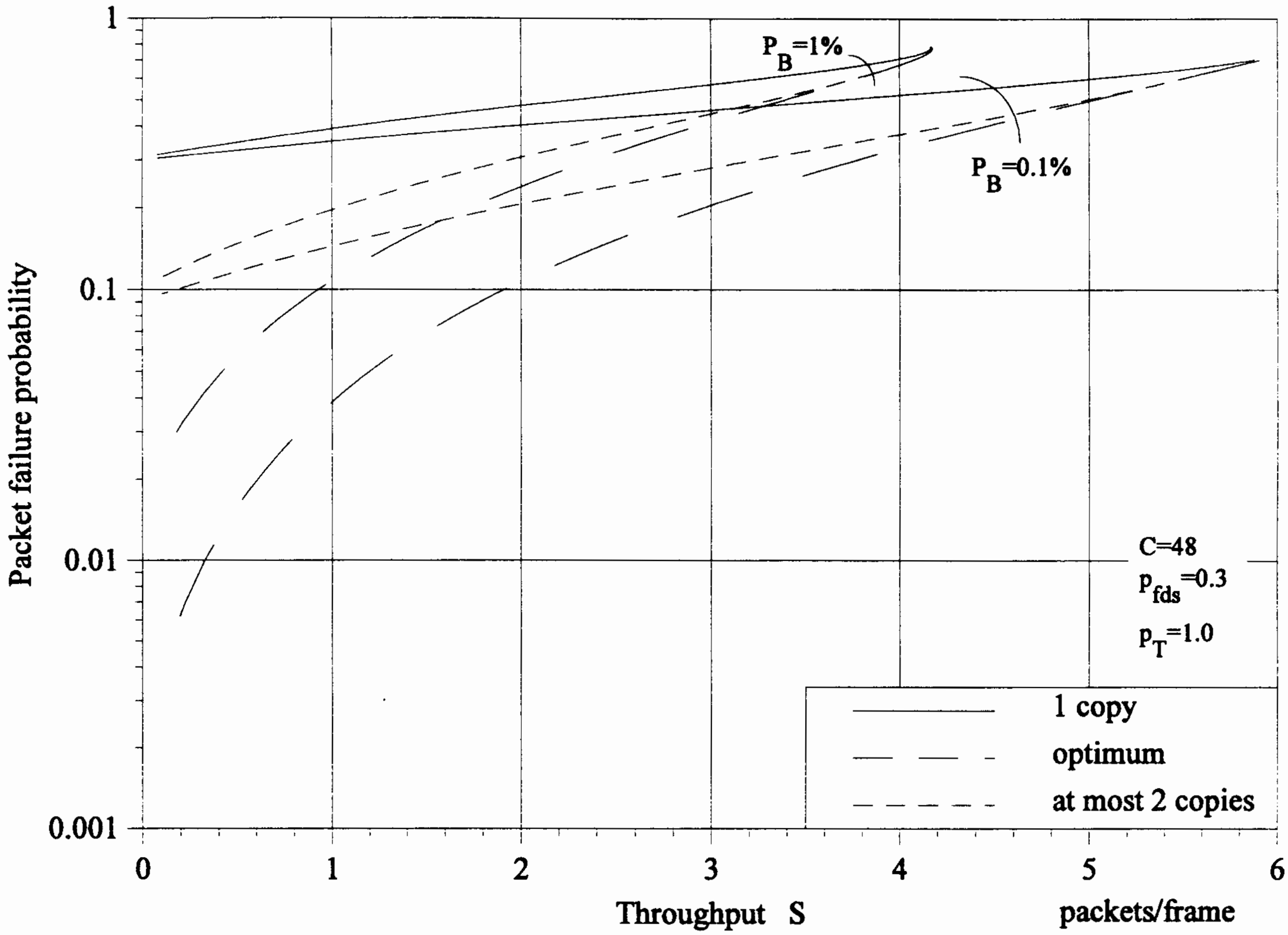

Fig.15 Packet failure probability for and integrated system in presence of uncorrelated fading at two different levels of voice traffic and for different diversity strategies.

C=48 p_{fds}=0.3 p_T=1.0

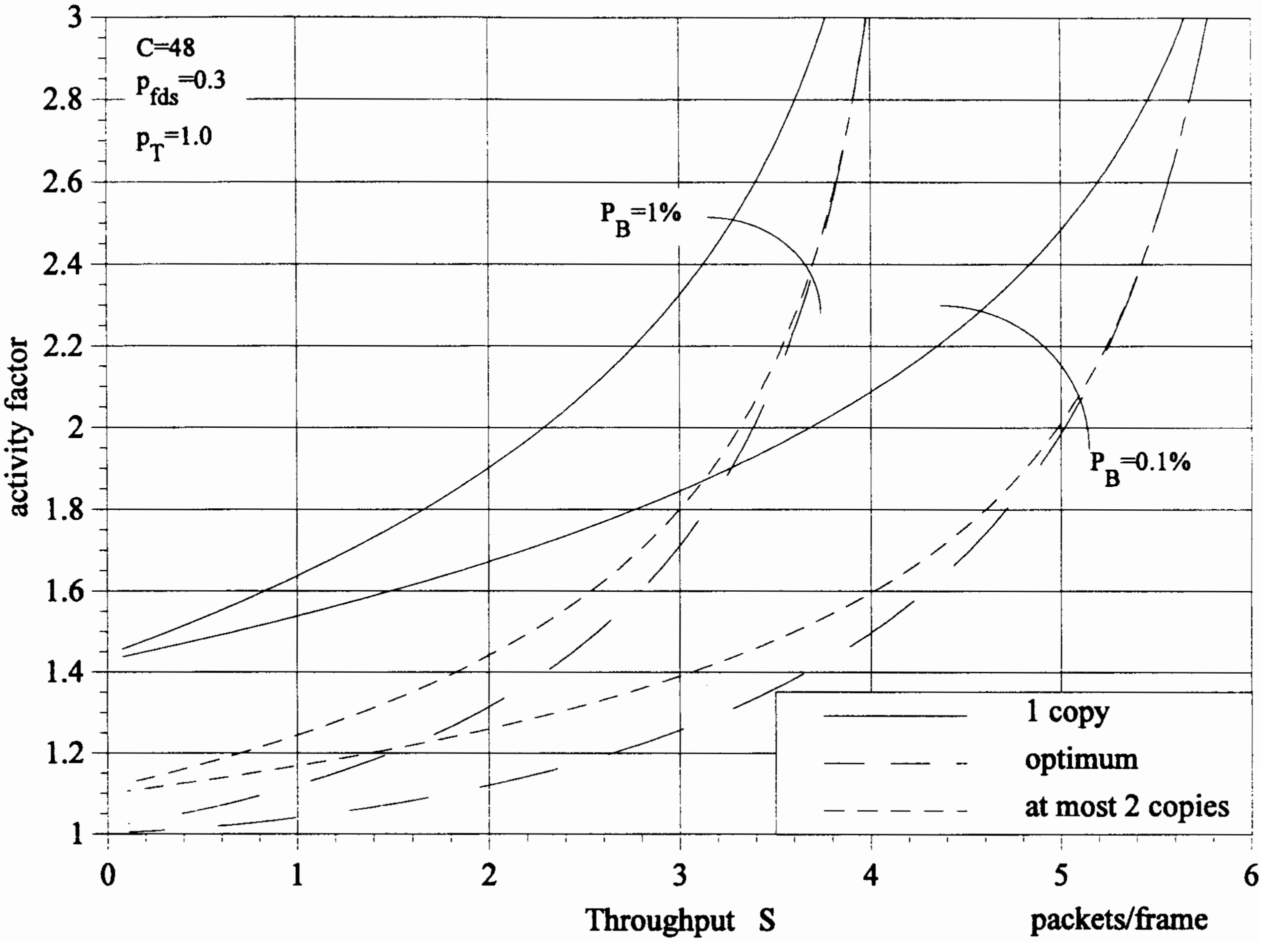

Fig.16 Activity factor for an integrated system in presence of uncorrelated fading at two different levels of voice traffic and for different diversity strategies.
C=48 p_{fds}=0.3 p_T=1.0

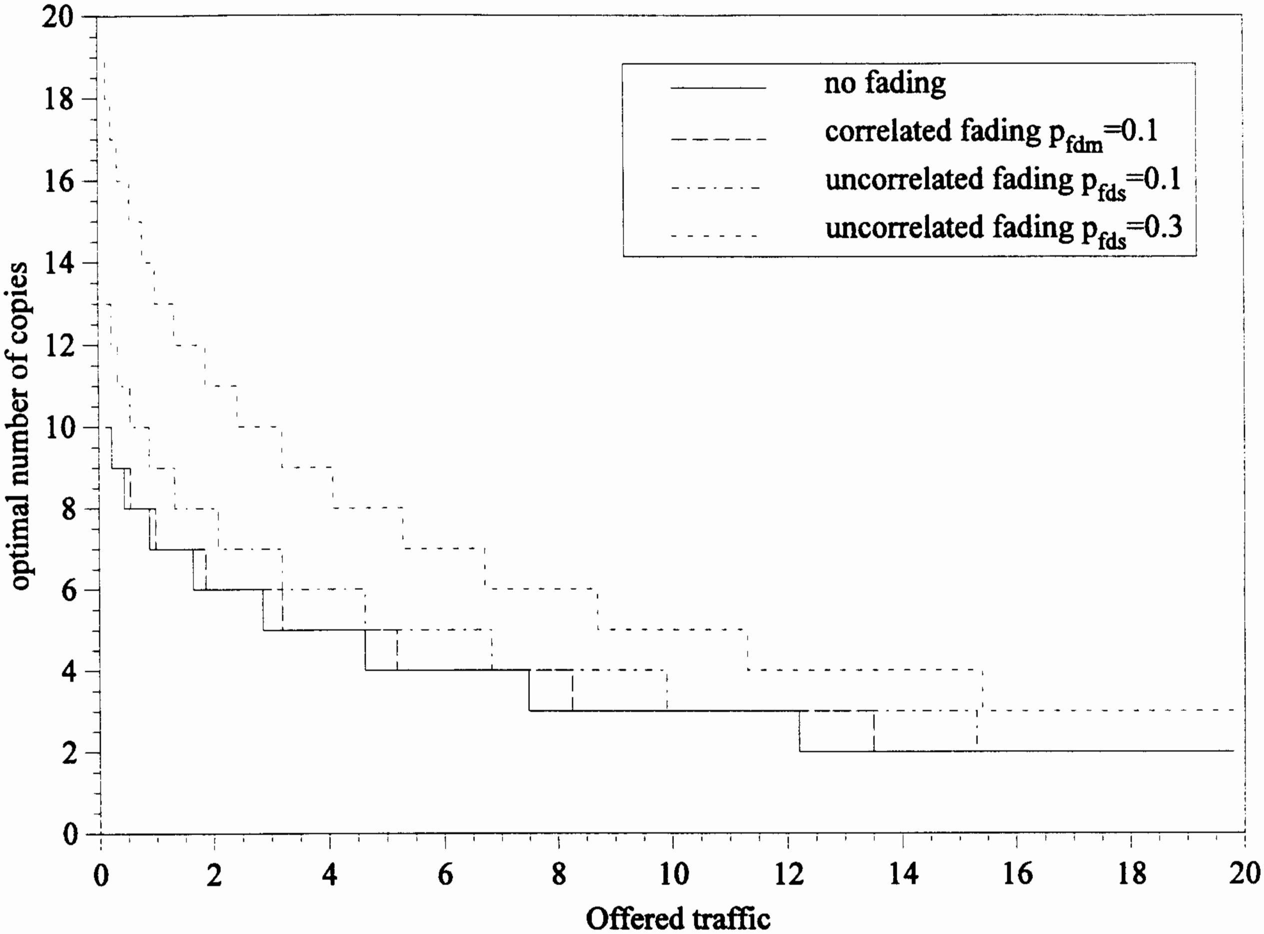

Fig.17 Effects of fading on the optimum number of copies.

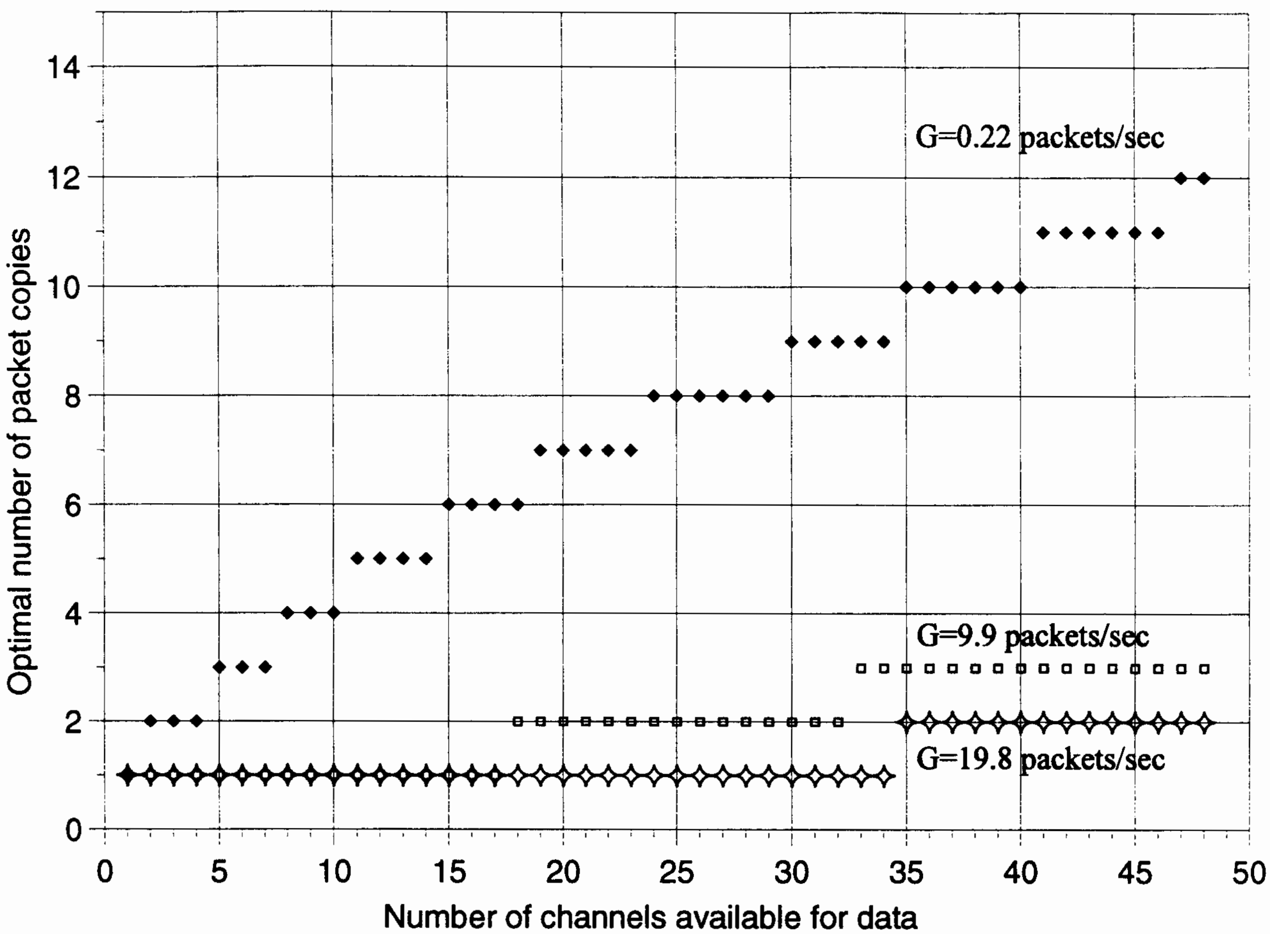

Fig.18 Dependence of the optimal number of packet copies on the number of available data channels at various offered data traffic intensities.
C=48 p_{fds}=0.1 p_T=1.0

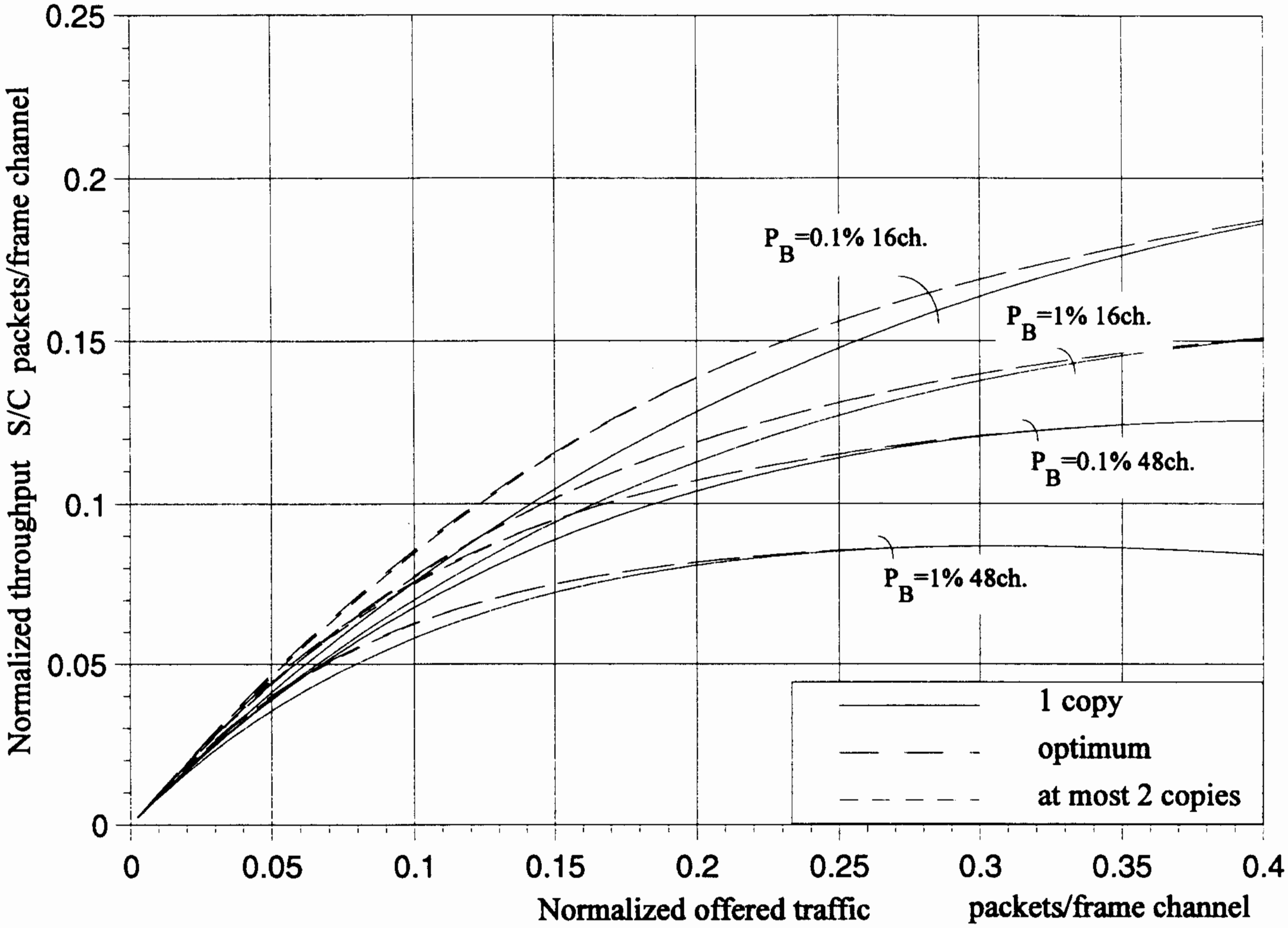

Fig.19 Dependence of data throughput on trunking efficiency of the voice subsystem. Uncorrelated fading p_{fds}=0.1, p_T=1.0.

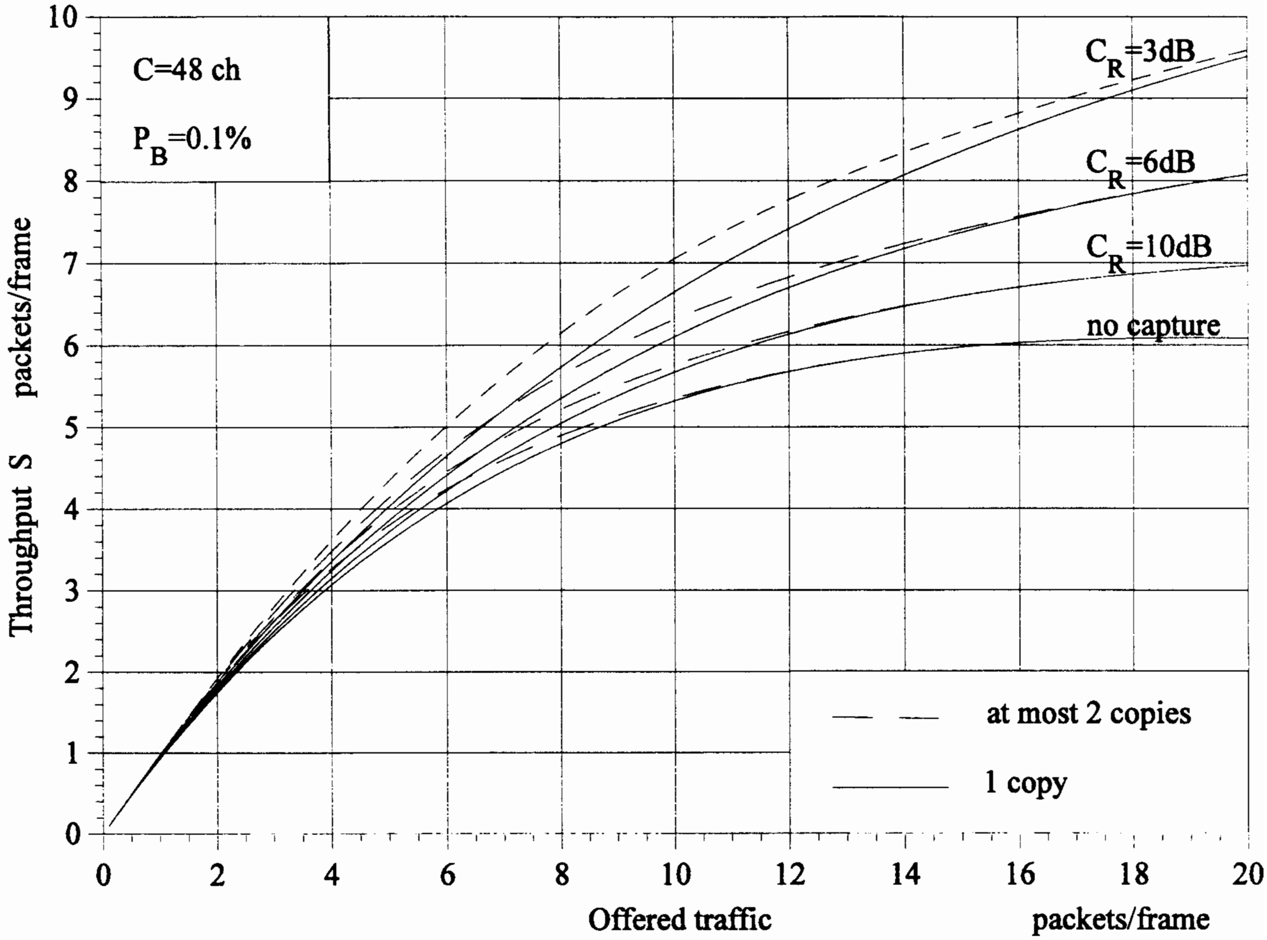

Fig.20 Comparison of data throughput for different capture ratios and diversity strategies.
C=48, voice call blocking P_B=0.1%, contention limited system.

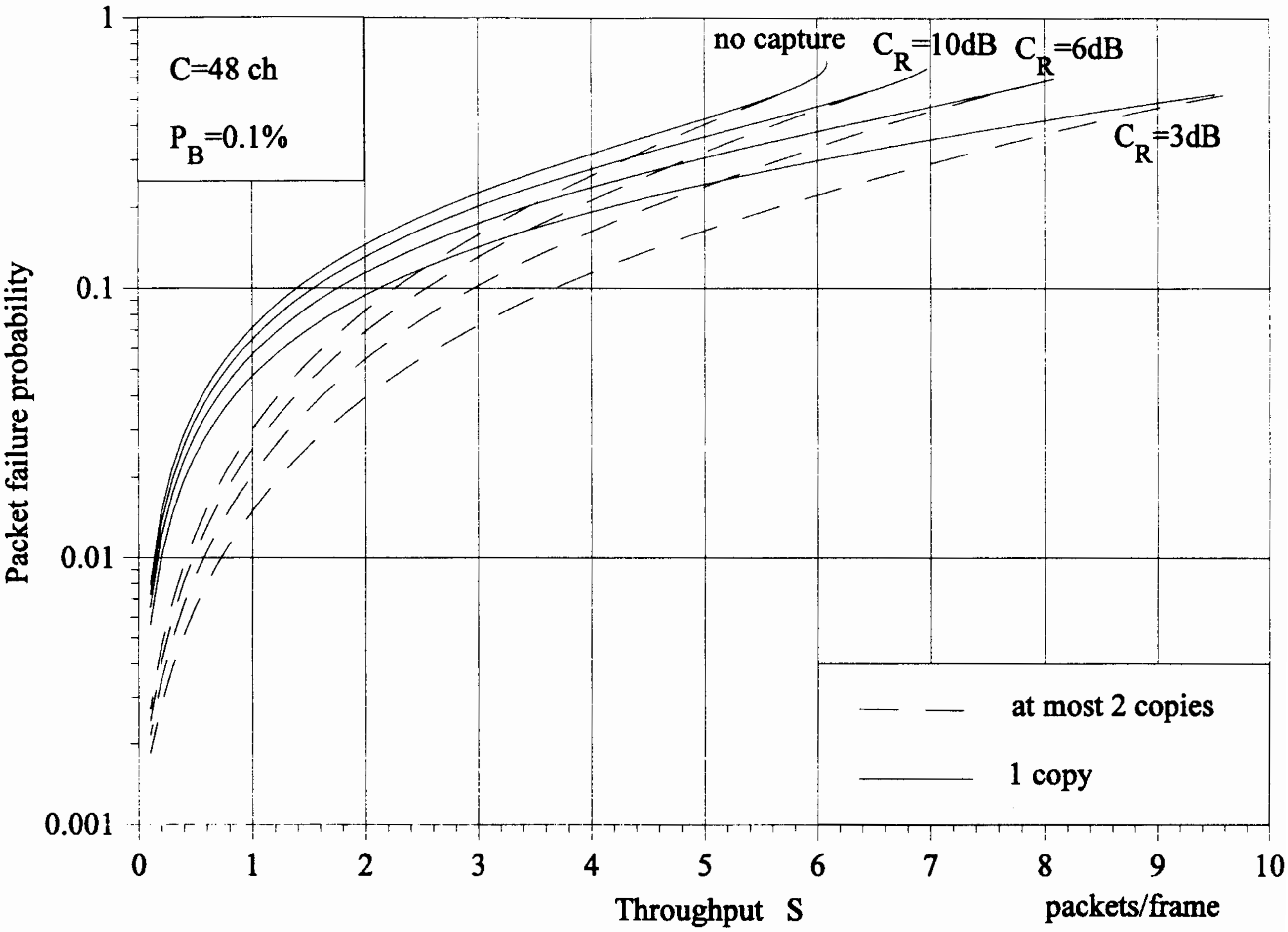

Fig.21 Packet failure probability for different capture ratios and diversity strategies. C=48, voice call blocking P_B=0.1%, contention limited system.

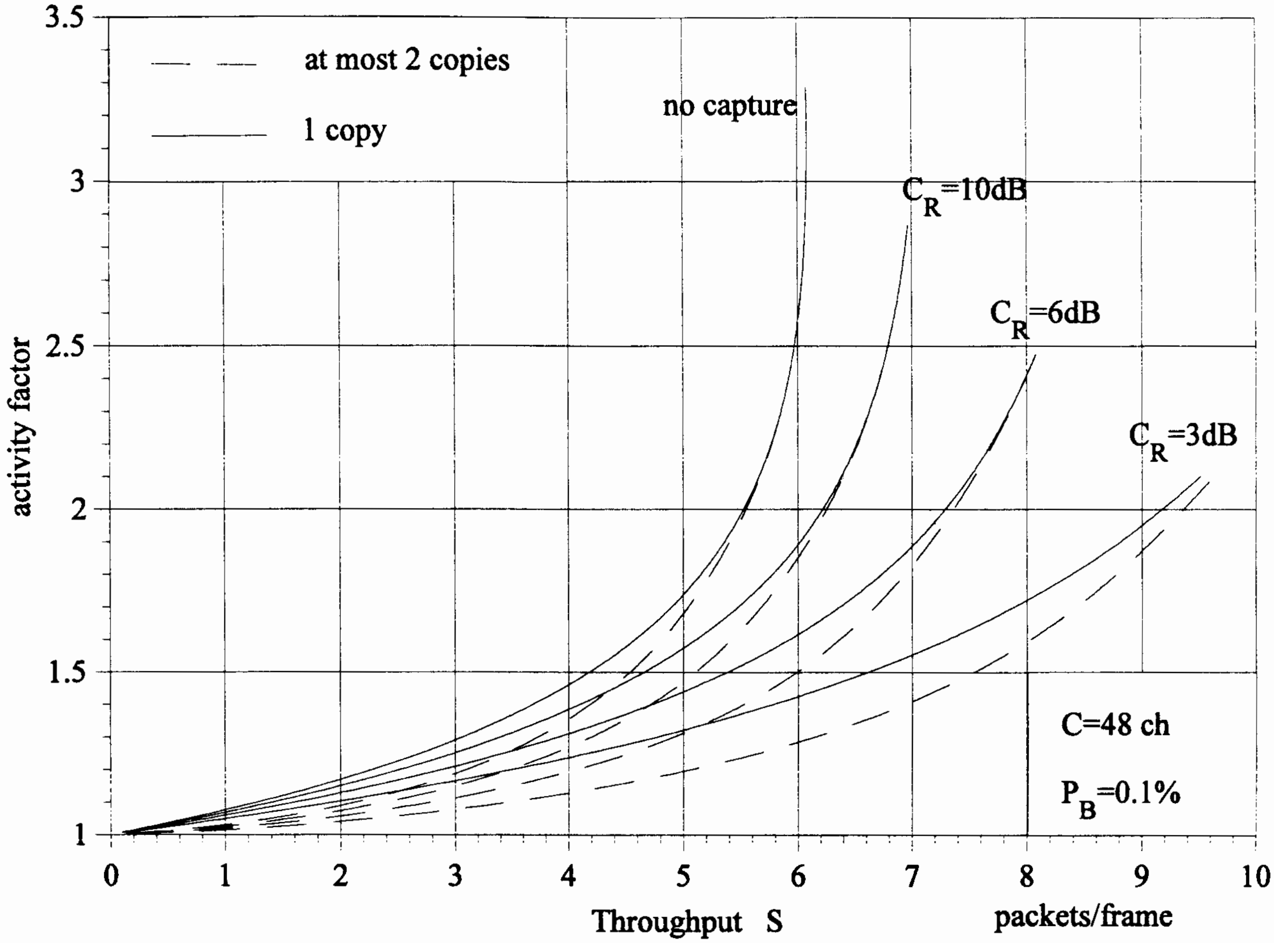

Fig.22 Comparison of activity factors for different capture ratios and diversity strategies.

C=48, voice call blocking P_B=0.1%, contention limited system.

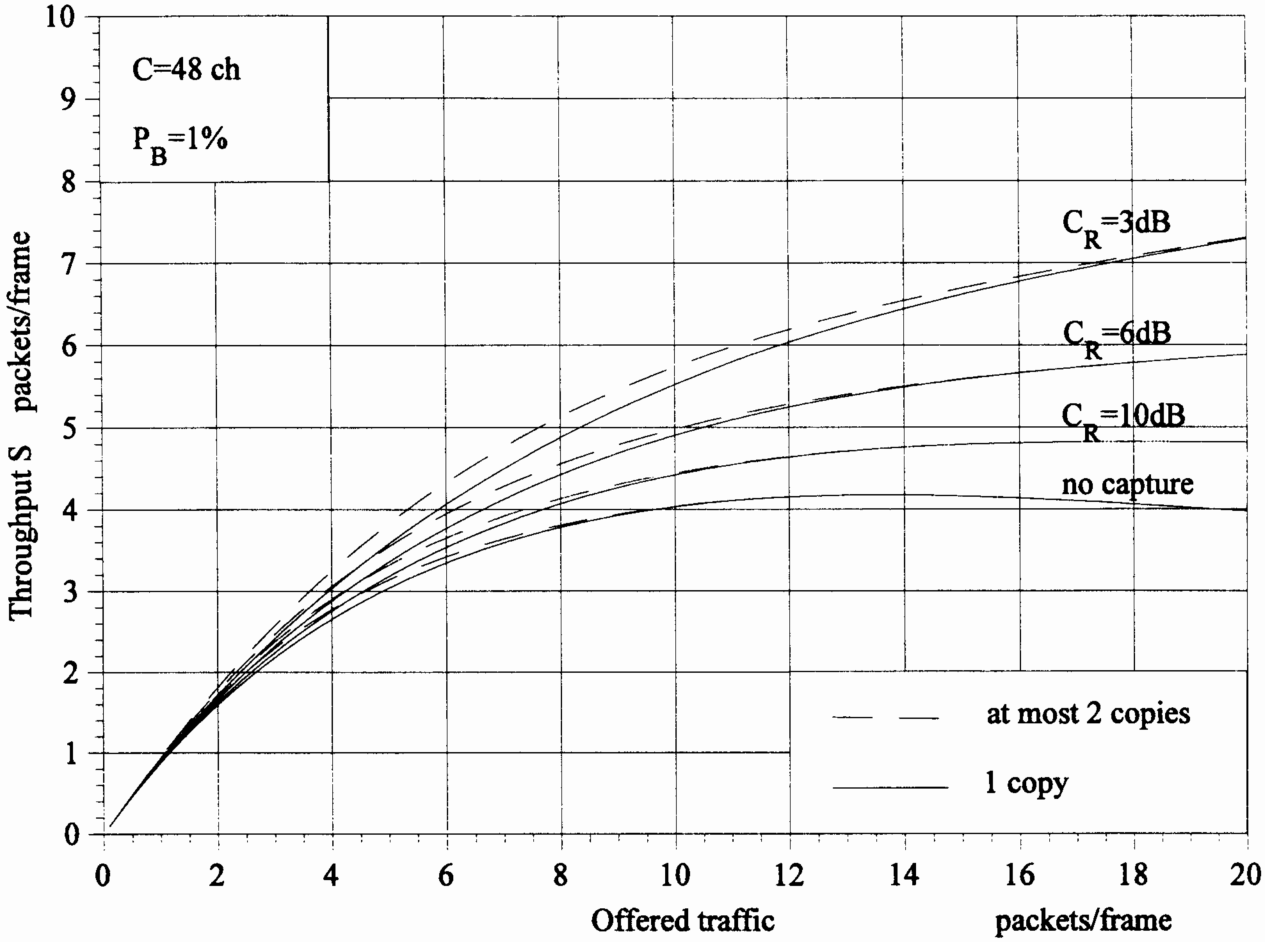

Fig.23 Comparison of data throughput for different capture ratios and diversity strategies.

C=48, voice call blocking P_B=1%, contention limited system.

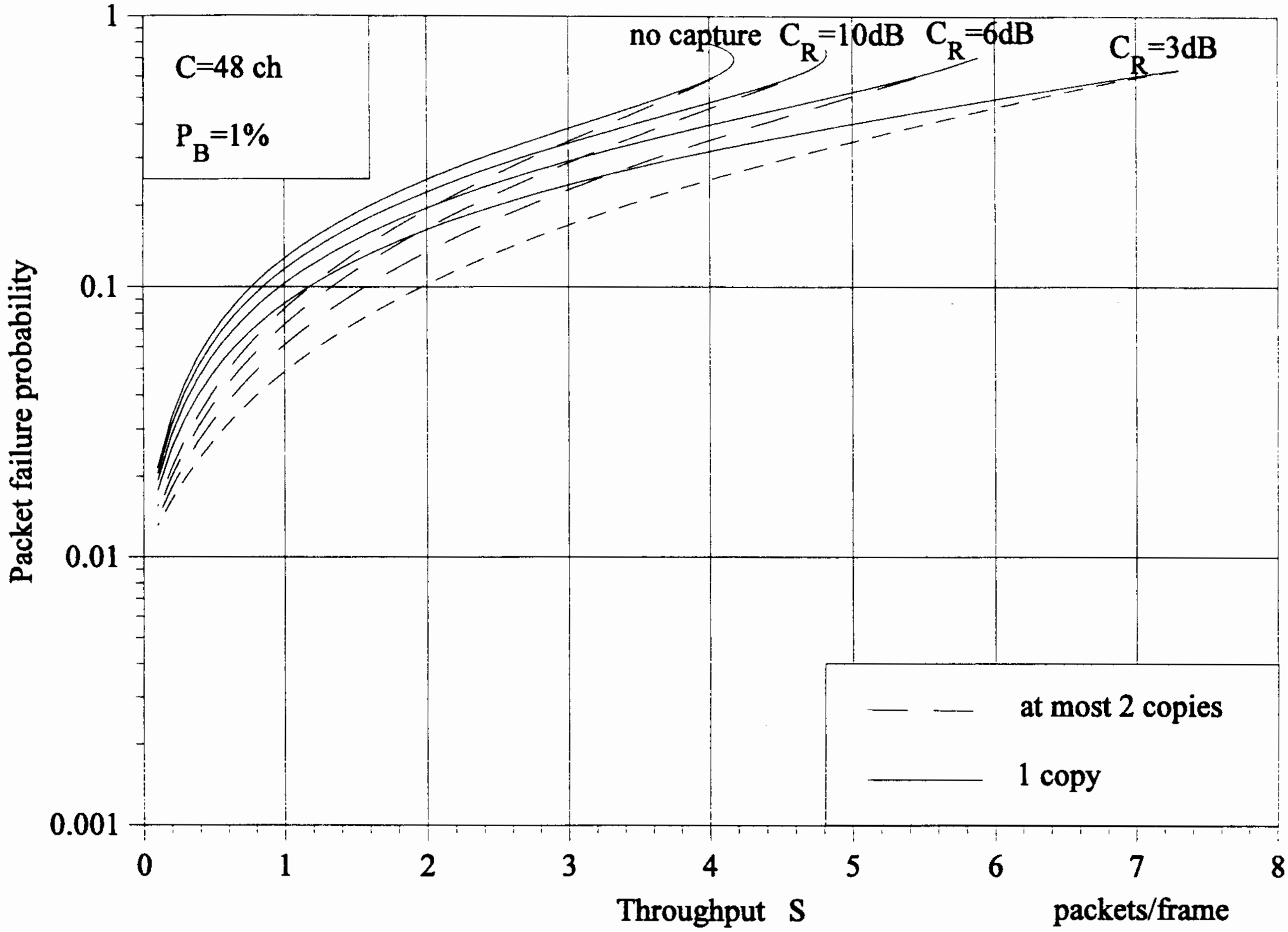

Fig.24 Packet failure probability for different capture ratios and diversity strategies. C=48, voice call blocking P_B=1%, contention limited system.

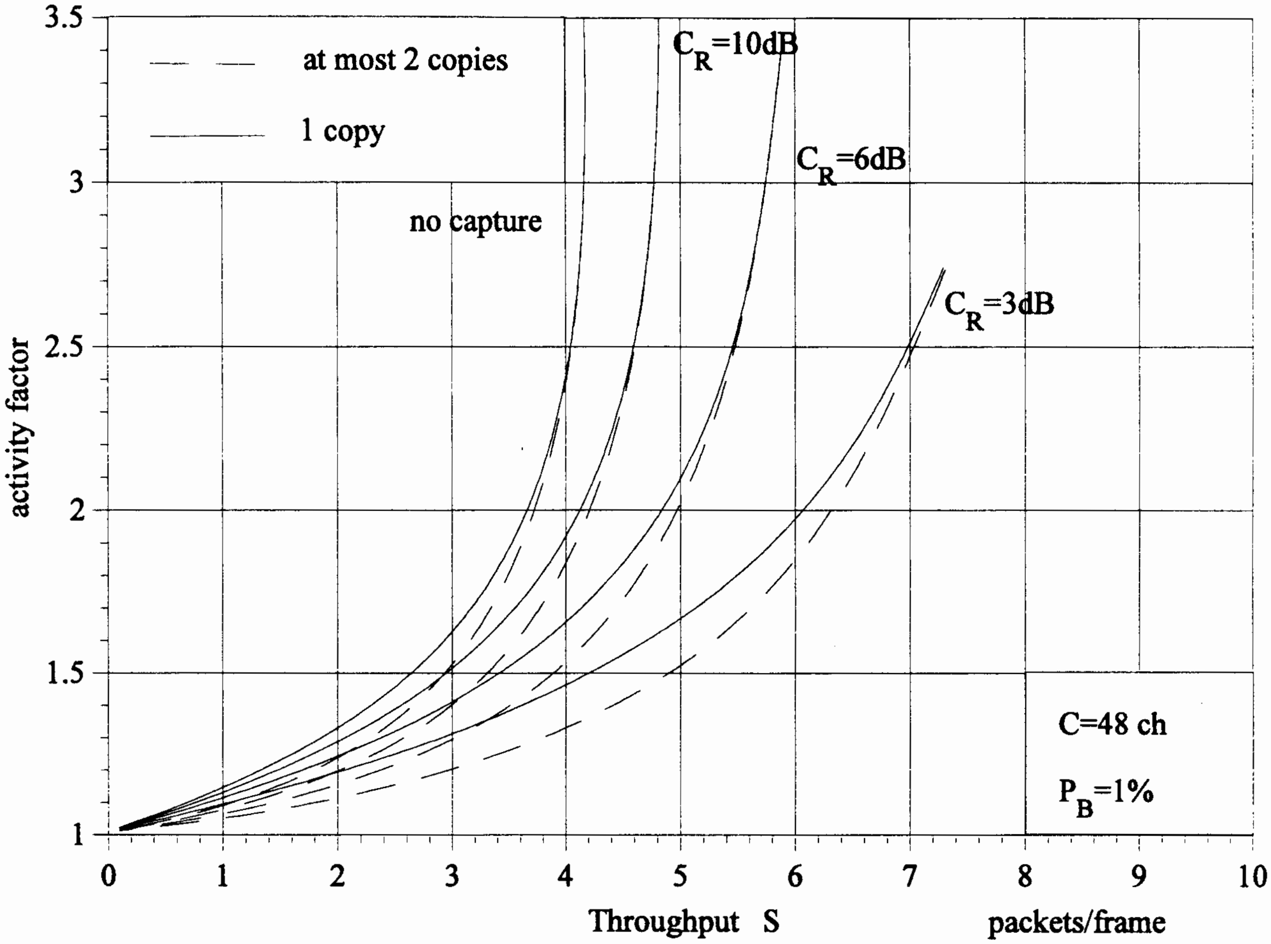

Fig.25 Comparison of activity factors for different capture ratios and diversity strategies.

C=48, voice call blocking P_B=1%, contention limited system.

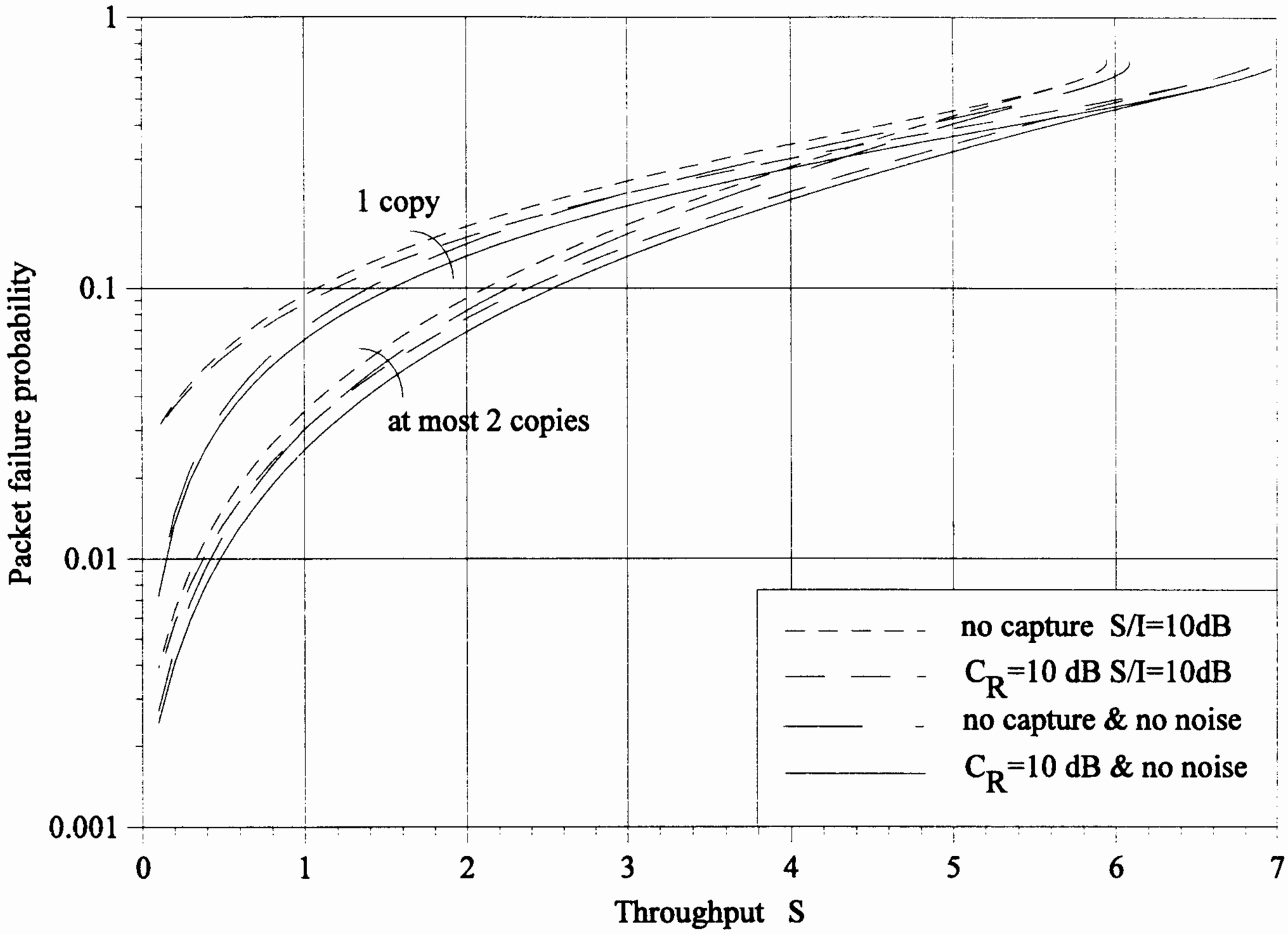

Fig. 26 Effect of background noise on system performance.
C=48, voice call blocking P_B=0.1%

APPENDIX A.

Let us denote by $P_{C_V}(n)$ the probability that n channels are occupied by voice sessions and by $P_{C_D}(n)$ the probability that n channels are available for data transmission. Clearly $P_{C_V}(n) = P_{C_D}(C-n)$.

From basic queuing theory we can obtain state probabilities of the M/M/C/C queue i.e. the probability distribution of the number of voice sessions

$$P_{C_V}(n) = P_{C_V}(0) \cdot \frac{1}{n!} \cdot \left(\frac{\lambda}{\mu}\right)^n \qquad 0 \le n \le C$$

where $P_{C_V}(0)$ is the normalization constant.

By a change of variables m=C-n where m denotes the number of free channels available for data we obtain

$$P_{C_D}(m) = P_{C_V}(0) \cdot \frac{1}{(C-m)!} \cdot \left(\frac{\lambda}{\mu}\right)^{C-m} = \underbrace{P_{C_V}(0) \cdot \frac{1}{C!} \cdot \left(\frac{\lambda}{\mu}\right)^{C}}_{P_{C_V}(C)} \cdot \frac{C!}{(C-m)!} \cdot \left(\frac{\mu}{\lambda}\right)^m =$$

$$= P_{C_V}(C) \frac{C!}{(C-m)!} \cdot \left(\frac{\mu}{\lambda}\right)^m = P_{C_D}(0) \frac{C!}{(C-m)!} \cdot \left(\frac{\mu}{\lambda}\right)^m$$

APPENDIX B.

Since m is not an independent r.v. and can be expressed as

$$m = \sum_{i=0}^{k(C_D)} j_i$$

therefore $P_{s|C_D}$ reduces to

$$P_{s|C_D} = \sum_{\underline{J}=0}^{\infty} P_{s|C_D,\underline{J}} \cdot P_{\underline{J}|m} \cdot P_m$$

Applying (11), (12) and (4) to the above equation we get

$$P_{s|C_D} = \sum_{j_0=0}^{\infty}\sum_{j_1=0}^{\infty}\cdots\sum_{j_{k(C_D)}=0}^{\infty}\sum_{n=1}^{k(C_D)}(-1)^{n+1}\cdot\binom{k(C_D)}{n}\cdot\left(1-p_{fds}\right)^n\cdot\prod_{i=0}^{k(C_D)}\left[\frac{\binom{C_D-n}{i}}{\binom{C_D}{i}}\right]^{j_i}\cdot$$

$$\binom{m}{j_0 j_1\ldots j_{k(C_D)}}\prod_{i=0}^{k(C_D)}\left[\binom{k(C_D)}{k(C_D)-i}\cdot\left(p_{fds}\right)^{k(C_D)-i}\left(1-p_{fds}\right)^i\right]^{j_i}\cdot\frac{\left(p_T\Lambda\tau\right)^m}{m!}\cdot e^{-p_T\Lambda\tau}$$

$$P_{s|C_D} = \sum_{n=1}^{k(C_D)}(-1)^{n+1}\cdot\binom{k(C_D)}{n}\cdot\left(1-p_{fds}\right)^n\cdot\sum_{j_0=0}^{\infty}\sum_{j_1=0}^{\infty}\cdots\sum_{j_{k(C_D)}=0}^{\infty}\binom{m}{j_0 j_1\ldots j_{k(C_D)}}\cdot$$

$$\prod_{i=0}^{k(C_D)}\left[\frac{\binom{C_D-n}{i}}{\binom{C_D}{i}}\binom{k(C_D)}{k(C_D)-i}\cdot\left(p_{fds}\right)^{k(C_D)-i}\left(1-p_{fds}\right)^i\cdot p_T\Lambda\tau\right]^{j_i}\cdot\frac{1}{m!}\cdot e^{-p_T\Lambda\tau}$$

$$P_{s|C_D} = \sum_{n=1}^{k(C_D)}(-1)^{n+1}\cdot\binom{k(C_D)}{n}\cdot\left(1-p_{fds}\right)^n\cdot\prod_{i=0}^{k(C_D)}\sum_{j_i=0}^{\infty}\frac{1}{j_i!}\cdot$$

$$\left[\frac{\binom{C_D-n}{i}}{\binom{C_D}{i}}\binom{k(C_D)}{k(C_D)-i}\cdot\left(p_{fds}\right)^{k(C_D)-i}\left(1-p_{fds}\right)^i\cdot p_T\Lambda\tau\right]^{j_i}\cdot e^{-p_T\Lambda\tau}$$

$$P_{s|C_D} = \sum_{n=1}^{k(C_D)}(-1)^{n+1}\cdot\binom{k(C_D)}{n}\cdot\left(1-p_{fds}\right)^n\cdot$$

$$\prod_{i=0}^{k(C_D)}\exp\left(\left[\frac{\binom{C_D-n}{i}}{\binom{C_D}{i}}\binom{k(C_D)}{k(C_D)-i}\cdot\left(p_{fds}\right)^{k(C_D)-i}\left(1-p_{fds}\right)^i\cdot p_T\Lambda\tau\right]\right)\cdot e^{-p_T\Lambda\tau}$$

Finally,

$$P_{s|C_D}=\sum_{n=1}^{k(C_D)}(-1)^{n+1}\cdot\binom{k(C_D)}{n}\cdot\left(1-p_{fds}\right)^n\cdot$$

$$\exp\left\{\left[\left(\sum_{i=0}^{k(C_D)}\frac{\binom{C_D-n}{i}}{\binom{C_D}{i}}\binom{k(C_D)}{k(C_D)-i}\cdot\left(p_{fds}\right)^{k(C_D)-i}\left(1-p_{fds}\right)^i\right)-1\right]p_T\Lambda\tau\right\}$$

Appendix C. Probability distribution of the number of interferers, p(n), for k=2 and k=3.

Let j denote the number of interfering users that have chosen both slot choices of user U and i denote the number of interfering users that have chosen only one of user's U slot choices. Then, the probability that there are $\underline{\mathbf{n}}=(n_1,n_2)$ interferers i.e. n_1 in slot 1 and n_2 in slot 2, can be found as

$$p(\underline{\mathbf{n}})=\sum_{m=\max(n_1,n_2)}^{\infty}e^{-\Lambda\tau}\frac{(\Lambda\tau)^m}{m!}\sum_{j=\max(0,n_1+n_2-m)}^{\min(n_1,n_2)}\binom{m}{i\quad j\quad m-i-j}\binom{i}{n_1-j}\binom{C-2}{k-1}^i\binom{C-2}{k}^{m-i-j}\Big/\binom{C}{k}^m=$$

$$\sum_{j=0}^{\min(n_1,n_2)}\sum_{m=\max(n_1,n_2)}^{\infty}e^{-\Lambda\tau}\frac{(\Lambda\tau)^m}{m!}\binom{m}{i\quad j\quad m-i-j}\binom{i}{n_1-j}\binom{C-2}{k-1}^i\binom{C-2}{k}^{m-i-j}\Big/\binom{C}{k}^m$$

Noting the following relationship

$$i+2j=n_1+n_2\Rightarrow i=n_1+n_2-2j$$

and substituting

$$m'=m-i-j=m-n_1-n_2+j$$

we can rewrite $p(\underline{\mathbf{n}})$ as

$$\sum_{j=0}^{\min(n_1,n_2)}\sum_{m'=0}^{\infty}e^{-\Lambda\tau}\frac{(\Lambda\tau)^{m'+n_1+n_2-j}}{m'!}\frac{1}{j!(n_1-j)(n_2-j)}\binom{C-2}{k-1}^{n_1+n_2-2j}\binom{C-2}{k}^{m'}\Big/\binom{C}{k}^{m'+n_1+n_2-j}$$

$$=\exp\left(\Lambda\tau\left[\frac{\binom{C-2}{k}}{\binom{C}{k}}-1\right]\right)\left[\frac{\Lambda\tau\binom{C-2}{k-1}}{\binom{C}{k}}\right]^{n_1+n_2}\sum_{j=0}^{\min(n_1,n_2)}\frac{1}{j!(n_1-j)(n_2-j)}\left[\frac{\Lambda\tau\binom{C-2}{k-1}}{\binom{C}{k}}\right]^{-j}\binom{C-2}{k-1}^{-j}$$

For the case of k=3, let us denote by i_{xy} the number of interfering users, who have chosen two slots, x and y, also chosen by user U. Moreover, let l_x denote the number of interfering users who have chosen slot x and let j denote the number of interfering users who have chosen all three slots also chosen by user U. Then, extending the approach taken for k=2, the probability distribution of the number of interferers, $\underline{\mathbf{n}}=(n_1,n_2,n_3)$, can be determined as

$$p(\underline{\mathbf{n}}) = \sum_{m=\max(n_1,n_2,n_3)}^{\infty} e^{-\Lambda\tau} \frac{(\Lambda\tau)^m}{m!} \sum_{j=0}^{u_1} \sum_{i_{12}=0}^{u_2} \sum_{i_{23}=0}^{u_3} \sum_{i_{13}=0}^{u_4} \binom{m}{j\; i_{12}\; i_{23}\; i_{13}\; l_1\; l_2\; l_3\; m-\Sigma} \binom{C-3}{k-1}^{i} \binom{C-3}{k-2}^{l} \binom{C-3}{k-3}^{m-\Sigma}$$

$$= \sum_{j=0}^{u_1} \sum_{i_{12}=0}^{u_2} \sum_{i_{23}=0}^{u_3} \sum_{i_{13}=0}^{u_4} \sum_{m=n_1+n_2+n_3-2j-i}^{\infty} e^{-\Lambda\tau} \frac{(\Lambda\tau)^m}{m!} \binom{m}{j\; i_{12}\; i_{23}\; i_{13}\; l_1\; l_2\; l_3\; m-\Sigma} \binom{C-3}{k-1}^{i} \binom{C-3}{k-2}^{l} \binom{C-3}{k-3}^{m-\Sigma} =$$

where

$$i = i_{12} + i_{23} + i_{13}$$
$$l = l_1 + l_2 + l_3$$
$$\Sigma = i + l + j$$

Noting the following relationships

$$n_1 = j + i_{12} + i_{13} + l_1 \qquad n_2 = j + i_{12} + i_{23} + l_2$$
$$n_3 = j + i_{13} + i_{23} + l_3$$

we can substitute

$$m' = m - \Sigma = m - (n_1 + n_2 + n_3) + 2j + i$$

and arrive at the following form of $p(\underline{\mathbf{n}})$

$$p(\underline{\mathbf{n}}) = \sum_{j=0}^{u_1} \sum_{i_{12}=0}^{u_2} \sum_{i_{23}=0}^{u_3} \sum_{i_{13}=0}^{u_4} e^{-\Lambda\tau} \sum_{m'=0}^{\infty} \frac{(\Lambda\tau)^{m'+n_1+n_2+n_3-2j-i}}{m'!\, j!\, i_{12}!\, i_{23}!\, i_{13}!\, l_1!\, l_2!\, l_3!} \binom{C-3}{k-1}^{i} \binom{C-3}{k-2}^{n_1+n_2+n_3-2j-i} \binom{C-3}{k-3}^{m'} =$$

$$\exp\left(\Lambda\tau \left[\frac{(C-5)(C-4)(C-3)}{(C-2)(C-1)C} - 1 \right] \right) \sum_{j=0}^{u_1} \sum_{i_{12}=0}^{u_2} \sum_{i_{23}=0}^{u_3} \sum_{i_{13}=0}^{u_4} \left(\frac{(\Lambda\tau)^{n_1+n_2+n_3-2j-i}}{j!\, i_{12}!\, i_{23}!\, i_{13}!\, l_1!\, l_2!\, l_3!} \right) \binom{C-3}{k-1}^{i} \left(\frac{\binom{C-3}{k-2}}{\binom{C}{k}} \right)^{n_1+n_2+n_3-2j-i}$$

where

$$u_1 = \min(n_1, n_2, n_3)$$
$$u_2 = \min(n_1 - j, n_2 - j)$$
$$u_3 = \min(n_2 - j - i_{12}, n_3 - j)$$
$$u_4 = \min(n_1 - j - i_{12}, n_3 - j - i_{23})$$

Appendix D. Determination of the average cochannel interference power.

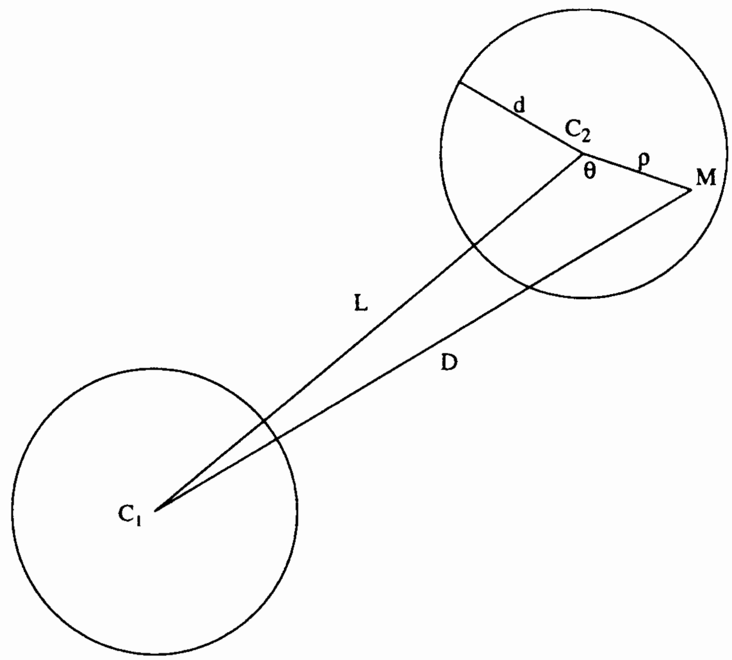

Figure A1. Geometry of the cochannel interference problem.

Consider the situation depicted in Fig. A1. We assume a center exited, non-sectored system. The cells are approximated by circular regions of radius d. Moreover, we assume a homogenous system in all aspects of the problem. This implies that all cells have the same radius and the average received power is the same at all base stations. We also assume fourth power propagation law and disregard the effects of shadowing. In the following we only consider the uplink as the more critical for system performance. A mobile located at point M in cell C2 will be received at the base station C2 with average power

$$\overline{R} = k_2 \cdot \overline{R_T} \Big/ \rho^4 \tag{25}$$

and at the base station C1 with average power

$$\overline{R_I} = \frac{k_1 \cdot \overline{R_T}}{D^4} = \frac{k_1}{k_2} \overline{R} \left(\frac{\rho}{D} \right)^4 \tag{26}$$

Distance D can be easily determined as

$$D^2 = L^2 + \rho^2 - 2L\rho\cos(\theta) \tag{27}$$

Denoting by $A(C_2)$ the area of a cell, the average interference power impinging on base C1 from a user located in cell C2 can be calculated as

$$\begin{aligned}\overline{P}_I &= \frac{k_2}{k_1}\overline{R}\frac{1}{A(C_2)}\iint_{C_2}\left(\frac{(X-x)^2+(Y-y)^2}{x^2+y^2}\right)^2 dxdy = \\ &= 2\frac{k_2}{k_1}\frac{\overline{R}}{\Pi d^2}\int_0^d\int_0^\Pi \frac{\rho^4}{\left(L^2+\rho^2-2L\rho\cos(\theta)\right)^2}\rho\, d\theta d\rho\end{aligned} \tag{28}$$

Normalizing the distances by substituting $x = \rho/L$ the average interference power generated by a single user is found to be

$$\begin{aligned}\overline{R}_I &= 2\frac{k_2}{k_1}\frac{\overline{R}}{\Pi d^2}\int_0^d\int_0^\Pi \frac{\rho^4}{\left(L^2+\rho^2-2L\rho\cos(\theta)\right)^2}\rho\, d\theta d\rho = \\ &= 2\frac{k_2}{k_1}\frac{\overline{R}}{\Pi\left(d/L\right)^2}\int_0^{d/L}\int_0^\Pi \frac{x^5}{\left(1+x^2-2x\cos(\theta)\right)^2} d\theta dx = 2\frac{k_2}{k_1}\frac{\overline{R}}{\Pi\left(d/L\right)^2}\int_0^{d/L}\int_{-\Pi/2}^{\Pi/2} \frac{x^5}{\left(1+x^2+2x\sin(\alpha)\right)^2} d\alpha dx = \\ &= 2\frac{k_2}{k_1}\frac{\overline{R}}{\Pi\left(d/L\right)^2}\int_0^{d/L}\frac{2x^5\left(1+x^2\right)}{\left(1-x^2\right)^3}\left(\arctan\left(\frac{1+x}{1-x}\right)+\arctan\left(\frac{1-x}{1+x}\right)\right)dx\end{aligned}$$

If we further note that the channel reuse distance can be determined from [6]

$$L = \sqrt{3K}\,d$$

where K is the cluster size in the channel reuse pattern we get

$$\overline{R}_I = 12\frac{k_2}{k_1}\frac{\overline{R}K}{\Pi}\int_0^{\frac{1}{\sqrt{3K}}}\frac{2x^5\left(1+x^2\right)}{\left(1-x^2\right)^3}\left(\arctan\left(\frac{1+x}{1-x}\right)+\arctan\left(\frac{1-x}{1+x}\right)\right)dx \tag{29}$$

Example numerical evaluation for K=7 and $k_1 = k_2$ results in $\overline{R_I} = \overline{R}\cdot 8.74\cdot 10^{-4}$.

The above results represent an average cochannel interference power generated by a single user. To find an average interference level we have to account for varying voice and data activity in the cochannel cells.

$$\overline{R}_{I_{TOTAL}} =$$

$$(K-1)\overline{R}_I \sum_{C_V=0}^{C} \left(\sum_{m=0}^{\infty} \frac{(\Lambda\tau)^m}{m!} e^{-\Lambda\tau} m \frac{k(C-C_V,\Lambda\tau)}{C-C_V} \frac{C-C_V}{C} P_{C_D}(C-C_V) + \frac{C_V}{C} P_{C_V}(C_V) \right) = \quad (30)$$

$$= \frac{(K-1)\overline{R}_I}{C} \sum_{C_V=0}^{C} \left(\Lambda\tau \cdot k(C-C_V,\Lambda\tau) + C_V \right) P_{C_V}(C_V)$$

Now the factor B in (18) can be determined from

$$B = 10^{\frac{S_N}{10}} \cdot \frac{\overline{R}_{I_{TOTAL}}}{\overline{R}} \tag{31}$$

where S_N is the desired minimum signal-to-interference level (in dB).

Glossary of symbols

τ	-	packet duration
λ	-	voice call origination rate ; offered voice traffic intensity
Λ	-	packet origination rate ; offered data traffic intensity
$1/\mu$	-	channel holding time for voice calls
C	-	total number of channels
C_V	-	number of channels in use by voice calls (a r.v.)
C_D	-	number of channels available for data transmission (a r.v.)
G	-	offered data traffic
S	-	data throughput
$k(C_D),k(C_D,G)$	-	number of packet copies in a transmission attempt
$p_T(C_D)$	-	probability that a terminal with a packet will attempt transmission
P_S	-	probability of packet success
$P_{s\mid r,C_D}$	-	conditional probability of packet success, conditioned on the number of users transmitting and the number of channels available for data transmission
$P_{s\mid C_D}$	-	conditional probability of packet success, conditioned on the number of channels available for data transmission
$P_{C_D}(m)$	-	probability that m channels are available for data transmission
$P_{C_V}(m)$	-	probability that m channels are in use by voice
p_{fdm}	-	probability of correlated fading
p_{fds}	-	probability of single packet fading in uncorrelated fading
$\underline{\mathbf{J}}$	-	interference vector in uncorrelated fading
$P_{\underline{\mathbf{J}}\mid m}$	-	probability that transmission attempts of m users will result in interference vector $\underline{\mathbf{J}}$
$P_{s\mid C_D,\underline{\mathbf{J}}}$	-	probability of packet success when other users' transmissions result in interference vector $\underline{\mathbf{J}}$ and there are C_D channels available for data transmissions
$\underline{\mathbf{n}}$	-	vector describing distribution of interfering packets on channels chosen by a particular user
$p_C(n)$	-	probability of packet capture when there are n other, simultaneous transmissions

$\overline{R}$	-	average received packet power
$f_{R_d}(\,)$	-	probability density of the power of the packet being captured
$f_{R_i}(\,)$	-	probability density of the power of interfering packets
B	-	background noise margin
z	-	capture ratio
P_F	-	probability of packet failure (in a transmission attempt)
R_a	-	activity factor
F	-	delay

Symbols used in Appendix D.

L	-	channel reuse distance
D	-	distance between an interferer and the base station
d	-	cell radius
θ	-	angle between C_1C_2 and C_2M
ρ	-	distance between an interferer and his own base station
$\overline{R}_I$	-	average interference power
$\overline{R}_T$	-	average transmitted power
k_1,k_2	-	propagation coefficients
A(C)	-	an area of a cell
K	-	cluster size
$\overline{R}_{I_{TOTAL}}$	-	average total received interference power
S_N	-	required signal-to-interference ratio