STATE UNIVERSITY OF NEW YORK AT STONY BROOK

REPORT NO. 43

AFCRL-66-654

ON INTERCHANNEL CORRELATION IN COMMUNICATION WITH FEEDBACK

by Patrick Barry

and Peter M. Dollard

College of Engineering
State University of New York
Stony Brook, New York 11790

Technical Report No. 43

Scientific Report No. 1 on

Contract No. AF 19(628)-3865

Project No. 5628

Task No. 562801

May, 1966

Prepared

for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
Bedford, Massachusetts

AFCRL -66-654

ON INTERCHANNEL CORRELATION IN COMMUNICATION WITH FEEDBACK

by Patrick Barry

and Peter M. Dollard

College of Engineering
State University of New York
Stony Brook, New York 11790

Technical Report No. 43

Scientific Report No. 1 on

Contract No. AF 19(628)-3865

Project No. 5628

Task No. 562801

May, 1966

Prepared

for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
Bedford, Massachusetts

NOTICE

ON INTERCHANNEL CORRELATION IN COMMUNICATION WITH FEEDBACK

Abstract

It has been firmly established in the literature that feedback offers a distinct improvement over conventional unidirectional communications systems in many cases, especially with channels subject to time varying statistics. Many of the analyses extant assume a noiseless, error-free feedback channel is available. Clearly this will not always be a valid assumption. Even in published results which include the effects of feedback errors, the possibility of interchannel correlation and its effects on performance have apparently never been considered.

This paper undertakes to show that at least in some cases, some of the system performance measures exhibit a significant sensitivity to interchannel correlation of fading.

With decision errors possible on the feedback channel, final errors of omission and insertion, as well as simple substitution errors, can occur. Occurrences of the different types of final errors will in general have a different relative disturbance effect on the output information. It is shown in several examples that correlation in general affects the rate of each error type differently, and that the functional dependences vary widely with other system characteristics. Thus the major conclusion is that in every case where correlation may exist, its possible effects should be examined for the particular case at hand, and if found to be significant should be taken in account both in analysis and synthesis.

ON INTERCHANNEL CORRELATION IN COMMUNICATION WITH FEEDBACK

I. Introduction

The concept of employing feedback procedures in conjunction with two-way communications systems offers an important practical solution to the problem of maintaining system reliability in the face of time varying channel statistics. The recent literature on the subject of communications with feedback has treated the evaluation of the various types of feedback procedures for different classes of systems, analysing the trade-off between rate and reliability which feedback affords, and developing more efficient encoding schemes to take advantage of the feedback. It has been firmly established by several authors[1-15] that decision and information feedback schemes offer a distinct improvement in performance over conventional unidirectional systems. However, in almost all cases the accompanying analyses assume a noiseless feedback channel. While this may be a valid approximation in some cases, it is certainly not valid, for example, in the case of a simultaneous two-way system, in which the feedback for one direction would be part of the forward transmission for the other direction. Thus both the feedback and feedforward information will be subject to errors at comparable rates and with comparable effect on overall system performance.

A complication in analysing the effects of feedback errors is the possibility of interchannel correlation of noise or fading. With many, perhaps most, two-way systems the two channels will be almost identical. The phenomena which give rise to fading or intense noise on one channel are likely to effect both channels simultaneously. Since these phenomena are frequently of long duration, there may be correlation between the occurrence of errors in a give unit of feed-forward information and their occurrence in the corresponding feed-back reply, even though the reply must clearly come later in time.

The most natural and commonly familiar feedback communication system is the ordinary telephone conversation. That the following situation is typical, is well known to those concerned with the psychology of telephone users. Talker A has difficulty hearing talker B. Therefore talker A's reaction is to shout. But then talker B hears A perfectly clearly, in fact too clearly. Therefore B speaks more and more softly while A becomes more and more desperate. Of course if <u>both</u> had difficulty, or <u>neither</u> had difficulty, the situation would not arise. The two-point moral is that correlation of fading can have a profound effect on feedback communications, and that at least in the telephone system, high positive correlation is so prevalent that users are conditioned to it and react accordingly, even though the vast majority of them (as laymen) have no least notion of the concept of correlation.

This paper undertakes an analysis of interchannel correlation of error occurrences, primarily with the intent of showing that the effect can be significant, and should be taken into account both in the analysis and synthesis of actual systems. We first consider the simplest possible system capable of exhibiting the salient effects. This is an uncoded binary decision feedback system. Then to show that the results carry over to systems of more practical interest, a decision feedback system employing a long block code is briefly analysed. The results show that of the various types of errors that can occur in a noisy feedback system, some are strongly dependent while others are but weakly dependent on correlation. With some the dependence has positive slope while with others the slope is negative. This raises the possibility of optimization through control on correlation by adjusting the frequency and/or time separation between the forward transmission and corresponding feedback reply.

II. Analysis of Uncoded Binary System

The system under consideration consists of a binary erasure channel for the forward channel and a binary channel (symmetric or unsymmetric depending on the decision region partition) for the feedback. The system characteristics are depicted in Figures 1a, b, c, with a "1" signal represented by the point +1 and the "0" signal by the point -1. With each decision, the receiver feeds back either a zero to indicate reject or a one to indicate acceptance of the previous forward transmission. Except for differences in output space partitions, both channels are assumed identical and subject to identical noise conditions. For simplicity the noise is assumed present in one of two states. In both states the noise is assumed zero mean gaussian. One is a low noise state with variance σ_1^2, the other a high noise state with a variance σ_2^2. The forward transmission of a single bit, coupled with the corresponding feedback reply is designated as a "total transmission". Let α_ν denote the state of the forward channel, β_ν the state of the feedback channel during the νth total transmission. α_ν and β_ν are identically distributed, correlated random variables, each with two possible "values" which we artifically designate as l for the low noise state and h for the high noise state.

There are four possible system states during the νth total transmission, designated

$$
\begin{aligned}
&s_1^\nu = (\alpha_\nu = l,\ \beta_\nu = l) \\
&s_2^\nu = (l,\ h) \\
(1) \qquad &s_3^\nu = (h,\ l) \\
&s_4^\nu = (h,\ h)
\end{aligned}
$$

Again for simplicity these are assumed independent of ν. Their probabilities of occurrence must satisfy the basic relations

$$P(s_1^\nu) + P(s_2^\nu) + P(s_3^\nu) + P(s_4^\nu) = 1$$

(2)
$$P(s_1^\nu) + P(s_2^\nu) = P(s_1^\nu) + P(s_3^\nu) = P(\alpha_\nu = 1) = P(\beta_\nu = 1)$$
$$P(s_2^\nu) + P(s_4^\nu) = P(s_3^\nu) + P(s_4^\nu) = P(\alpha_\nu = h) = P(\beta_\nu = h)$$

If (say) $P(\alpha_\nu=1)$ and any one of the four system state probabilities are specified, the remaining state probabilities are determined. Alternatively we may define a type of correlation coefficient as

(3)
$$\rho_{\alpha\beta} \simeq \frac{P(s_1^\nu) - P^2(\alpha_\nu=1)}{P(\alpha_\nu=1)[1-P(\alpha_\nu=1)]}$$

whose specification with $P(\alpha_\nu=1)$ determines the remaining state probabilities.*

Similarly if σ_1^2, σ_2^2, and the output decision space partitionings for both channels are specified, the probability of each decision event conditioned on the channel state can be calculated. On the forward channel we need distinguish only the decision events; "correct acceptance", "erroneous acceptance", and "null" or "reject", designated c, e, and r respectively. On the feedback channel we must distinguish four events; a correctly interpreted 1 and a correctly interpreted 0 (designated "1" and "0"), an error e_1 in which a 1 is read as a zero, and the converse error e_0. Typical decision space partitionings for a symmetric binary erasure forward channel and binary asymmetric feedback channel are depicted in Figure 1.

---

*Strictly speaking ρ is not "the" correlation coefficient since for simplicity we have ignored the shift in $\bar{\alpha}$ as $p(\alpha=1)$ is varied.

There are three distinct types of errors which can occur in the information sequence as finally assembled by the receiver. The standard type is the simple substitution error; the remaining types are the "spurious insertion" and "spurious deletion". The latter two result in nonconservation of message length through bit gains and losses, and therefore may be much more troublesome.

In this analysis it is assumed that the receiver infallibly recognizes the transmitter's failure to repeat or the presence of an unrequested repeat, provided the then-current bit is ultimately accepted correctly without intervening error. (Normally provision for this would be made through coding). An occurrence of the latter is simply ignored by the receiver, while the former requires the receiver to guess the missing bit with probability 1/2 of a correct guess. If the then-current bit is ultimately accepted erroneously or is not repeated when requested (and is guess erroneously), then a preceding unrequested repeat is assumed to result in spurious insertion while a previous failure to repeat results in spurious omission.

Since their dependence on correlation is different, we distinguish the two ways a simple substitution error can occur. Thus there are four error probabilities to consider; $P_I(e)$ - the probability of ultimate erroneous acceptance of any given bit, $P_{II}(e)$ - the probability of a recognised failure to repeat followed by erroneous guess, $P_{III}(e)$ - the probability of spurious omission, and $P_{IV}(e)$ - that of spurious insertion. They are computed as follows:

(4) $$P_I(e) = \frac{\overline{p(e)}}{1-p(r,0)}$$

(5) $$P_{II}(e) = \frac{1}{2}\,\frac{\overline{p(r,e_o)}}{1-\overline{p(r,0)}}$$

(6) $$P_{III}(e) = \overline{p(r,e_0)}[P_I(e) + P_{II}(e)]$$

(7) $$P_{IV}(e) = [\overline{p(c,e_1)} + \overline{p(e,e_1)}][P_I(e) + P_{II}(e)]$$

where for example, $\overline{p(e)}$ is the average probability of erroneous acceptance of any given forward transmission, $\overline{p(r,0)}$ is the joint average probability of a reject on the forward channel correctly interpreted as such on the feedback channel, etc. The expression for the latter is typical, viz.

(8) $$\overline{p(r,0)} = p(r|\alpha=1)p(0|\beta=1)P(s_1) + p(r|\alpha=1)\cdot p(0|\beta=h)P(s_2) + p(r|\alpha=h)p(0|\beta=1)P(s_3) + p(r|\alpha=h)p(0|\beta=h)P(s_4)$$

In addition to the above error probabilities, the information rate is also affected by interchannel correlation. Since final termination of transmission on any one information bit occurs when and only when the feedback channel reads a "1" (whether correctly or incorrectly) as a feedback reply on that bit, the average information transmission rate is simply

(9) $$R = \overline{p(c,1)} + \overline{p(e,1)} + \overline{p(r,e_0)}$$

Direct substitution of the relations exemplified by (8) into the five equations (4) - (7) and (9) yields straightforward but rather horrendous algebraic expressions for the desired performance functions. To reduce them to something more tractable we introduce some notational abbreviations and some simplifying approximations. Since correlation of fading will obviously not be significant unless there is substantial difference between performance in the good state and performance in the bad, we begin by assuming $p(e|\alpha=l)$ and $p(e_0|\beta=l)$ are both, for all practical purposes, equal to zero. Also if the use of feedback is to be worth the effort, then the criterion for acceptance on the forward channel must be made sufficiently stringent that $p(e|\alpha=h)$ is still very much less (by orders of magnitude) than unity. If this can be accomplished while maintaining a reasonably high value for $p(c|\alpha=l)$ then e_1 errors on the feedback channel will be much less disturbing than e_0 errors. This suggests that the feedback channel be made highly asymmetric to the extent that $p(e_0|\beta=h)$ is comparable in order of magnitude to $p(e|\alpha=h)$. With this in mind we retain only the terms of lowest non-vanishing order in these two factors. The abbreviated notation is as follows:

$$P(\alpha=l)=P(\beta=l)=a \quad ; \quad P(\alpha=h)=P(\beta=h)=1-a$$

$$P(s_1) = a^2 + \rho a(1-a); \; P(s_4) = (1-a)^2 + \rho a(1-a)$$

$$P(s_2) = P(s_3) = a(1-a)(1-\rho)$$

$$p(r|\alpha=l) = b \qquad p(r|\alpha=h) = 1 - \delta - \varepsilon \simeq 1$$

$$p(e|\alpha=l) = 0 \qquad p(e|\alpha=h) = \varepsilon$$

$p(c|\alpha=1) = 1-b \qquad p(c|\alpha=h) = \delta$

$p(e_o|\beta=1) = 0 \qquad p(e_o|\beta=h) = \gamma$*

$p(0|\beta=1) = 1 \qquad p(0|\beta=h) = 1 - \gamma \simeq 1$

$p(1|\beta=1) = 1-d \qquad p(1|\beta=h) = \eta$

$p(e_1|\beta=1) = d$* $\qquad p(e_1|\beta=h) = 1 - \eta \simeq 1$

Note that Greek letters have been used for all quantites which can generally be expected to be very small, while with the exception of ρ, Roman letters are used for the more moderate quantities. The desired algebraic expressions now reduce to:

$$P_I(e) \doteq \frac{\varepsilon(1-a)}{a(1-b)}$$

$$P_{II}(e) \doteq \frac{\gamma(1-a)}{2a(1-b)}[1-a(1-b)(1-\rho)]$$

(10)

$$P_{III}(e) \doteq \frac{\gamma(1-a)^2}{a(1-b)}[1-a(1-b)(1-\rho)][(\varepsilon+\frac{\gamma}{2}) - \frac{\gamma}{2}a(1-b)(1-\rho)]$$

$$P_{IV}(e) \doteq (1-a)[d+(1-a)(1-d)(1-\rho)][(\varepsilon+\frac{\gamma}{2}) - \frac{\gamma}{2}a(1-b)(1-\rho)]$$

$$R \doteq a(1-b)(1-d)[a+\rho(1-a)]$$

These expressions are still a bit messy for general interpretation. Two observations are worthy of note, however. First, the rate R is maximized when ρ is maximum. (In fact if one could conceive of negative correlation, the rate goes to zero at extreme negative correlation if $a \leq \frac{1}{2}$). Second, the probability of spurious insertion, $P_{IV}(e)$ goes to zero at $\rho=1$ if the good state is completely noise free such that $b = d = 0$, while the accompanying increase in $P_{III}(e)$ is of small moment since $P_{III}(e)$ in any case of second order in γ and ε.†

*Note that $d = b$, $\eta=\delta$ and $\gamma=\varepsilon$ if the partition of the decision space in the feedback receiver matches that of the feedforward receiver as depicted in Fig. 1.

†Actually, the approximation to $P_{IV}(e)$ goes to zero, but the exact expression reduces to second order in γ and ε.

With b = d = 0 and also assuming $\gamma=\varepsilon$, the total probability of all errors, to first order in ε, is given by:

$$(11)\quad P_I + P_{II} + P_{III} + P_{IV} = \frac{\varepsilon(1-a)}{2a}(3-a+\rho a)[1+a(1-a)(1-\rho)]$$

Unless $a\rightarrow 1$, this has negative slope for all ρ and in any case, is but weakly dependent on ρ. Therefore it is clear one would wish to maximize ρ in order to maximize R and minimize $P_{IV}(e)$.

III. Analysis of Block Coded Binary System.

We wish now to show that the salient effects illustrated by the simple example of the previous section carry over to systems of more practical interest. A simultaneous two-way system is considered with each channel employing a (n,k) random linear block code.* One of the k information digits in each block will be the feedback decision (1 or 0 for accept or reject) on an earlier block on the other channel. Similarly a second digit is designated as a "block label", alternately 1 and 0 for each block of k-2 true information bits as received from the source. This is to aid in the recognition of unrequested repeats. The channels are both assumed binary symmetric, again characterized by a high and a low noise state, the states occurring randomly with identical statistics on both channels and with interchannel correlation of noise states.

The decision to accept or reject will be based on a criterion of j or fewer apparent errors. That is, an apparent error pattern of

*More specifically, we assume the code has a generator matrix of the form $G=(I_k/P)$ where I_k is the kxk identity and P is a kx(n-k) parity check matrix each of whose elements is chosen by an independent coin toss.

j or fewer errors will be corrected; otherwise the block is rejected. However, we allow the cases where j is either greater or less, for purposes of reading the feedback digit than for reading the feedforward information digits. Two values, j = 0 and j = 2, are considered. Thus there are four possible combined decision criteria, NN, ND, DN and DD, where (for example) ND denotes no correction allowed for feedforward but double error correction allowed in acceptance for reading the feedback digit in the same block.

With a random linear block code it is considerably more difficult to correct all double errors than with a more systematic code. However, the following simple algorithm corrects all single errors and approximately 75% of all double errors:-As the information digits are received, the receiver generates its own parity check sequence, then adds it bit by bit mod 2 to the received parity check sequence. If the resulting n-k bit sequence is of weight zero, one or two, the receiver assumes all k information bits are correct. If it is of higher weight, the receiver attempts to match it to a row of the parity check matrix. When and if a match within a one digit error is discovered, the corresponding information digit is corrected. Otherwise the entire block is rejected. For computational purposes we assume this algorithm is used for the D mode correction.

For simplicity the noise state on either channel is assumed to change instantaneously and only at a division between blocks. In computing the results, we consider a (100,50) code with the two digit error probabilities assigned the values $p_{01} = .01$ and $p_{oh} = \frac{1}{2}$. With these values and any reasonably probable choice of the random code, the code performance will closely approximate that of the "best" (100, 50) code. In the noisy state the various decision event probabilities are computed by pure combinatorics independently of the particular code structure, while in the quiet state the fact that the minimum distance between code words will be somewhat less than the maximum minimum will have negligible effect on the computations.

Again there will be three basic types of error, simple substitution, omission, and insertion. (For brevity we will not distinguish between the different ways any one type may arise). Since unrequested repeats will clearly result in final errors less often than failures to repeat, the receiver should assume the feedback digit is zero whenever it cannot be read. We specify a somewhat simple-minded receiver, which always assumes that the most probable cause of repeated block labels in sequence (i.e. - unrequested repeat) is the actual cause, and therefore deletes the apparent repetition. The receiver could do better than this only with substantial increase in system complexity through more elaborate block labelling and addressing and/or comparisons with previously accepted information blocks. With this in mind a little reflection shows that omissions and insertions can only occur as double errors. Let $P_i(e)$ denote the average probability of simple substitution, $P_{ii}(e)$ that of double omission, and $P_{iii}(e)$ that of double insertion. They are given as follows:

$$(12) \qquad P_i(e) = \frac{\frac{1}{2}\,\overline{p(e)}}{1-\overline{p(r,0)}} + \frac{\frac{1}{2}\overline{p(c,e_1)}\;\overline{p(e,1)}}{[1-\overline{p(r,0)}][1-\overline{p(r,0)}-\overline{p(c,e_1)}]}$$

(13) $$P_{ii}(e) = \frac{\frac{1}{2}\,\overline{p(e.}}{1-p(r,0)} + \frac{p(r,e_0)}{1-p(r,0)}$$

(14) $$P_{iii}(e) = \frac{\frac{1}{2}\,\overline{p(c,e_1)}\;\overline{p(e,e_1)}}{[1-p(r,0)][1-p(r,0)-p(c,e_1)]}$$

where the notation on decision events follows that of the previous section, and the block label is assumed to be changed with probability $\frac{1}{2}$ whenever the forward transmission is accepted erroneously. The expression for rate R is identical to that of (9) above except for a multiplicative factor of (k-2/n.)

Using the same abbreviated notation and the same approximations as in the previous section, these reduce to

$$P_i(e) \doteq \frac{\varepsilon(1-a)}{2a(1-b)} \quad 1 + a(1-\rho)\frac{1-a+ad-\rho(1-a)(1-d)}{a+\rho(1-a)}$$

$$P_{ii}(e) \doteq \frac{(1-a)}{a(1-b)} \left\{\gamma[1-a(1-b)(1-\rho)] + \frac{\varepsilon}{2}\,a(1-d)(1-\rho)\right\}$$

(15) $$P_{iii}(e) \doteq \frac{\varepsilon(1-a)}{2a(1-b)(1-d)} \cdot \frac{[(1-a+ad)-\rho(1-a)(1-d)][(1-a+ad)+\rho a(1-d)]}{a+\rho(1-a)}$$

$$R \doteq \frac{k-1}{n}\,a(1-b)(1-d)[a+\rho(1-a)]$$

Again one could make a few general observations, such as the rate approaching zero and P_i and P_{iii} blowing up with extreme negative correlation (a highly unlikely circumstance, though perhaps not impossible, in a physical system) provided $a \leq \frac{1}{2}$. Also if d=0, then $P_{iii}(e) \to 0$ linearly as $\rho \to 1$. However the results are more readily interpretable for the numerical example proposed. For this example the parameters have the following sets of numerical values:

$\epsilon_N \doteq 10^{-15}$ $\qquad b_N \doteq 0.632$

$\epsilon_D \doteq 10^{-12}$ $\qquad b_D \doteq 0.127$

$\gamma_{N,D} = \frac{1}{2}\epsilon_{N,D}$ $\qquad d_{N,D} \doteq b_{N,D}$

where for example, ϵ_N is the value of ϵ with no error correction (N-mode) on the forward channel. Then for each of the four combined modes:

NN-Mode

$$P_i(e) = \frac{1-a}{a}(1.4\times10^{-15})[1 + a(1-\rho)\frac{1-.37a-.37\rho(1-a)}{a+\rho(1-a)}$$

(16)

$$P_{ii}(e) = \frac{1-a}{a}(1.4\times10^{-15})$$

$$P_{iii}(e) = \frac{1-a}{a}(3.7\times10^{-15}\ \frac{[1-.37a-.37\rho(1-a)](1-.37a+.37\rho a]}{a+\rho(1-a)}$$

$$R = .065\ a\ [a + \rho(1-a)]$$

ND-Mode

$$P_i(e) = \frac{1-a}{a}(1.4\times10^{-15})\ \ 1 + a(1-\rho)\frac{1-.87a-.87\rho(1-a)}{a+\rho(1-a)}$$

(17)

$$P_{ii}(e) = \frac{1-a}{a}(1.4\times10^{-12})[1-.37a + .37\rho a]$$

$$P_{iii}(e) = \frac{1-a}{a}(1.5\times10^{-15})\frac{[1-.87a-.87\rho(1-a)][1-.87a+.87\rho a]}{a+\rho(1-a)}$$

$$R = .155\ a[a + \rho(1-a)]$$

DN-Mode

$$P_i(e) = \frac{1-a}{a}(0.57\times10^{-12})\left\{1 + a(1-\rho)\ \frac{1-.37a-.37\rho(1-a)}{a+\rho(1-a)}\right\}$$

$$P_{ii}(e) = (1-a)(0.21\text{x}10^{-12})(1-\rho)$$

(18) $$P_{iii}(e) = \frac{1-a}{a}(1.6\text{x}10^{-12})\frac{[1-.37a-.37\rho(1-a)][1-.37a+.37\rho a]}{a+\rho(1-a)}$$

$$R = .155\ a[a + \rho(1-a)]$$

DD-Mode

$$P_i(e) = \frac{1-a}{a}(0.57\text{x}10^{-12})\ \ 1 + a(1-\rho)\frac{1-.87a-.87\rho(1-a)}{a+\rho(1-a)}$$

$$P_{ii}(e) = \frac{1-a}{a}(0.57\text{x}10^{-12})$$

(19) $$P_{iii}(e) = \frac{1-a}{a}(0.65\text{x}10^{-12})\frac{[1-.87a-.87\rho(1-a)][1-.87a+.87\rho a]}{a+\rho(1-a)}$$

$$R = .366\ a[a + \rho(1-a)]$$

To comb out the significant ρ-dependencies, let us first examine the desirable features of each combined decoding mode when $\rho = 0$. The NN-mode renders all three error probabilities so small that, barring an extremely small value of a, they are for all practical purposes zero. For example, if information blocks were delivered to the user at a rate of 1000 per second the mean time to first error occurrence would be on the order of three thousand years or more! Therefore one might readily hazard an increase in rate at the expense of a few orders of magnitude increase in the error probabilities. Adopting either the ND or DN mode will increase R by a factor of about 2.4. Again barring an extremely small value of a, both yield a comparable total probability of all errors, with ND slightly better. However, ND makes the type ii errors (which are surely the most disturbing), predominate by a wide margin while the DN mode makes the type iii errors predominant. Of greater interest is that in either case the DD mode achieves an additional increase in rate by a factor of again about 2.4, and an actual reduction in the total error probability over that for the DN mode.

Thus it would appear one should adopt the NN mode primarily for reasons of minimum system complexity, or the DD mode for a six-fold increase in rate at a cost of increased receiver complexity and a decrease in reliability to the point where it might begin to be of some concern (i.e.-mean time to first error occurrence on the order of three years for the case cited previously).

Now consider the ρ dependencies. If we ignore the remote possibility of negative correlation, then for all four decoding modes, $P_i(e)$ shows no significant dependence on ρ since the variation is by at most a factor of two from $\rho = 0$ to $\rho = 1$. This is true regardless of the values of a, b, d, etc. Note that $P_{ii}(e)$ is independent of ρ for the NN and DD modes. This is because the two terms in equation (13) for P_{ii} are both linear in ρ, one with positive slope, the other with negative slope, and the slopes cancel identically when b = d and $\gamma = \frac{1}{2}\varepsilon$. In the ND-mode, the (r,e_o) event is the dominant contributor (to the extent the other is ignored) and the slope is positive but the dependence is weak. In the DN mode it is the (e,1) event which dominates, the slope is negative and the dependence is strong in the sense that the probability of the dominant event goes to zero as $\rho \to 1$.

However, the possibility of a truly significant dependence on ρ arises in R and (since it is inversely proportional to R) in P_{iii} if a is very small, that is, if the channels are more often in a fade than out. For example, if a = 0.1, then in all four modes there is a tenfold increase in R as ρ varies from 0 to 1. In the NN and DN modes, the corresponding decrease in P_{iii} is magnified to fifteen-fold by the ρ dependence of the numerator, while in the ND and DD modes the magnification is to sixty-fold, for an almost two orders of magnitude decrease. Even at $\rho = \frac{1}{2}$ there is a better than fivefold increase in R, a sixfold decrease in P_{iii} for the NN and DN modes, and about an eleven-fold decrease in P_{iii} for the DN and DD modes, over the corresponding values at $\rho = 0$.

IV. Discussion

The examples analysed bear out two quite general conclusions, both of which one might have strongly suspected even in the absence of any specific analysis to show them, namely that correlation of fading in the two channels of a noisy feedback system is most likely to be significant if either the probability of experiencing good reception is extremely small (much less than one half), or that the quiet state is virtually noise free while the fading state borders on virtual blackout. Therefore, beyond verifying the intuitively obvious, perhaps the most striking conclusion one can draw from the analysis is that the system designer can adopt a very simple minded decision strategy, in particular one which is otherwise highly prone to insertion errors, and still expect the incidence of insertion errors to remain well within bounds provided fading on both channels exhibits high interchannel correlation. In fact he can offset the effects of an overly simple strategy by designing the channel pair to maximize any inherent tendency towards high correlation.

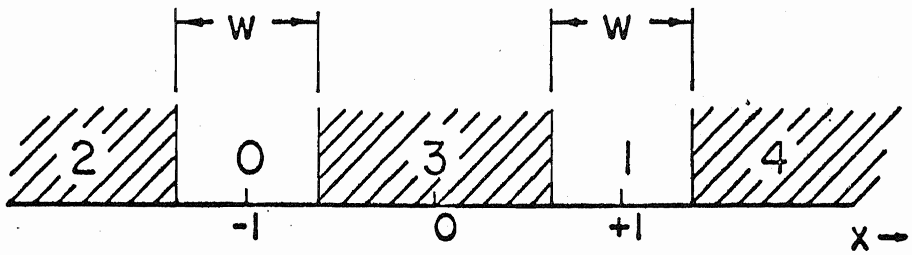
Of course there are many feedback schemes other than that of block coding for error detection with decision feedback, and under similar fading conditions on both channels one would expect correlation of fading to have similar, and perhaps more marked effects. We have chosen the block coded decision feedback system for three reasons. First, it is the simplest to analyse. Second, it conceptually encompasses certain other schemes such as sequential coding with decision feedback, which for practical procedural purposes would probably have to be treated as a long block code (with the repeat requests coming only at the end of each block and the entire block repeated each time there is an apparent repeat request) in order to avoid the necessity of exchanging an excessive amount of procedural information. Third, it (or the conceptually equivalent sequential code scheme) is very probably the best scheme to use under conditions of noisy feedback with severe fading or long burst noise.

Information feedback or compound feedback schemes would appear particularly susceptible to feedback errors and correlation, unless elaborate means are provided for the exchange of procedural information. Though we have not specifically analysed this case, perhaps we can illustrate the point by a return to the telephone example, and the following anecdote. While on a pleasure trip to Europe, J. B. Phogghorn, president of Z-company, gets wind of a business deal which could turn a tidy profit if approached in precisely the right way. He calls Q. G. Milquetoast, junior executive back at Z, and proceeds to give detailed instructions interspersed with numerous "Gotcha, J. B."s from Milquetoast. (Decision feedback). Milquetoast has indeed heard clearly, but J. B. wants to be sure, and asks him to read back the essential details. (Information feedback). Now the situation deteriorates rapidly. Phogghorn has been shouting and Milquetoast speaking softly. Up to this point, however, Milquetoast was transmitting at an extremely low data rate, and therefore has gotten through. His information feedback would still be at a low data rate by entropy measure _if Phogghorn expected_ Milquetoast to have received the message correctly. The trouble is, he _never_ expects Milquetoast to get things straight the first time, therefore the feedback is at a high rate, and with low signal-to-noise, the situation is hopeless. Milquetoast actually has the whole thing straight, but Phogghorn is unwilling to chance that without confirmation by clearly understandable information feedback. Again if _both_ channels were operating at high signal-to-noise everything would be fine, while if neither of them were, at least it would be the telephone system which bore the brunt of Phogghorn's ire.

FIGURE I

DECISION SCHEMES ON FORWARD AND FEEDBACK CHANNELS

Ia – FORWARD CHANNEL (BINARY ERASURE)

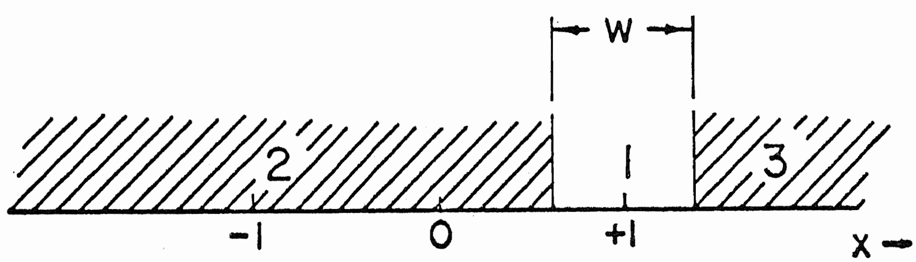


REGION 0 – ACCEPT AS "ZERO"

REGION 1 – ACCEPT AS "ONE"

REGION 2,3,4 – REJECT

Ib – FEEDBACK CHANNEL (BINARY ASYMMETRIC)



REGION 1 – READ "ACCEPTED PREVIOUS TRANSMISSION"

REGION 2,3 – READ "REJECTED PREVIOUS TRANSMISSION"

BIBLIOGRAPHY

1) Chang, S. S. L., "Improvement of two-way communication by feedback", 1961 International Convention Record, pt. 4, p. 88-104.

2) Chang, S. S. L., "Theory of information feedback systems", I.R.E. trans. P.G.I.T., vol. 2, p. 29-40, Sept. 1956.

3) Chang, S. S. L., Harris, B., Morgan, K. D., "Cumulative binary decision feedback systems", Proc. National Electronics Conference, Chicago, Ill.,1958.

4) Cowell, W. R., "Use of group codes in error detection and retransmission", I.R.E. trans. P.G.I.T., vol.-IT-7, July, 1961.

5) Harris, B., Hauptschein, A., Shwartz, L. S., "Optimum decision feedback systems", Convention Record, I.R.E., pt. 2, p. 3-10, 1957.

6) Harris, B., Morgan, K. C., "Binary symmetric decision feedback systems", Communication and Electronics, A.I.E.E., vol. 38, Sept. 1958.

7) Harris, B., Hauptschein, A., Morgan, K.C., Schwartz, L.S., "Binary communication feedback systems", Communication and Electronics, A.I.E.E., vol. 40, p. 960-969, Jan, 1959.

8) Horstein, M., "Sequential transmission of digital information with feedback", Sc. D. Thesis, Dept. of E.E., M.I.T., Cambridge, Mass., 1960.

9) Jacobs, I., "Optimal error detection codes for noiseless decision feedback", I.R.E. trans. on Info. Th., vol. IT-8, Oct, 1962.

10) Metzner, J. J., Morgan, K.C., "Coded feedback communication systems", A.I.E.E. trans., vol. 81, Communication and Electronics, Jan. 1962.

11) Metzner, J. J., Morgan, K. C., "Reliable fail-safe binary communication", N.Y.U. College of Engineering Research Division Second Scientific Report, Contract #19(604)-6168, July, 1960.

12) Schwartz, L. S., "Recent results in digital feedback communication systems", I.R.E. trans on Info. Th., vol. CS-9, March, 1961.

13) Shannon, C., "Two-way communication channels", Proc. of 4th Berkeley Symposium on Mathematical Statistics and Probability, Univ. of Calif. press, Berkeley, Calif., vol. 1, 0. 611-644, 1961.

14) Wozencraft, J. M., Horstein, M., "Coding for two-way channels", Proc. 4th London symposium on Info. Th., C. Cherry, Butterworths, Wash., D. C., 1961.

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)* College of Engineering State University of New York at Stony Brook Stony Brook, L.I. N.Y.	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

On Interchannel Correlation in Communication with Feedback

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Scientific Report Interim

5. AUTHOR(S) *(Last name, first name, initial)*

Barry, Patrick
Dollard, Peter M.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
May, 1966	21	15

8a. CONTRACT OR GRANT NO. AF k9(628)-3865	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Report No. 43 Scientific Report No. 1
b. PROJECT AND TASK NO. 5628-01	
c. DOD ELEMENT 614450 14	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*
d. DOD SUBELEMENT 681305	AFCRL-66-654

10. AVAILABILITY/LIMITATION NOTICES

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Hq.AFCRL, OAR (CRB) United States Air Force L.G.Hanscom Field,Bedford,Mass.

13. ABSTRACT

It has been firmly established in the literature that feedback offers a distinct improvement over conventional unidirectional communications systems in many cases, especially with channels subject to time varying statistics. Many of the analyses extant assume a noiseless, error-free feedback channel is available. Clearly this will not always be a valid assumption. Even in published results which include the effects of feedback errors, the possibility of interchannel correlation and its effects on performance have apparently never been considered. This paper undertakes to show that at least in some cases, some of the system performance measures exhibit a significant sensitivity to interchannel correlation of fading. With decision errors possible on the feedback channel, final errors of omission and insertion, as well as simple substitution errors, can occur. Occurrences of the different types of final errors will in general have a different relative disturbance effect on the output information. It is shown in several examples that correlation in general affects the rate of each error -type differently, and that the functional dependences vary widely with other system characteristics. Thus the major conclusion is that in every case where correlation may exist, its possible effects should be examined for the particular case at hand, and if found to be significant should be taken in account both in analysis and synthesis.

Unclassified

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Feedback Communications Noisy feedback Correlation of fading, interchannel Errors, types of Error rates						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization *(corporate author)* issuing the report.

2*a*. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2*b*. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year, or month, year. If more than one date appears on the report, use date of publication.

7*a*. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7*b*. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8*a*. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8*b*, 8*c*, & 8*d*. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9*a*. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9*b*. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers *(either by the originator or by the sponsor)*, also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring *(paying for)* the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as *(TS)*, *(S)*, *(C)*, or *(U)*.

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

Security Classification