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A Model for Teletraffic Performance and Channel Holding Time Characterization in Wireless Cellular Communications with General Session and Dwell Time Distributions

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Abstract

Channel holding times and user mobility are important topics in the study of wireless cellular communications. We present an approach to modeling user mobility and session time which enables the calculation of teletraffic performance characteristics and a characterization of holding time which agrees with published reports. The model allows both the dwell time and unencumbered session time to have general distributions. A derivation of the channel holding time distribution is given. We then show how the model's parameters can be chosen to fit empirical data including observations of channel holding time.

I. Introduction

Future wireless communication systems will be called upon to deliver a variety of services (voice, data, video) to a variety of users (pedestrians, vehicles, computer terminals). The interaction of mixed users and mixed

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services within the same system is the focus of this paper. We propose an analytical model, based on multi-dimensional birth-death (MDBD) processes [1] for the computation of system performance characteristics.

Recent work has focused on developing analytical models to compute performance measures for wireless systems with various features: Multiple calls on a single platform [2], multiple platform types [3], platforms with highly variable mobilities [4], and mixed services and platform types [5]. We extend this body of work for systems with mixed services and mixed platforms types existing in the same system to include very general classes of *unencumbered session times* and *dwell times*. In addition we present a calculation of *channel holding time* characteristics that is consistent with the MDBD characterization. The *unencumbered session time* is defined as the amount of time that a user "intends" to remain active when his/her call is initiated [3]. This is equivalent to the holding time in traditional wireline teletraffic theory, wherein premature call termination and active calls vacating channels are not issues. The *dwell time* for a mobile user is the duration of time a user spends proximate to a base station so that a two-way link of acceptable quality can be maintained. The definition of dwell time includes all propagation effects such as path loss, fading, multipath and shadowing.

Current trends in cellular/personal communication systems contemplate increasing system capacity and offering services such as data, video and electronic mail in addition to conventional voice service. One method of increasing system capacity is to use micro-cells or pico-cells to provide coverage to the region. Small cell systems, tend to have more irregular shapes and cell sizes are more variable throughout the coverage area. Early work [6], [7] used negative exponential models for dwell times. Generalizations based on sums of exponential variates were used subsequently in [3]. The large degree of variability in the size and shape of micro-cells suggest that a more general model for the dwell time is needed. In addition, service providers are offering many more services to customers. These include, but are not limited to: fax, video, electronic mail, and data. While the negative exponential distribution is commonly used to model the duration of voice telephone calls, this model may not be appropriate for other cellular services. Therefore, we also consider a broad model for unencumbered session times in this paper. An overview of other common teletraffic modeling techniques used in cellular communications is contained in [8] and the references therein.

We characterize unencumbered session time and dwell time as sums of hyperexponential variables. Unen-

cumbered session time and dwell time are each modeled as having a special probability density function (pdf) which we call a SOHYP (**Sum Of HYP**erexponentials). The SOHYP pdf can be used to approximate the behavior of any positive random variable [4], [9]. Its use allows one to consider a broad class of pdf's while retaining the Markovian properties that are required by the MDBD framework. The basic model developed here applies to connection oriented systems. These include those which use circuit switching, virtual circuit switching, as well as systems which employ admission control.

We define the *channel holding time* as the amount of time a call **holds a channel in a cell**. The holding time is not the same as the unencumbered session time since the platform supporting a call may leave the cell – and the channel used by the call in that cell will be relinquished. We also emphasize that the phrase "leaving a cell" is interpreted broadly. That is, it corresponds to the expiration of the platform's dwell time in the cell. Owing to propagation effects, even a stationary platform can have a finite dwell time. Modeling both the session and dwell time as SOHYP random variables leads to an analytically tractable characterization of the channel holding time.

We derive an expression for the channel holding time distribution in terms of the unencumbered session and dwell time (SOHYP) parameters and system parameters. The overall result is an analytical model that can predict system performance for a large class of unencumbered session time and dwell time distributions. Relevant performance measures include: holding time, blocking probability, hand-off failure probability, forced termination probability, hand-off activity factor and carried traffic [6]. The model is used to investigate the relationship between session and dwell time distribution on the channel holding time.

II. Model Description

We consider a geographical region covered by cells and traversed by G types of platforms which generally have different mobility characteristics, i.e., distinct dwell time pdfs.. The dwell time for each platform type, $T_D(g)$, is assumed to be SOHYP distributed. Platform types are indexed by $g = 1, 2, ..., G$. We also assume that there are W types of calls that the system can handle. Call types are distinguished by their unencumbered session time characteristics. Generally, the unencumbered session time of each distinct call type, $T_C(w)$, is

distributed according to a distinct SOHYP pdf. The call types are indexed by $w = 1, 2, \ldots, W$. At any time a mobile platform can support one of the W call types. The unencumbered session time and dwell time distributions are described in detail in the following section.

For convenience, in the current paper it is assumed that all call types require the same resources for service. Different resource requirements for different call types are considered in [5], however, general unencumbered session time and dwell time distributions were not considered. The generalization (to different resource requirements) considered in [5] can be easily applied to the model put forth in this paper. The state variable characterization is essentially the same as described here, but the state space must be restricted by the resource constraints. Thus no increase in the dimensionality will result.

Each cell has C channels assigned to it and can therefore support at most C calls simultaneously. A *cutoff priority* scheme is used to give hand-off calls priority over new calls. For this purpose, C_h channels in each cell are reserved for calls that arrive to the cell as hand-offs. Individual channels are not reserved, just the number. The overall effect is that new calls that arise in a cell will be blocked if more than $C - C_h$ channels are in use, while hand-off calls will be served if fewer than C channels are in use. Channel quotas for call types and platform types can also be enforced. At each cell no more than $J(w, g)$ channels can be occupied by w-type calls on g-type platforms.

The following system parameters are needed in the analysis of the system

- $\Lambda(w, g)$ is the arrival rate of w-type calls on g-type platforms.
- $v(g, 0)$ is the number of g-type platforms present in the cell
- The total rate at which w-type calls arrive on g-type platforms can be expressed as $\Lambda_n(w, g) = \Lambda(w, g) \cdot v(g, 0)$

In the state variable description that follows we assume an "infinite population model" for which $v(g, 0) >> C$. This is usually the case. A finite population model, which is essentially similar, can be constructed but the dimension of the state space is increased.

III. Dwell Time and Unencumbered Session Time Distributions

The dwell time random variable for a platform of type g, $T_D(g)$, is modeled as the sum of $N_D(g)$ statistically independent hyperexponentially distributed random variables denoted $T_D(g,i)$, where $i = 1, 2, ..., N_D(g)$. Each $T_D(g,i)$ constitutes one phase of $T_D(g)$. So $T_D(g)$ has $N_D(g)$ phases.

Consider now one phase of $T_D(g)$, for example, $T_D(g,i)$. The pdf of the random variable $T_D(g,i)$ is a weighted sum of negative exponential functions having parameter $\mu_D(g,i,k)$ and has the following form known as the hyperexponential density

$$f_{T_D(g,i)}(t) = \sum_{k=1}^{M_D(g,i)} \alpha_D(g,i,k)\mu_D(g,i,k)\exp(-\mu_D(g,i,k)t) \tag{1}$$

Where the $\alpha_D(g,i,k)$'s add to unity, $\sum_{k=1}^{M_D(g,i)} \alpha_D(g,i,k) = 1$. We see from (1) that the index k runs from 1 to $M_D(g,i)$. Each negative exponential function in (1) is referred as a *stage* of the density and in general we have $M_D(g,i)$ stages for each $T_D(g,i)$. We say that $T_D(g,i)$ is a $M_D(g,i) - stage$ hyperexponential random variable.

As stated before the dwell time for a g-type platform is a sum of $N_D(g)$ independent hyperexponential variates.

$$T_D(g) = \sum_{i=1}^{N_D(g)} T_D(g,i) \tag{2}$$

We can make use of the concepts of phases and stages in order to conceptualize platform movement. Each random variable, $T_D(g,i)$, is considered a phase of $T_D(g)$ and each phase can have any number of stages indexed by $k = 1, 2, ..., M_D(g,i)$. $M_D(g,i)$ is the number of stages in the i^{th} phase for a g-type platform. A platform can be thought of as completing a series of phases as it traverses a cell. For a g-type platform we have a total of $N_D(g)$ phases. The length of time spent in each phase is a hyperexponential random variable ($T_D(g,i)$) consisting of $M_D(g,i)$ stages. Furthermore, we can interpret the $\alpha_D(g,i,k)$ in (1) as the probability that a g-type platform "chooses" stage k when it enters phase i. When a g-type platform enters a cell it begins its dwell time in the first phase ($i = 1$) where it chooses a stage from the $M_D(g,1)$ that are available for the

first phase. The choice of stage k is made according to the probability $\alpha_D(g,1,k)$. After the completion of its first phase of dwell time the platform enters the second phase ($i=2$), where it "chooses" a stage k from the $M_D(g,2)$ stages available in the second phase. The choice is made according to the $\alpha_D(g,2,k)$ probabilities. This process continues until the platform completes its final phase of dwell time, ($i=N_D(g)$). Completion of the final phase corresponds to the platform exiting the current cell (moving out of communication range of the base station) and entering a neighboring cell.

To obtain an expression for the pdf of $T_D(g)$ we make use of Laplace Transforms. The Laplace Transform of the hyperexponential pdf in (1) is denoted as $f^*_{T_D(g,i)}(\xi)$ and is given by

$$f^*_{T_D(g,i)}(\xi) = \sum_{k=1}^{M_D(g,i)} \frac{\alpha_D(g,i,k)\mu_D(g,i,k)}{\mu_D(g,i,k)+\xi} \tag{3}$$

Since $T_D(g)$ is a sum of $N_D(g)$ independent hyperexponential random variables the Laplace Transform of its density is the product of the component transforms, that is

$$f^*_{T_D(g)}(\xi) = \prod_{i=1}^{N_D(g)} f^*_{T_D(g,i)}(\xi) = \prod_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} \frac{\alpha_D(g,i,k)\mu_D(g,i,k)}{\mu_D(g,i,k)+\xi} \tag{4}$$

Equation (4) can be inverted to find the density of $T_D(g)$, however, if we add the restriction that all the $\mu_D(g,i,k)$ are distinct then (4) has a simple partial fraction decomposition given by

$$f^*_{T_D(g)}(\xi) = \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} \frac{A_D(g,i,k)}{\mu_D(g,i,k)+\xi} \tag{5}$$

where the $A_D(g,i,k)$s are the coefficients of the partial fraction decomposition and are given by

$$A_D(g,i,k) = \alpha_D(g,i,k)\mu_D(g,i,k) \cdot \prod_{\substack{j=1 \\ j\neq i}}^{N_D(g)} \sum_{l=1}^{M_D(g,j)} \frac{\alpha_D(g,j,l)\mu_D(g,j,l)}{\mu_D(g,j,l)-\mu_D(g,i,k)} \tag{6}$$

We identify (5) as a sum of negative exponential transforms and easily invert it to find the density of the dwell time which is given below

$$f_{T_D(g)}(t) = \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} A_D(g,i,k) \cdot e^{-\mu_D(g,i,k)t} \tag{7}$$

The cumulative distribution function (cdf) for $T_D(g)$, denoted by $F_{T_D(g)}(t)$, is found by integrating (7)

$$F_{T_D(g)}(t) = 1 - \left(\sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} a_D(g,i,k) \cdot e^{-\mu_D(g,i,k)t} \right) \quad \text{where } a_D(g,i,k) = \frac{A_D(g,i,k)}{\mu_D(g,i,k)} \tag{8}$$

We note that the above derivation of the density assumes that the $\mu_D(g,i,k)$s are all distinct. That is, no stage of any phase is identical to any other stage of any other phase. This is equivalent to requiring that the Laplace Transform of the density, given in (4) does not have a repeated pole. Mathematically this is expressed as,

$$\mu_D(g,i,k) \neq \mu_D(g,j,l) \quad \text{for } i \neq j \tag{9}$$

This ensures that the $A_D(g,i,k)$s, given in (6), remain finite. A similar approach (using Laplace Transforms and partial fraction decomposition) can be used to find the functional form of the SOHYP pdf and cdf when a $\mu_D(g,i,k)$ parameter is repeated. We emphasize that the approach and formulation are valid even if (9) is not satisfied. It is only the specific functional form of (7) that would require generalization. It is not difficult to do this generalization but it is algebraically cumbersome to write the general formula.

The unencumbered session time for a w-type call is also modeled as a SOHYP random variable. That is the unencumbered session time random variable, $T_S(w)$, is a sum of $N_S(w)$ hyperexponential random variables. To differentiate between unencumbered session time and dwell time parameters, we denote session time parameters as $\alpha_S(w,p,q)$ and $\mu_S(w,p,q)$ where $p = 1,2,\ldots,N_S(w)$ is the number of phases of session time and $q = 1,2,\ldots,M_S(w,p)$ is the number of stages for the p^{th} stage. A similar interpretation of the SOHYP distribution as it applies to unencumbered session time can be made. In this case, a call of type w is considered to complete a sequence of phases and stages before its completion. The parameter $\alpha_S(w,p,q)$ has a similar interpretation. It is the probability of a w-type call "choosing" stage q when it enters phase p.

Following the same approach given for the dwell time, the pdf of $T_S(w)$ is

$$f_{T_s(w)}(t) = \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} A_S(w,p,q) \cdot e^{-\mu_S(w,p,q)t} \tag{10}$$

where

$$A_S(w,p,q) = \alpha_S(w,p,q)\mu_S(w,p,q) \cdot \prod_{\substack{j=1 \\ j \neq p}}^{N_S(w)} \sum_{l=1}^{M_S(w,j)} \frac{\alpha_S(w,j,l)\mu_S(w,j,l)}{\mu_S(w,j,l) - \mu_S(w,p,q)} \tag{11}$$

The corresponding cdf is

$$F_{T_S(w)}(t) = 1 - \left(\sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} a_S(w,p,q) \cdot e^{-\mu_S(w,p,q)t} \right) \quad \text{where } a_S(w,p,q) = \frac{A_S(w,p,q)}{\mu_S(w,p,q)} \tag{12}$$

Again, equations (10) - (12) assume that the transform of the pdf does not have a repeated pole. For reference we state the counterpart of (9) for the unencumbered session time

$$\mu_S(w,p,q) \neq \mu_S(w,j,l) \quad \text{for } p \neq j \tag{13}$$

Figure 1 shows an example of a SOHYP pdf with two phases, the first is a negative exponential phase and the second is a two stage hyperexponential. The value κ^2 in the figure is the squared coefficient of variation. This is defined as the ratio variance of a random variable to the square of its mean. As we see the SOHYP pdf can have coefficients of variation which are larger than unity. Properties of the SOHYP random variables are discussed in [4].

In addition, we would like to note that the model has special cases. Both the negative exponential distribution and the sum of negative exponentials distribution can be obtained by considering special cases of the SOHYP distribution. A SOHYP random variable with a single phase consisting of a single stage is a negative exponential distributed random variable. A SOHYP random variable with multiple phases where each phase consists of a single stage is the sum of negative exponentials discussed in [3]. Furthermore, if all the phases are identical then we have an Erlang distribution. The distributions mentioned above have squared coefficients

of variation which are less than or equal to unity. So we see that use of the SOHYP modeling approach is broadly applicable.

IV. State Description

The state of a *single* cell is a sequence of integers. We define the state variable, v_{wpqgik}, $\{w = 1, \ldots, W;$ $p = 1, \ldots, N_C(w);\ q = 1, \ldots, M_S(w,p);\ g = 1, \ldots, G;\ i = 1, \ldots, N_D(g);\ k = 1, \ldots, M_D(g,i)\}$ to be the number of w type calls in phase p and stage q of unencumbered session time that are active on g type platforms in phase i and stage k of dwell time. The length of the sequence of integers, that specify the state, is the dimension of the state space. This dimension depends on the number of call types, W, the number of platform types, G, and the number of phases and stages required to describe each SOHYP distribution. The state of a cell can be written in the form of an array. Each position in the array represents a unique set of unencumbered session time parameters and dwell time parameters. Consider the array shown below, if the value of the first element in the first row is 3, $v_{111111} = 3$, this corresponds to 3 type 1 calls in phase 1 stage 1 of session time on board type 1 platforms in phase 1 stage 1 of dwell time. Changes in horizontal and vertical positions in the array correspond to different platform mobility parameters and session parameters respectively.

$$
\begin{array}{ccc}
v_{111111}\cdots & v_{111gN_D(g)M_D(g,N_D(g))}\cdots & v_{111GN_D(G)M_D(G,N_D(G))} \\
v_{112111}\cdots & v_{112gN_D(g)M_D(g,N_D(g))}\cdots & v_{112GN_D(G)M_D(G,N_D(G))} \\
\vdots & & \vdots \\
v_{11M_S(1,1)111}\cdots & v_{11M_S(1,1)gN_D(g)M_D(g,N_D(g))}\cdots & v_{11M_S(1,1)GN_D(G)M_D(G,N_D(G))} \\
\vdots & & \vdots \\
v_{1N_S(1)M_S(1,N_S(1))111}\cdots & v_{1N_S(1)M_S(1,N_S(1))gN_D(g)M_D(g,N_D(g))}\cdots & v_{1N_S(1)M_S(1,N_S(1))GN_D(G)M_D(G,N_D(G))} \\
\vdots & & \vdots \\
v_{wN_S(w)M_S(w,N_S(w))111}\cdots & v_{1N_S(w)M_S(w,N_S(w))gN_D(g)M_D(g,N_D(g))}\cdots & v_{wN_S(w)M_S(w,N_S(w))GN_D(G)M_D(G,N_D(G))} \\
\vdots & & \vdots \\
v_{WN_S(W)M_S(W,N_S(W))111}\cdots & v_{WN_S(W)M_S(W,N_S(W))gN_D(g)M_D(g,N_D(g))}\cdots & v_{WN_S(W)M_S(W,N_S(W))GN_D(G)M_D(G,N_D(G))}
\end{array}
$$

The set of permissible states corresponds to all values of the array which satisfy all resource constraints [3]. It is convenient to order the states using the index s, $s = 0, 1, \ldots, s_{\max}$. The state variables v_{wpqgik} can be expressed as function of the state. The representation, $v(s, w, p, q, g, i, k)$ is the number of w-type calls in phase p stage q of session time on board g-type platforms in phase i stage k of dwell time when the cell is in state s. With this state variable description a number of characteristics can be determined for each state s. The number of channels in use by w-type calls on board g-type platforms is

$$j(s, w, g) = \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} v(s, w, p, q, g, i, k) \tag{14}$$

The number of channels in use by w-type calls is then

$$j(s, w) = \sum_{g=1}^{G} j(s, w, g) \tag{15}$$

and the total number of channels in use when the cell is in state s is

$$j(s) = \sum_{w=1}^{W} j(s, w). \tag{16}$$

A *permissible state* is any sequence of integers which satisfies the system constraints. That is $j(s) \leq C$ and $j(s, w, g) \leq J(w, g)$ for $w = 1, 2, \ldots, W$ and $g = 1, 2, \ldots, G$.

We note that a thorough formulation requires the identification of the **system state,** which is the **concatenation of all the cell states** in the region of coverage. Because of hand-off, the individual *cell state* transitions are coupled to one another. So a rigorous analysis requires the consideration of **all possible system states**. This approach, however, is not feasible because of the overwhelming dimensions and computational complexity involved. The approach put forth in [2] [3] [10] is used. Specifically, a single cell is isolated and its interaction with neighboring cells is represented with *averaged* hand-off arrival rates that are related to *averaged* hand-off departure rates. (For homogenous systems these rates must be equal. Nonhomogenous systems can be analyzed similarly [2] [3]). Formulation of the flow balance equations results in a system of

nonlinear equations to be solved for the cell state probabilities. Even with this simplification the number of states needed to describe a single cell can be formidable. An example with one call type and one platform be expressed as function of the state. The representation, $v(s, w, p, q, g, i, k)$ is the number of w-type calls in

Table1 : Number of cell states needed

C	4	6	8	10	12
number of states	714	5004	24309	92377	293929

$W = 1, N_S(1) = 2, M_S(1,1) = 2, M_S(1,2) = 1$
$G = 1, N_D(1) = 2, M_D(1,1) = 2, M_D(1,2) = 1$

The size of the state space grows quickly while the number of channels increases. Since we are usually dealing with personal communication systems, which use micro-cells or pico-cells, the number of channels per cell is normally low. However, for certain model parameters the number of states required can still be prohibitive.. Such situations can be handled by computing results for a system with fewer channels per cell and extrapolating results for the larger sized state space. This technique is described in [3]. It was shown that by keeping the offered load per channel constant performance characteristics for systems with a large number of channels could be extrapolated from results computed for smaller systems.

V. Driving processes and state transitions

For the system under consideration there are six driving processes that can be identified. These are: A) generation of new calls, B) completion of calls, C) arrival of hand-offs, D) departure of hand-offs, E) transition between session time phases, F) transition between dwell time phases. Since we consider multiple platform and call types, all of these processes are multidimensional. In any state, the new call arrival and hand-off arrival processes Poisson point processes. Markovian assumptions are made. These in addition to the SOHYP characterization of the state space allow the problem to be cast in the multidimensional birth-death framework [3], [10]

The solution for the cell state probabilities and the related performance measures follows the approach given in [3]. The identification of predecessor states and the corresponding state probability transition flows follows.

This leads to the flow balance equations, a set of nonlinear simultaneous equations which are then solved iteratively[2] [3] [10]. Details are given in [13]. The results of the solution yield the following: 1) Equilibrium state probabilities, $P(s)$; 2) Average hand-off call arrival rate (to a cell) of w-type calls on board g-type platforms, $\Lambda_h(w,g)$; 3) Average fraction of $\Lambda_h(w,g)$ that are in phase p stage q of unencumbered session time. Which is denoted $\Phi(w,p,q,g)$.

VI. Channel holding time

Channel holding time is defined as the amount of time that a call occupies a channel in a particular cell. This depends on several factors. A channel may be used by any of the various types of calls each having its own distinct session time distribution. In addition, a call may be on board any of the various platform types with each type having certain mobility characteristics as specified by a distinct dwell time distribution. Consider a call (of type w on board a platform of type g) which has just begun service in a cell. That is the call could have arrived as a new call or as a hand-off call. Calls which are new calls have an unencumbered session time phase index $p = 1$. Those which are hand-off calls have a dwell time phase index $i = 1$. Of all such calls, some will complete in the current cell. Others will be on board platforms which leave the cell before the call is completed. We will be interested in the overall channel holding time distribution (regardless of whether the call completes in the cell), as well as in the conditional channel holding time distribution (for those calls which complete in the current cell).

A. Overall Channel Holding Time Distribution

There is a distinct holding time distribution for each call type on board each platform type. For the system under consideration, we can identify $W \cdot G$ holding time distributions. That is each w-type call ($w = 1, 2, ...W$) on board a g-type platform ($g = 1, 2, ..., G$) gives rise to a distinct holding time distribution. We denote the holding time of a w-type call on board a g-type platform as $H(w,g)$. The holding time $H(w,g)$ depends on the session time of the particular call type, $T_S(w)$, the dwell time of the platform, $T_D(g)$ and the system performance.

In each cell, calls can be categorized as: 1) calls that arrive due to hand-offs from neighboring cells or 2)

new calls, which originate within the cell. We define $H_h(w,g)$ as the holding time of a w-type hand-off calls on g-type platforms and we denote $H_n(w,g)$ as the holding time of a w-type new call on a g-type platform. Throughout the remainder of this section we will assume the Laplace Transforms of the dwell time and session time pdfs not do not have repeated poles. That is, equations (9) and (13) hold. For the development that follows, it is also helpful to keep in mind the physical interpretation of SOHYP distributed random variables as they apply to unencumbered session time and dwell time. Specifically, a new call arising in a cell will always begin in phase 1 of unencumbered session time but can be on board a platform in any phase-stage of dwell time. A hand-off call entering a cell will always begin in phase 1 of dwell time but can be in any phase-stage of unencumbered session time.

A.1 New call holding time

A new call may begin service during any phase and stage of its platform's dwell time. It will remain active in the cell until the remaining dwell time expires or the call is completed (the unencumbered session time expires). For a g-type platform in phase i stage k we define $R_D(g,i,k)$ as the remaining dwell time of the platform. A g-type platform in phase i stage k must complete its current phase and stage and the $N_D(g)-i+1$ remaining phases before exiting the cell. So, $R_D(g,i,k)$ can be expressed as a sum of random variables

$$R_D(g,i,k) = T_D(g,i,k) + \sum_{j=i+1}^{N_D(g)} T_D(j) \tag{17}$$

$T_D(g,i,k)$ in (17) is a negative exponential random variable associated with the platform's k^{th} stage of phase i. That is the pdf of $T_D(g,i,k)$ is

$$f_{T_D(g,i,k)}(t) = \mu_D(g,i,k) \cdot e^{-\mu_D(g,i,k)t} \tag{18}$$

The components $T_D(j)$, where $j = i, i+1, ..., N_D(g)$, are the remaining hyperexponential phases of the dwell time. So we see that $R_D(g,i,k)$ as defined in (17) is also a SOHYP distributed random variable whose first phase ($T_D(g,i,k)$) is n.e.d. Therefore the pdf, $f_{R_D(g,i,k)}(t)$, can be written as a sum of negative exponential

functions as in (7). Computing the convolution of the component pdfs in (17) we obtain

$$f^*_{R_D(g,i,k)}(\xi) = \frac{\mu_D(g,i,k)}{\mu_D(g,i,k)+\xi} \prod_{j=i+1}^{N_D(g)} \sum_{l=1}^{M_D(g,j)} \frac{\mu_D(g,j,l)}{\mu_D(g,j,l)+\xi} \tag{19}$$

This can be factored as

$$f^*_{R_D(g,i,k)}(\xi) = \frac{B_D^{g,i,k}(i,k)}{\mu_D(g,i,k)+\xi} + \sum_{j=i+1}^{N_D(g)} \sum_{l=1}^{M_D(g,j)} \frac{B_D^{g,i,k}(j,l)}{\mu_D(g,j,l)+\xi} \tag{20}$$

The $B_D^{g,i,k}(j,l)$ are the coefficients of the partial fraction expansion of equation (19). The superscripts g,i,k are used to explicitly show that the coefficients are functions of the starting phase and stage. So for each starting phase i stage k, $R_D(g,i,k)$ has a distinct pdf. We can invert (20) to obtain

$$f_{R_D(g,i,k)}(t) = B_D^{g,i,k}(i,k)e^{-\mu_D(g,i,k)} + \sum_{j=i+1}^{N_D(g)} \sum_{l=1}^{M_D(g,j)} B_D^{g,i,k}(j,l)e^{-\mu_D(g,j,l)} \tag{21}$$

This can be written in a more compact form, which is similar to (7) and (10), by the following

$$f_{R_D(g,i,k)}(t) = \sum_{j=i}^{N_D(g)} \sum_{l=\begin{cases} k \text{ if } j=i \\ 1 \text{ if } j>i \end{cases}}^{M'_D(g,j)} B_D^{g,i,k}(j,l) \cdot e^{-\mu_D(g,j,l)t} \tag{22}$$

where

$$M'_D(g,j) = \begin{cases} k \text{ if } j = i \\ M_D(g,j) \text{ if } j > i \end{cases} \tag{23}$$

With the pdf of the remaining dwell time computed, we now define $H_n(w,g,i,k)$ as the holding time of a w-type new call which begins service on board a g-type platform in phase i stage k of dwell time. Since the call will either complete in the cell or leave the cell after $R_D(g,i,k)$ expires, we have

$$H_n(w,g,i,k) = \min(T_S(w), R_D(g,i,k)) \tag{24}$$

Since $T_S(w)$ and $R_D(g,i,k)$ are independent random variables, we have [11]

$$f_{H_n(w,g,i,k)}(t) = f_{T_S(w)}(t)\left(1 - F_{R_D(g,i,k)}(t)\right) + f_{R_D(g,i,k)}(t)\left(1 - F_{T_S(w)}(t)\right) \tag{25}$$

In (25),$F_{R_D(g,i,k)}(t)$, is the cdf of $R_D(g,i,k)$ and is easily obtained from (22).

The probability that a g-type platform is in phase i stage k is denoted by $\rho_n(g,i,k)$. This is given by

$$\rho_n(g,i,k) = \frac{\alpha_D(g,i,k)/\mu_D(g,i,k)}{\overline{T}_D(g)} \tag{26}$$

The holding time pdf for a new w-type call on board a g-type platform, $f_{H_n(w,g)}(t)$, is therefore given by

$$f_{H_n(w,g)}(t) = \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} \rho_n(g,i,k) \cdot f_{H_n(w,g,i,k)}(t) \tag{27}$$

A.2 Hand-off call holding time

The analysis of holding time for a hand-off call follows similar arguments. A hand-off call of any unencumbered session type, say w, may enter a cell in any phase p stage q of session time. It will hold its channel until its remaining unencumbered session time expires or until it completes its full dwell time, $T_D(g)$. We define $R_S(w,p,q)$ as the remaining unencumbered session time of a w-type call given that it is currently in phase p stage q. $R_S(w,p,q)$ is again a SOHYP random variable whose first phase is n.e.d.

$$R_S(w,p,q) = T_S(w,p,q) + \sum_{j=p+1}^{N_S(w)} T_S(j) \tag{28}$$

Its pdf is derived in the manner of (22) we give the following result

$$f_{R_S(w,p,q)}(t) = \sum_{j=p}^{N_S(w)} \sum_{l=\begin{cases} q \text{ if } j=p \\ 1 \text{ if } j>p \end{cases}}^{M'_S(w,j)} B_S^{w,p,q}(j,l) \cdot e^{-\mu_S(g,j,l)t} \tag{29}$$

where

$$M'_S(w,j) = \begin{cases} q \text{ if } j = p \\ M_S(w,j) \text{ if } j > p \end{cases} \tag{30}$$

We now have $H_h(w,p,q,g)$, the holding time of w-type hand-off calls in phase p stage q on board g-type platforms. A call that arrives at a cell as a hand-off will either complete, when its remaining unencumbered session time expires, or will leave the cell when its dwell time expires. Therefore, $H_h(w,p,q,g)$ is given by

$$H_h(w,p,q,g) = \min(R_S(w,p,q), T_D(g)) \tag{31}$$

Using the independence of $R_S(w,p,q)$ and $T_D(g)$ the pdf of $H_h(w,p,q,g)$ is

$$f_{H_h(w,p,q,g)}(t) = f_{R_S(w,p,q)}(t)\left(1 - F_{T_D(g)}(t)\right) + f_{T_D(g)}(t)\left(1 - F_{R_S(w,p,q)}(t)\right) \tag{32}$$

The fraction of w-type hand-off calls that arrive in phase p stage q on board g-type platforms is $\Phi(w,p,q,g)$. Note that this is the same fraction that appears in the iterative solution for the state probabilities discussed previously. So we can now write the pdf of $H_h(w,g)$ as

$$f_{H_h(w,g)}(t) = \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} \Phi(w,p,q,g) \cdot f_{H_h(w,p,q,g)}(t) \tag{33}$$

A.3 Total holding time and observations

With both hand-off and new call holding time specified we can develop an expression for the pdf of $H(w,g)$, the channel holding time of a w-type call on board a g-type platform. New calls of type w on board g-type platforms arrive with rate $\Lambda_n(w,g)$, such a call will be served with probability $1 - P_B(w,g)$. Where $P_B(w,g)$ is the blocking probability of a new w-type call which arises on a g-type platform. The computation of the blocking probability is discussed in the following section on performance measures. Hand-off calls of type w on board g type platforms arrive with rate $\Lambda_h(w,g)$. Such a call will be served with probability $1 - P_H(w,g)$. ($P_H(w,g)$is the hand-off failure probability for a w-type call on a g-type platform). It computation is also

discussed in the following section. The total call demand impinging on a cell the sum of the arrival rate of new and hand-off calls, $\Lambda_n(w,g)[1-P_B(w,g)]+\Lambda_h(w,g)[1-P_H(w,g)]$. The pdf of $H(w,g)$ is a mixture of the new and hand-off components expressed as follows

$$\begin{aligned} f_{H(w,g)}(t) &= \frac{\Lambda_n(w,g)[1-P_B(w,g)]}{\Lambda_n(w,g)[1-P_B(w,g)]+\Lambda_h(w,g)[1-P_H(w,g)]}\cdot f_{H_n(w,g)}(t) \\ &+\frac{\Lambda_h(w,g)[1-P_H(w,g)]}{\Lambda_n(w,g)[1-P_B(w,g)]+\Lambda_h(w,g)[1-P_H(w,g)]}\cdot f_{H_h(w,g)}(t) \end{aligned} \tag{34}$$

Equation (34)is the overall holding time pdf of w-type calls on board g-type platforms. The terms before the component pdfs are simply the fraction on calls that are present in the cell due to the arrival of new calls and hand-off calls respectively.

We emphasize that the channel holding time distribution depends on the session time SOHYP parameters, the dwell time SOHYP parameters and the system performance measures. The system performance measures are also a function of the session and dwell time parameters and the system characteristics (number of channels C, cutoff priority C_h, etc.). These dependencies are captured in (34) and the iterative solution to the state probabilities. We see from (34) that in general the computation of the channel holding time distribution requires the computation of other system performance measures such as blocking and hand-off failure probabilities and the hand-off arrival rates.

B. Channel holding time distribution for calls which complete

A call completing in its current cell corresponds to a given set of conditions. For example, if a w-type call enters the cell as a hand-off call on board a g-type platform and the unencumbered session time is currently in phase p stage q then the event $\{R_S(w,p,q) \leq T_D(g)\}$ will cause the call to complete in the current cell. That is, for hand-off calls, the remaining unencumbered session time, $R_S(w,p,q)$, is less than the dwell time $T_D(g)$. Let us also define, $ß_H(w,p,q,g)$, as the event that an active w-type hand-off call in phase p stage q of unencumbered session time on board a g-type platform completes. Mathematically this is

$$ß_H(w,p,q,g) = \{R_S(w,p,q) \leq T_D(g)\} \tag{35}$$

The corresponding event for new w-type calls on board g-type platforms in phase i stage k is denoted as $ß_N(w,g,i,k)$ and is given by

$$ß_N(w,g,i,k) = \{T_S(w) \leq R_D(g,i,k)\} \tag{36}$$

Again we note that there are two types of calls: hand-off calls and new calls. We will derive the conditional distribution, $F_{H_h}(t|ß_H(w,p,q,g))$. A $w-$type hand-off call on board a g-type platform which arrives in phase p stage q completes if its remaining session time, $R_S(w,p,q)$, is less than its dwell time $T_D(g)$. From the definition of $H_h(w,p,q.g)$ and the condition $\{R_S(w,p,q) \leq T_D(g)\}$we have

$$\begin{aligned} F_{H_h(w,p,q,g)}(t|ß_H(w,p,q,g)) &= \Pr\{H_h(w,p,q,g) \leq t|ß_H(w,p,q,g)\} \\ &= \Pr\{\min(R_S(w,p,q),T_D(g)) \leq t|ß_H(w,p,q,g)\} \\ &= \Pr\{R_S(w,p,q) \leq t|ß_H(w,p,q,g)\} \\ &= F_{R_S(w,p,q)}(t|R_S(w,p,q,g) \leq T_D(g)) \end{aligned} \tag{37}$$

So we conclude that the conditional distribution of completing w-type hand-off calls arriving in phase p stage q on board g-type platforms is equivalent to the conditional distribution of the remaining session time given that it is less than the dwell time. This conditional cdf is given by

$$F_{H_h(w,p,q,g)}(t|ß_H(w,p,q,g)) = \int_0^t \int_{\xi_1}^{\infty} f_{R_S(w,p,q)}(\xi_1) f_{T_D(g)}(\xi_2) d\xi_2 d\xi_1 \tag{38}$$

The event that a w-type hand-off call on board g-type platform completes is expressed as $ß_H(w,g) = \bigcup_{p=1}^{N_S(w)} \bigcup_{q=1}^{M_S(w,p)} ß_H(w,p,q,g)$. The conditional channel holding time distribution for a w-type hand-off call (which is in service in the cell) on board g-type platforms is then given by

$$F_{H_h(w,g)}(t|ß_H(w,g)) = \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} \Phi(w,p,q,g) \cdot F_{H_h(w,p,q,g)}(t|ß_H(w,p,q,g)) \tag{39}$$

A similar analysis is required for the conditional distribution of a new call which completes in the cell. The

overall conditional holding time is the combination of the new and hand-off components as in (34). The details of the calculation are given in [13].

VII. Other System Performance Measures

The approach outlined above was used to calculate equilibrium state probabilities. From these, various performance measures can be determined.

A. Blocking probability

The blocking probability of a new w-type call from a g-type platform is the average fraction of new w-type call attempts from g-type platforms which are denied access to a channel. Blocking of a new w-type call attempting to gain a channel from a g-type platform occurs if there are no channels available to serve the call or if the system's channel quotas are already full. We define the following disjoint sets

$$B_0 = \{s : C - C_h \leq j(s) \leq C\} \tag{40}$$

$$B_J = \{s : j(s) < C - C_h, j(s,w,g) = J(w,g)\} \tag{41}$$

Then the blocking probability for w-type calls on board g-type platforms can be written as

$$P_B(w,g) = \sum_{s \in B_0} P(s) + \sum_{s \in B_J} P(s) \tag{42}$$

B. Hand-off failure probability

The hand-off failure probability for w-type calls being served on g-type platforms is defined as the average fraction of hand-off attempts that are denied a channel. A hand-off attempt of a w-type call on board a g-type platform will fail if no channels are a available in the target cell (recall that hand-off attempts have access to all C channels) or if the cell's channel quota is already full. We have the following disjoint sets

$$H_0 = \{s : j(s) = C\} \tag{43}$$

$$H_J = \{s : j(s) < C, j(s,w,g) = J(w,g)\}$$

The hand-off failure probability for w-type calls served on g-type platforms is

$$P_H(w,g) = \sum_{s \in H_0} P(s) + \sum_{s \in H_J} P(s) \tag{44}$$

C. Carried traffic

The carried traffic per cell for each call and platform type is the average number of channels occupied by the calls from the given platform type. The carried traffic for w-type calls on board g-type platforms is

$$A_c(w,g) = \sum_{s=0}^{s_{max}} j(s,w,g) \cdot P(s) \tag{45}$$

the total carried traffic is given by

$$A_c = \sum_{w=1}^{W} \sum_{g=1}^{G} A_c(w,g) \tag{46}$$

D. Forced termination probability

The forced termination probability, $P_{FT}(w,g)$, is defined as the probability that a w-type call on board a g-type platform, which is initially admitted into the system, is subsequently interrupted due to a hand-off failure during its lifetime. For convenience the following discussion will consider that the dwell time and unencumbered session time pdfs can be written in the form of (7) and (10) respectively. A *new* w-type call can begin service during any phase i stage k of its originating platform's dwell time. In order for the call to generate a hand-off attempt, its session time must be greater than the platform's remaining dwell time. So for new a w-type call on board a g-type platform in phase i stage k we define $U(w,g,i,k)$ as the probability that the call will require a hand-off

$$\begin{aligned} U(w,g,i,k) &= \Pr\{R_D(g,i,k) \leq T_S(w)\} \\ &= \int_0^{\infty} \int_0^{\xi_2} f_{R_D(g,i,k)}(\xi_1) f_{T_S(w)}(\xi_2) d\xi_1 d\xi_2 \end{aligned} \tag{47}$$

$$= 1 - \sum_{j=i}^{N_D(g)} \sum_{l=\left\{\begin{smallmatrix} k \text{ if } j=i \\ 1 \text{ if } j>i \end{smallmatrix}\right.}^{M'_D(g,j)} \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} \frac{A_S(w,p,q) B_D^{g,i,k}(j,l)}{[\mu_D(g,j,l) + \mu_S(w,p,q)]\mu_D(g,j,l)}$$

The probability that a new w-type call on board a g-type platform requires a hand-off is given by

$$U(w,g) = \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g.i)} \rho_n(g,i,k) U(w,g,i,k) \tag{48}$$

where $\rho_n(g,i,k)$ is given by (26). A *hand-off* call enters its target cell in the supporting platform's first phase of dwell time. It will be in an arbitrary phase p stage q of its unencumbered session time, so for a hand-off call which enters its target cell in phase p stage q to generate a hand-off attempt the remaining unencumbered session time must be greater than the dwell time. It is assumed that as a platform moves from cell to cell the probability of a hand-off failure, $P_H(w,g)$, is independent of previous hand-off successes for the same call. This is reasonable because ordinarily there well be many *system* state transitions occurring between successive hand-off attempts of a single cell. We have the probability that a w-type hand-off call in phase p stage q of session time on board a g-type platform requires another hand-off is

$$\begin{aligned} V(w,p,q,g) &= \Pr\{T_D(g) \le R_C(w,p,q)\} \\ &= 1 - \sum_{j=i}^{N_S(w)} \sum_{l=\left\{\begin{smallmatrix} q \text{ if } j=p \\ 1 \text{ if } j>p \end{smallmatrix}\right.}^{M'_S(w,p)} \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} \frac{B_S^{w,p,q}(j,l) a_D(g,i,k)}{\mu_D(g,i,k) + \mu_S(w,j,l)} \end{aligned} \tag{49}$$

The probability that a w-type hand-off call will require another hand-off is

$$V(w,g) = \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} \Phi(w,p,q,g) V(w,p,q,g) \tag{50}$$

Where $\Phi(w,p,q,g)$ is the fraction of hand-offs of w-type calls that arrive in phase p stage q of session time on board g-type platforms.

The probability that the call is forced to terminate on its jth hand-off attempt is

$$Y_j(w,g) = U(w,g) \cdot P_H(w,g) \cdot \{V(w,g) \cdot (1 - P_H(w,g)\}^{j-1} \tag{51}$$

The forced termination probability of w-type calls on g-type platforms is

$$P_{FT}(w,g) = \sum_{j=1}^{\infty} Y_j(w,g) \tag{52}$$

This can be expressed in closed form as

$$P_{FT}(w,g) = \frac{U(w,g) \cdot P_H(w,g)}{1 - \psi(w,g)} \text{ , where } \psi(w,g) = V(w,g) \cdot (1 - P_H(w,g)) \tag{53}$$

E. Hand-off activity factor

The hand-off activity factor, $\eta(w,g)$, is defined as the expected number of hand-off attempts for nonblocked w-type calls on board g-type platforms. Let $\delta(w,g)$ be the probability that a w-type call on a g-type platform which requires a hand-off will not require another hand-off. Then

$$\delta(w,g) = P_H(w,g) + [1 - P_H(w,g)] \cdot [1 - V(w,g)] \tag{54}$$

We can write an expression for ,$\Gamma(w,g,j)$, which is the probability that a w-type call on board a g-type platform requires exactly j hand-offs before ending (either by completion or forced termination).

$$\Gamma(w,g,j) = U(w,g) \cdot \delta(w,g) \cdot [\psi(w,g)]^{j-1} \tag{55}$$

The hand-off activity factor is

$$\eta(w,g) = \sum_{j=1}^{\infty} j \cdot \Gamma(w,g,j) \tag{56}$$

This can be expressed in closed form as

$$\eta(w,g) = \frac{U(w,g)}{[1-\psi(w,g)]} \tag{57}$$

VIII. Discussion of results

A. Case study: Observed log-normal holding times

A study of a channel holding times for a cellular telephone system in Canada is reported in [12], where it is suggested that the observed holding times can be modeled using a log-normal distribution. We will show that a cellular system with SOHYP distributed dwell times, as described in section 2, produces a similar call holding time distribution. The log-normal result reported in [12] is actually the holding time distribution of calls that complete in the cell of interest. The authors state that they discarded some of the data they collected in order to smooth the resulting histogram. The cellular system they studied handled hand-off requests at discrete time points. They report seeing a large number of channel releases after 7 seconds, 17 seconds, 27 seconds,etc.. They attribute this to the call requiring a hand-off and waiting for the system to exchange signaling information between base stations. By discarding these data points the holding time of calls that released their channel because of hand-off were not considered. We assume that the remaining observed holding times are for those calls that *complete in the cell.* This corresponds to the development of conditional channel holding time given in section VII, B. In summary, we interpret the results reported in [12] as follows: For an active call, the conditional distribution of channel holding time given that the call completes in its current cell is approximately log-normal. Let ß denote the event that an active call completes in its current cell. The mathematical essence of [12] is

$$f_H(t|ß) \simeq f_L(t) = \left(1/\sigma t\sqrt{2\pi}\right) e^{-(\ln t-\beta)} \tag{58}$$

Equation (58) is the log-normal pdf with parameters σ and β. The corresponding cdf can be expressed in terms of the standard normal distribution, $\tilde{G}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\xi^2/2} d\xi$ [11]. That is,

$$F_L(t) = \tilde{G}\left(\frac{\ln t - \beta}{\sigma}\right) \tag{59}$$

We propose that a system in which the unencumbered session and dwell time distributions are SOHYP leads to a channel holding time distribution that is in agreement with (59). The procedure shown in figure 2 can be used to fit our calculated holding time distribution to observed holding time data. The derived holding time pdf given in (34) is holding of all calls whether they complete in the cell or exit the cell. To compare with (59) we need the conditional density that a call completes.

We can now fit the conditional channel holding time distribution of completing calls, $F_H(t|ß)$ to the log-normal cdf given in [12]. Ideally, $F_H(t|ß)$ should be fit to data. Since we do not have access to the raw data used in [12] we take a different approach. We find the parameters of $F_H(t|ß)$ that minimize the mean square error between the log-normal cdf and $F_H(t|ß)$. We define the mean square error, E, as

$$E = \int_0^\infty (F_L(t) - F_H(t|ß))^2 \cdot f_L(t) dt \tag{60}$$

We assume that the cellular system is homogenous with each base station having: $C = 10$ channels, no cut-off priority $C_h = 0$, new call origination rate $\Lambda = 2.75e - 4$ and the number of noncommunicating platforms is $v(1,0) = 100$. One platform type is assumed to be present and a single call type is assumed present $G = 1, W = 1$. For calculation purposes we use a two phase SOHYP for the , $N_D(1) = 2$, where the first phase has one stage, $M_D(1,1) = 1$, and the second phase has two stages, $M_D(1,2) = 2$. This allows four parameters, $\mu_D(1,1,1), \alpha_D(1,2,1), \mu_D(1,2,1), \mu_D(1,2,2)$, to be used for fitting to the log-normal holding time. The log-normal parameters (β,σ) are reported in [12]. For a small cell size located in downtown Vancouver the authors report $\beta = 3.37$ and $\sigma = 1.25$. We have assumed that the average dwell time is equal to thirty seconds, $\bar{T}_D(1) = 30.0$. This corresponds to automobiles traveling through cells with radius of a mile or less.

To hold the mean dwell time constant we set the $\mu_D(1,1,1)$ parameter to

$$\mu_D(1,1,1) = \left(\bar{T}_D(1) - \frac{\alpha_D(1,2,1)}{\mu_D(1,2,1)} - \frac{\alpha_D(1,2,2)}{\mu_D(1,2,2)}\right)\Big|_{\bar{T}_D(1)=30} \tag{61}$$

The authors of [12] do not mention unencumbered session time or anything about the total duration of calls in the system. We assume the unencumbered session time, $T_S(1)$, is n.e.d. and we allow the parameter $\mu_S(1,1,1)$ to be adjusted to the log-normal distribution.

Using the above assumptions, equation (60) can be minimized with respect to the parameters: $\mu(1,1,1)$, $\alpha_D(1,2,1), \mu_D(1,2,1), \mu_D(1,2,2)$. The problem of minimizing (60) is a multidimensional optimization. We modified and used the "Downhill Simplex Method" described in [14] to perform the minimization. To represent the system for the parameters $G = 1$, $W = 1$, $N_D(1) = 2$, $M_D(1,1) = 1$, $M_D(1,2) = 2$, three state variables and a total of 286 states were required. The fitting procedure was carried out on a SparcUltra 2 Workstation and results for figure 3 were obtained in approximately 10 hours. Table 2 gives the calculated dwell time parameters, hand-off arrival rate, unencumbered session time parameter and blocking and forced termination probabilities.

Table 2: System Performance and Fitted SOHYP Parameters

Parameter	$\mu_D(1,1,1)$	$\alpha_D(1,2,1)$	$\mu_D(1,2,1)$	$\mu_D(1,2,2)$	$\mu_S(1,1,1)$	$\Lambda_h(1,1)$	$P_B(1,1)$	$P_{FT}(1,1)$
Value	7.863	4.379×10^{-1}	1.458×10^{-2}	8.665	7.351×10^{-3}	0.1221	3.211×10^{-2}	1.831×10^{-2}

Figure 3 is a plot the log-normal cdf with the above parameters and ours based on SOHYP dwell times with parameters shown in Table 2. The parameters shown in Table 2 give a mean square error, as defined in (60), of 5.13E-04 between log-normal and our holding time distribution. We see that we have relatively good agreement with the observed cdf. In addition we are able to compute teletraffic performance measures during the fitting process. A smaller mean square error could be achieved by increasing the number of phases and stages of the SOHYP dwell time pdf. This would give us more parameters to adjust. The unencumbered session time does not have to be restricted to the negative exponential distribution but could have a more general SOHYP distribution as well. This also allows for the introduction of more parameters into the model.

We emphasize that the fit is to the log-normal cdf and not to the original data which is not available to us.

The curves shown in figure 3 are meant to suggest that SOHYP dwell and session times lead to reasonable channel holding time distributions. SOHYP based model of holding time has advantages since it allows the use of the MDBD framework for computation of system performance and the introduction of many parameters that can be used to fit to data.

B. SOHYP unencumbered session times, negative exponential dwell times

Figures 4 through 6 show the performance measures for a cellular system in which session times are SOHYP distributed. We consider a system with 20 channels ($C = 20$), a single call type and single platform type. The number of noncommunicating platforms per cell was taken as $v(1,0) = 400$ and the dwell time was assumed to be exponential with mean $\bar{T}_D(1) = 200$. The unencumbered session time was considered to be SOHYP with a mean of $\bar{T}_S(1) = 100$ and we varied the squared coefficient of variation, $\kappa_S^2(1)$ from 0.8 to 4.0. C_h was also varied from 0 to 4. Three state variables and 1771 states are required to represent this system. No quota constraints were considered for the system. The results are plotted for increasing new call arrival rate, $\Lambda(1,1)$.

We see from figure 4 that the blocking probability is extremely insensitive to the variance of the session time. Forced termination probabilities decrease slightly as the coefficient of variation increases. This is expected since shorter unencumbered session times can be expected and call complete more rapidly. The change is very slight and forced termination probability can be considered to be insensitive to the coefficient of variation. We also see in the figure the trade-off between blocking and forced termination probabilities. As C_h is increased hand-offs are protected at the expense of new call blocking.

Figure 5 shows carried traffic for the system. It was found that carried traffic is insensitive to session time coefficient of variation. A more interesting result is shown in figure 6. Here we see that the hand-off activity is dependent on the coefficient of variation. For large the hand-off activity decreases. This is because the pdf is spread out about its mean and shorter session times are more likely as $\kappa_S^2(1)$ increases. The effect of reserving channels for hand-off can be seen when demand is high. For $C_h = 0$ hand-off activity decreases with demand since calls requiring hand-offs are more likely to be terminated.

C. SOHYP unencumbered session times, SOHYP dwell times

Calculations were also performed for the case in which both the unencumbered session time and the dwell time random variables were SOHYP distributed. That is, $W = 1$, $N_S(1) = 2$, $M_S(1,1) = 1$, $M_S(1,2) = 2$, $G = 1$, $N_D(1) = 2$, $M_D(1,1) = 1$, $M_D(1,2) = 2$. The mean unencumbered session time was set at, $\bar{T}_S(1) = 100$ seconds while the mean dwell time was $\bar{T}_D(1) = 200$ seconds. We kept the squared coefficient of variation of unencumbered session time equal to that of dwell time and varied $\kappa_D^2(1) = \kappa_S^2(1) = 1, 2, 3, 4$. We considered cells with 10 channels ($C = 10$) and varied $C_h = 0, 1$. The number of noncomunicating platforms was taken to be $v(1,0) = 200$. For this parameter set 9 state variables and a total of 92378 states are required. The system performance was computed for increasing call arrival rate $\Lambda(1,1)$. Results are plotted in figures 7 through 9. Each set of results took approximately 7 hours to compute.

We see the effect of simultaneously increasing the coefficient of variation of both the unencumbered session and the dwell time form the figures. As seen in figure 7, coefficient of variation has little effect on blocking probability but as both the unencumbered session time and dwell time variation increase the forced termination probability decreases slightly. Figure 8 is a plot of the carried traffic. We see that carried traffic is insensitive to the changes in coefficient of variation. From figure 9 we see that increasing the coefficients of variation causes the expected number of hand-off attempts to decrease. Generally, the changes in the performance measures are slight as the coefficients of variation change. This may be a fortuitous observation. We can model both the unencumbered session time and dwell time as negative exponential random variables. Using negative exponential variates eases computation but, computed performance characteristics will be slightly inaccurate. Depending on the situation this inaccuracy may be acceptable.

IX. Conclusion

The MDBD framework has been extended to allow the consideration of a broad class of unencumbered session time and dwell time distributions. This is accomplished by representing the session time and dwell time as SOHYP distributed random variables and using an appropriate state variable description. The overall method enables the computation of theoretical performance characteristics for cellular communication

systems in which mixed call types and mixed platform types are simultaneously present. Another important issue in cellular communications is the distribution of channel holding time. We have developed the equations which define the channel holding time distribution and show its dependence on the session and dwell time parameters, as well as the system performance characteristics. So the framework can also be used to compute the distribution of channel holding time in a cell.

The SOHYP distribution allows a large number of parameters that can be adjusted to fit empirical data. An approach to fitting the parameters to channel holding time data has been discussed.

X. Acknowledgment

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APPENDIX

I. STATE PROBABILITY EQUATIONS AND SOLUTION

To determine the flow balance equations, we first identify the predecessor states for an arbitrary state s. In most cases the state s and it predecessor differ in a single state variable (dwell time and session time phase transitions each require that two state variables to change). We focus on the state variables that change when the cell enters state s from its predecessor. All other state variables are identical for the two states.

A. New call arrivals

A new w-type call that originates on a g-type platform must begin service in the first phase of session time. It can, however, arrive on a platform while it is in any phase i stage k ($i = 1, 2, ..., N_D(g)$; $k = 1, 2, ..., M_D(g, i)$). A transition into state s from state x_n due to an arrival of a w-type call in phase 1, stage q on board a g-type platform in phase i stage k will cause the state variable $v(x_n, w, 1, q, g, i, k)$ to be increased by one. Recall that a new call will be served if the number of channels in use is less than $C - C_h$. So a permissible state x_n is a predecessor of state s if $j(x_n) < C - C_h$and $j(x_n, w, g) < J(x_n, w, g)$ and the state variables are related by

$$v(x_n, w, 1, q, g, i, k) = v(s, w, 1, q, g, i, k) - 1 \tag{62}$$

Let $\Lambda_n(w, 1, q, g, i, k)$ be the average arrival rate of w-type calls in phase 1 stage q on board g-type platforms in phase i stage k. Of those new calls that arise on g-type platforms, the fraction of calls that arise during phase i stage k of dwell time is

$$\rho_n(g, i, k) = \frac{\alpha_D(g, i, k)/\mu_D(g, i, k)}{\overline{T}_D(g)} \tag{63}$$

Therefore

$$\Lambda_n(w, 1, q, g, i, k) = \rho_n(g, i, k) \cdot \Lambda_n(w, g) \cdot \alpha_S(w, 1, q) \tag{64}$$

We write the flow into state s from x_n as

$$\gamma_n(s, x_n) = \Lambda_n(w, 1, q, g, i, k) \tag{65}$$

B. Call completions

A w-type call will complete when its final unencumbered session time phase, $N_S(w)$, expires. Like new calls, a call completion can occur during any phase and stage of *dwell time.* Thus a transition into state s from state x_c due to a completion of a w-type call in phase $N_S(w)$ stage q on board a g-type platform in phase i stage k causes the state variable $v(x_c, w, N_S(w), q, g, i, k)$ to be decreased by one. So a permissible state x_c is a predecessor of state s if the following holds

$$v(x_c, w, N_S(w), q, g, i, k) = v(s, w, N_S(w), q, g, i, k) + 1 \tag{66}$$

The flow into s from x_c is

$$\gamma_c(s, x_c) = \mu_S(w, N_S(w), q) \cdot v\,(x_c, w, N_S(w), q, g, i, k) \tag{67}$$

C. Unencumbered Session time phase transitions

When a w-type call in phase $p-1$ stage q , $q = 1, 2, ..., M_S(w, p-1)$,completes its current phase and stage it enters phase p where it chooses stage $q', q' = 1, 2, ..., M_S(w, p)$, of its session time. Stage q' is chosen with probability $\alpha_S(w, p, q')$. This is a session time phase transition. Note that if $p - 1 = N_S(w)$ the call is already in its final phase and after the completion of this phase the call will terminate. So session time phase transitions occur only if $1 < p \leq N_S(w)$. Recall that a w-type call can be on board any g-type platform in phase i stage k of dwell time. A permissible state $x_{\phi c}$ is a predecessor of state s for w-type calls in phase p stage q' if

$$v(x_{\phi c}, w, p-1, q, g, i, k) \;=\; v(s, w, p, q, g, i, k) + 1 \tag{68}$$

$$v(x_{\phi c}, w, p, q', g, i, k) \quad = \quad v(s, w, p, q', g, i, k) - 1$$

Equation (68) simply states that the number of w-type calls in phase $p-1$ stage q when the cell is in state $x_{\phi c}$ is one more than when the cell is in state s. In addition, the number of w-type calls in phase p stage q' when the cell is in state $x_{\phi c}$ is one less than when the cell is in state s. The flow into state s from $x_{\phi c}$ is

$$\gamma_{\phi c}(s, x_{\phi c}) = \mu_S(w, p-1, q) \cdot v(x_{\phi c}, w, p-1, q, g, i, k) \cdot \alpha_S(w, p, q') \tag{69}$$

D. Hand-off arrivals

Arriving g-type platforms enter a cell in their first phase of dwell time but only those with calls in progress generate hand-off attempts. These (communicating) platforms can arrive with any w-type call in progress, and the call may be in any phase p stage q ($p = 1, 2, ..., N_S(w)$; $q = 1, 2, ..., M_S(w, p)$) of session time. A transition into state s from state x_h due to an arrival of a w-type call in phase p-stage q on board a g-type platform in phase 1 stage k will cause the state variable $v(x_h, w, p, q, g, 1, k)$ to be increased by one. Recall that calls arriving as hand-off have access to all C channels and will be serviced if the number of channel in use is less than C. Channel quotas still apply. So a permissible state x_h is a predecessor of state s if $j(x_n) < C$ and $j(x_n, w, g) < J(x_n, w, g)$ and the state variables are related

$$v(x_h, w, p, q, g, 1, k) = v(s, w, p, q, g, 1, k) - 1 \tag{70}$$

Let $\Lambda_h(w, g)$ be the average hand-off arrival rate of w-type calls on board g-type platforms, and let $\Phi(w, p, q, g)$ be the fraction of w-type hand-offs that arrive on g-type platforms in phase p stage q of session time. These parameters are assumed to be known for now. Their values will be subsequently calculated as part of an iterative solution algorithm. We can write the flow into state s from state x_h as

$$\gamma_h(s, x_h) = \Lambda_h(w, g) \cdot \Phi(w, p, q, g) \cdot \alpha_D(g, 1, k) \tag{71}$$

E. Hand-off departures

A hand-off departure occurs when a platform completes its final phase. A g-type platform may have any w-type call on board. The call can be in any phase and stage of its session time. Thus a permissible state x_d is a predecessor of state s for a hand-off departure of a w-type call in phase p stage q of session time on board a g-type platform in phase $N_D(g)$ stage k of dwell time if the state variables are related by

$$v(x_d, w, p, q, g, N_D(g), k) = v(s, w, p, q, g, N_D(g), k) + 1 \tag{72}$$

The corresponding flow is

$$\gamma_d(s, x_d) = \mu_D(g, N_D(g), k) \cdot v(x_d, w, p, q, g, N_D(g), k) \tag{73}$$

F. Dwell time phase transitions

Dwell time phase transition are similar to session time phase transitions. When a g-type call in phase $i-1$ stage k , $k = 1, 2, ..., M_D(g, i-1)$, completes its current phase and stage it enters the next phase i where it chooses stage $k', k' = 1, 2, ..., M_D(g, i)$, of its dwell time. Stage k' is chosen with probability $\alpha_D(g, i, k')$. Again note that dwell time phase transitions occur as long as $1 < i \leq N_D(h)$. Recall that a g-type platform may carry any w-type call in phase p stage q of dwell time. A permissible state $x_{\phi d}$ is a predecessor of state s for g-type calls in phase i stage k' if

$$\begin{aligned} v(x_{\phi d}, w, p, q, g, i-1, k) &= v(s, w, p, q, g, i-1, k) + 1 \\ v(x_{\phi d}, w, p, q, g, i, k') &= v(s, w, p, q, g, i, k') - 1 \end{aligned} \tag{74}$$

The flow into state s from $x_{\phi d}$ is

$$\gamma_{\phi d}(s, x_{\phi d}) = \mu_D(g, i-1, k) \cdot v(x_{\phi d}, w, p, q, g, i-1, k) \cdot \alpha_C(g, i, k') \tag{75}$$

G. Flow balance equations

The total **flow into state** s from any permissible state x can be expressed as the sum of the component flows due to each driving process.

$$Q(s,x) = \gamma_n(s,x) + \gamma_c(s,x) + \gamma_{\phi c}(s,x) + \gamma_h(s,x) + \gamma_d(s,x) + \gamma_{\phi d}(s,x) \quad \text{where } s \neq x \tag{76}$$

The flow out of state s is denoted as $Q(s,s)$ and is taken to be negative. This is found by

$$Q(s,s) = -\sum_{\substack{k=0 \\ k\neq s}}^{s_{\max}} Q(k,s) \tag{77}$$

The flow balance equations may be written using (76) and (77). These are a set of $s_{\max}+1$ simultaneous equations for the unknown state probabilities $P(s)$. They are of the following form

$$\sum_{j=0}^{s_{\max}} Q(i,j)P(j) = 0 \qquad i = 0,1,2,\ldots,s_{\max}-1$$

$$\sum_{j=0}^{s_{\max}} P(j) = 1 \tag{78}$$

The solution of (78) gives the equilibrium state probabilities, $P(s)$ $(s = 0,1,2,\ldots s_{\max})$.

H. Determination of hand-off arrival parameters

The hand-off arrival parameters ($\Lambda_h(w,g)$ and $\Phi(w,p,q,g)$) can be computed using the iterative method described in [3] [4]. The method relates the average hand-off *departure* rates to the average hand-off *arrival* rates. Specifically, let $\Delta_h(w,p,q,g)$ denote the average hand-off departure rate (from a cell) of w-type calls in phase p stage q of session time on board g-type platforms. $\Delta_h(w,p,q,g)$ can be computed by

$$\Delta_h(w,p,q,g) = \sum_{s=0}^{s_{\max}} \sum_{k=1}^{M_D(g,N_D(g))} v(s,w,p,q,g,N_D(g),k) \cdot \mu_D(g,N_D(g),k) \cdot P(s) \tag{79}$$

The average departure rate of w-type calls on board g-type platforms is

$$\Delta_h(w,g) = \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} \Delta_h(w,p,q,g) \tag{80}$$

We denote $\Phi'(w,p,q,g)$ as the fraction of departures that occur while the call is in phase p stage q this is given by

$$\Phi'(w,p,q,g) = \Delta_h(w,p,q,g)/\Delta_h(w,g) \tag{81}$$

The above equations are used to find the hand-off arrival parameters. Note that a hand-off departure of a w-type call in phase p stage q of session time on board a g-type platform corresponds to a hand-off arrival of a w-type call in phase p stage q on board a g-type platform in some other cell. For the system to be in statistical equilibrium the hand-off departure rates must equal the hand-off arrival rates (we assume the system is homogeneous). That is

$$\Phi(w,p,q,g) = \Phi'(w,p,q,g) \tag{82}$$

$$\Lambda_h(w,g) = \Delta_h(w,g) \tag{83}$$

Equations (82), (83) and (78) allow the use of the iterative algorithm described in [3] [4] to solve for the state probabilities, $P(s)$, and the hand-off arrival parameters, $\Phi(w,p,q,g)$ and $\Lambda_h(w,g)$.

II. Detailed Development of Channel Holding Time Distribution for Completed Calls

We derive the conditional channel holding time distribution of calls that complete in the current cell. This requires the computation of two distributions, one for calls that arise from hand-off requests and one for calls that arise from new call requests. We first consider the holding time of w-type hand-off calls in phase p stage q of unencumbered session time arriving on board g-type platforms. We repeated the defining equation below

$$\begin{aligned} F_{H_h(w,p,q,g)}(t|\beta_H(w,p,q,g)) &= \Pr\{H_h(w,p,q,g) \le t|\beta_H(w,p,q,g)\} \\ &= \Pr\{\min(R_S(w,p,q), T_D(g)) \le t|\beta_H(w,p,q,g)\} \end{aligned}$$

$$= \Pr\{R_S(w,p,q) \le t | ß_H(w,p,q,g)\}$$

$$= F_{R_S(w,p,q)}(t | R_S(w,p,q,g) \le T_D(g)) \tag{84}$$

Writing out the corresponding integral we have

$$F_{H_h(w,p,q,g)}(t|ß_H(w,p,q,g)) = \int_0^t \int_{\xi_1}^{\infty} f_{R_S(w,p,q)}(\xi_1) f_{T_D(g)}(\xi_2) d\xi_2 d\xi_1 \tag{85}$$

Carrying out the integration we have the following

$$\begin{aligned} F_{H_h(w,p,q,g)}(t|ß_H(w,p,q,g)) &= \int_0^t \left(1 - F_{T_D(g)}(\xi_2)\right) f_{R_S(w,p,q)}(\xi_2) d\xi_2 \\ &= \int_0^t \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} \sum_{j=p}^{N_S(w)} \sum_{l=\left\{\substack{q \text{ if } j=p \\ 1 \text{ if } j>p}\right.}^{M'_S(w,j)} a_D(g,i,k) B_S^{w,p,q}(j,l) e^{-[\mu_D(g,i,k)+\mu_S(w,j,l)]\xi_2} d\xi_2 \\ &= \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} \sum_{j=p}^{N_S(w)} \sum_{l=\left\{\substack{q \text{ if } j=p \\ 1 \text{ if } j>p}\right.}^{M'_S(w,j)} \frac{a_D(g,i,k) B_S^{w,p,q}(j,l)}{[\mu_D(g,i,k)+\mu_S(w,j,l)]} \left[1 - e^{-[\mu_D(g,i,k)+\mu_S(w,j,l)]t}\right] \end{aligned} \tag{86}$$

Recall that the fraction of w-type hand-off calls that arrive in phase p stage q of unencumbered session time on board g-type platforms is given by $\Phi(w,p,q,g)$. We can now express the conditional distribution of w-type hand-off calls on board g-type platforms that complete in their current cell, $F_{H_h(w,g)}(t|ß_H(w,g))$. This is given by

$$F_{H_h(w,g)}(t|ß_H(w,g)) = \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} \Phi(w,p,q,g) \cdot F_{H_h(w,p,q,g)}(t|ß_H(w,p,q,g)) \tag{87}$$

The conditional distribution of completing new calls follows a similar argument. New calls of type w can initiate service on any g-type platform while the platform is in phase i stage k of dwell time. The call will complete in its first cell if its unencumbered session time, $T_S(w)$, is less than the platform's remaining dwell time, $R_D(g,i,k)$. The definition of the conditional channel holding time for new w-type calls initiating service

on g-type platforms in phase i stage k of dwell time is

$$\begin{aligned} F_{H_n(w,g,i,k)}(t|ß_N(w,g,i,k)) &= \Pr\{H_n(w,g,i,k) \le t|ß_N(w,g,i,k)\} \\ &= \Pr\{\min(T_S(w), R_D(g,i,k)) \le t|ß_N(w,g,i,k)\} \\ &= \Pr\{T_S(w) \le t|ß_N(w,g,i,k)\} \\ &= F_{T_S(w)}(t|T_S(w) \le R_D(g,i,k,)) \end{aligned} \tag{88}$$

This can be computed by solving the following integral

$$F_{H_n(w,g,i,k)}(t|ß_N(w,g,i,k)) = \int_0^t \int_{\xi_1}^{\infty} f_{T_S(w)}(\xi_1) f_{R_D(g,i,k)}(\xi_2) d\xi_2 d\xi_1 \tag{89}$$

Carrying out the above integration we get the following result

$$F_{H_n(w,g,i,k)}(t|ß_N(w,g,i,k)) = \sum_{j=i}^{N_D(g)} \sum_{l=\left\{\begin{smallmatrix} k \text{ if } j=i \\ 1 \text{ if } j>i \end{smallmatrix}\right.}^{M'_D(g,i)} \sum_{p=1}^{N_S(w)} \sum_{q=1}^{M_S(w,p)} \frac{B_D^{g,i,k}(g,j,l) A_S(w,p,q)}{\mu_D(g,j,l)[\mu_D(g,j,l) + \mu_S(w,p,q)]} \left[1 - e^{-[\mu_D(g,j,l)+\mu_S(w,p,q)]t}\right. \tag{90}$$

Recall that the probability that a g-type platform in phase i stage k of dwell time is $\rho_n(g,i,k)$. So the conditional channel holding time distribution of new w-type calls on board g-type platforms is

$$F_{H_n(w,g)}(t|ß_N(w,g)) = \sum_{i=1}^{N_D(g)} \sum_{k=1}^{M_D(g,i)} \rho_n(g,i,k) F_{H_n(w,g,i,k)}(t|ß_N(w,g,i,k)) \tag{91}$$

To obtain the overall distribution of completing calls, $F_{H(w,g)}(t|ß)$, we make use of (34)

$$\begin{aligned} F_{H(w,g)}(t|ß) &= \frac{\Lambda_n(w,g)[1 - P_B(w,g)]}{\Lambda_n(w,g)[1 - P_B(w,g)] + \Lambda_h(w,g)[1 - P_H(w,g)]} \cdot F_{H_n(w,g)}(t|ß) \\ &+ \frac{\Lambda_h(w,g)[1 - P_H(w,g)]}{\Lambda_n(w,g)[1 - P_B(w,g)] + \Lambda_h(w,g)[1 - P_H(w,g)]} \cdot F_{H_h(w,g)}(t|ß) \end{aligned} \tag{92}$$

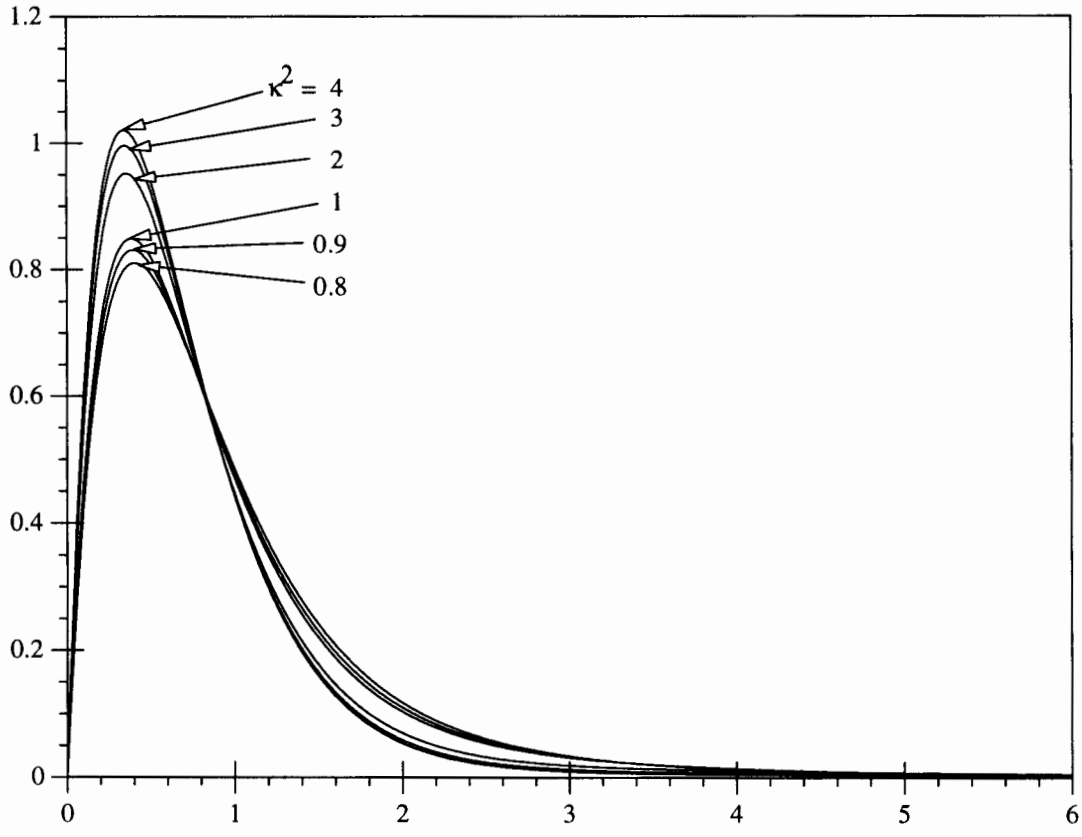


Figure 1: SOHYP pdfs with same mean (=1) and different coefficients of variation. SOHYP random variables with two phases. First phase is a negative exponential variate. Second phase is a hyperexponential variate with two stages.

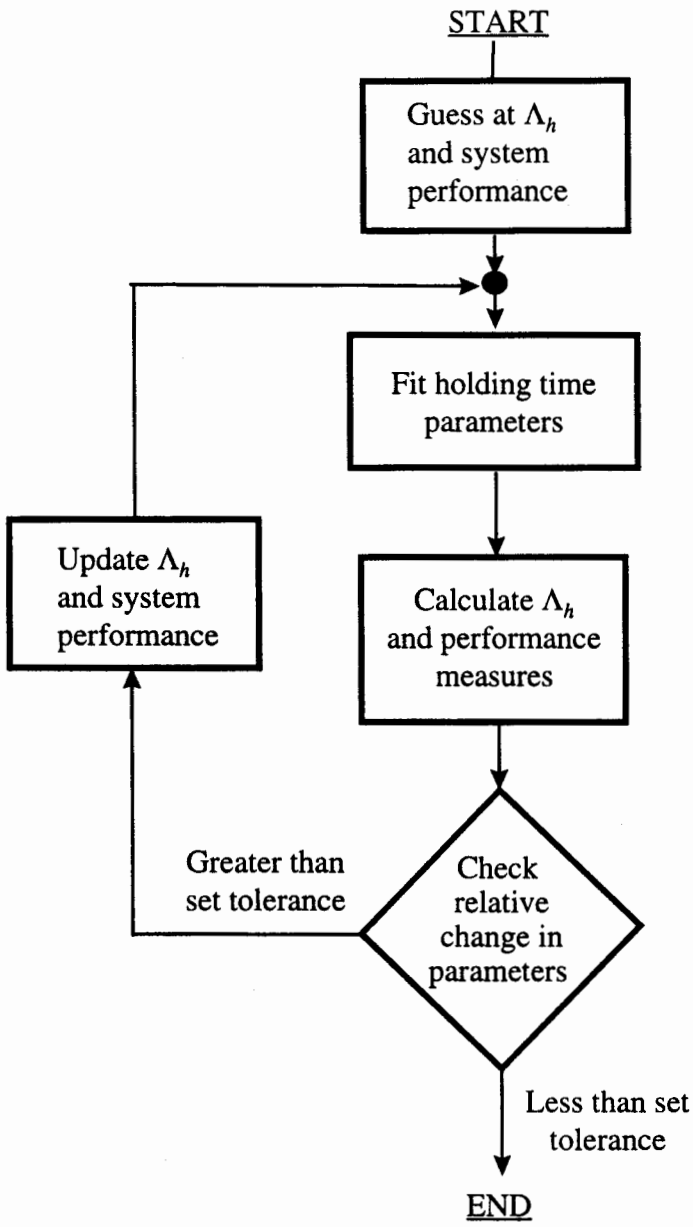


Figure 2: Procedure to fit channel holding time distribution to observed data

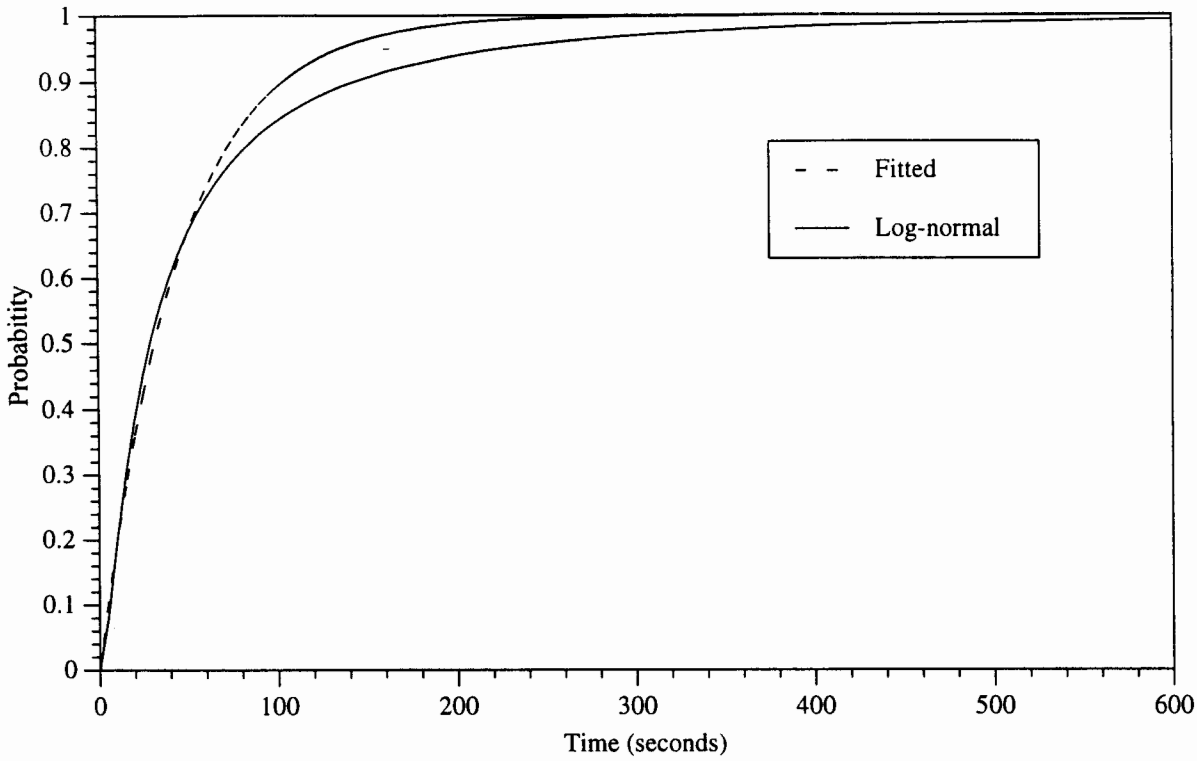
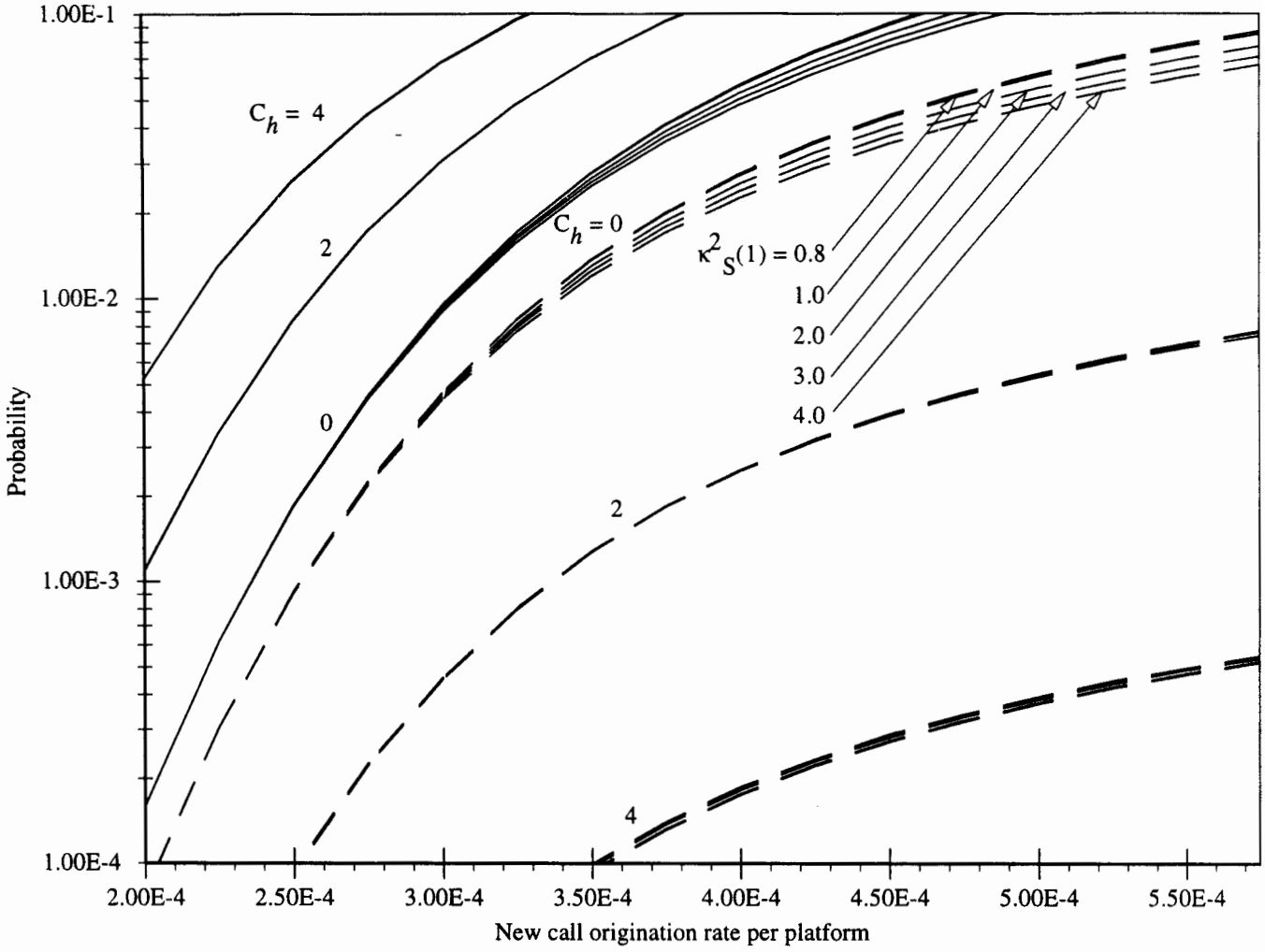


Figure 3: Log-normal and fitted cdfs



Mean dwell time = 200s, Mean session time =100s.

$G = 1, N_D(1) = 1, M_D(1,1) = 1$

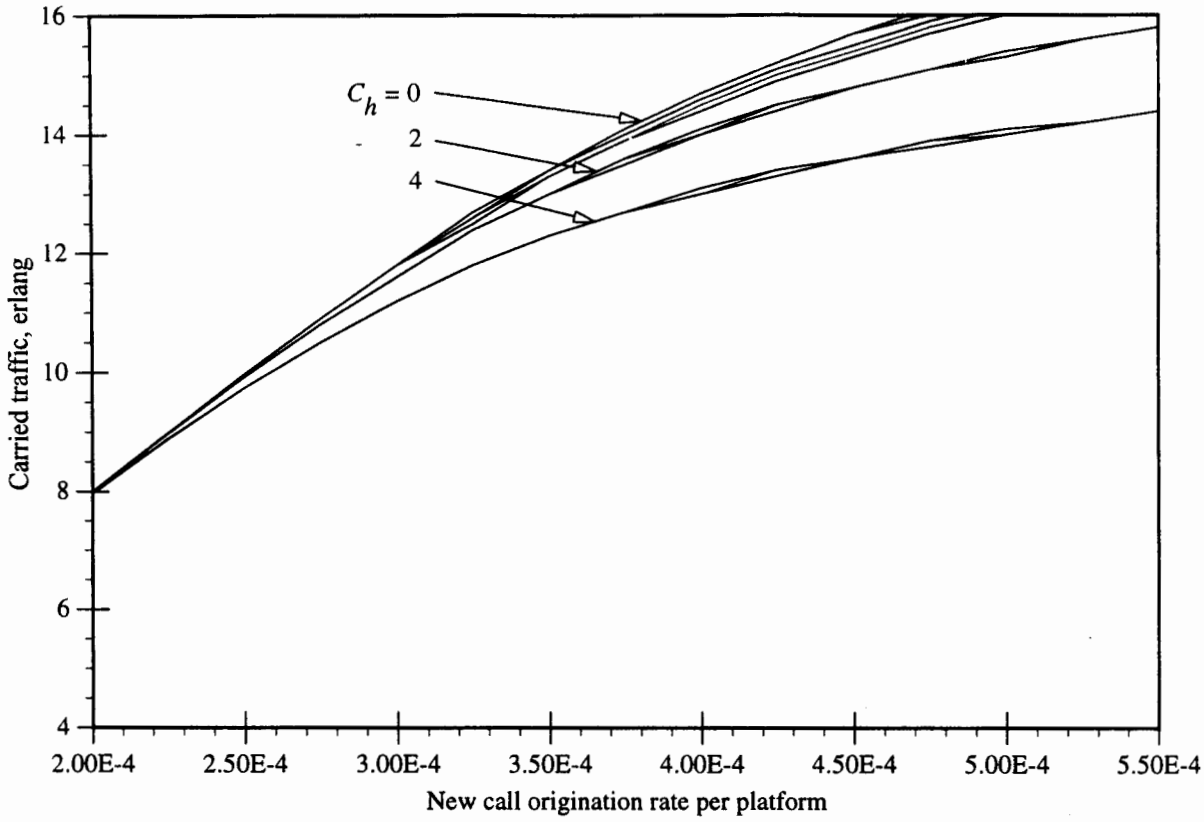
$W=1,\ N_S(1) = 1, M_S(1,1) = 1, M_S(1,2) = 2$

$C = 20, C_h = 0,2,4.\ \kappa^2_S(1) = 0.8, 1.0, 2.0, 3.0, 4.0,\ \nu(1,0)=400$

——— $P_B(1,1)$

— — $P_{FT}(1,1)$

Figure 4: Blocking and Forced termination probabilities



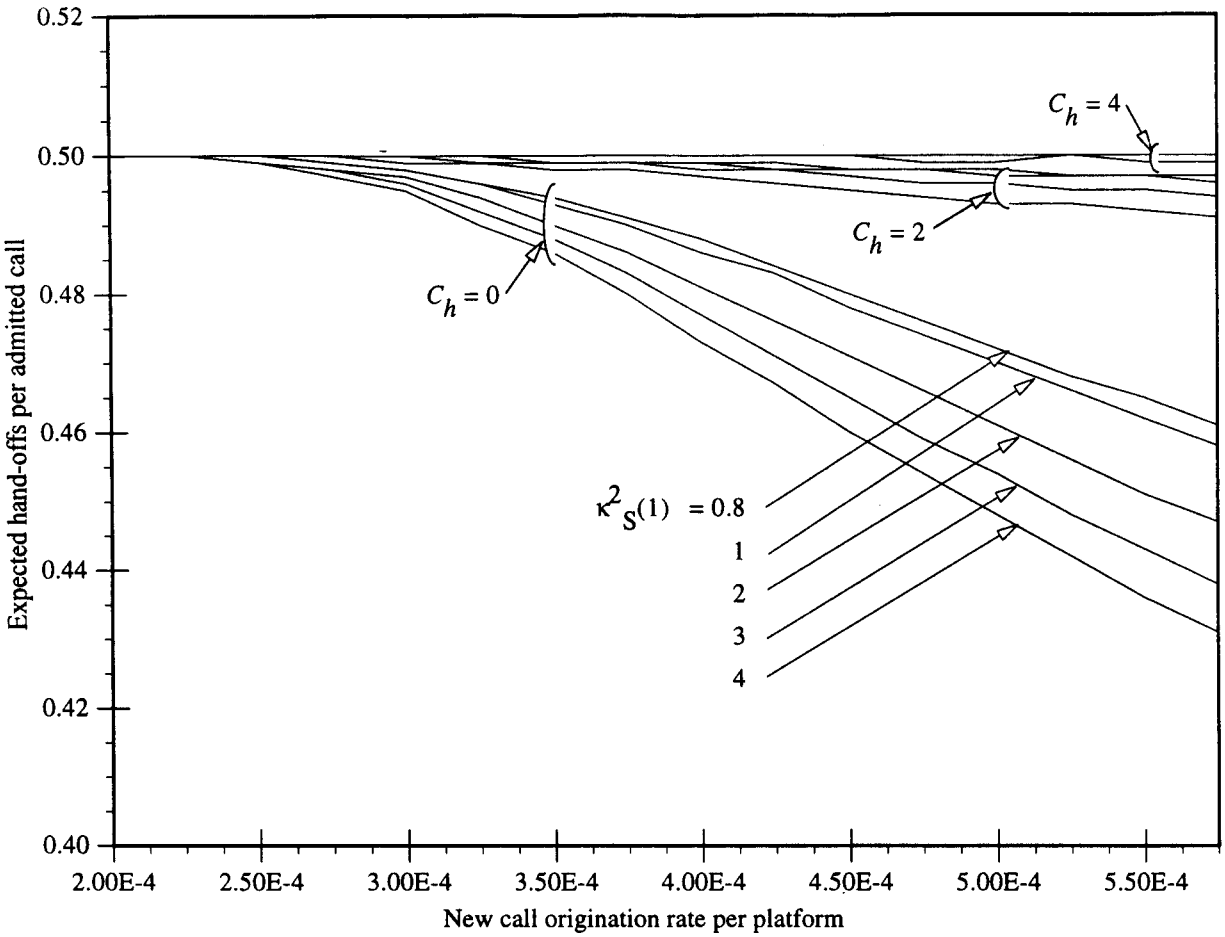
Mean dwell time = 200s, Mean session time =100s.

$G = 1, N_D(1) = 1, M_D(1,1) = 1$

$W=1,\ N_S(1) = 1, M_S(1,1) = 1, M_S(1,2) = 2$

$C = 20, C_h = 0,2,4.\ \kappa^2_S(1) = 0.8, 1.0, 2.0, 3.0, 4.0,\ \nu(1,0)=400$

Figure 5: Carried Traffic



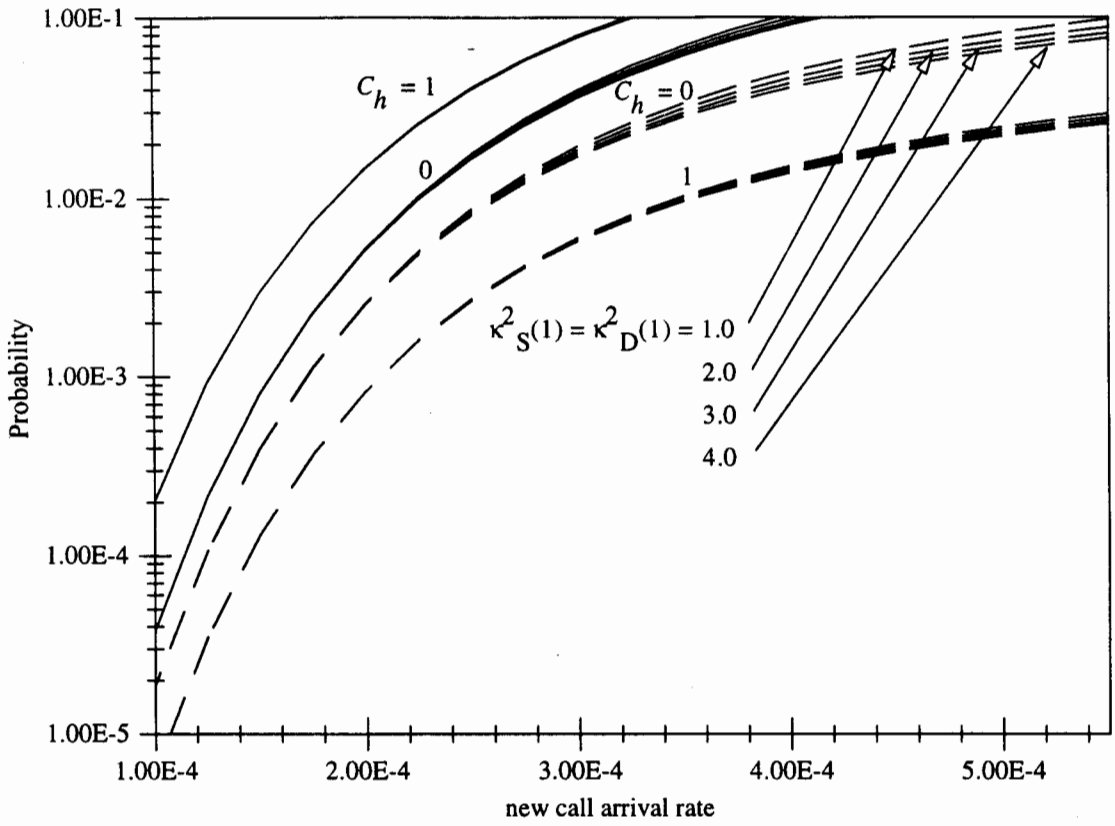
Mean dwell time = 200s, Mean session time =100s.

$G = 1, N_D(1) = 1, M_D(1,1) = 1$

$W=1,\ N_S(1) = 1, M_S(1,1) = 1, M_S(1,2) = 2$

$C = 20, C_h = 0,2,4.\ \kappa^2_S(1) = 0.8, 1.0, 2.0, 3.0, 4.0,\ \nu(1,0)=400$

Figure 6: Hand-off activity



Mean dwell time = 200s, Mean session time =100s.

$G = 1, N_D(1) = 1, M_D(1,1) = 1, M_D(1,1) = 2$

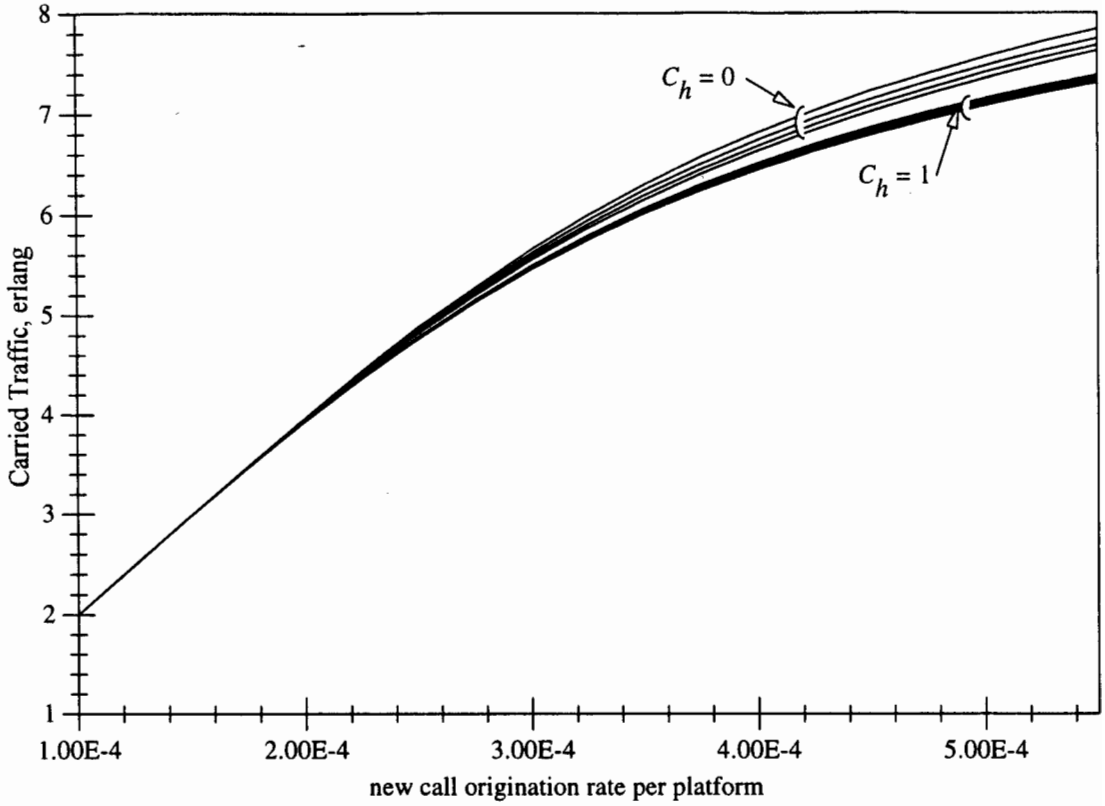
$W=1,\ N_S(1) = 1, M_S(1,1) = 1, M_S(1,2) = 2$

$C = 10, C_h = 0,1.\ \kappa^2_S(1) = \kappa^2_D(1) = 1.0, 2.0, 3.0, 4.0,\ \nu(1,0)=200$

——— $P_B(1,1)$

— — $P_{FT}(1,1)$

Figure 7: Blocking and Forced termination probabilities.
Unencumbered session time and dwell time are SOHYP distributed



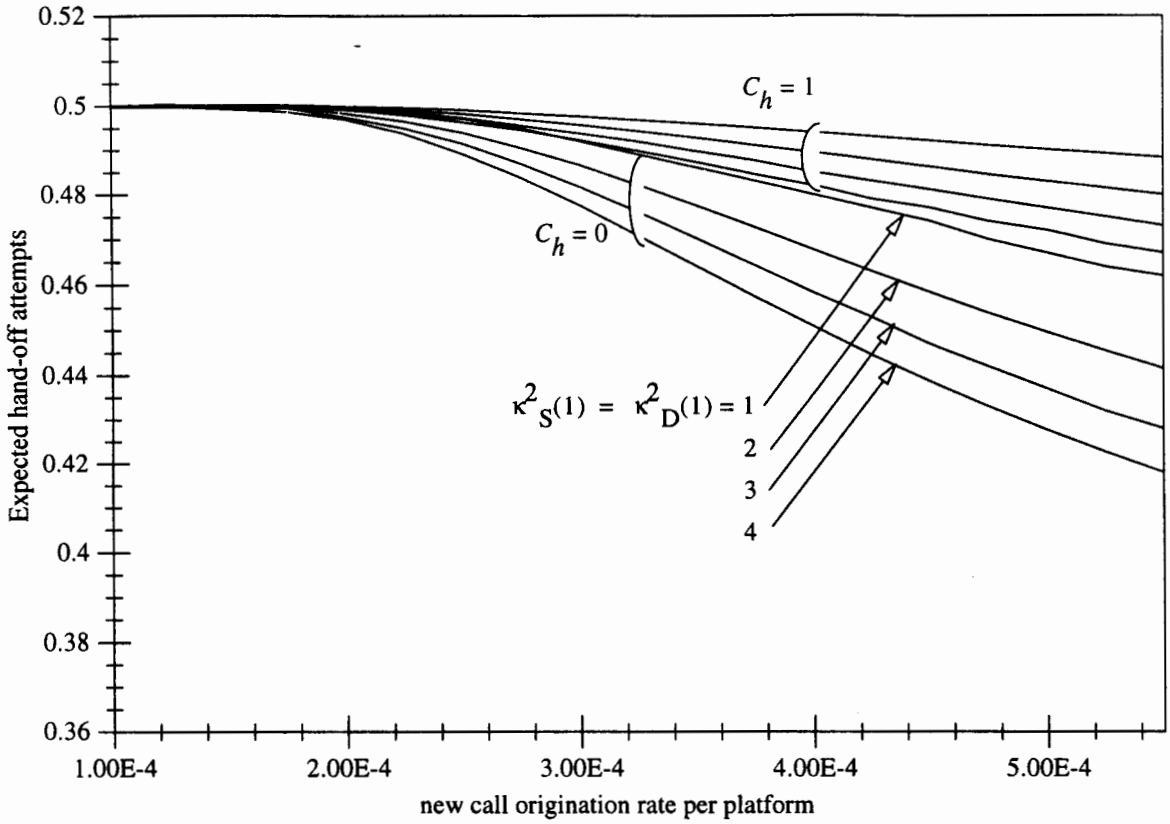
Mean dwell time = 200s, Mean session time =100s.

$G = 1, N_D(1) = 1, M_D(1,1) = 1, M_D(1,1) = 2$

$W=1,\ N_S(1) = 1, M_S(1,1) = 1, M_S(1,2) = 2$

$C = 10, C_h = 0,1.\ \kappa^2_S(1) = \kappa^2_D(1) = 1.0, 2.0, 3.0, 4.0,\ \nu(1,0)=200$

Figure 8: Carried Traffic. Unencumbered session time and dwell time are SOHYP distributed.



Mean dwell time = 200s, Mean session time =100s.

$G = 1, N_D(1) = 1, M_D(1,1) = 1, M_D(1,1) = 2$

$W=1,\ N_S(1) = 1, M_S(1,1) = 1, M_S(1,2) = 2$

$C = 10, C_h = 0,1.\ \kappa^2_S(1) = \kappa^2_D(1) = 1.0, 2.0, 3.0, 4.0,\ \nu(1,0)=200$

Figure 9: Hand-off activity. Unencumbered session time and dwell time are SOHYP distributed.