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OVERLAPPING COVERAGE WITH REUSE PARTITIONING IN MICROCELLULAR COMMUNICATION SYSTEMS

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Abstract

In cellular communication systems overlapping coverage areas of nearby base stations provide some users with access to more than one base. This can be used to improve teletraffic performance. Mobile users who are distant from base stations are helped most by this because they are more likely to be in communication range of other nearby bases. Reuse partitioning, on the other hand, tends to be most helpful to users that are close to base stations, because they can use channels from more partitions.

This paper considers the combined use of overlapping coverage and reuse partitioning so that ALL users gain some advantage. We develop an analytical model for such systems. Theoretical traffic performance characteristics are obtained and compared with those for fixed channel assignment schemes. Priority for hand-off calls is considered. Simulation results validate the analysis.

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1 Introduction

In cellular communication systems, it is usual to suppose that a mobile user is served by the base station which provides the best link quality. In many cases, however, a mobile user can establish a communication link of acceptable quality with more than one base. Succinctly, at many locations there is overlapping coverage, usually by nearby base stations [1]. This coverage overlap can be used to improve teletraffic performance characteristics. Several schemes that consider this have been suggested [2][3][4][5][6]. Generalized Fixed Channel Assignment (GFCA), a scheme which allows a call to be served by any of several nearby base stations, was considered in [2]. Directed retry, discussed in [3] and [4], allows a new call that cannot be served at one base to attempt access via a nearby alternate base. Load sharing is an enhancement of directed retry that allows calls in congested cells to be served by neighboring base stations. In [5] overlapping coverage for highway microcells was considered. The use of overlapping coverage with channel rearrangement was discussed in [6].

Reuse partitioning [7][8][9] can also improve traffic performance of fixed channel assignment (FCA). The method divides the channels into several disjoint partitions. These partitions are associated with different cluster sizes (or reuse factors). Channels are allocated to base stations according to these cluster sizes. To meet the same signal quality requirement, channels corresponding to smaller cluster sizes are used within a smaller area than that for channels associated with larger cluster sizes [7]. Since channels are reused more often for a smaller cluster size, there may be more channels available at a base in reuse partitioning than that in FCA. Therefore improved traffic performance can be obtained. Because there is a fixed relationship between channels and base stations, reuse partitioning is a fixed channel assignment scheme. In this paper the acronym FCA is used only to refer fixed channel assignment without utilizing overlapping coverage or reuse partitioning. That is, in FCA all channels are allocated using a single cluster size and overlapping coverage is not exploited.

When overlapping coverage exists in a system and is being used to provide enhanced access, users in overlapping areas may benefit. However this may be at the expense of users in non-overlapping areas who may encounter increased blocking or hand-off failure because of the higher channel occupancy of the system. Generally, overlapping areas tend to occur at the periphery of cells (i.e. distant from bases). In reuse partitioning, on the other hand, calls that are close to base stations can access channels from both the smaller cluster and larger cluster partitions. Those calls distant from bases cannot use channels from partitions of smaller cluster sizes. Such calls may in fact do more poorly than in FCA. When *both* overlapping coverage and reuse partitioning are used, they can complement one another.

We analyze teletraffic performance for this interesting combination and compare with FCA. Since users' mobility is important in small-cell environments, hand-off issues are considered. Cut-off priority for hand-off calls is utilized to reduce forced termination events at the cost of increased blocking. Performance exchanges between blocking and forced termination probability are also studied. Communication traffic performance and hand-off issues were considered in [10][11][12], but exploration of overlapping coverage or reuse partitioning was not considered. This paper is organized as follows. Section 2 presents the basic model for this scheme (that uses both overlapping coverage and reuse partitioning). Section 3 describes a state representation for analysis. Section 4 explains the driving processes and transition rates. Performance measures are discussed in Section 5. Numerical results and conclusions are given in Sections 6 and 7, respectively.

2 Model Description

2.1 System Layout

We consider a cellular system with omnidirectional base stations that are organized in a hexagonal pattern. Since overlapping coverage is considered, the term *cell* and *coverage* must be distinguished. The *cell* for any base is defined as the area where the nominal received signal power from that base is greater than that of any other base. With omnidirectional antennas, uniform propagation, flat terrain conditions, and base stations placed on a hexagonal grid, this corresponds to the usual layout with a base at the center of each hexagonal cell. This layout is shown in Figure 1. The *cell radius*, r, is defined as the distance from a base to a vertex of its own cell. The *coverage* of a base is the area in which users can establish a link with acceptable signal quality with that base. This area can be modeled by a circle with the center at the corresponding base station. The *coverage radius*, R, is defined as the distance from a base to its coverage boundary.

The coverage of a base is overlapped with coverages of neighboring bases. Thus calls can potentially access one, two, three or even more base stations depending on the platform location and the ratio of the coverage radius to the cell radius (R/r) [1]. For the interesting range of R/r, there are three kinds of regions, in which calls have potential access to one, two or three base stations. These regions which are shown in Figure 2 are denoted by A_1, A_2 and A_3 respectively. Thus region A_1 is the non-overlapping region while both A_2 and A_3 are overlapping regions. We consider (without loss of generality) the cell radius, r, to be normalized to unity. The coverage radius, R, is determined by the requirement of the link quality. Once R is found, the percentage of a *cell* that belongs to region A_1, A_2 or A_3 can be calculated [1]. They are denoted by p_1, p_2, and p_3 respectively. These relationships are discussed in subsequent sections.

2.2 Channel Assignment

We consider reuse partitioning that has two partitions of channels, denoted a and b. Systems with more partitions can be considered similarly. Channels of these two partitions are referred to as a-type and b-type channels respectively. These two partitions are used with different cluster sizes, N_a and N_b, to allocate channels to base stations. Channels of type-a are equally divided into N_a groups each with C_a channels. Similarly, b-type channels are equally divided into N_b groups each with C_b channels. Every base station is assigned one group of channels from each partition in such a way that co-channel interference is minimized. As a result, there are totally $C_a + C_b$ channels available at a base. Let C_T denote the total number of available channels and f denote the fraction of channels that are assigned to partition a. It follows that

$$f = \frac{N_a \cdot C_a}{C_T} \quad \text{and} \quad 1 - f = \frac{N_b \cdot C_b}{C_T} \quad . \tag{1}$$

Since N_a and N_b are different, we assume that N_a is the smaller one. Consequently a-type channels must be used in a smaller area. Channels of type-a are intended for use by users who can access only one base, which corresponds to users in region A_1. Channels of type-b can be used by all users. Figure 3 shows the channel assignment and the available channel groups in a specific region for such a system with $N_a = 3$ and $N_b = 7$. The three groups of a-type channels are labeled a_1, a_2, a_3 and the seven groups of b-type channels are labeled $b_1, b_2, \cdots, b_7$. Because of overlapping coverage, users in region A_2 can access two groups of channels from partition b. Similarly users in A_3 can access three groups of b-type channels.

In order to give priority to hand-off calls at each base, C_{ha} of the C_a a-type channels are reserved for hand-offs. Similarly, C_{hb} of the C_b b-type channels are reserved at a base. Specific channels are not reserved, only the numbers, C_{ha} and C_{hb}. As a result, new calls that arise in region A_1 can access $C_a - C_{ha}$ a-type channels and $C_b - C_{hb}$ b-type channels at the corresponding base. New calls that arise in region A_2 or A_3 can access $C_b - C_{hb}$ channels of b type at each potential base. New calls that arise in region A_1 are served by b-type channels only when no a-type channel is available. When more than one base has a channel available to serve a new call that arises in region A_2 or A_3, the one with the best link quality is chosen. Most likely, it is the nearest one under uniform propagation and flat terrain conditions.

Calls that use a-type channels are a-type calls and those that use b-type channels are b-type calls. An a-type call initiates a hand-off (a-type hand-off) when it leaves region A_1. As can be seen from Figure 3, an a-type hand-off may be an inter-base or an intra-base hand-off depending on which base continues its service. A b-type call initiates a hand-off (b-type hand-off) only when it leaves the coverage of the *serving base.* When a b-type call enters region A_1 of the *serving base*, it will not attempt to switch to an a-type channel. For example, referring to Figure 3, when a b-type call using a channel from group b_1 in an overlapping region moves into the center non-overlapping region, it still uses the same channel from group b_1. The reason for this is to minimize the number of hand-off attempts. Thus a b-type hand-off is always a inter-base hand-off because it leaves the coverage of the serving base and needs another base to continue. A b-type hand-off that enters region A_1 of a *neighboring base* can access all $C_a + C_b$ channels (a-type channels first) at that target base. A b-type hand-off that enters region A_2 can access all C_b channels of b type at each potential target base.

2.3 Determination of Coverage Radius

The received signal power is inversely proportional to the distance raised to an exponent, γ, which is called path loss exponent. Increasing coverage radius, R, increases the distance for b-type calls from the serving base, but also shortens the distance for a-type calls from the serving base. The worst case link quality of b-type calls occurs when they are on the coverage boundary of the serving base. Similarly, the worst case link quality of a-type calls occurs when they are on the edge of region A_1. Signal to interference ratio (SIR) is usually used to quantify link quality. The coverage radius, R, is determined such that the worst case SIR of a-type calls is equal to the worst case SIR of b-type calls. The motivation for this is to balance the worst case link quality for both types of calls. To be specific, consider Figure 1. The worst case SIR of a-type calls is denoted by $SIR_a(R, N_a, \gamma)$. This, for example, is SIR experienced by an a-type call at point X served by base A. This quantity is a function of 1) coverage radius, R; 2) cluster size of a-type channels, N_a; 3) path loss exponent, γ. Similarly, $SIR_b(R, N_b, \gamma)$ denotes the worst case SIR of b-type calls, which may be experienced by a b-type call at point X served by base C. It follows that the desired R is the one which satisfies

$$SIR_a(R, N_a, \gamma) = SIR_b(R, N_b, \gamma) \quad . \tag{2}$$

2.4 Teletraffic Model

The use of channel resources depends on teletraffic processes such as new call arrivals, call completions, a-type or b-type hand-off arrivals and a-type or b-type hand-off departures.

Mobile users are assumed to be uniformly distributed throughout the service area. We also assume that new call arrivals follow a Poisson point process with a rate Λ_n per cell and this rate is independent of the number of calls in progress. The unencumbered call duration, T, is defined as the time duration that a call would be served if it were not terminated prematurely. We assume T is a random variable with a negative exponential probability density function with mean $\overline{T}$ $(=\mu^{-1})$.

An a-type call can be served by the same base site and same channel as long as it is within region A_1. The time duration that an a-type call resides within region A_1 is defined as the dwell time, T_a. This is a random variable with a negative exponential distribution of mean $\overline{T_a}$ $(=\mu_a^{-1})$. Similarly, a b-type call can be served by the same base site and same channel as long as it is within the coverage of that base. The time duration that a b-type call resides within the coverage of the serving base is denoted by T_d, which is also assumed to be a negative exponentially distributed random variable with mean $\overline{T_d}$ $(={\mu_d}^{-1})$. We assume that $\overline{T_a}$ and $\overline{T_d}$ are proportional to the radius of the corresponding area. Although region A_1 is not circular, an equivalent radius (which is the radius of a circle that has the same area as A_1) can be used.

3 State Characterization

Two variables are needed to specify the state of a base. One is the number of a-type channels in use, the other is the number of b-type channels in use. A complete state representation for the whole system will be a string of base station states - two variables for each base. This state representation can keep track of all events that occur in the system. For example, it characterizes a b-type hand-off, which leaves the coverage of the serving base, is continuously served by another base; or an a-type call, which uses an a-type channel, switches to a b-type channel when it leaves region A_1; or a new call is served by a neighboring base due to high channel usage at the base in its own cell. However, the huge number of system states precludes pursuing this approach for most cases of interest. A simplified approach is to decouple a base from others by using average teletraffic demands related to neighboring bases. This is similar to the approach used in [11][12]. As a result, the state, s, is characterized from a given base by

$$a(s), b(s) \quad , \tag{3}$$

where $a(s)$ is the number of a-type channels in use in state s, and $b(s)$ is the number of b-type channels in use in state s. A permissible state must satisfy the condition that

$$0 \leq a(s) \leq C_a \quad \text{and} \quad 0 \leq b(s) \leq C_b \tag{4}$$

All permissible states are labeled from s=0 to s=s_{max}.

4 Driving Processes and Transition Rates

The state probabilities, $P(s)$, in statistical equilibrium are needed for determining the performance measures of interest. To calculate state probabilities, the state transitions and the corresponding transition rates must be identified and calculated. Due to overlapping coverage and reuse partitioning state transitions result from six driving processes: 1) new call arrivals 2) call completions 3) hand-off departures of a type 4) hand-off arrivals of a type 5) hand-off departures of b type 6) hand-off arrivals of b type. The transition rates from a current state s to next state s_n due to these driving processes are denoted by $r_n(s, s_n)$, $r_c(s, s_n)$, $r_{da}(s, s_n)$, $r_{ha}(s, s_n)$, $r_{db}(s, s_n)$ and $r_{hb}(s, s_n)$ respectively. All state transitions and corresponding transition rates are explained below.

4.1 New Call Arrivals

Mobile users are assumed to be uniformly distributed throughout the service area. The fraction of new calls that arise in region A_1 is p_1. Similarly, the fractions of new calls that arise in region A_2 and A_3 are p_2 and p_3 respectively. For a new call in region A_1, if there are less than $C_a - C_{ha}$ a-type channels in use at the corresponding base, this new call will be served by that base through an a-type channel. If no a-type channel is available, it can still be served by a b-type channel if there are less than $C_b - C_{hb}$ b-type channels in use. Otherwise, this new call is blocked. For a new call in region A_2 or A_3, it can only be served by a b-type channel. However, there are more than one base that can provide service. This new call is blocked only when there is no b-type channel available for a new call at any potential base. If more than one base can serve this new call, it will be served by the one that provides the best link quality. That is the nearest in the sense discussed in Section 2.2.

Consider the new call arrival demands on base A when base A is in states, s, in which $a(s) < C_a - C_{ha}$ or $b(s) < C_b - C_{hb}$. When base A is not in these states, there is no state transition due to new call arrivals. Base A serves new call arrivals from its own cell, which has the rate, Λ_n. In addition, due to overlapping coverage, base A can also serve new call arrivals from neighboring cells within its coverage under some circumstances. There are two kinds of new calls from neighboring cells that seek resources at base A. There are those that arise in region A_2 (area ILNW shown in Figure 1). Also there are those new calls that arise in region A_3 (area IJKL). New calls that arise in area ILNW and cannot be served by base C will make demands at base A. The probability that base C cannot serve these new calls is denoted by η_b. This is the probability that there are more than or equal to $C_b - C_{hb}$ b-type channels in use at a base. For now we proceed as if we know η_b. Actually this quantity is determined from the state probabilities that depend on the driving processes as

$$\eta_b = Prob\{s : b(s) \geq C_b - C_{hb}\} \quad . \tag{5}$$

This η_b is also the fraction of new calls that arise in area ILNW and make demands on base A. There are altogether six areas like area ILNW within the coverage of base A. The new call arrival demands on base A from these six areas is $p_2 \cdot \Lambda_n \cdot \eta_b$.

New calls in area IJKL that cannot be served by the nearest base (base C) still can be served if there are less than $C_b - C_{hb}$ b-type channels in use at either base A or D. Under the condition that base A has b-type channels available for new calls, the demands on base A from this area depend on whether or not base D has b-type channels available for new calls. When base D has no such channel, all new calls in area IJKL will be served by base A.

On the other hand, when base D has such channels, half of the new calls in area IJKL will be served by base A. (Another half will be served by base D.) Therefore given base C has no b-type channel available for new calls, the fraction of new calls in area IJKL that make demands on base A is denoted by κ_{nb}, which is the average over these two cases

$$\kappa_{nb} = \eta_b \cdot 1 + (1 - \eta_b) \cdot \frac{1}{2} \quad . \tag{6}$$

Moreover, there are twelve such areas within the coverage of base A. The new call arrival rate at base A from these twelve areas is $2 \cdot p_3 \cdot \Lambda_n \cdot \eta_b \cdot \kappa_{nb}$. Note that η_b is used to decouple base A from neighbors for new call arrivals from neighboring cells. In summary, the new call arrival rate of base A, $r_n(s, s_n)$, is

$$r_n(s, s_n) = \begin{cases} \Lambda_n \cdot p_1, & a(s) < C_a - C_{ha}; s_n = a(s) + 1, b(s) \\ \Lambda_n \cdot (1 - p_1 + p_2 \cdot \eta_b + 2p_3 \cdot \eta_b \cdot \kappa_{nb}), & a(s) < C_a - C_{ha}; b(s) < C_b - C_{hb}; \\ & s_n = a(s), b(s) + 1 \\ \Lambda_n \cdot (1 + p_2 \cdot \eta_b + 2p_3 \cdot \eta_b \cdot \kappa_{nb}), & a(s) \geq C_a - C_{ha}; b(s) < C_b - C_{hb}; \\ & s_n = a(s), b(s) + 1 \\ 0, & \text{otherwise}. \end{cases} \tag{7}$$

4.2 Call Completions

Upon a call completion the serving base will change state so that the value of either $a(s)$ or $b(s)$ is decreased by one depending on the type of the completed call. We assume that the unencumbered session duration has a negative exponential density function with mean $1/\mu$. As a result, μ is the call completion rate per call. The transition rate due to call completions, $r_c(s, s_n)$ is

$$r_c(s, s_n) = \begin{cases} a(s) \cdot \mu\,, & a(s) > 0\,;\ s_n = a(s) - 1, b(s) \\ b(s) \cdot \mu\,, & b(s) > 0\,;\ s_n = a(s), b(s) - 1 \\ 0\,, & \text{otherwise}. \end{cases} \tag{8}$$

4.3 Hand-off Departures of a Type

A hand-off departure of a type occurs when an a-type call leaves region A_1. There are three outcomes due to this departure: 1) If b-type channels are not fully occupied at the current serving base, this call will switch to a b-type channel but still be served by the same base. 2) If a b-type channel is not available at the current base but is available at the alternate target base, this call will be served by the alternate target base. For example, an a-type call served by base A leaves region A_1 by passing arc XY can be served by base B through a b-type channel. 3) Otherwise, this call will be forced to terminate. For the first case, state variable $a(s)$ is decreased by one and $b(s)$ is increased by one. For the remaining two cases, this call is no longer served by the base under consideration. Therefore only $a(s)$ is decreased by one. Recall from Section 2.4 that the time duration which an a-type call resides in region A_1 is defined as the dwell time, T_a, with mean $1/\mu_a$. Therefore μ_a is the hand-off departure rate per a-type call. As a result, the rate out of state s due to a-type hand-off departures, $r_{da}(s, s_n)$, is

$$r_{da}(s, s_n) = \begin{cases} a(s) \cdot \mu_a\,, & a(s) > 0;\ b(s) < C_b;\ s_n = a(s) - 1, b(s) + 1 \\ a(s) \cdot \mu_a\,, & a(s) > 0;\ b(s) = C_b;\ s_n = a(s) - 1, b(s) \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{9}$$

4.4 Hand-off Arrivals of a Type

Since any a-type hand-off arrival corresponds to an a-type hand-off departure in the system, the hand-off arrival rate is related to the hand-off departure rate. The a-type hand-off arrivals at a base may come from the base itself or from a neighboring base. The arrivals from the base itself are considered in Section 4.3. In this section we only consider the arrivals from a neighboring base. Consider those a-type hand-off arrivals from base B to base A. When there are i a-type calls in progress at base B, the total a-type hand-off departure rate is $i \cdot \mu_a$. Part of them will be accommodated by base B itself. Only when base B cannot serve them, they will make demands on neighboring bases. In addition, i can take values from 0 to C_a. We need to average over i to obtain the average rate. To do the average the probability that there are i a-type calls and C_b b-type calls in progress at a base is denoted by ϑ_i, which can be obtained from the state probabilities

$$\vartheta_i = Prob\{s : a(s) = i, b(s) = C_b\} \quad . \tag{10}$$

Note that only 1/6 of these departures will make demands on base A and there are totally six neighbors. These two factors cancel out each other. Finally the a-type hand-off arrival rate, $r_{ha}(s, s_n)$, can be written as follows

$$r_{ha}(s, s_n) = \begin{cases} \sum_{i=1}^{C_a} \vartheta_i \cdot i \cdot \mu_a \,, & b(s) < C_b \,;\, s_n = a(s), b(s) + 1 \\ 0 \,, & \text{otherwise} \,. \end{cases} \tag{11}$$

4.5 Hand-off Departures of b Type

A b-type hand-off departure occurs when a b-type call leaves the coverage of the current base. Recall from Section 2.4 that the dwell time, T_d, is the time during which a b-type call resides in the coverage of the serving base. The mean of T_d is $1/\mu_d$. Therefore μ_d is the hand-off departure rate per b-type call. A b-type hand-off departure will decrease the value of $b(s)$ by one. As a result, the rate out of state s due to b-type hand-off departures, $r_{db}(s, s_n)$, is

$$r_{db}(s, s_n) = \begin{cases} b(s) \cdot \mu_d \,, & b(s) > 0 \,;\, s_n = a(s), b(s) - 1 \\ 0 \,, & \text{otherwise} \,. \end{cases} \tag{12}$$

4.6 Hand-off Arrivals of b Type

There are two kinds of b-type hand-off arrivals. One is b-type hand-offs entering region A_1 of the target base. The other is b-type hand-offs entering region A_2 of target bases. Consider b-type hand-off arrivals that enter region A_1 of base A. They come from six neighbors. If one of the neighbors (say base B) has i b-type calls in progress, the total b-type hand-off departure rate of base B is $i \cdot \mu_d$. All these hand-offs occur uniformly distributed on the coverage boundary. The fraction of these hand-offs that enter region A_1 of base A is the ratio that the length of arc XY ($2\theta R$) to the circumference of coverage boundary ($2\pi R$), which is θ/π. Therefore, when there are i b-type calls in progress at base B, the resulting hand-off arrival demands that enter region A_1 of base A are $i \cdot \mu_d \cdot \theta/\pi$. This is denoted by $\Lambda_1(i)$

$$\Lambda_1(i) = i \cdot \mu_d \cdot \frac{\theta}{\pi} \quad . \tag{13}$$

The number of calls, i, at base B can take values from 0 to C_b. By averaging over i, we can calculate the average demands of this kind. To find the average, the probabilities that there are i b-type calls at base B must be determined. These probabilities are denoted by ξ_i and can be calculated from the state probabilities as

$$\xi_i = Prob\{s : b(s) = i\} \qquad i = 0, 1, 2, \ldots, C_b \quad . \tag{14}$$

This kind of hand-off arrivals will be served by a-type channels if a-type channels are not fully occupied. Otherwise, they can be served by b-type channels if b-type channels are not fully occupied.

When b-type hand-off calls leave base B by crossing arc WX (or YZ), base A and C (or base A and G) are the two potential target bases. These calls can be served only by b-type channels. Similarly, the fraction of b-type hand-offs that enter region A_2 of base A is α/π. For this kind of hand-off arrivals, we can calculate the fraction that make demands on base A using a similar method developed in Section 4.1. This fraction is denoted by κ_{hb} and can be calculated as

$$\kappa_{hb} = \nu_b \cdot 1 + (1 - \nu_b) \cdot \frac{1}{2} \quad , \tag{15}$$

where ν_b is the probability that all b-type channels are occupied at a base. Succinctly,

$$\nu_b = Prob\{s : b(s) = C_b\} \quad . \tag{16}$$

Therefore, when there are i b-type calls in progress at base B, the resulting hand-off arrival demands on base A that enter region A_2 of base A are $i \cdot \mu_d \cdot (\alpha/\pi) \cdot \kappa_{hb}$, which is denoted by $\Lambda_2(i)$

$$\Lambda_2(i) = i \cdot \mu_d \cdot \frac{\alpha}{\pi} \cdot \kappa_{hb} \quad . \tag{17}$$

The probabilities ξ_i are also used for averaging these demands over i. Finally, considering that there are six such neighbors, the rate $r_{hb}(s, s_n)$ is

$$r_{hb}(s,s_n) = \begin{cases} 6\sum_{i=1}^{C_b} \xi_i \cdot \Lambda_1(i)\,, & a(s) < C_a;\ s_n = a(s)+1, b(s) \\ 6\sum_{i=1}^{C_b} \xi_i \cdot \Lambda_2(i)\,, & a(s) < C_a;\ b(s) < C_b;\ s_n = a(s), b(s)+1 \\ 6\sum_{i=1}^{C_b} \xi_i \cdot [\Lambda_1(i) + \Lambda_2(i)]\,, & a(s) = C_a;\ b(s) < C_b;\ s_n = a(s), b(s)+1 \\ 0\,, & \text{otherwise}\,. \end{cases} \tag{18}$$

5 Performance Measures

There are four performance measures of interest: 1) blocking probability 2) forced termination probability 3) hand-off activity 4) carried traffic. They can be determined once the state probabilities are found. The state probabilities, $P(s)$, can be calculated by solving probability flow balance equations. Since the state probabilities are used to determine the average new call arrival and hand-off arrival rates, transition rates are functions of state probabilities. On the other hand, state probabilities are functions of transition rates. The result is a set of nonlinear equations, which can be solved for the state probabilities as in a manner similar to that described in [5][6][11][12]. A brief explanation is given in Appendix A.

5.1 Blocking Probability

The blocking probability is the average fraction of new calls that cannot gain access to a channel. New calls can arise in three kinds of regions in the system under study. New calls that arise in region A_1 are blocked when both a-type and b-type channels are not available at the corresponding base. Note that new calls can only access channels that are not reserved for hand-off calls. Thus the blocking probability for new calls in region A_1, P_{b1}, can be calculated from state probabilities as

$$P_{b1} = Prob\{s : a(s) \geq C_a - C_{ha} \ \text{ and } \ b(s) \geq C_b - C_{hb}\} \quad . \tag{19}$$

New calls that arise in region A_2 or A_3 can only be served by b-type channels. The probability that there are no b-type channels available for a new call at a single base is η_b, which is defined in equation (5). The blocking probability for new calls in region A_2 is ${\eta_b}^2$ because two potential bases can serve a new call in region A_2. Similarly, the blocking probability for new calls in region A_3 is ${\eta_b}^3$. Finally the overall blocking probability, P_B, is the average over these three regions, which is

$$P_B = p_1 \cdot P_{b1} + p_2 \cdot {\eta_b}^2 + p_3 \cdot {\eta_b}^3 \quad . \tag{20}$$

5.2 Forced Termination Probability

A forced termination occurs when a non-blocked call is interrupted due to a hand-off failure (including both a type and b type) during its lifetime. The forced termination probability, P_{FT}, is the average fraction of non-blocked calls that are forced to terminate. A signal flow graph and Mason's formula [13] are used to determine this probability. Similar applications appeared in [14]. The lifetime of a non-blocked call is represented by the signal flow graph shown in Figure 4. There are seven nodes, E_1, E_2, ... , E_7, to represent significant events in the lifetime of a call.

E_1: Beginning of a call that is served by the system.

E_2: Call uses an a-type channel.

E_3: Call uses a b-type channel.

E_4: Call initiates an a-type hand-off attempt.

E_5: Call initiates a b-type hand-off attempt.

E_6: Call is forced to terminate.

E_7: Call completes successfully.

The links between nodes are the possible transitions between various events and the arrows indicate the direction of transitions. The probability associated with each link is the transition probability from one event to another. In order to find all transition probabilities, one must first calculate the following probabilities: P_a, P_b, P_{d1}, P_{d2}, ν_b, β_a and β_b. The relationships between these probabilities and event transitions are shown in Figure 4. The detailed explanations and derivations for these probabilities are described in Appendix B. The forced termination probability then can be determined as the probability gain from node E_1 to node E_6 by using Mason's formula

$$P_{FT} = \frac{P_a P_{d2}[\nu_b{}^2(1-P_{d1}\beta_b) + (1-\nu_b{}^2)P_{d1}(1-\beta_a-\beta_b)] + P_b P_{d1}[(1-\beta_a-\beta_b)+\beta_a P_{d2}\nu_b{}^2]}{1-P_{d2}(1-\nu_b{}^2)P_{d1}\beta_a - P_{d1}\beta_b} \tag{21}$$

5.3 Hand-off Activity

Hand-off activity, H_A, is the expected number of hand-off attempts (they may be successful or not) that a non-blocked call will experience during its lifetime. We decompose H_A into two parts: the hand-off activity due to a-type hand-offs, Θ_a, and the hand-off activity due to b-type hand-offs, Θ_b. To calculate Θ_a, the following three probabilities need to be determined first: 1) the probability that a call will make its first a-type hand-off attempt, δ_a; 2) the probability that after a successful a-type hand-off a call will make another a-type hand-off attempt, δ_{na}; 3) the probability that after an a-type hand-off attempt a call will not make another a-type hand-off attempt, φ_a. We also use the signal flow graph and Mason's formula to determine these three probabilities. The details are given in Appendix C. Consequently, the probability that there are exactly i a-type hand-off attempts during the life of a call is $\delta_a[(1-\nu_b{}^2)\delta_{na}]^{i-1}\varphi_a$, where $1-\nu_b{}^2$ is the probability for an a-type hand-off to be successful. For example, if a call has exactly two a-type hand-off attempts, the following three events must occur: 1) it must initiate the first a-type hand-off and succeeds, 2) after the first a-type hand-off, it initiates another hand-off of same type, 3) then it will not make any more a-type hand-offs. It follows that the hand-off activity due to a-type hand-offs is

$$\Theta_a = \sum_{i=1}^{\infty} i \cdot \delta_a[(1-\nu_b{}^2)\delta_{na}]^{i-1}\varphi_a \quad . \tag{22}$$

This can be simplified to

$$\Theta_a = \delta_a\varphi_a/[1-(1-\nu_b{}^2)\delta_{na}]^2 \quad . \tag{23}$$

Similarly, to calculate Θ_b we first determine three probabilities: 1) the probability that a call will make its first b-type hand-off attempt, δ_b; 2) the probability that after a successful

b-type hand-off a call will make another b-type hand-off attempt, δ_{nb}; 3) the probability that after a b-type hand-off attempt a call will not make another b-type hand-off attempt, φ_b. The detailed calculations of these probabilities are also given in Appendix C. After these probabilities are found, the hand-off activity due to b-type hand-offs can be determined as

$$\Theta_b = \delta_b \varphi_b / [1 - (\beta_a + \beta_b)\delta_{nb}]^2 \quad , \tag{24}$$

where $\beta_a + \beta_b$ is the success probability of a b-type hand-off attempt. Finally, the hand-off activity, H_A, is the sum of Θ_a and Θ_b

$$H_A = \Theta_a + \Theta_b \quad . \tag{25}$$

5.4 Carried Traffic

The carried traffic, A_C, per base station is the average number of channels that are in use. When a base is in state s, the number of channels in use is $a(s) + b(s)$. By definition the carried traffic is

$$A_C = \sum_{s=0}^{s_{max}} [a(s) + b(s)] \cdot P(s) \quad . \tag{26}$$

6 Numerical Results

We use two sets of figures to show example numerical results. Figures 5 compare the performance measures between schemes of no coverage overlap, coverage overlap and coverage overlap with reuse partitioning (RP). The scheme of no coverage overlap corresponds to FCA. Figures 6 show the effect of cut-off priority for hand-off calls.

In Figures 5 the following parameters are used : $C_T = 180$, $N_a = 3$, $N_b = 9$, $\overline{T} = 100$ sec, $\overline{T_d} = 60$ sec, $C_{ha} = 0$, $C_{hb} = 0$ and $\gamma = 4$. We also consider various f for the scheme with reuse partitioning to assign different fractions of channels to partitions a and b. The relationship between C_a, C_b and f is in equation (1). Table 1 lists four values of f used in Figures 5 and corresponding C_a and C_b. For other two schemes the number of channels allocated to a base is 20 because total number of channels is 180 and cluster size is 9. From

f	0.05	0.10	0.15	0.20
C_a	3	6	9	12
C_b	19	18	17	16

Table 1: The relationship between C_a, C_b and f

equation (2) the coverage radius, R, is determined to be 1.1885. (Note that the cell radius, r, is normalized to unity.) For this coverage radius, the worst case SIR for both a-type and b-type calls is 16.9 dB. All these figures are plotted with respect to the offered traffic. In addition, simulation results are plotted in terms of 95% confidence interval for the scheme with reuse partitioning in which $f = 0.05$. Simulation results agree with analytical results well.

Figure 5.1 shows the blocking probability. Overlapping coverage improves this performance measure over FCA. However, significantly reduced blocking can be achieved by applying overlapping coverage with reuse partitioning. The improvement provided by overlapping coverage alone is not balanced over those three different regions. Region A_1 has the worst blocking situation because a call can only access a single base there. Reuse partitioning can allocate more channels for users in region A_1. This balances the blocking condition for calls in region A_1 and A_2. However, this should not be over balanced. When more and more channels are assigned to partition a, calls in region A_1 will eventually have better blocking performance than calls in region A_2. This is also not balanced and therefore increases the overall blocking probability. In Figure 5.1 blocking probability reduced when f increased up to f=0.15, which corresponds to $C_a = 9$ and $C_b = 17$. When f is further increased to 0.2, blocking probability increases.

Figure 5.2 shows forced termination probability. Basicly the trend is the same as for blocking probability. Forced termination probability is improved substantially by overlapping coverage combined with reuse partitioning. Since $\overline{T_d}$ is less than $\overline{T}$, on the average a non-blocked call will experience more than one hand-off during its lifetime. Thus forced termination probability is larger than blocking probability for the same offered traffic.

Figure 5.3 shows the hand-off activity for various schemes. The hand-off activity of coverage overlap scheme is less than FCA by the amount between 0.3 and 0.5 for the range of offered traffic shown. This is because the coverage of a base in coverage overlap scheme is larger than in FCA and mobile users generally stay longer with a serving base. Those coverage overlap with reuse partitioning schemes have more hand-off activities than the coverage overlap one because of the additional a-type hand-offs. However, the additional hand-offs are not many and therefore there will not be much additional work to benefit from reuse partitioning. Note that in order to show details the vertical axis in Figure 5.3 starts from 1.3 instead of 0.

Figure 5.4 shows the carried traffic. Coverage overlap scheme carries more traffic than FCA. Coverage overlap with reuse partitioning can further improve this performance. In addition, $f = 0.15$ is still the best one.

Figures 6 shows the effect of cut-off priority for hand-off calls. The four performance measures are shown for the combinations that $C_{ha} = 0$ or 2 and $C_{hb} = 0$ or 2. Other parameters are the same as those used for Figures 5. As we can see in these figures that C_{hb} has more impact than C_{ha}. As priority given to hand-off calls, forced termination probability reduced at the cost of increased blocking. The performance exchanges can be seen from these figures.

7 Conclusions

Reuse partitioning applied with overlapping coverage can complement each other and substantially improve overall blocking and forced termination probabilities, as well as carried traffic at the cost of minor increase of hand-off activity. Other cost for these benefits may be the slight degradation of link quality for users near the coverage boundary of a base. New call blocking can be exchanged for forced termination probability by applying the cut-off priority for hand-off calls. Whether or not this is favorable depends on the weights of the cost function. When the weights of blocking and forced termination are the same, this cut-off priority may not be favorable for the proposed scheme.

The modeling approach that decouples a base from its neighbors simplifies the analysis dramatically. Monte-Carlo simulation results validate the analytical model used.

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Appendix A

The state probabilities of equilibrium, $P(s)$, can be calculated by solving probability flow balance equations. From the rates given in Section 4, the transition rate from state s to any possible next state s_n can be found by

$$q(s, s_n) = r_n(s, s_n) + r_c(s, s_n) + r_{da}(s, s_n) + r_{ha}(s, s_n) + r_{db}(s, s_n) + r_{hb}(s, s_n) \quad , \qquad \text{(A.1)}$$

in which $s \neq s_n$. The total transition rate out of state s is denoted by $q(s, s)$ and is given by

$$q(s, s) = - \sum_{k=0, k\neq s}^{s_{max}} q(s, k) \quad , \qquad \text{(A.2)}$$

where the minus sign indicates the direction of the flow. In statistical equilibrium, the net probability flow into any state is zero. This can be written as

$$\sum_{i=0}^{s_{max}} P(i)q(i, j) = 0, \qquad j = 0, 1, 2, \ldots, s_{max} \quad . \qquad \text{(A.3)}$$

This provides $s_{max} + 1$ equations. Because of (A.2) one of these is redundant. The condition that the sum of all state probabilities is unity provides the additional equation. That is,

$$\sum_{s=0}^{s_{max}} P(s) = 1 \quad . \qquad \text{(A.4)}$$

As a result, we have $s_{max}+1$ simultaneous equations to determine $s_{max}+1$ state probabilities. This is a set of nonlinear equations. Gauss-Seidel numerical method is used to solve these equations.

Appendix B

The signal flow graph in Figure 4 describes the lifetime of a non-blocked call. The meanings of all nodes are explained in Section 5.2. The transition probabilities are determined below.

The probability that a non-blocked call uses an a-type channel is P_a; the probability that a non-blocked call uses a b-type channel is P_b. To calculate these two probabilities, we divide non-blocked new calls into three categories: 1) new calls that arise in region A_1 and are served by a-type channels, 2) new calls that arise in region A_1 and are served by b-type channels, 3) new calls that arise in region A_2 or A_3 and are served by b-type channels. For a new call in the first category, there must be less than $C_a - C_{ha}$ a-type channels in use when the new call originates in region A_1. Thus the fraction of new calls that belong to the first category, f_1, can be calculated as

$$f_1 = p_1 \cdot Prob\{s : a(s) < C_a - C_{ha}\} \quad . \tag{B.1}$$

Similarly, the fraction of new calls that belong to the second category, f_2, or the third category, f_3, can be calculated as follows

$$f_2 = p_1 \cdot Prob\{s : a(s) \geq C_a - C_{ha} \text{ and } b(s) < C_b - C_{hb}\} \quad , \tag{B.2}$$

$$f_3 = p_2(1 - {\eta_b}^2) + p_3(1 - {\eta_b}^3) \quad . \tag{B.3}$$

Where η_b is the probability that a base has no b-type channels to serve a new call and is defined in equation (5). From f_1, f_2 and f_3, P_a and P_b can be calculated as

$$P_a = \frac{f_1}{f_1 + f_2 + f_3} \quad , \qquad P_b = \frac{f_2 + f_3}{f_1 + f_2 + f_3} \quad . \tag{B.4}$$

The transition probability from node E_3 to node E_5 is P_{d1}, which is the probability that a b-type call will make a hand-off before it completes. Due to negative exponential assumption for the call duration and the dwell time. It follows that

$$P_{d1} = \frac{\mu_d}{\mu_d + \mu} \quad . \tag{B.5}$$

Similarly, the probability that an a-type call will make a hand-off before it completes is P_{d2}, which can be calculated as

$$P_{d2} = \frac{\mu_a}{\mu_a + \mu} \quad . \tag{B.6}$$

The quantity ν_b is the probability that b-type channels are fully occupied at a base and is defined in equation (16). Consider β_a and β_b. When a b-type hand-off is initiated, it has three possible outcomes: 1) It succeeds and continues service using an a-type channel. 2)

It succeeds and continues service using a b-type channel. 3) It is forced to terminate. The probability of the first and the second outcomes are β_a and β_b respectively. It follows that the probability of the third outcome is $1 - \beta_a - \beta_b$. To determine β_a, we defines an event that all a-type channels are occupied at a base. The probability of this event is denoted by ν_a, which can be calculated from the state probabilities as follows

$$\nu_a = Prob\{s : a(s) = C_a\} \quad . \tag{B.7}$$

Therefore $1 - \nu_a$ is the probability that a b-type hand-off which enters region A_1 of the target base can continue service using an a-type channel. In addition, the probability that a b-type hand-off enters region A_1 of the target base is $2\theta/(2\theta + \alpha)$. Thus, β_a can be determined as

$$\beta_a = \frac{2\theta}{2\theta + \alpha}(1 - \nu_a) \quad . \tag{B.8}$$

A b-type hand-off may get a b-type channel to continue its service when it enters region A_1 and there is no a-type channel available or when it enters region A_2. In a similar way, β_b is determined as

$$\beta_b = \frac{2\theta}{2\theta + \alpha}\nu_a(1 - \nu_b) + \frac{\alpha}{2\theta + \alpha}(1 - {\nu_b}^2) \quad . \tag{B.9}$$

Appendix C

To calculate the hand-off activity due to a-type hand-offs, Θ_a, one needs to first determine three probabilities, δ_a, δ_{na} and φ_a, as mentioned in Section 5.3. The signal flow graph in Figure 4 and Mason's formula are used to calculate these probabilities. The probability δ_a is the probability that a call will make its first a-type hand-off attempt, which is the gain from node E_1 to node E_4 after open the link $1 - {\nu_b}^2$. Using Mason's formula,

$$\delta_a = P_a P_{d2} + \frac{P_b P_{d1} \beta_a P_{d2}}{1 - P_{d1}\beta_b} \quad . \tag{C.1}$$

The probability δ_{na} is the probability that after a successful a-type hand-off a call will make another a-type hand-off attempt. It is the gain from node E_3 to node E_4 after open the link

$1-\nu_b{}^2$ and can be calculated as

$$\delta_{na} = \frac{P_{d1}\beta_a P_{d2}}{1-P_{d1}\beta_b} \quad . \tag{C.2}$$

The probability φ_a is the probability that after an a-type hand-off attempt a call will not make another a-type hand-off attempt. It is the gain from node E_4 to node E_6 or E_7 after open the link P_{d2}. From Mason's formula,

$$\varphi_a = \nu_b{}^2 + \frac{(1-\nu_b{}^2)[P_{d1}(1-\beta_a-\beta_b)+(1-P_{d1})+P_{d1}\beta_a(1-P_{d2})]}{1-P_{d1}\beta_b} \quad . \tag{C.3}$$

Similarly, to calculate the hand-off activity due to b-type hand-offs, Θ_b, one must first determine three probabilities, δ_b, δ_{nb} and φ_b. The probability δ_b is the probability that a call will make its first b-type hand-off attempt, which is the gain from node E_1 to node E_5 after open both links β_a and β_b. From Mason's formula, this is

$$\delta_b = P_b P_{d1} + P_a P_{d2}(1-\nu_b{}^2)P_{d1} \quad . \tag{C.4}$$

The probability δ_{nb} is the probability that a call will make another b-type hand-off after a successful b-type hand-off. After a successful b-type hand-off, the call may go to node E_2 or E_3 in the signal flow graph. Therefore δ_{nb} is the average over the gain from node E_3 to E_5 after open both links β_a and β_b and the gain from node E_2 to E_5 after open the same both links. In addition, the probability that a successful b-type hand-off goes to node E_2 is $\beta_a/(\beta_a+\beta_b)$. Consequently, the probability that this hand-off goes to node E_3 is $\beta_b/(\beta_a+\beta_b)$. It follows that δ_{nb} is

$$\delta_{nb} = \frac{\beta_a}{\beta_a+\beta_b}P_{d2}(1-\nu_b{}^2)P_{d1} + \frac{\beta_b}{\beta_a+\beta_b}P_{d1} \quad . \tag{C.5}$$

The probability φ_b is the probability that a call will not make any b-type hand-off after a b-type hand-off attempt. This is the gain from node E_5 to node E_6 or E_7 after open the link P_{d1}, which can be calculated as

$$\varphi_b = (1-\beta_a-\beta_b)+\beta_a P_{d2}\nu_b{}^2+\beta_b(1-P_{d1})+\beta_a(1-P_{d2})+\beta_a P_{d2}(1-\nu_b{}^2)(1-P_{d1}) \quad . \tag{C.6}$$

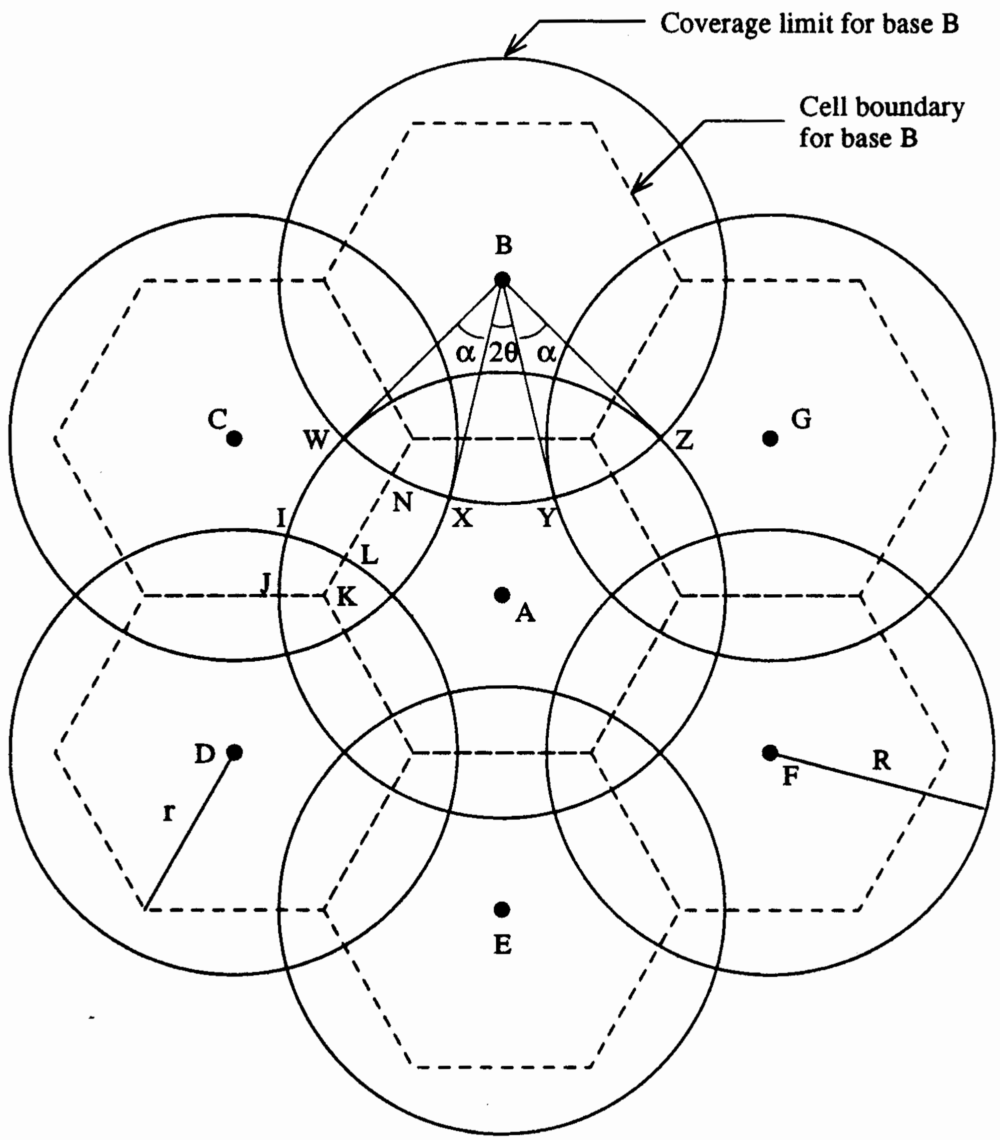

Figure 1: System layout for overlapping coverage

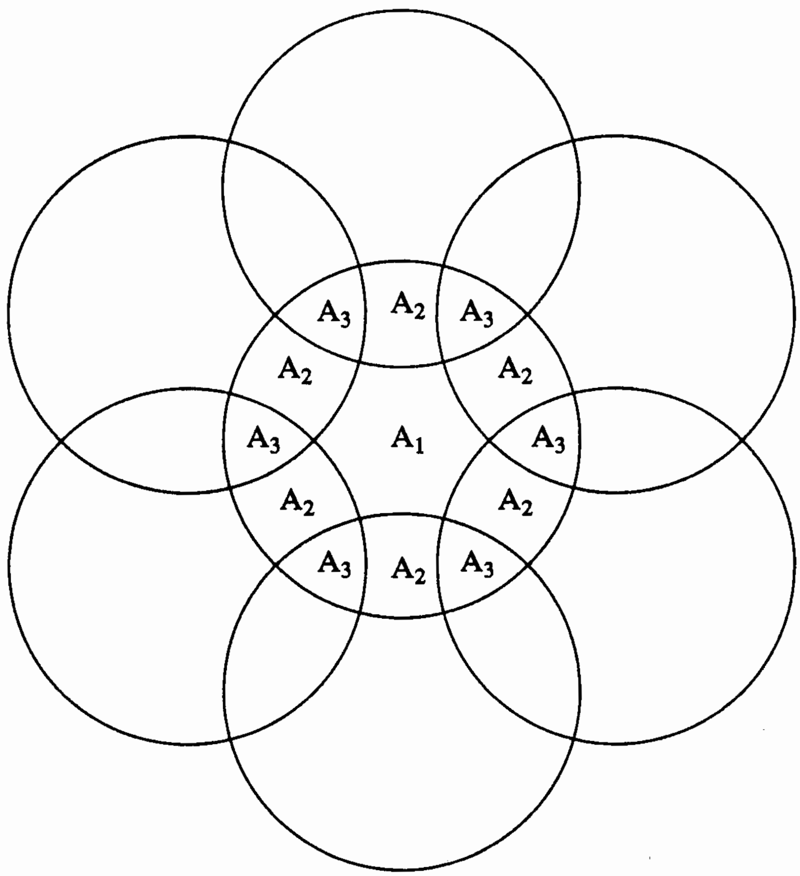

Figure 2: Three kinds of regions in the coverage of a base

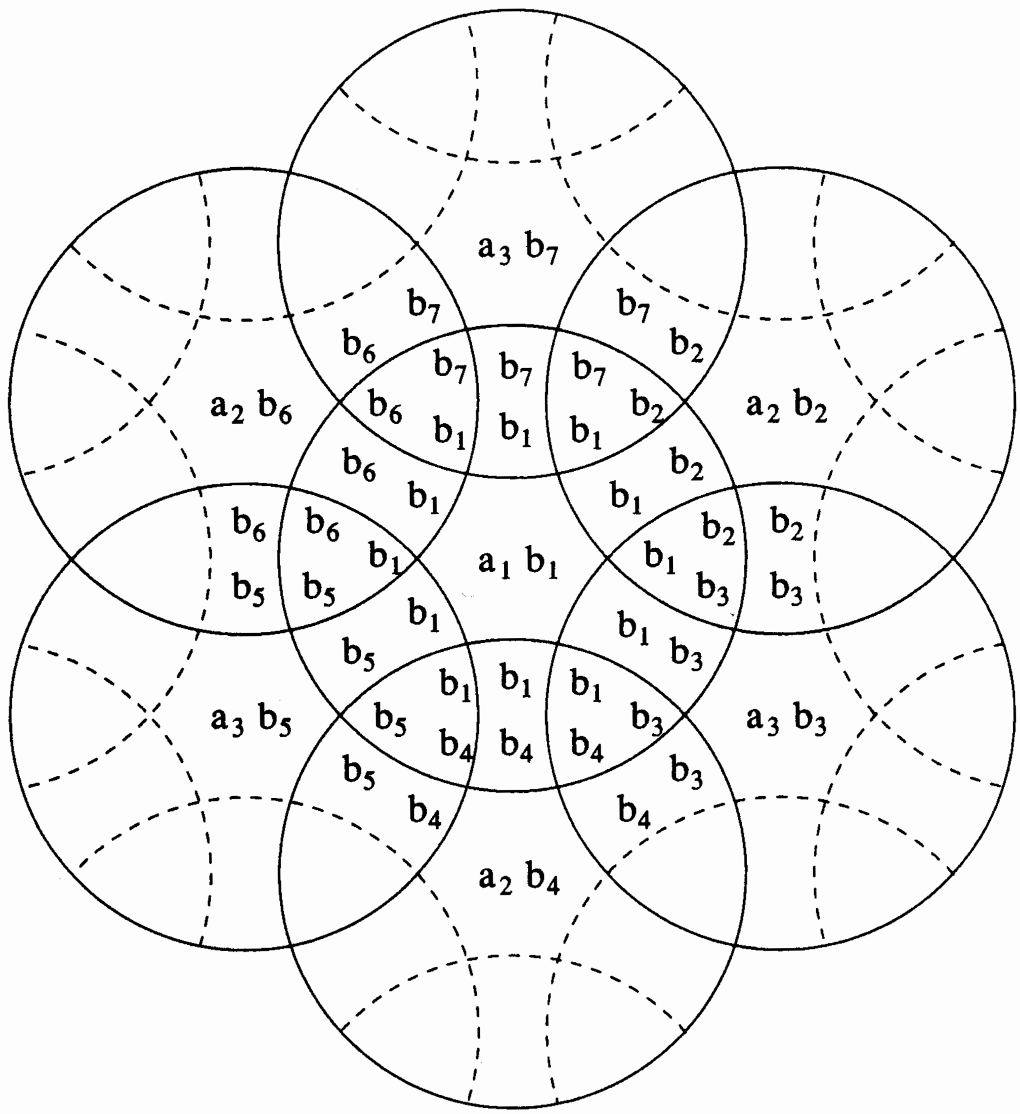

Figure 3: The relationship between channel groups and various regions for $N_a = 3$ and $N_b = 7$

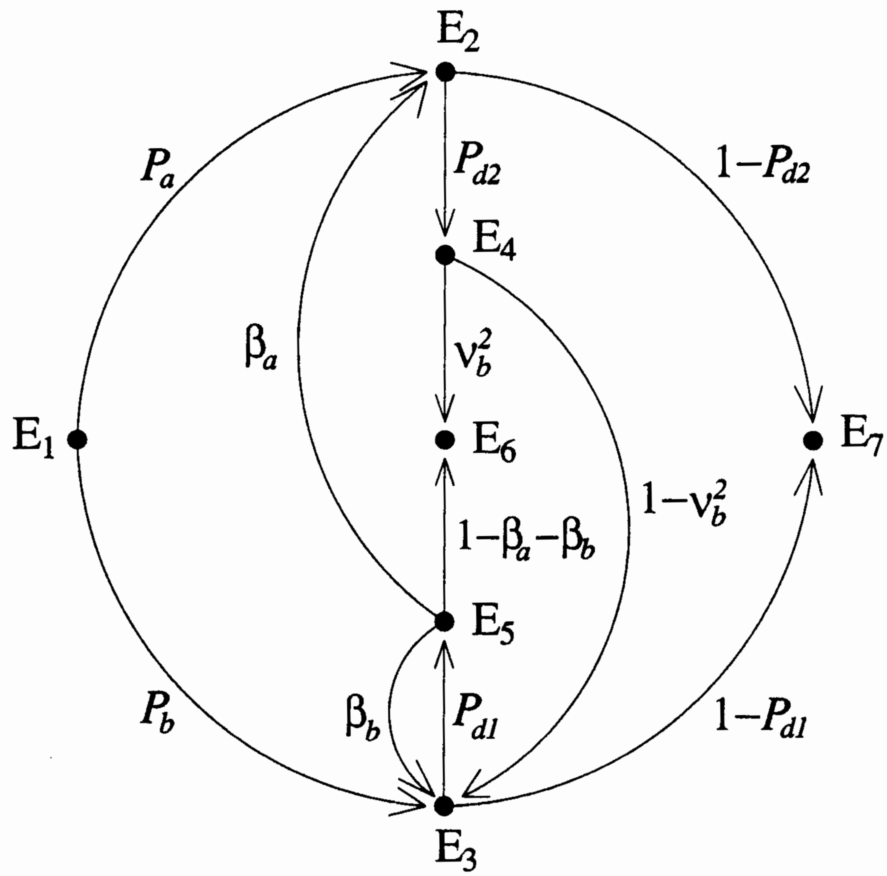

Figure 4: Signal flow graph for the lifetime of a non-blocked call

Fig. 5.1 Blocking Probability

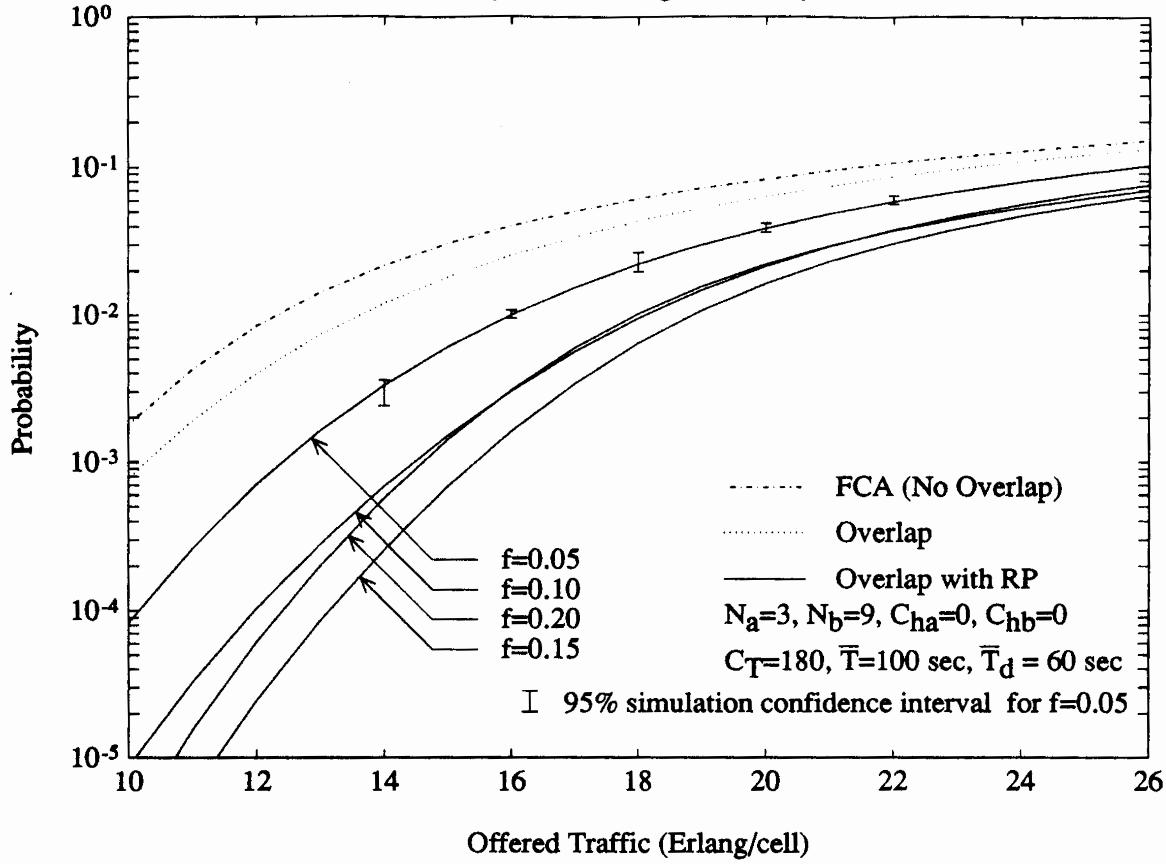

Fig. 5.2 Forced Termination Probability

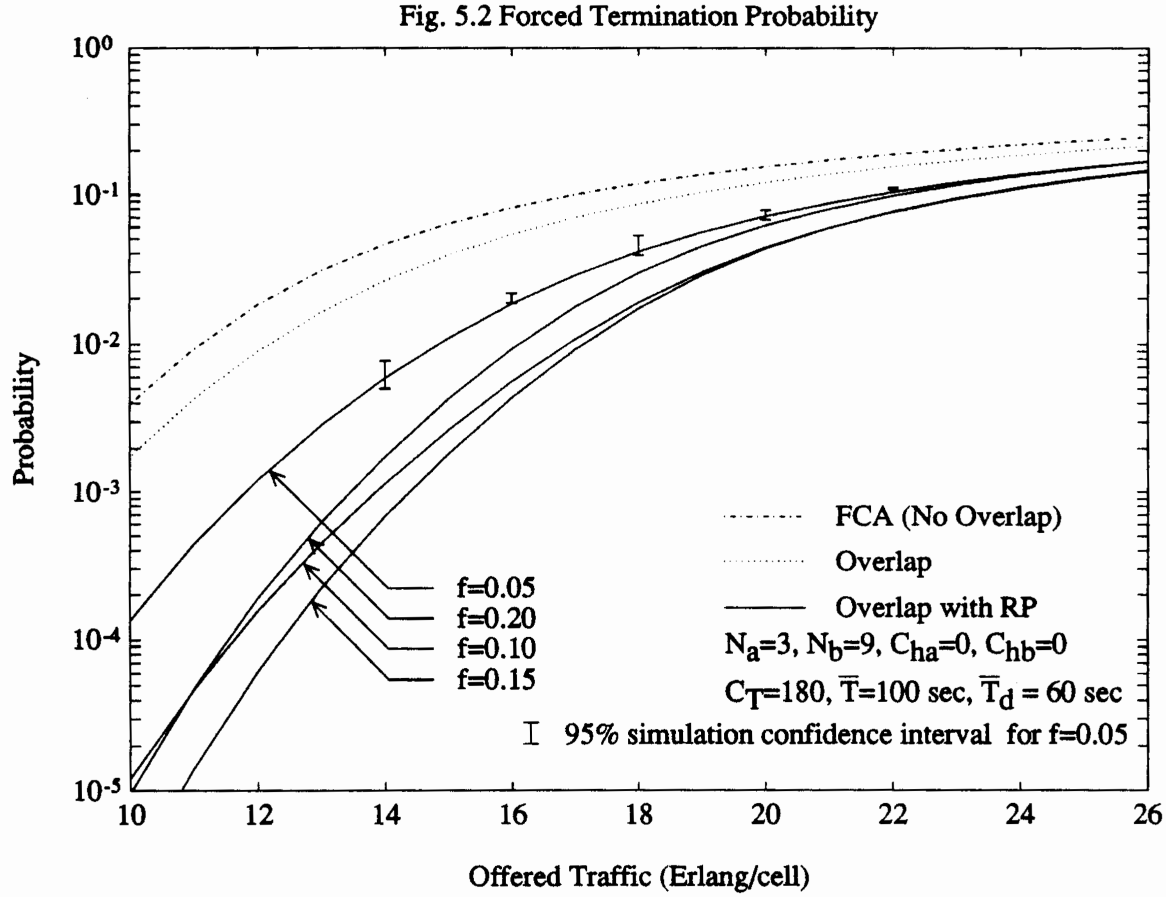

Fig. 5.3 Hand-off Activity

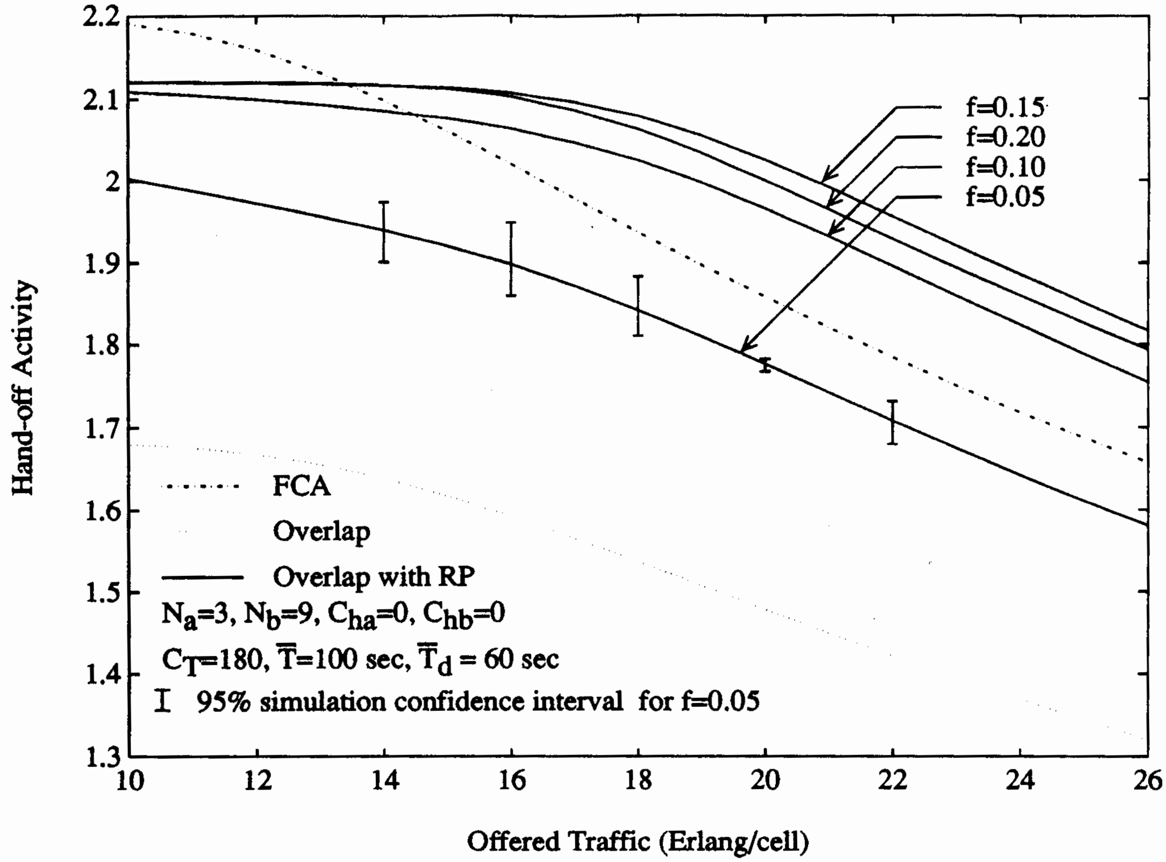

Fig. 5.4 Carried Traffic

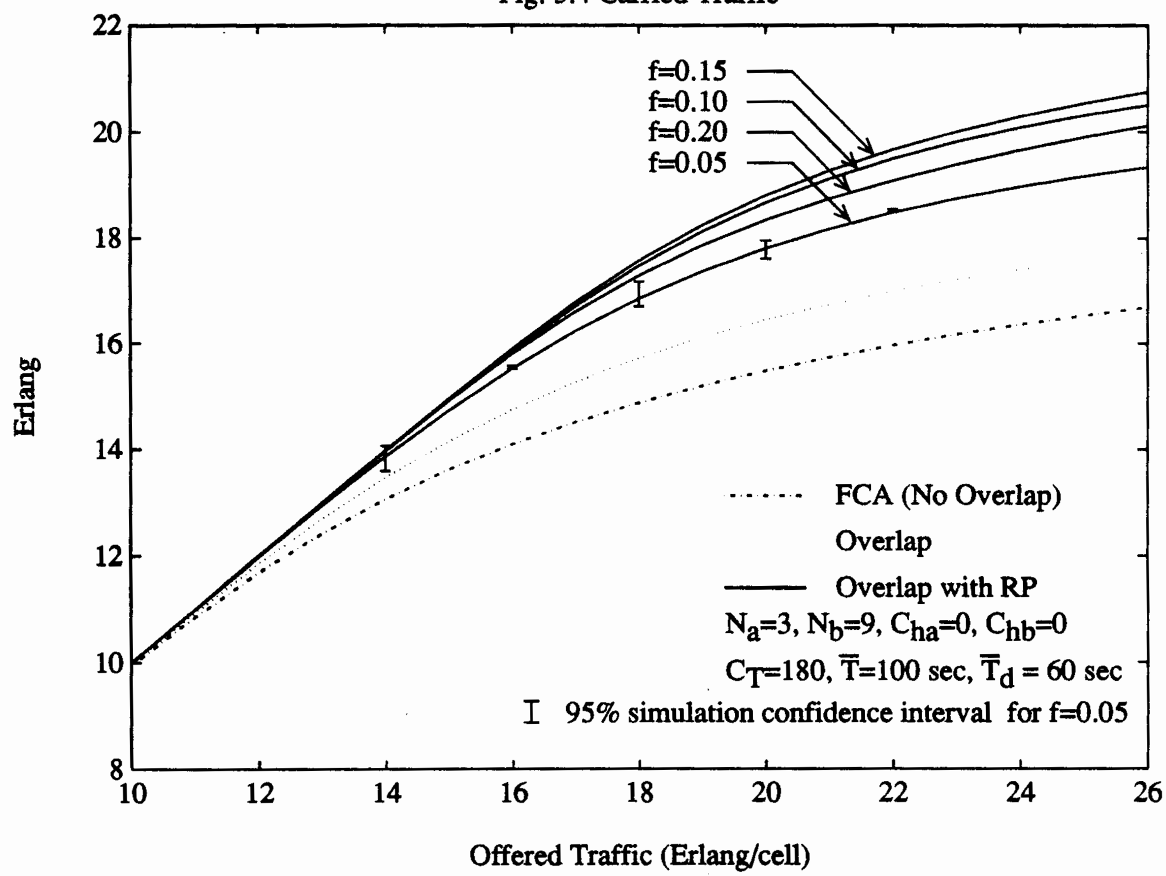

Fig. 6.1 Blocking Probability

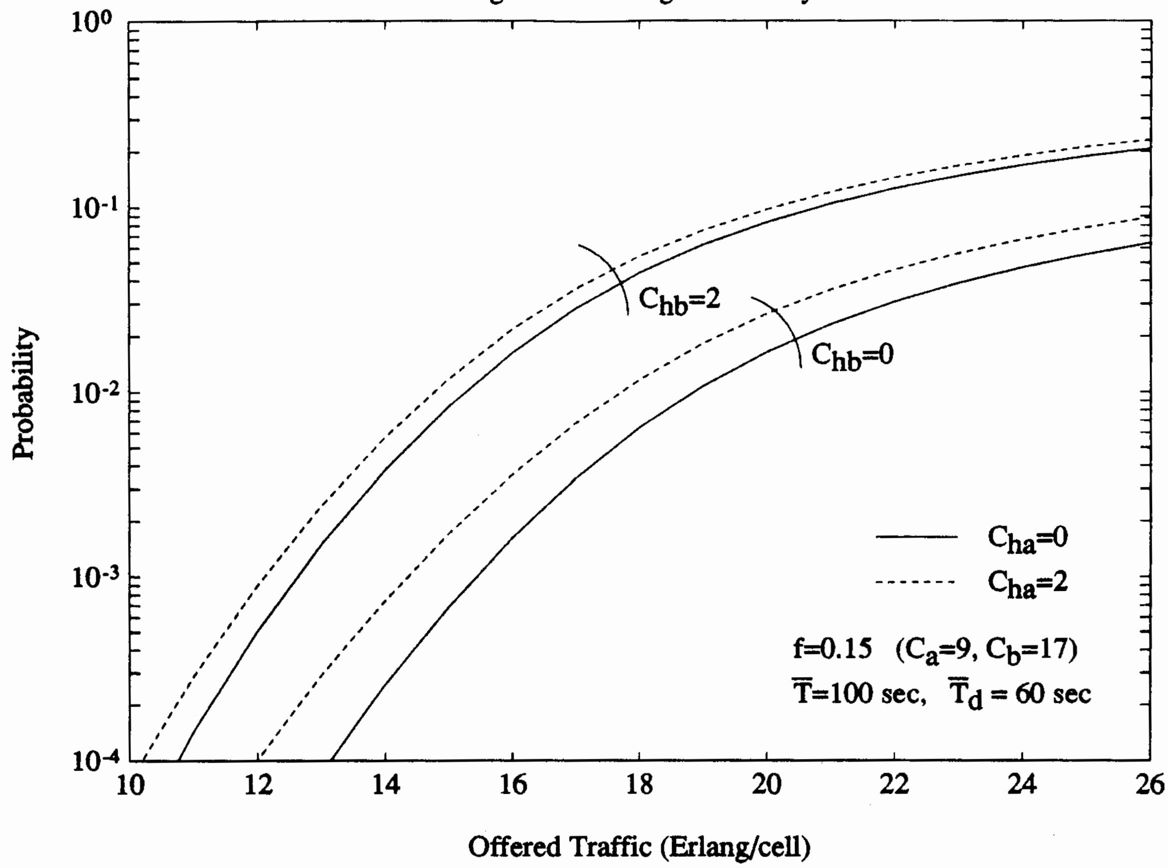

Fig. 6.2 Forced Termination Probability

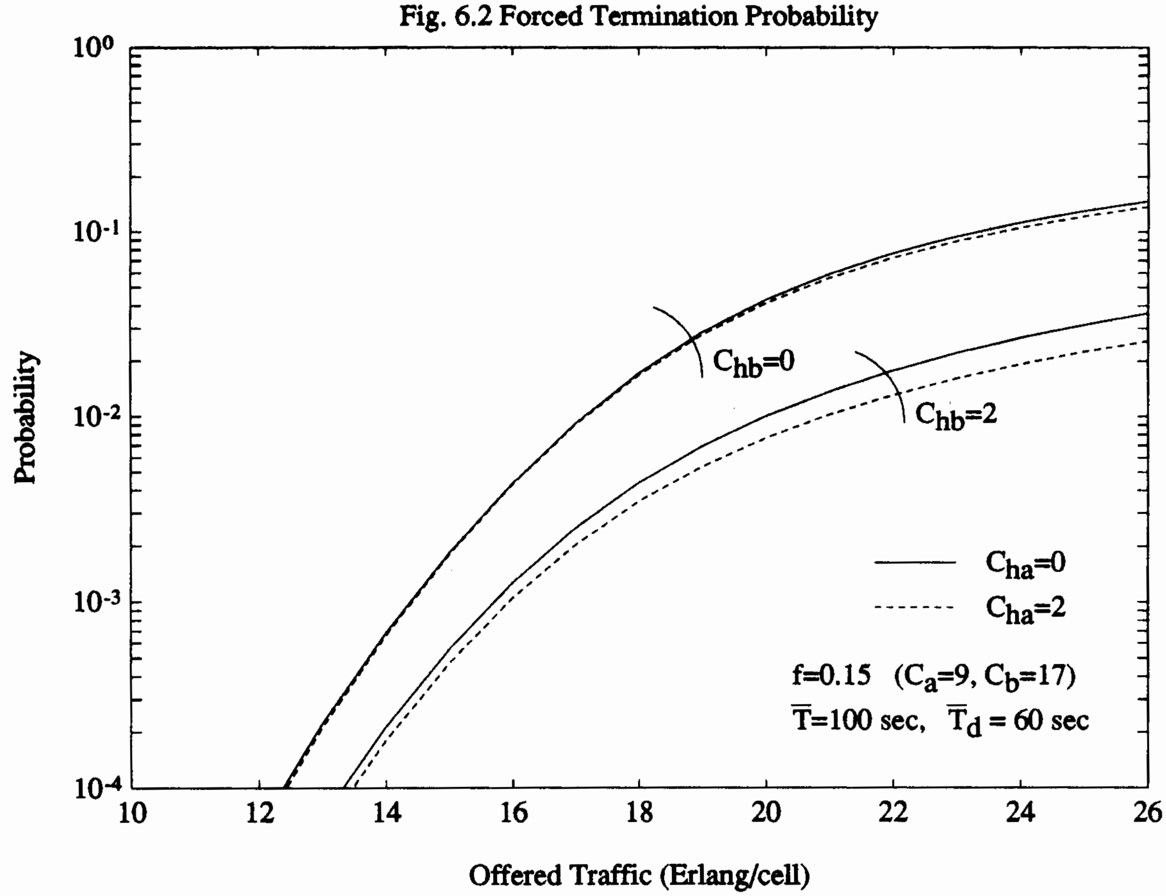

Fig. 6.3 Hand-off Activity

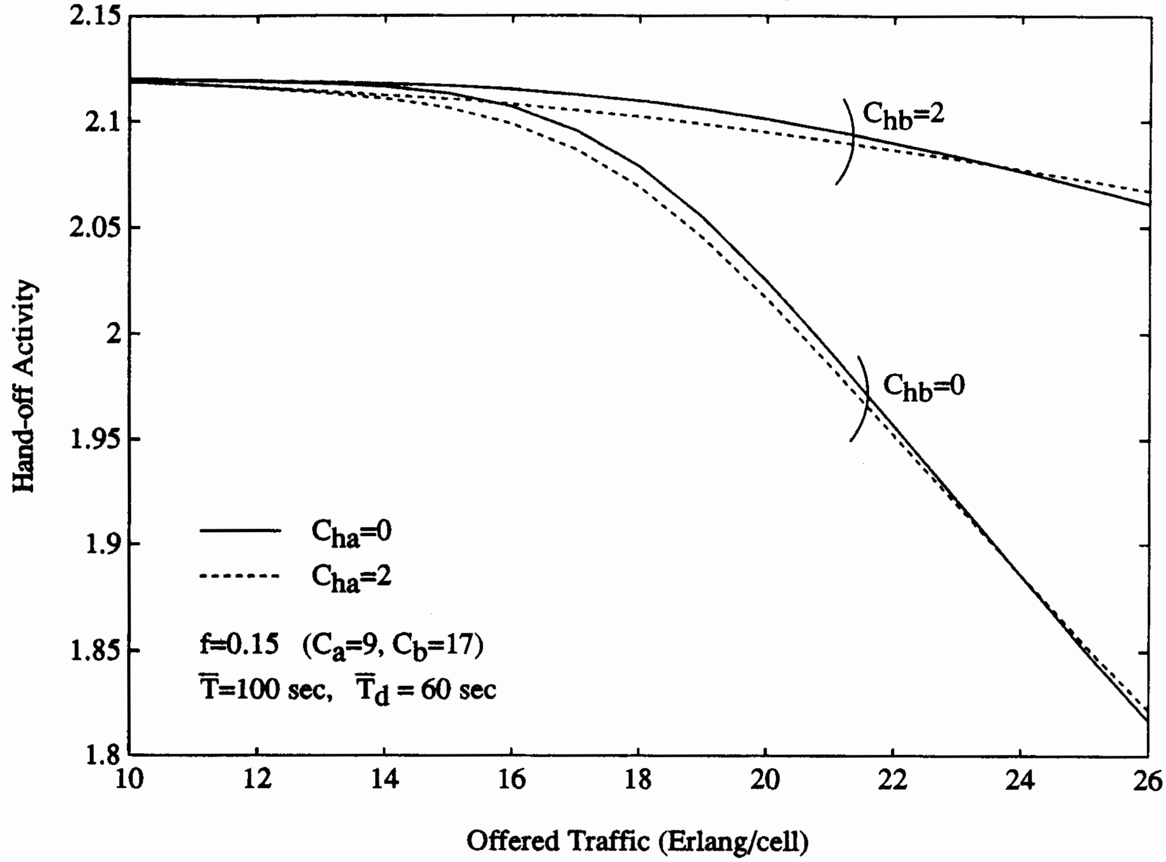

Fig. 6.4 Carried Traffic