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THE NONLINEAR HYDRODYNAMIC SLIDER BEARING

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Introduction

The classical theory of hydrodynamic lubrication is based on the assumption of negligible inertia forces. Consequently, the theory is a linear theory and corresponds mathematically to a flow with zero Reynolds number. In recent years, the nonlinear aspects of hydrodynamic lubrication have become of interest. This interest stems from the use of higher bearing speeds as well as low viscosity lubricants such as gases and liquid metals. Both of these effects contribute to an increased Reynolds number and therefore result in conditions for which the inertia effect may no longer be negligible.

The influence of the nonlinear inertia terms has been investigated in an approximate manner by Osterle, Saibel, et al. (references [2, 5, 6, 8, 10]).[1] The technique employed was to approximate the inertia terms by the average value across the film, thus making the inertia terms a function of longitudinal direction only. The equation of motion could then be integrated locally across the film and an expression for the pressure gradient obtained as a function of film thickness and total mass flow rate of lubricant.

Another approximate solution was obtained by Kahlert [9] who solved the equation of motion neglecting inertia and then approximated the inertia terms by averaging across the film, using the inertialess velocity expressions. In reference [5], a comparison is made between Kahlert's technique and the Osterle-Saibel approximation. The two methods agree reasonably well although Kahlert's corrections to the inertialess pressure distribution are generally larger than those predicted by the Osterle-Saibel method.

Milne [4] has considered a solution to the complete equations of motion in cylindrical coordinates without using the thin film approximation. His technique consists of a series expansion of the stream function with reciprocal kinematic viscosity as the expansion parameter. The zero order term in the series then corresponds to the inertialess case since $1/\nu = 0$ for negligible inertia.

In the present analysis, the influence of the nonlinear inertia terms in the slider bearing is investigated by a more exact series solution. The solution considers the local variation of the inertia terms across the film as well as in the direction of motion of the slider.

[1] Numbers in brackets designate References at end of paper.

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The Analysis

The governing equations for the slider bearing including inertia terms are

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} \qquad (1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

In the present analysis, a linear variation of film thickness is considered, and the geometry is illustrated in Fig. 1.

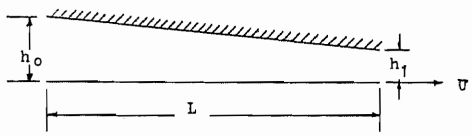

Fig. 1 Slider bearing geometry

Before proceeding to the solution of the complete equation of motion, it will be useful to summarize the results for the inertialess case. Neglecting the inertia terms in equation (1) and integrating the equation of motion across the film, subject to the boundary conditions

$$u(0) = U \qquad u(h) = 0 \qquad (3)$$

gives

$$u = -\frac{h^2}{2\mu}\frac{dp}{dx}\left(\eta - \frac{\eta^2}{2}\right) + U(1-\eta) \qquad (4)$$

where $\eta = y/h$. The total mass flow rate per unit depth in the Z direction is

$$m = \rho\int_0^h u\,dy = \rho h\int_0^1 u\,d\eta \qquad (5)$$

From equations (4) and (5) the following expression can be obtained which will be useful later

$$\frac{h^3}{\mu}\frac{dP}{dx} = -12\frac{m}{\rho} + 6Uh \qquad (6)$$

Integrating equation (6) over the length of the bearing subject to the boundary conditions

$$P(h = h_0) = P_0 \qquad P(h = h_1) = P_0 \qquad (7)$$

Nomenclature

- C_n = pressure expansion coefficients of equation (18)
- h = film thickness
- h_0 = h at $x = 0$
- h_1 = h at $x = L$
- $h' = \dfrac{dh}{dx} = \dfrac{h_1 - h_0}{L}$
- $K = \dfrac{\nu}{h'}$
- L = bearing length
- $\mathcal{L}$ = bearing load neglecting inertia
- $\mathcal{L}^*$ = bearing load including inertia
- m = total mass flow rate per unit depth in Z direction
- P = pressure
- P_0 = pressure at $x = 0$ and $x = L$
- u = longitudinal velocity
- U = plate velocity
- v = transverse velocity
- x = longitudinal coordinate
- y = transverse coordinate
- ρ = density
- μ = absolute viscosity
- $\eta = \dfrac{y}{h}$
- $\delta = h/h_0$
- Ψ = stream function
- ν = kinematic viscosity
- $\Phi = \dfrac{(Uh_1)}{\nu}\dfrac{(h_1)}{L}\left[\dfrac{\dfrac{h_0}{h_1}\left(\dfrac{h_0}{h_1} - 1\right)}{\left(\dfrac{h_0}{h_1} + 1\right)}\right]$

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gives

$$\frac{m}{\rho} = \frac{U}{2}\frac{h_0 h_1}{h_0 + h_1} \quad (8)$$

where h_0 is the film thickness at $x = 0$ and h_1 is the thickness at $x = L$.

From equations (4), (6), and (8), the velocity expression can be written as

$$u = -3U\left(1 - \frac{1}{h}\frac{h_0 h_1}{h_0 + h_1}\right)\left(\eta - \frac{\eta^2}{2}\right) + U(1 - \eta) \quad (9)$$

which is of the form

$$u = \frac{f(\eta)}{h} + g(\eta) \quad (10)$$

with

$$f(\eta) = 3U\frac{h_0 h_1}{h_0 + h_1}\left(\eta - \frac{\eta^2}{2}\right)$$

$$g(\eta) = U(1 - \eta) - 3U\left(\eta - \frac{\eta^2}{2}\right)$$

Defining the stream function Ψ such that

$$u = \frac{\partial \Psi}{\partial y} \qquad v = -\frac{\partial \Psi}{\partial x}$$

equation (10) suggests that a series solution for Ψ be assumed as

$$\Psi = K\sum_{n=0} f_n(\eta)\delta^n \quad (11)$$

where $\delta = h/h_0$, $\eta = y/h$, and K is a nondimensionalizing constant to be specified later.

Expressing the appropriate velocity expressions in terms of the stream function gives

$$u = \frac{K}{h}(f_0' + f_1'\delta + f_2'\delta^2 + f_3'\delta^3 + \ldots) \quad (12)$$

$$\frac{\partial u}{\partial y} = \frac{K}{h^2}(f_0'' + f_1''\delta + f_2''\delta^2 + f_3''\delta^3 + \ldots) \quad (13)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{K}{h^3}(f_0''' + f_1'''\delta + f_2'''\delta^2 + f_3'''\delta^3 + \ldots) \quad (14)$$

$$\frac{\partial u}{\partial x} = -\frac{Kh'}{h^2}(f_0' + f_1'\delta + f_2'\delta^2 + f_3'\delta^3 + \ldots) - \frac{K\eta h'}{h^2}(f_0'' + f_1''\delta + f_2''\delta^2 + \ldots) + \frac{Kh'}{h_0 h}(f_1' + 2f_2'\delta + 3f_3'\delta^2 + 4f_4'\delta^3 + \ldots) \quad (15)$$

$$v = \frac{K\eta h'}{h}(f_0' + f_1'\delta + f_2'\delta^2 + \ldots) - \frac{Kh'}{h_0}(f_1 + 2f_2\delta + 3f_3\delta^2 + \ldots) \quad (16)$$

where $h' = dh/dx =$ constant.

Substituting equations (12)–(16) into (1) and multiplying by $\frac{h^3}{\mu K}$ gives

$$-(f_0' + f_1'\delta + f_2'\delta^2 + \ldots)(f_0' + f_1'\delta + f_2'\delta^2 + \ldots) + (f_0' + f_1'\delta + f_2'\delta^2 + \ldots)(f_1'\delta + 2f_2'\delta^2 + 3f_3'\delta^3 + \ldots) - (f_0'' + f_1''\delta + f_2''\delta^2 + \ldots)(f_1\delta + 2f_2\delta^2 + 3f_3\delta^3 + \ldots) + \frac{h^3}{\mu K}\frac{\partial P}{\partial x} = f_0''' + f_1'''\delta + f_2'''\delta^2 + \ldots \quad (17)$$

Equation (6) for the inertialess case suggests that a series expansion of the pressure gradient be assumed in the form

$$\frac{h^3}{\mu}\frac{dP}{dx} = C_0 + C_1\delta + C_2\delta^2 + \ldots \quad (18)$$

where the C's are constants. Substituting equation (18) into (17) and equating coefficients of successive powers of δ to zero gives the series of equations

$$f_0''' + f_0'f_0' = \frac{1}{K}C_0 \quad (19)$$

$$f_1''' + f_0''f_1 + f_0'f_1' = \frac{1}{K}C_1 \quad (20)$$

$$f_2''' + 2f_0''f_2 + f_1''f_1 = \frac{1}{K}C_2 \quad (21)$$

$$f_3''' + f_0'f_3' + 3f_0''f_3 - f_1'f_2' + 2f_1''f_2 + f_1f_2'' = \frac{1}{K}C_3 \quad (22)$$

In writing equation (17), the constant K has been defined such that

$$\frac{K^2 h'\rho}{\mu K} = 1$$

or

$$K = \frac{\nu}{h'}$$

The series of equations for the f functions are each of third order and thus three constants of integration will appear. However, each equation for f involves one of the constants C of the pressure gradient expansion, and if C_n appearing in the f_n equation is considered to be a fourth unspecified constant, then four boundary conditions may be applied.

Except for the equation for f_0, each of the f_n equations is linear, and can be easily converted to a Volterra integral equation of the second kind.

Discussion of Boundary Conditions

The boundary conditions to be satisfied are

$$u(0) = U, \quad u(1) = 0, \quad v(0) = 0, \quad v(1) = 0, \quad m = \rho h\int_0^1 u\,d\eta \quad (23)$$

Referring to equations (12) and (16) shows that the following conditions will satisfy the boundary conditions of equation (23).

$$f_0(0) = 0, \quad f_0'(0) = 0, \quad f_0(1) = \frac{m}{\rho K}, \quad f_0'(1) = 0 \quad (24)$$

$$f_1(0) = 0, \quad f_1'(0) = \frac{h_0 U}{K}, \quad f_1(1) = 0, \quad f_1'(1) = 0 \quad (25)$$

$$f_n(0) = 0, \quad f_n'(0) = 0, \quad f_n(1) = 0, \quad f_n'(1) = 0; \quad n \geq 2 \quad (26)$$

The four boundary conditions for each f function will then determine the C_n of the pressure gradient expansion. The C_n will involve m as seen from equation (24). Substituting the C_n into equation (18) and integrating gives another constant of integration. This constant and the unknown constant m can be determined as in the inertialess case by applying the two boundary conditions on P

$$P(h = h_0) = P_0 \qquad P(h = h_1) = P_0 \quad (27)$$

The pressure distribution is thereby determined.

Solutions for f_0 and f_1. The first order inertia effects can be determined by considering only the first two terms of equation (18) by analogy with the corresponding inertialess expression given by equation (6). The problem is then to determine the constants C_0 and C_1 of equation (18) which requires a solution of equations (19) and (20) for f_0 and f_1, respectively.

From equation (19) we have

$$f_0''' + (f_0')^2 = \frac{1}{K} C_0 \tag{28}$$

with boundary conditions

$$f_0(0) = 0, \quad f_0'(0) = 0, \quad f_0(1) = \frac{m}{\rho K}, \quad f_0'(1) = 0$$

Because of the nonlinearity of the equation, an exact solution satisfying the boundary conditions cannot be given. However, an approximate solution can readily be obtained by successive approximations. If f_{0j} represents the jth order approximation and $g_j = (f_{0j}')^2$, the exact equation is replaced by the linear jth order approximation as

$$f_{0j}''' + g_{j-1} = \frac{1}{K} C_{0j} \tag{29}$$

Thus the zeroth order approximation will be the solution to

$$f_{00}''' = \frac{1}{K} C_{00}$$

the first order approximation the solution to

$$f_{01}''' + (f_{00}')^2 = \frac{1}{K} C_{01}$$

and so forth.

Integrating equation (29) three times between the limits of 0 and η and utilizing the boundary conditions $f_{0j}'(0) = 0 = f_{0j}(0)$ gives

$$f_{0j} + \frac{1}{2}\int_0^\eta (\eta - z)^2 g_{j-1}(z)dz = \frac{1}{K} C_{0j}\frac{\eta^3}{6} + A_j \frac{\eta^2}{2} \tag{30}$$

The constants C_{0j} and A_j are evaluated from the conditions $f_{0j}(1) = \frac{m}{\rho K}, f_{0j}'(1) = 0$ with the results

$$C_{0j} = -\frac{12m}{\rho} + 6K\int_0^1 (z)(1 - z)g_{j-1}(z)dz \tag{31a}$$

$$A_j = \frac{6m}{\rho K} + \int_0^1 (1 - z)(1 - 3z)g_{j-1}(z)dz \tag{31b}$$

$$f_{0j} = \frac{m}{\rho K}(3\eta^2 - 2\eta^3) + \eta^3\int_0^1 (z)(1 - z)g_{j-1}(z)dz$$
$$+ \frac{\eta^2}{2}\int_0^1 (1 - z)(1 - 3z)g_{j-1}(z)dz - \frac{1}{2}\int_0^\eta (\eta - z)^2 g_{j-1}(z)dz \tag{32}$$

$$f_{0j}' = \frac{6m}{\rho K}(\eta - \eta^2) + 3\eta^2\int_0^1 (z)(1 - z)g_{j-1}(z)dz$$
$$+ \eta\int_0^1 (1 - z)(1 - 3z)g_{j-1}(z)dz - \int_0^\eta (\eta - z)g_{j-1}(z)dz \tag{33}$$

Equations (31)–(33) are valid for $j \geq 1$ and the zeroth order results are

$$f_{00} = \frac{m}{\rho K}(3\eta^2 - 2\eta^3) \tag{34a}$$

$$C_{00} = -\frac{12m}{\rho} \tag{34b}$$

A comparison of equations (6), (18), and (31a) shows that the first term in the C_{0j} expression corresponds to the inertialess case. Equations (31a) and (33) are sufficient to determine C_{0j} to any desired accuracy by a process of successive substitutions. From an inspection of the equations, it is clear that C_{0j} will have the form

$$C_{0j} = -\frac{12m}{\rho} + \sum_{l=1}^{l=j} B_l\left(\frac{1}{K}\right)^l \tag{35}$$

where the B_l are constant. Since K was defined as

$$\frac{1}{K} = \frac{h'}{\nu}$$

the constant C_{0j} reduces to the inertialess case if $1/K = 0$. The condition $1/K = 0$ implies either $\nu = \infty$, the fictitious limiting case of zero Reynolds number, or $h' = 0$, the case of parallel surfaces.

Since the f_1 equation, (20), is linear and nonhomogeneous, it can readily be converted into a Volterra integral equation of the first kind by performing three integrations and applying the boundary conditions of equation (25). The result is

$$f_1 = \frac{h_0 U}{K}(\eta + \eta^3 - 2\eta^2) + \eta^3\int_0^1 (2z - 1)f_0'(z)f_1(z)dz$$
$$+ \eta^2\int_0^1 (2 - 3z)f_0'(z)f_1(z)dz - \int_0^\eta (\eta - z)f_0'(z)f_1(z)dz \tag{36a}$$

$$C_1 = 6h_0U + 6K\int_0^1 (2z - 1)f_0'(z)f_1(z)dz \tag{36b}$$

It is clear from an examination of equations (6), (18), and (36b) that the first term of the C_1 equation corresponds to the inertialess case. Using equations (33), (36a), and (36b) permits an evaluation of C_1 to any desired accuracy and it is clear from an inspection of the equations that C_1 will be expressible as a power series in $(1/K)$.

Pressure Distribution and Load Capacity. To evaluate the influence of inertia on pressure distribution equation (18) is integrated and compared with the integrated version of equation (6). The boundary conditions on pressure which are applied are

$$P(h = h_0) = P_0 \qquad P(h = h_1) = P_0 \tag{37}$$

In the present analysis, only the first two terms of equation (18) will be retained and the constants C_0 and C_1 will be evaluated to terms linear in $1/K$. This will give what might be termed first order inertia effects.

An examination of equations (31a) and (33) shows that

$$g_j = (f_{0j}')^2 = \frac{36}{K^2}\left(\frac{m}{\rho}\right)^2(\eta - \eta^2)^2 + 0\left(\frac{1}{K}\right)^3 \tag{38}$$

and that to evaluate C_{0j} to order $1/K$ requires the retention of terms to order $(1/K)^2$ in g_j. Equation (31a) then becomes

$$C_{0j} = -\frac{12m}{\rho} + \frac{(6)(36)}{K}\left(\frac{m}{\rho}\right)\int_0^1 (z)(1 - z)(z - z^2)^2dz$$
$$+ 0\left(\frac{1}{K}\right)^2 \tag{39}$$

Performing the integration gives

$$C_0 = -\frac{12m}{\rho} + \frac{54}{35K}\left(\frac{m}{\rho}\right)^2 \tag{40}$$

Similarly, an evaluation of C_1 to order $1/K$ requires terms up to order $(1/K)^2$ in $f_0'f_1$ and from equations (33) and (36a) we get

$$f_0'f_1 = \frac{6mh_0U}{\rho K^2}(\eta - \eta^2)(\eta - \eta^3 - 2\eta^2) + 0\left(\frac{1}{K}\right)^3 \quad (41)$$

Substituting equation (41) into (36*b*) and performing the integration gives

$$C_1 = 6h_0U - \frac{39h_0U}{35K}\left(\frac{m}{\rho}\right) \quad (42)$$

The quantity m/ρ and the constant of integration appearing when equations (6) and (18) are integrated are eliminated from the two boundary conditions of equation (37). The integration is facilitated if the transformation

$$\frac{d}{dx} = h'\frac{d}{dh}$$

is used.

The solution to equation (6) can be written

$$\frac{h'}{\mu}(P - P_0) = \frac{6Uh_1h_0}{h(h_1 + h_0)}\left\{\frac{1}{h_0}\left(\frac{h}{h_1} - 1\right) + \frac{1}{h_1}\left(\frac{h_1}{h} - 1\right)\right\} \quad (43)$$

and the solution to equation (18) is

$$\frac{h'}{\mu}(P - P_0) = \frac{C_1'h_1h_0}{h(h_1 + h_0)}\left\{\frac{1}{h_0}\left(\frac{h}{h_1} - 1\right) + \frac{1}{h_1}\left(\frac{h_1}{h} - 1\right)\right\} \quad (44)$$

where

$$C_1' = \frac{C_1}{h_0} = 6U - \frac{39U}{35K}\left(\frac{m}{\rho}\right) \quad (44a)$$

The constants C_0 and C_1' may be related by applying the boundary conditions to the integrated form of equation (18) with the result

$$\frac{C_0}{2} = -C_1'\frac{h_0h_1}{h_0 + h_1} \quad (45)$$

Equations (43) and (44) give the pressure distributions for the inertialess case and for the case of first order inertia effects, respectively. It is interesting to note that the equations differ only through the magnitudes of the coefficients $6U$ and C_1', respectively, and that $C_1' = 6U$ for $1/K = 0$. The first order inertia effects change the inertialess pressure distribution by a constant scale factor and the position of maximum pressure is the same in both cases. From the definition of $1/K$

$$\frac{1}{K} = \frac{h'}{\nu}$$

since $h' = dh/dx < 0$, $1/K < 0$ and therefore

$$C_1' > 6U \quad (46)$$

The effect of inertia results in higher pressures and consequently a greater load capacity than that predicted from the linear theory.

The quantity m/ρ may be determined from equations (40), (44), (45) and requires only the solution of a quadratic algebraic equation. Solving for the quantity $\frac{39}{35}\frac{U}{K}\frac{m}{\rho}$ gives

$$\frac{39U}{35K}\left(\frac{m}{\rho}\right) = \frac{13}{3}U - \frac{169}{210}U\Phi \pm \frac{13}{18}U\sqrt{36 + \frac{180}{35}\Phi + \frac{39}{35}\Phi^2} \quad (47)$$

where

$$\Phi = -\frac{U}{K}\frac{h_0h_1}{h_0 + h_1}$$

The negative sign must be chosen since the right side of equation (47) must vanish for $\Phi = 0$.

Neglecting the term quadratic in Φ and substituting equation (47) into (44*a*) gives

$$C_1' = 6U\left\{1 + \frac{13}{18}\left[\sqrt{1 + \frac{\Phi}{7}} - \left(1 - \frac{13}{70}\Phi\right)\right]\right\} \quad (48)$$

The parameter Φ can be rewritten as

$$\Phi = \left(\frac{Uh_1}{\nu}\right)\left(\frac{h_1}{L}\right)\frac{\frac{h_0}{h_1}\left(\frac{h_0}{h_1} - 1\right)}{\left(\frac{h_0}{h_1} + 1\right)} \quad (49)$$

which is the product of the Reynolds number based on h_1 and h_1/L.

The total load $\mathcal{L}$ is given by

$$\mathcal{L} = \int_0^L Pdx = \frac{1}{h'}\int_{h_0}^{h_1} Pdh \quad (50)$$

If $\mathcal{L}$ is the inertialess load and $\mathcal{L}^*$ is the load considering first order inertia effects, the percentage increase in load capacity due to the inertia effect can be written in the particularly simple form

$$\frac{\mathcal{L}^* - \mathcal{L}}{\mathcal{L}} = \frac{C_1' - 6U}{6U} = \frac{13}{18}\left[\sqrt{1 + \frac{\Phi}{7}} - \left(1 - \frac{13}{70}\Phi\right)\right] \quad (51)$$

where for simplicity P_0 has been taken as zero.

The quantity $(\mathcal{L}^* - \mathcal{L})/\mathcal{L}$ is shown in Fig. 2 for various values of $\Phi \leqq 1$. It is seen that the variation is essentially linear over

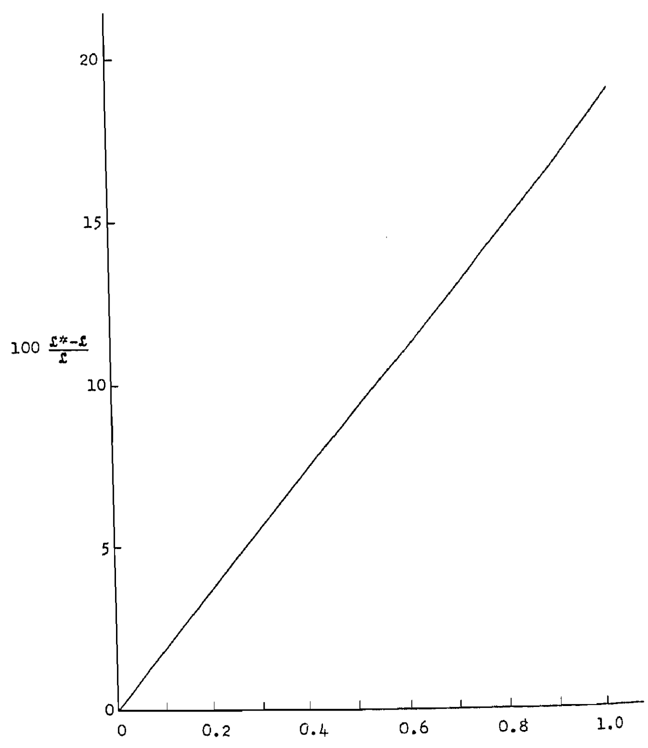

Fig. 2 Increase in load capacity considering first order inertia effect

the range considered. Also the inertialess load capacity is underestimated by 10 percent at a value of $\Phi = 0.55$.

Numerical Example. To demonstrate the range of Φ which can be encountered using different lubricants, the value of Φ for four lubricants will be calculated for the following assumed bearing conditions.

$$h_1 = 0.002 \text{ in.}, \quad U = 50 \text{ ft/sec}, \quad L = 12 \text{ in.}, \quad h_0/h_1 = 2$$

The property values are taken from reference [7] and the values of Φ are tabulated below.

Lubricant	ν, ft²/sec (200°F)	Φ
Air	0.239×10^{-3}	0.0464
Light oil	4.6×10^{-5}	0.242
Mercury	1×10^{-6}	11.1
Sodium	8.1×10^{-6}	1.37

The above examples show that the inertia effect for air would be negligible while the errors in load capacity for the light oil and sodium are approximately 5 percent and 19 percent, respectively. The error involved using mercury would be beyond the limit of validity of the first order correction which neglects the term quadratic in Φ in equation (47).

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