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THE INFLUENCE OF SOURCE HEIGHT ON SINGLE POINT DIFFUSION IN UNSTABLE WALL LAYERS

(Supplement to Reports No. 67 and 72)

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ABSTRACT

Equations describing the influence of a nonzero source height on the mean particle trajectory in slightly unstable and strongly unstable turbulent boundary layers are obtained. Typical results are presented for the slightly unstable boundary layer. The effect of a nonzero source height in this case is considerable as has been previously pointed out.

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1 Introduction

In previous reports Mandell and O'Brien (1966 a, b, and c) constructed an entirely Lagrangian similarity model to determine the mean trajectory and mean ground level concentration resulting from continuous sources located at the origin in both slightly unstable and strongly unstable turbulent boundary layers. In the present report the influence of a nonzero source height on the mean trajectory is considered. The procedure used to determine an approximate boundary condition at the source is similar to that adopted by Batchelor (1964) in his investigation of the neutrally stable wall layer.

The Influence of Source Height on Mean Trajectory

The Slightly Unstable Boundary Layer

In the neutral stratification analysis given by Batchelor, the time for a particle to forget its initial position is of the order $^h/u_*$ where h is the height of the source and u_* is the friction velocity. Batchelor then determines the order of magnitude of the integration constant in the trajectory equation from an approximate boundary condition which is obtained by using the logarithmic velocity profile, $u = \frac{u_*}{k} \log \frac{\overline{z}}{z_o}$, and the characteristic time, $\frac{h}{u_*}$. Thus

$$\overline{x} = \frac{h}{k} \log \frac{h}{z_o} \quad \text{when} \quad \overline{z} = h,$$

where k is von Karman's constant and z_o is a height characterizing surface roughness.

In the present case the characteristic time $\frac{h}{u_*}$ will again be used and since the logarithmic velocity profile is not valid, $u(h) = (\frac{d\overline{x}}{dt})_{\overline{z}=h}$ will be used to determine the boundary condition. Thus

$$\overline{x} = (\frac{d\overline{x}}{dt})_{\overline{z}=h} \frac{h}{u_*} \quad \text{when} \quad \overline{z} = h \tag{1}$$

The following equations are obtained from Mandell and O'Brien (1966a):

$$\overline{x}(\overline{z}) = \frac{\overline{z}}{kb} (\log \frac{\overline{z}}{z_o} - 1) - \frac{4}{3} \frac{\overline{z}\, b_1}{k\, b^{5/2}} (\frac{\overline{z}}{|L|})^{\frac{1}{2}} \log \frac{\overline{z}}{z_o} + A \tag{2}$$

where $b = .1$ and $b_1 = .0025$;

and $$\frac{d\overline{x}}{dt} = \frac{u_*}{k} (\log \frac{\overline{z}}{z_o} - \frac{4}{3} \frac{b_1}{b^{3/2}} (\frac{\overline{z}}{|L|})^{\frac{1}{2}}) \tag{3}$$

In the previous report the constant A was equal to zero for a source located at the origin.

A can be determined from equations (1), (2), and (3); and in dimensionless form, the equation for the mean particle trajectory is

$$bk\xi = \zeta (\log\zeta - 1) - \frac{4}{3} \frac{b_1}{b^{3/2}} |\alpha|^{\frac{1}{2}} \log \zeta \tag{4}$$

$$+ H \{(b - 1) \log H + 1 - \frac{4}{3} \frac{b_1}{b^{3/2}} |\alpha|^{\frac{1}{2}} H^{\frac{1}{2}} (b - \log H)\}$$

where $\xi = \frac{\bar{x}}{z_o}$, $\zeta = \frac{\bar{z}}{z_o}$, $|\alpha| = \frac{z_o}{|L|}$, and $H = \frac{h}{z_o}$.

2.2 The Strongly Unstable Boundary Layer

From dimensional reasoning the time for the particle to forget its initial position is of the order $\left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} \left(\frac{h}{c}\right)^{\frac{2}{3}}$ in the L = 0 case, where c_p, ρ_o, and T_o are reference values of specific heat, density, and temperature respectively; q is the heat flux, positive upward; g is the acceleration of gravity, and c is a constant introduced by Yaglom (1965). The L = 0 time scale will be used in evaluating the approximate boundary condition in this case. The boundary condition is

$$\bar{z} = h + \left(\frac{d\bar{z}}{dt}\right)_{\bar{z} = h} \left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} \left(\frac{h}{c}\right)^{\frac{2}{3}}$$

when

$$\bar{x} = \left(\frac{d\bar{x}}{dt}\right)_{\bar{z} = h} \left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} \left(\frac{h}{c}\right)^{\frac{2}{3}} \qquad (5).$$

The following equations are obtained from Mandell and O'Brien (1966b):

$$\left(\frac{d\bar{z}}{dt}\right)_{\bar{z} = h} = \frac{2}{3} \left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} c^{\frac{2}{3}} h^{\frac{1}{3}} \quad (6),$$

$$\left(\frac{d\bar{x}}{dt}\right)_{\bar{z} = h} = U_o - c_1 \left(\frac{qg}{c_p \rho_o T_o}\right)^{\frac{1}{3}} |L|^{\frac{1}{3}} \{\text{Log}\ [\left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} \frac{h^{\frac{2}{3}}}{c^{2/3} t_o}] -2\} \quad (7),$$

$$\bar{x}(\bar{z}) = U_o \left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} \left(\frac{\bar{z}}{c}\right)^{\frac{2}{3}} - c_1 \left(\frac{|L|}{\bar{z}}\right)^{\frac{1}{3}} \bar{z} \{\text{Log}\ [\left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} \frac{h^{\frac{2}{3}}}{c^{2/3} t_o}] -1\} + B \quad (8),$$

where U_o is a constant introduced by Yaglom and c_1, t_o, and B are constants. In the previous report B was equal to zero.

The constant B can be evaluated from equations (5) through (8) and the mean particle trajectory becomes

$$\bar{x}(\bar{z}) = \frac{U_o}{c^{2/3}} \left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} [\bar{z}^{\frac{2}{3}} - \frac{17}{20} h^{\frac{2}{3}}] - c_1 \left(\frac{|L|}{\bar{z}}\right)^{\frac{1}{3}} \bar{z} \{\text{Log}\ [\left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} \frac{\bar{z}^{\frac{2}{3}}}{c^{2/3} t_o}]-1\}$$

$$+ \frac{17}{20} c_1 \left(\frac{|L|}{\bar{z}}\right)^{\frac{1}{3}} h \{\text{Log}\ [\left(\frac{c_p \rho_o T_o}{qg}\right)^{\frac{1}{3}} \frac{h^{\frac{2}{3}}}{c^{2/3} t_o}] - \frac{3}{17}\} \quad (9).$$

Discussion

The influence of a nonzero source height on the strongly unstable boundary layer cannot be determined since the constants in equation (9) are unknown. The influence of the nonzero source height on slightly unstable stratification can be determined from equation (4). Typical values of $bk\xi$ are shown in the following table for $|\alpha| = 10^{-4}$ and three values of ζ:

	$bk\xi$					
	H = o		H = 50		H = 100	
ζ	$\|\alpha\| = o$	$\|\alpha\| = 10^{-4}$	$\|\alpha\| = o$	$\|\alpha\| = 10^{-4}$	$\|\alpha\| = o$	$\|\alpha\| = 10^{-4}$
100	360	356	234	221	--	--
500	2610	2530	2484	2405	2295	2220
1000	5910	5680	5784	5555	5595	5370

The above table shows that the difference between the trajectories for neutral layers and those for slightly unstable layers is slight. In both cases the effect of a nonzero source height is considerable as was previously pointed out by Cermak (1963).

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