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DOES A NETWORK OF MONOTONE RESISTORS HAVE AN OPERATING POINT?

A. H. Zemanian

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DOES A NETWORK OF MONOTONE RESISTORS HAVE AN OPERATING POINT? *

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Abstract — This work is a sequel to a prior work that established a procedure for determining whether or not a series-parallel network of continuous, strictly monotonically increasing resistors with restricted domains and ranges has an operating point. Herein we accomplish the same thing but this time for networks with arbitrary graphs; that is, the series-parallel restriction is now removed. This substantially expands the applicability of our result. Moreover, our procedure is highly efficient for the following reasons: We characterize each resistor by only four numbers ($-\infty$ and $+\infty$ allowed) determined by the asymptotes to which the resistor's characteristic curve approaches; it is only these numbers that are added or subtracted to test for the existence of an operating point. Furthermore, the numerical complexity of our procedure increases only linearly with the number of branches in the network. All this is accomplished without a computation of the operating point, which can be an onerous task that will perforce end in failure if the operating point does not exist.

1 Introduction

This is the third in a series of papers concerning a fast test for the existence and ratings compliance of the operating point of a network of monotone resistors. The first paper [4] was restricted to series-parallel networks; their resistor characteristics were taken to be continuous, strictly monotonically increasing functions of voltage or current, but their domains and ranges were allowed to be proper subintervals of the real line. Such restrictions on domains and ranges arise commonly in models of electronic devices and one-ports, as for example the Zener diode, but it leads to the difficulty that the network may not have

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any operating point — may indeed be senseless. A very rapid computational method was presented in [4] for ascertaining whether an operating point exists without actually attempting a computation of that operating point. The latter computation, if successful, is generally expensive and prolonged, as is commonly the case with Newton-Raphson, secant, or homotopy methods. The method of [4] accomplished its objective by using only the asymptotes of the characteristic curves. A determination of the nonexistence of an operating point could be followed by a reexamination or abandonment of the proposed design without wasting effort in trying to find the operating point.

The second paper [5] was also restricted to series-parallel networks with resistors of the type stated; it resolved the following question: In the event that an operating point exists, do all resistors operate within their ratings? In this case a manipulation of only the asymptotes of the characteristic curves cannot provide a solution. Instead each resistor was now represented by that segment of its characteristic curve lying between its two ratings points, and these segments were manipulated using some simplifications provided by the series-parallel structure to answer the second question quite rapidly. If ratings were found to be violated, a reexamination of the design would be mandated.

This present third paper is a sequel to the first one and assumes the same kind of resistors. It questions the existence of an operating point but this time for networks with arbitrary graphs. Graphs that are not of the series-parallel type have wheatstone bridges as subgraphs [2], and thus the series and parallel sums introduced in [4] are inadequate for examining such networks entirely. However, a fundamental result established in 1960 by Minty [3] is available for this purpose. Our procedure for a network having at least one embedded wheatstone bridge is first to reduce the network as much as possible by making as many series and parallel combinations as possible and then to exploit Minty's theorem for the reduced network. Once again, we can restrict our computations to asymptotes, and thus a very rapid determination of the existence or nonexistence of an operating point is achieved. In fact, the computational complexity of our method increases only linearly with the number of branches in the network.

The next natural question for the case of arbitrary networks is whether all resistors operate within their ratings when an operating point exists. Unfortunately, Minty's theorem cannot be used for finite segments of characteristic curves. How to resolve this last question rapidly without computing the operating point is presently an open problem.

2 Preliminary Considerations

In this paper the following is always assumed.

Conditions 2.1: **N** will denote a connected nonseparable network consisting of at least three nonlinear resistors whose voltage-current characteristics are continuous, strictly monotonically increasing curves that do not terminate at any finite point of the voltage-current plane (i.e., every characteristic curve extends infinitely in both directions). Moreover, every branch has an orientation with respect to which the positive values of voltage (drop) and current (flow) are measured.

We take the voltage axis as horizontal and the current axis as vertical and denote them as v-axis and i-axis respectively; the voltage-current plane is then the (v, i)-plane. Let W denote the characteristic curve of a particular resistor, and let V and I denote the projections of W upon the v-axis and i-axis respectively. Under Conditions 2.1, V and I are nonempty open intervals, which may be either finite, one-way infinite, or two-way infinite. We refer to the Cartesian product $V \times I$ as the *characteristic set* of the resistor or of W. If $\sup V < \infty$, then W is asymptotic to the vertical line $v = \sup V$. Similarly, W is asymptotic to $\inf V$ or $\sup I$ or $\inf I$ whenever any of the latter are finite. Moreover, $V \times I$ can be classified according to the nine types listed in [4, Section II] and illustrated herein in Fig. 1. Furthermore, if W passes through the origin of the (v, i)-plane, then the resistor or W itself is said to be ***passive***. If W does not pass through the origin of the (v, i)-plane, then the resistor implicitly contains a nonzero source, which can be represented either as a voltage source in series with a passive resistor or as a current source in parallel with a passive resistor.

With B denoting the number of branches in **N**, let us number the branches with the subscripts $j = 1, \ldots, B$. The same subscripts will be used for any entities belonging to a branch b_j, such as its voltage v_j, current i_j, or characteristic curve W_j. With $\mathbf{v} =$

$(v_1, \ldots, v_B)$ being the branch-voltage vector and $\mathbf{i} = (i_1, \ldots, i_B)$ being the branch-current vector, an *operating point* is a pair of vectors $(\mathbf{v}, \mathbf{i})$ such that $\mathbf{v}$ satisfies Kirchhoff's voltage law around every loop, $\mathbf{i}$ satisfies Kirchhoff's current law on every cutset, and $(v_j, i_j) \in W_j$ for every j. By exactly the same proof[1] as that for [4, Lemma 2.2], we have the following.

Theorem 2.2: Under Conditions 2.1, the network $\mathbf{N}$ has either a unique operating point or none at all.

Let V_j and I_j denote the projections of W_j on the v-axis and i-axis respectively for each branch b_j. V_j and I_j are open intervals, possibly proper subintervals of the real line. Also, Let L denote an oriented loop. With the usual definitions for sums and differences of intervals, we can assign to L the interval $\sum_{(L)} \pm V_j$, where $\sum_{(L)}$ denotes a summation over all the indices j for the branches b_j in L, the $+$ sign is used if the orientations of branch b_j and loop l agree, and the $-$ sign is used if those orientations disagree. Since each V_j is an open interval, so too is $\sum_{(L)} \pm V_j$. If the interval $\sum_{(L)} \pm V_j$ contains the point zero, then L is called *balanced*; otherwise, L is called *unbalanced*. Similarly, let C denote an oriented cutset. In this case, we assign to C the interval $\sum_{(C)} \pm I_j$, where $\sum_{(C)}$ denotes a summation over all the indices j of the branches b_j in C, the $+$ sign is used if the orientations of branch b_j and cutset C agree, and the $-$ sign is used if those orientations disagree. Here too, $\sum_{(C)} \pm I_j$ is an open interval. If the interval $\sum_{(C)} \pm I_j$ contains the point zero, then C is called *balanced*; otherwise, *unbalanced*. A result[2] due to Minty [3] can now be combined with Theorem 2.2 to obtain the following.

Theorem 2.3: Under Conditions 2.1, the network $\mathbf{N}$ has a unique operating point if and only if all the loops and all the cutsets of $\mathbf{N}$ are balanced.

With regard to series and parallel combinations of monotone resistors, we will use the same definition and results as those of [4] without repeating all of them here. Thus, $W_1 \diamond W_2$ denotes the series sum of two characteristic curves W_1 and W_2, and $W_1 \square W_2$ denotes their parallel sum. Furthermore, instead of manipulating an entire characteristic curve W, we shall manipulate only its asymptotes. As was mentioned before, the shapes of the charac-

[1] The proof is based upon a classical theorem due to Duffin [1]; unlike the present paper, Duffin's theorem requires that every characteristic curve define a bijection between the v-axis and i-axis.

[2] In particular, see Definition 4.2 on page 198 and the Corollary on page 210 of [3].

teristic curves having the properties stated in Conditions 2.1 can be classified into exactly nine types according to their characteristic sets[3] as shown in Fig. 1. We assign to every characteristic set S four asymptotes[4]: A_l, A_r, A_d, and A_u; at most two of them are finite with the others being either $-\infty$ or $+\infty$. We use the convention that, if W (or, equivalently, if its characteristic set) extends infinitely in a horizontal or vertical direction, then the asymptote in that direction is either $-\infty$ or $+\infty$. Thus, for each type of characteristic set shown in Fig. 1, the asymptotes take on finite or infinite values as listed in Table 1.

Finally, a path or loop embedded in **N** (and their branches as well) will be called *conformably oriented* if the orientations of all of the branches in the loop or path agree with some orientation of the loop or path. Similarly, a cutset or a parallel circuit embedded in **N** (and their branches too) will be called *conformably oriented* if all the branches therein have orientations pointing in the same direction from one side of the cutset or parallel circuit to the other side. For series and parallel circuits, conformable orientations are the same as the confluent orientations discussed in[4, page 48].

3 Fundamental Loops and Cutsets

Because of our assumptions that **N** is connected, nonseparable, and has at least three branches, the total number X of loops and cutsets in **N** is larger than the number of fundamental loops and fundamental cutsets for a given spanning tree **T** in **N**. Indeed, the latter number is simply the number B of branches in **N**, whereas the former number X is always larger than B. Moreover, X is usually considerably larger than B. For example, for the wheatstone bridge, $B = 6$ and $X = 14$. So, it will be advantageous to test only the fundamental loops and cutsets instead of all the loops and cutsets.

The following two lemmas justify doing so. We assume henceforth that a spanning tree **T** has been chosen arbitrarily and then fixed. Number the chords from 1 to $B - N$, where N is one less than the number of nodes in **N**, and number the tree branches from $B - N + 1$ to B. Also, number and orient each fundamental loop (resp. fundamental cutset) in accordance with the number and orientation of the unique chord (resp. tree branch) in

[3] Characteristic sets are defined in [4, page 46]

[4] l for "left," r for "right," d for "down," and u for "up."

that fundamental loop (resp. fundamental cutset). Let L be any oriented loop in $\mathbf{N}$ other than a fundamental loop. We will denote each loop by the algebraic sum of branches in that loop with a + or − sign attached to each branch in the loop in accordance with the conformity or nonconformity of the orientations of the loop and branch. Then, it is a fact that

$$L = \sum_{(L)}^{(c)} \pm L_j \tag{1}$$

Here, $\sum_{(L)}^{(c)}$ is a summation over the indices of the chords in L and a + (resp. −) sign is attached to the fundamental loop L_j for each chord in L if the orientations of L and the chord for L_j agree (resp. disagree). Indeed, adding or subtracting all the branches in those L_j in accordance with the signs resulting from the signs attached to the L_j and the signs assigned to the branches for the L_j, we obtain the correct representation $L = \sum_{(L)} \pm b_j$ for L in terms of its branches, where $\sum_{(L)}$ is a summation over all the indices of the branches in L. (Branches not in L cancel out.) In short, we can rewrite (1) as

$$L = \sum_{(L)}^{(c)} \pm \sum_{(L_j)} \pm b_k. \tag{2}$$

($\sum_{(L)}^{(c)}$ is a summation over the indices j for the chords in L, and $\sum_{(L_j)}$ is a summation over the indices k of all the branches in L_j.) This expression remains valid no matter what orientations are assigned to the branches and the loop L.

Now, let v_j be the branch voltage for b_j. Then, by (2), the algebraic sum of voltages around L is

$$\sum_{(L)} \pm v_j = \sum_{(L)}^{(c)} \pm \sum_{(L_j)} \pm v_k.$$

Next, replace each v_k by the voltage interval V_k assigned to b_k. We get the following voltage interval for L.

$$\sum_{(L)} \pm V_j = \sum_{(L)}^{(c)} \pm \sum_{(L_j)} \pm V_k. \tag{3}$$

Now, if the open interval $\sum_{(L_j)} \pm V_k$ for every fundamental loop L_j in the right-hand side of (3) contains the point zero, then so too does $\sum_{(L)} \pm V_j$. We have established the following

Lemma 3.1: If the algebraic sum $\sum_{(L_f)} \pm V_j$ of voltage intervals around every fundamental loop L_f contains the point zero, then the algebraic sum $\sum_{(L)} \pm V_j$ of voltage intervals around every loop L also contains the point zero.

Now, when examining any given fundamental loop L_f to see if its voltage interval contains zero (i.e., to see if it is balanced), we are free to choose the branch and loop orientations to suit ourselves. In particular, assign any orientation to L_f and then reorient (if need be) each branch b_j in L_f to make L_f conformably oriented. Let $\check{V}_j$ be the voltage interval of the so-oriented b_j. (This will require replacing V_j by $-V_j = \check{V}_j$ if the original orientation of b_j disagrees with that of L_f; in this case, $\inf \check{V}_j = \inf(-V_j) = -\sup V_j$ and $\sup \check{V}_j = \sup(-V_j) = -\inf V_j$.) Thus, L_f will be balanced if and only if $\sum_{(L_f)} \check{V}_j$ contains zero. Since every $\check{V}_j$ is an open interval, this will be so if and only if $\inf \sum_{(L_f)} \check{V}_j < 0$ and $\sup \sum_{(L_f)} \check{V}_j > 0$. Moreover, $\inf \sum_{(L_f)} \check{V}_j$ (resp. $\sup \sum_{(L_f)} \check{V}_j$) is obtained simply by adding the left-hand (resp. right-hand) asymptotes of the characteristic curves $\check{W}_j$ for the (conformably oriented) branches in L_f. We take the left-hand (resp. right-hand) asymptote of $\check{W}_j$ to be at $-\infty$ (resp. $+\infty$) if $\check{W}_j$ extends infinitely toward the left (resp. right).

Cutsets can be treated similarly. Again we choose and fix a spanning tree $\mathbf{T}$ and number the oriented branches as before. This assigns numbers and orientations to the fundamental cutsets as before. Now, let C be any oriented cutset in $\mathbf{N}$ other than a fundamental cutset. This time we have

$$C = \sum_{(C)}^{(t)} \pm C_j \tag{4}$$

where $\sum_{(C)}^{(t)}$ is a summation over the indices of the tree branches in C, a + or − sign is used in accordance with the conformity or nonconformity of the orientations of C and the tree branch for the fundamental cutset C_j, and the summation is interpreted as additions and subtractions of branches as before. Thus, by writing each C_j as an algebraic sum over the branches in C_j, (4) becomes

$$C = \sum_{(C)}^{(t)} \pm \sum_{(C_j)} \pm b_k, \tag{5}$$

no matter what orientations are chosen for the branches and for C. ($\sum_{(C)}^{(t)}$ is a summation over the indices j for the tree branches in C, and $\sum_{(C_j)}$ is a summation over the indices k

of all the branches in C_j.) With i_j being the branch current for b_j, the algebraic sum of the currents in C is

$$\sum_{(C)} \pm i_j = \sum_{(C)}^{(t)} \pm \sum_{(C_j)} \pm i_k.$$

Then, with i_k replaced by the current interval I_k for b_k, we get the following current interval for C.

$$\sum_{(C)} \pm I_j = \sum_{(C)}^{(t)} \pm \sum_{(C_j)} \pm I_k.$$

Again, if each open interval $\sum_{(C_j)} \pm I_k$ for the fundamental cutset C_j contains the point zero, then so too does $\sum_C \pm I_j$. From this, we get the following result.

Lemma 3.2: If the algebraic sum $\sum_{(C_f)} \pm I_j$ of current intervals for every fundamental cutset C_f contains the point zero, then the algebraic sum $\sum_C \pm I_j$ of current intervals for every cutset contains the point zero too.

Here too, we can check the fulfillment of the hypothesis of Lemma 3.2 simply as follows. When examining a particular fundamental cutset C_f, assign any orientation to C_f and orient all the branches b_j in C_f conformably. Under these possibly new orientations for the branches, let $\check{I}_j$ be the current interval for b_j. (If the orientation of a particular branch b_j becomes reversed, we have $\inf \check{I}_j = \inf(-I_j) = -\sup I_j$ and $\sup \check{I}_j = \sup(-I_j) = -\inf I_j$.) So, we need only check whether $\sum_{(C_f)} \check{I}_j$ contains zero to see if C_f is balanced, and this will truly be the case if the sum of the lower (resp. upper) asymptotes of the characteristic curves $\check{W}_j$ for the (conformably oriented) branches in C_f is less (resp. greater) than zero. As before, we take the lower (resp. upper) asymptotes of $\check{W}_j$ to be at $-\infty$ (resp. $+\infty$) if $\check{W}_j$ extends infinitely downward (resp. upward).

Combining all this with Theorem 2.2, we have the following result, given any spanning tree $\mathbf{T}$.

Theorem 3.3: The network $\mathbf{N}$ will have a unique operating point if and only if both of the following conditions are fulfilled.

(a) For every conformably oriented fundamental loop L_f, the sum of the left-hand (resp. right-hand) asymptotes of the characteristic curves $\check{W}_j$ for all the branches b_j in L_f is negative or $-\infty$ (resp. positive or $+\infty$).

(b) For every conformably oriented fundamental cutset C_f, the sum of the lower (resp. upper) asymptotes of the characteristic curves $\check{W}_j$ for all the branches b_j in C_f is negative or $-\infty$ (resp. positive or $+\infty$).

4 Series-Parallel Reductions

We can simplify our test for the existence of an operating point still further by first making as many series and parallel reductions of **N** as possible before checking the fundamental loops and cutsets. This is because checking the existence of a series or parallel sum requires the examination of just two branches [4], but checking the balancing of a fundamental loop or fundamental cutset requires the examination of, in general, many more branches. However, to justify this initial reduction of **N** through series and parallel combinations, we should check that the reduced network has an operating point if and only if the original network has one and that, if an operating point exists, then every branch in the reduced network operates with the same voltage and current as it does in the original network.

Consider any maximal series-parallel subnetwork N_{sp} of N. This is a one-port in that N_{sp} meets its complement in **N** at exactly two nodes, say, n_1 and n_2. Thus, the input resistance of $\mathbf{N}_{sp}$ can be viewed as the resistance of a branch in the reduced network. If any series or parallel sum for two branches in $\mathbf{N}_{sp}$ fails to exist, then $\mathbf{N}_{sp}$ has no sense, and therefore neither does **N** have an operating point. Conversely, if **N** has an operating point, then by definition every branch in $\mathbf{N}_{sp}$ has a voltage and current, and so too does the said one-port. The voltage and current at the terminals n_1 and n_2 of the one-port uniquely determine voltages and currents at all the branches within $\mathbf{N}_{sp}$. This is because any point (v, i) on the series sum $W_a \Diamond W_b$ for the characteristic curves of two branches b_a and b_b in series uniquely determines the voltage-current pairs $(v_a, i) \in W_a$ and $(v_b, i) \in W_b$ for those branches. Similarly, the point (v, i) on the parallel sum $W_a \Box W_b$ uniquely determines $(v, i_a) \in W_a$ and $(v, i_b) \in W_b$. Thus, given the voltage and current at the terminals n_1 and n_2, we can sequentially solve for all the branch voltages and currents in $\mathbf{N}_{sp}$. Conversely, those voltages and currents within $\mathbf{N}_{sp}$ uniquely determine the voltage and current at n_1 and n_2 through series and parallel sums. Thus, series and parallel combinations within $\mathbf{N}_{sp}$

do not disturb the terminal conditions at n_1 and n_2, and Kirchhoff's laws determine the undisturbed conditions on all the branches outside $\mathbf{N}_{sp}$. It follows that an operating point for the reduced network agrees with the operating point for the original network.

5 The Test

The procedure we shall use to test for the existence of an operating point for the network N satisfying Conditions 2.1 is first to reduce N as much as possible through a sequence of series and parallel combinations of resistors, checking for the success or failure of the series or parallel sum at each step of the sequence, and then to check the balancing of the fundamental loops and fundamental cutsets of the reduced network. Here then is the procedure stated as an algorithm. It is assumed that every branch of N has been assigned an initial orientation. Steps 1 through 4(c) repeat Procedure 8.1 of [4] and are justified as in that paper. Furthermore, only summations of asymptotes are involved in all the numerical steps of Procedure 5.1 (in particular, at Steps 3, 4(c), 7, and 9).

Procedure 5.1.

1) Search N for a series or parallel connection of two branches. If none exist, go to Step 5 below.

2) If the two branches are not conformably oriented, reverse the orientation and characteristic set of one of them.

3) Reduce the network by combining the two branches through a series sum or parallel sum of their characteristic set to obtain a single oriented branch with a determined characteristic set.

4) a) If that characteristic set is empty, stop. (An operating point does not exist.)

b) If that characteristic set is not empty and if the reduced network has at least three branches, return to Step 1.

c) If the reduced network has exactly two branches[5] with, say, the characteristic sets S_0 and S_1, change the orientation of one of them if need be to obtain the relative

[5]This implies that N is a series-parallel network

orientations shown in Fig. 2, and then determine whether $\hat{S}_0 \cap S_1$ is empty[6]. Then, stop. (**N** has an operating point if and only if $\hat{S}_0 \cap S_1$ is nonempty.)

5) Choose and fix a spanning tree in the reduced network $\mathbf{N}_r$. (Every branch of $\mathbf{N}_r$ now has a characteristic set and thereby four asymptotes as indicated in Table 1.)

6) Choose a chord different from all previously chosen chords, and thereby a fundamental loop L_f. If need be, change the orientations of some of the branches in L_f to make L_f conformably oriented.

7) Sum the left-hand asymptotes A_l for the (conformably oriented) branches in L_f and then sum their right-hand symptotes A_r to see if Condition (a) of Theorem 3.3 is fulfilled. If it is not fulfilled, stop. (An operating point does not exist.) If it is fulfilled and if some unchosen chords remain, return to Step 6. Otherwise, go to Step 8.

8) Choose a tree branch different from all previously chosen tree branches, and thereby a fundamental cutset C_f. If need be, change the orientations of some of the branches in C_f to make C_f conformably oriented.

9) Sum the downward asymptotes A_d for the (conformably oriented) branches of C_f and then sum their upward asymptotes A_u to see if Condition (b) of Theorem 3.3 is fulfilled. If it is not fulfilled, stop. (An operating point does not exist.) If it is fulfilled and if some unchosen tree branches remain, return to Step 8. Otherwise, go to Step 10.

10) Stop.

The next theorem states the main result of this paper. As always, **N** is assumed to satisfy Conditions 2.1.

Theorem 5.2: Apply Procedure 5.1 to the network **N**. If that procedure stops at Step 4(a) or Step 7 or Step 9, then **N** does not have an operating point. If that procedure stops at Step 4(c), then **N** has an operating point (and is a meaningful series-parallel network) if

[6] As was defined in [4, page 51], $\hat{S}_0 = \{(v, i) : (-v, i) \in S_0\}$

and only if $\hat{S}_0 \cap S_1$ is not empty. If that procedure reaches Step 10, then **N** has an operating point (and is not a series-parallel network). The operating point is unique if it exists.

The next corollary bounds the computational complexity of Procedure 5.1.

Corollary 5.3. The number of times Steps 3, 4(c), 7, and 9 are executed altogether is no larger than the number of branches in **N**.

6 Some Final Remarks

The conclusions of this paper are stated succinctly by Theorem 5.2 and Corollary 5.3. The numerical steps of Procedure 5.1 occur at Steps 3, 4(c), 7 and 9 and involve only summations of asymptotes, with just two summations of asymptotes occurring at each step (such a summation is trivial if any asymptote is $-\infty$ or $+\infty$). Thus, the numerical complexity of our test for the existence of an operating point increases only linearly with the number of branches and in this way is very efficient. In fact, if an operating point does not exist, this fact may be revealed well before the entire Procedure 5.1 is executed. Of course, this efficiency is achieved by sacrificing the determination of the operating point. However, it is well worth applying Procedure 5.1 before attempting to compute the operating point by the more computationally onerous standard methods for doing so. There is no point in undertaking such a computation if there is no operating point.

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TABLE 1

The Finite and Infinite Values for the Four Asymptotes of the Nine Types of Characteristic Curves Shown in Fig. 1.

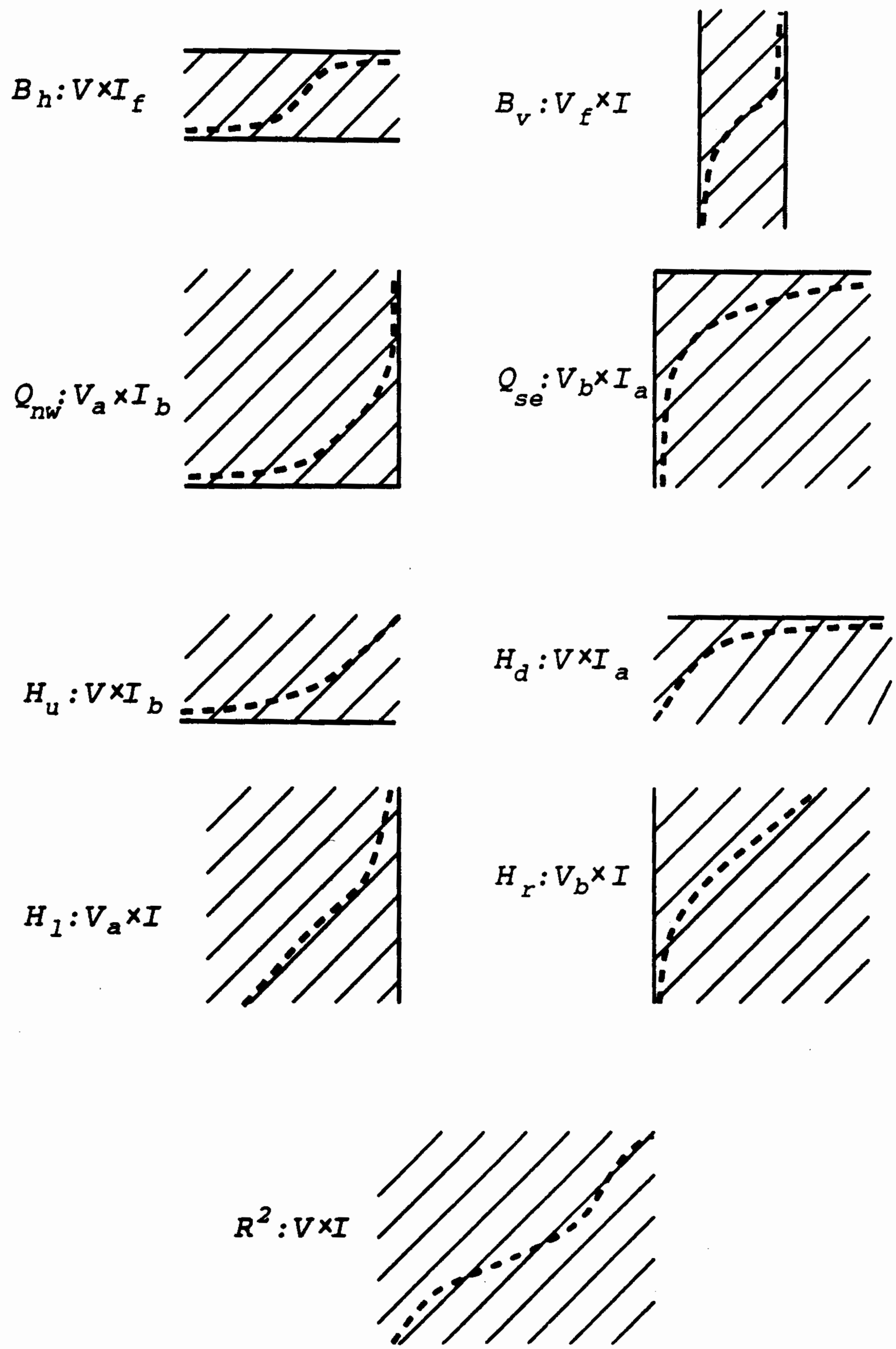
(In every case, $A_l < A_r$ and $A_d < A_u$.)

Characteristic Set	Symbol	A_l	A_r	A_d	A_u
Horizontal band	B_h	$-\infty$	$+\infty$	finite	finite
Vertical band	B_v	finite	finite	$-\infty$	$+\infty$
Northwest quadrant	Q_{nw}	$-\infty$	finite	finite	$+\infty$
Southeast quadrant	Q_{se}	finite	$+\infty$	$-\infty$	finite
Half-plane upward	H_u	$-\infty$	$+\infty$	finite	$+\infty$
Half-plane downward	H_d	$-\infty$	$+\infty$	$-\infty$	finite
Half-plane leftward	H_l	$-\infty$	finite	$-\infty$	$+\infty$
Half-plane rightward	H_r	finite	$+\infty$	$-\infty$	$+\infty$
The whole plane	R^2	$-\infty$	$+\infty$	$-\infty$	$+\infty$

Figure Captions

Fig. 1. The nine types of characteristic sets (shown crosshatched) and possible characteristic curves (shown dotted) within them.

Fig. 2. The relative orientations for the two-branch network of Step 4(c) of Procedure 5.1.



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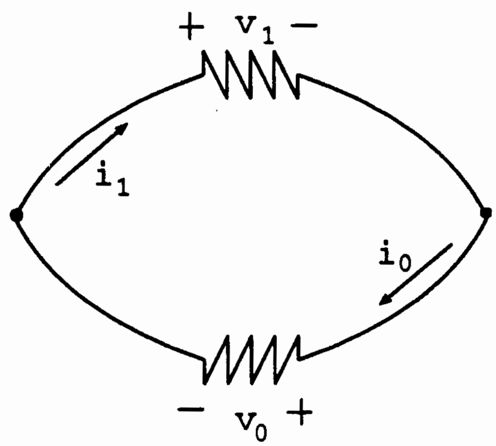

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