ESSAYS ON UNAWARENESS AND ITS APPLICATIONS

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Abstract of the Dissertation
Essays on unawareness and its applications
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This dissertation extends the standard framework used in game theory and information economics to incorporate unawareness of decision makers, i.e. the situation that they don't know that they lack relevant information. We start by revisiting a classic example in the game theory literature to demonstrate different implication of unawareness and imperfect information (impreciseness). Then we show that for games with incomplete information, cursed equilibrium, an innovative equilibrium concept which fits a broad range of experimental or field datasets, can be justified in the standard theoretical framework allowing unawareness. Namely, given any finite Bayesian game with a commonly known common prior, the set of cursed equilibria coincides a set of Bayesian Nash equilibria of the game augmented with players partially aware of other players' original information structure. Finally, we analyze an important stock market regulation on information disclosure. We show that if a professional investor knows that small investors are unaware of price relevant uncertainties, the regulation aimed at protecting small investors and improving market efficiency can however increase the cost of information acquisition. This result can explain an unexpected empirical finding on the impact of the regulation across different firms.

To my parents.

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Introduction

> *You don't get into trouble because of the things you don't know. It is the things you don't know you don't know that really get you into a mess.*
> *- unknown*

Intuitively, when we don't know a thing, it is possible that we still know our ignorance, yet it is also possible that we ignore our ignorance, and here comes *unawareness*.

Economists used to assume that the economic agents fully understand the game or the economic model as much as they do. For example, all agents have full knowledge about all possible states of nature, the identity, the possible actions and the informational types of all agents, even though they do not know the exact state of the world. This type of ignorance can be described by "impreciseness". However there are questions that can not be properly addressed by impreciseness only. Often it is natural to allow agents to be unaware about some information.

Unawareness and impreciseness have different effects on an agent's decision making. Impreciseness is equivalent to the notion of incompleteness of information, conventionally presented by an information partition of the state space. Thus taking all possibilities into consideration, an agent calculates the expected payoff function given her belief. When unawareness presents, the agent misinterprets the states, and her calculation may not exploit all relevant information. For example, the payoff function might have fewer arguments (say other agents' types) than it should.

The factor causes an agent to be unaware can be intrinsic bounded observability, like color blindness or blind-spot. It may also be intrinsic bounded resource as addressed by Simon (1955). Aragones et al. (2005) make a formal argument that when computational resources run out, individuals may arrive at imperfect descriptions of the world and therefore they may be unaware of the descriptions other people use. Other reasons like limited experience and education can also be valid.

Some researchers (Congleton, 2001) gave primary thoughts on unawareness that, since an unaware agent gathers data from a restricted domain, her estimates can be systematically biased. Although from the point of view of another agent or the outside analyst her choice is "irrational" or mistaken, she can be rational in the sense that she uses all the information she has to make the best choice. It is in the same spirit of conventional incomplete information model. Therefore, if rationality does not exclude impreciseness, it shall not exclude unawareness either. As we will see later, compared to other "behavioral regularities", unawareness is a much natural extension of current theory of incomplete information.

In the literature of game theory, Geanakoplos (1989) first suggests using nonpartitional information structures to model some fault in information processing related to unawareness. However Dekel et al. (1998) show that even nonpartitional models are not able to handle unawareness. The formal definition of awareness is first introduced in Modica and Rustichini (1994). Later on Modica and Rustichini (1999) and Halpern (2001) develop logical systems of unawareness. In Modica and Rustichini (1999), the model consists of an "objective" space, describing the world with the full vocabulary, and a "subjective" space for the sub-vocabulary of which the agent is aware. When an individual is unaware of an event, the states she considers as possible belong to a subjective space in which this event cannot be described.

Two semantic models are independently developed by Li (2006) and Heifetz et al. (2006). In the first paper, information consists of *awareness information*, which specifies the list of uncertainties of which the agent is aware of, and *factual information*, which specifies the resolution of uncertainties. The agent's understanding of the environment, the subjective state space, is restricted to those uncertainties of which she is aware of. In the second paper, there is a complete lattice of state-spaces with projections among them. The partial order of spaces indicates the strength of their expressive power. An agent at a state of one space may reside in a less-expressive subjective space, while her information structure about this space remains partitional. In both papers, an agent is aware of an event when the uncertainties it contains are within the expressive power of her subjective space, otherwise, she is unaware of it.

The three essays in this dissertation address unawareness in different contexts of game theory and information economics, with an emphasis on its applications.

We start by revisiting the barbecue problem, a classic example in the game theory literature, to demonstrate different implication of unawareness and imperfect information (impreciseness). The result implies that one agent's unawareness can actually simplifies some other agents' strategic computations, while the impreciseness can not.

In the second essay, we show that for games with incomplete information, cursed equilibria, an innovative equilibrium concept which fits a broad range of experimental or field datasets, can be justified in the standard theoretical framework allowing unawareness. Namely, given any finite Bayesian game with a commonly known common prior, the set of cursed equilibria coincides a set of Bayesian Nash equilibria of the game augmented with players partially aware of other players' original information structures. This framework also supports the intuition that, compared with zero cursedness, positive cursedness incurs lower complexity of strategic computation.

At last, we analyze an important stock market regulation called Regulation Fair Disclosure. It requires firms to replace private meetings with security industry professionals with public conferences. Under the standard symmetric awareness assumption, the regulation reduces the cost of capital and the cost of information acquisition. However, if a professional investor knows that small investors are unaware of price relevant uncertainties, the regulation aimed at protecting small investors can however increase the cost of information acquisition by the professional. This result can explain an unexpected empirical finding on the impact of the regulation across different firms.

Chapter 1

The Barbecue Problem with Unawareness

We revisit the classic barbecue problem (or dirty face game). There are several people, all of them with messy faces; each one can see other people's faces but not her own. It is known that, if some outsider makes it common knowledge that *at least one* individual has a messy face, then after a few rounds of observation and calculation, every intelligent individual will know that her face is messy. We introduce the notions of impreciseness (a coarser information partition) and unawareness (a smaller state space), and allow one individual to be imprecise about or unaware of the state of the faces of some other individuals. We show that the outcome is different under these two different assumptions. An unaware individual's action can provide better information to some other individuals, since there is less uncertainty in her mind. However, the impreciseness of an individual worsens the information revelation.

1.1 Introduction and the original problem

The barbecue problem[1], or dirty face game, exhibits a strong implication of common knowledge. In this paper we study what happens when information structure is slightly perturbed. In our version, one individual will be allowed to be either imprecise about or unaware of some relevant information. The key result is that

[1]The original version of the problem is published in Littlewood (1953). Different versions have been discussed in Game Theory (Myerson, 1991; Geanakoplos, 1992) and Artificial Intelligence (Halpern, Fagin, Moses and Vardi, 1995). The version we use is by Peter Vanderschraaf, available at http://plato.stanford.edu/entries/common-knowledge/.

the two alternative assumptions bring about quite different consequences. Hence it highlights the notion of unawareness as one special type of incomplete information.

In the original problem, N people join a picnic supper. At the end of the meal, everyone has barbecue sauce on her face. Everyone can see sauce on $N-1$ others' faces. No one would like her messy face if she knew it, but without knowing it, no one wipes her face. And those people do not want to tell anyone else that she has barbecue sauce on her face. Then the cook appears with a carton of ice cream. Amused by what he sees, the cook rings the dinner bell and makes the following statement: "At least one of you has barbecue sauce on her face. I will ring the dinner bell over and over, until someone wipes her face, then I will serve the ice cream." How will participants behave after the announcement?

Suppose $N = 2$. We describe all possible states of picnic participants by state space $\Omega_0 = \{11, 10, 01, 00\}$, where state 10 describes the state where individual 1's face is messy and individual 2's face is clean, state 01 is the opposite, state 11 is where both are messy, and state 00 is where both are clean.

Because every one sees all but her own face, individual 1's information structure is $\Pi_1 = \{\{11, 01\}, \{10, 00\}\}$, individual 2's is $\Pi_2 = \{\{11, 10\}, \{01, 00\}\}$. When the cook announces that "At least one of you has barbecue sauce on her face", no one objects, because everyone sees other's face messy at the real state 11. Then it is commonly known that the state 00 is impossible, and the state space is reduced to be $\Omega_1 = \{11, 10, 01\}$. Consequently, it is commonly known that the information structures are $\Pi_1 = \{\{11, 01\}, \{10\}\}$ and $\Pi_2 = \{\{11, 10\}, \{01\}\}$. However, nobody knows her face is messy or not.

Again the cook rings the bell. Then individual 1 knows state 01 is impossible, for otherwise individual 2 should wipe her face after the first ring. Same reasoning is conducted by individual 2 to rule out state 10. The state space is now $\Omega_2 = \{11\}$. They wipe there face simultaneously.

The general case follows by induction. Suppose that when $N = k$, each individual can determine that her face is messy after k rings. Then if $N = k + 1$, at the $k + 1$st ring, each of the $k + 1$ individuals will realize that she is messy. For if she were clean, it would be common knowledge for other people. Then they would have all realized their messiness and wiped themselves after the kth ring. Since it did not happen, at the $k + 1$st ring each messy person will conclude that someone besides other k people must also be messy, namely, herself.

1.2 Primary observation

The new element that we want to introduce could be generally called as ignorance. It can be either *impreciseness* or *unawareness*. The impreciseness case is where one individual does not see another individual's face but is aware of the possibility that she is either messy or clean, while the unawareness case is where she does not notice the existence of that individual. It is perhaps because there are too many people or she is just absent-minded. We want to compare the results under the two different assumptions.

We first consider the impreciseness case. When $N = 2$, the state space is $\Omega_0 = \{11, 10, 01, 00\}$. Individual 1's information structure is just $\Pi_1 = \Omega_0$ because she can not observe the other's face. Individual 2's information structure is unchanged: $\Pi_2 = \{\{11, 10\}, \{01, 00\}\}$. Suppose all above is common knowledge between the individuals, after the announcement by the cook, the state space becomes $\Omega_1 = \{11, 10, 01\}$ and information structures become $\Pi_1 = \Omega_1$, $\Pi_2 = \{\{11, 10\}, \{01\}\}$. Nobody wipes her face. After the second ring, individual 1 knows state 01 is not true. And two possibilities 11 and 10 remain. So is individual 2. Thus only individual 1 will wipe her face.

In the unawareness case, let individual 1 be unaware of the existence of individual 2. When $N = 2$, the state space for individual 1 is $S_0 = \{1x, 0x\}$, where x denotes her unawareness about the state of individual 2. The state space and information structure for individual 2 are unchanged. After the first ring, the simple-minded individual 1 thinks herself the only one the cook is talking about, then wipes her face at once. Individual 2, however, still can not tell whether the state is 11 or 10 from 1's move, so she does nothing.

What happens eventually depends on the rule used by the cook to stop ringing the bell. If the cook stops as soon as one individual wipes her face, then the face of individual 2 will remain dirty. If the cook keeps ringing as long as there is a dirty face, then in the next round 2 will wipe. We focus on the first case in this paper.

At first sight, the only difference lies in timing – individual 1 in the first case moves after second ring and in the second case moves after the first ring, individual 2 does nothing either way. But when N increases and there are some individuals not directly affected by individual 1's ignorance, as we show next, there are more distinctions.

1.3 Impreciseness

First let $N = 3$. The state space is $\Omega = \{111, 110, 101, 011, 100, 010, 001, 000\}$ and the true state is 111. In the impreciseness case, suppose individual 1 knows the existence of individual 3 but can not see her face, and everyone else's information structure is as in the original case. Individual 1, 2 and 3's information structures are:

Π_1: $\{\{111, 110, 010, 011\}, \{101, 100, 001, 000\}\}$

Π_2: $\{\{111, 101\}, \{011, 001\}, \{110, 100\}, \{010, 000\}\}$

Π_3: $\{\{111, 110\}, \{011, 010\}, \{101, 100\}, \{001, 000\}\}$

After the first ring, state 000 is eliminated. The state space is reduced to $\{111, 110, 101, 011, 100, 010, 001\}$, the information structures are:

Π_1: $\{\{111, 110, 010, 011\}, \{101, 100, 001\}\}$

Π_2: $\{\{111, 101\}, \{011, 001\}, \{110, 100\}, \{010\}\}$

Π_3: $\{\{111, 110\}, \{011, 010\}, \{101, 100\}, \{001\}\}$

No one moves. After the second ring, states 010 and 001 can be eliminated. The state space is now $\{111, 110, 101, 011, 100\}$, over which the information structures are:

Π_1: $\{\{111, 110, 011\}, \{101, 100\}\}$

Π_2: $\{\{111, 101\}, \{011\}, \{110, 100\}\}$

Π_3: $\{\{111, 110\}, \{011\}, \{101, 100\}\}$

Still, no one moves. After the third ring, state 011 and the set $\{101, 100\}$ are to be removed. It is common knowledge that the state space is $\{111, 110\}$. The information structures then are:

Π_1: $\{111, 110\}$

Π_2: $\{\{111\}, \{110\}\}$

Π_3: $\{111, 110\}$

At this point, individual 1 and 2 wipe their faces. But 3 stays uninformed because she thinks the possibility that it is still possible that her face is clean. We state now the general result.

Proposition 1.3.1. *For $N \geq 2$ individuals, suppose $i = 1$ can only observe the faces of individuals in the set $\{2, \ldots, N-M\}$ and is aware of the remaining M individuals' existence, while all other individuals observe the faces of everyone else. Let $T(i)$ be the number of rings that individual i needs to learn the true state of her face, with $T(i) = +\infty$ meaning that i never learns. Then*

1. $T(i) = N \quad$ *if* $i \in \{1, \ldots, N-M\}$;
2. $T(i) = +\infty \quad$ *otherwise.*

Proof. First following the examples we define the state space as $\Omega = \{\omega \equiv (t_1 t_2 \dots t_N) : t_i \in \{0,1\},\ \forall i \in \{1,\dots,N\}\}$. The information structure of individual i is defined by information function P_i. Given two states ω and $\omega' = (t'_1 t'_2 \dots t'_N)$, for every $i \in \{2,\dots,N\}$, $P_i(\omega') = P_i(\omega)$ if and only if $t'_j = t_j,\ \forall j \in \{1,\dots,N\} \backslash i$; for individual 1, $P_1(\omega') = P_1(\omega)$ if and only if $t'_j = t_j,\ \forall j \in \{2,\dots,N-M\}$.

Given that the true state is $(\underbrace{1\dots1}_{N})$ and the information structures, every individual i learns that her face is messy only when she can eliminate all states where $t_i = 0$.

After the first ring, it is common knowledge that the true state is not $(\underbrace{0\dots0}_{N})$. But no one moves after the first ring. After the second ring, it becomes common knowledge that the states where only one individual in set $\{2,\dots,N\}$ has a messy face, namely $\{(t_1 \dots t_i \dots t_N), i \in \{2,\dots,N\} : t_i = 1; t_j = 0, \forall j \in \{1,\dots,N\} \backslash i\}$ are impossible. But state $(\underbrace{10\dots0}_{N})$ is still possible because individual 1 can not observe the last M individuals.

After the third ring, it becomes common knowledge that the states where only two individuals in set $\{2,\dots,N\}$ have messy faces, namely $\{(t_1 \dots t_i \dots t_k \dots t_N), \{i,k\} \subset \{2,\dots,N\}, i \neq k : t_i = t_k = 1; t_j = 0, \forall j \in \{1,\dots,N\} \backslash \{i,k\}\}$ are impossible. For otherwise someone in $\{2,\dots,N\}$ wipes her face given her accumulated knowledge. But states like $(\underbrace{10\dots010\dots0}_{N})$ are still possible.

The information is not sufficient for any individual to learn, so the ring continues. After the Nth ring, it is common knowledge that state $(\underbrace{01\dots1}_{N})$ is impossible, which informs everyone that only the states where $t_1 = 1$ are possible; accordingly individual 1 learns. For everyone in $\{2,\dots,N-M\}$, 1's action also implies that her face is messy, for otherwise 1 should move one round earlier. So they also learn. But for everyone in $\{N-M+1,\dots,N\}$, the state where only she is clean can not be eliminated. She can not learn anyway. **QED.**

What is interesting is that even if individual 1 can not see some others' faces, he can still learn that his face is messy. At the same time all individuals $i > N - M$ can not learn at all since the information they need is jeopardized by individual 1's coarse information structure.

1.4 Unawareness

Again let $N = 3$. Individual 2 and 3 are normal while 1 is absentminded about 3's face. Hence the state space to 2 and 3 is still $\Omega = \{111, 110, 101, 011, 100, 010, 001, 000\}$, but to 1 it is $S = \{11x, 10x, 01x, 00x\}$, where x describes 1's unawareness about

3's face.

Individual 1, 2 and 3's information structures are:

Π_1: $\{\{11x, 01x\}, \{10x, 00x\}\}$

Π_2: $\{\{111, 101\}, \{011, 001\}, \{110, 100\}, \{010, 000\}\}$

Π_3 : $\{\{111, 110\}, \{011, 010\}, \{101, 100\}, \{001, 000\}\}$

Note that individual 2 and 3's information structures over Ω are not common knowledge because individual 1 does not have 3's information at all. Also in 1's mind, 2's information structure over S is:

Π_2 in 1's mind $\{\{11x, 10x\}, \{01x, 00x\}\}$.

After the first ring, because "at least one has sauce on her face", it becomes common knowledge to 2 and 3 that state 000 is impossible; but 1 considers that the common knowledge to 1 and 2 is that state $00x$ is impossible. The information structures are:

Π_1: $\{\{11x, 01x\}, \{10x\}\}$

Π_2: $\{\{111, 101\}, \{011, 001\}, \{110, 100\}, \{010\}\}$

Π_3: $\{\{111, 110\}, \{011, 010\}, \{101, 100\}, \{001\}\}$

Π_2 in 1's mind: $\{\{11x, 10x\}, \{01x\}\}$

Before the second ring, no one moves. After that, it becomes common knowledge to both 2 and 3 that states 001 and 010 are impossible. Meanwhile, 1 thinks that to her and 2, it is common knowledge that states $10x$ and $01x$ are not possible. Further, 2 and 3 realize that is common knowledge that state $10x$ is not possible for 1 didn't move. The information structures are:

Π_1: $\{11x\}$

Π_2: $\{\{111\}, \{011\}, \{110\}\}$

Π_3: $\{\{111, 110\}, \{011\}\}$

Π_2 in 1's mind: $\{11x\}$

Thus, both 1 and 2 learn after the second ring, 3 does not because she is still not sure. Now we state the general result.

Proposition 1.4.1. *For $N \geq 2$ individuals, suppose $i = 1$ is only aware of individuals in set $\{2, \ldots, N-M\}$. Let $T(i)$ be as in Proposition 1.3.1. Then*

1. $T(i) = N - M$ *if* $i \in \{1, \ldots, N-M\}$;
2. $T(i) = +\infty$ *otherwise.*

Proof. We define two types of states. An objective state completely describing the status is still $\omega = (t_1 t_2 \ldots t_N)$, where $t_i \in \{1, 0\}$ for all $i \in \{1, \ldots, N\}$. Given the same state ω, a subjective state in individual 1's mind is $\widetilde{\omega} = (t_1 t_2 \ldots t_{N-M} \underbrace{x \ldots x}_{M})$.

Then the information functions are defined as follows. Given states ω and $\omega' = (t'_1 t'_2 \ldots t'_N)$, for all $i \in \{2, \ldots, N\}$, $P_i(\omega') = P_i(\omega)$ if and only if $t'_j = t_j$, $\forall j \in \{1, \ldots, N\} \backslash i$; for individual 1, $P_1(\tilde{\omega}') = P_1(\tilde{\omega})$ if and only if $t'_j = t_j$, $\forall j \in \{2, \ldots, N-M\}$.

Given that the true state is $(\underbrace{1 \ldots 1}_{N})$ and the information structures, every individual i learns the fact only when she can eliminate all states where $t_i = 0$.

After the first ring, it is common knowledge to individuals $\{2, \ldots, N\}$ that state $(\underbrace{0 \ldots 0}_{N})$ is impossible. In 1's mind, it is common knowledge to $\{1, \ldots, N-M\}$ that state $(\underbrace{0 \ldots 0}_{N-M} \underbrace{x \ldots x}_{M})$ is impossible.

After the second ring, to $\{2, \ldots, N\}$ it becomes common knowledge that the states where only one individual in set $\{2, \ldots, N\}$ has a messy face, namely $\{(t_1 \ldots t_i \ldots t_N), i \in \{2, \ldots, N\} : t_i = 1; t_j = 0, \forall j \in \{1, \ldots, N\} \backslash i\}$ are impossible. At the same time, in 1's mind, it is common knowledge to $\{1, \ldots, N-M\}$ that states where only one individual in set $\{1, \ldots, N-M\}$ has a messy face, namely $\{(t_1 \ldots t_j \ldots t_{N-M} \underbrace{x \ldots x}_{M}), j \in \{1, \ldots, N-M\} : t_j = 1; t_k = 0, \forall k \in \{1, \ldots, N\} \backslash j\}$ are impossible.

After the third ring, to $\{2, \ldots, N\}$ it becomes common knowledge that the states where only two individuals in set $\{2, \ldots, N\}$ have messy faces, namely $\{(t_1 \ldots t_i \ldots t_l \ldots t_N), \{i, l\} \subset \{2, \ldots, N\}, i \neq l : t_i = t_k = 1; t_j = 0, j \in \{1, \ldots, N\} \backslash \{i, k\}\}$ are impossible. For otherwise someone in $\{2, \ldots, N\}$ wipes her face given her accumulated knowledge. At the same time, in 1's mind, it is common knowledge to $\{1, \ldots, N-M\}$ that states where only two individuals in set $\{1, \ldots, N-M\}$ have messy faces, namely $\{(t_1 \ldots t_j \ldots t_m \ldots t_{N-M} \underbrace{x \ldots x}_{M}), \{j, m\} \subset \{1, \ldots, N-M\}, j \neq m : t_j = t_m = 1; t_k = 0, \forall k \in \{1, \ldots, N\} \backslash \{j, m\}\}$ are impossible.

After the $N-M$th ring, to $\{2, \ldots, N\}$ it becomes common knowledge that the states where only $N-M$ individuals in set $\{2, \ldots, N\}$ have messy faces are impossible.

At the same time, in 1's mind, it is common knowledge to $\{1, \ldots, N-M\}$ that states where $N-M-1$ individuals in set $\{1, \ldots, N-M\}$ have messy faces are impossible. To 1, it implies that the state must be $(\underbrace{1 \ldots 1}_{N-M} \underbrace{x \ldots x}_{M})$. To everyone in $\{2, \ldots, N-M\}$, her face is messy too for otherwise 1 should have wiped earlier. Accordingly they all wipe their faces.

But the remaining M people already knew that the first $N-M$ people's faces were messy, they still need more information to figure out the true state. Therefore $T(i) = \infty$, for all $i \in \{N-M+1, \ldots, N\}$. **QED.**

Note that if we only consider the first $N-M$ individuals who exist in 1's mind, then the problem is the original barbecue problem with a small state space.

1.5 Conclusive remark

Original case	$T(i) = N, \quad \forall i \in \{1, \ldots, N\}$.
Impreciseness case	$T(i) = N \quad \text{if} \ \ i \in \{1, \ldots, N-M\}$; $T(i) = +\infty \quad \text{otherwise}$.
Unawareness case	$T(i) = N-M \quad \text{if} \ \ i \in \{1, \ldots, N-M\}$; $T(i) = +\infty \quad \text{otherwise}$.

Table 1.1: Comparison of three cases of barbecue problem. $N \geq 2$

Table 1.1 summarizes the original version and variations of the barbecue problem. Unawareness leads to a different consequence. With less awareness, individual 1 "learns" faster and provides information to some of others more quickly, because her state space is simpler than the objective one. This result can not be reached when the level of 1's impreciseness increases.

While there could be other more complicated variations of the Barbecue Problem, our objective is to highlight the subtlety in analyzing games with unawareness. [2] The idea implied by our example could be applied to other dynamic games where some individuals are not aware of all states. For example, they may be naive traders in

[2] There are several papers related to this work with strong focus on unawareness. Ewerhart (2001) analyzes the classic story of *Romeo and Juliet* to argue that unawareness is strategically inequivalent to uncertainty, which is similar to "impreciseness" in our notion. Feinberg (2005) generalizes the normal form game framework to allow unawareness of other players' actions or existence. A literature on the theory and application of unawareness is growing quickly. Interested readers can find *The Unawareness Bibliography* maintained by Burkhard C. Schipper at "http://www.econ.ucdavis.edu/faculty/schipper/unaw.htm".

speculative markets or unsophisticated bidders' in auctions of complex common value goods. It is also interesting to think of *The Emperor's New Clothes* by Hans Christian Andersen, in which only the innocent child who is not aware of the words that "the wonderful clothes is only invisible to man who is stupid." Sometimes, unawareness makes some people impatient, naive or bold, but other people better informed; also unexpected things can happen.

Chapter 2

Justifying Cursed Equilibria via Partial Awareness

We show that given any finite Bayesian game with a commonly-known common prior probability distribution, its set of cursed equilibria coincides a set of Bayesian Nash equilibria of an augmented game where players perceive other players types as if they are partially aware of others' original information structures. Consistent with the intuition that cursedness implies scarce computational resource, partial awareness is equivalent to a reduction of the complexity of players' strategic computation. This result also shows the potential of using unawareness to formulate imperfect strategic sophistication.

2.1 Introduction

In a game with incomplete information, if players have correct beliefs about other players' information and expect others rationally choose actions depending on their information, then the proper equilibrium concept is the standard Bayesian Nash equilibria. But when players are not sophisticated, one common mistake is to take other players' actions as unrelated with their private information. In reality, the way agents make decisions often lies between the two extremes. By a χ-weighted combination of the two situations, Eyster and Rabin (2005) introduce the concept of *cursed equilibria* to explain field or experimental data and do statistical inferences.

However, the notion of Bayesian Nash equilibrium itself is legitimate even when players have wrong (incompatible with the reality) or inconsistent (unable to be derived from a common prior) beliefs[1]. A large proportion of the economic literature assumes that prior beliefs must be common knowledge, so that there is no event that

[1] See Myerson (2004) for a discussion and Mertens and Zamir (1985) for a formal model.

some players know could happen while some don't. However, sometimes we might need to relax this assumption to make the Bayesian Nash framework more flexible. In this paper, we want to show that by properly choosing more general beliefs, a cursed equilibrium can be justified as a Bayesian Nash equilibrium in a more general sense.

The equivalence is not obvious whenever the weight of cursedness χ is positive. In the proof of the existence of cursed equilibria, Eyster and Rabin (2005) let every player have two possible payoffs at every state. With probability $1-\chi$, it depends on others' types, and with probability χ, it does not. In this virtual game, the Bayesian Nash equilibrium is a cursed equilibrium of the original game. Such formulation shows the existence of cursed equilibria but is not a very good justification within a Bayesian Nash framework, because it is difficult to argue that players' payoff functions are so versatile.

Using the idea of partial awareness, we want to justify cursed equilibrium within a Bayesian Nash framework . To see the intuition of awareness, suppose that a player's type is a state of mind determined by his information. In many realistic situations, the processing of the information is not perfect. This information can be a multidimensional signal and the player, as a receiver of the signal, may lack the ability to either perceive, or measure, or understand the variations in certain dimensions. Li (2006) considers a model in which the player is "aware" of and actually uses only some dimensions of the signal. Taking a different analogy, Heifetz et al. (2006) say that all information is expressed in some language and to express complex information, a language must be rich in its expressive power. For example, a language with a restricted vocabulary in general has less expressive power. In this sense, they define a player's awareness by the expressive power of the language that he uses.

Heifetz et al. (2007) provide a general framework formulating unawareness in games with incomplete information. In this paper, we focus on a specific situation that players may be partially aware of others' types, in the sense that the perceived types can be represented by a partition and the original types can be represented by another however finer partition, on the same set of states.

It is not very surprising that partial awareness can explain some imperfect strategic behaviors. What we show is that under certain assumptions the equilibria derived from partially aware types are exactly equivalent to the cursed equilibria.

It takes two steps to show the result. First we modify every player's types. While keeping every type's belief about the exogenous parameter, we let this type perceive an augmented set of other players' types by adding a state where all other players do not have any information. By assuming that the type believes others to play an averaged strategy at this additional state, we can show that a Bayesian Nash equilibrium is exactly a cursed equilibrium. Second we show that the augmented set of types imply that the perceived information structure is always worse than the original information structure in the sense of Blackwell condition. Therefore using the result of

Green and Stokey (1978), we can show that both the perceived information structure and the original information structure can be represented by two partitions of the same set of states with the same probability measure, and the partition representing the original information structure refines the other. This matches our definition of partial awareness.

A closely related work is Miettinen (2007). There he first defines the original set of states as a partition of an interval of measure one. Then he defines new partition of the interval. With the new partition, at every original state, he allows every player to be able to understand other players' type-dependent strategies with probability $1-\chi$, and not able to do so with probability χ. Therefore at every original state, the expected payoff function is just like the one in the virtual game in Eyster and Rabin (2005). He uses this idea and the concept of analogy based expectation equilibrium to provide a learning foundation of cursed equilibrium.

Roughly speaking, the analogy based expectation equilibrium allows players to partition other players' types and assume the strategies to be based on the members of the partition instead of those types. Since partial awareness can be considered as one possible reason for players to partition in a certain way, two ideas are quite similar. However, since the new partition in Miettinen (2007) has more members than the number of original states, he concludes that when players are partially (but not fully) cursed, they use more complex strategic computations. But in our framework, the partition representing the perceived information structure is always coarser than the one representing the original information structure, hence the implication is the opposite.

The paper is organized as follows. We set up the framework and review Bayesian Nash Equilibrium and cursed equilibrium in Section 2. A justification with expanded type spaces is shown in Section 3. We show the main result on partial awareness in Section 4. During the process, an example about a lemon market is demonstrated. In Section 5, we discuss a lemon market model with partial awareness without referring to cursed equilibrium. Conclusions are in Section 6.

2.2 Bayesian Nash equilibria and cursed equilibria

The game is a finite static game with incomplete information denoted by $\left\langle \Theta, T_i; q; A_i; u_i \right\rangle_{i=1}^{N}$. The set of players is $\{1, 2, \ldots, N\}$. The exogenous parameter is $\theta \in \Theta$. The space of player types is $T \equiv \times_{i=1}^{N} T_i$. A common prior probability distribution q puts positive measure on every state in $\Theta \times T$. Type t_i's belief on the parameter and other players' types (θ, t_{-i}) is given by $q(\theta, t_{-i}|t_i)$.

An action of Player i is $a_i \in A_i$, and A_i is the action set. All players' action profile is a vector $a \in A \equiv \times_{i=1}^{N} A_i$. The action profile space A is assumed to be fixed for all states. Player i's payoff function is $u_i : A \times \Theta \to \mathbf{R}$. It is also assumed that this information is common knowledge.

A mixed strategy σ_i for Player i specifies a probability distribution over actions for each type, $\sigma_i : T_i \to \Delta(A_i)$. Let $\sigma_i(a_i|t_i)$ be the probability that type t_i plays action a_i. A strategy profile is $\sigma(t) \equiv \times_{i=1}^N \sigma_i(t_i) : T \to \Delta(A)$. Let A_{-i} be the set of action profiles for players other than i, σ_{-i} be the strategy profile of players other than i, and $\sigma_{-i}(a_{-i}|t_{-i})$ be the probability that types $t_{-i} \in T_{-i}$ plays actions a_{-i} under strategy $\sigma_{-i}(t_{-i})$.

We review the definitions of Bayesian Nash equilibrium (Harsanyi, 1967-1968) and cursed equilibrium.

Definition 2.2.1. A strategy profile σ is a Bayesian Nash equilibrium if for each Player $i = 1, \ldots, N$, each type $t_i \in T_i$, and each a_i^* such that $\sigma_i(a_i^*|t_i) > 0$,

$$a_i^* \in \arg\max_{a_i \in A_i} \sum_{\theta \in \Theta} \sum_{t_{-i} \in T_{-i}} q(\theta, t_{-i}|t_i) \times \sum_{a_{-i} \in A_{-i}} \sigma_{-i}(a_{-i}|t_{-i}) u_i(a_i, a_{-i}; \theta). \tag{2.2.1}$$

In a cursed equilibrium (Eyster and Rabin, 2005), given other players' strategy σ_{-i}, Player i mistakenly believes that with probability $\chi \in [0, 1]$ other players play mixed strategies regardless their types, and these strategies average their true strategies over their types, which is

$$\bar{\sigma}_{-i}(a_{-i}|t_i) \equiv \sum_{t_{-i} \in T_{-i}} \sum_{\theta \in \Theta} q(\theta, t_{-i}|t_i) \sigma_{-i}(a_{-i}|t_{-i}). \tag{2.2.2}$$

Definition 2.2.2. A strategy profile σ is a χ-cursed equilibrium if for each Player i, each type $t_i \in T_i$, and each a_i^* such that $\sigma_i(a_i^*|\theta_i) > 0$,

$$a_i^* \in \arg\max_{a_i \in A_i} \sum_{\theta \in \Theta} \sum_{t_{-i} \in T_{-i}} q(\theta, t_{-i}|t_i) \times \sum_{a_{-i} \in A_{-i}} \left[\chi \bar{\sigma}_{-i}(a_{-i}|t_i) + (1 - \chi) \sigma_{-i}(a_{-i}|t_{-i})\right] \times u_i(a_i, a_{-i}; \theta). \tag{2.2.3}$$

When $\chi = 0$, χ-cursed equilibrium coincides with Bayesian Nash equilibrium. When $\chi = 1$, every player assumes that other players' strategy is completely unrelated with their types, namely players are *fully cursed*.

2.2.1 Lemon market: part 1

Consider the following example, taken from Eyster and Rabin (2005). There is a used car, a seller and a buyer. The exogenous parameter $v \in \{v_h, v_l\}$ determines the car's value. At state v_h, the value to the seller is $2,000$, the value to the buyer is $3,000$; at state v_l, the value to both is 0.

Ex ante, each state happens with probability $\frac{1}{2}$. Suppose the seller has a perfect signal $s \in \{g, b\}$ so that $Pr(v = v_h|s = g) = Pr(v = v_l|s = b) = 1$. The buyer has no information besides the prior probability distribution.

Then at a fixed price P, both sides are able to choose "deal" or "no deal". Trade happens only if both choose "deal".

Let $P = 1,000$. The seller sells only when $s = b$, and the buyer who knows this chooses "no deal". In the unique Bayesian Nash equilibrium, no trade happens.

In the cursed equilibrium, a χ-cursed buyer believes that with probability χ the seller sells with probability $\frac{1}{2}$ irrespective of the signal, the car's expected value is $3000[(1-\chi)0 + \chi\frac{1}{2}] = 1500\chi$. Hence, a buyer cursed with $\chi > \frac{2}{3}$ will buy. Also, the seller's strategy, selling whenever $s = b$, after being averaged over his types, is consistent with the buyer's belief.

2.3 An intermediate alternative justification

We let players to have types different from T. There will be a type $y_i \in Y_i$ corresponding to every type t_i, so that y_i shares t_i's belief on the parameter, but he believes that there is a state with positive probability that none of other players have information.

To formalize, we need, for every Player i, one set of types: (Y_i, p_i) and $N-1$ sets of types: $(Y_j^i, p_j^i), \forall j \neq i, j \in \{1, \ldots, N\}$. We define the relation among the type sets by two bijective mappings: $f_i : Y_i \to T_i$ and $f_j^i : Y_j^i \backslash y_{jx}^i \to T_j$, where y_{jx}^i denotes a special type related to the cursedness. Here Y_i is the set of new types of Player i, with exactly the same number of elements of set T_i, or $|Y_i| = |T_i|$. Not knowing that the types of Player j are in Y_j, Player i believes that Y_j^i is the set of types of Player j, and $|Y_j^i| = |T_j| + 1$.

The following assumptions are made regarding Player i's prior belief p_i.

1) The marginal probability distributions of p_i about y_i is equal to that of q about $f_i(y_i)$.

$$\sum_{\theta \in \Theta} \sum_{y_{-i}^i \in Y_j^i} p_i(\theta, y_i, y_{-i}^i) = \sum_{\theta \in \Theta} \sum_{t_{-i} \in T_{-i}} q(\theta, f_i(y_i), t_{-i}); \tag{2.3.1}$$

2) The conditional probability distribution of p_i on (θ, y_{-i}^i) given y_i is

$$p_i(\theta, y_{-i}^i | y_i) = \begin{cases} (1-\chi) q(\theta, f_{-i}^i(y_{-i}^i) | f_i(y_i)) & \text{if } y_{-i}^i \in \times_{j \neq i}(Y_j^i \backslash y_{jx}^i), \\ \chi \sum_{t_{-i}} q(\theta, t_{-i} | f_i(y_i)) & \text{if } y_{-i}^i = y_{-ix}^i, \\ 0 & \text{otherwise.} \end{cases} \tag{2.3.2}$$

Note that the second condition implies that Player i believes that either all other agents are cursed or none is cursed.

Player i also believes that j's belief p_j^i puts a positive measure on every state in $\Theta \times T_{-j} \times Y_j^i$: [2]

$$\sum_{\theta\in\Theta}\sum_{t_{-j}\in T_{-j}} p_j^i(\theta, t_{-j}, y_j^i) = \begin{cases} \chi & \text{if } y_j^i = y_{jx}^i, \\ (1-\chi)\sum_{\theta\in\Theta}\sum_{t_{-j}\in T_{-j}} q(\theta, t_{-j}, f_j^i(y_j^i)) & \text{if } y_j^i \neq y_{jx}^i. \end{cases} \tag{2.3.3}$$

And the conditional probability distribution of p_i on (θ, t_{-j}) given y_j^i is

$$p_j^i(\theta, t_{-j}|y_j^i) = \begin{cases} \sum_{t_j\in T_j} q(\theta, t_j, t_{-i}) & \text{if } y_j^i = y_{jx}^i, \\ q(\theta, t_{-j}|f_j^i(y_j^i)) & \text{if } y_j^i \in Y_j^i \backslash y_{jx}^i. \end{cases} \tag{2.3.4}$$

For Player i, denote the strategy of other players by a function of the perceived types of others, namely $\sigma'_{-i} : Y_{-i}^i \to \Delta(A_{-i})$. We assume that when the state y_{-ix}^i happens, Player i believes that other players play averaged strategies.

Assumption 2.3.1. *For every Player i, every type y_i believes that given type profile y_{-ix}^i, all other players play the strategy*

$$\bar{\sigma}'_{-i}(a_{-i}|y_i) = \frac{1}{1 - Prob\{y_{-i}^i = y_{-ix}^i|y_i\}}\sum_{\theta\in\Theta}\sum_{y_{-i}^i\in Y_{-i}^i\backslash y_{-ix}^i} p_i(\theta, y_{-i}^i|y_i)\sigma'_{-i}(a_{-i}|y_{-i}^i).$$

Lemma 2.3.2. *With Assumption 2.3.1 hold, the augmented game's set of Bayesian Nash equilibria coincides the set of cursed equilibria of the original game.*

Proof. First by Equation 2.3.2, for every y_i,

$$Prob\{\tilde{y}_{-i}^i = y_{-ix}^i|\tilde{y}_i = y_i\} = \sum_{\theta\in\Theta} p_i(\theta, y_{-ix}^i|y_i) = \sum_{\theta\in\Theta}\chi\sum_{t_{-i}} q(\theta, t_{-i}|f_i(t_i)) = \chi.$$

Then by this and Equation 2.3.2, Assumption 2.3.1 implies that

$$\bar{\sigma}'_{-i}(a_{-i}|y_i) = \sum_{\theta\in\Theta}\sum_{y_{-i}^i\in Y_{-i}^i\backslash y_{jx}^i} q(\theta, f_j^i(y_{-i}^i)|f_i(y_i))\sigma'_{-i}(a_{-i}|y_{-i}^i). \tag{2.3.5}$$

[2]Actually, this only matters in the next section, because to verify if a strategy profile is Bayesian Nash equilibria, only beliefs $\{p_i\}_{i=1}^N$ matters. We emphasize that with inconsistent beliefs it is generally not true that players could reason that others are rational. It is because players view others' information in a way that is inconsistent with what others believe.

Second by Definition 2.2.1, in a Bayesian Nash equilibrium $\sigma'' : \times_{i=1}^{N} Y_i \to \Delta(A)$[3], for each Player $i = 1, \ldots, N$, each type $y_i \in Y_i$, and each a_i^* such that $\sigma_i''(a_i^*|y_i) > 0$,

$$a_i^* \in \arg\max_{a_i \in A_i} \sum_{\theta\in\Theta} p_i(\theta, y_{-ix}^i|y_i) \sum_{a_{-i}\in A_{-i}} \sigma'_{-i}(a_{-i}|y_{-ix}^i) u_i(a;\theta) + \sum_{\theta\in\Theta} \sum_{y_{-i}^i \in Y_{-i}^i \backslash y_{-ix}^i} p_i(\theta, y_{-i}^i|y_i) \times \sum_{a_{-i}\in A_{-i}} \sigma'_{-i}(a_{-i}|y_{-i}^i) u_i(a;\theta).$$

The two components in the objective function are, first by Assumption 2.3.1 and Equation 2.3.2,

$$\sum_{\theta\in\Theta} p_i(\theta, y_{-ix}^i|y_i) \sum_{a_{-i}\in A_{-i}} \sigma'_{-i}(a_{-i}|y_{-ix}^i) u_i(a;\theta) = \sum_{\theta\in\Theta} \sum_{t_{-i}\in T_{-i}} \chi q(\theta, t_{-i}|f^i(y_i)) \sum_{a_{-i}\in A_{-i}} \bar{\sigma}'_{-i}(a_{-i}|y_i) u_i(a;\theta),$$

and secondly

$$\sum_{\theta\in\Theta} \sum_{y_{-i}^i \in Y_{-i}^i \backslash y_{-ix}^i} p_i(\theta, y_{-i}^i|y_i) \times \sum_{a_{-i}\in A_{-i}} \sigma'_{-i}(a_{-i}|y_{-i}^i) u_i(a;\theta) = \sum_{\theta\in\Theta} \sum_{y_{-i}^i \in Y_{-i}^i \backslash y_{-ix}^i} (1-\chi) q(\theta, f_{-i}^i(y_{-i}^i)|f_i(y_i)) \times \sum_{a_{-i}\in A_{-i}} \sigma'_{-i}(a_{-i}|y_{-i}^i) u_i(a;\theta).$$

Hence the objective function is equivalent to

$$\sum_{\theta\in\Theta} \sum_{y_{-i}^i \in Y_{-i}^i \backslash y_{-ix}^i} q(\theta, f_{-i}^i(y_{-i}^i)|f_i(y_i)) \times \sum_{a_{-i}\in A_{-i}} \left[\chi \bar{\sigma}'_{-i}(a_{-i}|y_i) + (1-\chi)\sigma'_{-i}(a_{-i}|y_{-i}^i)\right] \times u_i(a;\theta).$$

Compare it with the objective function in Definition 2.2.3, and in particular, compare Equation 2.3.5 and Equation 2.2.2. We see that the characterizations of cursed equilibrium and Bayesian Nash equilibrium are equivalent. The sets of two must be identical. **QED.**

2.3.1 Lemon market: part 2

Let the buyer believe that there is a new signal $s' \in \{x, g, b\}$ with prior probability $Pr(s' = x) = \chi$, $Pr(s' = g) = \frac{1}{2}(1-\chi)$, $Pr(s' = b) = \frac{1}{2}(1-\chi)$. The buyer believes

[3]Note that player j's true strategy $\sigma_j''(\cdot|y_j)$ is equivalent to $\sigma_j'(\cdot|y_j^i)$ when $f_j(y_j) = f_j^i(y_j^i)$.

that when the signal is g or b, it again conveys perfect information. But when it is x, the seller knows no information. This is

$$Pr(v = v_h|s' = g) = Pr(v = v_l|s' = b) = 1,$$
$$Pr(v = v_h|s' = x) = Pr(v = v_l|s' = x) = \frac{1}{2}.$$

Thus the buyer believes that the seller's expected value is 3000 at $s' = g$, 1000 at $s' = x$, and 0 at $s' = b$. The buyer expects that if the seller sells, the signal $s' \in \{x, b\}$; if the seller sells with .5 probability at $s' = x$, then the car's expected value conditional on a deal is

$$3000\frac{Pr(v = v_h, s' = x)0.5 + Pr(v = v_h, s' = b)}{0.5Pr(s' = x) + Pr(s' = b)} = \frac{3000\chi.5^2}{0.5(1-\chi) + .5\chi} = 1500\chi.$$

The buyer buys if $1500\chi > 1000$, or $\chi \geq \frac{2}{3}$, which is equivalent to the result in the first part of the example.

2.4 Partial awareness

The previous justification relies on adding artificial states. From the viewpoint of a modeler, there could be some cognitive reason for the deviation of the perception of players. We want to apply the idea of awareness on the relation between the perceived types and the original types. It requires taking the two sets of types as players' information structures and representing them by information partitions on the same set of states with the same prior belief. We show that the relation between the partitions matches the definition of partial awareness, therefore by Lemma 2.3.2 the main result follows.

We first explain what we mean by an information partition that represents a player's information structure. Generally speaking, consider a decision maker uncertain about the value of parameter $\phi \in \Phi$, where $\Phi = \{\phi_1, \ldots, \phi_K\}$ is a finite set. Let the prior probability distribution be $r = \{r_1, \ldots, r_K\}$.

An information structure has two alternative formalizations.

1. A set of types Y, and a prior probability distribution λ on $\Phi \times Y$. This is denoted by (Y, λ).

2. A set X, a probability distribution μ on $\Phi \times X$, and a partition P on X. This is denoted by (X, μ, P).

Definition 2.4.1. We say that (X, μ, P) represents (Y, λ) if there is a mapping:

$$\tau : P \to Y$$

such that

1) for each $y \in Y$, each $s \in \tau^{-1}(y)$, and all $\phi \in \Phi$,

$$\frac{\mu(\{\phi\} \times s)}{\mu(\Phi \times s)} = \lambda(\phi|y),$$

2) for each $y \in Y$,

$$\sum_{s \in \tau^{-1}(y)} \mu(\Phi \times s) = \lambda(\Phi, y).$$

Now we define partial awareness.

Definition 2.4.2. Player i is partially aware of Player j's original information structure if for all of his types, there is a set X_j, a probability measure μ_j on $\Theta \times X_j$, and two partitions of X_j: P_j and P'_j, such that

1) The information structure (X_j, μ_j, P_j) represents player j's original information structure (T_j, q);

2) The information structure (X_j, μ_j, P'_j) represents player j's information structure that i perceives, or (Y^i_j, p^i_j);

3) P_j is a refinement of P'_j.

Before we show the main result, the following definition and theorem are useful. Let (Y, q) and (Y', q') be two information structures about the value of same $\phi \in \Phi \equiv \{\phi_1, \ldots, \phi_K\}$, with $Y = \{y_1, \ldots, y_L\}$ and $Y' = \{y'_1, \ldots, y'_H\}$. Denote the conditional probability distribution $q(y|\phi_k)$, $k \in \{1, \ldots, K\}$, by a row vector π_k. Denote the conditional probability distribution $q'(y'|\phi_k)$ by a row vector π'_k.

Definition 2.4.3. Two information structures (Y, q) and (Y', q') satisfy Blackwell's condition if and only if there exists a Markov matrix B such that

$$\Pi' = \Pi B, \tag{2.4.1}$$

where $\Pi = (\pi_k(y_l))$, $l \in \{1, \ldots, L\}$ and $\Pi' = (\pi'_k(y'_h))$, $h \in \{1, \ldots, H\}$.

Theorem 2.4.1. Green and Stokey (1978) *If two information structures (Y, q) and (Y', q') satisfy Blackwell's condition 2.4.1, then there exists (X, μ, P, P') such that*

1. *(X, μ, P) represents (Y, q);*
2. *(X, μ, P') represents (Y', q');*
3. *P refines P'.*

The proof of the theorem is mainly about how to construct the elements (X, μ, P, P'). We will show that in the following subsection. But first we can present the following result.

Proposition 2.4.2. *Given that Player j's original types are (T_j, q) and Player i believes that Player j's types are (Y_j^i, p_j^i), Player i is partially aware of Player j's original information structure.*

Proof. 1. Conditional on every $\omega \in \Theta \times T_{-j}$, Player j's original type t_j's probability distribution is $q(t_j|\omega)$. Let the set T_j be indexed by $m \in \{1, \ldots, |T_j|\}$ and the set $\Theta \times T_{-j}$ be indexed by $n \in \{1, \ldots, |\Theta \times T_{-j}|\}$. We define a matrix Π_j such that an element $\pi_{nm} = q(t_{jm}|\omega_n)$.

2. Conditional on every $\omega \in \Theta \times T_{-j}$, by Conditions 2.3.3 and 2.3.4, the probability of type y_j^i given ω is

$$p_j^i(y_j^i|\omega) = \begin{cases} \chi & \text{if } y_j^i = y_{jx}^i, \\ (1-\chi)q(t_j|\omega) & \text{if } y_j^i \neq y_{jx}^i, \text{ and } f_j^i(y_j^i) = t_j. \end{cases} \tag{2.4.2}$$

Let the set Y_j^i be indexed by $m' \in \{1, \ldots, |Y_j^i|\}$. Define a matrix Π_j^i such that an element $\pi'_{nm'} = p_j^i(y_{jm'}^i|\omega_n)$. Properly arrange the indexes we will have that

$$\Pi_j^i = \begin{pmatrix} \chi I_{|\Theta \times T_{-j}| \times 1}, & (1-\chi)\Pi_j \end{pmatrix}.$$

Therefore there is a Markov matrix

$$B = \begin{pmatrix} \chi & 1-\chi & 0 & \ldots & 0 \\ \chi & 0 & 1-\chi & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \chi & 0 & 0 & \ldots & 1-\chi \end{pmatrix},$$

such that $\Pi_j^i = \Pi_j B$. Blackwell's condition 2.4.1 is satisfied. By Theorem 2.4.1, the result follows. **QED.**

Putting Lemma 2.3.2 and Proposition 2.4.2 together, we have the main result.

Theorem 2.4.3. *For any $\chi \in (0, 1]$, the set of cursed equilibria of a game coincides a set of Bayesian Nash equilibria of an augmented game where players are partially aware of other players' original information structures.*

Define the complexity of his strategic computation by the cardinality of the partitions of other players perceived by Player i. Because partition P_j refines P'_j, when Player i perceives P'_j rather than P_j, the complexity decreases.

Corollary 2.4.4. *The complexity of players' strategic computation decreases when the cursedness χ becomes positive.*

2.4.1 Constructing information partitions.

The construction uses the method of Green and Stokey (1978) shown in the proof of Theorem 2.4.1. For notational brevity, we omit the subscripts of (X_j, μ_j, P_j, P_j'). The set $X = T_j \times Y_j^i$. The partitions $P = \{(t_j, y_j^i) | t_j \in T_j, y_j^i \in Y_j^i\}$ and $P' = \{T_j \times \{y_j^i\} | y_j^i \in Y_j^i\}$. We see that P refines P'.

The probability measure

$$\mu\big(\{(\omega, x)\}\big) = \mu\big(\{(\omega_n, t_{jm}, y_{jm'}^i)\}\big) = q(\omega_n, t_{jm}) b_{mm'}.$$

where $b_{mm'}$ is the element of Markov matrix B.

Now we need to show that (X, μ, P) represents (T_j, q), i.e. they satisfy Definition 2.4.1. Take $\tau((t_j, y_j^i)) = t_j$, so that $\tau^{-1}(\bar{t}_j) = \{\{(\bar{t}_j, y_j'^i)\} | y_j'^i \in Y_j^i\}$. Then

$$\frac{\mu(\omega_n, t_{jm}, y_{jm'}^i)}{\mu(\Theta \times T_{-j} \times (t_{jm}, y_{jm'}^i))} = \frac{q(\omega_n, t_{jm}) b_{mm'}}{\sum_n q(\omega_n, t_{jm}) b_{mm'}} = q(\omega_n | t_{jm}).$$

This verifies the first condition in the definition.

Because $\sum_{m'} b_{mm'} = 1$ for every m,

$$\begin{aligned} \sum_{w \in \tau^{-1}(t_{jm})} \mu(\Theta \times T_{-j} \times w) &= \sum_n \sum_{m'} \mu(\omega_n, t_{jm}, y_{jm'}^i) \\ &= \sum_n \sum_{m'} q(\omega_n, t_{jm}) b_{mm'} \\ &= q(\Theta \times T_{-j}, t_{jm}). \end{aligned}$$

The second condition in the definition is also verified.

To show that (X, μ, P') represents (Y_j^i, p_j^i), define $\tau((t_j, y_j^i)) = y_j^i$ so that $\tau^{-1}(\bar{y}_j^i) = T_j \times \{\bar{y}_j^i\}$, then follow the same logic.

2.4.2 Lemon market: part 3

The probability distribution specified in part 2 implies that for the original types of the seller we have

$$\Pi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

and $\Pi_{vs} = Pr(s|v)$, where $v \in \{v_h, v_l\}$ is the row index and $s \in \{g, b\}$ is the column index.

For the perceived types of the seller, we have

$$\Pi' = \begin{pmatrix} \chi & 1-\chi & 0 \\ \chi & 0 & 1-\chi \end{pmatrix},$$

and $\pi'_{vs'} = Pr(s'|v)$, where $v \in \{v_h, v_l\}$ is the row index and $s' \in \{x, g, b\}$ is the column index.

The Markov matrix is $B = \Pi'$.

We construct a set $X = \{g, b\} \times \{x, g, b\}$ and partitions P and P', where $P = \{\{gg\}, \{gx\}, \{gb\}, \{bg\}, \{bx\}, \{bb\}\}$, and $P' = \{\{gg, bg\}, \{gx, bx\}, \{gb, bb\}\}$. It is easy to see that P refines P'. The measure μ is given as

$$\mu(v, s, s') = Pr(v) \times \Pi_{vs} \times \Pi'_{ss'}.$$

$\mu(v, s, s')$	$v = v_h$	$v = v_l$
$s = g, s' = x$	$\frac{\chi}{2}$	0
$s = g, s' = g$	$\frac{1-\chi}{2}$	0
$s = g, s' = b$	0	0
$s = b, s' = x$	0	$\frac{\chi}{2}$
$s = b, s' = g$	0	0
$s = b, s' = b$	0	$\frac{1-\chi}{2}$

Table 2.1: The probability distribution μ.

With a mapping $\tau(\{ss'\}) = s$, $\forall s \in \{g, b\}$ and $s' \in \{x, g, b\}$, we can check that (X, μ, P) is a representation of $s \in \{g, b\}$. A similar check for (X, μ, P') uses a mapping $\tau'(\{g, b\} \times \{s'\}) = s'$, $\forall s' \in \{x, g, b\}$.

2.5 An alternative lemon market model with partial awareness

To emphasize the idea we present this example in a reverse order. We define set $X = \{v_h, v_l\} \times \{g, b\} \times \{g', b'\}$, partition $P = \{\{gg'\}, \{gb'\}, \{bg'\}, \{bb'\}\}$, the partition $P' = \{\{gg', bg'\}, \{gb', bb'\}\}$. Hence P refines P'. Given $q \in (\frac{1}{2}, 1]$, the distribution μ is as

$\mu(v, s, s')$	$v = v_h$	$v = v_l$
$s = g, s' = g'$	$\frac{q}{2}$	0
$s = g, s' = b'$	$\frac{1-q}{2}$	0
$s = b, s' = g'$	0	$\frac{1-q}{2}$
$s = b, s' = b'$	0	$\frac{q}{2}$

Table 2.2: The probability distribution μ.

Thus if the buyer believes that the seller's information is represented by partition P', he knows that the seller could essentially have two types $s' \in \{g', b'\}$. Define a mapping $\tau' : P' \to \{g', b'\}$ so that $\tau'(\{gg', bg'\}) = g'$ and $\tau'(\{gb', bb'\}) = b'$. It implies that

$$Pr(s' = g') = \sum_{v \in \{v_h, v_l\}} \sum_{s \in \{g, b\}} \mu(v, s, s' = g') = \frac{1}{2};$$

$$Pr(s' = b') = \frac{1}{2};$$

$$Pr(v = v_h | s' = g') = \frac{\frac{q}{2}}{\frac{1}{2}} = q;$$

$$Pr(v = v_l | s' = b') = \frac{\frac{q}{2}}{\frac{1}{2}} = q;$$

Believing this, the buyer expects that the seller never sells at type $s' = g'$. But if $s' = b'$, he sells because $2000(1-q) \leq 1000$, the expected value of the car is $3000(1-q)$ to the buyer. So if $q < \frac{2}{3}$, he wants to buy in equilibrium. It is just like in a cursed equilibrium.

But the original types are given by partition P.

Define a mapping $\tau : P \to \{g, b\}$ so that $\tau(\{ss'\}) = s$. It implies that for every $s'' \in \tau^{-1}(s)$,

$$Pr(s'' = gg') = \sum_{v \in \{v_h, v_l\}} \mu(v, s = g, s' = g') = \frac{q}{2};$$

$$Pr(s'' = gb') = \sum_{v \in \{v_h, v_l\}} \mu(v, s = g, s' = b') = \frac{1-q}{2};$$

$$Pr(s'' = bg') = \sum_{v \in \{v_h, v_l\}} \mu(v, s = b, s' = g') = \frac{1-q}{2};$$

$$Pr(s'' = bb') = \sum_{v \in \{v_h, v_l\}} \mu(v, s = b, s' = b') = \frac{q}{2};$$

and

$$Pr(v = v_h | s'' = gg') = Pr(v = v_h | s'' = gb') = 1;$$
$$Pr(v = v_l | s'' = bg') = Pr(v = v_l | s'' = bb') = 1.$$

These imply that the seller has perfect information. And the buyer shall not trade in the equilibrium.

In addition, the true types induce

$$\Pi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

where $\pi_{11} = Pr(s = g|v = v_h)$, $\pi_{12} = Pr(s = b|v = v_h)$, $\pi_{21} = Pr(s = g|v = v_l)$, $\pi_{22} = Pr(s = b|v = v_l)$.

And the perceived types induce

$$\Pi' = \begin{pmatrix} q & 1-q \\ 1-q & q \end{pmatrix},$$

where $\pi'_{11} = Pr(s' = g|v = v_h)$, $\pi'_{12} = Pr(s' = b|v = v_h)$, $\pi'_{21} = Pr(s' = g|v = v_l)$, $\pi'_{22} = Pr(s' = b|v = v_l)$.

A Markov matrix $B = \Pi'$ satisfies Blackwell's condition $\Pi' = \Pi B$. The probability measure μ can be derived from Π and Π'.

2.6 Conclusions

In this paper we show the innovative cursed equilibrium concept is a special case of Bayesian Nash equilibrium with partial awareness. It justifies the first concept and suggests that the second concept is more general and may have potential to explain imperfect strategic sophistication from the cognitive aspects. Further applications in auction and trading can be promising.

Chapter 3

Fair Disclosure and Investor Asymmetric Awareness in Stock Markets

The US Security and Exchange Commission implemented Regulation Fair Disclosure in 2000, requiring that an issuer must make relevant information disclosed to any investor available to the general public in a fair manner. Focusing on firms that are affected by the regulation, we propose a model that characterizes the behavior of two types of investors—one professional investor and many small investors—in the regimes before and after the regulation, i.e., under selective disclosure and fair disclosure. In particular, we introduce the concept of awareness and allow investors to be aware of relevant information symmetrically or asymmetrically.

We show that with symmetric awareness, fair disclosure induces both a low cost of capital and a low cost of information, therefore making the market efficient. Also, the professional investor collects an equal level of information under fair disclosure than under selective disclosure. However when small investors are not fully aware, fair disclosure still induces a low cost of capital but may induce a high cost of information. The professional investor may deliberately collect less information under fair disclosure than under selective disclosure.

With asymmetric awareness, our theory produces predictions that match the empirical findings by Ahmed and Schneible Jr. (2004) and Gomes, Gorton and Madureira (2007). They find that small and complex firms are negatively affected by the regulation. We also show that fair disclosure improves the welfare of small investors when they are extremely unaware. Such results are not compatible with the standard symmetric awareness assumption.

3.1 Introduction

In stock markets, information is usually transmitted from issuers to investors through several different channels: 1. mandatory public disclosure by issuers; 2. voluntary public or private disclosure by issuers; 3. private acquisition by investors from sources other than the issuer, such as purchasing research reports from stock analysts, examining the firm's products or services, and consulting the firm's competitors.

While most small investors mainly rely on public disclosure, professional investors use all channels. In particular, some professional investors are selected by the issuer to receive material information, for example, through quarterly analyst conference calls. Many issuers favor such selective disclosure for practical reasons, such as concealing information from their competitors. However selective disclosure is viewed with suspicion by the regulatory authorities, since it creates a class of informationally privileged investors. Since selective disclosure creates information asymmetry, it also increases the cost of capital (Verrecchia, 2001).

To curtail selective disclosure, in October 2000, the U.S. Securities and Exchange Commission ratified *Regulation Fair Disclosure* (henceforth *the regulation*), also commonly referred to as Reg FD. This ruling requires that an issuer must make material information disclosed to securities market professionals or shareholders available to the general public *simultaneously* (for intentional disclosures), or *promptly* (for non-intentional disclosures). [1]

In this paper we model the regulation as requiring an issuer only to disclose information in a public forum where participants (professional investors and small investors) ask questions. This disclosure form is referred to as fair disclosure, in contrast to selective disclosure, where only professionals can ask questions.

At first glance, fair disclosure seems the best remedy for the information asymmetry caused by selective disclosure. It also reduces the incentive of private information acquisition, because information disclosed by the issuer usually has high quality and is relatively easy to collect. However practitioners have argued that the regulation has produced some undesirable side effects:

1. The ambiguous definition of material information makes issuers reluctant to

[1] The debate on the benefit and cost of Reg FD has never stopped since it was proposed by the SEC in December 1999. By the time it was adopted, "more than 6,000 comment letters, mostly from individual investors, were submitted in response to the Reg FD proposal. Individual investors and the media generally favored the proposed regulation, believing that it would level the playing field for the retail investor. Large brokerage firms, on the other hand, generally opposed the rule, predicting that it would lead to a chilling of the information flow from issuers to the marketplace. " Quote from the introduction of the SEC's *Special Study: Regulation Fair Disclosure Revisited.* Source link: http://www.sec.gov/news/studies/regfdstudy.htm.

provide "immaterial" information. [2]

2. Professionals may be unable to obtain information because of ineffective technology utilized in public communications. [3]

3. Professionals "with the most perception, intuition, or experience" [4] are not willing to share their insights with other investors under fair disclosure, so that less information can be revealed.

Using various approaches, many papers[5] empirically test the effects of the regulation. Although many results are mixed, some of them are relatively clear and related to this paper. We will discuss them later. However, there are only few theoretical analyses. In an interesting paper, Arya, Glover, Mittendorf and Narayanamoorthy (2005) show that with fair disclosure, certain kind of timing of disclosure can cause information cascades, which in term may heighten herding among analysts and leave investors worse off.

In this paper, we explore the third argument more in depth. There are two main reasons driving us. First, this argument was both less understood and less emphasized in practice. Second, the superior knowledge of professionals may imply either that uninformed small investors are just unable to acquire the information and aware of their ignorance, or that they are totally unaware of such information; we want to understand whether these assumptions produce different predictions and which one fits the facts better. To isolate it from the first two arguments and make the modeling simple, we assume that the issuer has to sincerely answer questions posed by investors under both disclosure forms, and that the technology used in public communications is effective.

We analyze the effect of the regulation on the cost of capital, the cost of information, the quantity of information collected by professionals, and the welfare of small investors. We show that the effect differs under different assumptions about investors' awareness. This result hence suggests reconsidering the traditional economic

[2]For one detailed example of opinions on this issue, see "Regulation FD - An Enforcement Perspective" - Remarks of Richard H. Walker, Director, Division of Enforcement, before the Compliance & Legal Division of the Securities Industry Association, New York, November 1, 2000. Source link: http://www.sec.gov/news/speech/spch415.htm.

[3]In August 2000, only 41.5 % of households had Internet access. The SEC has been revising the rule to ease the communications as the use of the Internet is growing.

[4]Words from the comment of the Association for Investment Management and Research to SEC regarding the regulation. Source link: http://www.cfainstitute.org/centre/issues/comment/2000/00disclosure.html.

[5]The following list of these papers is not meant to be complete: Soffer and Zhang (2001), Straser (2002), Sunder (2002), Zitzewitz (2002), Bailey et al. (2003), Heflin et al. (2003), Irani and Karamanou (2003), Ahmed and Schneible Jr. (2004), Bushee et al. (2004), Carnaghan and Sivakumar (2004), Irani (2004), Gadarowski and Sinha (2005), Griffin et al. (2005), Agrawal et al. (2006), Ferreira and Smith (2006), Francis et al. (2006), Gomes et al. (2007), Ke et al. (2006), Mohanram and Sunder (2006), Sidhu et al. (2006), Heflin et al. (undated).

approach to understanding the regulation; furthermore, it suggests that "asymmetric awareness" may offer important insights in certain fields of economic research, such as information disclosure where some agents are specialists.

We start by defining "asymmetric awareness" and discussing how it differs from "asymmetric information". In general, information may have many dimensions. For instance, information that affects a firm's value may regard general management, finance, marketing, technology, government regulation, and so on. In the literature on epistemic foundations of game theory, these dimensions are called awareness information, or simply awareness. Agents have asymmetric awareness if they have different dimensions of information. They are unaware of some information because either they are not aware of the existence of the information (this is what "unawareness" literally means), or they are not able to incorporate the information in their decision making—for instance, a layman can tell a technical term but can not properly use it. In many cases, imposing symmetric awareness makes an economic theory more appealing, especially for normative purposes.[6] However recent work in game theory (Modica and Rustichini, 1999; Feinberg, 2005; Li, 2006; Heifetz, Meier and Schipper, 2006) develops clear mathematical characterizations for awareness and enables us to understand asymmetric awareness in economic and game theoretical models.

The asymmetric information assumption in "symmetric awareness" models means that the *uninformed* agents know exactly how informed agents update their beliefs, though they don't have the same information. In short, they know that they don't know. On the contrary, with asymmetric awareness, *unaware* agents have no idea that other agents may update their beliefs upon certain pieces of information. In short, they don't know that they don't know. Such "ignorance about ignorance" is a consistency condition imposed on the belief hierarchies of agents in order to characterize unawareness.

Consider the following example. There are two agents A and B with different awareness from a set of dimensions about an object, $S = \{\text{color, size, shape}\}$. Agent A is aware of the set $S_A = \{\text{color, size}\}$ while agent B is aware of the set $S_B = \{\text{size, shape}\}$. Thus A is unaware that shape even is an issue, while B is unaware that the object may be of different colors. Moreover, when it comes to interactive awareness, since A is not aware of the shape, she considers that B is only aware of $S_B \cap S_A = \{\text{size}\}$, and she *believes* that this is common knowledge (Li, 2006; Heifetz et al., 2006).

Our main premise is that small investors are aware of fewer dimensions than professionals. This is supported by finance and accounting literature on small investors' behavior. For instance, Malmendier and Shanthikumar (forthcoming) find empirical

[6] A common opinion is that unawareness, like other behavioral assumptions or bounded rationality, has too little regularity to show interesting insights. Another difficulty is that unawareness can imply inconsistent preferences and make it ambiguous to define and calculate welfare.

evidences suggesting that small traders fail to account for the distortion in analyst stock recommendations, while large traders do not. For practical reasons, professionals usually take significant costs to gain not only more detailed information but also more dimensions of the information. Under fair disclosure, when professionals ask these questions about these dimensions of the information, small investors are very likely to know as much as professionals know. This potential free-riding certainly reduces the incentive of professionals to ask all the questions that they have. Before discussing our model, we invite readers to note a comment by one of the professionals six months after the regulation was implemented [7]:

> ... analysts, even if given an opportunity to ask all of his or her questions in a public forum, will not do so; at least buy side analysts will not do so. And this reflects the fact that the very questions posed by insightful, well-prepared and skilled analysts have value. At times, I would submit, even greater value than any particular answer that a company executive may provide.

In our model, we consider a general capital financing and stock trading scenario. There are one issuer, many small investors, one professional, and many market makers. The issuer raises capital from small investors in a primary market. Small investors expect to trade upon a potential liquidity shock in a secondary market, where the professional with his private information may trade for speculative purposes. Market makers set prices and execute orders.

Since uninformed market makers are aware of the private information of the professional, their prices reflect the information asymmetry and increase small investors' transaction costs. Having rational expectations, small investors demand from the issuer a return compensating both the initial investment and the transaction costs. This required return determines the cost of capital of the issuer.

The professional has several means to collect information at various costs. He can always privately acquire information, with a cost function increasing in the quantity of information; or, under selective disclosure, he can enter selective forums with a fixed cost as an entry fee; or, under fair disclosure, he can enter public forums at no cost.

We focus on an issuer who is affected by the regulation; namely under selective disclosure the professional uses the issuer's selective forums instead of private acquisition to acquire information. We define a stock market to be *efficient* if it maintains a low cost of capital and a low cost of information. Our first result is that, with symmetric awareness, the market is efficient under fair disclosure; but with asymmetric

[7]Comment by Eric Roiter, Sr., VP/General Counsel, Fidelity Management & Research, in Regulation FD Roundtable, on April 24, 2001.

Source link: http://www.sec.gov/news/studies/regfdconf.txt.ds.

awareness, although the cost of capital is low, the cost of information may be high, hence efficiency is not assured. The reason can be briefly explained as follows.

Suppose small investors have the same set of questions in mind, i.e., they are fully aware. Under fair disclosure, they can ask all the questions in public forums and acquire all information. The professional has no advantage even if he can find answers privately. This leads to zero cost of information and a low cost of capital due to symmetric information. Under selective disclosure, the professional incurs costs of information when he enters selective forums. Meanwhile, small investors with full awareness demand extra return for the information asymmetry, inducing a high cost of capital. Hence the market is efficient under fair disclosure.

What if small investors are not fully aware? Under fair disclosure, the cost of capital remains low because small investors can ask all the questions that they have and believe that information is symmetric. However the professional may not ask all questions that he has, and he may privately search for the answers about unasked questions. Therefore fair disclosure may still induce some cost of information. Under selective disclosure, small investors demand a high return due to the expected information asymmetry. The cost of capital is still high. But the cost of information may be lower than that under fair disclosure.

Our second result is on the quantity of information collected by the professional. Many empirical studies on the regulation have examined market or accounting data related to professionals' information. We find that with symmetric awareness, when the regime switches from selective disclosure to fair disclosure, the professional collects the same full information. However with asymmetric awareness, when the regime switches, the professional collects less information if the marginal cost of acquiring information is high enough.

To compare our results with empirical findings, we consider the scenario without the fund raising period so that the ownership share of small investors is fixed before and after the regulation. The model with asymmetric awareness matches the data better. It predicts that for firms which use selective disclosure more before the regulation (usually small firms and complex firms), professionals collect less information after the regulation. This is consistent with the findings on the effect of the regulation by Ahmed and Schneible Jr. (2004) and Gomes, Gorton and Madureira (2007). Ahmed and Schneible Jr. (2004) report that the information quality of average investors is worse for small firms and high tech firms; Gomes et al. (2007) report that small firms and complex firms suffered significant losses of analyst following.

In the law literature, without an analytical model, Thompson and King (2001) made an alternative argument on small firms' analyst following. Due to the regulation, if issuers generally provides less information, then analysts have to work harder for more information. If the benefit does not increase correspondingly, the total amount of information collected must be less than under selective disclosure. The analysts will follow fewer stocks and smaller stocks will most likely be ignored. This argument complements ours, since we do not assume that issuers do withhold information upon request, which is the first argument chilling effect argument as aforementioned. Also the empirical examinations of Reg FD (Sunder, 2002; Heflin et al., 2003; Choi, 2003) suggest that after the regulation, issuers in general disclose more information on many aspects about the stocks. Therefore the bottom line is that both asymmetric awareness and the hiding of issuers are causes of high cost of information. In addition, our result implies that even with corporative issuers, the disclosure may have negative impact on a group of firms.

At last we address the welfare of small investors. We show that if small investors are very unaware, and the professional does not share any information, then fair disclosure may reduce some losses, though not completely. In that sense, the regulation does help small investors. This effect of the regulation is not revealed with symmetric awareness, because then small investors with rational expectations always break even under both disclosure forms.

The remainder of this paper is organized as follows. In Section 3.2 we discuss related literature. In Section 3.3 we present the basic model and a few preliminary results. The symmetric awareness case is analyzed as a benchmark in Section 3.4. The asymmetric awareness case is analyzed in Section 3.5. In both cases, we first study selective disclosure then fair disclosure. In Section 3.6 we discuss the implications on the professional's information and small investors' welfare. Conclusions are in Section 3.7. All proofs are collected in Appendix 3.8.1. A generalized model allowing both selling and buying stocks is discussed in Appendix 3.8.2.

3.2 Related literature

Our paper is closely related to the literature on the theory of information disclosure in financial markets. The standard paradigm in that literature assumes fully aware and rational agents (see Verrecchia, 2001; Dye, 2001). In contrast, our model emphasizes that unawareness of some agents can have important effects, although we retain the rationality assumption. In the literature on auctions, Milgrom and Weber (1982) allow one bidder to acquire some information and compare the value of information with and without other bidders aware of this activity.

Our paper also contributes to a growing body of literature related to unawareness in different contexts. For example, Gabaix and Laibson (2006) study consumer product markets and Abreu and Brunnermeier (2003) study a stock market. In the first paper, the authors assume some consumers are myopic or unaware of negative attributes of products, and show that even if competitors have zero cost to educate consumers, the education does not happen in equilibrium. In the second paper, the authors assume that less sophisticated traders are unaware of the possibility of burst, and that rational arbitrageurs are not able to coordinate to correct the price because they become sequentially aware that the price has departed from fundamentals and they are not sure whether they learned that early or late relative to other rational arbitrageurs. In such settings the authors show that bubbles can be a unique equilibrium. In more abstract contexts, Modica, Rustichini and Tallon (1998) study the impact of unawareness in the theory of general equilibrium with pure exchange economies, and Kawamura (2005) extends the model to economies with production.

3.3 A stock market model

This section contains three parts. The first one introduces the agents, their goals, and the rules of the game. The second one discusses the information structures of agents. The third one presents some preliminary results. Our model is based on the model in Section 4 of Verrecchia (2001), which in turn draws on Diamond and Verrecchia (1991) and Baiman and Verrecchia (1996).

3.3.1 Time line

There are a monopoly issuer K, a measure one of small investors U, a professional investor I, and many market makers M. All agents are risk neutral and no investor has budget constraints. All this information is common knowledge.

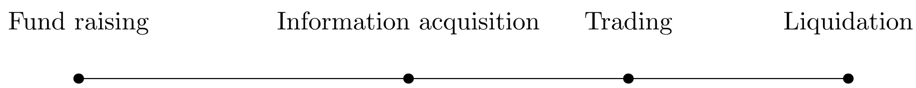

Figure 3.1: Time line of the game

The time line is shown in Figure 3.1. In the fund-raising period, the issuer raises a capital C for a risky project, whose return is a nonnegative real number $v \in \mathbb{V}$, as the realization of random variable V with expectation $E(V) > C$. Small investors are the only investors in the primary stock market. For the settings of this period, we assume that: 1. information is symmetric between the issuer and small investors; 2. the issuer offers an identical leave-it-or-take-it contract to every small investor, so that each small investor invests C and obtains R shares, $R \in [\frac{C}{E(V)}, 1]$.

In the information acquisition period, relevant information is generated and the professional can do the following: 1. privately acquire information from sources other than the issuer; 2. enter selective forums under selective disclosure; 3. enter public forums under fair disclosure. Small investors however are only able to acquire information under fair disclosure.

In the trading period, both the professional and small investors trade in a secondary market through competitive market makers. We assume that small investors may experience a negative liquidity shock with probability $q \in (0,1)$; when it happens, they always sell all the shares that they own. The professional trades on private information for profits; when he knows that the stock is over valued, he shorts r shares with a cap $N \leq \frac{C}{E(V)}$ [8]. For simplicity, we do not consider the case when the traders need to buy shares. In the appendix, we derive the same results in an extended model where both buying and selling are possible.

Market makers can not tell from whom they buy and do not acquire private information, but they anticipate the amount of the shares that small investors or the professional want to sell and the information structure of the professional. As market makers compete in a Bertrand style, they set a price λ, which is nonnegative, just to break even. In this way, the realized information of the professional is not revealed in the market equilibrium.

In the liquidation period, the value of the project is realized and the game ends.

3.3.2 Information structure

Denote the set of states of the world by $\mathbb{S}$. Its element s is the realization of a random variable S. The prior belief is $P \in \Delta\mathbb{S}$. A signal τ^θ is chosen from a family of partitions of $\mathbb{S}$, which is denoted by $\{\tau^\theta\}_{\theta\in\Theta}$, where set $\Theta = \{\underline{\theta}\} \cup [0, \bar{\theta}]$, $\underline{\theta} < 0$. At every s, given signal τ^θ, the realized value of signal is $\tau^\theta(s)$; an agent's posterior belief about S is $P(s|\tau^\theta(s)) \in \Delta\mathbb{S}$ and the posterior belief about v is $F(v|\tau^\theta(s)) \in \Delta\mathbb{V}$.

The set of all possible signals, $\{\tau^\theta\}_{\theta\in\Theta}$ is a fully ordered set with a larger θ referring to a finer signal. In other words, if $\theta > \theta'$, then partition τ^θ is finer than partition $\tau^{\theta'}$. Signal $\tau^{\underline{\theta}}$ is a null signal representing no information.

Before the trading period, the private information of the professional is denoted by τ^θ, the public information that small investors and market makers have is denoted by τ^η, $\eta \in \Theta$.

The issuer has the finest information $\bar{\tau} \equiv \tau^{\bar{\theta}}$ at no cost. Under either form of disclosure, the issuer commits to answer questions with the finest information that he has. It is equivalent to assume that given the issuer's information, he commits to disclose the best information that investors can possibly acquire by all other means.

Under selective disclosure, small investors can not collect information. When the professional chooses to enter the selective forum, the cost of information is F_s ("s" for selective forums), a fixed cost of entry which could be interpreted as an entry fee needed to gain the trust of the issuer. When the professional chooses to privately acquire, the cost is $C_p(\theta)$ ("p" for private acquisition), an increasing function of θ with $C_p(\underline{\theta}) = 0$.

[8]That is, he can only short no more than the minimum total amount of the shares.

Under fair disclosure, both small investors and the professional collect information at no cost when the issuer answers to questions posed by any investor. When the professional privately acquires τ^θ, the cost depends on both τ^θ and τ^η. Since the professional knows the public information, $\theta \geq \eta$. Then the cost is a function $C_f(\theta, \eta)$ ("f" for fair disclosure), which increases in θ. Since τ^η substitutes τ^θ, we assume that the cost $C_f(\theta, \eta) = 0$ if $\theta = \eta$ and decreases in η.

We define the cost of capital $C_c \equiv RE(V)$. We also define the cost of information C_i as the professional's cost. Under selective disclosure, if he uses selective forum, it is F_s; if he uses private acquisition, its $C_p(\theta)$. Under fair disclosure, it is $C_f(\theta, \eta)$.

Regarding awareness, the issuer, market makers and the professional are aware of $\mathbb{S}$ and all above are common knowledge among them. If awareness is symmetric, small investors are also aware of $\mathbb{S}$, and all above is common knowledge among all agents.

If awareness is asymmetric, small investors are certainly aware of V, but they are only aware of τ^θ with $\theta \in \{\underline{\theta}, 0\}$. In other words, they ignore signals τ^θ with $\theta \in (0, \bar{\theta}]$, which depends on some dimensions that they are unaware of. Using the language of game theory, it is convenient to assume that there are two versions of games g and G in agents' minds: small investors believe that game g is common knowledge to all; however, the issuer, market makers and the professional believe that game G is common knowledge to all except small investors; they also believe that it is common knowledge to all that small investors believe in g. [9]

3.3.3 Preliminary results

First we briefly restate the model. It is a three-stage game with observed actions. In the fund-raising period, the share of ownership R is determined by the monopoly issuer, who have the future periods in mind. Then all other agents know R. In the information acquisition period, the professional acquires θ, and at state s observes $\tau^\theta(s)$, while the public observes $\tau^\eta(s)$. Market makers also observe that the professional acquires θ. In the trading period, simultaneously, small investors sell R shares with probability q, the professional sells r shares, and market makers set price λ.

We begin with small investors and the determination of ownership share R. The optimal contract for the issuer is to keep small investors just break-even, namely every small investor earns zero expected profit.

For every small investor, in the case of a negative liquidity shock, he sells all shares he owns; his profit is $E(\lambda)R - C$, where $E(\lambda)$ is the expectation of the prices in the trading period. Otherwise, his profit is $E(V)R - C$. Since the shock will happen with probability q, his *ex ante* expected profit is

[9]See Feinberg (2005) for a formal model of games with incomplete awareness.

$$\pi_U := R[qE(\lambda) + (1-q)E(V)] - C. \qquad (3.3.1)$$

Now consider the trading period. We want to show that given information τ^θ and τ^η, there is a unique equilibrium where the price set by market makers and the quantity of shares traded by the professional are determined.

The professional's *ex post* profit is $\pi_I = r(\lambda - v)$. His decision problem is to maximize his interim expected profit from trade:

$$\max_{r\in[0,N]} E_\theta(\pi_I|s) := \int_{\mathbb{V}} r(\lambda - v)dF(v|\tau^\theta(s)). \qquad (3.3.2)$$

Note that because the professional can always choose not to trade, the optimal profit is nonnegative. Because the public information is not finer than the private information and λ depends on public information, the expression becomes

$$\int_{\mathbb{V}} r(\lambda - v)dF(v|\tau^\theta(s)) = r[\lambda - \int_{\mathbb{V}} vdF(v|\tau^\theta(s)] = r[\lambda - E_\theta(V|s)].$$

The optimal choices is, $r = N$ when the conditional expected value is $E_\theta(V|s) < \lambda$, and $r = 0$ otherwise.

Fact 1. *Given λ and τ^θ, define $d(\lambda,\theta) \equiv \{s|E_\theta(V|s) < \lambda\}$. An optimal trade rule of the professional is a function of s, λ, and θ.*

$$r(s;\lambda,\theta) = \begin{cases} N & \text{if } s \in d(\lambda,\theta) \\ 0 & \text{if else.} \end{cases}$$

When the professional uses the optimal trade rule, his interim expected profit conditional on the public information is:

$$E_\eta(\pi_I|s) = \int_{s'\in d(\lambda,\theta)} N(\lambda - E_\theta(V|s'))dP(s'|\tau^\eta(s)). \qquad (3.3.3)$$

Therefore $E_\eta(\pi_I|s)$ is determined by (θ,λ). It is useful to prove the following results.

Lemma 3.3.1. *Given any $s \in \mathbb{S}$, the profit function $\big[E_\eta(\pi_I|s)\big](\theta,\lambda)$ has the following properties:*

i It increases in θ. That is, if $\theta' > \theta$ (i.e., for all $s' \in \tau^\eta(s)$, $\tau^{\theta'}(s') \subset \tau^\theta(s')$), then $\big[E_\eta(\pi_I|s)\big](\theta',\lambda) \geq \big[E_\eta(\pi_I|s)\big](\theta,\lambda)$.

ii It increases in λ. That is, if $\lambda' > \lambda$, then $\big[E_\eta(\pi_I|s)\big](\theta,\lambda') \geq \big[E_\eta(\pi_I|s)\big](\theta,\lambda)$.

iii If the probability measure of the states where the conditional expectation of V is less than $x \in R_+$, or $G(x|s) \equiv \frac{Prob\{s'|s' \in \tau^\eta(s), E_\theta(V|s')<x\}}{Prob\{s'|s' \in \tau^\eta(s)\}}$, is smooth in x, then $E_\eta(\pi_I|s)$ is continuous in λ.

When the professional trades r shares and the market price is λ, market makers 's aggregate *ex post* profit is $\pi_M = (Rq+r)(v-\lambda)$. Because of the Bertrand competition and all market makers are homogeneous, given public information τ^η, equilibrium price is given by the highest λ (for they are competing for sell orders) which satisfies the zero profit condition:

$$E_\eta(\pi_M|s) := \int_{\mathbb{S}} \int_{\mathbb{V}} (Rq + r)(v - \lambda) dF(v|\tau^\theta(s')) dP(s'|\tau^\eta(s)) = 0. \tag{3.3.4}$$

By Condition 3.3.4 and Equation 3.3.3, the optimal pricing rule of market makers is given by

$$\lambda(s; R, \eta, \theta) = \max \left\{ \lambda \geq 0 \middle| Rq[E_\eta(V|s) - \lambda] = E_\eta(\pi_I|s) \right\}. \tag{3.3.5}$$

We prove the following Lemma in the Appendix.

Lemma 3.3.2. *The following is true for the optimal pricing rule $\lambda(s; R, \eta, \theta)$.*

i It exists and is unique.

ii It strictly increases in R, decreases in θ, and is continuous in R.

iii The ex ante expected price $[E(\lambda)](R, \eta, \theta) = \int_{s \in \mathbb{S}} \lambda(s; R, \eta, \theta) dP(s)$ strictly increases in R and decreases in θ.

Hence by Fact 1 and Lemma 3.3.2, given ownership share R, information τ^η and τ^θ, in the trading period, a unique pure strategy equilibrium $\big(\lambda(s; R, \eta, \theta), r(s; \eta, \theta)\big)_{s \in \mathbb{S}}$ exists. The equilibrium *ex ante* expected profit of the professional is

$$\Pi_I = \int_{\mathbb{S}} \int_{\mathbb{V}} r(s; \eta, \theta)(\lambda(s; R, \eta, \theta) - v) dF(v|\tau^\theta(s)) dP(s). \tag{3.3.6}$$

3.4 Symmetric awareness

Suppose that small investors are aware of $\mathbb{S}$, with belief P being common knowledge. First consider selective disclosure. Because there is no public information, namely $\eta = \underline{\theta}$, expected profit $E_\eta(\pi_I|s) = \Pi_I$ for all $s \in \mathbb{S}$, and the stock price is also constant. Denoted the price by $\lambda_1(\theta|R) \equiv [E(\lambda)](R, \underline{\theta}, \theta)$. Equation 3.3.5 implies that

$$\Pi_I = Rq\big(E(V) - \lambda_1(\theta|R)\big).$$

In the information acquisition period, adjusted by the costs of information, the professional's expected profit becomes the following: If he collects information through selective forums,

$$\Pi_I - F_s = Rq\big(E(V) - \lambda_1(\theta|R)\big) - F_s.$$

By Lemma 3.3.2, the optimal $\theta = \bar{\theta}$, for then $\lambda_1(\theta|R)$ is minimized. The optimal profit of the professional is

$$\Pi_{Is} = Rq\big(E(V) - \lambda_1(\bar{\theta}|R)\big) - F_s.$$

If he collects information through private acquisition,

$$\Pi_I - C_p(\theta) = Rq\big(E(V) - \lambda_1(\theta|R)\big) - C_p(\theta). \tag{3.4.1}$$

The optimal θ may not be $\bar{\theta}$ if the marginal cost is too high. Being a function of R, the optimal solution is denoted by $\hat{\theta}(R)$. The optimal profit of the professional is

$$\Pi_{Ip} = Rq\big(E(V) - \lambda_1(\hat{\theta}(R)|R)\big) - C_p(\hat{\theta}(R)).$$

Assume that for all $R \in [\frac{C}{E(V)}, 1]$, it is always profitable for the professional to privately collect information. Then when $\Pi_{Is} > \Pi_{Ip}$, or

$$Rq\lambda_1(\hat{\theta}(R)|R) + C_p(\hat{\theta}(R)) > Rq\lambda_1(\bar{\theta}|R) + F_s, \tag{3.4.2}$$

the professional chooses selective forums. When the equality holds, the professional is indifferent. Otherwise, the professional chooses private acquisition.

For the purpose of this paper, we want to restrict our attention to firms affected by the regulation. For such firms, professionals prefer using selective forums to acquire information about them. It is sufficient for Condition 3.4.2 to hold if the cost F_s is relatively low.

Lemma 3.4.1. *Denote $\hat{\theta}^* = \min\{\hat{\theta}(R)|R \in [\frac{C}{E(V)}, 1]\}$. If*

$$F_s < C_p(\hat{\theta}^*) \tag{3.4.3}$$

then Condition 3.4.2 holds for all $R \in [\frac{C}{E(V)}, 1]$.

For the rest of this paper, we assume Assumption 3.4.3 holds. Now we include the fund raising period when the ownership share is determined and show the existence of the equilibrium. By Definition 3.3.1, the issuer's optimal contract makes every small investor break-even, namely

$$\bar{R} \equiv \min\left\{R \in [\frac{C}{E(V)}, 1] \middle| R = \frac{C}{E(V)(1-q) + q\lambda_1(\bar{\theta}|R)}\right\}. \tag{3.4.4}$$

If the likelihood of liquidity shock is too large, it is possible that small investors require $R_e > 1$ when they expect a low stock price, which is not acceptable to the issuer. To ensure the initial investment, we assume that this likelihood is relatively small, or $q \leq 1 - \frac{C}{E(V)}$. Using the fixed point argument, we prove the existence and the uniqueness of the equilibrium. Here we summarize the results.

Proposition 3.4.2. *Denote an equilibrium by a vector* $\big(R_e, \eta_e, \theta_e, \lambda_e(s), r_e(s)\big)_{s\in\mathbb{S}}$. *Assume that* $q \leq 1 - \frac{C}{E(V)}$. *Under selective disclosure, a unique equilibrium exists and is*

$$\left(R_e = \bar{R},\ \ \eta_e = \underline{\theta},\ \ \theta_e = \bar{\theta},\ \ \lambda_e(s) = \lambda_1(\bar{\theta}|\bar{R}),\ \ r_e(s) = r(s; \lambda_1(\bar{\theta}|\bar{R}), \bar{\theta})\right)_{s\in\mathbb{S}}.$$

In this equilibrium, $C_c = \bar{R}E(V)$ *and* $C_i = F_s$.

Now consider fair disclosure. Since small investors have costless access to public forums, and the issuer answers questions sincerely, small investors can acquire as much information as they want to reduce their loss in trade. The professional's advantage in private acquisition becomes ineffective. In the Appendix, we prove that full information disclosure can happen in an equilibrium.

Proposition 3.4.3. *Under fair disclosure, there is an equilibrium where full information is disclosed, or*

$$\left(R_e = \frac{C}{E(V)},\ \ \eta_e = \theta_e = \bar{\theta},\ \ \lambda_e(s) = E(V|\bar{\tau}(s)),\ \ r_e(s) = 0\right)_{s\in\mathbb{S}}.$$

In this equilibrium, $C_c = C$ *and* $C_i = 0$.

In addition, there may be other equilibria where $\eta_e = \theta_e < \bar{\theta}$*, thus only partial information is disclosed, but it remains true that* $R_e = \frac{C}{E(V)}$, $C_c = C$ *and* $C_i = 0$.

Note that by Definitions 3.4.4, $\bar{R} \geq \frac{C}{E(V)}$. So the cost of capital is low under fair disclosure.

If small investors are not sure what cost it takes for the professional to privately acquire information, then requesting full information disclosure is a dominant strategy for them, because the professional will not have a chance to profit from acquiring private information. Also note that the investment is sure and it is not necessary that the possibility of shock q has to be small as assumed in Proposition 3.4.2. So investment is more likely to happen under fair disclosure.

Theorem 3.4.4. *Suppose awareness is symmetric. Fair disclosure is more efficient than selective disclosure. Namely, both the cost of the capital and the cost of information are greater under selective disclosure.*

Meanwhile, the professional's information does not change under fair disclosure.

Theorem 3.4.5. *Suppose awareness is symmetric. The professional collects the same full information under fair disclosure and under selective disclosure.*

3.5 Asymmetric awareness

Suppose that small investors are unaware of some dimensions of information. Thus they believe that only signal τ^θ, $\theta \in \{\underline{\theta}, 0\}$ matters. For them, the game is similar to the game in the previous section. The only difference is that they think the professional's information is given by $\dot{\theta} \in \{\underline{\theta}, 0\}$ and market makers believe so too. Here we use original notations with a dot to denote the variables in small investors' mind that are different from the fact.

First we consider selective disclosure. Denote small investors' expected price $\dot{\lambda}(\dot{\theta}|R) \equiv [E(\lambda)](R, \underline{\theta}, \dot{\theta})$. Then the profit of the professional when he chooses selective forums is

$$Rq\big(E(V) - \dot{\lambda}(\dot{\theta}|R)\big) - F_s.$$

By Lemma 3.3.2, the optimal $\dot{\theta}$ is 0. The profit of the professional when he privately acquires information is

$$Rq\big(E(V) - \dot{\lambda}(\dot{\theta}|R)\big) - C_p(\theta).$$

Assume that for all $R \in [\frac{C}{E(V)}, 1]$, it is always profitable for the professional to acquire signal τ^0 no matter it is by private acquisition or selective forums. Small investors believe that the professional collects τ^0 in the equilibrium.

Proposition 3.5.1. *Denote $\dot{\lambda}(R) \equiv [E(\lambda)](R, \underline{\theta}, 0)$. Assume that $q \leq 1 - \frac{C}{E(V)}$. Under selective disclosure, small investors believe that there is a unique equilibrium where the professional collects signal τ^0, or*

$$\Big(R_e = \tilde{R},\ \ \eta_e = \underline{\theta},\ \ \dot{\theta}_e = 0,\ \ \dot{\lambda}_e(s) = \dot{\lambda}(\tilde{R}),\ \ \dot{r}_e(s) = r\big(s; \dot{\lambda}(\tilde{R}), 0\big)\Big)_{s \in \mathbb{S}},$$

where $\tilde{R} = \min\left\{ R \in [\frac{C}{E(V)}, 1] \middle| R = \frac{C}{E(V)(1-q)+q\dot{\lambda}(R)} \right\}$.

The proof is omitted for it is similar to the proof of Proposition 3.4.2.

On the other hand, given that small investors choose $R_e = \tilde{R}$, the professional 's interim profit from trade is still given by Definition 3.3.3, with its properties stated in Lemma 3.3.1. Denote the expected price $\lambda_2(\theta) \equiv [E(\lambda)](\tilde{R}, \underline{\theta}, \theta)$.

If the professional chooses selective forums, his profit is

$$\tilde{R}q\big(E(V) - \lambda_2(\theta)\big) - F_s.$$

Then by Lemma 3.3.2, the optimal θ is $\bar{\theta}$. The optimal profit of the professional is

$$\Pi_{Is} = \tilde{R}q\big(E(V) - \lambda_2(\bar{\theta})\big) - F_s. \tag{3.5.1}$$

If the professional chooses private acquisition, his profit is

$$\tilde{R}q\big(E(V) - \lambda_2(\theta)\big) - C_p(\theta). \tag{3.5.2}$$

This profit function is similar to Function 3.4.1 with symmetric awareness but now R is fixed as $\tilde{R}$. Therefore, by Lemma 3.4.1, given Assumption 3.4.3, the professional always prefers using selective forums.

Assume that for the optimal profit 3.5.1 is positive. Then in equilibrium, the professional chooses selective forums. The result is stated in the following Proposition with the proof omitted.

Proposition 3.5.2. *Under selective disclosure, the professional chooses selective forums, or*

$$\left(R_e = \tilde{R},\ \ \eta_e = \underline{\theta},\ \ \theta_e = \bar{\theta},\ \ \lambda_e(s) = \lambda_2(\bar{\theta}),\ \ r_e = r(s; \lambda_2(\bar{\theta}), \bar{\theta})\right)_{s \in \mathbb{S}},$$

in which the costs $C_c = \tilde{R}E(V)$, $C_i = F_s$.

Suppose now fair disclosure is implemented. Small investors expect a game similar to the one with symmetric awareness. By an argument similar to the proof of Proposition 3.4.3, there is an equilibrium where small investors request the disclosure of τ^0, the full information in their minds, and set $R_e = \frac{C}{E(V)}$ in the fund raising period.

The professional can also request a disclosure, so that the index of public information is $\eta \geq 0$, and he also privately acquires information so that the index is $\theta \geq \eta$. Denote $\lambda_3(s; \eta, \theta) \equiv \lambda(s; \frac{C}{E(V)}, \eta, \theta)$, with its expectation denoted by $[E(\lambda_3)](\eta, \theta)$. The professional's *ex ante* expected profit function is

$$\frac{Cq}{E(V)}\big[E(V) - [E(\lambda_3)](\eta, \theta)\big] - C_f(\theta, \eta). \tag{3.5.3}$$

The equilibrium information $(\breve{\theta}, \breve{\eta})$ must maximize Equation 3.5.3.

$$\max_{0 \leq \eta \leq \theta \leq \bar{\theta}} \frac{Cq}{E(V)} \big[E(V) - [E(\lambda_3)](\eta, \theta)\big] - C_f(\theta, \eta). \tag{3.5.4}$$

Suppose everything is differentiable. Then there are two possibilities. One is that given any public information $\eta \geq 0$, the marginal cost of acquiring private information is no less than the marginal profit from trading on the private information, namely

$$\left.\frac{\partial C_f(\theta, \eta)}{\partial \theta}\right|_{\theta=\eta} \geq \frac{qC}{E(V)} \left.\frac{\partial [E(\lambda_3)](\eta, \theta)}{\partial \theta}\right|_{\theta=\eta} \tag{3.5.5}$$

then the professional does not have incentives to acquire more information. In equilibrium, $\breve{\theta} = \breve{\eta} = 0$, market makers do not charge transaction costs, the professional does not acquire private information and does not trade. The other possibility, is that there are some levels of public information $\eta > 0$, such that Condition 3.5.5 does not hold. Then in equilibrium, $\breve{\theta} > \breve{\eta} \geq 0$. In this equilibrium, the cost of information acquisition is $C_i = C_f(\breve{\theta}, \breve{\eta})$.

Proposition 3.5.3. *Under fair disclosure, there are two types of equilibria. If Condition 3.5.5 hold for all $\eta \geq 0$, then*

$$\left(R_e = \frac{C}{E(V)}, \;\; \eta_e = \theta_e = 0, \;\; \lambda_e(s) = E(V|\tau^0(s)), \;\; r_e(s) = 0\right)_{s \in \mathbb{S}},$$

In this equilibrium, $C_c = C$, and $C_i = 0$.

If not, then

$$\left(R_e = \frac{C}{E(V)}, \;\; \eta_e = \breve{\eta}, \;\; \theta_e = \breve{\theta}, \;\; \lambda_e(s) = \lambda_3(s; \breve{\eta}, \breve{\theta}), \;\; r_e(s) = r(s; \lambda_3(s; \breve{\eta}, \breve{\theta}), \breve{\theta})\right)_{s \in \mathbb{S}},$$

where $0 \leq \breve{\eta} < \breve{\theta} \leq \bar{\theta}$. In this equilibrium, $C_c = C$, and $C_i = C_f(\breve{\theta}, \breve{\eta})$.

To compare the costs under different disclosure forms, we first note that by definition, $\tilde{R} \geq \frac{C}{E(V)}$, so the cost of capital C_c is high under selective disclosure. Also we note that under selective disclosure $C_i = F_s$ and that under fair disclosure, in the second type of equilibrium, $C_i = C_f(\breve{\theta}, \breve{\eta})$. If F_s is small enough, for instance, F_s is smaller than $C_f(\breve{\theta}, \breve{\eta})$, then the cost of information under fair disclosure is also greater than under selective disclosure.

Fact 2. *Suppose awareness is asymmetric. The cost of capital is lower under fair disclosure than under selective disclosure. On the other hand, the cost of information may be higher than that under selective disclosure if the cost of entering selective forums is low.*

Now we compare the quantity of information collected by the professional. For the first type of equilibrium, it is clear that $\theta_e = 0 < \bar{\theta}$. Suppose for the second type of equilibrium the optimal solution is interior, $\breve{\theta} < \bar{\theta}$, then the professional does not collect full information. Hence his information becomes worse if he has collected it through selective disclosure.

Theorem 3.5.4. *Suppose awareness is asymmetric. If under fair disclosure, the equilibrium is of the first type or is of the second type with interior solution, the professional always collects less information than under selective disclosure, namely* $\breve{\theta} < \bar{\theta}$.

In the next section, we discuss the implication of this theorem.

3.6 Discussion

3.6.1 Professionals' information

Theorems 3.4.5 and 3.5.4 predict different effects of the regulation on the quantity of information acquired by professionals. These results are obtained in a framework including the fund raising period, hence the public investors' ownership share R is endogenous and can vary under different disclosure forms. The empirical examinations of this aspect, however, typically use the data of given firms before and after the regulation; in other words, R is nearly fixed under different disclosure forms. We discuss our result in this circumstance.

The result of Theorem 3.4.5 remains when R is fixed. Consider the information acquisition period. Under selective disclosure, whatever R is, the professional acquires full information through selective forums. Under fair disclosure, by requesting full information, small investors' loss can be minimized to zero, meanwhile the professional also have full information. In conclusion, after the regulation, in some equilibrium, the professional has the same information.

The result of Theorem 3.5.4 also remains when R is fixed. Consider the information acquisition period. Given R, if the professional acquires full information through selective forums before the regulation, then after the regulation, he does not acquire more information and probably less information because of the high marginal cost of private acquisition. We still have the following facts.

Fact 3. *For a given firm, i.e., for the same* R*, if the professional acquires information via selective forums before the regulation, then his information decreases after the regulation.*

Meanwhile, empirical studies in the literature on information disclosure tell us the following.

Assumption 2 *Before the regulation, for investors, the cost of privately acquiring information about small firms and complex firms is high. These firms' entry cost of selective forums is low.*

For example, Bushee, Matsumoto and Miller (2004) empirically find more complex firms were more likely to use selective forums in the pre-FD period.

Hence before the regulation for small firms and complex firms, professionals are more likely to acquire their information via selective forums. Putting things together, we have the following.

Theorem 3.6.1. *After the regulation, for small firms and complex firms, the professional collects less information.*

This prediction matches the finding by Ahmed and Schneible Jr. (2004) that Reg FD has worsened the information quality of average investors for some firms (particularly small, high tech firms). It is also related to the findings by Gomes et al. (2007). They report the following:[10]

1. Small firms on average lost 17 percent of their analyst following while big firms increased theirs by 7 percent, on average.[11]

2. More complex firms (using intangible assets as a proxy for complexity) overall are being more adversely affected than less complex firms. Regardless of size, more complex firms suffer a significantly larger loss of analyst following.

If the number of analysts following a stock is a proxy of the quantity of information acquired by professionals, our prediction is consistent with their findings. We should emphasize that the literature on analyst following, such as Bhushan (1989), suggests that the analyst following of a firm is a proxy for *the total expenditure* investors spend on information, which in turn depends on the demand and supply of information. In this paper we only consider a monopoly issuer, so there is no analysis about the professional 's choosing what firm's stock to trade, and firms' choosing the entry cost of selective disclosure. In addition, we do not model the role of analysts as producers of information. Although further study is needed to understand the full story, it is worth noting that such a simple theory featuring asymmetric awareness provides some clue. [12]

[10]The authors did not find satisfactory explanations and suggested that "Our cross-sectional results suggest that Reg FD had unintended consequences and that 'information' in financial markets may be more complicated than current finance theory admits."

[11]Survey data also support this finding: The ABA FD Task Force Survey, which surveyed securities attorneys about their clients' practices, reports that FD had bigger impact on small and midsize companies rather than large companies. Source link: http://www.sec.gov/news/studies/regfdconf.txt.

[12]Caveats: Mohanram and Sunder (2006) however find a different empirical result: In the post-FD period analysts *reduce* coverage for well followed firms, which mainly are big firms, and *increase* coverage of firms that were less followed prior to Reg. FD.

This seemingly contradicting result might be explained by different empirical methods applied in

3.6.2 Small investors' welfare

One of the regulators' major goals is to protect small investors. This issue is discussed in this subsection. We will compare the welfare gain under different disclosure forms by measuring the change of small investors' profits.

Since small investors are competitive and rational, when they have full awareness, their expected profits are zero. However, when their awareness is limited, although they think the expected profits are zero, the real expected profits are negative. With that said, when the loss under fair disclosure is less than that under selective disclosure, the protection of the regulation is effective.

Recall small investor's profit function in Definition 3.3.1 and the equilibrium behavior of the professional under different disclosure forms in Propositions 3.5.2 and 3.5.3. Under selective disclosure, since the professional chooses selective forums, the actual profit of each small investor small investors is

$$\pi_{Us} = \tilde{R}\Big[qE(\lambda)(\tilde{R}, \underline{\theta}, \bar{\theta}) + (1-q)E(V)\Big] - C.$$

Under fair disclosure, the actual profit of each small investors is

$$\pi_{Uf} = \frac{C}{E(V)}\Big[qE(\lambda)(\frac{C}{E(V)}, \breve{\eta}, \breve{\theta}) + (1-q)E(V)\Big] - C.$$

We prove the following in Appendix.

Theorem 3.6.2. *If small investors are completely unaware, namely signal τ^0 is null, and the professional does not request information via public forums after the regulation, namely $\eta = \underline{\theta}$, then*

$$\pi_{Uf} \geq \pi_{Us}.$$

Hence fair disclosure induces a lower loss than selective disclosure.

Although this result depends on very restrictive assumption, it has an interesting implication. That is, if small investors' are very unprepared and do not have any chance of free-riding, then fair disclosure may improve their welfare.

two papers. One difference is that Mohanram and Sunder (2006) only cover sample firms that had some analyst following in both the pre- and post-FD period; but Gomes et al. (2007) also use sample firms that totally lost analyst following after Reg. FD. However, we still need to be cautious about missing factors affecting their findings. Further investigation shall be taken along the process of our research.

3.7 Conclusions

In this paper we study Regulation Fair Disclosure, a ruling adopted by the SEC to forbid selective disclosure. Using a stock market model with four periods-—fund raising, information acquisition, trading, and liquidation—we analyze market participants' behavior when one professional can acquire information directly from an issuer in selective forums, i.e., under selective disclosure; or when he can not, i.e., under fair disclosure. Focusing on the issuer who would have used selective forums, we address the aspects including the cost of capital, the cost of information, the quantity of information acquired by the professional, and the welfare of small investors.

Our results depend on assumptions about investors' awareness. We find that, when all investors are equally aware of the relevant information, fair disclosure induces a low cost of capital and a low cost information, therefore making the market efficient. It also induces equally good information collected by the professional. When small investors are unaware, fair disclosure still induces a low cost of capital, but it may induce a high cost of information and less information collected by the professional.

Under the asymmetric awareness assumption, our theory gives predictions which match the empirical findings that the regulation has worsened the information quality of average investors for some firms (particularly small, high tech firms), and negatively affected the analyst following of small and complex firms. We also show that when small investors are extremely unaware, the regulation improves their welfare. Since the asymmetric awareness assumption is not standard in the literature, our analysis suggests an alternative approach to understand the regulation and perhaps other information disclosure related problems.

3.8 Appendix

3.8.1 Proofs

Proof of Lemma 3.3.1.

i. Suppose $\theta' > \theta$, by the definition in Fact 1, we denote the new optimal trade rule by $r(s;\lambda,\theta')$. Then first for each $\tau^{\theta'}(s')$, the optimal expected profit is

$$\begin{aligned}
[E_{\theta'}(\pi_I|s')](\theta',\lambda') &= \int r(s'';\lambda,\theta')[\lambda - E_{\theta'}(V|s'')]dP(s''|\tau^{\theta'}(s')) \\
&= \max_r \int r[\lambda - E_{\theta'}(V|s'')]dP(s''|\tau^{\theta'}(s')) \\
&\geq \int r(s'';\lambda,\theta)[\lambda - E_{\theta'}(V|s'')]dP(s''|\tau^{\theta'}(s')).
\end{aligned}$$

Because $\tau^{\theta'}$ is finer than τ^{θ}, for every s', $E_\theta(V|s') = \int E_{\theta'}(V|s'')dP(s''|\tau^\theta(s'))$ and $r(s'';\lambda,\theta) = r(s';\lambda,\theta)$ if $s'' \in \tau^{\theta'}(s')$. Now for each $\tau^\theta(s)$,

$$\begin{aligned}
[E_\theta(\pi_I|s')](\theta',\lambda') &= \int [E_{\theta'}(\pi_I|s')](\theta',\lambda')dP(s'|\tau^\theta(s)) \\
&\geq \int\int r(s'';\lambda,\theta)[\lambda - E_{\theta'}(V|s'')]dP(s''|\tau^{\theta'}(s'))dP(s'|\tau^\theta(s)) \\
&= \int r(s';\lambda,\theta)[\lambda - E_\theta(V|s')]dP(s'|\tau^\theta(s)) \\
&= [E_\theta(\pi_I|s')](\theta,\lambda).
\end{aligned}$$

Because $\theta' > \theta > \eta$, integrate both sides on $s' \in \tau^\eta(s)$, then the result follows.

ii. Suppose $\lambda' > \lambda$. By the definition in Fact 1, the new optimal trade rule is

$$r(s;\lambda',\theta) = \begin{cases} N & \text{if } s \in d(\lambda',\theta) \\ 0 & \text{if else.} \end{cases}$$

Again for simplicity we denote $d_1 \equiv d(\lambda',\theta)$, $d_2 \equiv d(\lambda,\theta)$. Note that $d_1 \supset d_2$, then

$$\begin{aligned}
[E_\eta(\pi_I|s)](\theta,\lambda') &= \int_{s'\in d_1} N\big[\lambda' - E_\theta(V|s')\big]dP(s'|\tau^\eta(s)) \\
&= \int_{s'\in d_2} N\big[\lambda' - E_\theta(V|s')\big]dP(s'|\tau^\eta(s)) \\
&\qquad + \int_{s'\in d_1\setminus d_2} N\big[\lambda' - E_\theta(V|s')\big]dP(s'|\tau^\eta(s)) \\
&\geq \int_{s'\in d_2} N\big[\lambda' - E_\theta(V|s')\big]dP(s'|\tau^\eta(s)) \\
&\geq \int_{s'\in d_2} N\big[\lambda - E_\theta(V|s')\big]dP(s'|\tau^\eta(s)) \\
&= [E_\eta(\pi_I|s)](\theta,\lambda).
\end{aligned}$$

Therefore $[E_\eta(\pi_I|s)](\theta,\lambda') \geq [E_\eta(\pi_I|s)](\theta,\lambda)$.

iii. Suppose the probability measure $G(x|s)$ is smooth in $x \in R_+$. Note that by its definition, $G(x|s) = \int_{s'\in d(x,\theta)} dP(s'|\tau^\eta(s))$. Then the function

$$\begin{aligned}
[E_\eta(\pi_I|s)](\theta,\lambda) &= \int_{s'\in d(\lambda,\theta)} N\big[\lambda - E_\theta(V|s')\big]dP(s'|\tau^\eta(s)) \\
&= N\lambda \int_{s'\in d(\lambda,\theta)} dP(s'|\tau^\eta(s)) - N\int_{s'\in d(\lambda,\theta)} E_\theta(V|s')dP(s'|\tau^\eta(s)) \\
&= N\lambda G(\lambda|s) - N\int_0^\lambda x dG(x|s).
\end{aligned}$$

As $G(x|s)$ is smooth, the second part in the last expression is well defined and continuous in λ. The function is continuous in λ because it is sum of two continuous functions.

QED.

Proof of Lemma 3.3.2.

i. Define function

$$f(\lambda) \equiv Rq[E_\eta(V|s) - \lambda] - E_\eta(\pi_I|s).$$

First $f(\lambda)$ is continuous in λ by the third claim in Lemma 3.3.1. Second when $\lambda = 0$, $d(\lambda, \theta) = \emptyset$, and $E_\eta(\pi_I|s) = 0$, then

$$f(0) = RqE_\eta(V|s) \geq 0.$$

Also when $\lambda = E_\eta(V|s)$,

$$f(E_\eta(V|s)) = -E_\eta(\pi_I|s) \leq 0.$$

Then by Intermediate Value Theorem, there is at least one $\lambda \in [0, E_\eta(V|s)]$ such that $f(\lambda) = 0$. Hence the pricing rule $\lambda(s; R, \eta, \theta)$ exists. Since $E_\eta(\pi_I|s)$ decreases in λ, $f(\lambda)$ is also strictly decreasing in λ, which implies the uniqueness.

ii. The monotonicity of $\lambda(s; R, \eta, \theta)$ can be shown by by contradictions. Suppose $\lambda(s; R, \eta, \theta)$ does not increase in R. Then if R increases, the LHS of

$$Rq[E_\eta(V|s) - \lambda(s; Rq, \eta, \theta)] = E_\eta(\pi_I|s)$$

would increase, while the RHS of it, an increasing function of λ, decreases. A contradiction.

Similarly we can show that if θ increases, $\lambda(s; R, \eta, \theta)$ can not increase, for otherwise the RHS of the equation increases but the LHS decreases, which is a contradiction.

For the continuity of function $k(R) \equiv \lambda(s; Rq, \eta, \theta)$ in R, we want to show that $\lim_{\epsilon \to 0} k(R + \epsilon) = k(R)$ for all $R \in \mathrm{R}_+$, where $k(R)$ is given by

$$Rq[E_\eta(V|s) - k(R)] = \big[E_\eta(\pi_I|s)\big](k(R)); \tag{3.8.1}$$

and $k(R + \epsilon)$ is given by

$$(R + \epsilon)q[E_\eta(V|s) - k(R + \epsilon)] = \big[E_\eta(\pi_I|s)\big](k(R + \epsilon)). \tag{3.8.2}$$

Subtracting Equation 3.8.1 from Equation 3.8.2, we have

$$\epsilon q[E_\eta(V|s) - k(R+\epsilon)] + Rq(k(R) - k(R+\epsilon)) = \big[E_\eta(\pi_I|s)\big](k(R+\epsilon)) - \big[E_\eta(\pi_I|s)\big](k(R)).$$

Suppose the continuity does not hold. Without loss of generality, we assume $k(R) - \lim_{\epsilon\to 0} k(R+\epsilon) > a > 0$, a being a constant. Then take the limit of both sides of the above equation. For the LHS,

$$\lim_{\epsilon\to 0}\big\{\epsilon q[E_\eta(V|s) - k(R+\epsilon)] + Rq(k(R) - k(R+\epsilon))\big\} > Rqa > 0;$$

for the RHS, because $E_\eta(\pi_I|s)$ is continuous and increases in λ,

$$\lim_{\epsilon\to 0}\big[E_\eta(\pi_I|s)\big](k(R+\epsilon)) - \big[E_\eta(\pi_I|s)\big]k(R) = \big[E_\eta(\pi_I|s)\big]\Big(\lim_{\epsilon\to 0} k(R+\epsilon)) - k(R)\Big) \leq 0.$$

A contradiction. Hence the continuity must hold.

iii. Following the last claim, the properties of $[E(\lambda)](R,\eta,\theta)$ are obtained by the definition $[E(\lambda)](R,\eta,\theta) = \int_{s\in\mathbb{S}} \lambda(s; R,\eta,\theta)dP(s)$.

QED.

Proof of Lemma 3.4.1

By definition, $\lambda_1(\bar\theta|R) = [E(\lambda)](R,\underline{\theta},\bar\theta)$ and $\lambda_1(\hat\theta(R)|R) = [E(\lambda)](R,\bar\theta,\hat\theta(R))$. Because $\bar\theta \geq \hat\theta(R)$, by Lemma 3.3.2,

$$\lambda_1(\bar\theta|R) \leq \lambda_1(\hat\theta(R)|R).$$

Then given that $F_s < C_p(\hat\theta^*)$ and C_p being increasing, by definition, for all $R \in [\frac{C}{E(V)}, 1]$,

$$\begin{aligned} Rq\lambda_1(\hat\theta(R)|R) + C_p(\hat\theta(R)) &\geq Rq\lambda_1(\hat\theta(R)|R) + C_p(\hat\theta^*) && (3.8.3)\\ &> Rq\lambda_1(\bar\theta|R) + F_s, && (3.8.4)\end{aligned}$$

which is Condition 3.4.2.

QED.

Proof of Proposition 3.4.2.

To ensure the existence of R, we use Brouwer's Fixed Point Theorem. Define function

$$g(R) = \frac{C}{E(V)(1-q) + q\lambda_1(\bar\theta|R)}.$$

Since $R \in [\frac{C}{E(V)}, 1]$, we want to show that $g(R) \in [\frac{C}{E(V)}, 1]$ too. Because $g(R)$ is monotone in $\lambda_1(\bar\theta|R)$, $\lambda_1(\bar\theta|R) \in [0, E(V)]$, and $q \leq 1 - \frac{C}{E(V)}$ as assumed, we have that

$$\begin{aligned} g(R) &\leq \frac{C}{E(V)(1-q)+0} \leq 1; \\ g(R) &\geq \frac{C}{E(V)(1-q)+qE(V)} = \frac{C}{E(V)}. \end{aligned}$$

Also by Claim 2 in Lemma 3.3.2, $\lambda_1(\bar{\theta}|R)$ is continuous in R, so $g(R)$ is also continuous in R. Then we can apply the Fixed Point Theorem. By the definition, $\bar{R}$ exists.

The cost of capital and the cost of information in each equilibrium follow by their definitions.

QED.

Proof of Proposition 3.4.3.

We want to show that it is an equilibrium that small investors request the disclosure of $\bar{\tau}$.

Suppose small investors commit to request $\bar{\tau}$. Since full information becomes public, all market makers and the professional know $\bar{\tau}$. At every s, the market makers' interim expected aggregate profit is

$$\begin{aligned} E_{\bar{\theta}}(\pi_M|s) &= Rq(E_{\bar{\theta}}(V|s)-\lambda)+r(E_{\bar{\theta}}(V|s)-\lambda) \\ &= [Rq+r](E_{\bar{\theta}}(V|s)-\lambda). \end{aligned}$$

If a market maker sets a price $\lambda < E_{\bar{\theta}}(V|s)$, then another market maker can take away the deal by a slightly higher price. That is, because of the competition, in equilibrium $\lambda_e(s) = E_{\bar{\theta}}(V|s)$. Therefore, by the optimal trade rule described in Fact 1, the professional will not trade.

For each small investors, regardless the liquidity shock, the *interim* expected profit is $E_{\bar{\theta}}(V|s)R - C$. Then the *ex ante* expected profit is $E(V)R - C$. The issuer must set $R_e = \frac{C}{E(V)}$.

Suppose there is one of small investors who commits not to request the disclosure of $\bar{\tau}$, then it does not matter because under fair disclosure, any information disclosed is public; as long as some other small investors request $\bar{\tau}$, he also knows it. So the ownership share is unchanged. Then the one who deviates does not gain. Neither does it matter how much information that the professional requests to disclose. Therefore, it is an equilibrium.

In this equilibrium $C_c = C$ and $C_i = 0$.

There may be some other equilibria where small investors only request disclosure of partial information, so that $\eta < \bar{\theta}$. For example, due to the high marginal cost of private acquisition the professional does not profit from privately collecting more information. Then information of all investors are symmetric, small investors do not worry about the transaction costs charged by market makers. In equilibrium, the share of ownership is still $R_e = \frac{C}{E(V)}$, and $C_c = C$, $C_i = 0$.

QED.

Proof of Theorem 3.6.2

Under selective disclosure, when small investors are fully unaware, they do not expect the professional to acquire any private information; then by Proposition 3.5.1, $\tilde{R} = \frac{C}{E(V)}$. If the professional chooses selective forums, the actual profit of each small investors is

$$\pi_{Us} = \frac{C}{E(V)}\Big[qE(\lambda)(\frac{C}{E(V)}q, \underline{\theta}, \bar{\theta}) + (1-q)E(V)\Big] - C.$$

Under fair disclosure, since the professional does not request information via public forums, $\breve{\eta} = \underline{\theta}$,

$$\pi_{Uf} = \frac{C}{E(V)}\Big[qE(\lambda)(\frac{Cq}{E(V)}, \underline{\theta}, \breve{\theta}) + (1-q)E(V)\Big] - C.$$

By Lemma 3.3.2, since $\breve{\theta} \le \bar{\theta}$,

$$E(\lambda)(\frac{C}{E(V)}q, \underline{\theta}, \bar{\theta}) \le E(\lambda)(\frac{C}{E(V)}q, \underline{\theta}, \breve{\theta}).$$

Thus $\pi_{Uf} \ge \pi_{Us}$.

QED.

3.8.2 Trade with both buy and sell orders

In this section we generalize the original model by allowing both selling and buying in the trading period. We will show that under selective disclosure and with symmetric awareness, there is a similar equilibrium where the professional acquires information from selective forums and small investors' ownership share reflects the adverse selection. We omit the derivation of other results since following the derivations in the original model, they are relatively easy to see.

The model

Now small investors could experience a negative liquidity shock or a positive one, which happen with probability q and p respectively, with $q, p \in (0, 1]$. We assume that small investors sell all their shares at a negative shock and double their shares at a positive shock. The professional can thus mimic them and profit from his private information by selling or buying. Market makers set price λ_q for every unit share sold by investors, and price λ_p for every unit share bought by investors. Note that $\lambda_q < \lambda_p$. Other notations follow the original model.

Then for a small investor, his *ex ante* profit function is

$$\pi_U = R\Big[qE(\lambda_q) - pE(\lambda_p) + (1-q+p)E(V)\Big] - C,$$

where $E(\lambda_i)$, $i \in \{q, p\}$ are the expected bid price and the expected ask price.

The professional's decision problem is

$$\max_{r_q, r_p \in [0,N]} E_\theta(\pi_I|s) := \int_{\mathbb{V}} r_q(\lambda_q - v)dP(v|\tau^\theta(s)) + \int_{\mathbb{V}} r_p(v - \lambda_p)dP(v|\tau^\theta(s))$$

Define sets $d(\lambda_q, \theta) \equiv \{s|E_\theta(V|s) < \lambda_q\}$ and $d(\lambda_p, \theta) \equiv \{s|E_\theta(V|s) > \lambda_p\}$. The new optimal trade rule is that for $i \in \{q, p\}$:

$$r_i(s; \lambda_i, \theta) = \begin{cases} N & \text{if } s \in d(\lambda_i, \theta) \\ 0 & \text{if else.} \end{cases}$$

Market makers know that the professional uses the optimal trade rule. If a sell order is executed for the professional, conditional on the public information, they expect the professional's interim expected profit to be

$$E_\eta(\pi_{Iq}|s) = \int_{s' \in d(\lambda_q, \theta)} N(\lambda_q - E_\theta(V|s))dF(s'|\tau^\eta(s)).$$

Similarly, if a buy order is executed for the professional, conditional on the public information, market makers expect the professional's interim expected profit to be

$$E_\eta(\pi_{Ip}|s) = \int_{s' \in d(\lambda_p, \theta)} N(E_\theta(V|s) - \lambda_p)dF(s'|\tau^\eta(s)).$$

All the properties states in Lemma 3.3.1 can be applied to both cases, additionally $E_\eta(\pi_{Ip}|s)$ decreases in λ_p.

For market makers, when executing a sell order, their aggregate profit is

$$E_\eta(\pi_{Mq}|s) := Rq[E_\eta(V|s) - \lambda_q] - E_\eta(\pi_{Iq}|s),$$

when executing a buy order, their aggregate profit is

$$E_\eta(\pi_{Mp}|s) := Rp[\lambda_p - E_\eta(V|s)] - E_\eta(\pi_{Ip}|s).$$

Due to the competition among market makers, in the equilibrium, the optimal pricing rule is given by

$$\lambda_q(s;R,\eta,\theta) = \max\left\{\lambda_q \geq 0 | Rq[E_\eta(V|s) - \lambda_q] = E_\eta(\pi_{Iq}|s)\right\};$$

$$\lambda_p(s;R,\eta,\theta) = \min\left\{\lambda_p \geq 0 | Rp[\lambda_p - E_\eta(V|s)] = E_\eta(\pi_{Ip}|s)\right\}. \quad (3.8.5)$$

Hence the bid and ask prices are determined in a similar way. The results in Lemma 3.3.2 can be applied to both. Note that to show the existence of $\lambda_p(s;R,\eta,\theta)$, we need to assume that the maximum expectation that the professional has, namely $\bar{v} = \max\{E_\theta(V|s)|\theta \in \Theta, s \in \mathbb{S}\}$ is finite, then show that for $\lambda_p > \bar{v}$, $f(\lambda_p) := Rq[\lambda_q - E_\eta(V|s)] - E_\eta(\pi_{Iq}|s) \geq 0$.

There is also a little change for λ_p since it strictly decreases in R and increases in θ.

Then we can show that the professional's information affects his profit from both selling and buying. Without considering the cost, more information is always better. And if F_s is small, the professional always prefers using selective forums under selective disclosure.

The equilibrium

For small investors, when they determine the ownership share R, they expect it to affect both λ_i, $i \in \{q,p\}$. The difference is that instead of λ, the focus is on $qE[\lambda_q] - pE[\lambda_p]$, which is the price spread adjusted by the probability of liquidity shocks. This expression is continuous in R. In order to use the fixed point argument, we need that

$$g(R) := \frac{C}{qE(\lambda_q) - pE(\lambda_p) + (1-q+p)E(V)} \in [\frac{C}{E(V)}, 1]$$

Function $g(R)$ is strictly decreasing in $qE(\lambda_q) - pE(\lambda_p)$. We can show that if

$$E(V)(1-q) + [E(V) - \bar{v}]\frac{p}{1+p} \geq C,^{13} \quad (3.8.6)$$

where $\bar{v} = \max\{E_\theta(V|s)|\theta \in \Theta, s \in \mathbb{S}\}$, then the range of the function is within $[\frac{C}{E(V)}, 1]$.

To see this, first, note that since $E(\lambda_q) \leq E(V)$ and $E(\lambda_p) \geq E(V)$, then for all R, $g(R) \geq \frac{C}{E(V)}$.

[13] It implies that both q and p are small.

Second, by Definition 3.8.5, $Rp[\lambda_p - E_\eta(V|s)] = E_\eta(\pi_{Ip}|s)$, so

$$\begin{aligned} Rp[E(\lambda_p) - E(V)] &= \int_{s\in d(\lambda_p,\theta)} N(E_\theta(V|s) - \lambda_p(s))dP(s) \\ &\leq \int_{s\in d(\lambda_p,\theta)} N(\bar{v} - \lambda_p(s))dP(s) \\ &\leq N(\bar{v} - E(\lambda_p)) \\ &\leq R(\bar{v} - E(\lambda_p)). \end{aligned}$$

The first inequality comes from the definition of $\bar{v}$. The second inequality comes from the fact that $\lambda_p(s) \leq \bar{v}$ for all $s \in \mathbb{S}$, otherwise $Rp[\lambda_p - E_\eta(V|s)] = E_\eta(\pi_{Ip}|s)$ can not hold[14]. The third inequality comes from the assumption that $N \leq \frac{C}{E(V)}$.

Therefore it is true that

$$E(\lambda_p) \leq \frac{\bar{v} + pE(V)}{1+p}.$$

Also note that $E(\lambda_q) \geq 0$. We have that

$$\begin{aligned} qE(\lambda_q) - pE(\lambda_p) + (1-q+p)E(V) &\geq 0 - p[\frac{\bar{v} + pE(V)}{1+p}] + (1-q+p)E(V) \\ &= E(V)(1-q) + [E(V) - \bar{v}]\frac{p}{1+p} \\ &\geq C. \end{aligned}$$

The last inequality comes from the assumption 3.8.6.

Then the Fixed Point Theorem is applied and an equilibrium R_e exists, namely

$$R_e = \min\left\{ R \in [\frac{C}{E(V)}, 1] \middle| R = \frac{C}{q\lambda_q(\bar{\theta}|R) - p\lambda_p(\bar{\theta}|R)) + (1-q+p)E(V)} \right\}.$$

where $\lambda_i(\bar{\theta}|R) = E(\lambda_i)$, $i \in \{q, p\}$ because the professional always chooses selective forums.

[14]If so, then $Rp[\lambda_p - E_\eta(V|s)] > 0$ and $E_\eta(\pi_{Ip}|s) = 0$.

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