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Traffic Performance and Mobility Modeling of Cellular Communications With Mixed Platforms and Highly Variable Mobilities

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Correction to CEAS Report No. 727 p. 20
'Traffic Performance and Mobility Modeling of Cellular Communications with Mixed Platforms and Highly Variable Mobilities,' by Philip V. Orlik and Stephen S. Rappaport

We wish to correct an error that appears in Section 7. It was correctly stated in Section 6 that handoff departures occur only when platforms complete the final phase (of dwell time). Thus the average handoff departure rate, $\Delta_h(g,i)$, as defined just above equation (42), has meaning only when $i=N(g)$. Equations (42) and (43) are therefore incorrect. They should be replaced with the following discussion.

Let $\Delta_h(g,k)$ be the average handoff departure rate of g-type platforms in ***stage k*** (not phase). This is given by

$$\Delta_h(g,k) = \sum_{s=0}^{S_{max}} v(s,g,i,k)\mu_D(g,i,k)p(s)\Big|_{i=N(g)} \qquad (42)$$

The average handoff departure rate of g-type platforms is then found by

$$\Delta_h(g) = \sum_{k=1}^{M(g,N(g))} \Delta_h(g,k) \qquad (43)$$

These were errors in explanation and did not affect any of the numerical results that we presented. We regret any inconvenience this may have caused.

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Abstract: In modeling teletraffic performance of mobile cellular networks some characteristic mobilities are assumed to be known. It is useful if these assumptions impose few restrictions and lead to analytically tractable models. Previous work has made use of the concept of dwell time — a random variable that describes the amount of time a platform remains in a cell, sector or microcell. The dwell time was characterized as a negative exponential variate or the sum of negative exponential variates. With a suitable state characterization this allows use of the memoryless property of negative exponential variates with the result that the problem of computing traffic performance characteristics can be cast in the framework of multi-dimensional birth-death processes. However, these assumptions restrict the dwell time coefficient of variation to be less than or equal to one. So if some mobile platform classes have mobility characteristics that are highly variable (dwell time standard deviation greater than the mean) the previous models may not be adequate. We present a new probability density function where the coefficient of variation can be larger than one but which nevertheless lends itself to analytical modeling using memoryless properties and multi-dimensional birth-death processes. This extends the previous framework to a broader class of mobilities. The approach allows computation of major teletraffic performance characteristics for cellular communications in which mobility issues are important. Multiple platform types and cut-off priority for hand-offs are considered. Computational issues are discussed and some theoretical performance measures are obtained to demonstrate the method and compare with previous work.

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1. Introduction

Development of analytically tractable models to compute performance characteristics of mobile wireless networks has been the thrust of recent work [1-3]. The most general models are based on multidimensional birth-death processes. An essential feature is the characterization of platform mobility by a *dwell time* random variable. The dwell time of a platform is a random variable that gives the amount of time that a platform can maintain a satisfactory communication link with a given network gateway. A platform's dwell time depends on many factors including speed, path, transmitting power, signal propagation and interference.

The coefficient of variation (denoted here by κ) of a random variable is defined as the ratio of its standard deviation to its mean. This paper extends earlier work [2] to include a class of dwell probability density functions (p.d.f.'s) with coefficients of variation greater than one. Previous work modeled the platform dwell time as the sum of statistically independent negative exponential random variables. This approach results in a dwell time r.v. with $\kappa < 1$. In the case of macrocellular systems, cell sizes are relatively large compared with shape and size variations. A dwell time variate with a coefficient of variation one or less seems to be a reasonable model for platform mobility. For microcellular systems, however, a dwell time variate with $\kappa<1$ has less intuitive appeal. Microcellular systems have small cells which lead to smaller mean dwell times for platforms traversing the system (at the same speeds). Also since the transmitting power is reduced obstacles such as buildings, trees, etc., will have a greater effect on cell size and shape. The result is that cells in a microcellular system tend to be less regular in shape and more variable throughout the coverage area. This large variation in cell size and shape coupled with the shorter mean dwell suggest that dwell time variates may have coefficients of variation that are greater than one.

The motivation here is to develop a more complete framework in which theoretical performance measures can be calculated for a wide range of coefficients of variation. Then an appropriate p.d.f. for the dwell time can be chosen from a family of models, and its parameters can be adjusted to fit empirical data. For example, if data shows that dwell times are concentrated around a mean value, then a p.d.f. with $\kappa < 1$ would be a suitable choice for a dwell time model. On the other hand, if dwell times are found to widely dispersed, then a more realistic p.d.f. with $\kappa > 1$ could be used as a dwell time model.

In the present paper we model the dwell time as a sum of statistically independent random variables, each distributed according to a hyperexponential p.d.f. The framework to compute traffic performance measures for a cellular system with platforms having such mobility characteristics is developed. Theoretical traffic performance characteristics are calculated for various values of κ (less than, equal, greater than unity). Results are compared. The discussion is presented for systems that have *mixed* platform types. That is, platforms having different mobility characteristics are present in the system at the same time.

The approach presented here focuses on the modeling of dwell time. The same approach and our new p.d.f. can also be modified to model channel *holding times in a cell.* Recall that holding time in a cell is the minimum of the remaining session duration and cell dwell time [2], [3]. Recent empirical studies [4] suggest that channel holding times within a cell have a log-normal distribution (when averaged over mobilities). The parameters of our p.d.f. can be chosen to fit a log-normal p.d.f. or any other similarly shaped p.d.f.. Work in deriving the relationship between platform dwell time and channel holding times is underway.

2. Model Description

The system description given here is similar to that of reference [2]. We consider a large geographical region covered by cells. Mobile platforms traverse the region and can each support a maximum of one call. We assume that in each cell there is one gateway with C channels assigned to it. A cut-off priority scheme is used so that C_h channels in each cell are reserved for hand-off calls. Specific channels are not reserved, just a fixed number. That is, new calls that arise is a cell will not be served if there are fewer than C_h idle channels.

We allow for multiple platform *types*, which are indexed by $g = 1,2,...,G$. Each platform type has distinct mobility characteristics. These are characterized by the statistical properties of the dwell time of a platform of the given type. The dwell time of a platform of type g is a random variable denoted by $T_D(g)$. At each gateway there may also be quotas assigned so that no more than $J(g)$ channels can be occupied by calls on g-type platforms at the same time. In the present paper it is assumed that the system is *homogeneous* and that hand-off detection and initiation is *perfect*. Therefore, all *valid* hand-off needs are detected and no unnecessary hand-off initiations are generated. Relaxation of these assumptions can be accommodated within the same framework [2].

3. Probability distributions for dwell times

3.1 Sum of negative exponentials

The first distribution we discuss results from summing independent negative exponential random variables. This was used to characterize dwell time in [2]. The random variable $T_D(g)$ is the sum of $N(g)$ statistically independent random variables denoted $T_D(g, i)$, ($i = 1,2,3,...,N(g)$) in which $T_D(g,i)$ has a negative exponential p.d.f. with mean $\overline{T}_D(g,i) = 1/\mu_D(g, i)$ and variance $\mathrm{VAR}[T_D(g,i)]=1/[\mu_D(g,i)]^2$. We define $N(g)$ as the number of dwell time phases for a platform of type g [2]. The mean and variance of $T_D(g)$ can be written as

$$\overline{T}_D(g) = \sum_{i=1}^{N(g)} \overline{T}_D(g,i) = \sum_{i=1}^{N(g)} \frac{1}{[\mu_D(g,i)]} \tag{1}$$

and

$$\text{VAR}[T_D(g)] = \sum_{i=1}^{N(g)} 1/[\mu_D(g,i)]^2 \tag{2}$$

The squared coefficient of variation (κ^2) is given by

$$\kappa^2[T_D(g)] = \frac{VAR[T_D(g)]}{[\overline{T}_D(g)]^2} = \frac{\sum_{i=1}^{N(g)} 1/[\mu_D(g,i)]^2}{\left[\sum_{i=1}^{N(g)} 1/\mu_D(g,i)\right]^2}. \tag{3}$$

Expanding the denominator in equation (3) we can obtain

$$\kappa^2[T_D(g)] = \frac{\sum_{i=1}^{N(g)} 1/[\mu_D(g,i)]^2}{\sum_{i=1}^{N(g)} 1/[\mu_D(g,i)]^2 + \text{other terms}} \leq 1 \tag{4}$$

Since all the terms on the right side of (4) are positive we have the result that $\kappa^2[T_D(g)]$ must be less than or equal to one. This result holds for any dwell time variate that is constructed using a sum of statistically independent negative exponentially distributed (n.e.d.) variates.

This formulation of the dwell time leads to an analytically tractable model. Although strictly, the individual dwell time components, $T_D(g,i)$ do not have any direct physical meaning, conceptually, one can suppose that a platform of type g enters a cell and completes $N(g)$ *phases* of dwell time. Completion of the last dwell time phase corresponds to leaving the cell (the completion of dwell time)[2]. If all $N(g)$ phases of dwell time are identically distributed, that is

$\mu_D(g,i)=\mu_D$ for $i = 1,2,\ldots,N(g)$, we have the special case of the Erlang distribution. The squared coefficient of variation reduces to $\kappa^2[T_D(g)]=1/N(g)$[2].

Figure 1 is a plot of the Erlang distribution for $N(g)$=1,2,3,4 and $\overline{T}_D(g)=1$. The reader will note that when the number of phases is one ($N(g)$=1), the Erlang distribution reduces to the n.e.d. with $\kappa=1$. Increasing $N(g)$ causes κ to decrease. The respective squared coefficient of variations are $\kappa^2 = 1,1/2,1/3,1/4$. The figure shows that as the number of phases ($N(g)$) increases the p.d.f. becomes more concentrated around its mean. Reference 2 gives a state characterization and a method to compute performance measures.

3.2 Hyperexponential

The hyperexponential p.d.f. is a weighted sum of exponential functions. The p.d.f. has the form

$$f_{T_D}(t) = \sum_{k=1}^{M}\alpha(k)\cdot\mu_D(k)\cdot\exp(-\mu_D(k)\cdot t) \tag{5}$$

where

$$\sum_{k=1}^{M}\alpha(k) = 1 \tag{6}$$

Equation (5) represents an *M stage* hyperexponential p.d.f., where $k=1,2,\ldots,M$. One can suppose that the p.d.f. (5) is that of a random variable generated in the following way. Choose a negative exponential random variable from a set of M possibilities. The probability of choosing the k^{th} n.e.d. random variable is given by the parameter $\alpha(k)$ ($k=1,2,\ldots,M$). The value of the random variable T_D is then a realization of the chosen n.e.d. random variable.

The mean of the hyperexponential is

$$\overline{T}_D = \sum_{k=1}^{M}\frac{\alpha(k)}{\mu_D(k)} \tag{7}$$

The variance, σ^2_{HYP}, of the M stage hyperexponential is given by

$$\sigma^2_{HYP} = \left[\sum_{k=1}^{M} \frac{2 \cdot \alpha(k)}{[\mu_D(k)]^2}\right] - (\overline{T}_D)^2 \qquad (8)$$

We can write the squared coefficient of variation as

$$\kappa^2 = \frac{\sigma^2_{HYP}}{(\overline{T}_D)^2} = \frac{2 \cdot \left[\sum_{k=1}^{M} \frac{\alpha(k)}{[\mu_D(k)]^2}\right] - (\overline{T}_D)^2}{(\overline{T}_D)^2} \qquad (9)$$

The sum in equation (9) can be thought of as the expectation of X^2, ($E[X^2]$), where X is a discrete random variable X which takes on values $\{1/\mu_D(1), 1/\mu_2(2), \ldots, 1/\mu_D(k), \ldots, 1/\mu_D(M)\}$ with respective probability of occurrence $\alpha(k)$. Since the variance of any random variable is non-negative we have $E[X^2] \geq E[X]\}^2$. We note that E[X] is identical to $(\overline{T}_D)$ in equation (7). The substitutions yield

$$\kappa^2 = \frac{2 \cdot \left[\sum_{k=1}^{M} \frac{\alpha(k)}{[\mu_D(k)]^2}\right] - (\overline{T}_D)^2}{(\overline{T}_D)^2} \geq \frac{2 \cdot \left[\sum_{k=1}^{M} \frac{\alpha(k)}{\mu_D(k)}\right]^2 - (\overline{T}_D)^2}{(\overline{T}_D)^2} = \frac{2 \cdot (\overline{T}_D)^2 - (\overline{T}_D)^2}{(\overline{T}_D)^2} = 1 \qquad (10)$$

Thus the coefficient of variation for a hyperexponential r.v. is greater or equal to one.

A state characterization can be given which allows multidimensional birth-death processes to be used to determine the state probabilities for a system with platform dwell times distributed according to the M stage hyperexponential p.d.f. One can conceptualize a platform that traverses the system. Each time the platform enters a new cell it chooses a stage from the M stages available for its dwell time in the new cell. The k^{th} stage is chosen with probability $\alpha(k)$. Upon completion of this stage the platform will enter another cell where it will again choose a new stage. It is important for the reader to notice the difference between the terms *phases* and *stages*.

If we set M equal to 2 the result is a two stage hyperexponential p.d.f. The variance of the 2 stage exponential can be expressed as

$$\sigma^2_{\mathrm{HYP}} = \frac{2\alpha(1)-\alpha^2(1)}{\mu_D(1)^2} + \frac{2\alpha(2)-\alpha^2(2)}{\mu_D(2)^2} - \frac{2\alpha(1)\cdot\alpha(2)}{\mu_D(1)\cdot\mu_D(2)} \tag{11}$$

With an appropriate choice of $\alpha(1)$, $\alpha(2)$, $\mu_D(1)$, and $\mu_D(2)$ the coefficient of variation for the hyperexponential p.d.f. can be set at any value greater than one. Figure 2. is a plot of several hyperexponential functions for κ^2=1,2,3,4 and mean equal 1. However, the hyperexponential's behavior near zero is intuitively unsatisfying for a dwell time model. This is because the probability density function is nonzero at the origin and may have too much of its weight below its mean. This has the effect of giving *extremely* short dwell times a high probability of occurrence. In addition, this low weighting near the origin will increase as κ increases. Extremely short dwell times are possible in microcellular systems but not to the extent that the hyperexponential favors them.

3.3 SOHYP

We seek a dwell time p.d.f., $f_{T_D(g)}(t)$, which has the following properties; $\kappa > 1$, $f_{T_D(g)}(0) = 0$, and which allows analytically tractable models to be constructed within the multidimensional birth-death process framework. For this purpose, consider a sum of statistically independent hyperexponential variates. The resulting p.d.f. we call the SOHYP (Sum Of HYPerexponentials). With the appropriate choice of parameters the corresponding random variable distributed on this p.d.f can have a coefficient of variation greater than one.

The dwell time for a g-type platform is a random variable $T_D(g)$. $T_D(g)$ is defined to be the sum of $N(g)$ statistically independent hyperexponentially distributed random variables denoted $T_D(g,i)$ where i=1,2,...,$N(g)$. We again will make use of the concept of *phases*. Each random variable, $T_D(g,i)$, is considered a phase of $T_D(g)$ and each phase can have any number of *stages* indexed by k=1,2,...,$M(g,i)$. $M(g,i)$ is the number of stages for a g-type platform in the i^{th} phase. From equation (5) we can find the mean of $T_D(g)$.

$$\overline{T}_D(g) = \sum_{i=1}^{N(g)} \sum_{k=1}^{M(g,i)} \frac{\alpha(g,i,k)}{\mu_D(g,i,k)} \qquad (12)$$

The variance, VAR[$T_D(g)$], is the sum of the variances of $T_D(g,i)$

$$\text{VAR}[T_D(g)] = \sum_{i=1}^{N(g)} \sigma^2_{\text{HYP}}(g,i) \qquad (13)$$

Conceptually one can suppose that a g-type platform traversing a cell completes a sequence of phases. When the platform enters a cell, it begins phase 1 of its dwell time. It chooses a dwell time stage for this phase from the $M(g,1)$ stages available for phase 1. The particular stage k is chosen with probability $\alpha(g,1,k)$. When the first phase is completed, the platform enters the second phase. It chooses a new dwell time stage from the $M(g,2)$ stages available for the second phase. The choice of stage k is made with probability $\alpha(g,2,k)$. The choice of stage is independent from one phase to the next. This process continues until the platform enters its *final* phase, $i=N(g)$. Completion of this last phase represents the platform exiting the cell (moving out of communication range of the base station).

This approach to dwell time modeling (using a sum of statistically independent random variables) has the desirable effect of forcing the p.d.f. of the dwell time variate ($f_{T_D(g)}(t)$) to zero at origin. In fact for an $N(g)$ phase SOHYP p.d.f., with $N(g) > 1$ we have $f_{T_D(g)}(0) \equiv 0$. In addition, for $N(g) > 2$ the first $N(g)$-2 derivatives are zero at the origin. To show this we make use of the Laplace Transform and its Initial Value Theorem [5]. $T_D(g)$ has been defined to be a sum of statistically independent hyperexponentially distributed random variables ($T_D(g,i)$). From probability theory we know that the p.d.f., $f_{T_D(g)}(t)$, is the convolution of the component p.d.f.s $f_{T_D(g,i)}(t)$ [6]. That is

$$f_{T_D(g)}(t) = f_{T_D(g,1)}(t) * f_{T_D(g,2)}(t) * \cdots * f_{T_D(g,N(g))}(t) \tag{14}$$

Taking the Laplace Transform of equation (14), with s as the transform variable, we obtain $L_{T_D(g)}(s)$ which has the following form

$$L_{T_D(g)}(s) = \prod_{i=1}^{N(g)} \sum_{k=1}^{M(g,i)} \frac{\alpha(g,i,k) \cdot \mu_D(g,i,k)}{s + \mu_D(g,i,k)} \tag{15}$$

We note that in general $L_{T_D(g)}(s)$ is a rational polynomial function of s which has the form $L_{T_D(g)}(s)$=N(s)/D(s) where N(s) and D(s) are the numerator and denominator respectively. From equation (15) we see that the degree of N(s) is 0 and the degree of D(s) is $N(g)$. Therefore, $L_{T_D(g)}(s)$ is a strictly proper rational function (that is the degree of the denominator is strictly greater than that of the numerator).

A useful property of the Laplace Transform states that for positive time functions ($f(t)$=0. for t<0) the transform of the j^{th} derivative of $f(t)$ is $s^j L(s)$, where $L(s)$ is the transform of $f(t)$ [5]. That is the transform of the j^{th} derivative of $f(t)$ is simply the transform of $f(t)$ multiplied by s^j. Then for $f_{T_D(g)}(t)$ we have

$$\frac{d^{(j)}}{dt^{(j)}} f_{T_D(g)}(t) = f^{(j)}_{T_D(g)}(t) \leftrightarrow s^j \cdot L_{T_D(g)}(s) \tag{16}$$

We can now write a general expression for the Laplace Transform of the j^{th} derivative of $f_{T_D(g)}(t)$ as

$$L^{(j)}_{T_D(g)}(s) = s^j L_{T_D(g)}(s) \tag{17}$$

The Initial Value Theorem [5] for the Laplace Transform states that the value of $f(0)$ is given by the limit as s tends to infinity of $sL(s)$. Applying the Initial Value Theorem to (17) we have

$$f^{(j)}_{T_D(g)}(0) = \lim_{s\to\infty} s \cdot L^{(j)}_{T_D(g)}(s) = \lim_{s\to\infty} s^{j+1} L_{T_D(g)}(s) \tag{18}$$

Recall that $L_{T_D(g)}(s)$ is a strictly proper rational polynomial function with numerator of degree 0 and denominator of degree $N(g)$. The limit in equation (18) will tend to zero as long as $s^{j+1} L_{T_D(g)}(s)$ is strictly proper. This is true for $j \le N(g)$-2. Therefore, the first $N(g)$-2 derivatives of $f_{T_D(g)}(t)$ are zero at the origin. For a $N(g)$ phase SOHYP p.d.f. this is expressed by the following

$$f_{T_D(g)}(0) = 0 \tag{19}$$

and

$$\frac{d^{(j)}}{dt^{(j)}} f_{T_D(g)}(0) = 0 \qquad \text{for } j = 0,1,2,\ldots,N(g)-2 \tag{20}$$

Figure 3 is a plot of several SOHYP p.d.f.s for $N(g)$ = 1,2,3,4, which demonstrates this behavior. Each of the curves was generated by convolving $N(g)$ identical hyperexponential functions.

Analytical expressions for the variance and other moments of an $N(g)$ phase SOHYP random variable can become extremely complex. However, the moment generating function ($\Phi_{SOHYP}(s)$) is easily obtained and it is given below

$$\Phi_{\text{SOHYP}}(s) = \prod_{i=1}^{N(g)} \sum_{k=1}^{M(g,i)} \frac{\alpha(g,i,k)\cdot \mu_D(g,i,k)}{\mu_D(g,i,k) - s} \tag{21}$$

The SOHYP's moments can be obtained from the above.

For the numerical calculations presented in this paper we have used a special two phase SOHYP ($N(g)$=2) although the formulation is more general. The first phase consists of a single stage ($M(g,1)=1$) and the second phase a two stage hyperexponential ($M(g,2)=2$). Thus for numerical purposes, we consider the dwell time of a platform to be a random variable which is the sum of an negative exponential variate and a 2-stage hyperexponential variate. The functional form of this special case of the p.d.f. can be found by taking the convolution of a negative exponential function with a hyperexponential function. The result is shown below

$$f_{T_D(g)}(t)=\left[\frac{\alpha(g,2,1)\cdot\mu_D(g,1,1)\cdot\mu_D(g,2,1)}{\mu_D(g,2,1)-\mu_D(g,1,1)}+\frac{\alpha(g,2,2)\cdot\mu_D(g,1,1)\cdot\mu_D(g,2,2)}{\mu_D(g,2,2)-\mu_D(g,1,1)}\right]e^{-\mu_D(g,1,1)t}+ \left[\frac{\alpha(g,2,1)\cdot\mu_D(g,1,1)\cdot\mu_D(g,2,1)}{\mu_D(g,1,1)-\mu_D(g,2,1)}\right]e^{-\mu_D(g,2,1)t}+\left[\frac{\alpha(g,2,2)\cdot\mu_D(g,1,1)\cdot\mu_D(g,2,2)}{\mu_D(g,1,1)-\mu_D(g,2,2)}\right]e^{-\mu_D(g,2,2)t} \quad (22)$$

This allows four parameters that can be adjusted to fit empirical data that may be available for *each* platform type g. The parameters are $\mu_D(g,1,1)$,$\alpha(g,2,1)$,$\mu_D(g,2,1)$, and $\mu_D(g,2,2)$. The reader should note that $\alpha(g,2,2)$ is not among the list of parameters since it is determined by equation (6), $\alpha(g,2,2)=1-\alpha(g,2,1)$. Figure 4 is a plot of this p.d.f. for $\kappa^2=1,2,3,4$ and a mean of one. Here we see that all the p.d.f.s in this family go through the origin. To the eye the shape appears similar to the Erlang-2 p.d.f. but it is much heavier on the tail. Moreover, it has a coefficient of variation that can be greater than unity in contrast to the Erlang-2 which is less than unity. SOHYP p.d.f.s with more than two phases stay even closer to the origin as the abscissa increases from zero. Since a SOHYP p.d.f. with N phases ($N\geq 2$) will go through the origin, as will its first N-2 derivatives. This behavior is similar to that of the Erlang p.d.f. with N phases.

4. Statement of example problem: Single call, hand-offs, cut-off priority, mixed platform types, SOHYP mobility characteristics.

The system has G types of mobile platforms, indexed by $g=1,2,\ldots,G$. Each platform can support at most one call at a time. Platform mobilities are characterized by random variables $T_D(g)$,($g=1,2,\ldots G$), each having a SOHYP distribution but with generally different parameters.

The number of *noncommunicating g-type platforms* in any cell is denoted by $v(g,0)$. It is assumed that $v(g,0)$ is much larger than the total number of calls that a cell can accommodate. In other words an infinite population model is assumed.

The new call origination rate for a non-communicating platform g-type platform is denoted $\Lambda(g)$. The total new call origination rate for g-type calls is denoted by $\Lambda_n(g)$ and is expressed as $\Lambda_n(g) = \Lambda(g)v(g,0)$.

Channel limit: Each gateway (or cell) can accommodate C calls.

Platform quotas: Each gateway can accommodate $J(g)$ g-type calls simultaneously, where $J(g) \leq C$.

Cut-off priority: At each gateway C_h channels are reserved for hand-off calls. New calls will be blocked if the number of channels in use is greater than C-C_h, while hand-off attempts will fail only if the number of channels in use is equal to C.

The holding time of a g-type call is a n.e.d. random variable $T(g)$ with mean $\overline{T}(g) = 1 / \mu(g)$.

The problem is to calculate performance measures including blocking probability, hand-off failure probability, forced termination probability and hand-off activity. The analysis is similar to that used in references [1] and [2]. The differences are in the definition of the state variables and in the equations that specify the state probability transition flows. What follows is the development of the state characterization, transition rate, and the probability flow balance equation that determine the state probabilities.

5. State Description

The state variables for the system are given by v_{gik} { g= 1,2,...,G, i=1,2,...,$N(g)$, k=1,2,...,$M(g,i)$} which represent the number of platforms of type g that are in phase i and stage k.

We consider a single cell. The state of a cell is a sequence of non-negative integers. The sequence is conveniently written as G n-tuples as follows

$$
\begin{array}{llll}
v_{111}\, v_{112} \cdots v_{11M(1,1)}, & v_{121}\, v_{122} \cdots v_{12M(1,2)}, & \cdots, & v_{1N(1)1}\, v_{1N(1)2} \cdots v_{1N(1)M(1,N(1))} \\
v_{211}\, v_{212} \cdots v_{21M(2,1)}, & v_{221}\, v_{222} \cdots v_{22M(2,2)}, & \cdots, & v_{2N(2)1}\, v_{2N(2)2} \cdots v_{2N(2)M(2,N(2))} \\
\vdots & & & \vdots \\
v_{g11}\, v_{g12} \cdots v_{g1M(g,1)}, & v_{g21}\, v_{g22} \cdots v_{g2M(g,2)}, & \cdots, & v_{gN(2)1}\, v_{gN(2)2} \cdots v_{gN(g)M(g,N(g))} \\
\vdots & & & \vdots \\
v_{G11}\, v_{G12} \cdots v_{G1M(G,1)}, & v_{G21}\, v_{G22} \cdots v_{G2M(G,2)}, & \cdots, & v_{GN(G)1}\, v_{GN(G)2} \cdots v_{GN(G)M(G,N(G))}
\end{array}
\tag{23}
$$

It is convenient to order the states using the index s=0,1,2,..s_{max}. The state variables can then be expressed as functions of s,g,i, and k. That is $v(s,g,i,k)$ represents the number of g-type platforms in phase i and stage k when the cell is in state s.

When a cell is in state s, a number of characteristics can be determined. The number of channels in use by g-type platforms is

$$
j(s,g) = \sum_{i=1}^{N(g)} \sum_{k=1}^{M(g,i)} v(s,g,i,k) \tag{24}
$$

The total number of channels in use when the cell is in state s is found by

$$
j(s) = \sum_{g=1}^{G} j(s,g) \tag{25}
$$

A sequence of integers is a permissible state if all constraints are satisfied. For this framework the channel limit (C) requires $j(s) \le C$, and platform quotas ($J(g)$) require $j(s,g) \le J(g)$ for g=1,2,...,G.

6. Driving processes and state transitions

For this model five driving processes can be identified. They are: $\{n\}$ the generation of new calls; $\{c\}$ the completion of calls; $\{h\}$ the arrival of communicating platforms at the cell of interest; $\{d\}$ the departure of communicating calls from the cell of interest; and $\{\phi\}$ the transition between dwell time phases. Since the model considers multiple platform types the above processes are all multidimensional.

In order to use a multidimensional birth-death formulation, Markovian assumptions about the driving processes are involved. The new call arrival process is assumed to follow a Poisson point process with state dependent means. Call session time has been assumed to be n.e.d. distributed, so the call completion process has the required memoryless property. The dwell time, however, has the SOHYP p.d.f., which itself does not have the memoryless property. However, by using the state description outlined in the previous section the dwell time is decomposed into its phases and stages. Each of these is n.e.d., so the memoryless property can be used [7]. The hand-off arrival process for each platform type is determined by the corresponding departure process. In considering the overall *system states* [1], [2] these are (system) state dependent Poisson processes. In focusing on an individual cell, however, each is represented as a Poisson process having a parameter determined by the decoupling assumption of $\{h\}$ and $\{d\}$ as described in [1], [2]. In comparison with dwell times that are n.e.d, the cost of this more appropriate model requires an increase in the size (dimension and number of states) of the state space and generally longer computation times.

It remains to specify all of the state transitions. For every state, the possible predecessor states must be identified. A predecessor state is any state which could have immediately given rise to the current state under each of the driving processes described above. Also, the state probability transition flows must be found. Then the flow balance equations can be determined and the state probabilities can be calculated [1,2,7].

In the paragraphs that follow three dummy indices z_1,z_2, and z_3 are used for convenience as follows: z_1=1,2,3,...,G; z_2=1,2,3...,$N(z_1)$;z_3=1,2,3,...,$M(z_1,z_2)$. Recall that G is the number of platform types, $N(z_1)$ is the number of phases of dwell time for a platform of type z_1, and $M(z_1,z_2)$ is the number of possible stages for a platform of type z_1 in phase z_2. In addition, when we discuss a state s it is implied that we are considering the actual sequence of integers indexed by s not the index itself.

6.1. New Call Arrivals

A transition into state s from state x_n due to a new call arrival on a g-type platform in phase i stage k will cause the state variable $v(x_n,g,i,k)$ to be increased by 1. Also a new call can only be served if the total number of channels in use is less than C-C_h. Therefore, a permissible state x_n is a predecessor of state s for new call arrivals on g-type platforms if $j(x_n) < C-C_h$, $j(x_n) < J(g)$ and the state variables are related by

$$\begin{aligned} v(x_n,g,i,k) &= v(s,g,i,k)-1 \\ v(x_n,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) \qquad z_1 \neq g \\ v(x_n,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) \qquad z_2 \neq i \\ v(x_n,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) \qquad z_3 \neq k \end{aligned} \tag{26}$$

The probability that a new call will arrive at a g-type platform while the platform is in phase i and stage k is given by

$$\rho_n(g,i,k) = \frac{\alpha(g,i,k)/\mu_D(g,i,k)}{\overline{T}_D(g)} \qquad , k=1,2,\ldots,M(g,i) \tag{27}$$

The new call arrival rate for calls on g-type platforms in phase i and stage k of dwell time can be expressed as

$$\Lambda_n(g,i,k) = \rho_n(g,i,k) \cdot \Lambda_n(g). \tag{28}$$

Finally the transition rate into state s from state x_n due to new call arrivals is

$$\gamma_n(s,x_n) = \Lambda_n(g,i,k) \tag{29}$$

6.2. Call Completion

A transition into state s due to a call completion on a g-type platform in stage k of phase i when the cell is in state x_c causes the state variable $v(x_c,g,i,k)$ to be decreased by 1. Therefore, a permissible state x_c is a predecessor of state s for call completion on g-type platforms in stage k of phase i if the state variables related by

$$\begin{aligned} v(x_c,g,i,k) &= v(s,g,i,k)+1 & & \\ v(x_c,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & z_1 &\neq g \\ v(x_c,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & z_2 &\neq i \\ v(x_c,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & z_3 &\neq k \end{aligned} \tag{30}$$

The corresponding transition rate is given by

$$\gamma_c(s,x_c) = \mu(g) \cdot v(x_c,g,i,k) \tag{31}$$

6.3. Hand-off Arrivals

When a g-type platform enters a cell from a neighboring cell, it will enter in its *first phase* of dwell time. Therefore a transition into state s from state x_h due to a hand-off arrival on a g-type platform will cause the state variable $v(x_h,g,1,k)$ to be incremented by 1. Since hand-off calls have access to all C channels in the target cell, a permissible state x_h is a predecessor of state s for hand-off arrivals on g-type platforms in phase i and stage k if $j(x_h)< C$, $j(x_h)<J(g)$, and the state variables are related by

$$\begin{aligned} v(x_h,g,1,k) &= v(s,g,1,k)-1 & & \\ v(x_h,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & z_1 &\neq g \\ v(x_h,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & z_2 &\neq i \\ v(x_h,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & z_3 &\neq k \end{aligned} \tag{32}$$

When the platform enters the target cell, it begins its first phase and chooses a stage k (k=1,2,...,$M(g,1)$) with probability $\alpha(g,1,k)$. Let Λ_h be the overall average hand-off arrival rate and F_g be the fraction of hand-off arrivals that occur on g-type platforms. The rate of hand-off arrivals of g-type platforms in phase 1 and stage k is given by

$$\Lambda_h(g,k) = \alpha(g,1,k) \cdot \Lambda_h \cdot F_g \tag{33}$$

in which $\alpha(g,1,k)$ is defined in equation (5). For now, the parameters Λ_h and F_g are assumed to be known, but they are actually functions of the state probabilities. Their values are determined in the iterative solution described later. The transition flow in given by

$$\gamma_h(s,x_h) = \Lambda_h(g,k) \tag{34}$$

6.4. Hand-off Departures

A hand-off departure of a g-type platform occurs when the platform completes its final phase ($i{=}N(g)$) of dwell time. Therefore, a transition into state s due to a hand-off departure of a g-type platform in stage k of its last ($i{=}N(g)$) phase when the cell is in state x_d, will cause the state variable $v(x_d,g,N(g),k)$ to be decreased by 1. Thus a permissible state x_d is a predecessor state of s for hand-off departures of g-type platforms in stage k of phase $N(g)$ if the state variables are related by

$$\begin{aligned} v(x_d,g,N(g),k) &= v(s,g,N(g),k)-1 & & \\ v(x_d,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & & z_1 \neq g \\ v(x_d,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & & z_2 \neq N(g) \\ v(x_d,z_1,z_2,z_3) &= v(s,z_1,z_2,z_3) & & z_3 \neq k \end{aligned} \tag{35}$$

The corresponding transition flow is given by

$$\gamma_d(s,x_d) = \mu_D(g,N(g),k) \cdot v(x_d,g,N(g),k) \tag{36}$$

6.5. Dwell time phase transitions

A transition into state s from state x_ϕ occurs when a platform, which is in phase i ($1 \leq i<N(g)$) and stage k, completes its current dwell time phase (i.e., stage k of phase i is completed). After completing phase i the platform begins phase i+1 where it chooses stage ξ ($\xi=1,2,\ldots,M(g,i+1)$) with probability $\alpha(g,i+1,\xi)$. Therefore, the transition from phase i stage k to phase i+1 stage ξ causes two state variables to change simultaneously. The state variable $v(x_\phi,g,i,k)$ will be decreased by 1 and the state variable $v(x_\phi,g,i+1,\xi)$ will be increased by 1. We

limit the discussion of phase transitions to $1 \le i < N(g)$ since a completion of phase $N(g)$ corresponds to a hand-off departure and has all ready been considered. Thus a permissible state x_ϕ is a predecessor state of s for dwell time phase transitions of g-type platforms from phase i and stage k to phase $i+1$ stage ξ if the state variables are related by

$$
\begin{aligned}
&v(x_\phi,g,i+1,k) = v(s,g,i+1,\xi)-1 \\
&v(x_\phi,g,i,k) \quad = v(s,g,i,\xi)+1 \\
&v(x_\phi,z_1,z_2,z_3) = v(s,z_1,z_2,z_3) \qquad z_1 \neq g \\
&v(x_\phi,z_1,z_2,z_3) = v(s,z_1,z_2,z_3) \qquad z_2 \neq i,\ i+1 \\
&v(x_\phi,z_1,z_2,z_3) = v(s,z_1,z_2,z_3) \qquad z_2 \neq k,\ \xi
\end{aligned}
\tag{37}
$$

The corresponding transition flow from state, x_ϕ to state, s is given by

$$\gamma_\phi(s,x_\phi) = \alpha(g,i+1,\xi)\cdot \mu_D(g,i,k)\cdot v(x_\phi,g,i,k) \tag{38}$$

7. Flow balance equations

The total transition flow into a state s from any permissible state x is the sum of the component flows due to the driving processes. This is expressed by the following

$$q(s,x) = \gamma_n(s,x) + \gamma_c(s,x) + \gamma_h(s,x) + \gamma_d(s,x) + \gamma_\phi(s,x) \tag{39}$$

where $q(s,x)$ is the total flow into state s from state x and $s \neq x$. Flow into a state is assigned a positive value and flow out of a state is assigned negative values. The total flow out of state s is therefore written as

$$q(s,s) = -\sum_{\substack{n=0 \\ n \neq s}}^{s_{max}} q(n,s) \tag{40}$$

When the system is in statistical equilibrium, the total flow into a state is equal to the total flow out of the state. The balance equations can be expressed as

$$\sum_{j=0}^{s_{max}} q(i,j)p(j) = 0 \qquad i = 0,1,2,\ldots s_{max}-1$$

$$\sum_{j=0}^{s_{max}} p(j) \tag{41}$$

The above are a set of s_{max} -1 simultaneous equations which can be solved for the unknown state probabilities $p(j)$.

Previously we assumed the parameters Λ_h (the average hand-off arrival rate) and F_g (the fraction of hand-offs that are g-type) where given. These are actually functions of the driving processes and must be determined from the processes themselves. The iterative approach described in [2] is used. We denote $\Delta_h(g,i)$ as the average hand-off departure rate of g-type platforms in phase i. This is given by

$$\Delta_h(g,i) = \sum_{s=0}^{s_{max}} \sum_{k=1}^{M(g,i)} \nu(s,g,i,k) \cdot \mu_D(g,i,k) \cdot p(s) \tag{42}$$

The average hand-off departure rate of g-type platforms is then found by

$$\Delta_h(g) = \sum_{i=1}^{N(g)} \Delta_h(g,i) \tag{43}$$

and the total average departure rate is

$$\Delta_h = \sum_{g=1}^{G} \Delta_h(g) \tag{44}$$

The fraction of hand-off departures that occur on g-type platforms is denoted F_g' and are given by

$$F_g' = \frac{\Delta_h(g)}{\Delta_h} \qquad g = 1,2,\ldots,G \tag{45}$$

A hand-off departure of a g-type platform from a cell causes a hand-off arrival of the same type in another cell. Then, for a homogeneous system in equilibrium, the average hand-off departure and arrival rates and the fraction of g-type arrivals and departures must be equal.

This is expressed in the following

$$F_g = F'_g$$

and (46)

$$\Lambda_h = \Delta_h$$

8. Computational procedure

As stated earlier the quantities F_g, Λ_h, and Δ_h which appear in the determination of the transition flows [$q(i,j)$s] in the flow balance equations depend on the unknown state probabilities $p(s)$ (s=0,1,2,...s_{max}). Consequently, the flow balance equations are actually a set of simultaneous non-linear equation. A solution for the state probabilities $p(s)$ and, Λ_h, and Δ_h can be obtained using an iterative approach[2]. For completeness of the present discussion, a brief outline of the solution procedure follows.

We define two function $Q_1(\Lambda_h)$ and $Q_2(\Lambda_h)$. Q_1 is a vector function which takes Λ_h as its argument and returns all the state probabilities $p(s)$; fractions of g-type hand-off departures F'_g; and the overall average hand-off departure rate Δ_h. Q_2 is a scalar function of Λ_h. For a given Λ_h the function calls $Q_1(\Lambda_h)$ and returns the difference $\Lambda_h - \Delta_h$.

The function Q_1 is implemented in the following steps.

Step 1: Given Λ_h, begin with guesses or previous values for F_g. Solve the flow balance equations for the state probabilities. We used a modified Gauss-Seidel algorithm to solve for the state probabilities, other methods for solving systems of linear equations can be used.

Step 2: Using the solution $[p(s)]$, determine the new fractions of g-type platform departures F'_g and compare with the previous values. If the relative error in any of the state probabilities $p(s)$ or fractions F_g exceeds some value (10^{-4} for accuracy of three significant figures) repeat *step 1* using the latest values for F_g. Otherwise for the given Λ_h, and solution $p(s)$, determine the hand-off departure rate Δ_h according to the formulas in the previous section. Return the latest values.

The final solution is obtained by finding the root of $Q_2 = \Lambda_h - \Delta_h = 0$. For this purpose we used a bisection algorithm. When the root of Q_2 is found, the average hand-off arrival and departure rates will be equal as required for a homogeneous system.

9. Performance measures

Once the state transition flows and state probabilities are found, various performance measures which are functions of the state probabilities can be calculated.

9.1. Carried traffic

The carried traffic per cell for each platform type is the average number of channels occupied by the calls from the given platform type. The carried traffic for g-type platforms is

$$A_c(g) = \sum_{s=0}^{s_{max}} j(s,g) \cdot p(s) \tag{47}$$

The total carried traffic for all platform types, i.e., the total carried traffic per cell is

$$A_c = \sum_{g=1}^{G} A_c(g) \tag{48}$$

9.2. Blocking probability

The blocking probability for a new call on a g-type platform is defined as the average fraction of new calls that arrive on a g-type platforms, but are denied access to a channel. A new call on a g-type platform will be blocked if: 1.) there are no channels available to serve the call or 2.) the number of g-type calls in progress is at the quota level $J(g)$. We define the following disjoint sets of states

$$B_0 = \{s: C\text{-}C_h \leq j(s) \leq C\}$$

$$B_g = \{s: j(s) < C\text{-}C_h, j(s,g) = J(g)\} \tag{49}$$

where $g = 1,2,\ldots,G$. The blocking probability for a g-type call is then given by

$$P_B(g) = \sum_{s \in B_0} p(s) + \sum_{s \in B_g} p(s) \tag{50}$$

9.3. Hand-off failure probability

The hand-off failure probability for g-type calls is defined as the average fraction of g-type hand-off attempts that are denied a channel. Hand-off attempts have access to all C channels in a cell, but may still be subject to channel quota constraints ($J(g)$). Therefore, we have the following disjoint sets of states, for which hand-off attempts will fail.

$$H_0 = \{s: j(s) = C\}$$

$$H_g = \{s: j(s) < C, j(s,g) = J(g)\} \tag{51}$$

The hand-off failure probability for a g-type hand-off can be written as

$$P_H(g) = \sum_{s \in H_0} p(s) + \sum_{s \in H_g} p(s) \tag{52}$$

9.4. Forced termination Probability

Forced termination probability for a g-type platform is denoted as $P_{FT}(g)$. It is defined as the probability that a g-type call that is not blocked is interrupted due to a hand-off failure during its lifetime. A *communicating* g-type platform in phase i (i=1,2,..,$N(g)$) must complete its *remaining* dwell time phases to require a hand-off. There are $N(g)$-i+1 remaining when a platform is in phase i. Recall that all phases are hyperexponentially distributed and phase i has k

(k=1,2,...,$M(g,i)$) stages. A platform selects a stage k for its i^{th} phase from the $M(g,i)$ stages available with probability $\alpha(g,i,k)$. Let $\pi_i(g|k)$ be the conditional probability that a communicating platform in phase i completes the current phase before its call is completed, given that it is in stage k.

$$\pi_i(g|k) = \frac{\alpha(g,i,k)\cdot\mu_D(g,i,k)}{\mu_D(g,i,k)+\mu(g)} \tag{53}$$

We now define $\pi_i(g)$ to be the probability that a communicating g-type platform completes phase i of dwell time before its call is completed. That is the platform completes its current stage of phase i.

$$\pi_i(g) = \sum_{k=1}^{M(g,i)} \frac{\alpha(g,i,k)\cdot\mu_D(g,i,k)}{\mu_D(g,i,k)+\mu(g)} \tag{54}$$

In order for a call being served on a g-type platform in phase i to generate a hand-off attempt $N(g)$-i+1 dwell time phases must be completed before the call completes. The probability that such a call will require a hand-off is

$$b(g,i) = \prod_{n=i}^{N(g)} \pi_n(g) \tag{55}$$

Note that right side of equation (55) is a product because each phase is independent of the others.

When a new call begins service on a g-type platform it may arrive during any phase of the platform's dwell time. The fraction of new calls that arrive on g-type platforms in phase i is given by

$$\rho_n(g,i) = \frac{\overline{T}_D(g,i)}{\overline{T}_D(g)} = \frac{\sum_{k=1}^{M(g,i)} \alpha(g,i,k)\Big/\mu_D(g,i,k)}{\overline{T}_D(g)} \tag{56}$$

We can now write the probability that a new call on a g-type platform requires a hand-off. This is given by

$$b(g) = \sum_{i=1}^{N(g)} \rho_n(g,i) \cdot b(g,i) \tag{57}$$

A call that is successfully handed off enters its target cell in the first phase of dwell time. Thus, the probability that a call on board a g-type platform, that has been handed off requires another hand-off is $b(g,1)$.

The probability that a call is forced to terminate on its k^{th} hand-off attempt is the probability of k-1 successful hand-offs followed by a hand-off failure on the k^{th} attempt. This is given by

$$\mathrm{Y}(g,k) = \mathrm{b}(g) \cdot \mathrm{P_H}(g) \cdot \{\mathrm{b}(g,1) \cdot [1\text{-}\mathrm{P_H}(g)]\}^{k-1} \tag{58}$$

The over all forced termination probability is

$$\mathrm{P_{FT}}(g) = \sum_{k=1}^{\infty} \mathrm{Y}(g,k) \tag{59}$$

We recognize the above as a geometric series and write the sum closed form as

$$\mathrm{P_{FT}}(g) = \frac{b(g) \cdot P_H(g)}{[1 - \Psi(g)]} \,, \qquad \text{where } \Psi(g) \equiv b(g,1) \cdot [1 - P_H(g)] \tag{60}$$

9.5. Hand-off activity factor

The hand-off activity factor, $\eta(g)$, is defined to be the expected number of hand-off attempts for nonblocked calls on g-type platforms. The derivation of the expression for $\eta(g)$ and the expression itself are identical to that given in [2]. The result is shown below

$$\eta(g) = \frac{b(g)}{[1 - \Psi(g)]} \tag{61}$$

Where $b(g)$ is defined in (57) and $\Psi(g)$ is defined in (60).

10. Discussion of results

Numerical results were generated using the method described here. Figure 5 is a plot of blocking probability and forced termination probability versus mean dwell time. The system considered has a single platform type, G=1, with two phases, $N(1)$=2. The first phase is n.e.d. and the second phase hyperexponential with two stages ($M(1,1)$=1, and $M(1,2)$=2). The mean unencumbered session time is assumed to be 100 seconds and the new call arrival rate per platform(Λ_n) is constant at 2.75E-04 calls/sec (approximately one call per hour per platform). Cells have 20 channels each (C=20) and the number of platforms in each cell was set at 400 ($\nu(1,0)$=400). No channel quotas are considered the effect of reserving channels for hand-offs is shown (C_h=0,2,4). The figure shows the effect of mean dwell time and variance.

We see that as the coefficient of variation increases the forced termination probability increases slightly for each C_h. This is expected since a large coefficient of variation, κ, corresponds to a large spread of dwell times around the mean, making it more likely that some calls require several hand-offs. Also shown in the figure is the decrease in forced termination as the dwell time increases. This is because calls on a platform with a large mean dwell time relative to the mean session time will require fewer hand-offs. The blocking probability is insensitive to both mean dwell time and coefficient of variation.

Figure 6 is again a plot of blocking and forced termination probabilities, however, for this figure the platform dwell time is distributed according to the Erlang p.d.f. The result shown in figure 5 were obtained using the method in reference 2. The parameters of the system are identical to those in figure 5., except that the number of phases varies from 1 to 4 ($N(g)$=1,2,3,4) which yields squared coefficients of variations κ^2=1,1/2,1/3,1/4. For this system the blocking probability is found to be insensitive to mean dwell time and variance. The forced termination probabilities increase as the coefficient of variation increases. It is seen from figures 4 and 5 that use of negative exponential variates for dwell time produce essentially the same blocking probabilities as more elaborate models. Forced termination probabilities, on the other hand, can be slightly optimistic.

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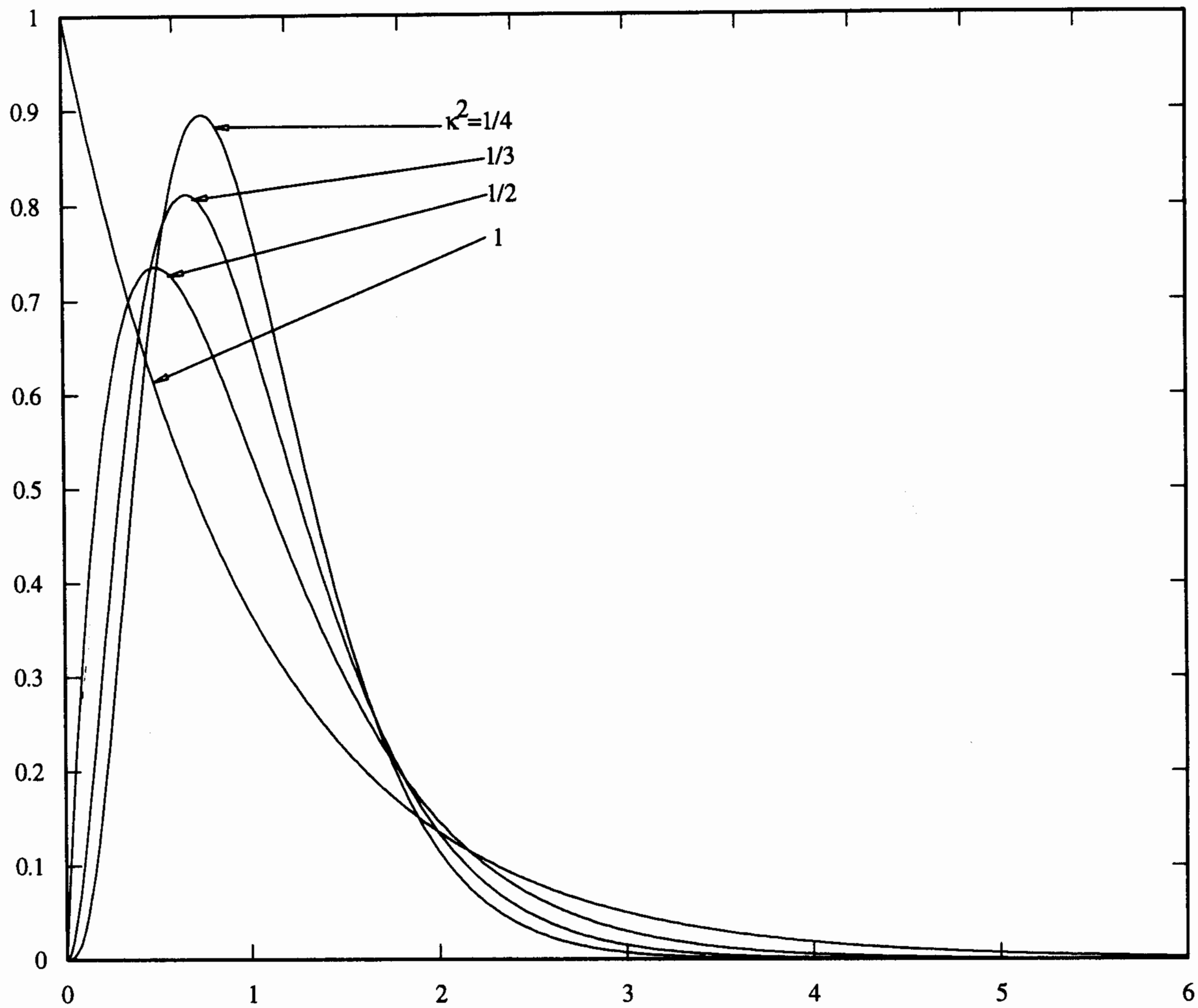

Figure 1 *Erlang p.d.f.s with mean = 1 and* κ^2=1,1/2,1/3,1/4

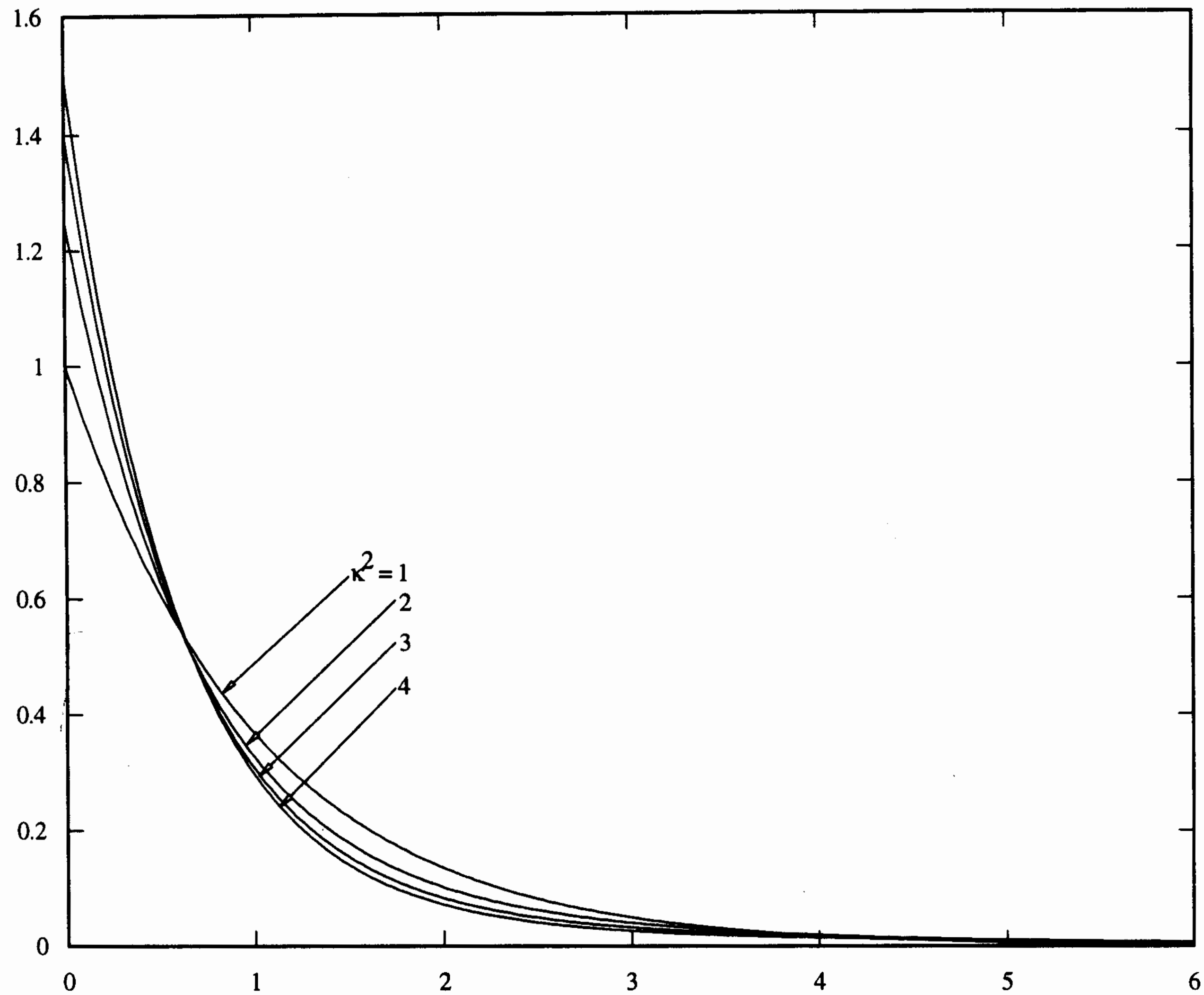

***Figure 2** Hyperexponential p.d.f. mean = 1 and κ^2 = 1,2,3,4*

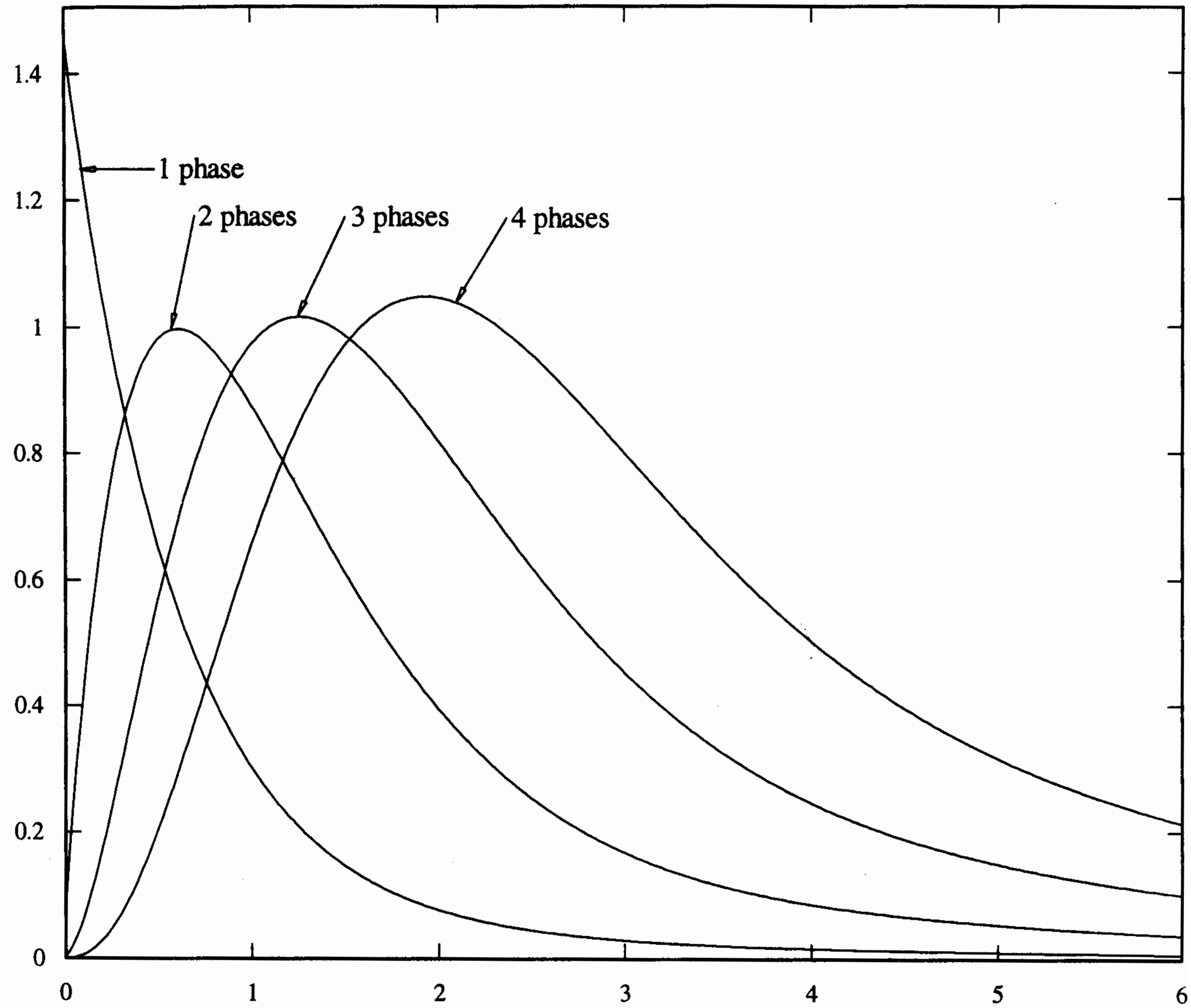

Figure 3 *SOHYP p.d.f. for N=1,2,3,4*

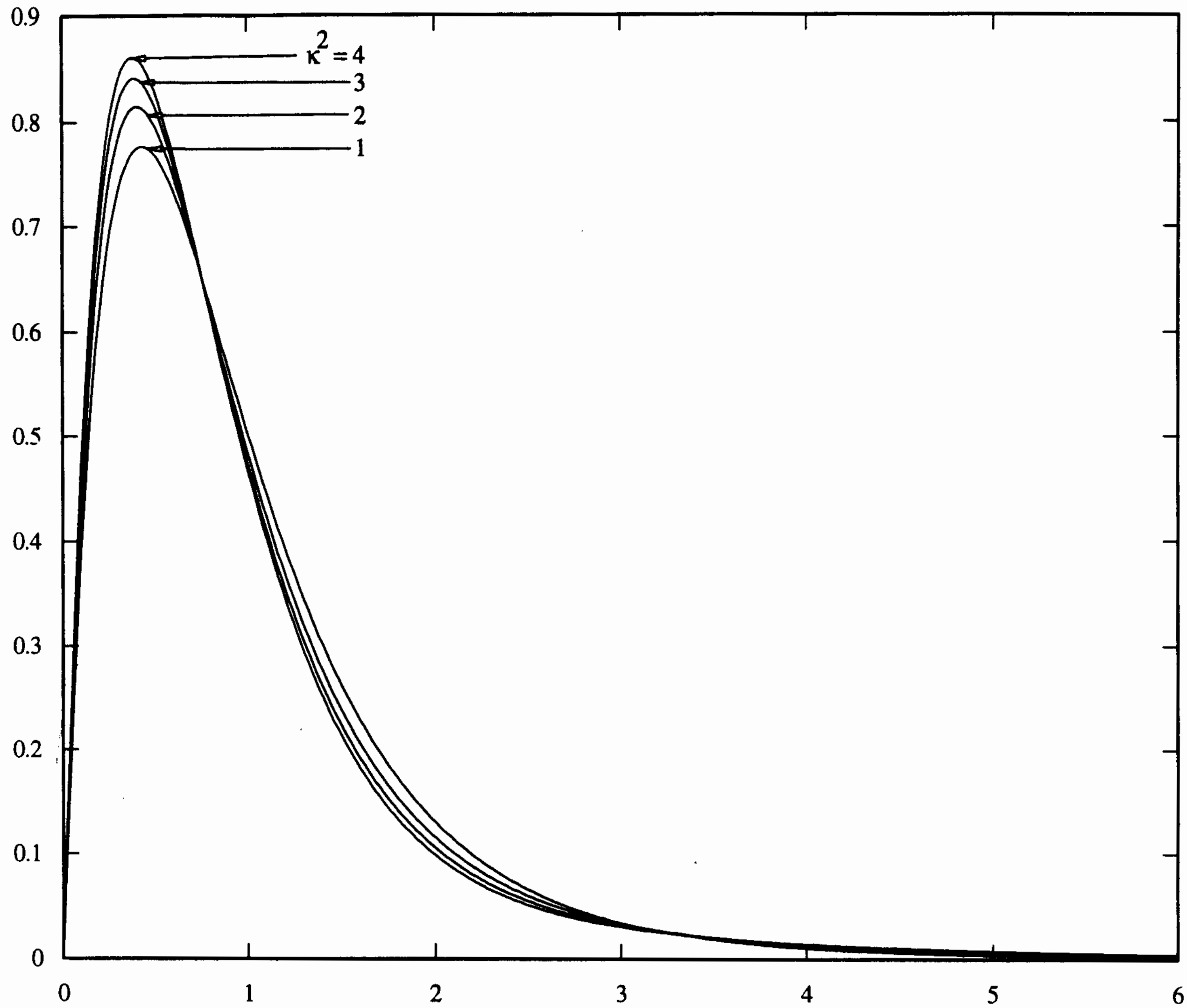

Figure 4 *2-phase SOHYP p.d.f., phase1=negative exponential, phase2=2 stage hyper-exponential. Mean = 1 and* κ^2=1,2,3,4

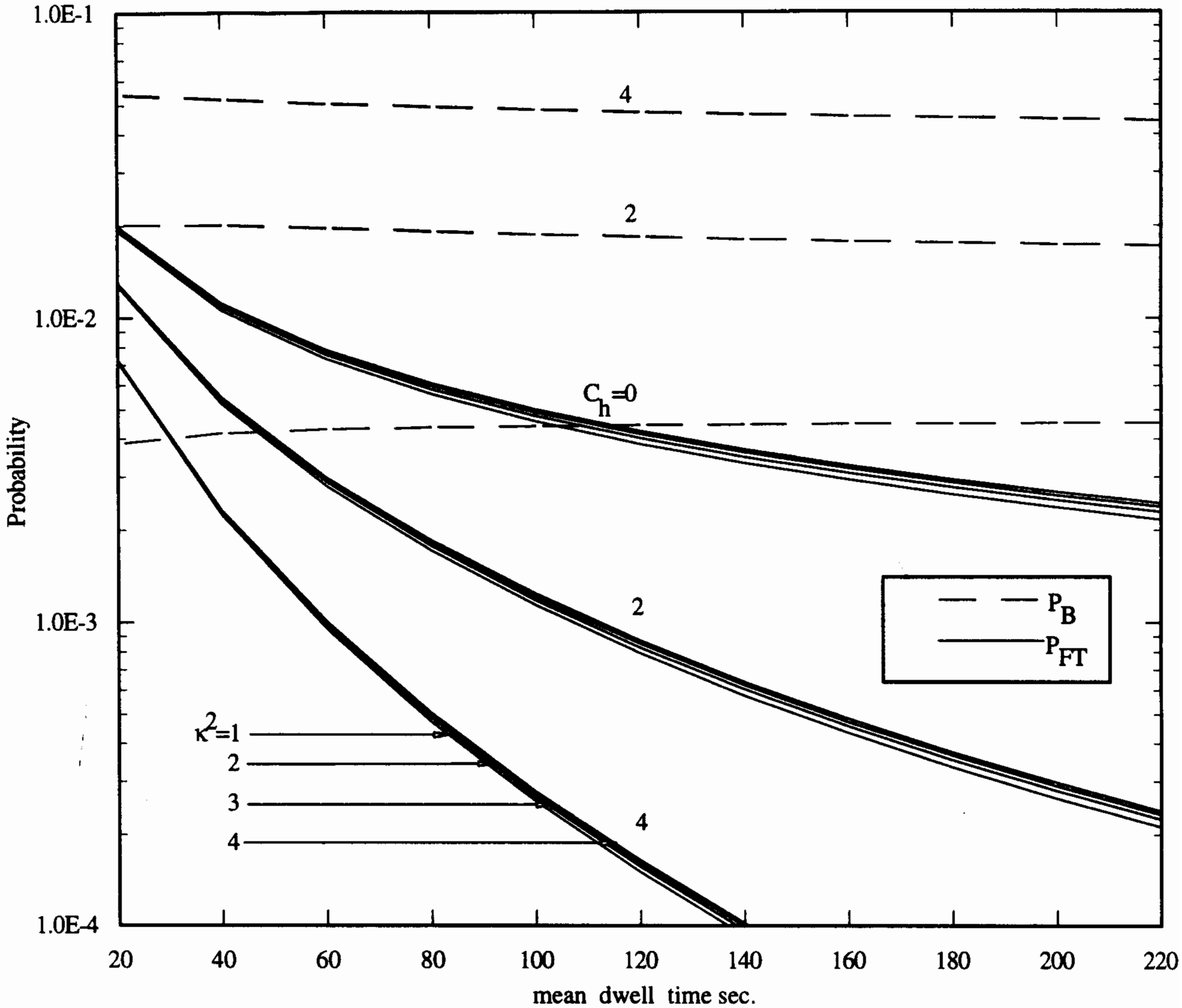

***Figure* 5** *Blocking and forced termination probabillities depend on dwell time mean and variance. Dwell times are SOHYP distributed*
C=20, C_h =0,2,4, G=1, $N(1)$=2, $M(1,1)$ = 1, $M(1,2)$= 2, $\nu(1,0)$=400, $\Lambda(1)$=2.75E-04
— — P_B
——— P_{FT}

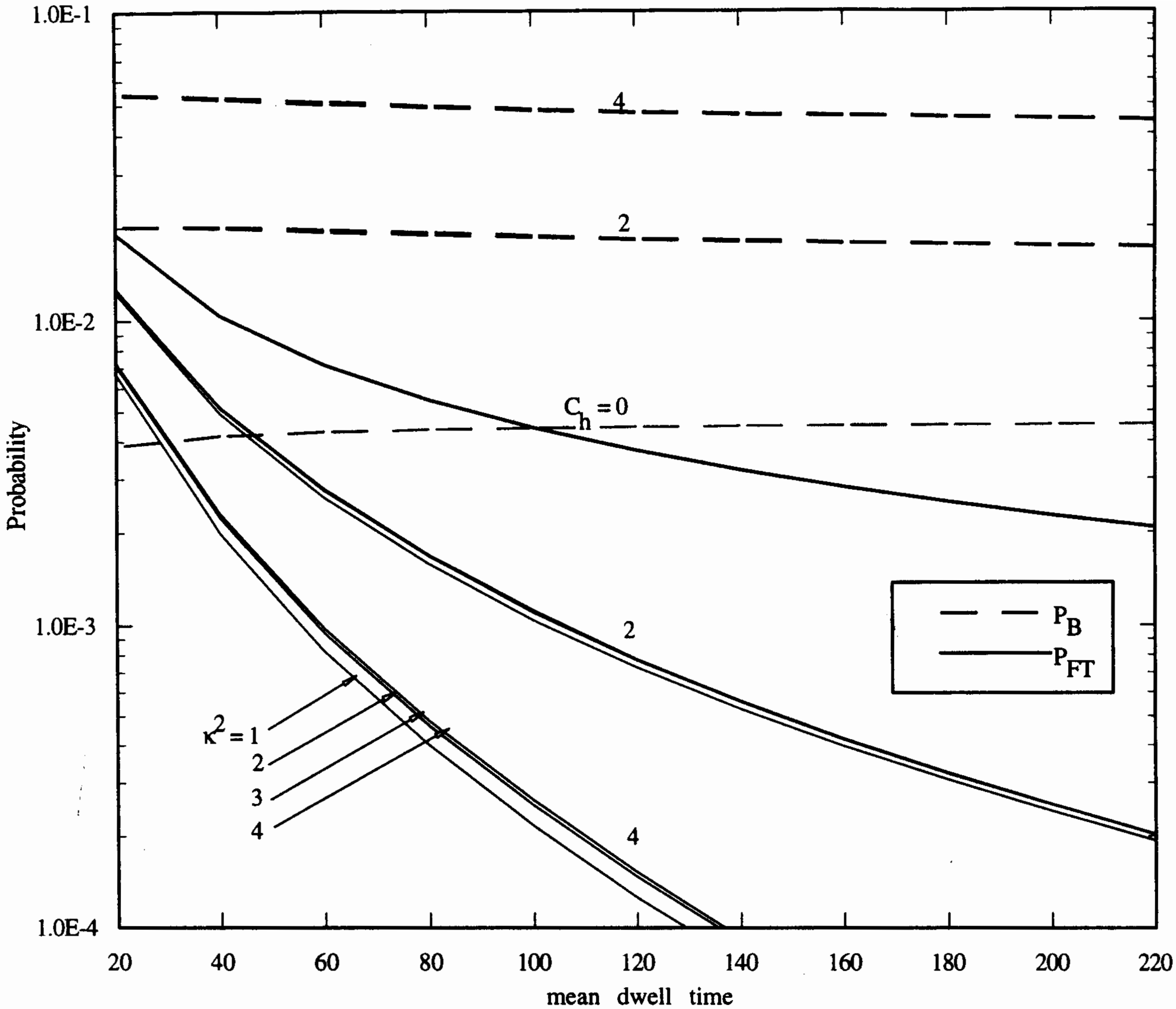

***Figure 6** Blocking and forced termination probabilities depend on dwell time mean and variance. Dwell times are Erlang distributed.*

$G = 1$, $N(1) = 1,2,3,4$, $C = 20$, $C_h = 0,2,4$, $\Lambda_n = 2.75E\text{-}4$, $\nu(1,0) = 400$, $T = 100$ s

— — P_B

—— P_{FT}