Three Essays on Social Security, Temptation and Individual Welfare

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Abstract of the Dissertation

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Essay 1: Outliving its Usefulness?

Mortality rates have improved at all ages. This paper examines the welfare-generating role of survivorship in a simple fully funded social security system, and finds that much of the power of social security to achieve individual welfare gains has been eroded by improved mortality. The paper also develops a measure of the utility of accidental bequests. The Addendum extends the analysis to a paygo social security system.

Essay 2: A Hybrid Reform

Heterogeneous ability/income which is correlated with longevity is incorporated into a large scale general equilibrium overlapping generations economy. I propose a hybrid refom of Social Security which includes (i) a paygo advanced old age government pension, and (ii) personal accounts to finance the early retirement ages. These policy reforms are studied in a life cycle consumer framework where households face temptation and self-control problems. Our analysis indicates that the economy with the hybrid reform is able to sustain higher levels of capital intensity, and therefore consumption, than either an economy with Social Security or an economy with no Social Security. In addition, the proposed hybrid reform reduces wealth inequality.

Essay 3: Consumer Surplus:

What accounts for the continued popularity of Social Security? Compensating variations and willingness-to-pay are analyzed by type of worker, to see if Social Security results in consumer surplus. Heterogeneous ability/income which is correlated with longevity is incorporated into a large scale general equilibrium overlapping generations economy. This model is used to measure welfare under both Social Security and No Social Security, in a variety of alternative experiments. We find consumer surplus under paygo Social Security when benefits start at older ages and in lower amounts. We find that with the Hybrid Reform, individual welfare increases with the social insurance program, and some agents are willing-to-pay more than the required tax, when analyzed in a temptation preferences framework.

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Preface

Three papers constitute my dissertation. They share a common focus on Social Security, a subject that has interested me going back to my professional career prior to my present academic life. As an actuarial student in my twenties, I studied the social insurance principles embedded in the design of Social Security, using a textbook written by Robert J. Myers, the first Actuary of the Social Security Administration; a legend and a hero among actuaries. He emphasized the balance in Social Security between the need for social adequacy and the need for individual equity. Too much emphasis on social adequacy may undermine popular support. Too much emphasis on individual equity may weaken the insurance mechanism. I embraced this balancing concept then, and continue to evaluate proposed Social Security reform ideas through this framework.

I gravitated naturally then to the topic of Social Security and its role in the macro-economy. I had heard of the research of Martin Feldstein long ago in the pages of The Wall Street Journal. As a graduate student, it was exciting to be able to read and (mostly) comprehend the academic papers and see the evolution of the discussion. But I was finally pulled me into this area of research by two papers I read in November, 2005. One was by Andrew Abel (1985), "Precautionary Savings and Accidental Bequests", in the American Economic Review. The other was by M.A. Milevsky (2005), "Real Longevity Insurance with a Deductible: Introduction to Advanced-Life Delayed Annuities", in the North American Actuarial Journal. (In 1993, I invented an entirely new actuarial mechanism for the payout phase of a variable immediate annuity. This accounts for my interest in the Milevsky paper.)

These were the background for the first paper in my dissertation, "Outliving its Usefulness? Longevity and Social Security". Abel's paper clearly showed that the insurance mechanism was the source of Social Security's ability to improve individual welfare. Milevsky's paper reminded me that improved mortality rates have made insurance gains in an annuity insignificant until much later ages are reached in life. This rather obvious point is the main idea in "Outliving its Usefulness?." The paper was a learning experience, following the model of Abel. In the Addendum to that paper (which I wrote this year), the analysis and flow of reasoning are more sure-footed. But I still remember the enjoyment of putting my own ideas into that first paper (which is a long way to say that you can save yourself some time and just read the Addendum).

The major takeaway from the first paper is that one way to make Social Security more effective at increasing welfare is to refocus it on advanced old ages. I combined this insight with the well-known finding (since Feldstein's 1974 paper) that Social Security leads to lower capital formation in the long run steady state. It seems obvious to me that if the Social Security mechanism is causing a reduction in capital formation, while providing no meaningful enhancement to individual welfare until one reaches advanced ages, then the thing to do is start Social Security at an advanced age, and replace Social Security in younger retirement years with a self-funded mandatory savings program. Hence the Hybrid Reform Proposal was born out of the main ideas of the first paper.

The second paper of the dissertation, "A Hybrid Reform: Social Security on Dual Power in the Presence of Temptation", analyzes the impact on the economy of Social Security and the Hybrid Reform Proposal, using an economic model with temptation preferences. When I started graduate school in 2002, I had no idea what economics was all about. The idea of a utility function was confusing to me. It took me some time to realize that its value is as a construct of a motive force, not with intrinsic value on its own. But in my quest to better understand utility, I learned about some fascinating ideas, such as time inconsistency, and temptation preferences. Temptation preferences caught my imagination, perhaps because they describe my life as a utility maximizer. I decided to use temptation preferences in my research.

There are basically two findings in "A Hybrid Reform". The main finding is that under temptation, economies with Social Security and economies with no Social Insurance program both fare poorly with respect to capital formation and resulting standards of living in the steady state. But an economy with the Hybrid Reform social insurance program is "immunized" from the worst effects of temptation, thanks to the mandatory savings aspect of its design. Capital formation and consumption are preserved at a high level in the steady state.

The second outcome picks up a thread of research I encountered in literature from the Cato Institute, a libertarian think tank, and advocate of eliminating Social Security altogether. They argue that Social Security increases inequality in society. Low income, young black males pay into the system, while rich old white females get the benefits. My paper provides some validation for their claim, and shows that the Hybrid Reform reduces the degree of wealth inequality in the economy, relative to the inequality in the Social Security economy.

Aside from the two main findings above, another main outcome for me was "breaking the code" of how to solve models with temptation. I explain the method in detail in the appendix to the Hybrid Reform essay, for the benefit of those who care to follow the temptation research path.

The third paper, "Social Security and Consumer Surplus", is a further exploration of the inequality theme. I am struck by how popular Social Security is, especially among low income groups, the very groups most harmed by its claim on their income. Perhaps low income workers get a lot more utility from the insurance value of Social Security. Perhaps they get so much more utility that they are happy to pay more than their fair share, even to subsidize high income workers.

From that thought I begin a journey of discovery, following that question into every corner of the model. Two findings from that paper stand out to me. First there is a significant drop-off in the marginal utility level of the first dollar of pension benefit and the marginal utility level of the last dollar of pension benefit. In some cases, Social Security creates consumer surplus at the first dollar of pension benefit, but it is all destroyed by the time the ultimate level of benefits is reached. The second finding that I found meaningful is that the Hybrid Reform, in an economy with temptation, does create consumer surplus. Newborns into the Hybrid Reform economy would be willing to give up some of their annual consumption to stay in that economy, rather than be in an economy with no social insurance. This is the Holy Grail, a government program which creates consumer surplus. If temptation preferences are a valid model of the economic decision-making environment, then the Hybrid Reform Proposal is the only one of the three models to produce consumer surplus.

Thanks to the editing suggestions of Dr. Eva Carceles-Poveda, my dissertation advisor, these essays are shorter, more focused and more readable than they once were. Dr. Carceles-Poveda has been a patient advisor, and generous with her time and support, for which I am very appreciative. Any remaining errors you encounter are, of course, my own.

The last six years of transition from the business world to academia could not have been possible without the overflowing, wonderful support of my wife, Ann. She is a "help meet" just for me.

Gordon C. Boronow
December 10, 2008
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Chapter 1

Outliving its Usefulness? Longevity and Social Security

Concurrent with the baby boom generation there has been a dramatic improvement, at all ages, in mortality rates in the United States. Among the many implications of this improvement in mortality is the obvious impact on the cost of the Social Security system. But this paper is not about the cost of Social Security, per se, but rather it is about the effect that improving mortality has had on the ability of a Social Security system to enhance welfare (utility) for the citizens of the United States. Using a simple two period model of an actuarially fair social security system of taxes and benefits, the gain in welfare from imposition of social security is today less than 40% of the corresponding gain in welfare using mortality rates appropriate to 1940.

Simple economic models often assume utility is derived from consumption, and ignore a bequest motive. This greatly simplifies the analysis, and a planned bequest can be thought of as a form of consumption. However, as the cost of Social Security increases, it becomes less acceptable to ignore utility from an accidental bequest. A formulation of a conservative basis for evaluating the utility from an accidental bequest is developed in this paper. There is a small but noticable effect from including utility from accidental bequests in the decision making process, based on the two period model used for analysis in this paper. In part the modest impact of the utility from bequests is due to the simplicity of the model. As the model works, accidental bequests can only occur during the working period, not the retirement period, thereby eliminating the possibility for accidental bequests during the period where they are most likely to occur. But this paper develops the concept and lays the groundwork for further analysis in this direction.

Finally, improvements in mortality have extended life expectencies, especially the period of retirement. This feature is built into the model and the result is a slight improvement in the ability of social security to produce welfare gains.

In the end, from the simple model of this paper, using mortality rates from the 2005 U.S. Life Table, we find that social security still generates welfare gains for our representative consumer. But these gains are little more than a third of the welfare gains produced in the same model, using mortatility from the 1940 U.S. Life Table.

This paper proceeds as follows: Section 1 presents some context for this paper in the vast literature on Social Security. Section 2 presents the two period model on which the paper is based. Section 3 analyzes the effect that improving mortality has had on the basic model of social security. Utility is from consumption only. Section 4 develops a basis for evaluating utility from accidental bequests. This utility is added to the model. There is an analysis based on an assumption that utility from consumption is independent of the utility from accidental bequests. However, it seems more economically sound to assume that utility from accidental bequests does affect the decision making process for consumption, so results are presented on that basis. (In results not presented in the paper, but pursued by the author, there was virtually no measurable difference in utility of consumption by including the utility from accidental bequest in the Euler Equation.) In Section 5 the model is adjusted again, this time to reflect increasing longevity during the retirement period. Finally in this same section, the model is reparameterized to use intuitively plausible factors. This serves as a modest check on the findings of the paper, and evaluate in a heuristic fashion the usefulness of social security as a welfare-raising vehicle for all citizens. [1]

The paper concludes with some ideas for extending this line of research.

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[1] The reader may wonder why intuitively plausible parameters were not used throughout the paper. It was to keep the mathematics as uncluttered as possible.

1.1 Context in the Literature

There is an extensive literature on Social Security and individual welfare. This paper focuses on a very small part of that literature having to do with the mechanism by which Social Security can enhance welfare. By pooling the risk of living too long, Social Security enhances welfare by providing annuities, which have not been supplied to individuals in the private markets. This market failure may be due to asymetric information, as discussed by Rothschild and Stiglitz (1976), or perhaps private annuities are not viable choices of optimizing individuals, as discussed by Friedman and Warshawsky (1985). Nonetheless, by providing a public pension, Social Security fills the vacuum left by the lack of private annuities.

Abel (1985) showed that an actuarially fair, fully funded Social Security system can generate increases in individual welfare. Moreover, he traces the source of that increase in welfare to the insurance mechanism. By redistributing the Social Security reserves of those who die to those who survive, the Social Security system assures a lifetime income in retirement. Precautionary savings can therefore be reduced and reallocated to utility (and welfare) producing consumption.

The question of bequests and its relative importance to understanding Social Security is sprinkled through the literature. Barro (1974) stressed the importance of an operative bequest motive in understanding the effect of government bonds, and resource allocation. Bequests have been recognized as an important feature in aggregate saving by Kotlikoff and Summers (1981). Abel (1985) studied the role of "accidental bequests" in the dispersion of intergenerational wealth transfer. He explicitly excluded a bequest motive in utility to focus on accidental bequests.

The author is intrigued by the possibility that bequests might be a meaningful source of utility. To the extent that Social Security "crowds out" accidental bequests, then perhaps including bequests in utility may reveal a diminution of individual welfare as a result of Social Security.

The actuarial literature contains many perspectives on the historical improvement in mortality rates in recent decades, in particular at older ages. A paper by Milevsky (2006) discusses the advantages of focusing attention of advanced ages when addressing the commercial development of private annuities. In this interesting take on the market failure of private annuities in general, he points out the limited scope for survivorship gains which can be redistributed to survivors until one reaches advanced old ages. This prompted the author to speculate on whether Social Security had outlived its usefulness.

1.2 The Basic Model

This paper uses essentially the same model of social security used by Abel (1985) to study the implications of precautionary savings and accidental bequests. That model is a two period model, with a social security framework which is fully funded. While Abel was focused on the distribution of receiving accidental bequests, the focus of this paper is on the implications of improving mortality rates. So Abel's model can be simplified further in this paper by eliminating consideration of bequests that the consumer receives in their lifetime. [2]

Consider an economy with many identical consumers, and a single commodity. The length of life is either one period or two periods. Each consumer works during the first period and earns a fixed labor income Y , consumes an amount c_1, and pays a tax T. Whatever wealth from period 1 that is not consumed earns a fixed gross return of R. (Throughout the paper, in the numerical computations, Y is normalized to 1.) There is a probability p that the consumer survives to the second period, retirement, during which the consumer does not work. (Abel used p to represent the probability of dying in period 1. This paper uses instead the actuarial notational convention that p is the probability of survival, and that $q = 1 - p$ is the probabillity of dying.) In period 2, survivors receive a (lump sum) social security benefit of S , consume an amount c_2, and die with certainty, optimally leaving no unplanned bequests in period 2.

[2] This is equivalent to assuming a 100% tax on estates.

1.2.1 The Individual's Problem

The consumer chooses c_1 and c_2 to maximize the following expected utility function:

$$U_c = u(c_1) + p\delta u(c_2)$$
$$u(c) = (1-\gamma)^{-1} \cdot c^{(1-\gamma)}$$

where δ is a time preference parameter, and γ is a parameter of the constant elasticity of substitution between first period and second period consumption.

Implicit in this formulation of the utility function is the assumption that there is no utility from leaving an unplanned (aka accidental) bequest. This is a key assumption of the model, for as we shall see shortly, the welfare enhancing outcome that results from introducing a social security system lies in converting accidental bequests which have no utility, into higher consumption which produces utility.

The consumer faces the constraint that $c_2 = R(Y - T - c_1) + S$.

The first order condition with respect to c_1 is:

$$u'(c_1) = Rp\delta \cdot u'(c_2)$$

We obtain the following solutions for optimal consumption, c_1 and c_2 :

$$c_1 = a(Y - T + SR^{-1}) = aI$$
$$c_2 = R(1-a)(Y - T + SR^{-1}) = R(1-a)I$$

where:

$$I = Y - T + SR^{-1}$$
$$a = R/(R + (Rp\delta)^{1/\gamma})$$

Note that I is the period 1 value of lifetime income, so that optimal consumption is a linear function of total lifetime income, for both c_1 and c_2. Also note that $0 < a < 1$, and that a , the share of lifetime income optimally consumed in period 1, depends on the parameters of the model, i.e. R, p, δ, and γ. As Abel points out, the greater the parameter of relative risk aversion (γ), the more willing the consumer is to smooth consumption.

Table 1 shows the value of a, the share of lifetime income optimally consumed in period 1, for various illustrative cases. For the computations, we use $R = 1.56$ (i.e. $1.02^{22.5}$, where 22.5 is one half the working lifetime of a male aged 20 who will retire at age 65), $\delta = .635$ (i.e. $.98^{22.5}$) , and γ is usually parameterized to equal 2, and occasionally, to show a difference, 6. The probability of survival, p, is parameterized to equal the probability of survivng from age 20 to 65, according to the United States Life Tables in 1940, 1970 and 2005. Table 1 also shows the value of a^*, the optimal share of lifetime income consumed in period one when survival is certain. Note that as $p \to 1, a \to a^* = \frac{R}{R+(R\delta)^{1/\gamma}}$

Table 1. Period 1 Optimal Consumption Share ($S = .1Y$).

Life Table	p	$\gamma = 2$ a	$\gamma = 6$ a
1940	.607	.6682	.6294
1970	.669	.6571	.6256
2005	.796	.6371	.6188
a^*	1.0	.6105	.6097

We see that consumption is smoother between period 1 and period 2 when $\gamma = 6$, than when $\gamma = 2$, as expected. We also note that as probability of survival increases in the more recent Life Tables, the optimal share decreases towards the share corresponding to certain survival. In the 1940 Life Table, optimal period 1 consumption share is 9.5% greater than the optimal share under survival certainty. In the 2005 Life Table, that gain in share is reduced to only 4.4%.

The model of this paper is a fully funded model of social security that is actuarially fair. That is to say that the taxes in period 1, together with the investment return R, are sufficient to pay benefits S to the survivors in period 2. Thus:

$$
\begin{aligned}
T &= pSR^{-1} \\
I &= Y - T + SR^{-1} \\
I &= Y + (1-p)SR^{-1} = Y + \frac{qT}{p}
\end{aligned}
$$

Lifetime income is increased by the taxes paid by those who die in the first period and divided among those who survive. These survivorship benefits, or tontine benefits, raise expected lifetime income and thereby consumption for all. As a result of the higher consumption, Abel and others have found that the introduction of social security raises individual welfare for all.

In the absence of social security, the consumer would save assets to consume in period 2. But in the event of death, no utility would be realized from those assets. By taxing away some of those assets, and providing a benefit contingent on survivial to period 2, the consumer is able to increase consumption in period 1 and thereby gain some utility from what otherwise would be lost to an accidental bequest.

However, if accidental bequests have utility, social security reduces potential utility from an accidental bequest, since the tax confiscates some or all of the accidental bequest. In Section 4 we consider this possibillity.

1.3 The Impact of Improving Mortality

There has been a very significant improvement in mortality at all ages over the period since Social Security came into existence. For our purposes, using the simple model described above, this improvement can be captured in two aspects. First, more workers are living into retirement. Second, they are living longer in retirement. Their benefits must stretch further than originally planned.

Table 2 presents relevent data on these two aspects of improving longevity.

Table 2. Indicators of Improving Mortality

Life Table	Probability that Male 20 Survives to Age 65	Life Expectency Male, Aged 65
1940	.606	11.92
1970	.669	13.14
2005	.796	16.05

In this section of the paper we analyze the impact of more workers living to reach retirement. Later we will modify the model to analyze the impact of a longer retirement period.

The dramatic increase in survival rates has an important impact on the dynamics of the social security system. Most obvious perhaps is that the actuarially fair tax rate that applied in 1940 is inadequate for 2005. See Table 3 below . But there are also important implications for the pattern of consumption between c_1 and c_2 , and the fact that Lifetime Income is falling due to fewer bequests spread among more survivors ($I = Y + \frac{qT}{p}$). These are implications for the ability of social security to enhance welfare, due only to the improvement in mortality rates.

Table 3. Impact of Improved Mortality

Life Table	Lifetime Income (I)	Actuarially Fair Tax	Share Soc. Sec. Benefit Funded by: Redistributed Bequests	Tax + Return
1940	1.0253	3.88%	39.5%	60.5%
1970	1.0212	4.29	33.1	66.9
2005	1.0131	5.10	20.4	79.6

Note how the improvement in survival to retirement has reduced the share of the social security benefit that is funded by redistributing the taxes paid by the non-survivors. From nearly 40% in 1940 to only 20% in 2005, increased survivor probabilities put a greater burden on the tax rate to provide for the social security benefit. It is the redistribution mechanism which gives Social Security the capability to enhance welfare. Improved mortality makes that mechanism less effective, since fewer redistributed bequests results in reduced Lifetime Income.

In this section, we analyze the effect of changing mortality on the overall utility derived from the social security system. We start with the utility function: $U_c = u(c_1) + p\delta u(c_2)$. We are interested in $\partial U_c/\partial p$ .To derive the partial derivative of utility from consumption with respect to mortality, we first need some other derivatives, which are interesting in their own right.

We start with the effect changes in p have on a . In Table 1 we noted that as p increases, a decreases, thereby decreasing optimal period 1 consumption. We see this analytically from the expression for $\partial a/\partial p$:

$$\partial a/\partial p = -a^2\delta$$

This term is clearly always negative. As the probability of survival to period 2 increases, the consumer will reduce period 1 consumption.

Next we consider $\partial c_1/\partial p$ and $\partial c_2/\partial p$, the change in consumption pattern as survival probabilities change.

Using CRRA utility, the derivative of optimal first period consumption with respect to mortality is given by:

$$\partial c_1/\partial p = -aSR^{-1} + \partial a/\partial p \cdot I$$

The first term, $-aSR^{-1}$, is negative, and comes from the result that fewer deaths prior to retirement reduces expected lifetime income ($I = Y + (1-p)SR^{-1}$). The partial derivative of lifetime income with respect to mortality is $-SR^{-1}$, and the share of that reduced lifetime income that affects first period consumption c_1, is $a(-SR^{-1})$.

The second term is also negative. It derives from the fact that $\partial a/\partial p$ is negative, as we observed just above. The total of the two terms is therefore negative. Table 4 shows the values of c_1 and $\partial c_1/\partial p$ for illustrative examples.

Table 4. Change in Optimal Period 1 Consumption

Life	$S=0$		$S=.1$	
Table	c_1	$\partial c_1/\partial p$	c_1	$\partial c_1/\partial p$
1940	.6682	$-.0672$	.6850	$-.1117$
1970	.6571	$-.0754$	.6710	$-.1191$
2005	.6373	$-.0920$	.6456	$-.1341$
$p=1$	.6105		.6105	

There are a couple interesting results to point out from Table 4. The case where $S=0$ is of course the case without social security. Therefore there are no taxes and no tontine effects, and the first term of the derivative drops out (i.e. $-aSR^{-1}=0$). The remaining term is the effect of the declining value of a , the share of lifetime income optimally consumed in period 1. It is also interesting to note that the case where $S=0$ and the case where $S=.1$, converge to the same value of c_1 as p approaches 1. This is not only a nice check that our equations are programmed correctly, it confirms our intuition that the individual welfare gains from social security diminish to zero as mortality improves. If all workers survive to retirement, there are no welfare gains in this simple model. The workers will have outlived the usefulness of social security.

Now consider c_2 , optimal consumption in period 2. The partial derivative of c_2 with respect to probability of survival is:

$$\partial c_2/\partial p = -(1-a)S + \partial(1-a)/\partial p \cdot RI$$

The first term is negative and is again the direct effect of the decrease in lifetime income due to smaller tontine effects. The amount of the decrease in lifetime income from improving mortality is $-SR^{-1}$. The share that affects optimal period 2 consumption is $R(1-a)(-SR^{-1}) = -(1-a)S$.

The second term is the indirect effect of the change in $1-a$, the share of lifetime income (accumulated to period 2) that is optimally consumed . Since $\partial(1-a)/\partial p = -\partial a/\partial p$, then this term is positive. The net effect is ambiguous due to offsetting signs. If the shifting balance of smoothing consumption dominates the effect of decreasing lifetime resources, then the $\partial c_2/\partial p$ will be positive. But if the effect of decreasing lifetime resources is greater than the smoothing effect change, then $\partial c_2/\partial p$ will be negative. Table 5 finds an example where the sign of $\partial c_2/\partial p$ changes to negative in the case where $S = .3$ and $\gamma = 6$. Note again that as $p \to 1$, the outcome of the social security system approaches the outcome without social security.

Table 5. Change in Optimal Period 2 Consumption

	$\gamma = 2$				$\gamma = 6$	
Life	$S = 0$		$S = .1$		$S = .3$	
Table	c_2	$\partial c_2/\partial p$	c_2	$\partial c_2/\partial p$	c_2	$\partial c_2/\partial p$
1940	.5177	.1048	.5308	.0743	.6219	$-.0716$
1970	.5349	.1176	.5463	.0858	.6213	$-.0690$
2005	.5659	.1435	.5733	.1091	.6181	$-.0636$
$p = 1$	.6076		.6076		.6088	

It should be noted that $\gamma = 6$ would be considered an extremely high level of resistance to intertemptoral substitution, so that more normal levels would find that the change in c_2 from a change in survival would most likely be positively correlated, due to the shifting of consumption from period 1 to period 2. The columns in Table 5 where $S = 0$, show only the effect of shifting consumption shares, since the first term (the effect of decreased tax redistributions) is zero.

Since the focus of this paper is on the effect that improving mortality has had and will continue to have on the ability of social security to enhance welfare for all, let's consider $\partial U_c/\partial p$.

$$\begin{aligned} U_c &= u(c_1) + p\delta u(c_2) \\ &\implies \partial U_c/\partial p = u'(c_1)\cdot\partial c_1/\partial p + \delta u(c_2) + p\delta u'(c_2)\cdot\partial c_2/\partial p \\ &\implies \partial U_c/\partial p = \delta u(c_2) && \text{((a))} \\ &\quad +u'(c_1)(-aSR^{-1}) + p\delta u'(c_2)(-(1-a)S) && \text{((b))} \\ &\quad +u'(c_1)\cdot\partial a/\partial p\cdot I + p\delta u'(c_2)\cdot\partial(1-a)/\partial p\cdot RI && \text{((c))} \end{aligned}$$

Line (a) above, represents the change in utility from living long enough to consume c_2. This increases utility. Line (b) represents the effect on marginal utility from the direct change in c_1 and c_2 that derive from a reduced lifetime income (due to smaller tontine benefits). Line (b) always represents a decrease in utility. Line (c) represents the indirect effect on marginal utility of the shifts in consumption of lifetime income due to mortality-induced change in a. The first term in line (c) always decreases utility and the second term always increases utility. The net effect on line (c) is ambiguous, but we expect that the effect of lines (a) and (b) will dominate, making the sign of the overall derivative negative. Certainly this is the case in Table 6 below, which presents examples with no Social Security ($S = 0$), and Social Security examples with baseline elasticity of intertemporal substitution ($\gamma = 2$), and inelastic intertemporal substitution ($\gamma = 6$).

Table 6. Impact of Improved Mortality on Utillity from Consumption

	$\gamma = 2$				$\gamma = 6$	
Life	$S = 0$		$S = .1$		$S = .3$	
Table	Uc	$\partial Uc/\partial p$	Uc	$\partial Uc/\partial p$	Uc	$\partial Uc/\partial p$
1940	−2.2400	−1.2266	−2.1848	−1.3330	−2.8393	−2.6239
1970	−2.3160	−1.1871	−2.2679	−1.3048	−3.0046	−2.6246
2005	−2.4625	−1.2222	−2.4307	−1.2615	−3.3395	−2.6560

1.4 Accidental Bequests

We now relax the assumption that assigns no utility to accidental bequests. A small contribution of the paper is a method to find a minimum value for the utility of an accidental bequest, and to include that value into the model of social security.

The minimum value for the utility of an accidental bequest is based on the well documented reluctance of consumers to use non-pension assets to buy private annuities.[3] This reluctance is sometimes attributed to a failure of the market to provide product choices, or unfair pricing of private annuities, or a desire for control on the part of the consumer. This paper assumes that the reason consumers do not buy private annuities is that the utility from consumption of the somewhat higher income possible via annuitization is less than the utility from an accidental bequest. The consumer has evaluated the option of a higher income against the inability to pass assets to their heirs in the case of an untimely death. Overwhelmingly, the consumer opts to not annuitize. The clearest and simplest explanation is that the consumer prefers the utility of an accidental bequest to the utility of somewhat higher consumption.

If that is the case, we can derive a minimum value for the utility of an accidental bequest as the difference between the utility from consumption that would be possible with a life annuity, and the utility from consumption that would be possible with an annuity certain (i.e., without life contingencies). In a two period model, that would translate mathematically as follows:

$$\widehat{u}(A) \geq [u(c_x) - u(c_{\neg})] \cdot (1 + \delta p)$$

where $\widehat{u}(A)$ =the utillity from an accidental bequest of A

$u(z)$ = the utility from consumption of amount z

$c_x = A/(1 + pR^{-1})$; this is the level of consumption possible if A is converted

[3] Pension assets are often used to provide annuities, but these choices are made by employers, not by employees.

to a life annuity.

$c_{\neg} = A/(1+R^{-1})$; this is the level of consumption possible if A is converted to an annuity certain.

$(1+\delta p)$ sums the discounted utility value of over periods 1 and 2.

Let the utility of accidental bequests be defined as equal to the above minimum value. There can only be an accidental bequest in period 1, since death is certain after period 2. In period 1, the wealth available to annuitize is $Y-T$, which is $Y-pSR^{-1}$ when the system is actuarially fair. Thus,

$$\widehat{u}(Y-pSR^{-1}) = [u((Y-pSR^{-1})/(1+pR^{-1})) - u((Y-pSR^{-1})/(1+R^{-1}))] \cdot (1+\delta p)$$

We can now modifiy the model to include the utililty from accidental bequests as follows:

$$\begin{aligned} U &= U_c + U_b \\ U &= u(c_1) + p\delta u(c_2) + q\widehat{u}(Y-pSR^{-1}) \end{aligned}$$

Under this definition of U_b, U_b and U_c are independent. Below we will challenge this setup, but for now it allows us to analyze U_b in a somewhat simpler setting.

What is the impact of improving mortality on U_b? As the probability of an untimely death diminishes, so does the opportunity for an accidental bequest. This is confirmed analytically in the derivative of U_b with respect to mortality, $\partial U_b/\partial p$.

$$U_b = q\widehat{u}(Y-pSR^{-1}) = (1-p)\widehat{u}(Y-pSR^{-1})$$

$$\begin{aligned} &\Longrightarrow \quad \partial U_b/\partial p = (1-p)\widehat{u}'(Y-pSR^{-1})(-SR^{-1}) - \widehat{u}(Y-pSR^{-1}) \\ &\Longrightarrow \quad \partial U_b/\partial p = q\widehat{u}'(Y-pSR^{-1})(-SR^{-1}) - \widehat{u}(Y-pSR^{-1}) \end{aligned}$$

The first term on the right hand side represents the change in marginal utility from accidental bequests due to the decrease in wealth that is exposed to the risk of accidental bequest. It is always negative. Wealth decreases because taxes must increase as mortality improves. The marginal tax increase is SR^{-1} , so the reduction in marginal utility is qSR^{-1} times the rate of marginal utility at that point, $\widehat{u}'(Y - pSR^{-1})$. The second term represents the direct change in utility due to the fact that the probability of leaving a bequest is decreasing. Since utility of an accidental bequest is defined as the difference between the utility of consuming c_x and the utility of consuming $c_{\neg}$ (a lesser amount), the second term is always negative. So $\partial U_b / \partial p$ is always negative.

It does not seem economically sound however, that U_c and U_b should be independent. Certainly the prospect of achieving utility from an accidental bequest ought to have an effect on decisions about c_1 and c_2 . To accomplish this, the definition of $\widehat{u}(A)$ is modified slightly to anchor it to the utility of consuming c_1 and c_2. Let: $c_x = A/(1 + pR^{-1})$ and $c_{\neg} = A/(1 + R^{-1})$ as before. Define $d = c_x - c_{\neg}$, the extra periodic income that is possible from annuitization. Then, define $\widehat{u}(A)$ as follows:

$\widehat{u}(A) = u(c_1 + d) - u(c_1) + p\delta(u(c_2 + d) - u(c_2)).$

This definition more accurately reflects the fact that the utility of an accidental bequest is not only a function of the size of the bequest, but also a function of the level of consumption of the consumer. Unfortunately, now that U_c and U_b are no longer independent, analysis becomes more difficult. We fall back on numerical methods to find derivatives and solve the Euler Equation.

Our model objective is now to maximize:

$$
\begin{aligned}
U &= U_c + U_b = u(c_1) + p\delta u(c_2) + q\widehat{u}(Y - pSR^{-1}) \\
&= u(c_1) + p\delta u(c_2) + q\{u(c_1 + d) - u(c_1) + p\delta(u(c_2 + d) - u(c_2))\}
\end{aligned}
$$

where:

$$\begin{aligned} u(c) &= (1-\gamma)^{-1} \cdot c^{(1-\gamma)} \\ d &= c_x - c_\neg \\ T &= pSR^{-1} \\ c_x &= (Y-T)/(1+pR^{-1}) \\ c_\neg &= (Y-T)/(1+R^{-1}) \end{aligned}$$

and subject to the budget constraint:

$$c_2 = R(Y - T - c_1) + S$$

The first order condition yields the following Euler Equation:

$$0 = u'(c_1) - Rp\delta u'(c_2) + q\{u'(c_1 + d) - u'(c_1) - Rp\delta(u'(c_2 + d) - u'(c_2)\}$$

The first two terms are the marginal utility from consumption and the third term is the marginal utility from an accidental bequest. The first order condition is that the marginal utility from consumption must balance the marginal utility from an accidental bequest. Using CRRA utility and imposing the budget constraint, we get :

$$\begin{aligned} 0 &= (c_1)^{-1/\gamma} - Rp\delta(R(Y - T - c_1) + S)^{-1/\gamma} + q\{(c_1 + d)^{-1/\gamma} - (c_1)^{-1/\gamma} \ldots \\ &\quad -Rp\delta[(R(Y - T - c_1) + S + d)^{-1/\gamma} - (R(Y - T - c_1) + S)^{-1/\gamma}]\} \end{aligned}$$

We solve the above for c_1, and then solve for c_2, U_c, and U_b. Results are displayed in Tables 7, 8, and 9.

Table 7. Optimal Period 1 Consumption ($S = .1, \gamma = 2$)

Life Table	U_c c_1	$U_c + U_b$ c_1
1940	.6850	.6880
1970	.6710	.6727
2005	.6456	.6456

Table 7 shows the change in consumption as a result of including utility from accidental bequests is very small.

Table 8. Total Utility ($S = .1, \gamma = 2$)

Life Table	U_c	U_b	$U_c + U_b$
1940	-2.1849	.1253	-2.0596
1970	-2.2679	.0917	-2.1762
2005	-2.4307	.0372	-2.3935

In Table 8, the components of total utility are shown separately. We see the rapid decline in U_b as mortality improves. This amplifies the decrease in total utility, reinforcing the decrease in utility from consumption as survival probabilities improve.

Table 9. Bequest Utility and Total Utility at Various Benefit Levels ($\gamma = 2$)

Life	U_b			$U_c + U_b$		
Table	$S = 0$	$S = .1$	$S = .3$	$S = 0$	$S = .1$	$S = .3$
1940	.1357	.1253	.1066	-2.1044	-2.0596	-1.9757
1970	.0991	.0917	.0781	-2.2170	-2.1762	-2.0993
2005	.0400	.0372	.0319	-2.4224	-2.3935	-2.3376

Table 9 presents the effect of increasing benefits on utility from bequests and total utility. As expected, U_b decreases as the tax and benefit level increases. But the model shows increasing total utility as the tax and benefits increase, even with decreasing utility from accidental bequests taken into account. The utility from higher lifetime consumable income dominates the loss of utility from lower bequests that are associated with higher Social Security benefits.

1.5 Increasing Life Expectency

Improvements in mortality rates in the United States means more than just more workers reaching retirement age and fewer redistributed tax bequests. Life expectencies have also increased, meaning that period 2 in 2005 is relatively "longer" than it was in 1940. We modify our two period model to build in the effect of increasing longevity. Let $e_0 = 1$ represent the adjustment for life expectency corresponding to the 1940 Table. Life expectency for a male aged 65 increased from 11.92 in 1940 to 16.05 in the 2005 Table. Let $e_1 = 11.92/16.05 = .7427$. Then we will define "effective c_2" (i.e. c_2 adjusted for longevity) in 2005 to be $e_1 \cdot c_2$. What we mean is that relative to c_1 (which represents consumption over the comparatively unchanged working period), c_2 in 2005 is only 74.27% as effective as c_2 relative to c_1 in 1940. It is less effective because it has to be stretched over a longer retirement. It is perhaps more intuitively clear if we recall that the model is as if the consumer received benefits as a lump sum, not a lifetime annuity. Thus it is clear that the benefits and indeed all of c_2 have to stretch further.

We make an assumption that improvements in mortality before retirement are related to increases in longevity once in retirement. We make the simplifying assumption that the relationship is a linear function of p. Define an extension function,

$$e(p) = e_b - e_m p$$

where the subscripts b and m refer to the y-intercept b and the slope m in the point-slope version of a linear equation. For the model, using data from 1940 and 2005 U.S. Life Tables, we solve for the linear relationship which produces the values $\{e_0, e_1\}$, when p takes on the values $\{.607, .796\}$, respectively (see Table 1). Thus $e(p) = 1.8264 - 1.3614p$. We use $e(p)$ to adjust the effectiveness of c_2 as improving mortality extends the lifetime over which c_2 must be stretched.

Consider first the model with utililty only from consumption. Choose c_1 and c_2 to maximize:

$$U = U_c = u(c_1) + \delta p u(e \cdot c_2)$$

subject to:

$$\begin{aligned} c_2 &= R(Y - T - c_1) + S \\ T &= pSR^{-1} \\ e &= e_b - e_m p \end{aligned}$$

The first order condition gives us:

$$u'(c_1) = eR\delta p u'(e \cdot c_2)$$

Imposing the constraint and using CRRA utility, we get a result parallel to the result in Section 3:

$$\begin{aligned} c_1 &= aI \\ c_2 &= R(1 - a)I \end{aligned}$$

where

$$\begin{aligned} a &= eR/(eR + (eR\delta p)^{1/\gamma}) \\ I &= Y + qSR^{-1} \end{aligned}$$

Table 10 presents the results of the model with the adjustment for longevity.

Table 10. Total Utility with Extended Longevity

Life Table	$S = 0$	$S = .1$	$S = .3$
1940	−2.1044	−2.0596	−1.9757
1970	−2.2853	−2.2433	−2.1640
2005	−2.7048	−2.6725	−2.6101

As before, utility decreases as survival probabilites improve. Also, utility increases as taxes and benefits increase.

Finally, we consider a setup of the model parameters which have more intuitive meaning. In 1940, a worker could expect to work 3.34 years for every year in retirement. So, weighting period one consumption by 1/3.34, and period 2 consumption by $e(p)$, we get a version of the model which provides some minimal insight into the reasonableness of the results. Tables 11 and 12 present results based on this intuitive setup.

Table 11. Model with Intuitive Setup ($\gamma = 2$)

Life	$S = 0$					
Table	I	T	c_1	c_2	U_c	$U_c + U_b$
1940	1.0	0	.7881	.3305	−5.4022	−5.3444
1970	1.0	0	.7715	.3565	−5.6297	−5.5848
2005	1.0	0	.7356	.4125	−6.1788	−6.1578
	$S = .1$					
1940	1.0253	3.88%	.8079	.3390	−5.2691	−5.2162
1970	1.0212	4.29%	.7878	.3642	−5.5127	−5.4714
2005	1.0131	5.10%	.7452	.4178	−6.0991	−6.0796

In Table 11, the basic components of the model are shown for the case without Social Security ($S = 0$), and the case with Social Security ($S = .1$). Note that under Social Security, consumption and total welfare increase, although the increase in Lifetime Income (I) under Social Security relative to no Social Security decreases with improving survival probability.

Table 12. Diminution in Welfare per tax dollar

Life	Gain in Welfare			Gain in Period 1 Consumption		
Table	utils	T	utils/T	$\%\Delta c_1$	T	$\%\Delta c_1/T$
1940	.128	3.88%	.033	2.54	3.88%	.65%
1970	.113	4.29	.026	2.07	4.29	.48
2005	.078	5.10	.015	1.22	5.10	.24

Table 12 tells us that in 2005, the individual welfare gain from Social Security in utils per unit of tax is only 46% of what the improvement in individual welfare had been in 1940. The increase in period one consumption per unit of tax is 37% of what it was in

1940. Both results show that while Social Security still delivers individual welfare gains, there is diminished effectiveness at enhancing individual welfare.

1.6 Conclusion

In the end, from the simple model of this paper, using mortality rates from the 2005 U.S. Life Table, we find that social security still generates welfare gains for our representative consumer. But Social Security with the longevity of today is much reduced from the effectiveness of the welfare gains produced in the same model, using mortatility from the 1940 U.S. Life Table. Moreover, there has been a significant shift in the means of funding benefits, away from gains from survivorship in a tontine system (which produce welfare gains), to a reliance on investment returns on accumulated taxes (which are welfare nuetral). In the 1940 version of the model, the sources for funding the social security benefit were 38.8% from the tax itself, 21.7% from an investment return on the tax, and 39.5% from the tontine effects of redistributing the taxes and returns from those who die before retirement to those who survive to retirement. By the 2005 version, the breakdown by source was 51% from the tax itself, 28.6% from the investment return on the tax, and only 20.4% from the tontine effects of survivorship.

Of course the model does not reflect the way Social Security works in actual practice. Social Security is not a funded, actuarially fair system, but a pay-as-you-go (paygo) system in which the taxes are presently inadequate to meet the promised benefits in the future. A goal of this paper is simply to shed light on the role that mortality has played in creating welfare gains, so that the power of mortality to create welfare gains can again be harnessed in the reforms that are inevitable. A welfare enhancing reform might be possible if the Social Security mechanism design provided the opportunity again for survivorship gains. This means that the age at which benefits are received would need to be increased so that the survivor probability is reduced, and tontine gains are possible.

A further goal of the paper is to explore the role that utility from accidental bequests might play in a reform of social Security. Social Security taxes resources that otherwise might produce accidental bequests, thereby reducing any utility that they might generate. Perhaps a future reform could be designed to minimize the loss of utility from accidental bequests, and thereby increase the chances of a welfare enhancing reform. This paper is inconclusive with respect to accidental bequests. A method for evaluating the utility from accidental bequests is developed, but outcomes of the model with utility from accidental bequests are not significantly different than outcomes without such utility. The improvement in longevity reduces the likelihood of an accidental bequest, and makes this factor even less significant.

Social Security is a paygo system. My future direction of research will shift to the paygo design. The model of this paper ignored the role of bequests received by consumers in their lifetime, but received bequests are an important part of wealth transmission in society, and cannot be ignored. Future research will include received bequests as well in the analysis of Social Security. The role of bequests, both utility of bequests left by consumers and the impact of bequests received on wealth transmission and equality of wealth distribution , are believed by the author to be a key part of Social Security reform design. Future research will include this dimension of Social Security reform analysis.

Chapter 2

Outliving its Usefulness? Addendum

Boronow (2006) used a simple two period model of a fully funded social security system to analyze the impact of improving mortality rates on the efficacy of Social Security to enhance indivdual welfare. It is not surprising that social security is more expensive as longevity increases. But, as seen in Boronow (2006), it is also the case that lower mortality rates reduce the scope of social security to capture survivorship gains and enhance individual welfare. A limitation of the 2006 analysis is that the model of social security used a fully funded design, while the actual system is a pay-as-you-go (paygo) system. This addendum extends the analysis to a paygo model. Another limitation of the earlier paper is that received bequests were ignored in the model. The model in this addendum includes received bequests.

A fully funded social security system relies on survivorship (tontine) gains to enhance individual welfare. A paygo system depends on both survivorship and population growth relative to capital returns to enhance individual welfare. A paygo system will destroy welfare if the return to survivorship and population growth is less than returns to capital. As mortality rates improve and more workers survive to retirement, the question of whether welfare is enhanced or destroyed under a paygo system increasingly depends on the population growth rate.

The role of received bequests in the analysis of social security is both central and complex. Insights are more easily gained in social security models without bequests, but these insights must be analyzed to see how they are affected by bequests. In simple models such as the one employed in Boronow (2006), there is one homogeneous agent. In this setting received bequests do not significantly alter the welfare enhancing mechanism of social security. That is a finding in Abel (1985), and it is also the case in this addendum for the fully funded model. But it is not the case in models with heterogeneous agents. With heterogeneous agents, received bequests play a key role in transmitting wealth inequality. In this addendum, received bequests are added to the simple two period model, to gain understanding of how they affect the dynamics of the model. In future research we can build on this further in models with heterogeneity in income and bequests.

Finally, purely for the sake of clarity of presentation, the model in the addendum treats increased longevity during the retirement period slightly differently than the 2006 paper. In that paper, there was an assumed relationship between the improvement in the probability of reaching retirement age and increased longevity in retirement. As retirement longevity increased, period 2 consumption was "stretched" to cover the longer period. This addendum treats these two different aspects of improving mortality independently. A retirement longevity factor is introduced to weight the second period utility in proportion to retirement longevity. This is not a significant change. While the two aspects of improved mortality are not totally independent, it is not too far from truth, and it simplifies the analysis and understanding to treat them as such.

This addendum is organized as follows: First, the two period model is modified to accomodate a paygo design for social security. Then received bequests are included. Finally, we analyze the model for the dynamics of key components with respect to survival rates and with respect to a longevity factor. The addendum concludes with a summary of findings.

2.1 Addendum Two-Period Model

Consider an economy with many identical consumers, and a single commodity. The length of life is either one period or two periods. Each consumer works during the first period and earns a fixed labor income Y, consumes an amount c_1, and pays a tax T. Whatever wealth from period 1 that is not consumed earns a fixed gross return of R. There is a probability p that the consumer survives to the second period, retirement, during which the consumer does not work. The expected length of the retirement period, relative to the length of the working period is denoted by σ, and this factor is also used to weight utility in period 2 relative to utility in period 1. In period 2, survivors receive a (lump sum) social security benefit of $S \cdot \sigma$, consume an amount c_2, and die with certainty, optimally leaving no unplanned bequests in period 2. Each generation has G children, who enter the model as new workers in period 2. We begin with a preliminary model in which received bequests are still not included. Later we will add received bequests to the model.

2.1.1 The Individual's Problem (without Received Bequests)

The consumer chooses c_1 and c_2 to maximize the following expected utility function:

$$U = u(c_1) + p\delta\sigma u(c_2)$$

where δ is a time preference parameter, and

$$u(c) = \frac{c^{(1-\gamma)}}{1-\gamma}$$

.

The consumer faces the budget constraint:

$$c_2 = R(Y - T - c_1) + S\sigma$$

The first order condition with respect to c_1 is:

$$u'(c_1) = Rp\delta\sigma \cdot u'(c_2)$$

We obtain the following solutions for optimal consumption, c_1 and c_2 :

$$\begin{aligned} c_1 &= a(Y - T + \frac{S\sigma}{R}) \\ c_1 &= aI \end{aligned} \tag{2.1}$$

$$\begin{aligned} c_2 &= R(1-a)(Y - T + \frac{S\sigma}{R}) \\ c_2 &= R(1-a)I \end{aligned} \tag{2.2}$$

where

$$I = Y - T + \frac{S\sigma}{R} \tag{2.3}$$

$$a = \frac{R}{R + (Rp\delta\sigma)^{1/\gamma}} \tag{2.4}$$

These are intuitive outcomes. Consumable lifetime income (I) is equal to aftertax earnings plus the period one value of the social security benefits. Optimal period one consumption is a proportion of lifetime income based on utility parameters, and period two consumption is the remaining income, accumulated to the next period.

Fully Funded

In the mechanism design where social security benefits are fully funded, taxes earn a return sufficient to pay benefits to the survivors ($TR = pS\sigma$). Substituting for $S\sigma$ in equation (3) above, we get:

$$I = Y + \frac{1-p}{p} \cdot T$$

In words, lifetime consumable income (I) equals earned income (Y) plus the social security taxes paid by those who die before retirement, divided among those who survive to retirement ($\frac{1-p}{p} \cdot T$). It is evident that any increase in individual welfare under fully funded social security derives from survivorship. If all workers survived to retirement, then social security could add nothing to their lifetime consumable income and hence to their individual welfare.

Paygo

In the design wherein social security benefits are funded by a paygo system, taxes paid by the children of the old generation pay for the benefits to the survivors ($TG = pS\sigma$).

Substituting for $S\sigma$ in the equation for lifetime consumable income (3), we get:

$$I = Y - T + \frac{TG}{p} \cdot \frac{1}{R}$$

In words, lifetime consumable income equals earned income less the taxes to provide benefits to the current old, plus the present value ($\frac{1}{R}$) of taxes paid by all the children of the present generation (TG), divided among the survivors to retirement. It is evident that survivorship again plays a key role in creating individual welfare gains under social security, even with a paygo system. With a little algebra we get:

$$I = Y + S\sigma \cdot (\frac{1}{R} - \frac{1}{G} + \frac{1-p}{G})$$

It is clear that if survivorship approaches 100%, then welfare gains from survivorship dissipate, leaving only the difference between capital returns (R) and returns from population growth (G) to produce individual welfare gains (or losses) under paygo.

Whichever method is used to fund social security benefits, social security generates individual welfare gains via the mechanism of survivorship. In the fully funded model, all individual welfare gains derive from survivorship; under paygo, welfare gains come from survivorship, and possible gains or losses come from financing via population growth rather than capital returns.

2.1.2 The Individual's Problem (with Received Bequests)

Let us introduce received bequests into the addendum model. Following a common technique, we assume that assets at death are confiscated by the government and redistributed equally to new workers in the next generation. Let B represent the amount of bequest received by a worker, and B_{-1} represent the amount of bequest received by a worker in the prior generation.

The consumer chooses c_1 and c_2 to maximize the following expected utility function:

$$U = u(c_1) + p\delta\sigma u(c_2)$$

subject to:

$$\begin{aligned} c_2 &= (B + Y - T - c_1)R + S\sigma \\ B &= (B_{-1} + Y - T - c_1)R \cdot \frac{(1-p)}{G} \\ u(c) &= \frac{c^{(1-\gamma)}}{1-\gamma} \\ T &= \frac{pS\sigma}{R} \text{ (fully funded)} \\ T &= \frac{pS\sigma}{G} \text{ (paygo)} \end{aligned}$$

In the steady state, $B = B_{-1}$. Solving for B:

$$\begin{aligned} B &= \frac{(Y - T - c_1)R \cdot \frac{(1-p)}{G}}{1 - R \cdot \frac{(1-p)}{G}} \\ B &= (Y - T - c_1) \cdot q \\ q &\equiv \frac{R \cdot \frac{(1-p)}{G}}{1 - R \cdot \frac{(1-p)}{G}} \end{aligned} \tag{2.5}$$

The term denoted by q gives the intuition of the "rate of bequests", in that the amount of bequests is q times the ending period 1 savings from income.

Define the gross bequest factor Q :

$$Q \equiv 1 + q = \frac{G}{G - (1-p)R} \tag{2.6}$$

. The factor Q "grosses up" whatever it is multiplied by to give the value in the steady state after an infinite series of generational bequests.[1] Substituting for B in the budget constraint, we get:

$$c_2 = (Y - T - c_1)QR + S\sigma$$

The intuition here is that the resources available for consumption in period 2 are after-tax earnings less period 1 consumption, "grossed up" by aquired wealth through bequests, and accumulated to period 2, plus the social security benefit.

[1] Throughout this paper it will be assumed that $G - (1-p)R > 0$.

The first order condition with respect to c_1 is:

$$u'(c_1) = QRp\delta\sigma \cdot u'(c_2)$$

We obtain the following solutions for optimal consumption, c_1 and c_2 :

$$\begin{aligned} c_1 &= a[(Y-T)Q + \frac{S\sigma}{R}] \\ c_1 &= aI \end{aligned} \tag{2.7}$$

$$\begin{aligned} c_2 &= R(1-aQ)[(Y-T)Q + \frac{S\sigma}{R}] \\ c_2 &= R(1-aQ)I \\ c_2 &= R(I - c_1 Q) \end{aligned} \tag{2.8}$$

where

$$I = (Y-T)Q + \frac{S\sigma}{R} \tag{2.9}$$

$$a = \frac{R}{QR + (QRp\delta\sigma)^{1/\gamma}} \tag{2.10}$$

Formally, the optimal solutions for the model with received bequests (equations (7) and (8)) look similar to the solution for the model without bequests (equations (1) and (2)), except that the gross bequest factor Q appears in key places to account for generational bequests on the outcome of the problem. Lifetime consumable income (I) is after-tax earnings, grossed up to account for bequests, plus the present value of social security benefits. Consumption in period 2 is the difference between lifetime income (accumulated to period 2) and the amount consumed in period 1, grossed up to account for the effect of bequests that won't happen on consumed wealth (and accumulated to period 2). Since Q appears in the first order condition, the optimal proportion of wealth to consume (a), also contains Q in its formulation.

Fully Funded

In the model design wherein social security benefits are fully funded, taxes earn a return sufficient to pay benefits to the survivors ($TR = pS\sigma$). Substituting for $S\sigma$ in equation (9) above, we get:

$$I = YQ + \frac{1 - pQ}{p} \cdot T$$

In words, lifetime consumable income (I) equals earned income times the gross bequest factor (YQ) plus the social security taxes paid by all workers less the taxes paid by those who survived, (which are multiplied by the gross bequest factor to account for the fact that taxes paid by survivors are not available to be increased over the generations by bequests), divided among those who survive to retirement. It is evident that the increase in individual welfare under fully funded social security is still derived from survivorship. If all workers survived to retirement (i.e., $p = 1$), then fully funded social security could add nothing to their lifetime consumable income ($Q \rightarrow 1$ as $p \rightarrow 1$). This is essentially the same outcome we obtained earlier in the simplified fully funded model without received bequests.

Paygo

In a paygo system, taxes paid by the children of the old generation pay for the benefits to the survivors ($TG = pS\sigma$). Substituting for $S\sigma$ in the equation for lifetime consumable income, we get:

$$I = (Y - T)Q + \frac{TG}{p} \cdot \frac{1}{R} \tag{2.11}$$

In words, lifetime consumable income (I) equals earned after-tax income grossed up to account for generations of bequests ($(Y - T)Q$), plus the present value of taxes paid by the children of the present generation ($\frac{TG}{R}$), divided among the survivors ($\frac{1}{p}$). Survivorship again plays a key role in creating individual welfare gains under social security, even with a paygo system. But, with a little algebra we get:

$$I = YQ + \frac{S\sigma}{R} \cdot [\frac{G - R}{G - (1-p)R}]$$

Here we see that with paygo financing of social security and assuming an equal redistribution of bequests, the gain or loss comes from population growth less capital return. For example, if population growth G is greater than R, the expression in brackets is positive, and lifetime income is enhanced by social security. But if G is less than R, than the expression is negative and lifetime income is reduced by social security. Survivorship amplifies the effect of the gain or loss on lifetime consumable income, through the terms in the denominator. A high rate of survivorship makes the denominator larger than if there were a low rate of survivorship. Thus high survivorship amplifies the gains or losses from $G - R$ less than low survivorship. In the paygo model, lifetime consumable income (I) is best when short-lived generations have many children. It is worst when long-lived generations have few children.

The chosen model mechanism for distributing bequests (equal shares to new workers) algebraically hides the role of survivorship in adding to lifetime income under a paygo design. An alternative bequest mechansim also based on equation (11), would have survivorship explicitly add to lifetime income by distributing taxes paid by the generation's children to the surviviors. Survivorship does matter.

With the optimal solutions in hand, we turn our attention to the dynamics of the system. In particular, we are interested in what happens when survivorship increases, and when longevity increases.

2.2 Dynamics of the Model with Bequests

2.2.1 Fully Funded Model

Dynamics of Increasing Survivorship

What is the effect on lifetime consumable income and on the pattern of consumption when the probability of surviving to retirement increases? For insight, we consider the two period model with received bequests, and look at derivatives of key components. For somewhat more simplicity in the analysis, we utilize log utility (i.e. $\gamma = 1$).

We may as well start with the obvious; taxes.

$$\frac{\partial T}{\partial p} = \frac{\partial[\frac{pS\sigma}{R}]}{\partial p} = \frac{S\sigma}{R} \tag{2.12}$$

Clearly, an improvement in survivorship means an increase in taxes. While each generation fully funds their own benefits, each worker does not fully fund her own benefit. Instead she relies in part on redistribution of taxes paid by those who do not survive to retirement. As there are fewer non-survivors, taxes have to increase to make up the shortfall, and increase the self-funded portion of the social security benefit.

We next consider the derivative of the gross bequest factor Q.

$$\begin{aligned} Q &= \frac{G}{G-(1-p)R} \\ \frac{\partial Q}{\partial p} &= \frac{-GR}{[G-(1-p)R]^2} = -Q^2\frac{R}{G} \end{aligned} \tag{2.13}$$

The derivative is negative, so that Q decreases when survivorship improves. This makes sense, since there will be fewer bequests to build up over the generations.

We next consider the derivative of lifetime income ($\frac{\partial I}{\partial p}$):

$$\begin{aligned} I &= (Y-T)Q + \frac{S\sigma}{R} \\ &\Rightarrow \frac{\partial I}{\partial p} = -(Y-T)Q^2\frac{R}{G} - \frac{S\sigma}{R}Q \end{aligned} \tag{2.14}$$

This derivative is negative. Lifetime income decreases when survivorship improves. This is from the effect of fewer bequests which build up over generations, and because taxes paid by non-survivors do not spread as far when there are more survivors and fewer non-survivors.

What is the effect on the distribution of consumption between periods 1 and 2? Consider $\frac{\partial a}{\partial p}$. (Under log utility, $a = \frac{1}{Q+Qp\delta\sigma} = \frac{1}{Q}\frac{1}{1+p\delta\sigma}$)

$$\frac{\partial a}{\partial p} = \frac{1}{Q}\frac{-\delta\sigma}{(1+p\delta\sigma)^2} + \frac{R}{G}\frac{1}{(1+p\delta\sigma)} \tag{2.15}$$

There are two terms on the right hand side of equation (15). The first term is the effect of shifting consumption between the two periods. It is negative, meaning that as survivorship increases, a smaller portion of wealth is consumed in period 1. The second term is positive. It is the income effect, as lifetime income decreases in response to greater survivorship. As lifetime income decreases, a greater share is consumed in period 1. The net effect depends on the relative returns from capital and population. If $\frac{R}{G} > \frac{1}{Q}\frac{\delta\sigma}{(1+p\delta\sigma)}$, then $\frac{\partial a}{\partial p} > 0$. Otherwise, $\frac{\partial a}{\partial p} < 0$.

Can we say anything about the dynamics of consumption?

$$\begin{aligned}
\frac{\partial c_1}{\partial p} &= \frac{\partial aI}{\partial p} = a\frac{\partial I}{\partial p} + I\frac{\partial a}{\partial p} \\
\frac{\partial c_1}{\partial p} &= a[-(Y-T)Q^2\frac{R}{G} - \frac{S\sigma}{R}Q] + I[\frac{1}{Q}\frac{-\delta\sigma}{(1+p\delta\sigma)^2} + \frac{R}{G}\frac{1}{(1+p\delta\sigma)}]
\end{aligned}$$

The first bracket expression on the right hand side of the above equation is negative, making the first term negatve. This is the effect from decreased lifetime income as survivorship increases. The sign of the second bracket expression is ambiguous; it is the effect of the change on a. But further analysis reveals:

$$\frac{\partial c_1}{\partial p} = \frac{-\sigma}{1+p\delta\sigma}[S(\frac{1}{R} - \frac{1}{G}) + c_1\delta] \tag{2.16}$$

If the term in brackets is negative, then $\frac{\partial c_1}{\partial p}$ is positive. But for the term in brackets to be negative, then it must be that $\frac{1}{G} > \frac{1}{R} + \frac{c_1\delta}{S}$. Keeping in mind the interpretation of the terms in $\frac{c_1\delta}{S}$, it is unlikely that this condition is feasible. Thus, we can expect that $\frac{\partial c_1}{\partial p}$ is negative, and period 1 consumption decreases as survivorship increases.

If $\frac{\partial c_1}{\partial p}$ is negative, we might expect $\frac{\partial c_2}{\partial p}$ to be positive. But this is more ambiguous than the case with period one consumption.

$$\frac{\partial c_2}{\partial p} = \frac{\partial R(I - aIQ)}{\partial p} = \frac{\partial RI(1 - \frac{1}{1+p\delta\sigma})}{\partial p}$$
$$\frac{\partial c_2}{\partial p} = R\{\frac{\partial I}{\partial p}(1 - \frac{1}{(1+p\delta\sigma)}) + I\frac{\delta\sigma}{(1+p\delta\sigma)^2}\}$$

The first term in the braces in the above equation is the income effect from decreasing lifetime income as survivorship increases. It is negative. The second term in the braces is the effect of a shifting pattern of consumption as survivorship increases. It is positive. The net effect is ambiguous. But further analysis produces:

$$\frac{\partial c_2}{\partial p} = \frac{R\delta\sigma}{(1+p\delta\sigma)} \cdot \frac{1}{(G-(1-p)R)} \cdot \{(Y-T)Q(G-R) + TG\} \tag{2.17}$$

This expression for $\frac{\partial c_2}{\partial p}$ is positive if the expression in braces is positive. That will be the case if $G > R$. It will also be true if $R > G$ and $\frac{R-G}{G} < \frac{T}{(Y-T)Q}$. Otherwise, if these two conditions do not hold, $\frac{\partial c_2}{\partial p}$ is negative. The key condition is that if the percentage by which the return on captial exceeds the return on population growth is less than the ratio of the social security tax to the after-tax period 1 wealth (before consumption in period 1), then $\frac{\partial c_2}{\partial p}$ is positive, and c_2 increases as p increases. Interestingly, even in the fully funded design, population growth rates enter the steady state analysis through the mechanism of generational bequests, and their effect on the accumulation of wealth.

Dynamics of Improvement in Longevity

In the last 50 years there has not only been a significant improvement in survivorship **to** retirement, there has been a significant increase in the longevity of those **in** retirement. What effect does an increase in longevity have in our two period model?

We will consider the derivatives of T, Q, I, a, c_1, and c_2 with respect to longevity (σ).

Consider $\frac{\partial T}{\partial \sigma}$.

$$\frac{\partial T}{\partial \sigma} = \frac{\partial[\frac{pS\sigma}{R}]}{\partial \sigma} = \frac{pS}{R} \tag{2.18}$$

No surprise here; taxes have to increase as longevity increases.

For the gross bequest factor Q, the derivative is 0, since bequests in this model only depend on pre-retirement wealth. Retirement wealth is assumed to be fully consumed.[2] Therefore, bequests are independent of retirement longevity.

Consider $\frac{\partial I}{\partial \sigma}$.

$$\begin{aligned} I &= (Y-T)Q + \frac{S\sigma}{R} = YQ + \sigma(\frac{S}{R} - \frac{pSQ}{R}) \\ \Rightarrow & \frac{\partial I}{\partial \sigma} = \frac{S}{R} - \frac{pS}{R}Q \end{aligned} \tag{2.19}$$

The first term on the right hand side of equation (19) is the effect of increasing longevity on the social security benefit, and the second term is the effect on taxes. Since taxes go up with longevity, this term must reduce lifetime income and is therefore negative. The social security benefit increases proportionately with longevity, so this term is positive. The change in lifetime income is the net effect of the change from higher social security benefits less the change from higher taxes. Further analysis finds:

$$\frac{\partial I}{\partial \sigma} = \frac{S}{R}[\frac{(1-p)(G-R)}{G-(1-p)R}]$$

If $G > R$, then $\frac{\partial I}{\partial \sigma} > 0$. If $G < R$, then $\frac{\partial I}{\partial \sigma} < 0$. If $G > R$, then the second term (the effect on taxes) in (19) has a smaller absolute value, since bequests have to be shared among a greater number of children. So the first term (the effect on social security benefits) determines the sign of the derivative in that case. Just the opposite occurs when population growth is less than the return on capital.

Now consider $\frac{\partial a}{\partial \sigma}$:

$$\frac{\partial a}{\partial \sigma} = \frac{1}{Q}\frac{-p\delta}{(1+p\delta\sigma)^2} \tag{2.20}$$

This term is always negative. As longevity increases, the proportion of resources consumed shifts from period 1 to period 2.

[2] This is equivalent to assuming that retirees hold all their wealth in the form of a life annuity.

This suggests that period 1 consumption is likely to decrease with increased longevity.

$$\begin{aligned}\frac{\partial c_1}{\partial \sigma} &= a\frac{\partial I}{\partial \sigma} + I\frac{\partial a}{\partial \sigma} \\ \frac{\partial c_1}{\partial \sigma} &= a\frac{S}{R}[\frac{(1-p)(G-R)}{G-(1-p)R}] + \frac{I}{Q}\frac{-p\delta}{(1+p\delta\sigma)^2}\end{aligned}$$

Further analysis produces:

$$\frac{\partial c_1}{\partial \sigma} = \frac{1}{(1+p\delta\sigma)}[\frac{S}{R}\frac{(1-p)(G-R)}{G} - c_1 p\delta] \tag{2.21}$$

Thus $\frac{\partial c_1}{\partial \sigma} > 0$ if the term is brackets is positive. However, this requires $(1-p)\frac{G-R}{G} > \frac{c_1 p\delta R}{S}$. With usual parameterizations, this condition is unlikely to be met, and it clearly fails when $R > G$. We expect therefore, that $\frac{\partial c_1}{\partial \sigma} < 0$, and period 1 consumption decreases with increased longevity.

Finally, consider $\frac{\partial c_2}{\partial \sigma}$:

$$\begin{aligned}\frac{\partial c_2}{\partial \sigma} &= \frac{\partial R(I - aIQ)}{\partial \sigma} = R\frac{\partial I}{\partial \sigma} - RQ\frac{\partial aI}{\partial \sigma} \\ \frac{\partial c_2}{\partial \sigma} &= R\{\frac{\partial I}{\partial \sigma}(1 - \frac{1}{(1+p\delta\sigma)}) + IQ\frac{p\delta}{(1+p\delta\sigma)^2}\} \\ \frac{\partial c_2}{\partial \sigma} &= \frac{RQp\delta}{1+p\delta\sigma}[\frac{S\sigma}{R}\frac{(1-p)(G-R)}{G} + c_1]\end{aligned} \tag{2.22}$$

If $G > R$, then $\frac{\partial c_2}{\partial \sigma} > 0$, and period 2 consumption increases with longevity. If $G < R$, then it is still possible for $\frac{\partial c_2}{\partial \sigma}$ to be positive, if $(1-p)\frac{(R-G)}{G} < \frac{c_1}{S}R$. Under usual parameterizations, this is likely to be the case. So we expect that $\frac{\partial c_2}{\partial \sigma} > 0$.

2.2.2 The Paygo Model

Dynamics of Increasing Survivorship

We now take up the model with a Paygo design for financing social security. Therefore, $T = \frac{pS\sigma}{G}$. We want to examine the dynamics of the model, as the chances of surviving to retirement increase.

The derivation of Q and a is the same for paygo as for the fully funded model, and the derivatives are the same as in equations (13) and (15), respectively.

We expect taxes must increase with survivorship, and so they do, as seen below.

$$\frac{\partial T}{\partial p} = \frac{\partial[\frac{pS\sigma}{G}]}{\partial p} = \frac{S\sigma}{G} \tag{2.23}$$

We next consider the derivative of lifetime income ($\frac{\partial I}{\partial p}$):

$$\begin{aligned} I &= (Y-T)Q + \frac{S\sigma}{R} \\ &\Rightarrow \frac{\partial I}{\partial p} = \frac{\partial YQ}{\partial p} - \frac{\partial TQ}{\partial p} \\ &\Rightarrow \frac{\partial I}{\partial p} = -(Y-T)Q^2\frac{R}{G} - \frac{S\sigma}{G}Q \end{aligned} \tag{2.24}$$

The derivative is negative. Lifetime income decreases when survivorship improves. This is from the effect of fewer bequests which build up over generations, and because taxes increase proportionally with survivorship.

Next we consider the dynamics of consumption.

$$\begin{aligned} \frac{\partial c_1}{\partial p} &= \frac{\partial aI}{\partial p} = a\frac{\partial I}{\partial p} + I\frac{\partial a}{\partial p} \\ \frac{\partial c_1}{\partial p} &= a[-(Y-T)Q^2\frac{R}{G} - \frac{S\sigma}{G}Q] + I[\frac{1}{Q}\frac{-\delta\sigma}{(1+p\delta\sigma)^2} + \frac{R}{G}\frac{1}{(1+p\delta\sigma)}] \end{aligned}$$

The first bracket expression on the right hand side of the equation above is negative. This is the effect on period 1 consumption from decreased lifetime income (as survivorship increases). The sign of the second bracket expression is ambiguous; it is the effect from the change on a as surviviorship increases. But further analysis reveals:

$$\frac{\partial c_1}{\partial p} = \frac{-1}{1+p\delta\sigma}[c_1\delta\sigma] \tag{2.25}$$

Thus, $\frac{\partial c_1}{\partial p}$ is negative, and period 1 consumption decreases as survivorship increases.

If $\frac{\partial c_1}{\partial p}$ is negative, we might expect $\frac{\partial c_2}{\partial p}$ to be positive.

$$\frac{\partial c_2}{\partial p} = \frac{\partial R(I - aIQ)}{\partial p} = \frac{\partial RI(1 - \frac{1}{1+p\delta\sigma})}{\partial p}$$
$$\frac{\partial c_2}{\partial p} = R\{\frac{\partial I}{\partial p}(1 - \frac{1}{(1+p\delta\sigma)}) + I\frac{\delta\sigma}{(1+p\delta\sigma)^2}\}$$

Just as in the fully funded case, the first term in the braces in the equation above is the income effect from decreasing lifetime income as survivorship increases. It is negative. The second term in the braces is the effect of a shifting pattern of consumption as survivorship increases. It is positive. The net effect is ambiguous. But with some further effort we get:

$$\frac{\partial c_2}{\partial p} = \frac{(Y-T)QR}{(1+p\delta\sigma)} \frac{\delta\sigma}{(G-(1-p)R)}[a - pR] + \frac{S\sigma}{(1+p\delta\sigma)} \frac{\delta\sigma}{(G-(1-p)R)}[a - pR] \quad (2.26)$$

This expression for $\frac{\partial c_2}{\partial p}$ is positive if the expression in brackets is positive. That will be the case if $a > pR$. Otherwise, $\frac{\partial c_2}{\partial p}$ is negative.

Dynamics of Improvement in Longevity

Now we consider the dynamics of increased longevity in the Paygo model, treating it as indepedent of survivorship to retirement.

As was the case with survivorship, the derivative of Q with respect to longevity is 0, and the derivative of a with respect to longevity is the same as in the fully funded model (equation (20)).

We will consider the derivatives of T, I, c_1, and c_2 with respect to longevity (σ).

Consider $\frac{\partial T}{\partial \sigma}$.

$$\frac{\partial T}{\partial \sigma} = \frac{\partial [\frac{pS\sigma}{G}]}{\partial \sigma} = \frac{pS}{G} \quad (2.27)$$

Taxes have to increase as longevity increases.

Consider $\frac{\partial I}{\partial \sigma}$.

$$\begin{aligned} I &= (Y-T)Q + \frac{S\sigma}{R} = YQ + \sigma(\frac{S}{R} - \frac{pSQ}{G}) \\ &\Rightarrow \frac{\partial I}{\partial \sigma} = \frac{S}{R} - \frac{pSQ}{G} \end{aligned}$$

The first term on the right hand side of the equation above is the marginal effect of longevity on the social security benefit, and the second term is the marginal effect of taxes. Since taxes go up with longevity, the second term must reduce lifetime income and is therefore negative. The social security benefit increases proportionately with longevity, so this term is positive. Further analysis finds:

$$\frac{\partial I}{\partial \sigma} = \frac{S}{R}[\frac{G-R}{G-(1-p)R}] \tag{2.28}$$

If $G > R$, then $\frac{\partial I}{\partial \sigma} > 0$. If $G < R$, then $\frac{\partial I}{\partial \sigma} < 0$. Thus we find that despite differences in financing, the dynamics of lifetime income with respect to longevity are dependent on the relationship between the return on captial and the population growth in both the fully funded and paygo models.

From equation (20) we know that that $\frac{\partial a}{\partial \sigma} < 0$. So we expect period 1 consumption is likely to decrease with increased longevity.

$$\begin{aligned} \frac{\partial c_1}{\partial \sigma} &= a\frac{\partial I}{\partial \sigma} + I\frac{\partial a}{\partial \sigma} \\ \frac{\partial c_1}{\partial \sigma} &= a\frac{S}{R}[\frac{(G-R)}{G-(1-p)R}] + \frac{I}{Q}\frac{-p\delta}{(1+p\delta\sigma)^2} \end{aligned}$$

The first term on the right hand side is the marginal net effect of increased benefits and taxes, as longiveity increases. It is positive or negative as $G > R$ or not. The second term is the marginal effect of the shifting proportion of consumption between period 1 and period 2 as longevity increases. The shift is always from period 1 to period 2, reducing period 1 consumption.

Further analysis produces:

$$\frac{\partial c_1}{\partial \sigma} = \frac{1}{(1+p\delta\sigma)}[\frac{S}{R}\frac{(G-R)}{G} - c_1 p\delta] \tag{2.29}$$

Thus $\frac{\partial c_1}{\partial \sigma} > 0$ if the term is brackets is positive. This requires $\frac{G-R}{G} > \frac{c_1 p\delta R}{S}$. With usual parameterizations, this condition is unlikely to be met, and it clearly fails when $R > G$. We expect therefore, that $\frac{\partial c_1}{\partial \sigma} < 0$, and period 1 consumption decreases with increased longevity.

Finally, consider $\frac{\partial c_2}{\partial \sigma}$:

$$\begin{aligned}
\frac{\partial c_2}{\partial \sigma} &= \frac{\partial R(I - aIQ)}{\partial \sigma} = R\frac{\partial I}{\partial \sigma} - RQ\frac{\partial aI}{\partial \sigma} \\
\frac{\partial c_2}{\partial \sigma} &= R\{\frac{\partial I}{\partial \sigma}(1 - \frac{1}{(1+p\delta\sigma)}) + IQ\frac{p\delta}{(1+p\delta\sigma)^2}\} \\
\frac{\partial c_2}{\partial \sigma} &= \frac{RQp\delta}{1+p\delta\sigma}[\frac{S}{R}\frac{(G-R)}{G} + c_1 Q]
\end{aligned} \tag{2.30}$$

If $G > R$, then the term in brackets is positive, and $\frac{\partial c_2}{\partial \sigma} > 0$. Period 2 consumption increases with longevity. If $G < R$, then it is still possible for $\frac{\partial c_2}{\partial \sigma}$ to be positive, if $\frac{(R-G)}{G} < \frac{c_1}{S}RQ$. This seems likely.

2.3 Conclusion

This addendum to the 2006 paper by Boronow extends the two period analysis of social security beyond just a fully funded financing model to paygo financing, and to include the effect of received bequests into the steady state analysis. We find that the key role played by survivorship in enhancing individual welfare in the fully funded model is not changed when received bequests are incorporated into the model. But for the paygo model, we find that there are two sources for individual welfare gains or loss. Survivorship leads to welfare gains, and the difference between financing through population growth and financing through capital return can lead to welfare gains or losses. We also find that under a simple homogeneous agent model, in which bequests are redistributed equally, the sign of the welfare gain (or loss) is determined by the financing gain, with low survivorship acting to amplify the gain or loss more than high rates of survivorship.

From an analysis of dynamics with respect to survivorship to retirement, we find that in both fully funded and paygo models lifetime income is reduced as survivorship increases.

This is due directly to higher taxes, and indirectly through fewer generational bequests.

Increasing survivorship also causes a shift in the proportion of lifetime income consumed in each period. Regardless of the financing method, the pattern of consumption depends on the relationship between population growth G, and capital return R. A greater likelihood of survival to period 2 causes an increase in saving, but the lower lifetime income that results from fewer bequests results in a decrease in saving. The net effect depends on the interplay of G and R.

With respect to the dynamics of longevity in retirement, we find that once again it depends on the relationship between population growth G and capital return R. Lifetime income increases with longevity when $G > R$, and lifetime income decreases with longevity when $G < R$. This is true regardless of financing method. The reason is that longevity increases lead to tax increases which reduce wealth that is left to bequests, which is shared among G heirs. If G is relatively large, then the loss of bequest wealth from higher taxes is relatively small. So the gain from higher benefits (due to longevity increase) outweighs the loss due to higher taxes. If G is relatively small, then the loss of bequest wealth is felt more keenly, and the loss on taxes offsets the gain on benefits.

We also find that increasing longevity causes an unambiguous shift in the proportion of lifetime income that is consumed from period 1 to period 2.

The extension of the model to include bequests and paygo financing reaffirms a finding of the 2006 paper, that survivorship is the key driver in creating individual welfare gains. With paygo another source of welfare gains and losses is introduced in the difference between population growth and capital returns. This relationship also plays a role in the dynamics of the model as longevity increases.

Are there any implications for policy makers in this simple two period model? There are at least two relevent findings.

1. The share of Social Security benefit derived from redistributing tontine gains to survivors has unambiguously decreased as mortality rates have improved. This shifts the burden of Social Security directly onto taxation. A reform which significantly increased

the age at which benefits are paid would reverse this trend and relieve taxes of some of the burden of Social Security. Lifetime income and welfare would increase as a result.

2. When population growth rate G is greater than capital return R, than paygo financing offers a dynamic channel for increasing welfare. However, when G is less than R, lifetime income is reduced and welfare is destroyed. In the future, as population growth rates decline, it may make sense to reduce the paygo aspects of Social Security in favor of funding to reduce the destruction of lifetime income.

Chapter 3

A Hybrid Reform: Social Security on Dual Power in the Presence of Temptation

One of the most important issues studied in the macroeconomics literature is Social Security reform and its subsequent effect on the macroeconomy.[1] As a result, we have a sound understanding of many issues related to Social Security, and the mechanisms by which Social Security affects the national economy. It is an indication of the inherent complexity of Social Security in the political economy, that even with this broad understanding there is still not a corresponding consensus as to the best way to address the coming financial meltdown in Social Security as the Baby Boomer generation reaches retirement.

There are two basic strategies for reforming Social Security, and many intermediate mixtures of these two. One strategy is for individuals to take responsibility for their own retirement. A pure version on this theme would be total elimination of Social Security. Another variation would require each individual to build up a Personal Security Account (PSA) through mandated contributions. In either variation, the distinguishing characteristic is individual responsibility, ownership and freedom of action for their own retirement income. The other basic approach is for the government to continue to have responsibility for providing a retirement pension to qualified workers. Given the current actuarial

[1]For this paper, Social Security will only refer to old age pension benefits and exclude disability and survivor benefits.

imbalance, reform proposals under this approach necessarily involve some variation of tax increase, or benefit reduction to "fix" Social Security. But the distinguishing characteristic of this approach is government action with no choice or responsibility on the part of the individual.

This paper postulates a "hybrid" reform, one of many possible intermediate mixtures between the two basic approaches. The Hybrid Reform proposal is motivated by the possibility that the optimal solution to Social Security is not a corner solution, but instead takes design features from each of the two basic approaches, based on what we already have learned.

From the present government-based system, the Hybrid Reform proposal retains a government provided pension, but one which starts at an advanced old age (in particular, at age 80). We know that in a partial equilibrium setting, a government-provided pension can be a welfare-enhancing benefit for individuals (Abel 1985). It reduces retirement income uncertainty, and protects citizens when they are not necessarily able to fend for themselves. But, as discussed below in the related literature, we have also learned that the reduction in income uncertainty in a life cycle model leads to lower precautionary savings, and less capital formation in the economy. In general equilibrium models of the economy, lower rates of savings lead to lower levels of output, wages and individual welfare.

From the individual responsibility approach, The Hybrid Reform proposal incorporates a mandatory Personal Security Account (PSA) to accumulate assets that will provide retirement income in the years before the government pension starts. By eschewing (in part) intergenerational transfers in favor of accumulated assets, the Hybrid Reform results in increased saving (and greater capital formation), which in equilibrium leads to a higher standard of living. An approach that relies on individual responsibility has problems too, such as the free rider problem, whereby some people may not save for retirement in the expectation that society will not let them starve. In this paper, this is managed by making contributions to the PSA mandatory in nature.

The peculiar design of the Hybrid Reform, which has the pension start at a much later age than we are accustomed to, deserves a comment as to its motivation. It is not uncommon in the group insurance businesses, that larger companies self-insure their group health insurance benefits. If they do so, they also purchase additional insurance coverage against the tail of the claim distribution, as a matter of risk management. In this way, they are fairly certain what their costs will be, and they are not exposed to an unlimited risk. Similarly, the early retirement years of individuals are such a candidate for self-insurance. Since life expectancy is higher (Boronow (2006)), we can self-insure the early retirement years with a reasonable degree of certainty. In addition, we can insure for the late retirement years, which have a high degree of uncertainty, with a public pension. By self-insuring, we gain more freedom of choice over the optimal use of our resources, and hope to enhance welfare.

The paper compares first the stationary state in economies under Social Security, No Social Security, and the proposed Hybrid Reform assuming standard preferences. Next, the same analysis is conducted under temptation preferences, as developed by Gul and Pesendorfer (2000). In particular, we use the formulation of Krusell, Kuruşçu and Smith (2005), assuming that agents are tempted by a higher level of current consumption relative to future consumption. The framework for our analysis is a large-scale general equilibrium overlapping generations (OLG) model. This setting, in various permutations has been used extensively since Auerback and Kotlikoff (1987) first analyzed labor supply and capital stock with a 55-period deterministic OLG model.[2] Like Fuster, Imrohoroğlu, Imrohoroğlu (2002), we assume that agents are endowed with heterogeneous ability and correlated mortality.

Our main results can be summarized as follows. First, the paper makes the argument that the Hybrid Reform economy (HRE), using the dual power of a government provided pension and mandatory, self-funded Personal Security Accounts (PSA), does a better job of protecting the macroeconomy and enhancing welfare in the long run than either the Social Security economy (SSE), which has only a government pension, or the No Social Security

[2] Imrohoroğlu, Imrohoroğlu and Joines (1998a) discuss the main features of the model, used in many papers over the last twenty years.

economy (NSSE), which has only voluntary savings. This takes into account the effects of temptation preferences, which induce agents to struggle with self-control issues. The argument that HRE produces a better outcome in the long run is based on the fact that aggregate welfare is higher for the HRE model under standard preferences (i.e. corresponding to agents with perfect foresight), while this welfare level is sustained under temptation preferences (i.e. corresponding to agents with myopic foresight). In contrast, the SSE model leads to the lowest long run welfare in the three model economies under standard preferences, and long run welfare decreases even more with temptation preferences. Finally, the NSSE model leads to the highest long run welfare with perfect foresight, but welfare falls even more rapidly than in the SSE in the presence of temptation. In sum, only the HRE model sustains the level of savings and individual welfare with temptation preferences.

Second, this paper shows that the Hybrid Reform economy generates less wealth inequality than does a Social Security economy in a setup in which ability and income are correlated with mortality. In such a situation, applying a uniform tax rate leads to reverse redistribution in the government pension system, since low income workers subsidize the pensions of high income workers, who have longer life expectancies. With the dramatic reduction in the tax rate required for a pay-as-you-go (paygo) pension under the Hybrid Reform, relative to the tax rate that is needed under Social Security, reverse redistribution is reduced, and therefore wealth inequality, relative to the Social Security equilibrium. Also, the presence of mandatory savings (PSA) implies that low income workers will accumulate wealth, and high income workers may substitute out some voluntary savings, thereby reducing wealth inequality further.

3.1 Related Literature

Our paper contributes to several strands of literature. First, it contributes to the literature on unfunded public pension systems. A significant part of this literature has revealed that the overall welfare effect of introducing such a system crucially depend on the importance of two opposing effects: a higher intergenerational risk sharing and a lower capital intensity.

On the one hand, an unfunded social security system reallocates the impact of shocks across generations, reducing the consumption risk of the old aged relative to the risk they would face with private markets (Bohn 1999). This provides a welfare improvement for all generations alive and for the ones to be born in the future. On the other hand, such a system redistributes income away from younger agents with lower marginal propensities to consume, toward older agents with higher marginal propensities to consume. This lowers aggregate savings and aggregate capital formation (Feldstein 1974, Diamond 1977). The so-called "crowding-out" effect on capital from unfunded social security has been noted by many researchers over the years (see, for example, Auerbach and Kotlikoff 1987; Imrohoroğlu, Imrohoroğlu and Joines 1998b and 1999). This crowding out effect arises in life-cycle models, in which social security substitutes for precautionary savings to guard against an uncertain length of life. As in our setting, the net effect of lower capital intensity (due to crowding-out) is that agents would be better off in the long run if they were born into an economy without Social Security.[3]

Recently, Fuster, Imrohoroğlu and Imrohoroğlu (2005) also study the impact of mandatory Personal Security Accounts (PSA) in an economy with a dynastic framework. Under this assumption, the majority of households are better off with mandatory PSA than with paygo Social Security.[4] As in our paper, the authors find that wealth inequality increases when Social Security is introduced. In particular, low ability workers are disadvantaged by the mandatory PSA when it is not owned by them but merely funded during their working lifetime, while they are better off when the PSA is owned. One of the differences between our work and the one in Fuster et al (2005) is that they analyze a dynastic framework

[3] Subsequently, Fuster (1999) and Fuster, Imrohoroğlu and Imrohoroğlu (2002) have analyzed Social Security in a dynastic framework. In this setting, the household can undo the effect of Social Security through its altruistic motives, so that there is much less crowding-out.

[4] In Fuster et al (2005), the mandatory PSA was simply a mechanism to achieve funded Social Security. They also present results with an alternative lump-sum-at-retirement PSA, which is closer in spirit to the PSA of this paper. With the alternative PSA, the agent actually owns the PSA, and can bequeath it to heirs.

while we study a life cycle framework. Given this, we obtain a larger crowding-out effect in capital formation. In addition, we study a hybrid reform and we also analyze the impact of temptation preferences.

Related to the issue on inequality, much of the literature on Social Security has studied frameworks where mortality is independent of income. On the other hand, there is empirical evidence that low income is associated with high mortality rates and vice versa. Fuster (1999) and Fuster, Imrohoroğlu, Imrohoroğlu (2002, 2005) analyze a setup in which ability (and therefore income) is correlated with longevity. As we noted above, they find that Social Security increases wealth inequality. Gokhale et al (2001) also show that this correlation turns a paygo social security system into a "money pump", in the sense that it pumps wealth from working poor men to retired rich women. As discussed before, the present paper also finds that Social Security increases wealth inequality. With lower capital formation under Social Security, wages are depressed and capital returns are elevated. This hurts low income workers more than high income workers. Social Security also induces low income workers to reduce savings and bequests more significantly than do high income workers. However, capital formation is increased under the Hybrid Reform specified in this paper, thereby increasing wages and reducing interest rates. Furthermore, since workers own their PSA under the proposed reform, wealth increases directly for low income workers, so that, unlike Social Security, wealth inequality is reduced under the hybrid reform.

Second, our paper contributes to the literature on time inconsistency and self control. Typically, in much of the Social Security literature, the usual assumption is that households have standard preferences that exhibit a consumption smoothing motive. But it is empirically observed that preference-reversals can occur as time horizons change. Of two prizes in the distant future, the subject will choose the larger and later prize. But as the time to receive the prize draws closer, they would prefer the smaller but earlier prize. An excellent and fascinating survey of this literature is found in O'Donoghue and Rabin (1999).

Out of this literature two approaches towards modelling behavior with such preference reversals have developed. One approach is to model time inconsistent preferences with quasi-geometric discounting (Phelps and Pollak 1968; Laibson 1997). Imrohoroğlu, Imrohoroğlu and Joines (2003) analyzed Social Security under quasi-geometric (aka hyperbolic) discounting, and found that Social Security is a weak commitment device. The second approach to modelling behavior which exhibit preference reversals is to use temptation preferences (Gul and Pesendorfer 2001, 2004). Under temptation preferences, the agent has to exercise costly self-control to make the "right" choice. The choice can reverse if self-control becomes too costly. The advantage of this approach is that preferences are consistent over time, and can be formulated recursively. This naturally commends temptation preferences to the many problems which are approached with recursive models. A recent paper by Kumru and Thanopoulos (2007) used the Gul and Pesendorfer temptation preferences to study Social Security. They also find that Social Security is a commitment device that can increase welfare for agents afflicted with self-control problems.

This paper differs from Kumru and Thanopoulos in several important aspects. First, it extends the temptation preference analysis to agents which have heterogeneous ability and correlated mortality. Unlike Kumru and Thanopoulos (2007), this paper uses the formulation of Krusell, Kuruşçu and Smith (2005), assuming that an agent is tempted by a higher level of current consumption relative to future consumption, but is not tempted by changes to rankings of future consumption. Thus, agents would not necessarily be tempted to consume all their wealth currently. Instead, they would be tempted to shift some consumption to the present period, but otherwise leave future consumption rankings unchanged. In contrast, in the setup of Kumru and Thanopoulos, agents are tempted to consume all their wealth in the current period. Finally, this paper analyzes a hybrid Reform and finds it to be, unlike Social Security, an effective commitment device.

The paper is organized as follows. Section 2 presents the overlapping generations model which is used throughout this paper. The model is presented under standard expected utility preferences and temptation preferences. Section 3 discusses the calibration and solution method. Section 4 presents the results of the various experiments in both partial and general equilibrium and the welfare analysis. Finally, Section 6 summarizes and concludes.

3.2 The Overlapping Generations Model

The OLG model used in this paper is based on the 65-period model used by Imrohoroğlu, Imrohoroğlu and Joines (1997). It has been modified in two significant dimensions. First, the model has been extended to 85 periods to enable it to better capture advanced age dynamics. Second, we have introduced two types of workers, distinguished by their labor efficiency and correlated mortality rates, in order to analyze the effect that heterogeneous ability with correlated mortality has on a social security economy, and on alternative reform economies. Fuster, Imrohoroğlu and Imrohoroğlu (2002) also study a model with heterogeneous ability types, and this paper borrows from their setup. While their model is a dynastic setup, this paper uses a life-cycle model.

3.2.1 Demographics

Time is discrete, and each period represents one year. Age 1 corresponds to real age 21. The oldest possible age is age J , where $J = 85$ (real age 105). We assume that death is certain thereafter.

There are two types of agents indexed by z, where $z \in \{1, 2\}$. An agent's type is revealed at birth and this determines lifetime ability, which can be either high ($z = 1$) or low ($z = 2$). The realization of ability follows a first-order Markov process with transition matrix Π:

$$\begin{aligned} \Pi(z, z') &= [\pi_{ij}]; i, j \in \{1, 2\} \\ \pi_{ij} &= \Pr(z' = j|z = i). \end{aligned}$$

where z is the ability type of the parent and z' is the ability type of the child. It is assumed that the transition probabilities, π_{ij}, are such that there is a resulting stationary distribution of ability types, Λ, where $\lambda(z) \in \Lambda$, and $\lambda(1) + \lambda(2) = 1$.

The ability type determines the endowment of efficiency units an agent receives. In a given period, the cross-sectional labor efficiency $\varepsilon_j(z)$ is indexed by ability type z and age j. Without loss of generality, we assume throughout this paper that the rate of technological growth is zero. Under this assumption, the longitudinal efficiency units of a particular agent equal the cross-sectional efficiency factors, $\varepsilon_j^l(z) = \varepsilon_j(z)$.

Agents have uncertain lifetimes. Survival probabilities are correlated with ability type, so that high ability agents have longer expected lifetimes than low ability agents. Thus survival rates are indexed by age and type. The probability that an agent age j and ability type z survives to age $j+1$ is denoted by $\psi_j(z)$. The probability that an agent age j and ability type z survives to age $j+t$ is denoted by $\Psi_{j,t}(z)$, where:

$$\begin{aligned} \Psi_{j,t}(z) &= 1, \text{ if } t = 0 \\ \Psi_{j,t}(z) &= \Pi_{s=1}^{t}\psi_{j+s-1}(z), \text{ if } t > 0. \end{aligned}$$

Like much of the social security literature, this paper analyzes the steady states of a stationary population distribution, with time invariant cohort shares. Let ρ be the assumed constant rate of growth in population. Then, the cohort share of a new agent of type 1 relative to a new agent of type 2 is equal to $\frac{\lambda(1)}{\lambda(2)}$. That is, the size of the newborn type cohorts, relative to each other is determined by the Markov process stationary distribution of types. Thereafter, relative cohort shares are a result of the population growth rate and survival probabilities. Letting $\mu_j(z)$ denote the cohort share for an agent of age j and ability type z, and letting N denote the total population when the newborn cohort is indexed to one (1), for newborns ($j = 1$):

$$\begin{aligned} \mu_1(z) &= \lambda(z) \cdot [1/N], \text{ where} \qquad (1) \\ N &= \sum_{z=1}^{2}\sum_{t=0}^{J}\lambda(z)\cdot(1+\rho)^{-t}\cdot\Psi_{1,t}(z). \end{aligned}$$

Each new cohort is $(1+\rho)$ times as large as the preceding cohort, and each cohort survives to the next period according to the corresponding age and ability type, $\psi_j(z)$. Thus for $j = 1, 2, ...J-1$:

$$\mu_{j+1}(z) = \mu_j(z) \cdot \frac{\psi_j(z)}{1+\rho}. \qquad (2)$$

Finally, the sum of all cohorts must equal 100% so that,

$$\sum_{z=1}^{2}\sum_{j=1}^{J}\mu_j(z) = 1. \tag{3}$$

Given the Markov process, survival rates, and population growth rate, $\Pi, \psi_j(z)$ and ρ, respectively, the above relationships uniquely determine the time invariant cohort shares, $\{\mu_j(z)\}$.

3.2.2 Technology and Factor Prices

There is a single good in the economy, produced by one or more firms using a constant returns to scale Cobb-Douglas production function:

$$Y = AK^{1-\alpha} \cdot L^{\alpha}, \text{ where } \alpha \in (0,1).$$

Total factor productivity A is normalized to 1. The labor share is α and K and L are aggregate capital and labor supplied as inputs. Capital is assumed to depreciate at the constant rate δ. Therefore, in a competitive equilibrium, we get factor prices for capital and labor:

$$\begin{aligned} r &= (1-\alpha) \cdot K^{-\alpha} \cdot L^{\alpha} - \delta \\ w &= \alpha \cdot K^{1-\alpha} \cdot L^{\alpha-1} \end{aligned} \tag{4}$$

K represents the aggregate asset holdings over the population in a given period. The size of L is determined by the workers up to retirement age j^*. Workers are assumed to supply labor inelastically to age j^*, and do not work thereafter. The actual supply of efficient labor depends on the ability type of agents in the working age population.

$$L = \sum_{z=1}^{2}\sum_{j=1}^{j^*-1}\varepsilon_j(z) \cdot \mu_j(z). \tag{5}$$

3.2.3 Government Policy and Social Security

This paper will analyze three model economies that differ in their approach to Government Policy and Social Security. There is a No Social Security economy (NSSE), a Social Security economy (SSE), and a Hybrid Reform economy (HRE).

No Social Security Model Economy

Government policy in the NSSE model is straightforward. There is no social security tax, and there are no social security benefits. Each worker must prepare for retirement income by saving during their working years, to build up a retirement nest egg.

Social Security Model Economy

In the SSE model, there is a social security program that provides a public pension to retirees. In this simplified model, the pension is a flat percentage θ of the average lifetime earnings. The social security benefit, denoted by b^{SS}, is given by:

$$b^{SS}(z) = \frac{\theta}{j^* - 1} \cdot \sum_{j=1}^{j^*-1} w\varepsilon_j(z) \tag{6}$$

The role of the government is to collect a tax on labor income to exactly provide the social security pension to retirees. The necessary tax rate, τ_{SS}, in this pay-as-you-go model is:

$$\tau_{SS} = \frac{\sum_{z=1}^{2} \sum_{j=j^*}^{J} b^{SS}(z) \cdot \mu_j(z)}{w \cdot \{\sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z)\}} \tag{7}$$

The numerator is the total benefit paid under social security and the denominator is the total wage base over which the tax is applied in a given period.

Hybrid Reform Model Economy

The HRE model economy has a social security pension that starts at an advanced old age (AOA), i.e., later than the retirement age. Also, the HRE includes a mandatory personal security account (PSA), owned by the worker, in which mandatory contributions during the agent's working years are accumulated to provide a source for retirement income in the retirement years prior to the start of the public pension. The HRE model is called the Hybrid Reform economy, since the combination of personal accounts and an advanced old age public pension is a hybrid of the present system and a pure private accounts system.

The Hybrid Reform economy provides a public pension starting at age j^b, the benefits start age ($j^* < j^b < J$). The benefit amount is the same as in the social security economy. However, since benefits start at age j^b, rather than j^*, the tax to pay for the benefits is lower.

Let τ_{AOA} denote the tax rate needed to provide advanced old age benefits in the HRE. Then:

$$\tau_{AOA} = \frac{\sum_{z=1}^{2} \sum_{j=jb}^{J} b^{SS}(z) \cdot \mu_j(z)}{w \cdot \{\sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z)\}} \tag{8}$$

There is also a mandatory tax, τ_{PSA}, to accumulate a Personal Security Account (PSA). The PSA is a self-insurance mechanism which provides income for the years between retirement age, j^*, and the start of the public pension at advanced old age j^b.

There are many possible choices as to the specification of the self-insurance mechanism. We assume that contributions to the PSA are mandatory. It is a contribution of this paper that a mandatory funded pension is a superior commitment device to the present pay-as-you-go social security system. The level of funding is chosen so that the rate of contributions to the PSA plus the tax rate to provide the advanced old age public pension are the same in total as the tax rate to provide the public pension in the SSE model. Thus, $\tau_{SS} = \tau_{AOA} + \tau_{PSA}$.

It is assumed that contributions grow with interest equal to the return on capital r, and there are no taxes on that capital return. Let $a_j^{PSA}(z)$ denote the assets at the end of the period in a PSA owned by an agent of age j and type z. Define $a_0^{PSA}(z) = 0$ to be the initial balance in a newborn worker's PSA.Then during the agent's working years:

$$a_j^{PSA}(z) = \sum_{h=1}^{j} [\tau_{PSA} \cdot w \cdot \varepsilon_h(z)(1+r)^{j-h}], \text{ for } j < j^* \tag{9}$$

At retirement, the agent begins to deplete the PSA according to a pro-rata formula. In the event of death, the remaining PSA balance is bequeathed to the agent's heirs.[5] Each period from retirement age to the benefits start age, a pro-rata portion of the PSA is made available to the agent as income in that period. Let $b_j^{PSA}(z)$ denote the income benefit from the PSA to an agent of age j and type z. Then:

$$\begin{aligned} b_j^{PSA}(z) &= 0, \quad \text{for } j < j^* \\ b_j^{PSA}(z) &= \frac{a_{j-1}^{PSA}(z)}{j^b - j}, \text{ for } j = j^*, j^* + 1, ... j^b - 1 \\ b_j^{PSA}(z) &= 0, \quad \text{for } j \geq j^b \end{aligned} \tag{10}$$

The remaining assets in the PSA accumulate with interest at rate r. Thus:

$$\begin{aligned} a_j^{PSA}(z) &= [a_{j-1}^{PSA}(z) - b_j^{PSA}(z)] \cdot (1 + r), \text{ for } j = j^*, j^* + 1, ... j^b - 1 \\ a_j^{PSA}(z) &= 0, \quad \text{for } j \geq j^b \end{aligned} \tag{11}$$

It should be recognized that $a_j^{PSA}(z) = 0$ in the SSE and NSSE models.

3.2.4 Constraints and Bequests

During the working years, the agent receives after-tax labor income based on their age/ability profile of labor efficiency. Upon retirement in the SSE model, the agent receives social security benefits. In the NSSE model, there are no social security benefits. In the HRE model, the retired agent receives distributions from the PSA each year from age j^* to age $j^b - 1$, and then receives a public pension from advanced old age j^b to the end of life.

Each period, the agent must choose the amount of consumption and the amount of voluntary saving. Savings earn the rate of return on capital r. We assume that agents are subject to a no borrowing constraint.

[5] Fuster, Imrohoroğlu, and Imrohoroğlu study mandatory annuitization of a PSA which is not able to be bequeathed by the worker.

Because lifetimes are uncertain, many agents will die with positive amounts of assets (ak a accidental bequests). In the HRE model, agents younger than j^b will also have positive amounts of PSA assets. It is not uncommon for researchers to assume that these unintended bequests are confiscated and redistributed equally (per effective worker) to the survivors. This paper modifies that assumption somewhat. In this paper, accidental bequests are redistributed to surviving agents, in such a way that each type of agent receives an equal share based on the expected bequest of that agent, given their ability type. The amount of the bequest distributed to agents of ability type z is denoted by $\xi(z)$.

We can now describe the budget constraint faced by an agent of age j and ability type z, which is given by:

$$c_j(z) + a_{j+1}(z) = [a_j(z) + \xi(z)] \cdot (1 + r) + Q_j(z) \tag{12}$$

The left hand side of the equation is the allocation of that period's wealth to consumption and savings, while the right hand side is the total of the resources available from prior savings and returns, bequests, wages and benefits (if any) from the social insurance mechanism. In all economies, $a_1(z) = 0$ and $a_{J+1}(z) = 0$. Further, for the three model economies, Q_j is defined as follows. In the No Social Security economy (NSSE):

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \ \text{ for } j < j^* \\ Q_j(z) &= 0 \ \text{ for } j^* \leq j \end{aligned}$$

In the Social Security economy (SSE):

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \cdot (1 - \tau_{SS}) \text{ for } j < j^* \\ Q_j(z) &= b^{SS}(z) \text{ for } j^* \leq j \end{aligned}$$

In the Hybrid Reform economy (HRE):

$$
\begin{aligned}
Q_j(z) &= w \cdot \varepsilon_j(z) \cdot (1 - \tau_{AOA} - \tau_{PSA}) \;\text{ for } j < j^* \\
Q_j(z) &= b_j^{PSA}(z) \cdot (1 + r) \text{ for } j^* \leq j < j^b \\
Q_j(z) &= b^{SS}(z) \;\text{ for } j^b \leq j
\end{aligned}
$$

In all economies, households face a borrowing constraint:

$$a_j(z) \geq 0, \;\forall\; j$$

The bequest $\xi(z)$ is defined below.

Expected Bequests

Since ability type determines labor earnings, then the average size of an accidental bequest differs by type. It is reasonable to assume that children receive the accidental bequest left by the parent. The problem is how to allocate accidental bequests to agents of type 1 and type 2 so that the allocation is consistent with a presumption that the bequest stay in the family. To do this, let $Beq(z)$ denote the average bequest of agents of type z that die in a given period:

$$Beq(z) = \frac{\sum_{j=1}^{J} \{[a_j(z) + a_j^{PSA}(z)] \cdot \mu_j(z) \cdot (1 - \psi_j(z))\}}{\sum_{j=1}^{J} \mu_j(z) \cdot (1 - \psi_j(z))} \tag{13}$$

The numerator is the sum of assets owned by the type z agents who die in a given period, while the denominator is the number of such agents.

Recall that $\pi_{ij} \in \Pi$ is the probability that a parent of type i has a child of type j, and that $\lambda(z)$ is the probability that a newborn is type z. It turns out that the probability that a child of type z has a parent of a given type produces exactly the same Markov probability matrix Π. We use this fact to construct the ratio of the conditional expected bequest for agents of type z to the overall average bequest to agents of all types. We then use that ratio to allocate accidental bequests as follows:

$$
\begin{aligned}
\xi(z) &= \frac{\pi_{z1} \cdot Beq(1) + \pi_{z2} \cdot Beq(2)}{\sum_{z=1}^{2} \lambda(z) \cdot (\pi_{z1} \cdot Beq(1) + \pi_{z2} \cdot Beq(2))} \cdot Beq \qquad (14) \\
\text{where } Beq &= \sum_{z=1}^{2} \sum_{j=1}^{J} \{[a_j(z) + a_j^{PSA}(z)] \cdot \mu_j(z) \cdot (1 - \psi_j(z))\}
\end{aligned}
$$

Despite the extra complexity of this approach, there is a purpose in the specification of accidental bequests in such a way that it takes account of the likely ability type of the parent of the agent. That purpose is to build into the model, at least to this degree, the intergenerational effect of bequests under the alternative security regimes, when mortality is correlated with ability/income.[6]

3.2.5 Preferences and Individual Optimization Problem

A contribution of this paper is to analyze how the three model economies respond under temptation preferences, and to contrast those results to the results obtained under standard preferences. Standard preferences are defined below. These are followed by the specification of temptation preferences, as in the setup of Krusell, Kuruşçu and Smith, in which self-control over the interaction of normative and temptation utility produces overall utility. An expanded discussion of temptation preferences, as developed by Gul and Pesendorfer (2001), is presented in Appendix A.

To streamline notation, we let a denote $a_j(z)$, where age and type are defined by the context of the usage. Likewise, we will make the same notational shortcut for $\tilde{a}', c, \tilde{c}$, Q and ξ, where the tilda refers to the fact that the temptation choice for $\widetilde{c}$ and $\widetilde{a}'$ is from a different optimization than the normative choice c and a'. Also, we use the notational convention that the prime symbol denotes the next period value.

Standard Preferences

Standard preferences are defined over a lifetime sequence of consumption $\{c_j(z)\}_{j=1}^{J}$. The individual agent's objective for an agent age j is to maximize expected discounted lifetime

[6] In our experiments, the allocation of accidental bequests according to expectations, rather than equally across effective workers, did not result in significant differences. However, in possible extensions of this paper in which accidental bequests do give utility, this method of allocating bequests may prove to be more relevent to the outcome.

utility:

$$U_j = \sum_{t=0}^{J-j} \beta^t [\Psi_{j,t}(z)] \cdot u(c_{j+t}(z)) \tag{15}$$

where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, γ is the constant coefficient of relative risk aversion and β is the discount factor. The term in brackets is the probability of survival to age $j+t$ for an agent of age j and type z. Throughout this paper, these preferences are referred to as standard preferences.

An agent of age j and type z starts a given period with initial asset holdings a. The individual's dynamic problem is to choose how much to consume now c, and how much to save for future consumption a', in order to maximize the Bellman equation:

$$W_j(a) = \max_{c,a'} \{u(c) + \beta\psi_j(z)W_{j+1}(a')\} \tag{19}$$

subject to the budget constraint in (12), the borrowing constraint, the initial and optimality conditions, and taking the factor prices as given.

Temptation Preferences

Let U denote normative utility and V denote temptation utility. The normative utility is given by equation (15). In value function format, normative utility is:

$$U_j(c, a, a')) = u(c) + \beta\psi_j(z)W_{j+1}(a')$$

Further, temptation utility is specified in terms of the felicity function u, and two parameters, a strength parameter σ and a future discount parameter φ:

$$V_j(\widetilde{c}, a, \widetilde{a}')) = \sigma[u(\widetilde{c}) + \varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')] \tag{17}$$

Putting this all together, an agent with temptation preferences maximizes:

$$\begin{aligned} W_j(a) &= \max_{c,a'} \left\{U_j(c, a, a') + V_j(c, a, a')\right\} - \max_{\widetilde{c},\widetilde{a}'} V_j(\widetilde{c}, a, \widetilde{a}')) \\ &= \max_{c,a'} \{(1+\sigma)u(c) + (1+\sigma\varphi)\beta\psi_j(z)W_{j+1}(a')\} \\ &\quad - \max_{\widetilde{c},\widetilde{a}'} \sigma\{u(\widetilde{c}) + \varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')\} \end{aligned} \tag{18}$$

subject to the same conditions as under standard preferences. Note that c and $\widetilde{c}$ are given by:

$$c = (a+\xi)\cdot(1+r)+Q-a'$$
$$\widetilde{c} = (a+\xi)\cdot(1+r)+Q-\widetilde{a}'$$

3.2.6 The Steady State Equilibrium

Let $D=\{d_1,d_2,...,d_m\}$ represent the discrete set of values that asset holdings are permitted to take. The feasible set for an age j agent of type z and asset holdings a is denoted by $\Omega(j,a,z)$. The possible choices for a using standard preferences and for a and $\tilde{a}$ with temptation preferences satisfy $a'\in\Omega(j,a,z)$, $\tilde{a}'\in\Omega(j,a,z)$, $a'\geq 0$, $\tilde{a}'\geq 0$ and the budget constraints.

A steady state equilibrium for a set of policy parameters $\{\theta,\tau_{SS},\tau_{AOA},\tau_{PSA}\}$ is a collection of value functions $\{W_j(a)\}$; decision rules $R^c_{a,j,z}: D\times\{1,2,...,J\}\times\{1,2\}\rightarrow\mathbb{R}_+$ and $R^{a'}_{a,j,z}: D\times\{1,2,...,J\}\times\{1,2\}\rightarrow D$; a stationary distribution of types of newborns, $\{\lambda(1),\lambda(2)\}$; a time invariant distribution of agents by type, $\{\mu_j(z)|\forall j\in\{1,2,...J\},\forall z\in\{1,2\}\}$; a set of prices for capital and labor $\{r,w\}$; and a set of lump sum transfers of accidental bequests to agents $\{\xi(z)\}$; such that

1. Given factor prices, government policy and the lump sum transfers, the decision rules solve the individual optimization problem.

2. Factor prices solve the optimization problem of the firm

3. Markets clear, implying that:

$$K=\sum_{z=1}^{2}\sum_{j=1}^{J}[a_j(z)+\xi(z)+a^{PSA}_{j-1}(z)]\cdot\mu_j(z) \tag{21}$$

$$L=\sum_{z=1}^{2}\sum_{j=1}^{j^*-1}[\varepsilon_j(z)\cdot\mu_j(z)] \tag{22}$$

$$Y=\sum_{z=1}^{2}\sum_{j=1}^{J}\mu_j(z)\cdot[c_j(z)+a_{j+1}(z)+a^{PSA}_j(z)-(1-\delta)\cdot(a_j(z)+\xi(z)+a^{PSA}_{j-1}(z)\ )] \tag{23}$$

Thus, aggregate capital is the sum of individual asset holdings, aggregate labor is the sum of effective workers and output equals aggregate consumption plus the increase in aggregate capital.

4. The invariant population distribution conditions are satisfied.

5. Government pensions are fully financed by the labor tax.

6. The lump sum transfers satisfy:

$$\sum_{z=1}^{2}\sum_{j=1}^{J}\xi(z)\cdot\mu_j(z)=\sum_{z=1}^{2}\sum_{j=1}^{J}\{[a_j(z)+a_j^{PSA}(z)]\cdot\mu_j(z)\cdot(1-\psi_j(z))\} \tag{24}$$

3.2.7 Calibration

Demographic and Labor Parameters

The model is calibrated following some of the previous literature. Each period represents one year. The probabilities that make up the transition matrix Π are taken from the calibration used by Fuster, Imrohoroğlu and Imrohoroğlu (2002). As they explained, the values of Π were chosen to match 1991 Bureau of the Census data on the proportion of college graduates in the work force, and to match an observed correlation between the permanent component of income of parents and children, based on estimates by Zimmerman (1992) and Solon (1992). Thus:

$$\Pi=\begin{bmatrix}\pi(1,1) & \pi(1,2)\\ \pi(2,1) & \pi(2,2)\end{bmatrix}=\begin{bmatrix}.57 & .43\\ .17 & .83\end{bmatrix}$$

This transition matrix produces a stationary distribution of types for newborns $\Lambda = [\lambda(1)\ \lambda(2)] = [.2833\ .7167]$. Therefore, the proportion of newborns with high ability is 28.33% and the proportion of low ability is 71.67%.

Mortality is assumed to be different between high ability agents and low ability agents. The survival rates are developed in three steps. First, the study begins with the same mortality rates used by Imrohoroğlu, Imrohoroğlu and Joines (1999) in their 65 period model, which are based on mortality rates from the Social Security Administration. The present paper extends the mortality rates 20 more periods, again based on mortality rates from the

Social Security Administration. The aggregate mortality rates are then split into two sets of mortality rates; one for the high ability workers and one for the low ability workers. The method used to split aggregate mortality rates by type is explained in Appendix B.

The model also splits the aggregate labor efficiency factors into two sets, one for each type. The aggregate factors are the age-related factors used by Imrohoroğlu, Imrohoroğlu and Joines (1999), based on research by Hansen (1993). The aggregate efficiency factors are then split into type specific efficiency factors in a manner similar to that used to split the mortality rates. The details are explained in Appendix C. Labor is supplied inelastically to retirement age, which is fixed at $j^* = 45$, corresponding to real age 65. Thereafter, $\bar{\varepsilon}_j = 0$.

The population growth rate ρ is assumed to be a constant and equal to 1.2%, using the calibration by Fuster, Imrohoroğlu and Imrohoroğlu (2002). This corresponds to the average annual population growth rate of the United States over a fifty year period.

Technology Parameters

The model uses a Cobb-Douglas production function with constant returns to scale, $Y = AK^{1-\alpha}L^{\alpha}$. Total factor productivity A is normalized to one. The labor's share α is set to 0.64. These values are often used in a simple model, as they approximate the observed patterns in the US over a long period. The value of L is determined by the demographic assumptions.

Depreciation is set at a constant rate of 6.9%, following Imrohoroğlu, Imrohoroğlu and Joines (1999), in which they calculated their technology parameters based on annual US data since 1954. The rate of exogenous technological growth is set to zero.

Government Policy and Social Security

The replacement rate θ that is used to model social security is 40%, roughly comparable to the average level of benefits of Social Security. The age at which benefits start, j^b, is 45 (corresponding to real age 65) for the Social Security economy, and 60 (real age 80) for the Hybrid Reform economy.

Preferences

The parameters that specify standard preferences in the model are the coefficient of relative risk aversion, γ, and the time preference discount rate, β. The coefficient of relative risk aversion, γ, is set to 2, and the time preference discount rate, β, is set to .978. These values match the values used by Imrohoroğlu, Imrohoroğlu and Joines (1999), which were chosen to produce a capital intensity ratio close to 2.5, which is the empirical average in the US since 1954, according to their analysis. In this paper, under the SSE model, this parameterization produces a capital intensity of 2.9. While higher than the Imrohoroğlu et al model result, it is closer to the wealth output ratios reported by Cooley and Prescott (1995) and Laitner (1992), which were 3.32 and 3.15 respectively.

Temptation preferences are specified by the Temptation strength parameter, denoted by σ, and the temptation future discount factor, denoted by φ. Laibson Reppetto and Tobacman (1998) find that values for the hyperbolic discount factor in a model of hyperbolic discounting can be as low as .6, and estimate a value of 0.7. The temptation future discount factor is comparable to the hyperbolic discount factor. Imrohoroğlu, Imrohoroğlu and Joines (2000) use 0.85 and 0.9 for experiments with hyperbolic discount factors. This paper uses 0.7 and 0.9 as experimental values for the temptation future discount factor.

For the temptation strength parameter, this paper uses increasing degrees of temptation strength, with σ taking on the values 0.1, 0.2, and 0.3. [7]

Summary of Calibration and Resulting Tax Rates

Table 1 presents a summary of the calibration parameters.

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[7] These will be referred to as weak, moderate and strong temptation respectively. As noted in the paper by Imrohoroğlu, Imrohorouglu and Joines (2000), in which they studied Social Security with a model using hyperbolic discounting in the utility function, as σ approached .3, their model became more difficult to work with. That was also the case in this paper. To find the equilibrium for the cases where $\sigma = .3$, it was necessary to increase the density of grid points in the state space by 67% over the density that was adequate for the cases where $\sigma = .1$. For consistency, all experiments were carried out at the higher grid density.

Table 1. Calibration Parameters

Demographics		
Population growth rate	ρ	.012
Maximum survival age	J	85 (real age 105)
Retirement age	j^*	45(real age 65)
Conditional survival probabilities	$\bar{\psi}(j)$	based on SSA Tables
Low ability mortality factor		112.6%
Efficiency profile	$\bar{\varepsilon}(j)$	Hansen (1993)
High ability efficiency factor		150%
Production		
Labor share	α	.64
Total factor productivity	A	1
Depreciation	δ	.069
Government Policy		
Benefits start age	j^*	45 (Social Security Economy)
	j^b	60 (Hybrid Reform Economy)
Replacement rate	θ	40%
Preferences		
Relative risk aversion	γ	2
Time preference factor	β	.978
Temptation strength parameter	σ	(.1, .2, .3)
Yemptation future discount rate	φ	various (.9, .7)

One of the distinguishing characteristics of the three alternative economic models is the tax rate which is needed to finance their respective social insurance programs. The rates are shown below. While technically a result of the model, they are a result of the invariant demographics and benefit design, not a result of economic behavior. Note that tax rates are unaffected by temptation preferences.

Table 2. Social InsuranceTax Rates

	NSSE	SSE	HRE
Tax to provide Paygo Pension	0	8.77%	2.11%
Tax to fund PSA	0	0	6.66%

3.2.8 Solution Method

The model is solved using a recursive method applied on a discretized state space. The solution we seek is a steady state of the economy. Starting from an initial guess as to the value of aggregate capital, K, and a guess as to the value of aggregate bequests, B, the solution algorithm computes optimal saving and consumption decisions for all agents in the invariant population distribution for a given period. New aggregate capital and aggregate bequests are calculated, given the optimal policies, and the new values for K and B are compared to the starting values. If they differ by more than a tolerance amount defined in advance, the starting guess is updated, and the algorithm is repeated. The process repeats until the aggregate capital and aggregate bequests reach a steady state.

To solve the individual problem under standard preferences, we can use a backwards induction algorithm, starting in the last period of life. To solve the problem with temptation preferences, consider the individual problem:

$$\begin{aligned} W_j(a) &= \max_{c,a'}\{(1+\sigma)u(c) + (1+\sigma\varphi)\beta\psi_j(z)W_{j+1}(a')\} \\ &\quad - \max_{\widetilde{c},\widetilde{a}'}\sigma\{u(\widetilde{c}) + \varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')\} \end{aligned} \tag{20}$$

Since this problem involves two maximization problems, there will be two first order conditions needed. The first is the condition that leads to the solution of the first maximization term. This is the problem that determines the action the agent will take:

$$(1+\sigma)u'(c) = (1+\sigma\varphi)\beta\psi_j\frac{\partial}{\partial a'}[W_{j+1}(a')] \tag{26}$$

The second condition solves the second maximization problem, and it determines the disutility of self-control that will be expended by the agent. It affects his overall utility, but only indirectly the action chosen:

$$u'(\widetilde{c}) = \varphi\beta\psi_j \frac{\partial}{\partial\widetilde{a}'}[W_{j+1}(\widetilde{a}')] \tag{27}$$

The solution to the problem is a set of decision rules, $g(a)$ and $\widetilde{g}(a)$ which determine the choices a' and $\widetilde{a}'$, respectively. This solution is also obtained using backward induction, solving the first order conditions in each period to find the optimal choice for a' and $\tilde{a}'$. The CRRA form of the felicity function makes the first order conditions analytically tractable. The details of the backward induction solution with temptation preferences and a CRRA felicity function are presented in Appendix D.

3.3 Numerical Results

3.3.1 Partial Equilibrium

Our first experiments analyze, in a partial equilibrium, the effect that each of the social insurance programs has on consumption (as a proxy for welfare), when introduced into a steady state economy with no social security. The social insurance programs are No Social Security, Social Security and Hybrid Reform. In the partial equilibrium analysis we make the important assumption that factor prices do not deviate from their steady state levels.

Table 3. Partial Equilibrium: Increase in Consumption

	SSE	SSE	SSE	HRE	HRE	HRE
	Overall	High	Low	Overall	High	Low
Correlated	.29%	.59%	.05%	1.73%	1.93%	1.56%
Uncorrelated	.58%	0.50%	0.64%	1.90%	1.81%	1.97%

Starting from the NSSE steady state, we obtain the known result (Abel 1985) that a Social Security program can improve individual welfare when factor prices do not change. Consumption increases by .29% overall when Social Security is introduced into the economy. But even more dramatic is the 1.73% overall increase in consumption that results from the introduction of the Hybrid Reform program. What is behind this difference? In both the

Social Security program and in the Hybrid Reform program, the worker pays a 2.11% payroll tax and receives a lifetime pension from age 80. The two programs differ with respect to the remaining 6.66% payroll tax, and pension benefits from age 65 to 80. In the case of Social Security, the additional 6.66% payroll tax is used to pay pension benefits to retirees between ages 65 and 80. In the case of the Hybrid Reform program, the 6.66% tax is accumulated in a PSA which is then depleted to provide an income benefit between age 65 and 80.

Evidently, based on the results in Table 3, potential consumption is reduced by using paygo taxes to provide pension benefits to younger aged retirees. We know from Samuelson (1959) and Abel (1985) that the rate of return on Social Security taxes will be the rate of population growth plus mortality gains from survivorship (tontine gains). But, in our world, in which workers routinely live to age 80, there are not meaningful tontine gains to be had prior to age 80. If, for clarity of argument's sake, we set the small pre-80 tontine gains to zero, then the worker in the SSE is giving up the opportunity to earn 3.33% (the equilibrium return under NSSE) in a PSA savings account, in return for a 1.2% (population growth rate) return on his paygo tax. This accounts for the lower consumption under SS than under the Hybrid Reform, in this partial equilibrium experiment.

The second finding from the partial equilibrium analysis is that in a world where mortality is correlated with ability/income, the benefits of social security accrue mainly to high income workers, who enjoy longer expected lifetimes. Table 3 breaks out the increase in consumption between high ability/high income worker and low ability/low income worker. It presents these results under the assumption that mortality is correlated with income, and also under the assumption that mortality is uncorrelated with income. When mortality is correlated with ability, Table 3 shows that almost all the welfare gain from introducing Social Security into a NSSE steady state economy accrues to the high income worker. In contrast, when mortality is not correlated to ability/income, the increase in consumption from adding social security is distributed quite evenly between the two classes of workers, even slightly favoring low income workers. The reason that welfare gains accrue to high income workers is the result of charging all workers a uniform tax rate, while low workers will receive fewer benefits due to shorter lives. This unwanted outcome is referred to as reverse redistribution in this paper.[8]

When the Hybrid Reform is introduced, we see in Table 3 the same pattern of welfare gains by type of worker that we see with Social Security. If mortality is correlated with income, then high income workers benefit more than low income workers. If mortality is uncorrelated, then low income workers benefit relatively more than high income workers, from a reallocation of 6.6% payroll taxes to saving in a PSA. This pattern is to be expected, as the Hybrid Reform design is also subject to reverse redistribution. But since the Hybrid Reform pension covers a significantly smaller and later period of life, the tax is correspondingly low. Thus, while the paygo portion of the Hybrid Reform design means that reverse redistribution effects are present, they are small enough that investment gains in the PSA, which help low income workers relatively more than high income workers, and can offset some of the unequal effects. This is not the case with Social Security.

Third, we also find the well known result that, in a life cycle model, social security results in less capital formation than in a model without social security. The paygo pension is a substitute for precautionary savings (Barro 1974). Table 4 presents results which show this effect in a partial equilibrium, as workers make different choices about saving and consumption.

Table 4. Partial Equilibrium: Change in Key Indicators

	NSSE	SSE	HRE
% Change in Aggregate Savings	0%	−56.6%	−18.1%
% Change in Bequests	0%	−63.4%	−37.1%

Table 4 shows a dramatic reduction in savings when Social Security is introduced. Workers reduce their savings in response to the pension. But in the case of the Hybrid Reform,

[8] In the real Social Security System, the benefit formula is skewed in favor of low income workers, which offsets to some degree the reverse subsidy for those workers who live to receive benefits. Historically, this was deemed to be a better solution than progressive tax rates, which might undermine public support for the whole system. Even with the skewed benefits, there is unremediated reverse redistribution.

workers maintain most of their savings, reducing aggregate savings by only 18.1%, compared to 56.6% in the case of Social Security. This is not surprising, since they have to self fund retirement income prior to age 80.

But do extra savings add to individual welfare, or is it merely a case of dying richer? Table 4 gives insight into that question also. Starting from the NSSE steady state, we see that introducing a lifetime pension provision reduces bequests in both the SSE and HRE scenarios. In the SSE scenario, bequests (wealth at death) are reduced 63.4%, while aggregate savings (capital) are reduced 56.6%. On average, a 1% reduction in wealth at death in the SSE scenario corresponds to a .89% reduction in capital (i.e. $56.7 \div 63.4$). The same 1% reduction in wealth at death corresponds to only a .49% reduction in aggregate savings (capital) in the Hybrid Reform scenario ($18.1 \div 37.1$). In this sense, the Hybrid Reform is "bequest efficient", meaning that it is able to reduce wealth at death (which represents lost potential consumption) without reducing capital to the same degree. The reason for the bequest efficiency is simply that workers are highly likely to live to age 80, and realize (consume) the gains from self insuring, rather than dying prior to age 80 and leaving a bequest. This is a motivating insight into the self-insurance design of the Hybrid Reform.

In summary, our partial equilibrium analysis reveals that the Hybrid Reform is able to increase individual welfare relatively more than Social Security, when introduced into a steady state economy with no social security. It also reveals that the Hybrid Reform is able to distribute individual welfare gains more evenly by type of worker. And it shows the mechanism for these results to be closely tied to its efficiency with respect to converting savings into increased capital and consumption, not just into larger bequests. We now consider the Hybrid Reform in a general equilibrium setting, where factor prices adjust to changes in the economy brought by the alternative social insurance programs.

3.3.2 General Equilibrium Analysis

Steady State Equilibrium

In contrast to the partial equilibrium setting, in which factor prices are fixed, when factor prices are free to adjust to Social Security, the drop off in savings behavior in a standard preferences life-cycle model leads to lower capital, lower output and lower consumption per worker. A contribution of this paper is to examine general equilibrium outcomes when agents are subject to temptation preferences.

It is an empirical question as to what extent, if any, an economy is subject to issues of temptation and self-control. This paper takes an agnostic position and merely analyzes the impact of temptation preferences, without trying to quantify the presence of temptation and self-control problems. We want to know, assuming temptation preferences, what the impact on the equilibrium outcomes in the SSE, NSSE and HRE alternative economies will be.

Table 5 presents the equilibrium values for key economic indicators in the three model economies under standard preferences. The equilibrium values for key economic indicators for agents under moderate strength temptation preferences ($\sigma = .2$, $\varphi = .7$) are presented in Table 6.

Table 5. Steady State Values for Key Indicators (Standard Pref.)

		NSSE	SSE	HRE	SSE %NSSE	HRE %NSSE
Capital	K	5.7208	4.204	5.0913	73.5	89.0
Output	Y	1.6261	1.4554	1.5592	89.5	95.9
Consumption	C	1.1632	1.1152	1.1472	95.9	98.6
Welfare	W_1	−30.266	−33.960	−31.920		
Bequests	Beq	.11129	.07603	.08008	68.3	72.0
Wage	w	1.2986	1.1623	1.2452	89.5	95.9
Interest Rate	r	3.33%	5.56%	4.13%	167.0	124.0

Beyond the result that in general equilibrium, welfare in the long run (measured in terms of consumption) under Social Security is worse than under No Social Security, there are several other outcomes embedded in the results presented in Tables 5 and 6 which receive comment.

First, under standard preferences, Table 5 shows that capital, output and consumption in the HRE model are also worse than the corresponding outcomes in the NSSE model (89.0%, 95.9% and 98.6% respectively). However the crowding-out effect of social security on capital formation is not as severe in the HRE model (capital is 89.0% of NSSE vs. 73.5% in the SSE model). In other words, for the same total payroll tax, by allocating some of the payroll tax to a self-insured mandatory PSA, the HRE model economy maintains capital formation, leading to levels of consumption which are 98.6% of the NSSE result, as compared to 95.9% under SSE. The value function also indicates the welfare ranking among the three alternative models is consistent with the other indicators.

Table 6. Steady State Values for Key Indicators (moderate tempt.)

		NSSE	SSE	HRE	SSE %NSSE	HRE %NSSE
Capital	K	2.8134	2.3138	4.0101	82.2 %	142.5 %
Output	Y	1.2595	1.1739	1.4308	93.2	113.6
Consumption	C	1.0319	.98674	1.1063	95.6	107.2
Welfare	W_1	−35.712	−39.815	−33.639		
Bequests	Beq	.07017	.05221	.07072	74.4	100.8
Wage	w	1.0058	.93747	1.1427	93.2	113.6
Interest Rate	r	9.22%	11.36%	5.95%	123.2	64.5

Second, we see that temptation preferences have a strong negative impact on capital formation in the three model economies. Table 6 reflects that capital in the NSSE steady state is only 49% of the level of steady state capital under standard preferences. For the SSE and HRE model economies, the corresponding level of capital is 55% and 79% respectively. Agents are tempted to consume now, rather than save for future consumption. The SSE drop in capital is slightly less severe than the NSSE drop in capital, showing that Social Security is a commitment device, although a weak one.[9]

We also see that the ranking of the alternatives according to individual welfare is also different under temptation. The effectiveness of HRE as a commitment device is evident in that the value function for HRE is highest of the three under temptation.

Another important aspect reflected in Tables 5 and 6 is the bequest efficient design of the Hybrid Reform that was highlighted in the partial equilibrium setting. Under standard preferences, capital in the HRE steady state is 21.1% greater than in SSE, while bequests are only 5.3% greater. Under temptation the corresponding figures are 73.3% and 35.5%. A contribution of this paper is that there is an advantage to the macroeconomy and to individual welfare, from a bequest efficient Hybrid Reform.

Results from experiments under various levels of temptation are shown in the section on sensitivity analysis but one of the important findings is presented here. When we increase the degree of temptation, after an initial negative effect on capital formation, the mandatory PSA serves as an effective commitment device and insulates the HRE economy from further deterioration in capital, output, wages, and consumption. This is not the case for the SSE economy, which continues to see greater deterioration of capital at stronger levels of temptation. Table 7 illustrates the insulation effect of the Hybrid Reform compared to SSE.

Table 7. Effective Commitment Device?: Hybrid Reform vs. Social Security

	HRE Capital	HRE Consumption	SSE Capital	SSE Consumption
Standard Pref.	100 %	100 %	100 %	100 %
Weak Tempt.	80.4	96.8	70.9	93.7
Mod. Tempt.	78.8	96.4	55.0	88.5
Strong Tempt.	78.5	96.4	45.2	84.3

[9] This result is consistent with a result obtained by Imrohoroğlu, Imrohoroğlu and Joines (2000), in their analysis of Social Security under hyperbolic discounting.

At low levels of temptation, the agent in the HRE economy is able to reduce voluntary saving for more consumption, and we notice the reduction in capital in Table 7 relative standard preferences. But at stronger levels of temptation, the agent is constrained from further reductions in saving by the mandatory PSA. This allows the economy to maintain capital, even if agents reduce voluntary savings to zero. This suggests that there is potentially significant value to a mandatory PSA, not only in increasing individual welfare (consumption), but also in immunizing the macroeconomy from adverse effects of temptation preferences on capital formation.

Individual Welfare, Reverse Redistribution and Wealth Inequality

In partial equilibrium, we saw that welfare gains under Social Security were captured by high income workers at the expense of low income workers. This same result is also evident in general equilibrium. Table 8 shows the change in individual welfare, relative to the welfare under the NSSE steady state. To show this change, we use the device of measuring the compensating variation; that is the amount of annual consumption a newborn would give up to be born into the NSSE economy, rather than the alternative economy.

Table 8. Individual Welfare by Type of Worker

	NSSE	NSSE	SSE	SSE	HRE	HRE
Standard Pref.	High	Low	High	Low	High	Low
Compensating Var.	0	0	10.0 %	12.8 %	4.3 %	5.8 %
Moderate Tempt.						
Compensating Var.	0	0	10.3 %	11.8 %	−5.0 %	−6.0 %

There are a couple observations from this table. First, reverse redistribution in SSE model persists in general equilibrium with standard preferences and with temptation preferences. Under standard preferences, high income workers in SSE have compensating variation of 10.0%, while low income workers have a compensating variation of 12.8%; 28% greater. Under temptation preferences, the differences between high income and low income welfare, relative to standard preferences, persist. One might have speculated that under temptation low income workers in SSE would not be as adversely affected, due to the commitment device of Social Security. And this appears to be the case, since low income workers compensating variation is now only 14.6% greater than high income worker's.

In the HRE model under standard preferences, we see that low income workers would pay up to 5.8% of annual consumption to be born into the NSSE economy, but that under temptation preferences, they would want to be paid 6.0% of annual consumption to be born into the NSSE economy. A similar outcome also holds for high income workers. Clearly, HRE is an effective commitment device against temptation.

Second, in the HRE model, the reverse redistribution that is evident under standard preferences disappears when temptation is taken into the model. Under temptation, the high income worker is more adversely affected, and the low income worker is relatively better off. It is certainly the case that the high income worker under standard preferences would have greater voluntary savings. Since voluntary savings are likely to be consumed under temptation, the high income worker would be adversely affected more than low income workers who would have little voluntary savings to consume. Furthermore, the scarcity of capital will produce a higher return on the PSA, which benefits both types of workers proportionately. So we see a reversal of positions under temptation. This is not to say that the reverse subsidy is gone. It is still present, inherent in the single tax rate design for a public pension. But the impact of the reverse subsidy is overwhelmed by the adverse effect of temptation, which hits high income workers harder than low income workers.

Table 10. Wealth Inequality in the Alternative Economies

	NSSE	SSE	HRE
Standard Preferences			
Wealth Gini Coefficient	.5481	.5116	.4812
% Wealth Owned by High Ability	48.8%	50.8%	48.5%
% Wealth Owned by Low Ability	51.2%	49.2%	51.5%
Moderate Temptation			
Wealth Gini Coefficient	.6257	.5721	.4811
% Wealth Owned by High Ability	49.7%	50.6%	45.0%
% Wealth Owned by Low Ability	50.3%	49.4%	55.0%

We observe in Table 10 that the wealth Gini coefficient indicates wealth inequality is improved in the SSE steady state over the NSSE steady state, and it improves even more in the HRE steady state. In both cases, the improvement arises from the result that a paygo pension improves the wealth of poor elderly households.

When wealth is aggregated across ages by type, we see a somewhat different story. In the NSSE economy, high ability agents own 48.8% of the aggregate wealth, while low ability workers own 51.2% of the wealth. (All the economies have the same invariant population distribution, in which 29.7% of the population are high ability and 70.3% are low ability). Under SSE, high ability agents increase their ownership of wealth to 50.8%, while low ability agents drop to 49.2%. This appears to contradict the finding of the Gini coefficient. But the two outcomes are consistent, taking into account that the poorest households are largely elderly, who are disproportionately high ability since they have a life expectancy which is five years longer than low ability workers.

Under temptation, wealth inequality is increased even further, except under HRE, where wealth inequality is decreased. Temptation has a bigger impact on high income agents, who have more available resources to consume. Temptation raises interest rates in all economies, as capital becomes more scarce. The low income worker in HRE is able to benefit from the higher returns, because of the mandatory PSA. A significant outcome is that under the

HRE, not only does the Gini coefficient demonstrate improved wealth distribution, but there is improvement in the share of wealth owned by low ability agents over both the NSSE model and the SSE model.

Our findings from the general equilibrium analysis can be summarized as follows. (i) Under standard preferences, NSSE produces the highest levels of capital, output and consumption. (ii) When agents have temptation preferences, both NSSE and SSE economies suffer significant reductions in capital, output and consumption. Only the HRE is able to sustain the macroeconomy under temptation, as the mandatory PSA is an effective commitment device. (iii) When mortality is correlated with income, then Social Security increases wealth inequality as the rich capture much of the benefit of social security. However, wealth inequality is reduced in HRE even below the levels of NSSE economy. This is especially apparent under temptation preferences. (iv) The Hybrid Reform under standard preferences achieves capital formation levels close to that of NSSE, and under temptation it exceeds the capital formation of the NSSE model economy. The HRE also avoids most of the problem of reverse redistribution by means of the self-insurance mechanism, while it retains the social welfare benefits of a government guaranteed income at the advanced ages of life.

3.4 Conclusions

Even under standard preferences, there is a good case to be made for the second best welfare alternative for the long run, a Hybrid Reform proposal. As we have seen, the Hybrid Reform imposes a mandatory funding mechanism which improves capital formation in the economy. Ownership of the PSA is in the hands of the worker, which eliminates reverse redistribution with respect to the mandatory funding portion of the payroll tax. The HRE combination of funded self-insurance for early retirement benefits, and an unfunded public pension for advanced old age provides for greater capital formation, with better equity between workers of different income levels, while still providing an old age pension and avoiding the free rider problem inherent in NSSE.

But it is under temptation preferences that the Hybrid Reform really outperforms either of the two other model economies. In both the SSE and NSSE economies, temptation preferences produce a significant depressive effect on capital formation, output, and consumption. While SSE was slightly less sensitive to temptation than the NSSE economy, their differences were overwhelmed by their similarity in response to temptation preferences.

Unlike the NSSE and SSE economies, the HRE model economy contains an effective commitment mechanism, the mandatory PSA, which immunizes capital formation, output and consumption from the worst effects of temptation preferences. As a result, the long run welfare analysis showed advantages to workers in the HRE economy, while also providing insulation to the macroeconomy from the effects of temptation. Harnessing the dual power of a government provided pension at advanced old ages, and a self-insured mandatory PSA during younger retirement ages, the HRE is able to restore capital formation and immunize the economy from the deleterious effects of temptation preferences on voluntary savings.

As much of the literature, the present work has provided a steady state analysis and the results should therefore be interpreted with care. Clearly, it would be interesting to analyze the transition path to establish conclusive results. This is a very important issue that we leave for further research.

Chapter 4

Social Security and Consumer Surplus

For over thirty years, economists have studied the effect of Social Security on capital formation. Consistently, studies that employed a life-cycle consumer model have shown that agents reduce saving in the economy with Social Security. Lower savings leads to lower levels of capital, output, consumption and welfare in a general equilibrium steady state. [1] A notable exception to the conclusion that Social Security reduces steady state welfare is when the decision maker is the head of a multi-generation family unit (Fuster 1999). In this dynastic setting, Social Security, which performs the type of intergenerational transfers a dynastic decision maker would affect in the absence of Social Security, is effectively nullified by the actions of the decision maker. Hence in a dynastic setup, there is little difference in the steady state of the economy with or without Social Security. Whether a dynastic setup or a life-cycle model is more valid is not known. What is clear is that if a life cycle model is valid, then Social Security reduces steady state capital, output and welfare, and if a dynastic setup is valid, then Social Security is cancelled out by the multi-generation household decision maker.

It is natural to wonder then, why Social Security remains such a popular public program. Perhaps it is simply a result of the schedule of taxes, which are back loaded to increase in the future when the heaviest burden of retirements will occur. Perhaps people are myopic, and

[1] This has been shown in two period models, in overlapping generation models with standard preferences, with time inconsistent preferences and with temptation preferences.

they do not appreciate the future costs yet to be borne. But assuming this is not the case, and that people are fully aware of the rising tax burden ahead, then what accounts for the continued popularity of Social Security? The question at the heart of this paper is whether or not the present system creates consumer surplus, so that todays workers are willing to pay for the true costs of Social Security. We will seek to find consumer surplus in various experiments that include bequests in the utility of workers, a progressive (kinked) benefit formula, increasing the age at which benefits start to be received, and alternative elasticities of intertemporal substitution. We will then consider whether workers with temptation preferences, (i.e., self-control issues), would realize consumer surplus with Social Security.

It seems an odd question, given that workers have happily paid into the Social Security system for over 70 years. Yet for much of the history of Social Security, benefits have expanded faster than the costs to current workers, so that there have been "first generation windfalls" to workers already in the system for the period up to 1977. Since the overhaul of the system in connection with the 1983 Social Security reform commission, benefit expansion has stopped, and the changes to Social Security in 1983 and in 1996 have been in the direction of reduced benefits and higher taxes. This is clearly in the future again as the social security system is reformed to correct actuarial imbalances. Is there evidence that enough welfare is generated in the system to make these coming tax increases (or benefit reductions) politically palitable?

The necessity of the impending reform of Social Security has prompted a variety of Social Security reform proposals. One particular reform proposal was analyzed in an earlier paper by the author (Boronow 2007). That paper proposed a "hybrid" reform, that takes design features from two basic approaches.

From the present government-based system, the Hybrid Reform proposal retains a government provided paygo pension, but one which starts at an advanced old age (in particular, at age 80). We know that in a partial equilibrium setting, a government-provided pension can be a welfare-enhancing benefit for individuals (Abel 1985). Such a pension reduces retirement income uncertainty, and protects citizens when they are not necessarily able to fend for themselves.

From an individual responsibility approach, the Hybrid Reform proposal incorporates a mandatory Personal Security Account (PSA) to accumulate assets that will provide retirement income in the years before the government pension starts. By eschewing (in part) intergenerational transfers in favor of accumulated assets, the Hybrid Reform results in increased saving (and greater capital formation), which in equilibrium leads to a higher standard of living.

The objective of the paper is to illuminate the channels in Social Security design through which welfare can be improved as future reforms are considered. Our benchmark economies are an economy with No Social Security (NSSE) and a social security economy with a flat 40% replacement rate pension benefit (SSE-40). When we analyze social security under temptation, we will also consider an economy with the Hybrid Reform pension system (HRE).

The findings of the paper can be summarized as follows:

1. Low income workers are willing-to-pay more for Social Security benefits relative to the fair value of those benefits than high income workers are willing-to-pay. But this alone is not enough (under the parameterization of the model used in this paper) to account for the popularity of Social Security.

2. Diminishing marginal utility of pension levels dominates improved individual welfare emanating from reduced uncertainty in retirement income under Social Security. Thus, willingness-to-pay decreases as the level of Social Security benefits increases.

3. When pension benefits start at a later age, individuals have an increasing willingness-to-pay the required tax.

4. Including bequests in utility does not make a significant difference in results (assuming a "joy of giving" specification for bequests).

5. Risk aversion may increase or decrease willingness-to-pay, depending on the return on capital relative to the demographic return on survivorship. When the return on capital is large enough, relative to the return on survivorship, higher risk aversion results in a

decreasing willingness to pay. But when the return on survivorship is high, relative to the return on capital, higher risk aversion results in an increasing willingness to pay.

6. In a framework with temptation, a social security mechanism with an effective commitment device raises willingness-to-pay. But in the absence of an effective commitment device, temptation reduces willingness-to-pay even further.

The paper is organized as follows. We start by considering a simple two period model of Social Security, to gain intuition into the effect of our experiments. The model and its calibration are presented in Sections 2 and 3. Results are in Section 4. After some closing remarks, the Appendix contains details of the results of the experiments and a brief explanation of temptation preferences.

4.1 Two Period Model of Social Security

We start our analysis with a simple two period model in which factor prices are givien. With a simple model we can use the tools of calculus to gain insight into the dynamics of the model in a way that would be intractable in the larger model that we will use in the rest of this paper.

In this two period model, an agent lives up to two periods, with the probability of surviving to period two denoted by ψ. The agent receives a bequest (B) and a wage (w) in period one. The agent pays a tax on wages at rate τ to fund a social security benefit in period two. The size of the social security benefit is $\theta \cdot w$. Savings earn a gross return R. The agent has a time preference discount factor of β. The agent must choose consumption in each of the two periods $\{c_1, c_2\}$, in such a way as to maximize utility. We assume isoelastic utility. The agent's problem is:

$$\max\{u(c_1) + \beta\psi u(c_2)\}$$

$$\text{subject to:} \qquad u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

$$c_2 = [B + w(1-\tau) - c_1]R + \theta w$$

$$\tau w = \psi\theta w$$

The first order condition is:

$$\frac{c_2}{c_1} = (\beta\psi R)^{\frac{1}{\gamma}}$$

Solving for c_1 (and hence for c_2 also):

$$c_1 = \frac{BR + w[(1-\tau)R + \frac{\tau}{\psi}]}{R + (\beta\psi R)^{\frac{1}{\gamma}}}$$

With this result we examine the effect on saving of several experiments. We start with an experiment in which the probability of survival to retirement increases, as has been the case since the start of the Social Security system in 1935. We look to the calculus for help and consider $\frac{\partial c_1}{\partial \psi}$.

$$\begin{aligned}\frac{\partial c_1}{\partial \psi} &= \{BR + w[(1-\tau)R + \frac{\tau}{\psi}]\}\{\frac{-1}{[R + (\beta\psi R)^{\frac{1}{\gamma}}]^2}\}\{\beta R\frac{1}{\gamma}(\beta\psi R)^{\frac{1}{\gamma}}\} + ... \\ &\quad \frac{1}{R + (\beta\psi R)^{\frac{1}{\gamma}}}[\frac{-wR}{\psi^2}]\end{aligned}$$

The derivative is always negative, so that as the probability of survival (ψ) increases, consumption in period one decreases, and savings increase. The reverse is also true; as the probability of surviving to the second period decreases, then period one consumption will increase and savings decrease. This example illustrates both the strength and limitations of a simple model. We gain insight that improving mortatility makes the insurance aspect of social security less effective (decreases consumption, requires more savings), but it does not take into account the impact of increased longevity on social security.

Suppose we increase the social security benefit in period two. What effect does that have on savings and consumption? In the model, increasing benefits is equivalent to increasing

τ, so we look to the derivative with respect to τ.

$$\frac{\partial c_1}{\partial \tau} = \frac{1}{R + (\beta\psi R)^{\frac{1}{\gamma}}}[\frac{w}{\psi}] - \frac{1}{R + (\beta\psi R)^{\frac{1}{\gamma}}}[wR]$$

In this case, $\frac{\partial c_1}{\partial \tau}$ is positive if $\frac{1}{\psi} > R$, and $\frac{\partial c_1}{\partial \tau}$ is negative if $\frac{1}{\psi} < R$. (The "return on survivorship" is $\frac{1}{\psi}$, while the return on savings is R.) Thus, if the age at which the benefits are received is old enough so that the survivorship return is greater than the return on savings, i.e. $\frac{1}{\psi} > R$, then $\frac{\partial c_1}{\partial \tau}$ will be positive, and an increase in taxes (and benefits) will also result in an increase in c_1, and a decrease in savings. This is a happy result, whereby increased taxes lead to greater consumption in period one and in period two. But if the return on survivorship is less than the return on savings, $\frac{1}{\psi} < R$, then $\frac{\partial c_1}{\partial \tau}$ will be negative, and an increase in taxes will reduce c_1. The return on survivorship is a function of demographics. In an economy with high savings, returns on savings will be relatively low. In a capital starved economy, returns on savings will be relatively high. So the question of whether an increase in period two benefits will increase welfare depends on the capital intensity of the economy.

Finally, we are interested in the effect that different degrees of risk aversion might have in the model. Since returns are certain in this model, the risk aversion parameter γ is more appropriately interpreted as the inverse of the constant elasticity of intertemporal substitution. When γ is large, the inverse is small, and the agent is less willing to shift consumption between periods to improve utility. But when γ is small and the inverse is large, then the agent is more willing to shift consumption between periods. We consider again the derivative, this time with respect to $\frac{1}{\gamma}$, the elasticity of intertemporal substitution.

$$\frac{\partial c_1}{\partial \frac{1}{\gamma}} = \{BR + w[(1-\tau)R + \frac{\tau}{\psi}]\}\{\frac{-1}{[R + (\beta\psi R)^{\frac{1}{\gamma}}]^2}\}\{(\beta\psi R)^{\frac{1}{\gamma}} \log(\beta\psi R)\}$$

In this case, the key to the sign of the derivative is the term $\log(\beta\psi R)$. If $\log(\beta\psi R) < 0$, then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} > 0$, and if $\log(\beta\psi R) > 0$, then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} < 0$. Assuming that $0 < \beta < 1$, we have $\log(\beta\psi R) > 0 \iff \beta\psi R > 1 \iff R > \frac{1}{\beta\psi}$. If $R > \frac{1}{\beta\psi}$, (which would be the case if the age at which benefits are received is not too old), then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} < 0$. Thus as $\frac{1}{\gamma}$ increases (becomes more elastic), c_1 decreases and savings increases. But if $R < \frac{1}{\beta\psi}$ (which could happen if the age at which benefits are received is old enough), then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} > 0$, and then consumption increases as $\frac{1}{\gamma}$ increases, and savings decreases. This insight will be helpful when considering the complex interaction of mortality, interest and preferences with respect to intertemporal choices.

4.2 The Overlapping Generations Model

The OLG model used in this paper is essentially the same as the one used in an earlier paper (Boronow 2007), with certain changes as needed to explore various alternative experiments. This paper includes bequests in the utlity function as an experimental option. Also, the model includes a kinked benefit formula matching the formula used by the Social Security Administration, as an alternative experiment to the simple average replacement ratio used in the earlier paper. Finally, while the earlier paper studied the effect of temptation preferences on steady state outcomes of model economies, this paper usually uses standard preferences in its various experiments.[2] We do come back to temptation preferences as another experimental framework eventually.

The specific details of the model are presented in Boronow (2007). A breif description is included here for convenience of the reader.

4.2.1 Demographics

The demographic setup is exactly the same as that used in Boronow (2007). The reader who is familiar with that setup can skip ahead.

Time is discrete, and each period represents one year. Age 1 corresponds to real age 21. The oldest possible age is age J , where $J = 85$ (real age 105). We assume that death is certain thereafter.

There are two types of agents indexed by z, where $z \in \{1, 2\}$. An agent's type is revealed

[2] One further technical difference is that this model is solved on a uniform grid with 5000 points. The earlier paper used a non-uniform grid with 7401 points.

at birth and this determines lifetime ability, which can be either high ($z = 1$) or low ($z = 2$). The realization of ability follows a first-order Markov process with transition matrix Π:

$$\begin{aligned} \Pi(z, z') &= [\pi_{ij}]; i, j \in \{1, 2\} \\ \pi_{ij} &= \Pr(z' = j | z = i). \end{aligned}$$

where z is the ability type of the parent and z' is the ability type of the child. It is assumed that the transition probabilities, π_{ij}, are such that there is a resulting stationary distribution of ability types, Λ, where $\lambda(z) \in \Lambda$, and $\lambda(1) + \lambda(2) = 1$.

The ability type determines the endowment of efficiency units an agent receives. In a given period, the cross-sectional labor efficiency $\varepsilon_j(z)$ is indexed by ability type z and age j. Without loss of generality, we assume throughout this paper that the rate of technological growth is zero. Under this assumption, the longitudinal efficiency units of a particular agent equal the cross-sectional efficiency factors, $\varepsilon_j^l(z) = \varepsilon_j(z)$.

Agents have uncertain lifetimes. Survival probabilities are correlated with ability type, so that high ability agents have longer expected lifetimes than low ability agents. Thus survival rates are indexed by age and type. The probability that an agent age j and ability type z survives to age $j + 1$ is denoted by $\psi_j(z)$. The probability that an agent age j and ability type z survives to age $j + t$ is denoted by $\Psi_{j,t}(z)$, where:

$$\begin{aligned} \Psi_{j,t}(z) &= 1, \quad \text{if } t = 0 \\ \Psi_{j,t}(z) &= \Pi_{s=1}^{t} \psi_{j+s-1}(z), \ \text{if } t > 0. \end{aligned}$$

Like much of the social security literature, this paper analyzes the steady states of a stationary population distribution, with time invariant cohort shares. Let ρ be the assumed constant rate of growth in population. Then, the cohort share of a new agent of type 1 relative to a new agent of type 2 is equal to $\frac{\lambda(1)}{\lambda(2)}$. That is, the size of the newborn type cohorts, relative to each other is determined by the Markov process stationary distribution of types. Thereafter, relative cohort shares are a result of the population growth rate and survival probabilities. Letting $\mu_j(z)$ denote the cohort share for an agent of age j and ability type z, and letting N denote the total population when the newborn cohort is indexed to one (1), for newborns ($j = 1$):

$$\begin{aligned} \mu_1(z) &= \lambda(z) \cdot [1/N], \text{ where} \\ N &= \sum_{z=1}^{2} \sum_{t=0}^{J} \lambda(z) \cdot (1+\rho)^{-t} \cdot \Psi_{1,t}(z). \end{aligned} \tag{1}$$

Each new cohort is $(1 + \rho)$ times as large as the preceding cohort, and each cohort survives to the next period according to the corresponding age and ability type, $\psi_j(z)$. Thus for $j = 1, 2, ...J - 1$:

$$\mu_{j+1}(z) = \mu_j(z) \cdot \frac{\psi_j(z)}{1+\rho}. \tag{2}$$

Finally, the sum of all cohorts must equal 100% so that,

$$\sum_{z=1}^{2} \sum_{j=1}^{J} \mu_j(z) = 1. \tag{3}$$

Given the Markov process, survival rates, and population growth rate, $\Pi, \psi_j(z)$ and ρ, respectively, the above relationships uniquely determine the time invariant cohort shares, $\{\mu_j(z)\}$.

4.2.2 Technology and Factor Prices

This part of the model is also exacly the same as in Boronow (2007).

There is a single good in the economy, produced by one or more firms using a constant returns to scale Cobb-Douglas production function:

$$Y = AK^{1-\alpha} \cdot L^{\alpha}, \text{ where } \alpha \in (0, 1).$$

Total factor productivity A is normalized to 1. The labor share is α and K and L are aggregate capital and labor supplied as inputs. Capital is assumed to depreciate at the constant rate δ. Therefore, in a competitive equilibrium, we get factor prices for capital and labor:

$$\begin{aligned} r &= (1-\alpha)\cdot K^{-\alpha}\cdot L^{\alpha}-\delta \\ w &= \alpha\cdot K^{1-\alpha}\cdot L^{\alpha-1} \end{aligned} \tag{4}$$

K represents the aggregate asset holdings over the population in a given period. The size of L is determined by the workers up to retirement age j^*. Workers are assumed to supply labor inelastically to age j^*, and do not work thereafter. The actual supply of efficient labor depends on the ability type of agents in the working age population.

$$L = \sum_{z=1}^{2}\sum_{j=1}^{j^*-1} \varepsilon_j(z)\cdot\mu_j(z). \tag{5}$$

4.2.3 Government Policy and Social Security

This paper will analyze model economies that differ in their approach to Government Policy and Social Security. Our benchmark economies are a No Social Security economy (NSSE), and a Social Security economy with a 40% replacement rate benefit (SSE-40). We will conduct experiments in which the benefit formula is kinked to match the actual formula used by the Social Security Administration. This economy is designated as SSE. Other experiments that affect policy will include starting benefits at ages 70, 75 and 80. These economies will be designated as SSE-40@70, and so on, to indicate the 40% benefit formula and later starting age. (Note that even though the pension starts at a later age, the worker retires at the same age 65.) Finally, we will look at experiments under temptation preferences using the Hybrid Reform proposal of Boronow (2007), in which a mandatory personal security account is combined with a paygo pension that starts at an advanced age. The Hybrid Reform economy will be designated as HRE. A detailed description of HRE is in Boronow (2007).

No Social Security Model Economy

Government policy in the NSSE model is straightforward. There is no social security tax, and there are no social security benefits. Each worker must prepare for retirement income by saving during their working years, to build up a retirement nest egg.

Social Security Model Economy

In the SSE and SSE-40 models, there is a social security program that provides a public pension to retirees. Average lifetime earnings for a worker of ability type z, denoted by $w^{SS}(z)$, is given by:

$$w^{SS}(z) = \frac{1}{j^* - 1} \cdot \sum_{j=1}^{j^*-1} w\varepsilon_j(z) \tag{6}$$

In the SSE-40 model, the social security benefit is simply $b^{SS}(z) = \theta * w^{SS}(z)$, where θ is a flat percentage of average lifetime earnings.

But in the SSE model, the social security benefit, $b^{SS}(z)$, is determined as a formula of average lifetime earnings, where the percentage decreases as the average lifetime earnings increase. Let $wbar$ denote the average lifetime earnings over all types of workers. Then for low income workers ($z = 2$),

$$b^{SS}(2) = .9 * (.2 * wbar) + .33 * (w^{SS}(2) - .2 * wbar)$$

For high income workers ($z = 1$),

$$b^{SS}(1) = .9 * (.2 * wbar) + .33 * ((1.25 - .2) * wbar) + .15 * (w^{SS}(1) - 1.25 * wbar)$$

The set of breakpoints (20% and 125%), together with the factors .9, .33, .15, are collectively referred to as policy choice θ.

The role of the government is to collect a tax on labor income to exactly provide the social security pension to retirees. The necessary tax rate, τ_{SS}, in this pay-as-you-go model is:

$$\tau_{SS} = \frac{\sum_{z=1}^{2} \sum_{j=j^*}^{J} b^{SS}(z) \cdot \mu_j(z)}{w \cdot \{\sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z)\}} \tag{7}$$

The numerator is the total benefit paid under social security and the denominator is the total wage base over which the tax is applied in a given period.

Hybrid Reform Economy

The Hybrid Reform proposal is described in detail in Boronow (2007).

The HRE model economy has a social security pension that starts at an advanced old age (AOA) that is later than the retirement age. Also, the HRE includes a mandatory personal security account (PSA), owned by the worker, in which mandatory contributions during the agent's working years are accumulated to provide a source for retirement income in the retirement years prior to the start of the public pension. The HRE model is called the Hybrid Reform economy, since the combination of personal accounts and an advanced old age public pension is a hybrid of the present system and a pure private accounts system.

The Hybrid Reform economy provides a public pension starting at age j^b, the benefits start age ($j^* < j^b < J$). The benefit amount is the same replacement rate percentage as in SSE-40. However, since benefits start at age j^b, rather than j^*, the tax to pay for the benefits is lower. (In one set of experiments, we use the kinked benefit formula in a Hybrid Reform system. The tax calculation is adjusted appropriately.)

Let τ_{AOA} denote the tax rate needed to provide advanced old age benefits in the HRE. Then:

$$\tau_{AOA} = \frac{\sum_{z=1}^{2} \sum_{j=jb}^{J} b^{SS}(z) \cdot \mu_j(z)}{w \cdot \{\sum_{z=1}^{2} \sum_{j=1}^{j^*-1} \varepsilon_j(z) \cdot \mu_j(z)\}} \tag{7HRE}$$

There is also a mandatory tax, τ_{PSA}, to accumulate a Personal Security Account (PSA). The PSA is a self-insurance mechanism which provides income for the years between retirement age, j^*, and the start of the public pension at advanced old age j^b. The design of the Hybrid Reform as detailed n Boronow (2007) specifies a pro rata depletion of the PSA during the early retirement years. The income amount taken from the PSA during the period from retirement to the start of the old age pension is denoted by $b_j^{PSA}(z)$. Assets in the PSA are bequeathable.

4.2.4 Constraints and Bequests

Constraints and Bequests are the same as in Boronow (2007), except that this paper includes certain experiments with bequests in utility, which changes the optimality constraint at the end of life.

Each period, the agent must choose the amount of consumption and the amount of voluntary saving. Savings earn the rate of return on capital r. Agents are subject to a no borrowing constraint.

Because lifetimes are uncertain, many agents will die with positive amounts of assets (aka accidental bequests). Accidental bequests are redistributed to surviving agents, in such a way that each type of agent receives an equal share based on the expected bequest of that agent, given their ability type. The amount of the bequest distributed to agents of ability type z is denoted by $\xi(z)$.

The budget constraint faced by an agent of age j and ability type z is given by:

$$c_j(z) + a_{j+1}(z) = [a_j(z) + \xi(z)] \cdot (1 + r) + Q_j(z) \tag{12}$$

where Q_j is defined as follows.

In the No Social Security economy:

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \ \text{ for } j < j^* \\ Q_j(z) &= 0 \ \text{ for } j^* \leq j \end{aligned}$$

In the Social Security economies:

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \cdot (1 - \tau_{SS}) \text{ for } j < j^* \\ Q_j(z) &= b^{SS}(z) \text{ for } j^* \leq j \end{aligned}$$

In the Hybrid Reform economy:

$$\begin{aligned} Q_j(z) &= w \cdot \varepsilon_j(z) \cdot (1 - \tau_{AOA} - \tau_{PSA}) \ \text{ for } j < j^* \\ Q_j(z) &= b_j^{PSA}(z) \cdot (1 + r) \text{ for } j^* \leq j < j^b \\ Q_j(z) &= b^{SS}(z) \ \text{ for } j^b \leq j \end{aligned}$$

In all economies, newborn agents enter with zero assets ($a_1(z) = 0$). In the final period, optimality requires that $a_{J+1}(z) = \omega^{\frac{1}{\gamma}} c_J$. (The parameter ω is the strength of the bequest motive, as defined later.) When bequests are not in the utility function, the optimality condition becomes $a_{J+1}(z) = 0$.

In all economies, households face a borrowing constraint:

$$a_j(z) \geq 0, \ \forall \ j$$

The bequest $\xi(z)$ is described in detail in Boronow (2007).

4.2.5 Preferences and Individual Optimization Problem

In the various experiments, our benchmark utility is standard preferences based on consumption. But we also experiment with preferences which include bequests in utility. We utilize the "joy of giving" specification with a parameter to capture the strength of bequest motive. We otherwise use the same isoelastic form of utility for bequests that we use for consumption. Some experiments use the same temptation preferences that are used in the earlier paper (Boronow 2007). These are described in detail in that paper, and briefly in Appendix B of this paper.

To streamline notation, let a denote $a_j(z)$, where age and type are defined by the context of the usage. Likewise, we use the same notational shortcut for c, Q and ξ. Following notational convention, the prime symbol denotes the next period value.

Standard preferences are defined over a lifetime sequence of consumption $\{c_j(z)\}_{j=1}^{J}$. In the case where only consumption is in utility (CIU), the individual agent's objective for an agent age j is to maximize expected discounted lifetime utility:

$$U_j = \sum_{t=0}^{J-j} \beta^t [\Psi_{j,t}(z)] \cdot u(c_{j+t}(z)) \tag{15}$$

where $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, γ is the constant coefficient of relative risk aversion and β is the discount factor. The term in brackets is the probability of survival to age $j + t$ for an agent of type z.

For the case where consumption and bequests are in the utility function (CBIU), the expected discounted lifetime utility is:

$$U_j = \sum_{t=0}^{J-j} \beta^t [\Psi_{j,t}(z)] \cdot u(c_{j+t}(z), a'_{j+t}(z)) \tag{16}$$

$$\begin{aligned} \textit{where} \;\; u(c) &= \frac{c^{1-\gamma}}{1-\gamma} + (1-\psi)\omega \frac{a'^{1-\gamma}}{1-\gamma} \\ a' &= (a+\xi)(1+r) + Q - c \end{aligned}$$

As noted above, the parameter ω represents the strength of the bequest motive.

The Individual's Problem

An agent of age j and type z starts a given period with initial asset holdings a. The individual's dynamic problem is to choose how much to consume now c, and how much to save for future consumption a', in order to maximize the Bellman equation:

$$W_j(a) = \max_{c,a'} \{u(c, a') + \beta \psi_j(z) W_{j+1}(a')\} \tag{17}$$

subject to the budget constraint, the borrowing constraint, the initial and optimality conditions, and taking the factor prices as given.

4.2.6 The Steady State Equilibrium

Let $D = \{d_1, d_2, ..., d_m\}$ represent the discrete set of values that asset holdings are permitted to take. The feasible set for an age j agent of type z and asset holdings a is denoted by $\Omega(j, a, z)$. The possible choices for a satisfy $a' \in \Omega(j, a, z)$, $a' \geq 0$, and the budget constraints.

A steady state equilibrium for a set of policy parameters $\{\theta, \tau_{SS}\}$ is a collection of value functions $\{W_j(a)\}$; decision rules $R^c_{a,j,z} : D \times \{1, 2, ..., J\} \times \{1, 2\} \to \mathbb{R}_+$ and $R^{a'}_{a,j,z} : D \times \{1, 2, ..., J\} \times \{1, 2\} \to D$; a stationary distribution of types of newborns, $\{\lambda(1), \lambda(2)\}$; a time invariant distribution of agents by type, $\{\mu_j(z) | \forall j \in \{1, 2, ...J\}, \forall z \in \{1, 2\}\}$; a set of prices for capital and labor $\{r, w\}$; and a set of lump sum transfers of accidental bequests to agents $\{\xi(z)\}$; such that

1. Given factor prices, government policy and the lump sum transfers, the decision rules solve the individual optimization problem.

2. Factor prices solve the optimization problem of the firm

3. Markets clear, implying that:

$$K = \sum_{z=1}^{2} \sum_{j=1}^{J} [a_j(z) + \xi(z)] \cdot \mu_j(z) \tag{18}$$

$$L = \sum_{z=1}^{2} \sum_{j=1}^{j^*-1} [\varepsilon_j(z) \cdot \mu_j(z)] \tag{19}$$

$$Y = \sum_{z=1}^{2} \sum_{j=1}^{J} \mu_j(z) \cdot [c_j(z) + a_{j+1}(z) - (1-\delta) \cdot (a_j(z) + \xi(z)] \tag{20}$$

   Thus, aggregate capital is the sum of individual asset holdings, aggregate labor is the sum of effective workers and output equals aggregate consumption plus the increase in aggregate capital.

4. The invariant population distribution conditions are satisfied.

5. Government pensions are fully financed by the labor tax.

6. The lump sum transfers satisfy:

$$\sum_{z=1}^{2} \sum_{j=1}^{J} \xi(z) \cdot \mu_j(z) = \sum_{z=1}^{2} \sum_{j=1}^{J} \{a_j(z) \cdot \mu_j(z) \cdot (1 - \psi_j(z))\} \tag{21}$$

4.3 Calibration and Solution Method

4.3.1 Calibration

The calibration of the model follows Boronow (2007). The parameter values are shown in Table 1.

Table 1. Calibration Parameters

Demographics		
Population growth rate	ρ	.012
Maximum survival age	J	105 (model period 85)
Retirement age	j^*	65 (model period 45)
Conditional survival probabilities	$\bar{\psi}(j)$	based on SSA Tables
Low ability mortality factor		112.6%
Efficiency profile	$\bar{\varepsilon}(j)$	Hansen (1993)
High ability efficiency factor		150%
Production and Preferences		
Labor share	α	.64
Total factor productivity	A	1
Depreciation	δ	.069
Relative risk aversion	γ	2 (except where noted)
Strength of "joy of giving"	ω	$\{0, 2\}$ as noted
Discount rate	β	.978
Government Policy		
Benefits start age	j^*	age 65 (except where noted)
Replacement Rate Parameters	θ	40% or $\{.2, 1.25, 90\%, 33\%, 15\%\}$

4.3.2 Solution Method

The model is solved using a recursive method applied on a discretized state space. The solution we seek is a steady state of the economy. Starting from an initial guess as to the value of aggregate capital, K, and a guess as to the value of aggregate bequests, B, the solution algorithm computes optimal saving and consumption decisions for all agents in the invariant population distribution for a given period. New aggregate capital and aggregate bequests are calculated, given the optimal policies, and the new values for K and B are compared to the starting values. If they differ by more than a tolerance amount defined in advance, the starting guess is updated, and the algorithm is repeated. The process repeats until the aggregate capital and aggregate bequests reach a steady state.

The solution to the problem is a set of decision rules, $g(a)$ which determine the choices $\{c, a'\}$ for every age and type. The solution is obtained using backward induction.

4.4 Numerical Results

4.4.1 Benchmark Economies

Our analysis is usually from the general equilibrium steady states of the benchmark economies. Some salient features of those steady states are presented in Table 2 for the benchmark economies NSSE and SSE-40.

Table 2. Key Indicators in Steady State

	NSSE	SSE-40
Capital (K)	5.757	4.205
Output (Y)	1.630	1.456
Capital Intensity (K/Y)	3.53	2.89
Wages (w)	1.302	1.162
Return on Capital (r)	3.29%	5.56%
Soc. Sec. Tax (τ_{SS})	0	8.77%

The NSSE and SSE-40 economies differ in capital formation, as previously discussed. This is evident in Table 2, and as a result, output and wages are lower in the SSE-40 steady state than in the NSSE steady state. Returns on capital are correspondingly higher.

A result from Boronow (2007) is that when income and longevity are correlated, low income workers subsidize the old age pension of high income workers (excluding disability and survivor's benefits) under SSE-40. In the model SSE-40, the tax rate for high income workers would be 10.04% and for low income workers it would be 7.81%, if it were possible to charge each type of worker a separate fair tax rate. We will refer to these rates as the hypothetical fair tax rates. The difference between these hypothetical fair tax rates, 2.23%, is a measure of the subsidy. Is it possible that for high income workers the subsidy is enough to gain their support for the program? For low income workers, is the utility of social security large enough to offset the cost of the subsidy to garner their support?

To answer this question and others that we will pose in this paper, we look at two standard ways to measure changes in individual welfare; compensating variation in consumption, and willingness-to-pay. Compensating variation in consumption gives us the perspective of the overall average change in individual welfare, relative to the benchmark economy with no social security. Willingness-to-pay gives us the marginal change in individual welfare at that specification of the experimental situation. Combined, we get a picture of the overall impact as well as the direction and steepness of the marginal impact of the experiment on individual welfare.

Compensating Variation in Consumption

Compensating variation (CV) in consumption in our setting is the additional annual consumption that a newborn would require to be indifferent between an economy with or without social security.

Table 3. Compensating Variation for SSE-40

CV: High Income	10.07%
CV: Low Income	12.87%

The level of compensating variations is due to the impact of Social Security on capital formation, which directly affects the wages of both high and low income workers. High income workers benefit disproportionately more than low income workers from higher interest rates that exist in the low capital intensity SSE economy, hence they require a smaller CV than low income workers. But in any case, the average impact is such that rational agents without a CV would prefer no social security. The subsidy to high income workers is not large enough to gain their support, and the utility to low income workers is not large enough to gain their support, according to our model.

Willingness-to-Pay

The other measure of change in individual welfare used in this paper is marginal willingness-to-pay. In this analysis, we measure the willingness-to-pay of high and low income workers when faced with an offer for a marginal additional pension benefit of 1% of lifetime average wage. In the NSSE model prior to the offer there is no pension, so the 1% addition is added to 0, to produce a 1% pension benefit level. In the SSE-40 model with a flat 40% replacement rate, the extra 1% brings the pension total to 41%.

Willingness-to-pay is measured by the increase in tax which would make the agent indifferent to the offer of a higher pension (i.e., result in zero net change in the value function of a newborn in that economy).

Table 4. Willingness-to-pay for Marginal Increase in Public Pension

	NSSE	SSE-40
WTP: High Income	.214%	.087%
WTP: Low Income	.200%	.083%
Ratio to Hypothetical Fair Tax		
High Income	85.3%	34.7%
Low Income	102.5%	42.3%
Ratio to Required Tax Rate		
High Income	97.6%	39.6%
Low Income	91.2%	37.6%

There are a number of observations to be made from this table. First, the low income worker is willing to pay almost as much as the high income worker is willing to pay. Recall that the hypothetical fair tax rate of the low income worker is only 77.8% that of the high income worker. Thus, when willingness-to-pay of low income workers is compared to their hypothetical fair tax rate, we see they are willing to pay 102.5% of hypothetical fair value

(NSSE). High income workers are willing to pay 85.3% of hypothetical fair value (NSSE). Thus there is a small amount of margin for low income workers to pay more than fair value and still be indifferent, while for high income workers the subsidy must close the gap that exists between WTP and fair value. When we look at the ratio of WTP to the required aggregate tax rate, we see that the subsidy comes very close in the case of NSSE to closing the gap for high income workers (97.6%), but that the WTP of low income workers is only 91.2% of the required tax, leaving them unwilling to pay the tax. So the hypothesis that the subsidy of the high income worker and utility of insurance for the low income worker might result in consumer surplus from a paygo public pension is not supported, even at the margin in the NSSE case.

Another observation is the significant drop in willingness-to-pay between the first dollar of pension benefits (as seen in the NSSE model), and the $41st$ percentage level benefit in SSE-40. As noted above, for the first 1% average wage pension benefit, the WTP is 85.3%/102.5% of hypothetical fair value (high/low respectively). Yet for the $41st$ percentage level benefit , WTP drops to 34.7%/42.3% of hypothetical fair value. There is clearly a drop in marginal utility going on here. Also, the worker paying the tax faces a higher opportunity cost when interest rates are high, as they are in the SSE economy. It strongly implies that policy makers looking to modify the present Social Security system should not look to increasing replacement rates. It may also help to explain the relatively mild public response to benefit cuts that were made to Social Security in 1983 and 1996 when benefits that had previously not been subject to income tax were included into taxable income.

4.4.2 Bequests in the Utility Function

When the experimental setup is changed to include bequests are in utility, agents consume less and save more, increasing steady state capital, output and wages, and reducing interest rates, relative to the steady state under the specification without bequests in the utility. But with respect to the question of consumer surplus and social security, the differences between results with bequests in utility and results without bequests in utility are not dramatic. For sake of clarity, we will present results only from the CIU framework. Details of results from the CBIU framework are in Appendix A, and analysis of those results is in Appendix C.

4.4.3 Progressive (Kinked) Pension Benefit Formula

Next we compare results of social security economy with a progressive kinked benefit formula used by the Social Security Administration to the results of the SSE-40 model economy. Table 5 presents a steady state comparison.

Table 5. Key Indicators in Steady State

	SSE-40	SSE
Capital (K)	4.205	4.142
Output (Y)	1.456	1.448
Consumption	1.115	1.113
Capital Intensity (K/Y)	2.89	2.86
Wages (w)	1.162	1.156
Return on Capital (r)	5.56%	5.68%
Soc. Sec. Tax (τ_{SS})	8.77%	9.32%
Hypotheitical Fair Tax: High Inc.	10.04%	9.43%
Hypotheitical Fair Tax: Low Inc.	7.81%	9.23%

As one would expect, the progressive formula significantly reduces the extent of the reverse subsidy. Where the difference in hypothetical fair tax rates under SSE-40 is 2.23%, the difference with the kinked formula is only 0.20%. Replacement rates for low income workers are higher than in the SSE-40 model and conversely, replacement rates are lower for high income workers. Low income workers reduce precautionary saving even more, and high income workers increase savings over the SSE-40 level. On balance, the capital intensity of the SSE model steady state is slightly less than the SSE-40 model, leading to slightly lower wages, output, and consumption. There is also greater wealth inequality in

SSE than SSE-40, as high and low income workers adjust their savings.[3]

Compensating Variation in Consumption

Table 6 presents the outcome with respect to compensating variation.

Table 6. Compensating Variation Comparison

	SSE-40	SSE	Difference
CV: High Income	10.07%	10.99%	.92%
CV: Low Income	12.87%	13.69%	.82%

High income workers pay a greater tax and receive less benefits under SSE than under SSE-40, so it is not surprising that their compensating variation would be higher. Low income workers, with the kinked benefit formula, receive a bigger slice of this somewhat smaller pie. But they also end up with a slightly smaller average annual consumption than in the SSE-40 model (see Appendix A Table 15A), and a higher level CV.

A major difference between the SSE and SSE-40 models is that the reverse subsidy is greatly reduced in the SSE model. Yet that does not seem to affect the difference in compensating variation between high income and low income worker. At most, the increase in CV is slightly less for low income workers than for high income workers. Instead, the difference in CV remains, most likely due to the increased wealth inequality in the SSE model. This favors high income workers who are better able to sustain consumption from returns on capital, thus offsetting the reduction in the reverse subsidy.

Willingness-to-Pay

Willingness-to-pay under the kinked benefit formula experiment is compared in Table 7.

[3] Table 15A in Appendix A shows the increase in wealth inequality most clearly in the relative change in bequests between high and low income workers in the two experimental economies.

Table 7. Willingness-to-pay for Marginal Increase in Public Pension

	SSE-40	SSE
WTP: High Income	.087%	.084%
WTP: Low Income	.083%	.078%
Ratio to Hypothetical Fair Tax		
High Income	34.7%	33.4%
Low Income	42.3%	39.9%

For high income workers, there is a small decrease in WTP from the SSE-40 model. We might have expected a significant drop in WTP, since the high income benefit replacement rate drops to about 37%, while the tax rate increases from 8.77% to 9.32%. Yet with lower benefits and higher taxes, the high income WTP only drops from 34.7% of fair value to 33.4% of fair value, a drop of less than 4%. Higher marginal utility experienced at the lower benefit level offsets to some degree the 6.2% higher tax rate and 8% lower benefits. For low income workers, the kinked benefit formula results in a higher replacement rate (47%), but also higher taxes. At the higher level of benefits, marginal utility is lower, resulting in a decrease in WTP from 42.2% to 39.9% of the hypothetical fair tax.

So while a kinked benefit reduces the reverse subsidy, it does not reduce inequality nor does it increase consumer surplus.

4.4.4 Increase in Starting Age for the Public Pension

Consider next the effect of increasing the age at which pension benefits begin. Here we seek to take advantage of the known result (Abel, 1985) that it is the insurance mechanism of Social Security which generates individual welfare gains. We expect that when pension benefits start at an older age, the insurance mechanism will be more intensive, with a correspondingly greater effect on welfare and WTP.

Compensating Variation in Consumption

Table 8 presents the compensating variation in consumption for the SSE-40 model at alternative benefit starting ages.

Table 8. Compensating Variation by Pension Benefit Starting Age (SSE-40)

Age	65	70	75	80
CV: High Income	10.07%	7.64%	5.51%	3.68%
CV: Low Income	12.87%	10.18%	7.64%	5.34%
Difference in CV	2.80%	2.54%	2.13%	1.66%

As the benefit starting age increases, the levels of compensating variation decrease. This is not surprising, since as the benefit starting age increases, the model economy becomes more like a NSSE economy. The difference in compensating variation between high and low income workers decreases also. It may be due to the decrease in wealth inequality as low income workers increase their savings at higher benefit starting ages. This means that even low income workers can take better advantage of the higher interest rates that result in a SSE-40 economy, relative to a NSSE economy. Yet another explanation for the gap between high income and low income CV might be that high income workers value longevity insurance more highly than low income workers as the starting age increases, given their higher probability of living to an advanced age.

Willingness-to-Pay with Advanced Starting Ages

Table 9A presents willingness-to-pay results in a NSSE economy, and Table 9B presents willingness-to-pay in a SSE-40 economy, for alternative pension benefit starting ages.

Table 9A. WTP by Pension Benefit Starting Age (NSSE)

Pension Starting Age	65	70	75	80
WTP: High Income	.2140%	.1700%	.1324%	.1006%
WTP: Low Income	.2000%	.1565%	.1196%	.0883%
Ratio to Hypothetical Fair Tax				
High Income	85.3%	95.6%	113.4%	146.2%
Low Income	102.5%	121.2%	154.5%	218.0%
Ratio to Required SS Tax				
High Income	97.6%	113.3%	140.3%	190.5%
Low Income	91.2%	104.3%	126.7%	167.2%

In an economy without Social Security, WTP for the first 1% benefit level decreases with an increasing starting age, consistent with the decreasing value of a pension that has a later starting age. When related to the hypothetical fair tax however, we see an increasing willingness-to-pay. In other words, the utility of the insurance decreases more slowly than the risk-neutral cost of the hypothetical fair tax. WTP relative to fair tax for high income workers increases from 85.3% to 146.2% over the range of ages, an increase of 171%. For low income workers, the corresponding increase is from 102.5% to 218.0%, or an increase of 213%.

The creation of consumer surplus depends on the actual required tax, not the hypothetical fair tax. WTP relative to the required social security tax rate, also increases for both types of workers. For high income workers, the WTP relative to the required tax rate increases from 97.6% to 190.5%. The corresponding increase for low income workers is from 91.2% to 167.2%. Both types of workers are willing to pay more than the required tax when the benefits starting age is 70 or higher. At these ages, the first 1% level of benefits generates positive consumer surplus!

What happens when the base level of benefits is 40% as in the SSE-40 model? Table 9B presents the results.

Table 9B. WTP by Pension Benefit Starting Age (SSE-40)

Pension Starting Age	65	70	75	80
WTP: High Income	.0869%	.0728%	.0610%	.0495%
WTP: Low Income	.0825%	.0678%	.0545%	.0414%
Ratio to Hypothetical Fair Tax				
High Income	34.7%	41.0%	52.3%	71.9%
Low Income	42.3%	52.6%	70.4%	102.1%
Ratio to Required SS Tax				
High Income	39.6%	48.5%	64.6%	93.8%
Low Income	37.6%	45.1%	57.7%	78.4%

WTP drops steadily from the first 1% of average lifetime wage pension to the $41st$% of average lifetime wage pension. At starting age 80, where WTP is greatest, a high income worker has a WTP of .1006% for the first 1% level benefit (Table 9A), but a WTP of only .0495% for the forty-first 1% level benefit (Table 9B). Low income workers have a similar outcome. In no case does WTP exceed the required SS tax rate. Diminshing marginal utility of higher pension benefits means that workers would not support an increase in benefits and taxes, even at advanced starting ages.

Since workers are willing to pay more than the tax for the first 1% pension, but not at a 41% level pension, what is the level of benefits at which willingness to pay first falls below the required tax? Table 10 presents the result.

Table 10. Optimal Pension Level by Starting Age (SSE-40)

Pension Starting Age		65	70	75	80
Optimal Benefit Level:	High Income	0	6%	16%	37%
Optimal Benefit Level:	Low Income	0	1%	9%	25%

Raising the benefit level above these amounts results in the destruction of consumer surplus. If not for the reverse subsidy, low income workers would have a higher optimal benefit level and high income workers a lower optimal benefit level. But clearly, increasing the pension start age is one path to greater consumer surplus (or less destruction of individual welfare).

4.4.5 Alternative Risk Aversion Parameters

Perhaps a more risk averse worker (i.e. a lower elasticity of intertemporal substitution) would be willing to pay more than the required tax for social security? Once again the answer is maybe for the first 1% level of benefits, but not for the forty-first 1% level of benefits. There is a further interesting angle to this problem. At low levels of capital intensity (such as in the SSE-40 steady state), a higher elasticity of intertemporal substitution (i.e., a lower level of risk aversion) results in greater WTP than at lower elasticities. But at high levels of capital intensity (such as in the NSSE steady state), a higher elasticity of intertemporal substitution (lower level of risk aversion) results in diminished WTP than at lower elasticities. The two period model showed that this complex interaction is traceable to returns on survivorship relative to returns on capital. In the SSE-40 case, there is a high return on capital, relative to the return on survivorship from Social Security. As elasticity is increased, savings increase, reducing interest rates, and raises WTP (the alternative to saving). In the NSSE case, interest rates are low. Increased elasticity leads to reduced savings, which raises interest rates and lowers WTP.

The numerical results and analysis of the overlapping generation model, which involve the balancing of returns from investment against returns from survivorship, are shown in Appendix C.

4.4.6 Welfare Under Temptation

Research by Imrohoroğlu, Imrohoroğlu, and Joines (2003), Kumru and Thanopoulos (2007), and Boronow (2007) all present evidence that Social Security is a weak commitment device. But is even a weak commitment device enough to raise welfare so that Social Security creates consumer surplus? To evaluate that hypothesis, we return to the model of Temptation in the earlier paper by Boronow (2007), to compare welfare outcomes for the various model economies.

Under standard preferences in which agents know and act according to their best interests, there is no need for a commitment device. However, if agents are subject to temptation preferences, they make choices which are not in their best long run interests. We use the model of choices under temptation used by Gul and Pesendorfer (2001) and Krusell, Kuruscu, and Smith (2005) and Boronow (2007). For a more complete description of the temptation model used here and in Boronow (2007), see the earlier paper. A brief description of the temptation model is in Appendix B. As in the 2007 paper, results are presented for three different specifications of temptation (labeled weak, moderate and strong), corresponding to increasing the temptation stength parameter.

Compensating Variation under Temptation Preferences

The model is computed for the case where utility is from consumption (CIU). [4]

[4] The algorithm to compute results for the temptation model with CBIU is available from the author, but results were not produced in view of the modest difference between CIU outcomes and CBIU outcomes observed with standard preferences.

Table 11. Compensating Variation under Temptation

	High Income	Low Income	Difference
SSE-40 @ Age 65			
Standard Preferences	9.59%	12.77%	3.18%
Weak Temptation	10.71	12.72	2.01
Moderate Temptation	10.14	11.88	1.74
Strong Temptation	9.61	11.28	1.67
SSE-40 @ Age 80			
Standard Preferences	3.41%	5.23%	1.82%
Weak Temptation	4.22	5.06	.84
Moderate Temptation	3.49	4.34	.85
Strong Temptation	3.18	4.05	.87
HRE			
Standard Preferences	4.07%	5.84%	1.77%
Weak Temptation	1.94	1.08	−0.86
Moderate Temptation	−4.66	−6.31	−1.65
Strong Temptation	−10.21	−12.06	−1.85

In the SSE-40@65 case, the subsidy from low income to high income workers, and the high interest rates in the SSE-40 economy (relative to the NSSE economy) both benefit high income workers. Hence the CV for low income workers is greater than the CV for high income workers. But we notice that the difference in CV between low income workers and high income workers gets smaller as the temptation setup gets stronger. The difference is 3.18% under standard preferences but only 1.67% under the strongest specification of temptation. This indicates that the impact of temptation on high income workers is relatively greater than the impact on low income workers, perhaps because only high income workers have meaningful savings. (Outcomes under temptation are shown in Appendix A.)

Under the SSE-40@85 model, both types of workers save for the early retirement years. Therefore, both types of workers should be affected by temptation. We observe that the difference in CV between high income and low income workers is flat, or even slightly increasing in strength of temptation. This suggests that both types of workers are impacted by temptation in approximately the same degree.

Another notable feature of SSE-40@65 and SSE-40@85 models in Table 11 is that compensating variations decrease as the strength of temptation increases. This is evidence of the commitment value in social security. As temptation is stronger, the agent values the freedom of the NSSE economy less highly. This is true for both high and low income workers.

There is a different situation for the HRE model, however. For both types of agents, their CV is positive under standard preferences, indicating the agents prefer the NSSE model to the HRE model. But under temptation, the CV is smaller (weak specification) and eventually is negative (moderate and strong specifications), indicating that agents prefer the HRE model to the NSSE model, when strong enough temptation is present. This is evidence that the PSA is an effective commitment device, preserving savings, capital, wages and welfare under temptation. (Recall that the only difference between the HRE model and the SSE-40@85 model is that the HRE mandates the accumulation and depletion of a PSA device, whereas the SSE-40@85 leaves the saving/consumption decision to individual utility optimization.)

Not only is the HRE model preferred to the NSSE model, but the usual pattern where the CV of low income workers is greater than high income workers is reversed under temptation. High income workers, having more voluntary savings, are more adversely affected by temptation than low income workers. With the HRE model, low income workers benefit from interest returns on their mandatory PSA, while high income workers reduce voluntary savings due to temptation. CVs under temptation indicate low income workers are relatively happier with HRE than are high income workers.

Willingness-to-Pay under Temptation Preferences

Does this mean that willingness to pay is also higher under HRE than under NSSE or SSE economies, in the framework of temptation preferences? Table 12 presents results about willingness to pay as a percentage of the actuarially required tax.

Table 12. WTP and Temptation Preferences

(as % of the required tax)

	NSSE	SSE-40@65	SSE-40@80	HRE
High Income Worker				
Standard Preferences	105.3%	39.1%	93.8%	98.9%
Weak Temptation	87.1	36.5	88.8	113.5
Moderate Temptation	76.3	33.7	78.9	118.9
Strong Temptation	68.1	31.7	72.6	130.2
Low Income Worker				
Standard Preferences	87.0%	37.9%	78.4%	78.3%
Weak Temptation	79.2	35.7	73.5	69.5
Moderate Temptation	69.0	33.0	68.3	70.9
Strong Temptation	60.4	30.9	62.9	76.4

In the NSSE model under standard preferences, the usual result is obtained, that there is a high willingness to pay for the first marginal dollar of pension. If the commitment device embedded in a paygo public pension had a high utility value, we would expect to see WTP increase with the strength parameter of temptation. But when temptation is introduced in the setup, WTP decreases as temptation strength is increased. So the commitment device of a paygo pension under temptation is not effective, even at the point of the first marginal 1% level of benefits. In fact, temptation makes the popularity of social security even harder to explain. In the results for the SSE-40@65 model, the deteriorating effect of temptation reduce WTP to about 31% of the actuarially required tax , from about 38% without temptation. Starting social security benefits at an advanced age increases WTP, but the pattern under temptation is still the same, as seen in the SSE-40@85 model results. When benefits start at age 80, WTP for high income workers is 93.8% under standard preferences, dropping to 72.8% under temptation preferences. For low income workers the drop-off in WTP is 78.4% to 62.9%. Clearly, temptation is not increasing the effecacy of social security to improve individual welfare.

But under HRE, the picture is different. Under standard preferences, WTP is about the same as the SSE-40@80 model. This makes sense, since the mandatory savings under HRE is approximately the same as the voluntary savings that would take place under the SSE-40@80 model in the absence of temptation. But when temptation is included in the setup, we find that WTP increases as temptation strength increases. High income workers, who are subsidized by low income workers, are willing to pay more than the tax rate. Low income workers are willing to pay more than their fair share, but less than the required tax rate which includes the subsidy. Even so, they are willing to pay 70% or more of the tax rate, getting close to the situation where both types of agents are willing to pay the required tax. Perhaps both types of agents would be willing to pay the required tax, if the HRE economy were using a kinked benefit formula, which would reduce the subsidy.

Willingness to Pay under Temptation with Kinked Benefit Formula Consider an experiment where the HRE public pension has a kinked benefit formula which matches the one used by the Social Security Administration. The tax to pay for the public pension has to increase from 2.11% (for a flat 40% benefit) to 2.21% (for the kinked formula). In this experiment, we retain the mandatory PSA contribution rate at the same 6.66% level that was used in the HRE experiments based on a flat 40% benefit.[5] Table 13 presents the results as it relates to willingness-to-pay.

[5] A detailed description of the HRE design is in Boronow (2007). The combined tax for the public pension and the PSA account in that paper is constrained to 8.77%, the same level of tax in the baseline Social Security design. This produces a 2.11% tax for the paygo pension and a mandatory 6.60% PSA contribution rate.

Table 13. WTP in an HRE Model with a Kinked Benefit Formula

(as % of the required tax)

	HRE (40%)	HRE (kinked)
High Income Worker		
Standard Preferences	98.9%	97.5%
Weak Temptation	113.5	116.3%
Strong Temptation	130.2	134.1%
Low Income Worker		
Standard Preferences	78.3%	73.9%
Weak Temptation	69.5	62.3%
Strong Temptation	76.4	66.9%

A kinked benefit formula does not cause both types of workers to be willing to pay more than the required tax. High income workers are not much different with respect to Willingness to pay, but low income workers are willing to pay even less than under a flat 40% benefit formula. Under the kinked benefit formula, high income workers receive a lower level of benefits, so their marginal willingness to pay is actually higher at the lower level of benefits, moving up the diminishing marginal utility curve. This effect offsets lost utility from eliminating much of the subsidy from low income workers. For low income workers, the opposite is true. The kinked formula removes much of the subsidy, improving willingness to pay, but the higher level of benefits for low income workers under a kinked formula moves them further down the diminishing marginal utility curve. The net result is a WTP that is even lower than the marginal WTP under a flat benefit formula, just as we saw in Table 7 for the SSE model. As seen in Table 13, the temptation framework does not make a meaningful difference when comparing results under a flat benefit formula or a kinked benefit formula.

Nonetheless, the direction for reform that will increase individual welfare seems to lie in starting benefits at a more advanced age, and including an effective commitment device to sustain capital formation. As a final experiment, we consider the effect on willingness to pay when the contribution rate to the PSA account is increased by one percentage point. Will an increase in contribution rate to the effective commitment device of a mandatory PSA result in a higher willingness to pay the tax for the public pension? Table 14 presents the results, for the case under "strong" temptation.

Table 14. WTP by PSA Contribution Rates

(as % of the required tax)

Mandatory PSA Contribution Rate	6.60%	7.60%
High Income Worker		
Strong Temptation	130%	134%
Low Income Worker		
Strong Temptation	76%	80%

When the PSA contribution rate is increased from 6.6% to 7.6%, there is a slight increase in the willingness to pay for the tax which provides the advanced old age pension, under the framework of "strong" temptation. But the effect is not strong enough to raise willingness to pay to the point where both tyes of workers are willing to pay more than the tax required to provide the pension. That elusive outcome still requires a reduction in the level of pension benefits.

4.5 Conclusions

When income is correlated with longevity, then low income workers will have a shorter expected lifetime than high income workers. Yet they pay the same tax rate as high income workers under Social Security. Thus there is an inherent disadvantage for low income workers (partially remediated in the pension benefit formula). We also know that concave utility functions assign greater utility from insurance to low income workers than to high income workers. Could a subsidy which appeals to high income workers and insurance which appeals to low income workers offer an explanation as to why Social Security remains a popular program at all income levels? We examined this hypothesis under a progressive benefit formula (to reduce the subsidy), under alternative ages at which pensions begin (to increase the intensity of the insurance element), and under alternative parameters for risk aversion (which changes the concavity of the utility function). Finally, we examined the hypothesis under a setup in which agents are subject to temptation preferences. We obtained tantalizing hints of happy voters in an economy with no Social Security faced with the introduction of marginal pension benefits. But, in almost every case the effects of diminishing marginal utility of increased paygo pension benefits reduced overall individual welfare to levels well below what would explain continued support for Social Security.

We did find that certain changes to the present system could lead to positive consumer surplus. Increasing the age at which the public pension starts, and reducing the level of the pension benefit could produce a positive consumer surplus. Also, the Hybrid Reform proposal produced increasing welfare under temptation, as measured by the compensating variation in consumption. For temptation that is sufficiently strong, agents prefer the Hybrid Reform social insurance mechanism to the economy with no social security. Also, the Hybrid Reform produced the highest ratio of the marginal willingness-to-pay to the required tax. Under temptation, high income agents were willing to pay more than the required tax, and low income workers were willing to pay most of the required tax. So, while we were not able to find a welfare explanation for the popularity of social security, we did find support for the argument that if agents are indeed subject to temptation preferences, then the HRE commitment device would enhance individual welfare above the level with no social security or with the present social security.

It is hardly necessary to remind ourselves that these results are only indications of outcomes based on a simplistic model of agents behavior. Key assumptions, such as the strength of temptation preferences are not based on econometric analysis. The findings of this paper are only qualitative findings. Nonetheless, we can see directional indicators for possible future reforms of the social security system.

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Appendix

I. A Hybrid Reform: Social Security on Dual Power in the Presence of Temptation

Appendix A: Temptation Preferences

This paper analyzes the three model economies using temptation preferences, as presented by Gul and Pesendorfer in a series of articles in 2001, 2002, and 2004. The axiomatic development of a utility function that represents temptation preferences is presented in Gul and Pesendorfer (2001). Below is a summary of the intuition of temptation preferences.

The setting consists of an individual representative agent who will choose today a set of alternatives B, among which she will choose a consumption lottery next period. Assuming that the agent is an expected utility maximizer, the next period the agent chooses lottery $p \in B$ which solves $\max_{p\in B} \int u(p)dp$. At time 0, our agent prefers the set of alternatives B to the set of alternatives B' when $\max_{p\in B} \int u(p)dp \geq \max_{p\in B'} \int u(p)dp$. In the theory of expected utility, a standard axiom is that $B \succsim B' \Rightarrow B \sim B \cup B'$ (Kreps (1979)). This axiom rules out situations where the agent may benefit from or be harmed by the addition of alternatives in B'. The idea of temptation preferences is that the agent may strictly prefer B to $B \cup B'$, knowing that B' contains temptations to which she will be vulnerable in time 1. Thus, in the Gul and Pesendorfer development of temptation preferences, the standard axiom is relaxed to the 'set betweenness' axiom:

$$B \succsim B' \Rightarrow B \succsim B \cup B' \succsim B'$$

With the 'set betweenness' axiom, if $B \succsim B \cup B'$ it is possible for the agent at time 0 to strictly prefer the smaller set B of time 1 choices, to the larger set $B \cup B'$ which contains tempting choices that the agent would rather not face at time 1. The idea here is that the presence of temptations at time 1 are the reason for the preference for commitment to a choice set at time 0. To avoid the temptation, at time 0 our agent strictly prefers the set of alternatives B. In fact, this is the criteria by which a temptation is identified.

As Gul and Pesendorfer (2001) show, the set betweenness axiom together with the other axioms that otherwise lead to standard expected utility, lead instead to the utility representation of temptation preferences. The utility representation is as follows. Let $u(p)$ and $v(p)$ be von Neumann-Morgenstern expected utility functions. Intuitively, u is the utility function that is not affected by temptation, and v is the utility function that is affected by temptation. Then temptation preferences are represented by the function:

$$U(B) = \max_{p \in B} \int (u(p) + v(p))dp - \max_{\widetilde{p}' \in B} \int v(\widetilde{p})d\widetilde{p}$$

Note that when the agent has only one choice, then the function u is the agent's utility; the v terms net to zero. Thus Gul and Pesendorfer refer to $u(p)$ as 'the commitment utility', since it represents utility when one is committed to p (i.e., $B = \{p\}$). It is worth noting here that an agent with perfect foresight would also have utility u, and the standard assumption $B \sim B \cup B'$ would apply. An agent with perfect foresight is not vulnerable to temptation; for such a decision-maker $v = u$ and therefore the utility formulation above collapses to the usual expected utility function. Thus other authors refer to u as 'normative' utility, which is the terminology used in this paper.

The utility function v represents the temptation utility. In the model presented by Gul and Pesendorfer (2002), the agent maximizes $u + v$, arriving at the optimal compromise, with the expenditure of self-control with disutility

$$v(c) - \max_{\widetilde{p} \in B} \int v(\widetilde{p})d\widetilde{p}$$

where c is $\arg\max_{p \in B} \int (u(p) + v(p))dp$.

As Gul and Pesendorfer (2002) point out, temptation is closely related to the literature on time-inconsistent preferences. Time-inconsistent preferences lead to a solution strategy in which multiple selves are players in a game to find an equilibrium which is consistent. Krusell, Kuruşçu and Smith (2005) show that the Phelps-Pollack form of time-inconsistent preferences are equivalent to temptation preferences under a setup where the strength of the temptation goes to infinity.

An important feature of temptation preferences, and the reason they are useful in this analysis of Social Security, is that unlike the Phelps-Pollack formulation of time inconsistent preferences, temptation preferences are amenable to a dynamic setting. In the finite horizon case, the setting is that an agent chooses a consumption lottery from a set of alternatives B_t, and chooses a decision problem set B_{t+1} for the next period. Formally,

$$W_{t-1}(B_t) = \max_{p \in B_t} \int [u_t(p) + W_t(p, B_{t+1}) + V_t(p, B_{t+1})]dp - \max_{\widetilde{p} \in B_t} \int V_t(\widetilde{p}, B_{t+1})d\widetilde{p}$$

In this representation, W_{t-1} represents the value function of the agent over the period t choices; prior to period t, the agent is committed to choose from among the choices in B_t. The normative (commitment) utility is $u_t + W_t$, and the temptation utility is V_t. In the final period (period T), the decision problem set is a singleton set, typically $\{0\}$ or the empty set (i.e., $B_{T+1} = \{0\}$).

$$W_{T-1}(B_T) = \max_{p \in B_T} \int [u_T(p) + 0 + V_T(p)]dp - \max_{\widetilde{p} \in B_T} \int V_T(\widetilde{p})d\widetilde{p}$$

Following the approach of Krusell, Kuruşçu and Smith (2005), this paper uses a finite horizon, discrete time dynamic setting for temptation preferences.

Appendix B. Method to Split Mortality Rates by Type

The rates are split as follows. Low ability workers are assumed to have mortality rates that are a constant multiple of the aggregate mortality rate. Let x denote this constant multiple ($x > 1$). Let $\bar{\psi}_j$ denote the aggregate survival rate at age j, based on the Social Security Administration table. Then $(1 - \psi_j(2)) = x(1 - \bar{\psi}_j)$. We then solve for $\psi_j(1)$ so that the aggregate rate $\bar{\psi}(j)$ equals the weighted average of $\psi_j(1)$ and $\psi_j(2)$. The weights used are the age dependent proportion of the cohort by type. Since the mortality rates differ there is a gradually increasing proportion of higher ability agents as the cohort ages. We develop $\psi_j(1)$ recursively, starting with $j = 1$:

For $j = 1$:

$$
\begin{aligned}
1 - \psi_1(2) &= x(1 - \bar{\psi}_1) \\
1 - \psi_1(1) &= \frac{1 - \bar{\psi}_1 - p_2 \cdot [1 - \psi_1(2)]}{p_1}
\end{aligned}
$$

where

$$
\begin{aligned}
p_1 &= \frac{\lambda(1)}{\lambda(1) + \lambda(2)} \\
p_2 &= \frac{\lambda(2)}{\lambda(1) + \lambda(2)}
\end{aligned}
$$

For $j = 2$:

$$
\begin{aligned}
1 - \psi_2(2) &= x(1 - \bar{\psi}_2) \\
1 - \psi_2(1) &= \frac{1 - \bar{\psi}_2 - p_2 \cdot [1 - \psi_2(2)]}{p_1}
\end{aligned}
$$

where

$$
\begin{aligned}
p_1 &= \frac{\lambda(1) \cdot \Psi_{1,j-1}(1)}{\lambda(1) \cdot \Psi_{1,j-1}(1) + \lambda(2) \cdot \Psi_{1,j-1}(2)} \\
p_2 &= \frac{\lambda(2) \cdot \Psi_{1,j-1}(2)}{\lambda(1) \cdot \Psi_{1,j-1}(1) + \lambda(2) \cdot \Psi_{1,j-1}(2)}
\end{aligned}
$$

For a general j until $j = J = 85$, where $\psi_J(z) = 0$:

$$
\begin{aligned}
1 - \psi_j(2) &= x(1 - \bar{\psi}_j) \\
1 - \psi_j(1) &= \frac{1 - \bar{\psi}_j - p_2 \cdot [1 - \psi_j(2)]}{p_1}
\end{aligned}
$$

where

$$
\begin{aligned}
p_1 &= \frac{\lambda(1) \cdot \Psi_{1,j-1}(1)}{\lambda(1) \cdot \Psi_{1,j-1}(1) + \lambda(2) \cdot \Psi_{1,j-1}(2)} \\
p_2 &= \frac{\lambda(2) \cdot \Psi_{1,j-1}(2)}{\lambda(1) \cdot \Psi_{1,j-1}(1) + \lambda(2) \cdot \Psi_{1,j-1}(2)}
\end{aligned}
$$

With this method of splitting the aggregate mortality rate, x is chosen so that the expected lifetime of high ability workers (college graduates represent high ability workers) is five years longer than the expected lifetime of low ability workers. This same five year differential was used by Fuster, Imrohoroğlu and Imrohorglu (2002), based on research by Elo and Preston (1996), who found a five year differential in life expectancies between college graduates and non-graduates. It can be found that $x = 1.126$ produces the desired outcome. The model results in a life expectancy of the high ability group that corresponds to real expected life of 76.1 years, and for the low ability group a life expectancy of 71.1.

Appendix C. Method to Split the Efficiency Factors by Type

First, based on data from the Bureau of the Census and the Bureau of Labor Statistics, the model assumes that the high ability group (college graduates) earns 150% of the aggregate wage. Thus, if $\bar{\varepsilon}_j$ denotes the aggregate labor efficiency factor at age j, then $\varepsilon_j(1) = 1.5 \cdot \bar{\varepsilon}_j$ for $j < j^*$. Efficiency factors for low ability workers are then determined so that the weighted average of the two types match the aggregate factor at each age. The weights p_1 and p_2 (i.e. the proportion of the surviving cohort at a given age j that is type 1 and 2 , respectively), which are used to reproduce the average efficiency factor, are the same weights used earlier when splitting the aggregate mortality rates. For $j = 1, 2, ...j^* - 1$:

$$
\begin{aligned}
\varepsilon_j(1) &= 1.5 \cdot \bar{\varepsilon}_j \\
\varepsilon_j(2) &= \frac{\bar{\varepsilon}_j - p_1 \cdot \varepsilon_j(1)}{p_2}
\end{aligned}
$$

where

$$
\begin{aligned}
p_1 &= \frac{\lambda(1) \cdot \Psi_{1,j-1}(1)}{\lambda(1) \cdot \Psi_{1,j-1}(1) + \lambda(2) \cdot \Psi_{1,j-1}(2)} \\
p_2 &= \frac{\lambda(2) \cdot \Psi_{1,j-1}(2)}{\lambda(1) \cdot \Psi_{1,j-1}(1) + \lambda(2) \cdot \Psi_{1,j-1}(2)}
\end{aligned}
$$

Appendix D. Solution Method

The agent is indexed at the start of each period according to age, asset holdings, and type, (j, a, z). Age is naturally discretized by $j \in \{1, 2, ...J\}$. Type is either high ability ($z = 1$) or low ability $(z = 2)$. It remains to discretize the state space for asset holdings. Let $D = \{d_1, d_2, ..., d_m\}$ represent the discrete set of values that asset holdings are permitted to take. With the calibration described here, a reasonable corresponding grid of values for possible asset holdings is on the interval $[0, 32]$. To get a sufficient smoothness without adding computational time, the model uses a greater concentration of grid points on the interval $[0, 12]$. This analysis uses 6001 uniform grid points for asset holdings on the interval $[0, 12]$, and 1400 uniform grid points for the interval $(12, 32]$.

The feasible set for an age j agent of type z and asset holdings a is denoted by $\Omega(j, a, z)$. The possible choices, a in the experiments using standard preferences, and a and $\tilde{a}$ for temptation preferences, satisfy $a' \in \Omega(j, a, z)$, $\tilde{a}' \in \Omega(j, a, z)$, $a' \geq 0$, $\tilde{a}' \geq 0$, and the budget constraints are satisfied.

The details of the backward induction solution with Temptation preferences and a CRRA felicity function follow.

The Last Period. Starting with the last period, the problem is greatly simplified by the assumption that there is no utility from bequests. The optimal decision for both normative and temptation utility is therefore to consume everything. The value function is therefore:

$$\begin{aligned} W_J(a) &= u(c_J) \\ \text{where } c_J &= (a + \xi_J)(1 + r) + Q_J \end{aligned}$$

Anticipating the soon-to-be-needed derivative, we note that $\frac{\partial}{\partial a}[W_J(a)] = u'(c_T)(1 + r)$.

The Next to Last Period. In the next to last period, the value function is:

$$\begin{aligned} W_{J-1}(a) &= \max_{c,a}\{(1+\sigma)u(c_{J-1})+(1+\sigma\varphi)\beta\psi_{J-1}(z)W_J(a')\} \\ &\quad -\max_{\widetilde{c},\widetilde{a}'}\sigma\{u(\widetilde{c}_{J-1})+\varphi\beta\psi_{J-1}(z)W_J(\widetilde{a}')\} \\ \text{where } c_{J-1} &= (a+\xi_{J-1})(1+r)+Q_{J-1}-a' \\ \widetilde{c}_{J-1} &= (a+\xi_{J-1})(1+r)+Q_{J-1}-\widetilde{a}' \end{aligned}$$

There are two first order conditions:

$$(1+\sigma)u'(c_{J-1})=(1+\sigma\varphi)\beta\psi_{J-1}\frac{\partial}{\partial a'}[W_J(a')]$$

$$u'(\widetilde{c}_{J-1})=\varphi\beta\psi_{J-1}\frac{\partial}{\partial\widetilde{a}'}[W_J(\widetilde{a}')]$$

Substituting for the derivative of the last period value function, we get the following Euler equations:

$$(1+\sigma)u'(c_{J-1})=(1+\sigma\varphi)\beta\psi_{J-1}u'(c_J)(1+r) \quad \text{(Euler 1)}$$

$$u'(\widetilde{c}_{J-1})=\varphi\beta\psi_{J-1}u'(\widetilde{c}_J)(1+r) \quad \text{(Euler 2)}$$

In Euler equation 1, c_J is a function of a_J. But $a_J = a' = g(a_{J-1})$, so then c_J is a function of a'. Note that c_{J-1} is also a function of a'. So the Euler equation itself is an implicit function of a'. Likewise, in Euler equation 2, $\widetilde{c}_J$ is a function of $\widetilde{a}' = \widetilde{g}(a_{J-1})$, and $\widetilde{c}_{J-1}$ is also a function of $\widetilde{a}'$. So Euler equation two is an implicit function of $\widetilde{a}'$. Thanks to the mathematically tractable nature of the CRRA form of the felicity function, we can solve these equations for a'. Using the CRRA form for $u(c)$, Euler equation 1 becomes:

$$(1+\sigma)\cdot(c_{J-1})^{-\gamma}=(1+\sigma\varphi)\beta\psi_{J-1}\cdot(c_J)^{-\gamma}\cdot(1+r)$$

Substituting for c_{J-1} and for c_J we get:

$$\begin{aligned} (a_{J-1}+\xi_{J-1})(1+r)+Q_{J-1}-a' &= [\frac{(1+\sigma\varphi)}{(1+\sigma)}\beta\psi_{J-1}(1+r)]^{\frac{-1}{\gamma}}\cdot ... \\ &\quad ((a'+\xi_J)(1+r)+Q_J) \end{aligned}$$

Likewise for Euler equation 2:

$$(a_{J-1}+\xi_{J-1})(1+r)+Q_{J-1}-\widetilde{a}' = [\sigma\varphi\beta\psi_{J-1}(1+r)]^{\frac{-1}{\gamma}}\ldots$$
$$\cdot((\widetilde{a}'+\xi_J)(1+r)+Q_J)$$

Given the state variable a, the two Euler equations can be solved for a' and $\widetilde{a}'$, respectively. (Note that Q is not dependent on a.) We thus get solutions for $g(a)=a'$, and for $\widetilde{g}(a)=\widetilde{a}'$. Again anticipating the need for the derivative of the value function, we get:

$$\begin{aligned}
\frac{\partial}{\partial a}[W_{J-1}(a)] &= (1+\sigma)\cdot u'(c_{J-1})(1+r)-\sigma u'(\widetilde{c}_{J-1})(1+r)\\
\text{where } c_{J-1} &= (a+\xi)(1+r)+Q_{J-1}-g(a)\\
\widetilde{c}_{J-1} &= (a+\xi)(1+r)+Q_{J-1}-\widetilde{g}(a)
\end{aligned}$$

The Next to Next to Last Period. Moving backwards to the next to next to last period, the value function is:

$$\begin{aligned}
W_{J-2}(a) &= \max_{c,a}\{(1+\sigma)u(c_{J-2})+(1+\sigma\varphi)\beta\psi_{J-2}W_{J-1}(a')\}\ldots\\
&\quad -\max_{\widetilde{c},\widetilde{a}'}\sigma\{u(\widetilde{c}_{J-2})+\varphi\beta\psi_{J-2}W_{J-1}(\widetilde{a}')\}
\end{aligned}$$

$$\begin{aligned}
\text{where } c_{J-2} &= (a+\xi)(1+r)+Q_{J-2}-a'\\
\widetilde{c}_{J-2} &= (a+\xi)(1+r)+Q_{J-2}-\widetilde{a}'
\end{aligned}$$

There are two first order conditions. The first Euler equation is:

$$(1+\sigma)u'(c_{J-2}) = (1+\sigma\varphi)\beta\psi_{J-2}\frac{\partial}{\partial a'}[W_{J-1}(a')] \Rightarrow \qquad \text{(Euler 1)}$$

$$\begin{aligned}
(1+\sigma)u'(c_{J-2}) &= (1+\sigma\varphi)\beta\psi_{J-2}\cdot(1+\sigma)\cdot u'(c_{J-1})(1+r)-\sigma u'(\widetilde{c}_{J-1})(1+r)\\
\text{where } c_{J-1} &= (a'+\xi)(1+r)+Q_{J-1}-g(a')\\
\widetilde{c}_{J-1} &= (a'+\xi)(1+r)+Q_{J-1}-\widetilde{g}(a')
\end{aligned}$$

Euler equation 2 is:

$$\begin{aligned} u'(\widetilde{c}_{J-2}) &= \varphi\beta\psi_{J-2}\frac{\partial}{\partial\widetilde{a}'}[W_{J-1}(\widetilde{a}')] \Rightarrow \qquad \text{(Euler 2)} \\ u'(\widetilde{c}_{J-2}) &= \varphi\beta\psi_{J-2}\cdot(1+\sigma)\cdot u'(\widehat{c}_{J-1})(1+r) - \sigma u'(\widetilde{\widehat{c}}_{J-1})(1+r) \\ \text{where } \widehat{c}_{J-1} &= (\widetilde{a}'+\xi)(1+r)+Q_{J-1}-g(\widetilde{a}') \\ \widetilde{\widehat{c}}_{J-1} &= (\widetilde{a}'+\xi)(1+r)+Q_{J-1}-\widetilde{g}(\widetilde{a}') \end{aligned}$$

As we did in the next to last period, we substitute for the derivative of the next period value function, and use the CRRA form of the felicity to arrive at a solvable set of Euler equations.

$$\begin{aligned} (a_{J-2}+\xi_{J-2})(1+r)+Q_{J-2}-a' &= [\frac{(1+\sigma\varphi)}{(1+\sigma)}\beta\psi_{J-2}(1+r)]^{\frac{-1}{\gamma}}... \\ &\quad \cdot((a'+\xi_{J-1})(1+r)+Q_{J-1}-g(a')) \end{aligned}$$

$$\begin{aligned} (a_{J-2}+\xi_{J-2})(1+r)+Q_{J-2}-\widetilde{a}' &= [\sigma\varphi\beta\psi_{J-2}(1+r)]^{\frac{-1}{\gamma}}... \\ &\quad \cdot((\widetilde{a}'+\xi_{J-1})(1+r)+Q_{J-1})-\widetilde{g}(\widetilde{a}') \end{aligned}$$

Exactly as before, Euler equation 1 is an implicit function of a', . Likewise, Euler equation two is an implicit function of $\widetilde{a}'$. Again, thanks to the mathematically tractable nature of the CRRA form of the felicity function, we can solve these equations for a' and $\widetilde{a}'$.

The problem is solved by continuing to work backwards, solving the Euler equations for the decision rules at each period in the same manner as above. Since our model is a discrete model, it chooses the value $d_i \in D$ closest to the analytical values for a' and $\widetilde{a}'$.

Given the initial guess as to the level of aggregate capita, K,and aggregate bequests, B, in the model economy, the model solves the individual's dynamic problem using the backwards induction algorithm. The optimal decision rules for consumption and saving result in a particular computed value for aggregate capital and aggregate bequests in the economy at the end of the period. If the ending values for K' and B', matches the starting values, then the economy is in a steady state. If not, the solution algorithm iterates with updated starting values for K and B. With standard preferences, we are assured of a steady state solution, since the update algorithm is constructed so that Blackwell's sufficiency conditions are satisfied. While the Temptation preferences do not satisfy Blackwell's sufficiency conditions, we proceed carefully along this algorithm and reach a steady state nonetheless.

Appendix E. Sensitivity to Temptation Parameters

Table E1. Key Indicators under Temptation (σ, φ)

	St. Pref.	$(.1, .9)$	$(.2, .9)$	$(.3, .9)$	$(.1, .7)$	$(.2, .7)$	$(.3, .7)$
Capital							
NSSE	5.720	4.9764	4.4323	4.0593	3.8296	2.8134	2.235
SSE	4.204	3.7214	3.3731	3.1338	2.9801	2.3138	1.898
HRE	5.091	4.5713	4.2674	4.1331	4.0959	4.0101	3.996
Welfare							
NSSE	−30.25	−31.05	−31.82	−32.48	−32.96	−35.71	−38.15
SSE	−33.96	−34.95	−35.82	−36.52	−37.01	−39.81	−42.31
HRE	−31.92	−32.68	−33.14	−33.30	−33.40	−33.63	−33.73
Comp. Var.	High in.						
NSSE	0	0	0	0	0	0	0
SSE %	10.0	10.30	10.7	10.79	11.2	10.3	9.78
HRE %	4.28	4.15	3.8	3.0	1.78	−5.02	−10.61
Comp. Var.	Low in.						
NSSE	0	0	0	0	0	0	0
SSE %	12.8	13.1	13.0	12.82	12.5	11.76	11.15
HRE %	5.8	5.51	4.22	2.43	1.26	−5.98	−11.82
Consumption							
NSSE	1.163	1.143	1.124	1.108	1.097	1.031	.9785
SSE	1.115	1.091	1.071	1.055	1.044	.9867	.9396
HRE	1.147	1.130	1.117	1.112	1.110	1.106	1.105

Table E1 provides some further results under a range of alternative settings for the temptation parameters. They are presented here to illustrate the sensitivity of results to the strength parameter (σ) and future discount parameter (φ) values.

II. Social Security and Consumer Surplus

Appendix A: Steady State Values

Table 15A. Key Indicators ($\gamma = 2$)

		NSSE	SSE-40	SSE	NSSE	SSE-40	SSE
Preferences		CIU	CIU	CIU	CBIU	CBIU	CBIU
Capital	K	5.757	4.205	4.142	6.078	4.465	4.407
Output	Y	1.630	1.456	1.448	1.662	1.487	1.480
Consumption	C	1.164	1.115	1.113	1.170	1.126	1.124
Cons: High	$C_{(1)}$	.503	.494	.492	.504	.496	.493
Cons: Low	$C_{(2)}$	.661	.622	.621	.666	.630	.631
Wage	w	1.302	1.162	1.156	1.327	1.188	1.182
Interest Rate	r	3.29%	5.56%	5.68%	2.94%	5.09%	5.19%
Bequest	Beq	.1116	.0762	.0742	.1164	.0854	.0838
Beq: High	$Beq_{(1)}$	.0488	.0343	.0363	.0503	.0377	.0392
Beq: Low	$Beq_{(2)}$	.0628	.0419	.0379	.0661	.0477	.0446

Table 15B. Key Indicators at Alternative Benefit Ages ($\gamma = 2$, SSE-40, CIU)

		Age 65	Age 70	Age 75	Age 80
Preferences		CIU	CIU	CIU	CIU
Capital	K	4.205	4.422	4.660	4.914
Output	Y	1.456	1.482	1.510	1.540
Consumption	C	1.115	1.124	1.133	1.142
Cons: High	$C_{(1)}$	.494	.496	.498	.500
Cons: Low	$C_{(2)}$	.622	.628	.635	.642
Wage	w	1.162	1.184	1.206	1.230
Interest Rate	r	5.56%	5.16%	4.77%	4.38%
Bequest	Beq	.0762	.0754	.0769	.0810
Beq: High	$Beq_{(1)}$	.0343	.0335	.0333	.0342
Beq: Low	$Beq_{(2)}$	.0419	.0419	.0436	.0468

Table 15C. Key Indicators at Alternative Benefit Ages ($\gamma = 2$, SSE-40, CBIU)

		Age 65	Age 70	Age 75	Age 80
Preferences		CBIU	CBIU	CBIU	CBIU
Capital	K	4.465	4.711	4.980	5.270
Output	Y	1.487	1.516	1.547	1.579
Consumption	C	1.126	1.135	1.144	1.152
Cons: High	$C_{(1)}$	.496	.498	.500	.502
Cons: Low	$C_{(2)}$	.630	.637	.644	.650
Wage	w	1.188	1.211	1.235	1.261
Interest Rate	r	5.09%	4.69%	4.28%	3.88%
Bequest	Beq	.0854	.0848	.0866	.0911
Beq: High	$Beq_{(1)}$	.0377	.0368	.0368	.0379
Beq: Low	$Beq_{(2)}$	.0477	.0480	.0498	.0532

Table 15D. Key Indicators at Alternative Elasticities of Intertemporal Substituion (NSSE, CIU)

		log util.	$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
Preferences		CIU	CIU	CIU	CIU	CIU
Capital	K	5.511	5.597	5.757	6.268	7.003
Output	Y	1.604	1.613	1.630	1.681	1.749
Consumption	C	1.159	1.161	1.164	1.173	1.182
Wage	w	1.281	1.288	1.302	1.342	1.397
Interest Rate	r	3.58%	3.48%	3.29%	2.75%	2.09%
Bequest	Beq	.0898	.1020	.1116	.1280	.1435

Table 15E. Key Indicators at Alternative Elasticities of Intertemporal Substituion (NSSE, CBIU)

		$\frac{1}{\gamma}=\frac{2}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{4}$
Preferences		CBIU	CBIU	CBIU	CBIU
Capital	K	5.923	6.078	6.724	7.648
Output	Y	1.646	1.662	1.724	1.805
Consumption	C	1.167	1.170	1.179	1.186
Wage	w	1.315	1.327	1.377	1.442
Interest Rate	r	3.11%	2.94%	2.33%	1.60%
Bequest	Beq	.1100	.1164	.1320	.1490

Table 15F. Key Indicators at Alternative Elasticities of Intertemporal Substituion (SSE-40, CIU)

		$\frac{1}{\gamma} = \frac{2}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{4}$
Preferences		CIU	CIU	CIU	CIU
Capital	K	4.435	4.205	3.822	3.507
Output	Y	1.484	1.456	1.406	1.363
Consumption	C	1.125	1.115	1.097	1.080
Wage	w	1.185	1.162	1.123	1.089
Interest Rate	r	5.14%	5.56%	6.35%	7.10%
Bequest	Beq	.0718	.0762	.0810	.0827

Table 15G. Key Indicators at Alternative Elasticities of Intertemporal Substituion (SSE, CIU)

		$\frac{1}{\gamma} = \frac{2}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{4}$
Preferences		CIU	CIU	CIU	CIU
Capital	K	4.387	4.142	3.738	3.410
Output	Y	1.478	1.448	1.395	1.350
Consumption	C	1.123	1.113	1.093	1.074
Wage	w	1.180	1.156	1.114	1.078
Interest Rate	r	5.23%	5.68%	6.537%	7.35%
Bequest	Beq	.0699	.0742	.0787	.0800

Table 15H. Key Indicators at Alternative Elasticities of Intertemporal Substituion (SSE, CBIU)

		$\frac{1}{\gamma} = \frac{2}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{4}$
Preferences		CBIU	CBIU	CBIU	CBIU
Capital	K	4.986	4.407	4.033	3.800
Output	Y	1.515	1.480	1.434	1.403
Consumption	C	1.135	1.124	1.108	1.096
Wage	w	1.21	1.182	1.145	1.121
Interest Rate	r	4.71%	5.19%	5.90%	6.40%
Bequest	Beq	.0853	.0838	.0832	.0833

Table 15I. Key Indicators under Temptation (NSSE, CIU)

Preferences		Standard	Weak Tempt.	Mod. Tempt.	Str. Tempt.
Capital	K	5.720	3.830	2.813	2.235
Output	Y	1.626	1.407	1.260	1.159
Consumption	C	1.163	1.097	1.032	.979
Wage	w	1.299	1.124	1.006	.926
Interest Rate	r	3.33%	6.33%	9.22%	11.77%
Bequest	Beq	.1113	.0865	.0702	.0598

Table 15J. Key Indicators under Temptation (SSE-40, CIU)

Preferences		Standard	Weak Tempt.	Mod. Tempt.	Str. Tempt.
Capital	K	4.204	2.980	2.314	1.898
Output	Y	1.455	1.286	1.174	1.093
Consumption	C	1.115	1.044	.987	.940
Wage	w	1.162	1.027	.937	.873
Interest Rate	r	5.56%	8.63%	11.36%	13.83%
Bequest	Beq	.0760	.0614	.0522	.0456

Table 15K. Key Indicators under Temptation (SSE-40@Age85, CIU)

Preferences		Standard	Weak Tempt.	Mod. Tempt.	Str. Tempt.
Capital	K	4.914	3.387	2.589	2.094
Output	Y	1.540	1.347	1.222	1.132
Consumption	C	1.142	1.072	1.023	.963
Wage	w	1.230	1.075	.976	.904
Interest Rate	r	4.38%	7.41	10.10	12.57
Bequest	Beq	.0810	.0647	.0548	.0476

Table 15L. Key Indicators under Temptation (HRE, CIU)

Preferences		Standard	Weak Tempt.	Mod. Tempt.	Str. Tempt.
Capital	K	5.091	4.096	4.010	3.996
Output	Y	1.559	1.442	1.431	1.429
Consumption	C	1.147	1.110	1.106	1.105
Wage	w	1.245	1.152	1.143	1.141
Interest Rate	r	4.13%	5.77%	5.95%	5.97%
Bequest	Beq	.0801	.0711	.0707	.0696

Table 15M. Key Indicators under Temptation (HRE-kinked, CIU)

Preferences		Standard	Weak Tempt.	Str. Tempt.
Capital	K	5.079	4.096	3.994
Consumption	C	1.147	1.110	1.106
Wage	w	1.244	1.151	1.141
Interest Rate	r	4.14%	5.78%	5.98%
Bequest	Beq	.0788	.0707	.0693

Table 15N. Key Indicators by PSA Contribution Rate

(Strong Temptation, HRE, CIU)

Mandatory PSA Contribution Rate		6.60%	7.60%
Capital	K	3.996	4.234
Output	Y	1.429	1.459
Consumption	C	1.105	1.116
Wage	w	1.141	1.165
Interest Rate	r	5.97%	5.51%
Bequest	Beq	.0696	.0711

Appendix B: Temptation

Intuition

Temptation preferences are developed by Gul and Pesendorfer in a series of articles in 2001, 2002, and 2004. Below is a summary of the intuition of temptation preferences.

The setting consists of an individual representative agent who will choose today a set of alternatives B, among which she will choose a consumption lottery next period. Assuming that the agent is an expected utility maximizer, the next period the agent chooses lottery $p \in B$ which solves $\max_{p \in B} \int u(p)dp$. At time 0, our agent prefers the set of alternatives B to the set of alternatives B' when $\max_{p \in B} \int u(p)dp \geq \max_{p \in B'} \int u(p)dp$. In the theory of expected utility, a standard axiom is that $B \succsim B' \Rightarrow B \sim B \cup B'$ (Kreps (1979)). This axiom rules out situations where the agent may benefit from or be harmed by the addition of alternatives in B'. The idea of temptation preferences is that the agent may strictly prefer B to $B \cup B'$, knowing that B' contains temptations to which she will be vulnerable in time 1. Thus, in the Gul and Pesendorfer development of temptation preferences, the standard axiom is relaxed to the 'set betweenness' axiom:

$$B \succsim B' \Rightarrow B \succsim B \cup B' \succsim B'$$

With the 'set betweenness' axiom, if $B \succsim B \cup B'$ it is possible for the agent at time 0 to strictly prefer the smaller set B of time 1 choices, to the larger set $B \cup B'$ which contains tempting choices that the agent would rather not face at time 1. The idea here is that the presence of temptations at time 1 are the reason for the preference for commitment to a choice set at time 0. To avoid the temptation, at time 0 our agent strictly prefers the set of alternatives B. In fact, this is the criteria by which a temptation is identified.

As Gul and Pesendorfer (2001) show, the set betweenness axiom together with the other axioms that otherwise lead to standard expected utility, lead instead to the utility representation of temptation preferences. The utility representation is as follows. Let $u(p)$ and $v(p)$ be von Neumann-Morgenstern expected utility functions. Intuitively, u is the utility function that is not affected by temptation, and v is the utility function that is affected by temptation. Then temptation preferences are represented by the function:

$$U(B) = \max_{p \in B} \int (u(p) + v(p))dp - \max_{\widetilde{p}' \in B} \int v(\widetilde{p})d\widetilde{p}$$

Note that when the agent has only one choice, then the function u is the agent's utility; the v terms net to zero. Thus Gul and Pesendorfer refer to $u(p)$ as 'the commitment utility', since it represents utility when one is committed to p (i.e., $B = \{p\}$). It is worth noting here that an agent with perfect foresight would also have utility u, and the standard assumption $B \sim B \cup B'$ would apply. An agent with perfect foresight is not vulnerable to temptation; for such a decision-maker $v = u$ and therefore the utility formulation above collapses to the usual expected utility function. Thus other authors refer to u as 'normative' utility, which is the terminology used in this paper.

The utility function v represents the temptation utility. In the model presented by Gul and Pesendorfer (2002), the agent maximizes $u + v$, arriving at the optimal compromise, with the expenditure of self-control with disutility

$$v(c) - \max_{\widetilde{p} \in B} \int v(\widetilde{p})d\widetilde{p}$$

where c is $\arg\max_{p \in B} \int (u(p) + v(p))dp$.

As Gul and Pesendorfer (2002) point out, temptation is closely related to the literature on time-inconsistent preferences. Time-inconsistent preferences lead to a solution strategy in which multiple selves are players in a game to find an equilibrium which is consistent. Krusell, Kuruşçu and Smith (2005) show that the Phelps-Pollack form of time-inconsistent preferences are equivalent to temptation preferences under a setup where the strength of the temptation goes to infinity.

An important feature of temptation preferences, and the reason they are useful in this analysis of Social Security, is that unlike the Phelps-Pollack formulation of time inconsistent preferences, temptation preferences are amenable to a dynamic setting. In the finite horizon case, the setting is that an agent chooses a consumption lottery from a set of alternatives B_t, and chooses a decision problem set B_{t+1} for the next period. Formally,

$$W_{t-1}(B_t) = \max_{p \in B_t} \int [u_t(p) + W_t(p, B_{t+1}) + V_t(p, B_{t+1})]dp - \max_{\widetilde{p} \in B_t} \int V_t(\widetilde{p}, B_{t+1})d\widetilde{p}$$

In this representation, W_{t-1} represents the value function of the agent over the period t choices; prior to period t, the agent is committed to choose from among the choices in B_t. The normative (commitment) utility is $u_t + W_t$, and the temptation utility is V_t. In the final period (period T), the decision problem set is a singleton set, typically $\{0\}$ or the empty set (i.e., $B_{T+1} = \{0\}$).

$$W_{T-1}(B_T) = \max_{p \in B_T} \int [u_T(p) + 0 + V_T(p)]dp - \max_{\widetilde{p} \in B_T} \int V_T(\widetilde{p})d\widetilde{p}$$

Setup

This paper uses the same temptation model as Boronow (2007), which uses the setup of Krusell, Kuruşçu and Smith (2005).

Let U denote normative utility and V denote temptation utility. The normative utility is given by:

$$U_j = \sum_{t=0}^{J-j} \beta^t [\Psi_{j,t}(z)] \cdot u(c_{j+t}(z))$$

Temptation utility is specified in terms of the felicity function u, and two parameters, a strength parameter σ and a future discount parameter φ:

$$V_j(\widetilde{c}, a, \widetilde{a}')) = \sigma[u(\widetilde{c}) + \varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')]$$

Putting this all together, an agent with temptation preferences maximizes:

$$\begin{aligned} W_j(a) &= \max_{c,a'} \left\{U_j(c, a, a') + V_j(c, a, a')\right\} - \max_{\widetilde{c},\widetilde{a}'} V_j(\widetilde{c}, a, \widetilde{a}')) \\ &= \max_{c,a'} \{(1+\sigma)u(c) + (1+\sigma\varphi)\beta\psi_j(z)W_{j+1}(a')\} \\ &\quad - \max_{\widetilde{c},\widetilde{a}'} \sigma\{u(\widetilde{c}) + \varphi\beta\psi_j(z)W_{j+1}(\widetilde{a}')\} \end{aligned}$$

subject to the same conditions as under standard preferences. Note that c and $\widetilde{c}$ are given by:

$$\begin{aligned} c &= (a+\xi)\cdot(1+r)+Q-a' \\ \widetilde{c} &= (a+\xi)\cdot(1+r)+Q-\widetilde{a}' \end{aligned}$$

In the model used in this paper, the temptation parameters (σ, φ) are parameterized as $(.1, .7), (.2, .7)$, and $(.3, .7)$, representing weak, moderate and strong temptation respectively.

Appendix C: More Results

Bequests in the Utility Function

Table C1 presents results for the benchmark economies NSSE and SSE-40 for two different settings. In the first setting, utility is from consumption (CIU). In the second setting, utility is from consumption and bequests (CBIU).

Table C1. Key Indicators in Steady State

	NSSE	SSE-40	NSSE	SSE-40
Utility Specification	CIU	CIU	CBIU	CBIU
Capital (K)	5.757	4.205	6.078	4.465
Output (Y)	1.630	1.456	1.662	1.487
Capital Intensity (K/Y)	3.53	2.89	3.66	3.00
Wages (w)	1.302	1.162	1.327	1.188
Return on Capital (r)	3.29%	5.56%	2.94%	5.09%
Soc. Sec. Tax (τ_{SS})	0	8.77%	0	8.77%

With bequests in the utility, agents consume less and save more, increasing steady state capital, output and wages, and reducing interest rates, relative to the steady state under the specification without bequests in the utility. The social security tax rate depends on public policy specification and demographic assumptions, not on economic optimization, so it is identical in both the CIU and CBIU frameworks.

Compensating Variation in Consumption

CV results shown in Table C2 are for standard preferences and for bequests in utility.

Table C2. Compensating Variation for SSE-40

Utility Specification	CIU	CBIU
CV: High Income	10.07%	9.15%
CV: Low Income	12.87%	11.83%

When bequests are in the utillty, there is a smaller difference in compensating variation between high and low income workers. One might have expected this. We know (Boronow 2007) that in the CIU life cycle model, Social Security induces low income workers to reduce savings more than high income workers, which would reduce bequests from low income workers more than hgh income workers. Therefore, one might expect that bequests in utility have a larger behavioral effect on low income workers. And indeed there is a decrease in the compensating variation gap between high and low income workers. The relative difference in behavioral response can be noticed in that under CIU, bequests from high income workers were 45.1% of the total bequests, while under the CBIU model, the share of bequests from high income workers drops to 44.1%. (Details are in Table 15A of Appendix A.)

We also notice that with CBIU the level of compensating variation decreases for both high and low income workers. CBIU induces less consumption and greater savings, which raises captial formation in the steady state, leading to higher steady state levels of capital, output, wages and consumption, and therefore lower levels of compensating variations.

Willingness-to-Pay

Table C3 presents WTP results when utility is based on consumption only in utility (CIU), and when utility is based on consumption and bequests (CBIU).

Table C3. Willingness-to-pay for Marginal Increase in Public Pension

	NSSE	SSE-40	NSSE	SSE-40
Utility Specification	CIU	CIU	CBIU	CBIU
WTP: High Income	.214%	.087%	.219%	.092%
WTP: Low Income	.200%	.083%	.198%	.084%
Ratio to Hypothetical Fair Tax				
High Income	85.3%	34.7%	87.4%	36.8%
Low Income	102.5%	42.3%	101.5%	43.1%

An observation from Table C3 is that when bequests are in utility, there is an ambiguous effect on WTP. For high income workers, there is a noticeable increase in WTP. For low income workers, there is a decreased WTP for the first dollar of public pension (the NSSE model), but an increased WTP for additional pension benefits in the SSE models. With CBIU, there is a higher steady state level of capital, than with CIU. Both types of workers benefit from higher wages that result. In the models with Social Security, capital intensity is low and interest rates are hgh. High income workers can maintain consumption and bequests from high returns on savings. But low income workers cannot benefit as much from high interest rates, and must achieve their bequest objective by reducing consumption. Hence in the NSSE case, they have a lower WTP, but in the SSE-40 case they have a higher WTP, relative to the CIU framework.

Alternative Risk Aversion Parameters

Partial Equilibrium Setting

Consider a partial equilibrium setting, where factor prices are given . Tables C4 and C5 present the outcomes for agents with alternative elasticities, as they optimize savings and consumption decisions in a partial equilibrium setting. Table C4 uses factor prices from the NSSE steady state when $\gamma = 2$. Table C5 uses factor prices from the SSE steady state when $\gamma = 2$.

Table C4. Alternative Elasticities of Intertemporal Substitution

(NSSE Partial Equilibrium, $r = 2.944\%$)

Elasticity	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{2}{3}$
Savings (K')	6.699	6.078	5.778
Consumption	1.162	1.170	1.175

We notice that as $\frac{1}{\gamma}$ increases, savings decreases. Consumption increases as less is being saved. This reminds us of the situation in the two period model, where, when $R < \frac{1}{\beta\psi}$, the sign of the derivative of c_1 with respect to $\frac{1}{\gamma}$ was positive. Thus consumption moved in the same direction as $\frac{1}{\gamma}$ and savings moved in the opposite direction. That is precisely what we see in Table C4. Agents are facing a low interest rate. For agents with higher elasticities, the risk of dying before the return is realized reduces the expected return enough that they reduce their savings and increase consumption. In this partial equilibrium, $R\beta = 1.006787$, so ψ has to be less than .993 for the condition to be satisfied. For high ability agents, the probability of survival to the next period (ψ) is less than .993 for ages older than age 57. For low ability agents, the condition is satisfied for ages older than age 51. In either case, the condition is satisfied well before retirement, at ages where the agent is making significant savings decisions. Thus we might expect savings to decrease, based on intuition from the two period model.

Consider next a partial equilibrium where there is social security, and factor prices are taken from the SSE steady state, when $\gamma = 2$.

Table C5. Alternative Elasticities of Intertemporal Substitution

(SSE Partial Equilibrium, $r = 5.194\%$)

Elasticity	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{2}{3}$
Savings (K')	3.803	4.406	5.279
Consumption	1.108	1.124	1.144

Here we see the opposite effect from what we observed in the NSSE steady state. As elasticity increases, savings also increases. This is reminiscent of the case in the two period model where $R > \frac{1}{\beta\psi}$. Recall that when $R > \frac{1}{\beta\psi}$, then $\frac{\partial c_1}{\partial \frac{1}{\gamma}} < 0$. In the two period model as $\frac{1}{\gamma}$ increases, c_1 decreases, which causes savings to increase. In Table C5 we also see savings increase as $\frac{1}{\gamma}$ increases. (Consumption also increases, which is counter-intuitive.See explanation in the note.[6]) Here agents face high interest rates, so agents with greater elasticities shift consumption to take advantage of the high returns. In the SSE steady state, $R\beta = 1.0288$. Thus to satisfy the condition $R > \frac{1}{\beta\psi}$, ψ must be greater than .972. For high ability workers, this is the situation for ages up to age 72. For low ability

[6] As noted above, average annual consumption increased along with increased saving and higher elasticity in the SSE setting. One would expect consumption to decrease if savings increase. However, in the SSE steady state, interest rates are high enough that increased savings earns interest that funds enough consumption to offset the consumption lost to higher savings. Likewise, as $\frac{1}{\gamma}$ falls, savings decreases and consumption increases. But the interest not earned on savings that does not occur is enough to offset the extra consumption that otherwise would have gone to savings. Overall there is a decrease in consumption.

workers, $R > \frac{1}{\beta\psi}$ up to age 67. Thus during the working years, when agents are making significant savings decisions, the condition is satisfied.

General Equilibrium

It does not necessarily follow that this pattern holds in a general equilibrium setting, but it does give added intuition. In Tables C6 and C7 we turn our attention to a general equilibrium analysis, allowing factor prices to be determined by the steady state levels of capital. We extend the analysis to also consider the case $\gamma = 4$.

Table C6. Key Indicators with Alternative Risk Aversion Parameters

(NSSE,CIU)

Risk Aversion Parameter	$\frac{1}{\gamma} = \frac{1}{4}$	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{2}{3}$
Capital	7.003	6.269	5.757	5.597
Consumption	1.182	1.173	1.164	1.161
Wage	1.397	1.342	1.302	1.288
Interest Rate	2.09%	2.75%	3.29%	3.48%

Table C7. Key Indicators with Alternative Risk Aversion Parameters

(SSE-40,CIU)

Risk Aversion Parameter	$\frac{1}{\gamma} = \frac{1}{4}$	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{2}{3}$
Capital	3.507	3.822	4.205	4.435
Consumption	1.080	1.097	1.115	1.125
Wage	1.089	1.123	1.162	1.185
Interest Rate	7.10%	6.35%	5.56%	5.14%

In the NSSE model of Table C6, an increasing elasticity of intertemptoral substitution ($\frac{1}{\gamma}$) causes a decrease in saving, wages and consumption. In Table C7, under the SSE-40 model, an increase in the elasticity of intertemporal substitution causes an increase in saving, wages and consumption. This result was also obtained by Imrohoroglu, Imrohoroglu and Joines (2003). They commented, "a low elasticity of substitution also raises the costs of social security, which takes the form of lower steady-state capital and lifetime consumption". They also commented, "The smaller the elasticity of substitution, the greater the reduction in the saving of workers when government attempts to reallocate consumption toward retirement years through the payroll tax." Based on the intuition of the two period and partial equilibrium models, we trace the source of the response to the relationship between the return on investment, the probabilility of survival and the time preference parameter.

To gain further insight, we consider the compensating variation in compensation. Table C8 presents the compensating variation for agents born into the SSE-40 model under alternative values for the elasticity of intertemptoral substitution.

Table C8. Compensating Variation and Elasticity of Intertemptoral Substitution

Risk Aversion Parameter	$\frac{1}{\gamma}=\frac{1}{4}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{2}{3}$
Utility Specification	CIU	CIU	CIU	CIU
CV: High Income	25.5%	16.9%	10.1%	7.5%
CV: Low Income	29.0%	19.6%	12.9%	10.2%

We saw in Tables C6 and C7 that in the NSSE economy, consumption and capital decrease with a increase in $\frac{1}{\gamma}$, while the opposite occurs in the SSE-40 economy. Thus we would expect a decrease in compensating variation as $\frac{1}{\gamma}$ increases. As Table C8 shows, the compensating variation decreases significantly over this range of parameters.

Tables C9 and C10 present willingness-to-pay under alternative elasticities of substitution for the two benchmark economies.

Table C9. WTP and Elasticity of Intertemptoral Substitution

(NSSE, CIU)

Risk Aversion Parameter	$\frac{1}{\gamma}=\frac{1}{4}$	$\frac{1}{\gamma}=\frac{1}{3}$	$\frac{1}{\gamma}=\frac{1}{2}$	$\frac{1}{\gamma}=\frac{2}{3}$
WTP: High Income	.2644%	.2329%	.2140%	.2070%
WTP: Low Income	.2579%	.2238%	.2000%	.1920%
Ratio to Required SS Tax				
High Income	120.6%	106.2%	97.6%	94.4%
Low Income	117.6%	102.1%	91.2%	87.6%

Table C10. WTP and Risk Aversion Parameters

(SSE-40, CIU)

Risk Aversion Parameter	$\frac{1}{\gamma} = \frac{1}{4}$	$\frac{1}{\gamma} = \frac{1}{3}$	$\frac{1}{\gamma} = \frac{1}{2}$	$\frac{1}{\gamma} = \frac{2}{3}$
WTP: High Income	.0287%	.0535%	.0869%	.1062%
WTP: Low Income	.0290%	.0523%	.0825%	.1000%
Ratio to Required SS Tax				
High Income	13.1%	24.4%	39.6%	48.4%
Low Income	13.2%	23.8%	37.6%	45.6%

In Table C9, WTP decreases as $\frac{1}{\gamma}$ increases. Low returns on investment make the returns to survivorship relatively more attractive and raises WTP generally. But as elasticity increases, and agents reduce savings, capital becomes dearer and interest rates increase, reducing WTP. However, under the SSE-40 model, we find that WTP increases as $\frac{1}{\gamma}$ increases. Here returns on investment are more attractive than returns on survivorship, thus decreasing WTP. This is especially the case for capital starved steady state when $\frac{1}{\gamma} = \frac{1}{4}$. As elasticity increases, savings increase and interest rates fall. At lower interest rates, the return on survivorship gains in relative attractiveness, raising the WTP. But in no case are agents willing to pay the required tax.