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On the Hand-off Arrival Process in Cellular Communications

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Abstract: Hand-offs in cellular communication systems cause interactions among cells that can be modeled using multi-dimensional birth-death process approaches and the concept of *system state*. However, *exact* numerical calculation of traffic performance characteristics is hindered by unmanageably large system state spaces even for systems of modest size. Previous analytical models get around the difficulty by isolating a *cell of interest* and invoking a Poisson process assumption for hand-off arrivals to the cell. Interactions among cells are characterized by relating the mean hand-off and departure rates from cells. The current paper seeks to explore the interactions in more detail. Two additional approximate analytical models are developed for this purpose. Each of these is more complicated than the simple Poisson process model, but is analytically tractable – at least for small system sizes. One model isolates a cluster of cells (rather than just the cell of interest) from the system and invokes a Poisson process assumption for cells on the cluster periphery. Performance is calculated for the central cell. The second model also isolates a cluster of cells surrounding the cell of interest, but uses an *equivalent* two-state Markov Modulated Poisson Process (MMPP) to characterize hand-off arrival processes to the cell of interest from each of the neighboring cells. Poisson hand-off arrivals to cells on the cluster periphery are assumed. This approach has fewer states than the cluster approach. Finally we present the *exact solution* for a regional coverage area consisting of a single seven-cell cluster. Teletraffic performance characteristics are computed for each modeling technique and are compared. It was found that all are in close agreement with the original "single isolated *cell*, Poisson hand-off arrival model," which requires the least states.

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I. Introduction

Computing the traffic performance characteristics of cellular communication systems has been the focus of much research reported in the literature. Specifically, an analytical model based on multidimensional birth death processes has previously been developed [1]. The basic methodology set forth in [1] and [2] requires the representation of a cell by a suitable state description. Since individual cells interact with neighboring cells through the hand-off process, a complete representation would require consideration of system states – concatenations of the states of all the cells in the region of coverage. The size of the *system state space* is so overwhelmingly large, that some reasonable assumptions and approximations are required for analytical tractability.

If one assumes that the hand-off arrival process to a cell follows a Poisson point process, relating the mean hand-off arrival rates and departure rates (to/from) a cell can capture the interactions between a cell and its neighbors. The detailed dependence of instantaneous hand-off arrival rates to a cell on knowledge of the states of neighboring cells is obviated. This eventually allows decoupling an individual cell from the system with the result that only the state of an individual cell (rather than the state of the system) must be considered. The size of the *cell state space* for cases of practical interest is quite manageable and performance characteristics can be computed. In the current paper we consider several potentially more thorough models of a cell's interaction with it neighbors. First, we use an approximation, in which a *cluster of cells* surrounding the cell of interest is considered, and hand-off arrivals to the *cells on the periphery of the cluster* are assumed to be Poisson. We call this the "isolated cluster, Poisson hand-off arrival model." This approach requires the consideration of *cluster states* (i.e., concatenations of the states of all cells in the cluster).

To reduce the number of *cluster states* that must be considered, we next consider a similar model in which again a cluster of cells surrounding the neighbors of the cell of interest is isolated. But for the purpose of determining the hand-off arrival process to the cell of interest, each of the neighboring cells is assumed to generate hand-off arrivals according to a two-state Markov-Modulated Poisson Process. We call this the "two-state MMPP neighbor model." Finally, a cellular system whose coverage region consists of a

single seven-cell cluster is solved *exactly*. Performance characteristics for the three models are computed and compared with the "single isolated cell, Poisson hand-off arrival model" proposed in [1].

Two related studies of the hand-off arrival process appear in [6] and [7]. In [6] the hand-off arrival process is shown to be Poisson for a *nonblocking* system and the authors conclude that the Poisson approximation for a blocking system yields reasonable results. In [7] the authors analyze hand-off traffic between cells using a moment matching technique and a "virtual cell." The "virtual cell" is a mathematical abstraction that is used to determine the offered traffic to neighboring cells. It is similar to the use of an infinite server queue to model overflow traffic from a queuing system. In this way, the authors are able to match the mean and variance of the hand-off arrival process. The current work is different in that we analyze an entire cluster. The hand-off process between cells in the cluster is completely determined by the resulting birth-death equations. In addition, our MMPP approximation allows higher order moments to be considered and does not require the use of a virtual cell as the hand-off process is again accounted for in the birth-death equations that are developed.

This paper is organized as follows: In section II we describe the system under consideration and the "single isolated cell, Poisson hand-off arrival model" that was used in [1] [2]. In section III we describe "isolated cluster, Poisson hand-off arrival model" and the procedure to compute cluster state probabilities. In section IV we present the "two-state MMPP neighbor model. We conclude the paper with a discussion of the calculation of relevant performance measures,(in section V), and a comparison of the (models being discussed) and the exact solution of a seven-cell system (section VI).

II. System Description

We consider a cellular communication system in which each cell has a total of C channels available for calls. Some number, C_h , of these channels is reserved for arriving hand-off calls, but *specific* channels *are not* reserved. Therefore, new calls arising within a cell will be served if the total number of calls already in progress is less than C-C_h, but a hand-off call be served unless all C channels are occupied. For convenience in demonstrating the approach we assume that the system has hexagonal geometry and that

the cellular system is homogeneous. That is, all cells are identical and have the same statistical behavior.

Mobile platforms traverse the region and generate call demands. Each platform can have at most one call at a time. The total number of noncommunicating platforms in each cell is denoted by N_0. Each platform generates calls according to a Poisson process with intensity, L. We assume that $N_0 >> C$ so that each cell receives new call arrivals at rate of $L_n = L * N_0$ regardless of the number of calls already in progress.

The unencumbered session time is a random variable, denoted by T and is assumed to have a negative exponential distribution. Its pdf is given by

$$f_T(t) = m_S \exp(-m_S t) . \quad (1)$$

The mean of T is $\bar{T} = 1/m_S$. The dwell time of a platform is denoted by T_D. We also assume that T_D follows a negative exponential distribution. Its pdf is given by

$$f_{T_D}(t) = m_D \exp(-m_D t) . \quad (2)$$

The negative exponential assumptions for T and T_D can be relaxed and a development following [3] can be used. However, for simplicity in the current discussion we will assume T and T_D are negative exponential random variables. The problem is to compute the performance characteristics for the system described above.

A thorough approach would be to consider *system states* [1]. This would be the concatenation of all the cell states. From this description of the system the solution for the equlibrium *system state probabilities* may proceed. Unfortunately, this approach is unfeasible in most situations. The number of states required to describe an entire system of cells is normally very large and only small systems with a few cells can be considered in this manner.

To circumvent this difficulty it was proposed in [1][2] that a single cell be considered. The interaction of this cell with its neighbors (due to hand-off) is represented by relating the *average hand-off departure rate* to the *average hand-off arrival rate*. This technique has allowed many interesting systems to be studied.

The solution method presented in [1] and [2] can be used for the problem stated above. One would proceed as follows:

The state of a single cell can be defined as the number of channels currently in use. We denoted the state with the variable v, note that v takes integer values in the set $\{0,1,\ldots,C\}$. Following the approach set forth in [1] we can identify the driving processes. These are the random events that cause the state of a cell to change. We have the following driving processes: 1) Generation of new calls 2) Completion of calls 3) Arrival of hand-off calls 4) Departure of hand-off calls. We can identify the state transitions and write the flow balance equations. The iterative procedure described in [1] can then be used to solve for the state probabilities, $P(s)$, (s= 0,1,…,C), and the average hand-off arrival rate, $?_h$.

As described in [1] the state transitions in a cell are coupled with the state transitions of its neighbors. This is because a hand-off arrival to the cell of interest corresponds to a hand-off departure from one of its neighbors. A rigorous solution would require the computation of the *system state probabilities*. The method proposed in [1] and [2] assumes that the hand-off arrivals comprise a Poisson point process with mean arrival rate Λ_h. This allows the state transitions of a cell to be decoupled from the states of its neighboring cells. The result is that only a single cell has to be considered and the state space is of manageable size for many reasonable parameters of interest. Here we will call this approach the original "single isolated cell, Poisson hand-off arrival model."

III. Isolated Cluster, Poisson Hand-off Arrival Model

A. State description

We consider a cluster of seven cells, as shown in Figure 1. The *state of the cluster* is a string of integers, which specifies the number of channels in use in each cell. For simplicity, we do not consider limit or quota constraints here [1]. So a state is represented by a 7-tuple of integers, $v_0\, v_1\, v_2\, v_3\, v_4\, v_5\, v_6$ in which $v_i \in \{0,1,2,\ldots,C\}$ for i=0,1,…6. The cell of interest has index i=0 and is located at the center of the cluster. The states can be ordered lexicographically with a state index s, s=0,1,…,s_{max}, so that state s=0 corresponds to the all zeros sequence (0 0 0 0 0 0 0) and state $s=s_{max}$ corresponds to the sequence ($C\ C\ C\ C\ C\ C\ C$), although any ordering will suffice. The state of the cluster can then be written so that each state variable is expressed as a

function of the index *s*. That is, the string, ($v_0(s)$, $v_1(s)$, $v_2(s)$, $v_3(s)$, $v_4(s)$, $v_5(s)$, $v_6(s)$) gives the state variable values when the cluster is in state, *s*. The number of occupied channels in cell i of the cluster when the cluster is in state *s, is* $v_i(s)$.

B. Driving Processes

Five driving processes can be identified which cause the state of the cluster to change. These are:

1. New call arrivals - newly generated calls that arise within cells of the cluster.
2. Call completions - calls that are completed within cells of the cluster.
3. Hand-offs within the cluster - calls that are on platforms that move from some cell of the cluster to some other cell of the cluster.
4. Hand-off departures - calls that are on platforms that move from some cell of the cluster to some other cell that is outside the cluster.
5. Hand-off arrivals – calls that are on platforms that move form some cell outside the cluster to some other cell that is in the cluster.

A *predecessor* state for state *s* is any state *x* that can immediately give rise to state *s* upon occurrence of an event in any of the driving processes. For each driving process and any given state, s, the *predecessor* state can be found.

1. New call arrivals.

A cluster state x_n is a predecessor state for state *s* due to new call arrivals in the i^{th} cell if the state variables have the following relation

$$v_i(s) = v_i(x_n) + 1 \qquad (3)$$

$$v_j(s) = v_j(x_n) \qquad \text{for} \quad i \neq j \qquad (4)$$

Recall that a new call will be served only if the number of occupied channels in the center cell is less than C-C_h. So the above transition will take place only if $v_i(x_n) < C$-C_h. The transition rate into cluster state *s* from cluster state x_n is denoted as $g_n(x_n,s)$ this is given by

$$g_n(x_n, s) = \Lambda_n \qquad (5)$$

2. Call completions

When a call completes in the i^{th} cell, the state variable v_i will decrease by one. A state x_c is a predecessor of state s for call completions if the state variables have the following relationships

$$v_i(s) = v_i(x_c) - 1 \quad (6)$$

$$v_j(s) = v_j(x_c) \qquad \text{for} \quad i \neq j \quad (7)$$

The transition rate into s from x_c is given by

$$g_c(x_c, s) = v_0(x_c) \cdot m_S \quad (8)$$

3. Hand-offs within the cluster

A call that departs cell *i*, destined for cell *j* can experience one of two possible outcomes. The destination, cell *j*, may have an available channel for the call, in this case the call is served and the state variable v_i will decrease by one while the state variable v_j will increase by one. One the other hand, the destination cell may have all of its *C* channels in use. In this case, the state variable v_j will remain unchanged while v_i will decrease by one.

Generally, we will say that two cells, *i* and *j*, are neighbors if it is possible for a call to hand-off from one cell to the other. Since hand-offs are determined by power measurements, the two cells need not necessarily be physically adjacent. Usually, and in the simple model they are. Not all cells in the cluster are neighbors of a particular cell *i.* In the usual simple hexagonal case shown in Figure 1, the indices of neighboring cells are related as follows: If i=0 then cells j=1,2,3,4,5,6 are all neighbors of cell 0. However, if *i* is different from 0 then cell *j* is a neighbor of cell *i* if any one of the following holds: a) j=0, b) |i-j|=1, c) |i-j|=5. The last condition allows for cells 6 and 1 to be neighbors.

For the case in which the hand-off *is served* in the target cell, the following relationships must hold between a state and its predecessor state. A cluster state x_d, is a predecessor of cluster state *s* due to a hand-off departure from cell *i* destined for neighbor *j* if the following relationships hold

$$v_i(s) = v_i(x_d) + 1 \quad (9)$$

$$v_j(s) = v_j(x_d) - 1 \tag{10}$$

$$v_k(s) = v_k(x_d) \qquad \text{for } k \neq i, j \tag{11}$$

For the case in which the target cell, *j* is fully occupied the following relationships must hold:

$$v_i(s) = v_i(x_d) + 1 \tag{12}$$

$$v_j(s) = v_j(x_d) = C \tag{13}$$

$$v_k(s) = v_k(x_d) \qquad \text{for } k \neq i, j \tag{14}$$

For convenience we assume that a hand-off departure *from a cell* is equally likely to target any of its (six, for the usual hexagonal case) neighboring cells. For either case described above the transition rate into cluster state *s* from cluster state x_d is given by

$$g_d(x_d, s) = \frac{1}{6} v_i(x_d) \cdot m_D \tag{15}$$

4. Hand-off departures

A platform in any of the outer cells (cells 1,2,3,4,5,6) may depart its current cell and leave the cluster. If a call is in progress on board the platform then a hand-off departure from the cluster is generated. In this scenario, only one state variable is involved. A hand-off departing cell *i (i=1,2,3,4,5,6),* destined for a cell outside the cluster will cause the state variable v_i to decrease by one. Therefore, a state x_o is a predecessor state for state *s* due to a hand-off departure from cell *i* to a cell outside the cluster if the following relationships between the state variables hold:

$$v_i(s) = v_i(x_o) - 1 \tag{16}$$

$$v_j(s) = v_j(x_o) \qquad \text{for} \quad j \neq i \tag{17}$$

For the geometry shown in Figure 1, one half of the hand-off departures from a cell on the periphery of the cluster will be targeted to cells that are outside the cluster. Thus the flow into state *s* from state x_o is given by

$$g_o(x_o, s) = \frac{1}{2} m_D \cdot v_i(x_o) \qquad \text{for } i = 1,2,3,4,5,6\,. \tag{18}$$

5. Hand-off arrivals to the cluster

An outer cell, say cell i (i=1,2,3,4,5,6), may receive hand-off arrivals from cells that are outside the cluster. If this occurs the state variable v_i will increase by one. A cluster state x_h is a predecessor of cluster state s if the following relationships hold

$$v_i(s) = v_i(x_h) + 1 \tag{19}$$

$$v_j(s) = v_j(x_h) \qquad \text{for} \quad j \neq i \tag{20}$$

To determine the transition rate into state s from state x_h, we let Λ_h denote the average rate of hand-off arrivals *to the cluster*. For now we assume that Λ_h is given but subsequently we will solve for this quantity. If there are six cells on the periphery of the cluster, a particular cell in the outer ring will receive 1/6 of the total number of hand-offs entering the cluster. Thus the transition rate into state s from state x_h due to a hand-off arrival from outside the cluster is

$$g_h(x_h, s) = \frac{1}{6} \cdot \Lambda_h \tag{21}$$

.

C. Balance Equations and Solution

We can write transition rate from cluster state x into cluster state s. This is simply the sum of all component flows under each driving process. Let $q(x,s)$ be the transition rate state x to state s (for $x \neq s$). This is given by

$$q(x,s) = g_n(x,s) + g_c(x,s) + g_d(x,s) + g_o(x,s) + g_h(x,s) \tag{22}$$

Note that if cluster state x is *not* a predecessor state of s under any of the driving processes then $q(x,s)$=0. The total flow out of state s (s=*0,1,2,...,*s_{max}), can be expressed as

$$q(s,s) = -\sum_{\substack{k=0 \\ k \neq s}}^{s_{max}} q(s,k) \tag{23}$$

In order to find the equilibrium cluster state probabilities, we write the flow balance equations for the cluster states. These comprise the following set of simultaneous equations for the unknown cluster state probabilities, $P(s)$.

$$\sum_{i=0}^{s_{max}} P(i) \cdot q(i,j) = 0 \qquad , \qquad j = 0,1,2,...,s_{max} \tag{24}$$

$$\sum_{j=0}^{s_{max}} P(j) = 1 \tag{25}$$

The above equations express that in equilibrium the *net* probability flow into a state is zero and the sum of the probabilities is unity.

D. Determination of the hand-off arrival rate

The solution of the cluster state probabilities, $P(s)$ s=0,1,2,…,s_{max}, requires knowledge of the hand-off arrival rate in the Markov chains representing the neighbors. In the derivation of the flow balance equations we assumed that Λ_h (the average hand-off arrival rate to the cluster) was known. However, Λ_h is a function of the cluster state probabilities and the system parameters. Its value can be determined by noting that for a homogeneous system in statistical equilibrium, the average hand-off *departure rate from a cluster* must equal the *average hand-off arrival rate to the cluster*. The value Λ_h can be determined by an iterative computation. From the cluster state probabilities we can compute the average hand-off departure rate. Let Δ_h be the average hand-off departure rate from the cluster it is found from the following

$$\Delta_h = \sum_{s=0}^{s_{max}} \sum_{i=1}^{6} \frac{1}{2} v_i(s) \cdot m_D \cdot P(s) \tag{26}$$

As stated before, when the system is in statistical equilibrium the average hand-off departure rate from a cluster is equal to the average hand-off arrival rate to a cluster. This is expressed mathematically by the following

$$\Lambda_h = \Delta_h \tag{27}$$

Equations (31)-(34) can be used in the iterative procedure described in [1]. The following steps are used to compute the cluster state probabilities $P(s)$ and the hand-off arrival rate to the cluster, Λ_h:

1. Make an initial guess at Λ_h.
2. Solve the flow balance equations ((24) and (25))
3. Compute Δ_h from equation (26)
4. Update Λ_h=Δ_h.
5. Goto step 2.

The above procedure is repeated until the relative change in the state probabilities, $P(s)$, and the hand-off arrival rate, Λ_h, is within a set tolerance.

The method above produces the cluster state probabilities, $P(s)$, which can be used to compute performance characteristics such as blocking probability, hand-off failure probability, carried traffic, hand-off activity, and forced termination probability. Before discussing the performance measures we next discuss an additional approximation that can reduce the number of states needed to describe a cluster.

IV. Two-State MMPP Neighbor Model

In order to use the *isolated cluster, Poisson hand-off arrival model* to compute cluster state probabilities, the number of occupied channels in each cell of the cluster is used to describe the state of the cluster. This can lead to an extremely large number of cluster states. In general, the number of states needed for a cluster with seven cells and C channels per cell is $(C+1)^7$. As an example , if each cell depicted in Figure 1, had only five channels, the total number of states needed to describe the cluster is $6^7 = 279{,}936$. We see that the size of the state space can quickly become overwhelming.

We consider approximating each of the cells that are neighbors of the (center) cell of interest, a *two* state Markov Chain. This approximation technique is based on work presented in [4] and [5]. Following the notation of [5] a two state MMPP is parameterized by an infinitesimal generator matrix $\mathbf{Q}^{(2)}$ and by $\mathrm{L}^{(2)}$, both are 2x2 matrices and have the form

$$\mathbf{Q}^{(2)} = \begin{bmatrix} -r_0 & r_0 \\ r_1 & -r_1 \end{bmatrix} \tag{28}$$

$$\mathrm{L}^{(2)} = \begin{bmatrix} l_0 & 0 \\ 0 & l_1 \end{bmatrix} \tag{29}$$

Figure 2 shows a two state Markov chain in which the flow out of state 0 is r_0 and the flow out of state 1 is r_1. The λ_0 and λ_1 beside state 0 and state 1 are meant to remind the reader that when the chain is in state 0 Poisson arrivals occur at rate Λ_0 and when the chain is in state 1 Poisson arrivals occur at rate λ_1. We also define the vector $\mathbf{P}=[P_0, P_1]$ as the equilibrium state probabilities for the 2 state Markov chain. The vector $\mathbf{P}$ is the

solution to the balance equation $\mathbf{PQ}^{(2)}=\mathbf{0}$ and the normalization equation $P_0+P_1=1$. This leads to the following expressions for the equilibrium state probabilities

$$P_0 = \frac{r_1}{r_0 + r_1} \tag{30}$$

$$P_1 = \frac{r_0}{r_0 + r_1} \tag{31}$$

We will need expressions for the moments of the MMPP's arrival rate for use in the approximation. The n^{th} moment of the arrival rate of the a two state MMPP, denoted by m^n, is found by the following

$$m^n=\mathbf{P}[\mathrm{L}^{(2)}]^n\mathbf{1} \tag{32}$$

Where **1** is a 2x1 column vector of ones. From the above we can compute the variance of the arrival rate as $s=m^2-(m)^2$. As in [4] and [5] we define a time constant for the process as

$$t_c = \frac{1}{s}\int_0^\infty r(t)dt \tag{33}$$

In the above equation $r(t)$ is the covariance function of the arrival rate. If $l(t)$ represents the instantaneous arrival rate of the MMPP at time t then $r(t)$ is defined by $r(t)=\mathrm{E}\{(l(t)-m)(l(t+t)-m)\}$. For a general MMPP with infinitesimal generator **Q,** rate matrix L and equilibrium probability vector **P,** the covariance function is given by

$$r(t)=\mathbf{P}\mathrm{L}(\exp(\mathbf{Q}t)-\mathbf{1P})\mathrm{L}\mathbf{1} \tag{34}$$

where **1** is an appropriately dimensioned column vector of ones. (see [4] and [5] for details). For the two state MMPP we have the following expression for the time constant

$$t_c = \frac{1}{s}\int_0^\infty r(t)dt$$
$$= \frac{1}{s}\left[\begin{bmatrix} P_0 & P_1 \end{bmatrix} \begin{bmatrix} l_0 & 0 \\ 0 & l_1 \end{bmatrix} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} P_0 & P_1 \end{bmatrix} - \begin{bmatrix} -r_0 & r_0 \\ r_1 & -r_1 \end{bmatrix} \right)^{-1} \begin{bmatrix} l_0 & 0 \\ 0 & l_1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - m^2 \right] \quad (35)$$

Our approximation of the neighbors is based on fitting the mean, m, variance, s, third moment, m^3 and the time constant, t_c, of the MMPPs to appropriate values which are determined from the hand-off departure process from the center cell.

A. Cluster State Description

Consider figure 1 again, we wish to use the two-state MMPP discussed in the previous section. In this case we describe the state of the cluster (shown in Figure 1) as the number of occupied channels in the center cell along with the state of the MMPP in each neighbor (either 0 or 1). We will denote the state of the cluster as $(v_0,a_1,a_2,a_3,a_4,a_5,a_6)$. The state variable v_0 is the number of occupied channels in the center cell and it takes values in the set $\{0,1,\ldots,C\}$. The state variable a_i, i=1,2,…,6, denotes the state of the MMPP describing the i^{th} neighbor. Each a_i takes on values in the set $\{0,1\}$. We can again order the states using the index s, s=0,1,…,s_{max}. In this case the state s=0 is the all zero sequence (0,0,0,0,0,0,0) while state s=s_{max} is the sequence (C,1,1,1,1,1,1). We note that modeling the neighbors with two state MMPPs greatly reduces the number of states required to describe the cluster. In general the number of cluster states needed is $(C+1)*2^6$. For a system with five channels in each cell the number of cluster states is $6*2^6$=384. Recall that for the same number of channels the number of cluster states needed in the corresponding *isolated cluster, Poisson hand-off arrival model* is 6^7= 279,936. When using the *two-state MMPP neighbor model*, we will let $v_0(s)$ denote the number of occupied channels in the center cell when the cluster is in state s, and we will let $a_i(s)$ denote the state of the i^{th} neighbor's MMPP when the cluster is in state s.

B. Predecessors and Rate equations for 2-state Approximation

A similar approach to finding the cluster state probabilities is used. Specifically, we find all permissible states and their predecessor's under each of the driving processes. For this case we can identify 5 driving processes, these are

1. New call arrivals in the center cell.
2. Call completions in the center cell.
3. Hand-off departures from the center cell.
4. Hand-off arrivals to the center cell.
5. Changes in the state of a neighbor's MMPP.

The resulting flow balance equations are then solved iteratively.

Recall that we have reduced the number of states used to represent the neighbors of the cell of interest. Therefore, we cannot say that a hand-off departure from the center will cause a change in the state of the MMPP in the destination cell. Likewise, a hand-off arrival to the center cell from the i^{th} neighbor will not necessarily decrease the state variable a_i. This is the major difference between the 2-state MMPP approximation and the model discussed earlier.

1. New call arrivals to the center cell.

A cluster state x_n is a predecessor state for state s due to new call arrivals in the center cell if the state variables have the following relation

$$v_0(s) = v_0(x_n) + 1 \tag{36}$$

$$a_i(s) = a_i(x_n) \qquad for \quad i = 1,2,...,6 \tag{37}$$

Recall that a new call will be served only if the number of occupied channels in the center cell is less than C-C_h. So the above transition will take place only if $v_0(x_n) < C$-C_h. The transition rate into cluster state s from cluster state x_n is denoted as $g_n(x_n,s)$ this is given by

$$g_n(x_n, s) = \Lambda_n \tag{38}$$

2. Call completions in the center cell

A cluster state x_c is a predecessor of state s due to call completions in the center cell if the following holds

$$v_0(s) = v_0(x_c) - 1 \tag{39}$$

$$a_i(s) = a_i(x_c) \qquad for \quad i = 1,2,...,6 \tag{40}$$

The flow into state s from state x_c is given by

$$g_c(x_c, s) = v_0(x_c) \cdot m_S \tag{41}$$

Where $v_0(x_c)$ is the number of occupied channels in the center cell when the cluster is in state x_c.

3. Hand-off departures from the center cell

A cluster state x_d, is a predecessor of cluster state s due to a hand-off departure from the center cell if the following hold

$$v_0(s) = v_0(x_d) - 1 \tag{42}$$

$$a_i(s) = a_i(x_d) \qquad for \quad i = 1,2,...,6 \tag{43}$$

The transition rate is given by

$$g_d(x_d, s) = v_0(x_d) \cdot m_D \tag{44}$$

4. Hand-off arrivals to the center cell

A hand-off arrival will be served in the center cell when the cluster is in state s if the state variable $v_0(s)$ is less then C. A state x_h is a predecessor for state s due to a hand-off arrival to the center cell if the state variables are related by

$$v_0(s) = v_0(x_h) + 1 \tag{45}$$

$$a_i(s) = a_i(x_h) \qquad for \quad i = 1,2,...,6 \tag{46}$$

Recall that each neighbor's MMPP can be in one of two states (either 0 or 1). When a neighbor's MMPP is in state i it generates hand-offs at rate l_i. So the flow into cluster state s from cluster state x_h due to hand-off arrivals to the center cell is given by

$$g_h(x_h, s) = \frac{1}{6} \sum_{i=1}^{6} l_{a_i(x_h)} \tag{47}$$

5. Changes in the state on the neighboring MMPPs.

From Figure 2 we see that for any cluster state x, the state variables $a_i(x)$ may either increase from 0 to 1 (if $a_i(x)=0$) or decrease from 1 to 0 (if $a_i(x)=1$). We call these *right* transitions and *left* transitions respectively.

A state x_r is a predecessor for state s due to a right transition in the i^{th} neighbor if the following holds

$$a_i(s)=1 \quad \text{and} \quad a_i(x_r)=0 \tag{48}$$

$$a_j(s) = a_j(x_r) \quad \text{if } j \neq i \quad \text{and } v_0(s) = v_0(x_r) \tag{49}$$

The rate of flow into cluster state s from cluster state x_r is

$$g_r(x_r, s) = r_0 \tag{50}$$

A state x_l is a predecessor for state s due to a left transition in the i^{th} neighbor if the following holds

$$a_i(s)=0 \quad \text{and} \quad a_i(x_l)=1 \tag{51}$$

$$a_j(s) = a_j(x_l) \quad \text{if } j \neq i \quad \text{and } v_0(s) = v_0(x_l) \tag{52}$$

The rate of flow into cluster state s from cluster state x_l is

$$g_l(x_l, s) = r_1 \tag{53}$$

As before we can write the total flow out of cluster state x into cluster state s. In this case we write

$$q(x,s) = g_n(x,s) + g_c(x,s) + g_d(x,s) + g_h(x,s) + g_r(x,s) + g_l(x,s) \tag{54}$$

We also have the same balance and normalization equations as (24) and (25). The balance equations can be solved to find the cluster state probabilities for the system in which each neighbor of the cell of interest is modeled as a two-state state MMPP.

C. Solution of Equilibrium State Probabilities

Again our development has assumed that the two-state MMPP parameters are known. Specifically, we assumed that l_0, l_1, r_0, and r_1 were known when we developed

the previous flow balance equations. However, these parameters must be related to the hand-off departure process from the center cell. We choose l_0, l_1, r_0, and r_1 so that the first three moments and the time constant (defined in (35)) of the MMPP arrival rate are equivalent to those of the hand-off departure process from the center cell. The moments and the time constant are chosen to be equivalent because the system has been assumed to be homogeneous, therefore, the hand-off departure and arrival processes should be identical.

From the cluster state probabilities we can compute the moments of the hand-off departure rate from the center cell. The i^{th} moment is denoted by $\overline{\Delta_h^i}$ and it is given by

$$\overline{\Delta_h^i} = \sum_{k=0}^{s_{\max}} \left(v_0(k) \cdot m_D\right)^i \cdot P(k) \tag{55}$$

The variance of the hand-off departure rate is given by

$$\mathrm{VAR}(\Delta_h) = \overline{\Delta_h^2} - \left(\overline{\Delta_h}\right)^2 \tag{56}$$

Computing the time constant of the hand-off departure rate is slightly more involved. When the cluster is in state s hand-off departures from the center cell occur at a rate of $v_0(s)*m_D$. So we see that hand-off departures from the center cell form an MMPP as well. We can define the covariance function, $r(t)$, as in (34) we repeat it here

$$r(t)=\mathbf{P}\mathrm{L}(\exp(\mathbf{Q}t)\text{-}\mathbf{1P})\mathrm{L}\mathbf{1} \tag{57}$$

In the equation above **P** is the equilibrium cluster state probabilities, **Q** is the $(s_{max}+1)\times(s_{max}+1)$ transition rate matrix. That is, **Q** has elements $q(x,s)$ denoting the flow into cluster state s from cluster state x. These are simply the coefficients of the balance equations and are given by (54). The diagonal elements of **Q**, $q(s,s)$, are found using (23). The matrix L is an $(s_{max}+1)\times(s_{max}+1)$ diagonal matrix that gives the hand-off departure rate from the center cell for each cluster state. It has the following form

$$\mathrm{L}=\begin{bmatrix} v_0(0)*m_D & 0 & \cdots & 0 \\ 0 & v_0(1)*m_D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v_0(s_{\max})*m_D \end{bmatrix} \tag{58}$$

We denote the time constant for the hand-off departure process from the center cell as T it can be computed from the following

$$\mathrm{T}=\frac{1}{\mathrm{VAR}(\Delta_h)}[\mathbf{P}\mathrm{L}(\mathbf{1P\text{-}Q})^{-1}\mathrm{L}\mathbf{1}\text{-}\left(\overline{\Delta_h}\right)^2] \tag{59}$$

The reader is referred to [4] and [5] for details of the MMPP modeling that was adapted for the present paper.

For a homogeneous system we must have the following relationships for the moments of the hand-off departure process from the center cell and the hand-off arrival process.

$$m=\overline{\Delta_h} \tag{60}$$

$$s=\mathrm{VAR}(\Delta_h) \tag{61}$$

$$m^3=\overline{\Delta_h^3} \tag{62}$$

$$t_c=\mathrm{T} \tag{63}$$

We see from equations (60)-(63) that we must choose the 2-state MMPP parameters (l_0, l_1, r_0, and r_1) so that the moments and the time constant match those of the hand-off departure process from the center cell. The equations to compute l_0, l_1, r_0, and r_1 from a set of desired moments and time constant ($\overline{\Delta_h}$,VAR(Δ_h), $\overline{\Delta_h^3}$, and T) are given in [4], we repeat them here

$$r_0=\frac{1}{\mathrm{T}(1+h)} \tag{64}$$

$$r_0=\frac{h}{\mathrm{T}(1+h)} \tag{65}$$

$$l_0=\overline{\Delta_h}+\sqrt{\mathrm{VAR}(\Delta_h)/h} \tag{66}$$

$$l_1 = \overline{\Delta_h} - \sqrt{\mathrm{VAR}(\Delta_h) \cdot h} \tag{67}$$

where

$$h = 1 + \frac{d}{2}\left[d - \sqrt{4 + d^2}\right] \tag{68}$$

and

$$d = \frac{\overline{\Delta_h^3} - 3\overline{\Delta_h}\mathrm{VAR}(\Delta_h) - \left(\overline{\Delta_h}\right)^3}{\left[\mathrm{VAR}(\Delta_h)\right]^{3/2}} \tag{69}$$

Using equations (64)-(69) and applying them to the problem on hand, we have a method to compute the two state MMPP parameters so that the mean, variance, third moment and the time constant are equivalent to those of the hand-off departure process from the center cell. To compute the equilibrium cluster state probabilities, $P(s)$, we used the following algorithm

1. Make an initial guess at the MMPP parameters l_0, l_1, r_0, and r_1.
2. Solve resulting flow balance equations for $P(s)$.
3. Compute hand-off departure statistics ($\overline{\Delta_h}$,VAR(Δ_h), $\overline{\Delta_h^3}$, and T) using (55), (56), and (59).
4. Update MMPP parameters using (64)-(69).
5. Goto step 2.

The above process is stopped when the relative change in the MMPP parameters is less than a set tolerance.

V. Seven-Cell System – Exact Solution

For comparison we compute the performance of a simple seven-cell system. The analysis is very similar to that used for the isolated cluster with Poisson hand-off arrivals (to the cluster) that was discussed earlier. In this case however, the *entire* system consists of just seven cells (see Fig. 1). This implies that we have no hand-off arrivals into the cluster, since there are no cells outside the cluster that are servicing calls.

To describe the system we require $(C+1)^7$ system states and we identify the following driving processes:

1. New call arrivals - newly generated calls that arise within cells of the cluster.
2. Call completions - calls that are satisfactorily completed.
3. Hand-offs within the cluster - calls that are on platforms that move from some to another cell.
4. Hand-off departures - calls that are on platforms that move from some cell of the system to some other cell that is outside the system.

Note that these processes are identical to the first four driving processes of the isolated cluster model. In this case, the hand-off departure process (number 4 in the above list) represents the departure of calls from the system/coverage area. The identification of predecessor states and transition rates flows is essentially similar to that already described for the isolated cluster model. The resulting flow-balance equations can then be solved to find the system state probabilities. The flow-balance equations are of the same form as (24) and (25), but the transition rate from state i into state j is given by

$$q(i, j) = g_n(i, j) + g_c(i, j) + g_d(i, j) + g_o(i, j) \qquad (70)$$

The only difference between (70) and (22) is that the component due to hand-off arrivals to the cluster ($g_h(i, j)$) is not present. The solution for the state probabilities requires the solution of the probability flow balance equations, (24) and (25). Note that the solution does not require an iterative procedure like to ones described above. This is because all cell-to-cell interactions are accounted for by using system states.

VI. Performance Measures

Using the modeling techniques described above we can compute the equilibrium cluster state probabilities, $P(s)$ were $s=0,1,\ldots,s_{max}$. We can compute various performance characteristics from the cluster state probabilities. We will be concerned with the performance in the center cell of the cluster. All the following discussions will assume that we are measuring the performance of the center cell in the cluster. Since we have assumed a homogeneous system, all the other cells in the system should have the same performance characteristics.

A. Blocking probability

The blocking probability is defined as the fraction of new call attempts that are denied access to a channel. Recall that a new call will be blocked if the number of occupied channels in the cell is greater than $C - C_h$. So for a given cluster state s, a new call attempt will be blocked if $v_0(s) \geq C - C_h$. We can compute blocking probability as follows

$$P_B = \sum_{\{s:v_0(s)\geq C-C_h\}} P(s) \tag{71}$$

B. Hand-off failure probability

The hand-off failure probability is the fraction of hand-off attempts that fail to capture a channel in their target cell. A hand-off attempt will fail if the number of occupied channels is equal to C. We can compute the hand-off failure probability as

$$P_H = \frac{\sum_{\{s:v_0(s)=C\}} \sum_{i=1}^{6} \frac{1}{6} \cdot v_i(s) \cdot m_D \cdot P(s)}{\sum_{s=0}^{s_{\max}} \sum_{i=1}^{6} \frac{1}{6} \cdot v_i(s) \cdot m_D \cdot P(s)} \tag{72}$$

The above is the hand-off failure probability for the general cluster model. For the two-state MMPP model we have the following

$$P_H = \frac{\sum_{\{s:v_0(s)=C\}} \sum_{i=1}^{6} \frac{1}{6} \cdot l_{a_i(s)} \cdot P(s)}{\sum_{s=0}^{s_{\max}} \sum_{i=1}^{6} \frac{1}{6} \cdot l_{a_i(s)} \cdot P(s)} \tag{73}$$

C. Forced termination Probability

The forced termination probability is defined as the probability that a call that initially gains access to the system is interrupted due to a hand-off failure during its lifetime. We denote the forced termination probability by P_{FT}. Let D be the probability that a active call will require a hand-off before its unencumbered session time expires.

Since both T and T_D are negative exponential random variables $D = m_D / (m_D + m)$. The probability that a call is force to terminate on its k^{th} hand-off attempt is denoted as $Y_k = D^k*(1-P_H)^{k-1}*P_H$. The forced termination probability is then given by

$$P_{FT} = \sum_{k=1}^{\infty} Y_k = \sum_{k=1}^{\infty} \frac{P_H}{(1-P_H)} \cdot [D(1-P_H)]^k = \frac{P_H D}{1-D(1-P_H)} \tag{74}$$

VII. Discussion of Results

Numerical results were obtained using the models described above. For all the reported results we set the mean unencumbered session duration to 100s ($\overline{T} = 100$) and the mean dwell time was set to 200s ($\overline{T_D} = 200$). Figure 3 is a plot of blocking probability versus new call arrival rate per platform (Λ). The number of channels per cell was set at 5 (C=5), and the number of noncommunicating platforms per cell was 50 ($n_0 = 50$). The probabilities where computed using the approximate models developed in this paper and by the "single isolated cell, Poisson hand-off arrival model" discussed in [1]. In addition, exact results for a seven-cell system were computed.

We see that over the range of arrival rates shown, the "single isolated cell, Poisson hand-off arrival model" and the "more general cluster model" yield nearly identical results. When the "2-state MMPP neighbor model" is used the blocking probability is seen to be slightly higher. The blocking probability for the seven-cell system is lower than the other three models over the entire range of arrival rates. This result is expected since the cells surrounding the center cell carry less traffic, due to the lack of hand-off arrivals. Therefore, they will tend to offer fewer hand-offs to the center cell.

Figure 4 is a plot of hand-off failure probability for the same parameter values given for figure 3. Again we see very close agreement between the "isolated cell" and the "isolated cluster" models. The "2-state MMPP neighbor model" produces a larger hand-off failure probability. As expected the hand-off failure probability is lowest when we consider a seven-cell system.

From figures 3 and 4 we notice that as the call demand increases the differences between the three approximate models decreases. This is because the MMPP arrival streams are beginning to behave like Poisson streams. As the call demand increases all the cells in the system will tend to have more channels in use. The MMPP from each of the neighboring cells will tend to remain in its highest (largest arrival rate offered to the center cell) state. In the limit, the MMPPs/neighbors will *always* be in a single state representing a heavily loaded neighbor. Thus the MMPP streams reduce to simple Poisson arrival streams. We also note that the results are reported for a relatively few channels. We would expect that there would be a greater difference in computed performance among the three models if the number of channels per cell were increases.

Figure 5 demonstrates the behavior of the two-state MMPP model and the simple Poisson model as the number of channels per cell increases. We have plotted the blocking probability versus the number of channels per cell. We define χ as the *offered traffic per channel per cell*. This is given by $c = {\Lambda \cdot n_0}/{m \cdot C}$. To compare the models as C increases we wish to keep χ fixed for all values of C. This is achieved by letting the new call arrival rate per platform vary according to

$$\Lambda = \frac{c \cdot m \cdot C}{n_0} \tag{74}$$

For the system performance shown in Figure 5 we have let $n_0 = 200$. We have also let C vary from 3 to 19 and χ vary from 0.07 to 0.9. As expected we see that for low new call offered traffic (χ) the two models produce significantly different blocking probabilities. However, for higher new call offered traffic the difference is less. We also see that as the number of channels per cell increases the difference between the two blocking probabilities also increases. Also note that for all values of χ the curves have a downward slope as C increases. We are observing the well-known queueing result that a large number of servers are more efficient than a small number for the same offered load per server.

Figure 6 shows the hand-off failure probability for the same system as Figure 5. We see that for a lightly loaded system the 2-state MMPP model produces higher hand-off failure probabilities.

VII. Conclusions

We have discussed two new approximate models to compute the performance characteristics of cellular communication systems. Both consider a cluster of cells, in which the hand-off arrival process to the center cell from a neighboring cell is modeled as an MMPP. The first model considered a cluster of seven cells and assumed that hand-off arrivals *to the cluster* followed a Poisson point process. The second model considered a cluster of seven cells where the neighbors of the center cell were modeled using 2-state MMPPs. The differences in performance characteristics generated via the various models are small.

The greatest differences occur at low loading. The MMPP models differ from the isolated cell, Poisson model, with the MMPP models and show higher probabilities of call blocking and forced termination. Results for the cluster model lie between the isolated cell model and the two-state MMPP model. Under heavy traffic loading the two models produce results that are even closer to the "single isolated cell, Poisson hand-off arrival model." In the range of most interest, P_B=10^{-2}-10^{-3}, the difference between the three models is small. These results provide additional evidence that the "single isolated cell, Poisson hand-off arrival model," [1], (which requires the fewest number of states) can be used with increased confidence.

VIII. References

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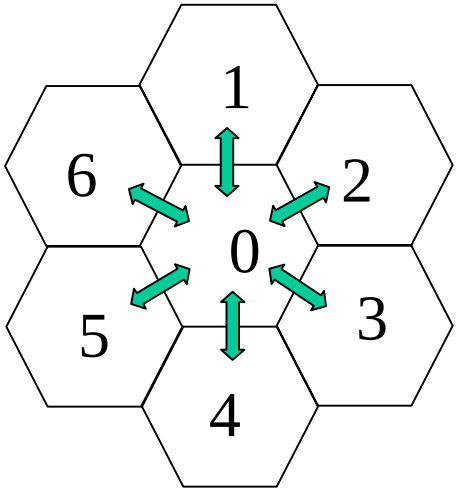

Figure 1: Cluster of seven cells.
Arrows represent hand-off departures and arrivals to center cell.

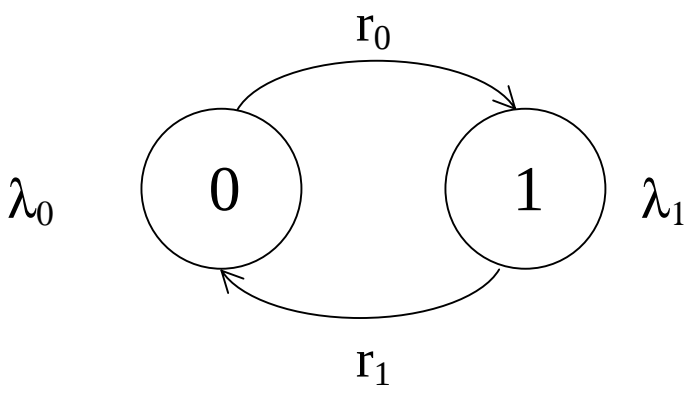

Figure 2: Two-state Markov modulated Poisson Process.

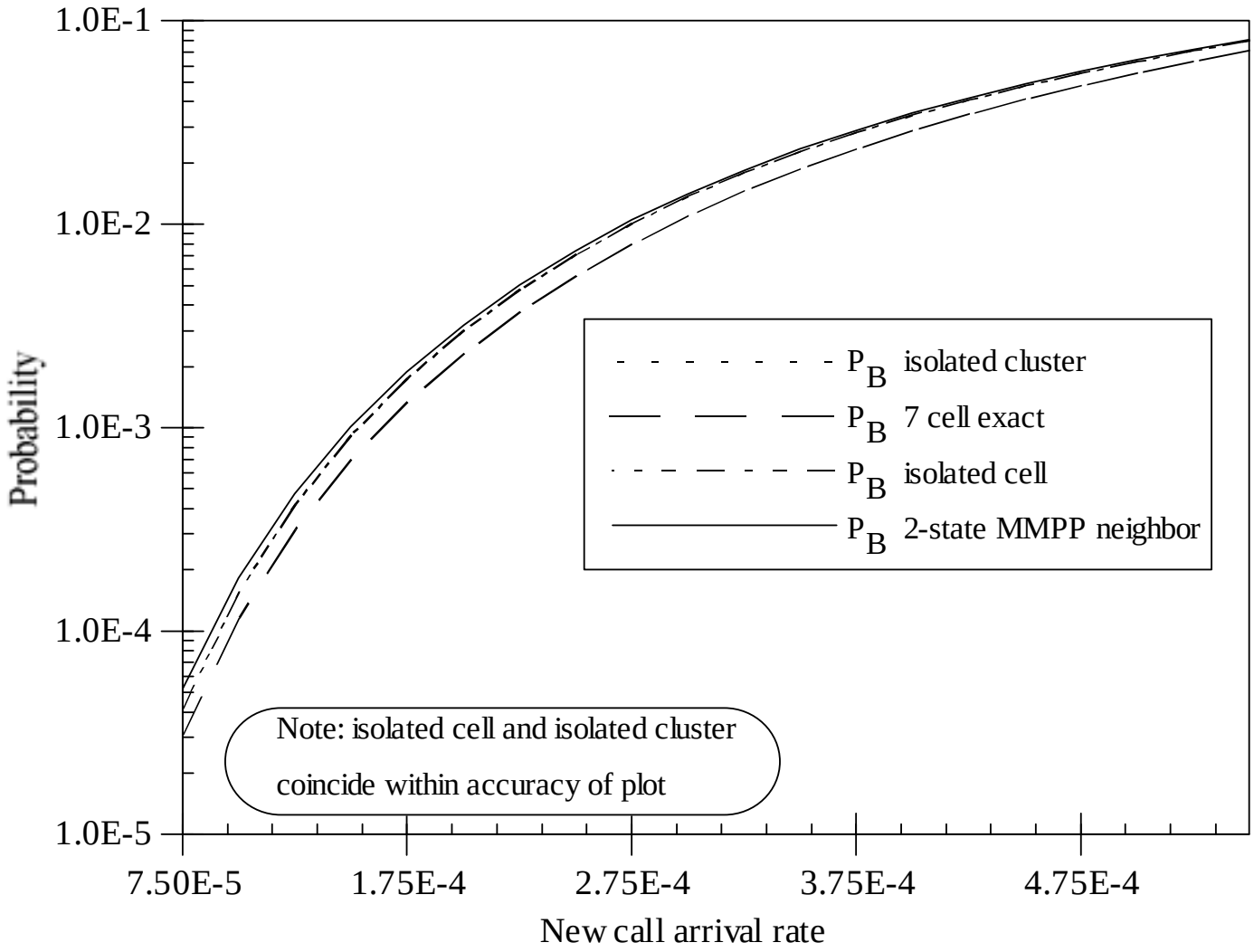

Figure 3: Blocking probabilities. $\overline{T} = 100s$, $\overline{T_D} = 100s$, $C = 5$, $\nu_0 = 50$.

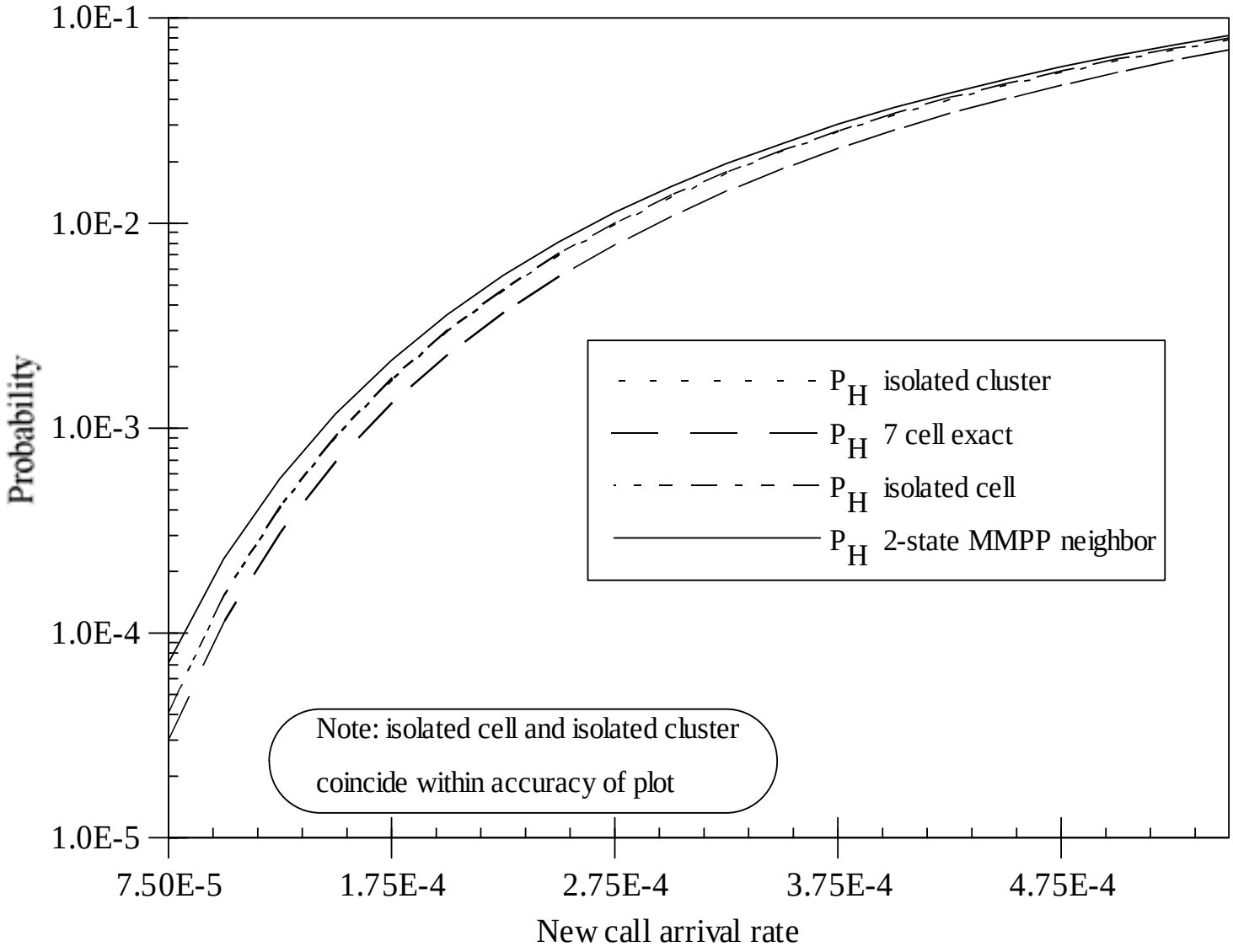

Figure 4: Hand-off failure probabilities. $\overline{T} = 100s$, $\overline{T_D} = 100s$, C = 5, ν_0 = 50.

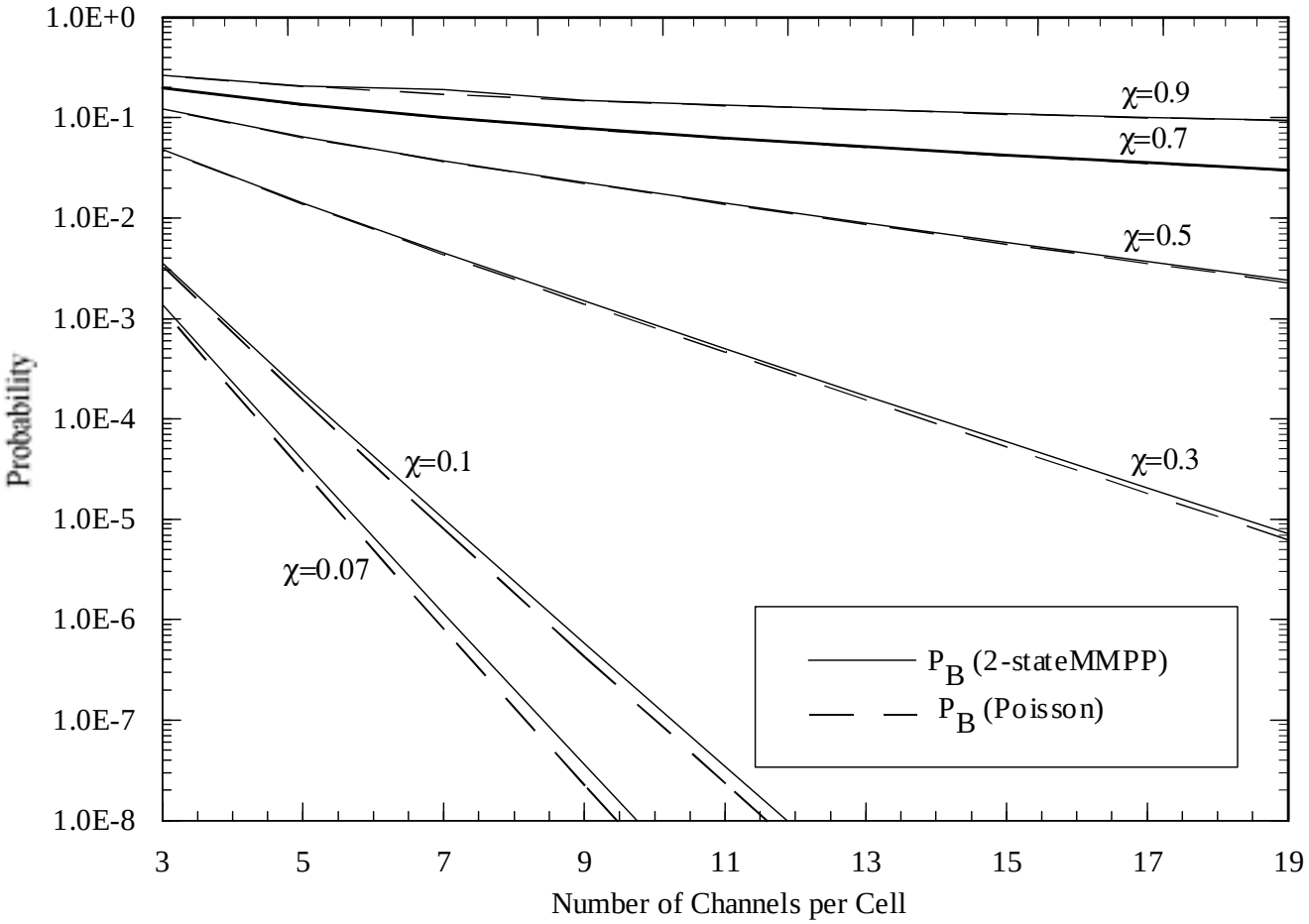

Figure 5: Blocking probabilities. $\overline{T} = 100s$, $\overline{T_D} = 100s$, ν_0 = 50.

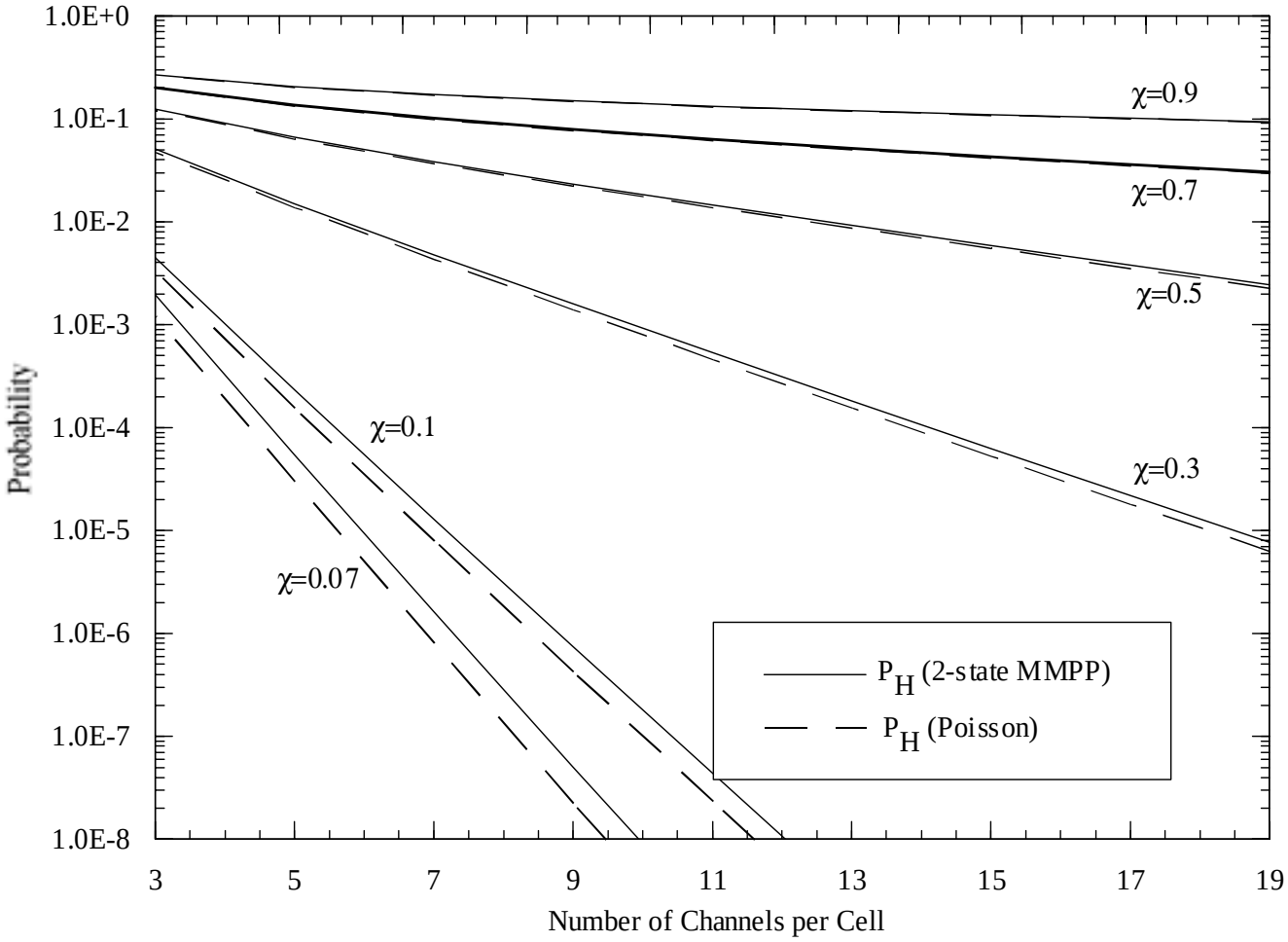

Figure 6: Hand-off failure probabilities. $\overline{T} = 100s, \overline{T_D} = 100s$, ν_0 = 50.