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X-RAY IMAGING WITH MULTIPLE-PINHOLE CAMERAS USING A POSTERIORI HOLOGRAPHIC IMAGE SYNTHESIS

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A new form of "extended-source" Fourier-transform holography may be used to synthesize into a single image the multiplicity of N identical images obtained by a multiple-pinhole camera in X-ray astronomy, and thus to provide a holographic solution for achieving the considerable $\sqrt{N}$ signal-to-noise ratio gain conceived by R. H. Dicke.

A number of recent publications [1-4] have dealt with the well-known difficulties of constructing or even of conceiving adequately perfect *conventional* focusing systems (e.g. mirrors and lenses) for imaging in the X-ray and gamma-ray domain. Following independent suggestions by Dicke [1] and Underwood [2], Stroke [3] recently proposed that "extended-source" Fourier-transform holography (see ref. [5] pp. 127-137] may be used to synthesize into a single image the multiplicity of N identical images obtained by a multiple-pinhole camera, and thus to provide a holographic solution for achieving the considerable $\sqrt{N}$ signal-to-noise ratio gain compared to previously used single-pinhole cameras. An *indirect* holographic verification of the principle [3] involved was given in our ref. [3], together with background and references. Because the new image synthesis method involves additional theoretical considerations as well as delicate experimental parameters [6] it appeared essential to us to attempt to *directly* verify the new holographic image synthesis arrangement, before applying it to the actual X-ray experiments now under way [7]. The results of our experiments are given in figs. 1 and 2. The method in its present form is now directly applicable to the X-ray image-synthesis problem.

For the sake of clarity, we shall limit our theoretical description to that essential for the comprehension of the experiment. More detailed theoretical and experimental background may be found in refs. [5,6,8]. For simplicity we give the necessary theory in the form of description of the figures.

Fig. 1 is a reproduction of the carefully-aligned, computer-generated pair of functions $g(x',y') = \iint f(x,y)\,h(x'-x, y'-y)\,\mathrm{d}x\,\mathrm{d}y = f \otimes h$ and $h(x,y)$, where $f(x,y)$, equal here to a horizontal letter "H", is the single image which would be obtained if one of the N (here equal to 2 500) pinholes had been used alone, and $h(x,y)$ is the "spread function" [5] formed by the array of randomly arranged N pinholes [1]. In other words, the spread function $h(x,y)$ is the *image* of a single "star" point formed by the multiple pinhole camera, and $g(x,y)$ is the image of the object of astrophysical interest (e.g. star field, galaxy, nebula) formed by the camera.

Our holographic image synthesis methods consists of the following steps (described here as a model). First, we record an "extended-source" Fourier-transform hologram [5] of $g(x,y)$ using a lens, with $h(x,y)$ being the "extended source". The hologram exposure (intensity) is given by

$$|G|^2 + |H|^2 + GH^* + G^*H = |G|^2 + |H|^2 + FHH^* + F^*H^*H$$

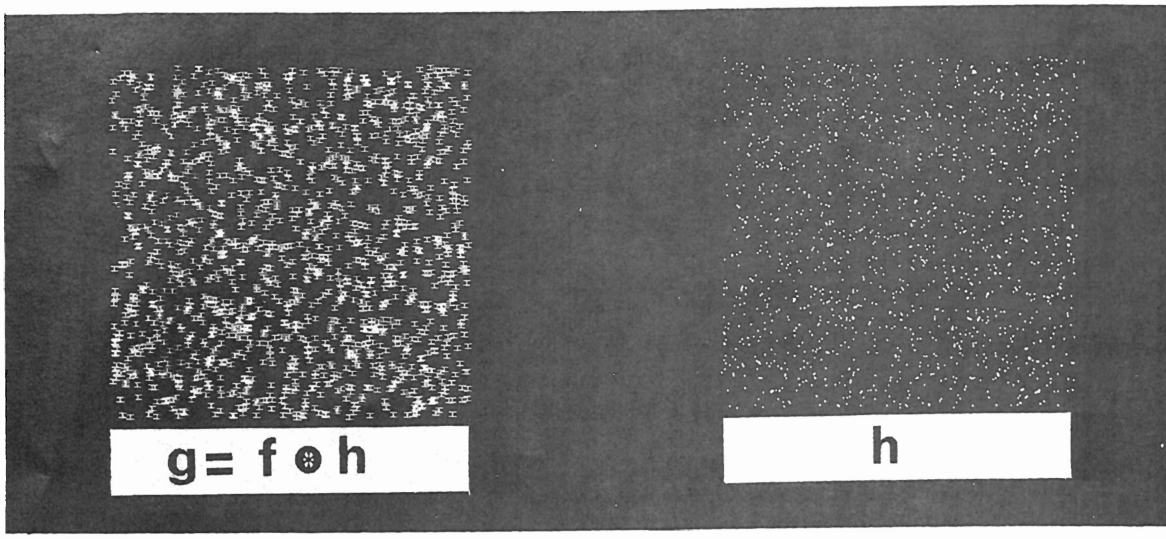

Fig. 1. Functions $g(x,y)$ and $h(x,y)$ used to record the Fourier-transform hologram for the purpose of synthesizi the single image $f(x,y)$ according to text. Dimensions of $h(x,y)$: 2 500 50-micron diameter pinholes, randomly di posed in a 18×18 mm square. Dimensions of $g(x,y)$: 2 500 "horizontal" letters "H" disposed in the same arrang ment as $h(x,y)$ within an 18×18 mm square. Height of "H": 300 microns; width of "bars": 50 microns (same as pinhole diameter); separation of "bars": 200 microns. Distance between center of g and center of h: 31 mm. Fo length of Fourier-transform hologram recording lens: 1 069 mm. No diffusors were used in the hologram record with g and h placed in the front focal plane of the lens, but a density of about 3.02 was placed in front of g to help approximately equalizing the two scattered fields.

where

$$G(u,v) = \iint g(x,y)\ \exp[2\pi i(ux+vy)]\ dx dy$$

in normalized form [5], and similarly for F and H, in relation to f and h, noting that

$$\iint f \otimes h\ \exp[2\pi i(ux+vy)]\ dx dy = FH\ .$$

The synthesized single image $f(x,y)$, shown in fig. 2, is extracted from the hologram by Fourier transformation using a lens, in such a way that the hologram is placed in the front focal plane of the Fourier-transforming lens, and is illuminated with a collimated plane wave, incident normally onto it in the usual manner (see e.g. ref. [5]). Indeed, Fourier transformation of the term FHH^* of the hologram will be equal to $f(x,y)$ in the cases where $h * h^* = \delta$ (a delta function), to which corresponds the condition $HH^* = 1$ (unity), that is the condition deliberately selected for this purpose! (This is the condition which would also be essential for the success of the "image integration" method [4] using *lensless* Fourier-transform hologram recording after ref. [9], although the condition was not indicated in ref. [4].)

Two main parameters are essential [3,6] for the purpose of holographically synthesizing a

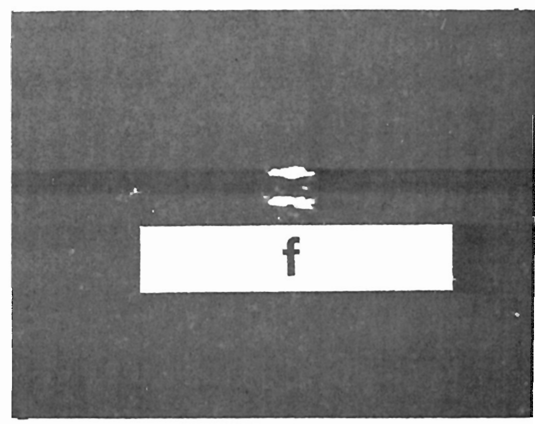

Fig. 2. Synthesized single image $f(x,y)$ of the "hor zontal" letter "H" reconstructed by holographic Fo ier transformation from the hologram recorded ac ing to fig. 1 and text. The image shown is an enlar ment, recorded in the best focus for the "horizont bars of the "horizontal" letter "H" obtainable with 1 069 mm lens used. (The "vertical" bar of the "ho zontal" letter "H" clearly appears in other photos, focused at a distance where the "horizontal" bars shown here are less sharply imaged!). A positive tact print of the hologram recorded according to fi was used in the reconstruction.

single image $f(x, y)$ from the convolution photograph $g(x, y)$ as we describe:

1. The spatial auto-correlation function of the spread function must be very sharply peaked, i.e. we must have $h * h^* = \iint h(x, y)\, h(x+x', y+y') \times \mathrm{d}x\mathrm{d}y \cong \delta$ (a delta function).

2. The photograph $g(x, y)$ and the photograph $h(x, y)$ (or a suitable mask equal to it) must be carefully aligned in the Fourier-transform hologram recording arrangement [5] in such a way that no rotation component exists between the array $h(x, y)$, on the one hand, and, on the other, the identical array of the centres of gravity of the multiple images of $g(x, y)$ (it being understood that all images in $g(x, y)$ are oriented in the same direction, and are identical to each other according to the equation $g = f \otimes h$).

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REFERENCES

[1] R.H.Dicke, Astrophys. J. 153 (1968) L101.
[2] J.H.Underwood, Science 159 (1968) 383, and private communication.
[3] G.W.Stroke, Phys. Letters 28A (1968) 252.
[4] H.J.Einighammer, unpublished paper, presented at 5th Intern. Congr. on X-ray optics and microanalysis, Tübingen (9-14 Sept. 1968) [private communication from Dr. Goetz K.Oertel to G.W.Stroke, 4 February 1969]. See also Naturwissenschaften 55 (1968) 295.
[5] G.W.Stroke, An introduction to coherent optics and holography, 2nd ed. (Academic Press, New York, 1969).
[6] G.W.Stroke, Opt. Acta, in print.
[7] R.B.Hoover, Astrophys. J., in preparation.
[8] G.S.Hayat, M. Sc. Thesis, Electrical Sciences Dept., Sate University of New York at Stony Brook (1969), unpublished.
[9] G.W.Stroke, Appl. Phys. Letters 6 (1965) 201.