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Report No. 41

APPLICATION OF GENERALIZED FUNCTION THEORY TO NETWORK REALIZABILITY THEORY AND TIME DOMAIN SYNTHESIS OF POSITIVE-REAL FUNCTIONS

by

A. H. Zemanian

Contract No. AF 19(628)-2981
Project No. 5628
Task No. 562806

FINAL REPORT

May 31, 1965
Period covered: June 1, 1963 to May 31, 1965

Prepared for

Air Force Cambridge Research Laboratories
Office of Aerospace Research
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UNITED STATES AIR FORCE

BEDFORD, MASSACHUSETTS

ABSTRACT

This is a summary of the work performed under contract AF 19(628)-2981 during the period from June 1, 1963 to May 31, 1965. All significant results have previously been reported in scientific reports and journal publications. The present report summarizes and compares these results.

List of Contributors to the Research Herein

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List of Previous Publications Produced Under

Contract AF19(629)-2981

Scientific Reports:

1. A. H. Zemanian, "A Time-Domain Characterization of Rational Positive-Real Matrices." First Scientific Report, AFCRL-63-390, College of Engineering Tech. Rep. 12, State University of New York at Stony Brook; August 5, 1963.

2. A. H. Zemanian, "A Time-Domain Characterization of Positive-Real Matrices." Second Scientific Report, AFCRL-63-391, College of Engineering Tech. Rep. 13, State University of New York at Stony Brook; August 16, 1963.

3. A. H. Zemanian, "The Time-Domain Synthesis of Distributions." Third Scientific Report, AFCRL-64-191, College of Engineering Tech. Rep. 19, State University of New York at Stony Brook; February 1, 1964.

4. A. H. Zemanian, "The Distributional Laplace and Mellin Transformations." Fourth Scientific Report, AFCRL-64-685, College of Engineering Tech. Rep. 26, State University of New York at Stony Brook; August 15, 1964.

5. A. H. Zemanian, "Orthonormal Series Expansion of Certain Distributions and Distributional Transform Calculus." Fifth Scientific Report, AFCRL-64-995, College of Engineering Tech. Rep. 22, State University of New York at Stony Brook; November 15, 1964.

Papers:

1. A. H. Zemanian, "The Time-Domain Synthesis of Distributions." Proceedings of the First Allerton Conference on Circuit Theory, University of Illinois; 1963.

2. A. H. Zemanian, "The Approximation of Distributions by the Impulse Responses of RLC Two-ports." Proceedings of the International Conference of Microwaves, Circuit Theory, and Information Theory; Tokyo; September, 1964.

3. A. H. Zemanian, "The Time-Domain Synthesis of Distributions." IEEE Transactions on Circuit Theory, Vol. CT-11, pp 487-493; December, 1964.

4. A. H. Zemanian, "A Characterization of the Inverse Laplace Transforms of Rational Positive-Real Functions." J. Soc. Indust. Appl. Math., accepted for publication.

5. A. H. Zemanian, "Some Convergence Properties of Exponential Series Expansions of Distributions." J. Soc. Math. Anal. Appl., accepted for publication.

6. H. Konig and A. H. Zemanian, "Necessary and Sufficient Conditions for a Matrix Distribution to Have a Positive-Real Laplace Transform." J. Soc. Indust. Appl. Math., accepted for publication.

7. A. H. Zemanian, "The Distributional Laplace and Mellin Transformations." J. Soc. Indust. Appl. Math., accepted for publication.

8. A. H. Zemanian, "Inversion Formulas for the Distributional Laplace Transformation." J. Soc. Indust. Appl. Math., accepted for publication.

A Summary of Research Results:

This is a report on the research results obtained during the period of June 1, 1963 to May 31, 1965 under the contract AF 19(628)-2981, sponsored by the Air Force Cambridge Research Laboratories, Office of Aerospace Research, United States Air Force, Bedford Massachusetts. This contract has been extended to the period of June 1, 1965 to May 31, 1967. All significant results obtained so far have been reported in the Scientific Reports and research papers listed in the preceding section. Here we shall summarize and correlate these results.

Our research efforts can be classified under the following three categories.

1. The time-domain significance of positive-reality.
2. The time-domain synthesis of distributions.
3. The distributional generalization of the Laplace and Mellin transformation.

We shall discuss each of these separately.

1. The Time-Domain Significance of Positive-Reality.

A necessary and sufficient time-domain characterization of a positive-real function or positive-real matrix has been obtained. Scientific Report (1) presents the development for the special case where the positive-real function or matrix is rational. It also appears in the Journal of the Society for Industrial and Applied Mathematics as paper (4). Scientific Report (2) gives the development in the most general case where there are no restrictions on the positive-real function or matrix. In the latter case our main conclusions can be stated as follows:

Theorem: Let $w(t)$ be an $n \times n$ matrix distribution and let $W(s)$ be its Laplace transform. The necessary and sufficient conditions for $W(s)$ to be positive-real are the following:

(i) $w(t) = A\delta^{(1)}(t) + w_0(t)$ where A is a real symmetric nonnegative-definite constant matrix, $\delta^{(1)}(t)$ is the first derivative of the Dirac delta functional, and $w_0(t) = d^2 g/dt^2$ where the elements of the matrix are continuous functions whose supports are contained in $0 \le t < \infty$.

(ii) For every $n \times 1$ constant vector y, $y^* [w(t) + w^T(-t)] y$ is a nonnegative-definite distribution, where w^T is the transpose of w, and y^* = complex-conjugate of y^T.

It is expected that this most general result will also appear in the Journal of the Society for Industrial and Applied Mathematics.

Actually, the nonnegative definiteness of condition (ii) is a reflection of the passivity of a system whose impulse response matrix is $w(t)$. Thus, this gives a concise and yet completely general link between passivity and positive-reality.

2. The Time-Domain Synthesis of Distributions.

In this area we have developed for the first time a method for the time-domain synthesis of a distribution. This method, which is based upon a Fourier series technique, is described in Scientific Report (3). Various aspects of it are published in Papers (1) - (3), (5). An outstanding feature of this technique is that, if the Laplace transform of a given distribution is known, a realizable approximating signal can be written down without any computation; it only requires values of the Laplace transform at various points in its region of convergence. Since ordinary locally integrable functions are special cases of distributions, the technique is significant for the customary time-domain synthesis problem as well.

More recently, our investigations in this area have developed along a somewhat different line. In particular, we have generalized the Lee-Kautz-Huggins time-domain synthesis procedure which uses an orthonormal series expansion technique. Our generalization now allows one to synthesize a distributional signal in the time-domain, whereas the Lee-Kautz-Huggins procedure was previously retricted to ordinary functions. In order to accomplish this, we had to develop a method for expanding distributions into orthonormal series, something that apparently had not as yet been done in mathematics. This mathematically new result is described in Scientific Report (5). At the present time a paper on the generalization of the Lee-Kautz-Huggins procedure is being prepared for submission to the IEEE Transactions on Circuit Theory.

3. The Distributional Generalization of the Laplace and Mellin Transformation.

Another problem we have undertaken to solve is that of analyzing in a distributional way the Euler-Cauchy type of time-varying networks. These are networks that are characterized by differential equations of the form

$$a_n t^n D^n y + a_{n-1} t^{n-1} D^{n-1} y + \dots + a_o y = f(t) \qquad (t>o)$$

where $D = d/dt$. Physically, this means that the resistors of the circuit remain constant whereas the inductors and capacitors vary linearly with time. A natural way of analyzing such net-

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works is by using the Mellin transformation. Thus, our first problem was to generalize this transformation so that it could be used for distributions. This was accomplished and reported in Scientific Report (4). It is expected that a condensed version of this report will appear in the Journal of the Society for Industrial and Applied Mathematics. We are now in the process of applying this transformation to the distributional analysis of Euler-Cauchy networks.

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13. ABSTRACT

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