Finding highly probable(robust) paths in the presence of uncertain weather

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Abstract of the Thesis
Finding highly probable(robust) paths in the presence of uncertain weather
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Uncertain weather conditions is one of the biggest challenges faced by air traffic route management. The US airspace is constructed of virtual freeways and all flights have predetermined routes on which to fly, which have originated over years of work and experience. These flight routes are sometimes marred by bad weather conditions which force the pilots to take on alternative routes. It is not very difficult to come up which these alternates if the conditions are known in advance but the real challenge comes when they come up unexpectedly enroute.

Stochastic weather conditions such as turbulance and icing are very difficult to predict. For example the most reliable way to know that turbulance exists in a particular region is throug Pilot Reports. The National Center for Atmospheric Research (NCAR) uses mathematical models to get a rough estimate of these weather conditions. These conditions can be quite severe at times and can badly affect flight conditions resulting in unhappy customers.

In this thesis I propose various problems, solutions and future directions of research dealing with stochastic weather conditions. As these conditions are not known with certainty, routes have to be planned which have a high probability of success and are robust to small variations.

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1 Introduction

In Air Traffic Management, finding optimal routes through areas constrained by weather conditions is a primary problem. The ATM group at Stony Brook has been working on several problems dealing with Air Traffic Management. The group worked on finding capacity of regions with convective weather constraint. The problem was proved to be polynomially solvable, the capacity being just the min cut. Later the problem of finding the capacity of a region with two types of constraint (hard and soft) and two types of aircraft (one which have to avoid both and one which can pass through the soft constraints) was proved to be NP-hard. In this report we deal with non-convective weather, example of such weather conditions are turbulence and icing. Non-convective weather is very difficult to predict with certainty. The only known method to know that turbulence exists with certainty is PiReps (Pilot Reports).

The National Center for Atmospheric Research (NCAR) uses different mathematical models to predict non-convective weather. Depending on the method they assign a confidence factor(probability) of the prediction being correct. Now instead of deterministic constraints we have probabilistic constrains. Now we want to find paths with have a high probability of being clear. We also consider finding "robust" routes, i.e. routes which behave well even with small changes in condition, e.g. a sudden unexpected weather phenomenon.

The organization of the rest of the thesis is as follows. We talk about related work in area of probabilistic graphs, weighted regions and path planning in section 2. In section 3 we define highly probable path and present some hardness results and open problems. In section 4 we define some problems related to robust routes.

2 Related Work

2.1 Probabilistic graphs

A probabilistic graph is one where there is a probability of existence associated with every edge of the graph. For example in our domain of Air Traffic Management the edges might be paths on which a flight is allowed to travel but some edges might be blocked or unavailable due to certain weather conditions. The availability of an edge depends upon the probability of the weather condition occurring. Various formulations on probabilistic graphs have been studied for quite some time. One of the main problems people have tried to solve is whether the graph is connected or whether two given nodes are connected.

Valiant showed that evaluating the probability that two given nodes in a probabilistic graph are connected is NP hard [1]. Later Provan and Ball proved that other problems such as evaluating the probability that a probabilistic graph is connected, and approximating the probability that a given probabilistic graph is connected, and approximating the probability that two vertices in a probabilistic graph are connected are NP hard [2] . Krager gave a randomized polynomial time approximation scheme for the probability that a given network will fail [3, 4].This was done using the enumeration of the approximately minimum cuts in the network which has been dealt with in [5, 6, 7].Another problem studied for probabilistic graphs is to determine the most reliable source in a given network or graph. Linear time algorithms exists for this problem when the graph is a tree [8] or a series parallel graph [9].

Next is the class of problems where a given graph has weights on edges which are random variables. Frank [15] and Mirchandani [16] studied the problem of determining the probability distribution of the shortest path length in a stochastic network where link travel times are random variables but not time dependent. Loui [17], Mirchandani and Soroush [18], and Murthy and Sarkar [19] studied the variations of the shortest path problems in stochastic networks by considering different types of cost functions. It was found that if the objective is to identify the expected shortest path (or linear cost function), then the problem simply reduces to a deterministic shortest path problem in a network where the random link travel times are replaced by their expected values. Therefore, the existing standard shortest path algorithms still can be used to find the expected shortest paths in a static and stochastic network.

Hall [20] first investigated the shortest path problem in a transportation network where the link travel times are random and time-dependent and demonstrated that the standard shortest path algorithm may fail to find the expected shortest path in these networks. An optimal dynamic programming based algorithm was proposed to find the shortest paths and this algorithm was demonstrated on a small transit network example. The paper only considers the case where link travel times are modeled as discrete-time stochastic processes. Liping Fu and L.R.Rilett [21] extend the shortest path problem in dynamic and stochastic networks (DSSPP) where link travel times are defined as continuous time stochastic processes. An extensive analysis of the properties associated with the DSSPP is provided and a heuristic algorithm based on the k-short- est path algorithm is proposed. Dimitri P. Baertsekas and John N. Tsitsiklis [22] look at shortest path problem in a graph where both the edge cost and the existence of an edge are random variables. Such problems have being dealt with using the general theory of Markovian decision problems.

2.2 Weighted Region

In the weighted region problem the plane is subdivided into polygonal regions each of which has an associated weight α specifying the "cost per unit distance" of traveling in that region. The objective is to find a path in the plane that minimizes total cost according to a weighted Euclidean metric. For example, in our domain we might have weather conditions which a flight could pass through but with a higher cost, regions without any obstacles will have lesser cost and regions with weather conditions one has to avoid will have cost infinity.

Mitchell and Papadimitriou [10] use Snell Law and "continuous Dijkstra" method to give an optimal-path map for any given source point s. The time complexity of their algorithm is O($n^8 M$). Here M is a function of various input parameters. Mata and Mitchell [11] presented another approximation algorithm based on discretization, the "path-net" algorithm. An approximate optimal path is computed after constructing a "pathnet graph" Lanthier et al [12] and Aleksandrov et al [13] use a different approach in which they discretize the polygonal subdivision by placing m Steiner points along the edges of polygonal regions and then try to compute an optimal path in the resulting discrete graph using Dijkstra's algorithm. Reif and Sun [14] introduce a wavefront like algorithm to compute optimal path in the discrete setting more efficiently than the above methods.

2.3 Path Planning in the Robotics Community

Real-time collision-free robot-path planning is a fundamentally important issue in robotics. There are many studies on robot-path planning using various approaches, such as the grid-based A* algorithm [23], [24], road maps (Voronoi diagrams and visibility graphs) [25], [26], cell decomposition [27],[28], and artificial potential field [29],[30]. Some of the previous approaches use global methods to search the possible paths in the workspace, normally deal with static environments only, and are computationally expensive when the environment is complex.

Some neural-network models ([31],[32]) were proposed to generate real-time robot trajectories through learning. These models deal with static environments only and assume that there are no obstacles in the workspace. Zalama et al. [33] proposed a neural-network model for a mobile robot navigation, which can generate dynamic collision-free trajectories through unsupervised learning. Glasius et al. [34] proposed a neural-network model for realtime trajectory generation with obstacle avoidance in a non stationary environment. It is rigorously proven that the generated trajectory does not suffer from undesired local minima and is globally optimal in a stationary environment. However, these models require that the robot dynamics be faster than the target and obstacle dynamics, and have difficulty dealing with fast changing environments. Yang and Meng [35], [36] proposed a new neural-network approach to the dynamic collision-free trajectory generation for robots in dynamic environments.

3 Highly Probable Paths

3.1 Ensemble version

Weather prediction is a difficult branch of science. Scientists over the years have built many mathematical models to predict weather. Depending on the method there is a confidence factor associated with the prediction. In this version of the problem we assume that NCAR uses various means to generate several weather maps. Each map has 2 regions, regions through which a flight can go and regions through which a flight cannot go (free space and obstacles). We assume each condition has some probability of occurring and that one and only one of these conditions occur (therefore the sum of all probabilities equal one). Given these ensembles we now have to find "highly probable" paths through the region.

3.1.1 One Highly Probable Path

Let P be a simple polygon; let s, t be two edges on its boundary. A *path* starts at edge s and ends at edge t and has non negligible thickness. A *forecast* is a set of disjoint (open) simple polygons inside P.

Let $\mathcal{F}_1, \ldots, \mathcal{F}_F$ be a set of forecasts. Each forecast $f = 1 \ldots F$ has an associated "probability" p_f of being the true weather prediction; $\sum p_f = 1$. Only one forecast will become the true weather. For a path π its *survival probability*, p_π, is 1 minus the sum of the probabilities of the forecasts that the path intersects:

$$p_\pi = 1 - \sum_{f : \pi \cap \mathcal{F}_f \neq \emptyset} p_f .$$

We want to find a path whose p_π is maximized.

Hardness Results

We prove here that the graph version of the problem is NP-hard.

Problem Definition: Given a graph G and sets of edges $E_1, \ldots, E_F$ each having an associated color, find a path from vertex s to vertex t using the minimum number of colors.

Theorem 1. *Finding a path in graph G using the minimum number of colors is NP-hard.*

Proof. We try to prove that the decision version of the problem, does there exist a path from s to t using at most k colors is NP-complete. Given an instance of 3SAT with n variables we create an instance of our problem such that the 3SAT instance is satisfiable iff in the instance of our problem there exists a path using at most n colors.

The graph contains a chain of n parallel channels (two channels per variable) corresponding to the variable and its negation, followed by a chain of m channels (three channels per clause); refer to Figure 1. The channels in a clause represent its 3 literals.

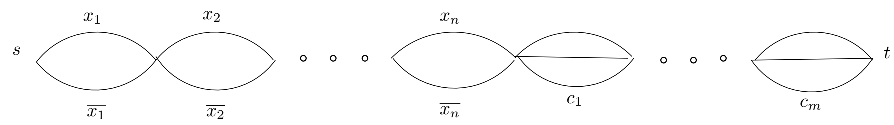

Figure 1: There are 2 channels per variable and 3 channels per clause.

We give every literal (a variable or its negation) a different color. A channel, representing a literal ℓ, has the same color corresponding to the literal ℓ.

Since any s to t path must go through each of the n variables gadgets, it uses at least n colors. The way the path goes through the variable gadgets determines the truth assignment for the variables. The path uses only n colors iff the path may go through all clauses without hitting any new color, i.e., if in each clause there exists a true literal. □

We can easily reduce the graph problem to our geometric problem. We consider the planar embedding of the graph. We thicken the edges to the size of the path width. If a particular color s occurs in k edges, then there is a forecast with polygons at all those edges. All the free space can consist of all the colors, so that no path can pass through the free space. Therefore any path in this construction will be a path in the graph.

3.1.2 Two Highly Probable Paths

In some cases one path cannot have a high survival probability. Imagine an instance where two slightly overlapping forecasts (each occurring with probability .5) together block the entire polygon P; any s to t path has a survival probability of only .5. But if we plan two paths, with one route piercing only one forecast, then it is with probability 1 that one of the two paths would survive no matter which forecast becomes true.

Given the above idea, for two s to t paths, π_1 and π_2, their overall survival probability can be computed by 1 minus the sum of the probabilities of the forecasts that they both intersect:

$$p_{\pi_1,\pi_2} = 1 - \sum_{f:\pi_1 \cap \mathcal{F}_f \neq \emptyset, \text{ and } \pi_2 \cap \mathcal{F}_f \neq \emptyset} p_f.$$

We want to find paths π_1 and π_2 such that p_{π_1,π_2} is maximized.

Hardness Results

We again prove here that the graph version of the problem is NP-hard.

Problem Definition: Given a graph G and sets of edges $E_1, \ldots, E_F$ each having an associated color. Find two paths from vertex s to vertex t such that the number of colors overlapping in the two paths is minimized.

Theorem 2. *Finding two paths in graph G such that they do not contain more than k colors in common is NP-hard.*

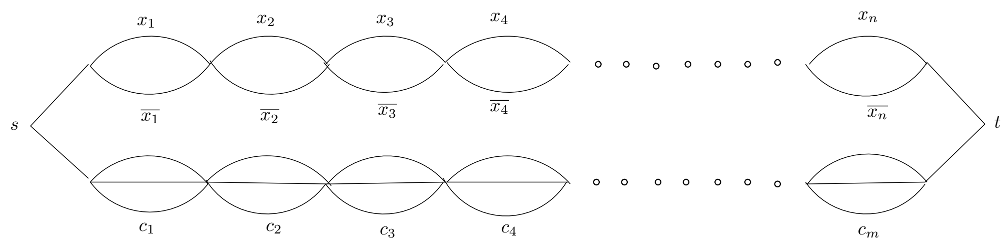

Figure 2: There are 2 channels per variable and 3 channels per clause.

Proof. We try to prove that the decision version of the problem, does there exist two paths from s to t having at most k colors in common is NP-complete. Given an instance of 3SAT with n variables we create an instance of our problem such that the 3SAT instance is satisfiable iff in the instance of our problem there exists two paths using no common colors. The graph contains two chains, a chain of parallel channels (two channels per variable) corresponding to the variable and its negation and a chain of channels (three channels per clause); refer to Figure 2. The channels in a clause represent its 3 literals.

We give every literal (a variable or its negation) a different color. A channel, representing a literal ℓ, has the same color corresponding to the negation of the literal ℓ.

An s to t path through the variable channel must go through each of the n variables gadgets and therefore use n colors. The way the path goes through the variable gadgets determines the truth assignment for the variables. The second path does not use any color that the first path uses iff it does not pass through variables whose negation is assigned true. □

3.1.3 Capacity through the region

The above two problems help us understand the complexity involved in finding a high probable path through uncertain weather. But finding just one path is not very useful. We are actually concerned about the Capacity(number of lanes) of the region.

Problem Definition: Let P be a simple polygon; and s, t be two edges of P. A *path* starts at edge s and ends at edge t and has non negligible thickness. A *forecast* is a simple polygon inside P.

Let $\mathcal{F}_1, \ldots, \mathcal{F}_F$ be a set of forecasts. Each forecast $f = 1 \ldots F$ has an associated "probability" p_f of being the true weather prediction; $\sum p_f = 1$. Only one forecast will become the true weather. For a path π its *survival probability*, p_π, is 1 minus the sum of the probabilities of the forecasts that the path intersects:

$$p_\pi = 1 - \sum_{f:\pi\cap\mathcal{F}_f\neq\emptyset} p_f.$$

Determine the number of paths that can be routed through this domain such that each path has survival probability $> p$.

3.2 Simpler models of the ensemble version

To better understand the "highly probable path" problem we try looking at some simpler versions of the problem. We still stick to the ensemble version but with some constraints.

Forecasts as single polygons

Problem Definition: Let P be a simple polygon; and s, t be two edges of P. A *path* starts at edge s and ends at edge t and has non negligible thickness. A *forecast* is a simple polygon inside P.

Let $\mathcal{F}_1, \ldots, \mathcal{F}_F$ be a set of forecasts. Each forecast $f = 1 \ldots F$ has an associated "probability" p_f of being the true weather prediction; $\sum p_f = 1$. Only one forecast will become the true weather. For a path π its *survival probability*, p_π, is 1 minus the sum of the probabilities of the forecasts that the path intersects:

$$p_\pi = 1 - \sum_{f:\pi \cap \mathcal{F}_f \neq \emptyset} p_f.$$

Find a path whose p_π is maximized.

In this problem each forecast is a single polygon instead of a set of polygons. Note that our previous hardness proof is not valid in this case.

Forecasts as translates of the same polygon

Problem Definition: Let P be a simple polygon; and s, t be two edges of P.

Let $\mathcal{F}_1, \ldots, \mathcal{F}_F$ be a set of forecasts each of which is a translate of the same polygon S. Each forecast $f = 1 \ldots F$ has an associated "probability" p_f of being the true weather prediction; $\sum p_f = 1$. Only one forecast will become the true weather. For a path π its *survival probability*, p_π, is 1 minus the sum of the probabilities of the forecasts that the path intersects:

$$p_\pi = 1 - \sum_{f:\pi \cap \mathcal{F}_f \neq \emptyset} p_f.$$

Find a path whose p_π is maximized.

Note that this is not an unreasonable assumption. We can have a moving wind whose speed we are unsure of, though we know the severity and shape of the obstacle, we are unsure of its exact location. Such a condition will give rise to exactly this problem.

Special Cases:
Version 1: Given n line segments in the plane and two point s and t, find a path from s to t crossing the minimum number of line segments.
Version 2: Given n unit disks in the plane and two points s and t, find a path from s to t

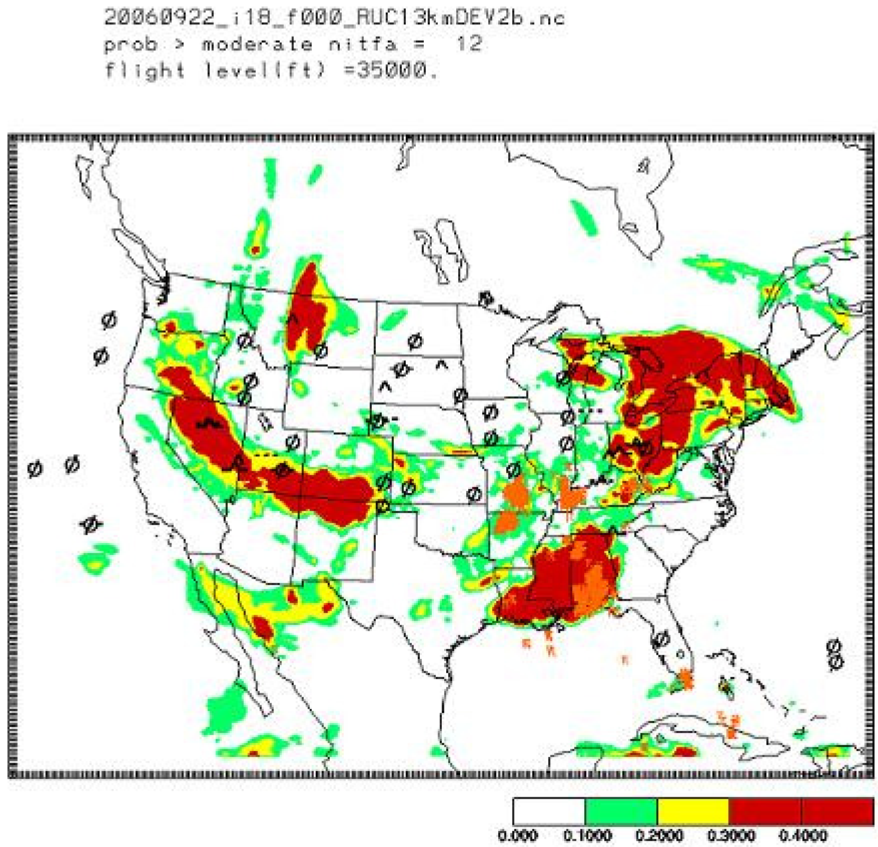

Figure 3: Weather map of moderate or higher turbulence.

crossing the minimum number of disks.

Version 3: Let P be a convex polygon. Given n translates of P in a plane and two points s and t, find a path from s to t crossing the minimum number of polygons.

3.3 GTG version

We are given a map as shown in the Figure 3. It is a map of moderate or higher turbulence. Different colors define different probabilities of there being moderate or higher turbulence. Our objective is to find a “highly probable path” through this region.

Problem Definition: Let P be a simple polygon; let s, t be two edges of P. Let $\S_1, \ldots, \S_F$ be regions inside P with each region occurring with probability p_f. Our problem is to find a path in this region which is clear with probability $\geq p$.

One major question that occurs here is how do we define the probability of a path being clear. We define four different models to answer this question.

Model A: Ensemble Model

We can model this problem as our ensemble problem. Lets take for example the map in Figure 3. The probabilities are in increments of .1, therefore we convert this in 10 ensembles. As none of the region has probability more than .5 of being present, we have five ensembles which contain nothing, in other words no polygons. The other five ensembles are as follows: The red region is present in all five, the orange region in first four, the yellow in first three, green in first two and the white in first one. Each ensemble has probability .1 of occurring. We ask the same question, is it possible to get a path with survival probability at least p with the survival probability of a path defined as 1 minus the sum of the probabilities of the forecasts that the path intersects.

In this model as with the ensemble case we make the assumption that one and only one of the ensembles occurs.

Model B: Independent Model

In this model we consider each pixel to be independent, i.e. each pixel does a coin toss based on its probability and either the region is blocked or not. It does not depend on the existence or absence of the others. Let S be the set of pixels that exist and the path passes through them. So the probability of a path being clear is $\prod_{p_i \in S}(1 - p_i)$.

This is a very weak model as it will hardly give us any good path, for example even if the path passed through 5 pixel which have .1 probability of occuring, the survival probability of the path drops to .59.

Model C: Weighted Region Model

In this model we consider every region has a cost associated with it. The cost of a region depends on its color code, a region with high probability of occurrence will have higher cost associated with it, thus making travel through that region costly. In the simplest case the cost of a region can be the probability of it occurring. Now our problem boils down to finding a low cost path through this weighted region.

The weights assigned cannot be directly proportional to the severity of the weather condition, as the air traffic community mostly think in terms of binary(go or no-go), they are not comfortable with the idea of traveling a short distance is bad weather as opposed to traveling a very long distance in moderate conditions(which can very well be the result of weighted regions with proportional weights).

Though as mentioned in related work section there is a lot of work done on finding shortest paths through weighted region, they are mostly thin paths (poly lines), whereas what we require are thick (non negligible thickness) paths.

Model D: Poisson Process Model

Model A and B are two ends of a spectrum, Model A being a completely correlated model whereas Model B being completely independent model both of which are not very reasonable considering that our obstacles are weather conditions. We define another model in which we consider weather to be independent in general but having some correlation with nearby weather conditions. We can view this problem as a 2-D poisson process.

We can define every condition to be a poisson process with a rate (λ) which is proportional to the probability of that event occurring. Let there be points splattered on the region according to the poisson rate. Note that the regions that have high probability of occurring will have denser points. We consider every point to be the center of a disk with radius r, thus denoting that if that point is affected every region within radius r will be. Now for every path we construct we could calculate the number of disk it intersects. We could run the poisson process many times to get an idea of how a path behaves, or what is the survival probability of a path on an average.

4 Robust Paths and Contingency Planning

4.1 Delta Bomb

In Air Traffic Management we care about generating paths which are "robust", i.e. we want paths which are not affected much by small changes in conditions.

There can be weather conditions which was not known or which pop up suddenly, in such cases a flight must be able to make a detor which is not much longer than the normal route. We define these unexpected weather conditions as an adversary dropping bombs (circular disks of certain radius) on our path. We must be able to plan paths which are not affected much by this change.

1-Delta Bomb

Problem Definition: Given a polygonal domain P with obstacles (polygons within P), we want to find a thick (non negligible width) path (avoiding the obstacles) from source s to sink t (s and t are edges of P), such that if a bomb (disk of radius δ) were to fall on our path, the reroute would not be more than a parameter k.

There should be a reveal time to the bomb's location, we can say we know the bomb is there just before touching it, just giving us enough time to make a detor.

One idea here is to avoid corridors (regions blocked on both sides by constraints) of small width, as the only reroute possible once we enter them is to trace back.

Lets look at Figure 4. The top route in green is an original route and the yellow shows the reroute when the blue bomb was dropped. This reroute is within our constraints. But if we take the bottom green route, it passes through a long corridor, if the adversary plans to drop the bomb at the end, the only reroute possible is to traverse back the whole length and join the top green route (the reroute is shown in yellow).

ℓ-Delta Bomb

We can enhance the earlier problem by allowing the adversary to drop more than 1 bomb.

Problem Definition: Given a polygonal domain P with obstacles (polygons within P), we want to find a thick (non negligible width) path (avoiding the obstacles) from source s to sink t (s and t are edges of P), such that if ℓ bombs (disk of radius) δ) were to fall on our path, the reroute would not be more than a parameter k.

Constraint Enlargement

The above problems do a nice job of defining the robustness of a route but it is not very usual for weather conditions to pop up at places which was predicted to be absolutely clear. A more reasonable assumption is that weather conditions which are already present move

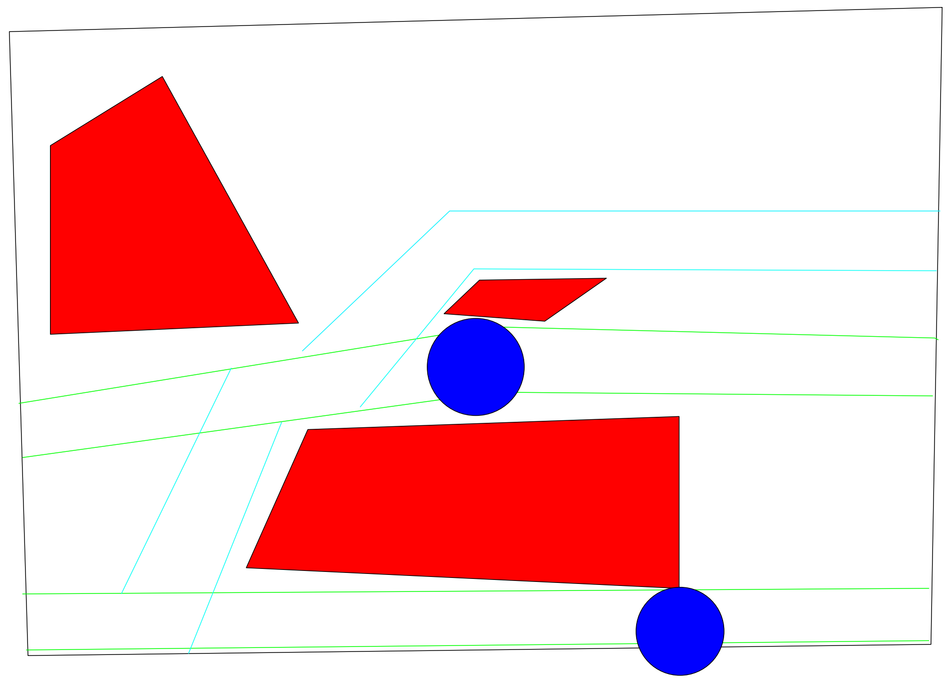

Figure 4: Original paths shown in green and the reroutes in yellow.

a bit or get enlarged. We now consider the problem from constraint enlargement point of view.

Problem Definition: Given a polygonal domain P with obstacles (polygons within P), we want to find a thick (non negligible width) path (avoiding the obstacles) from source s to sink t (s and t are edges of P), such that if an adversary enlarged some constraint by a factor δ (the boundary of the constraint expanded by δ units on each side), the reroute would not be more than a parameter k.

It is open weather these problems are hard or polynomially solvable.

4.2 Wiggle Room

In this problem we try to create some sort of an holding pattern, a path along which a flight can stay for a certain amount of time in case of some unfavorable events, along our path. The requirements of these holding patterns differ. Along some routes we might want equi-spaced holding patterns and along others we might want them only at the junction of some critical weather scenarios.

Well spaced holding pattern

Problem Definition: Given a polygonal domain P with obstacles (polygons within P), we want to find a thick (non negligible width) path (avoiding the obstacles) from source

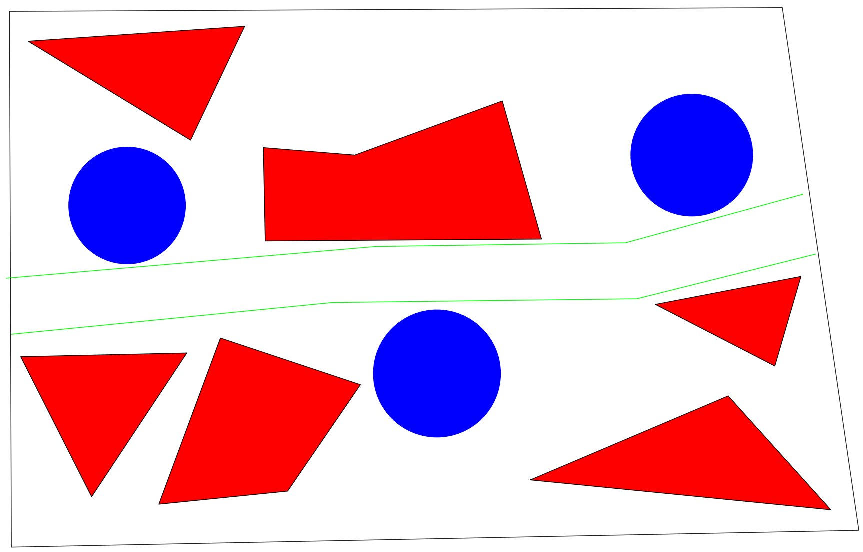

Figure 5: Holding pattern shown in blue at regular intervals.

s to sink t (s and t are edges of P), such that we are able to construct holding patterns (circular disks of radius r), along the path such that the distance between the patterns does not exceed a parameter k.

It should seem obvious that the holding patterns should also avoid obstacles. The holding patterns can be of different shapes, they could be circular disks, triangles or trapezoids. It does not seem the shape of the holding pattern affects the complexity of the problem.

Refer to Figure 5, we have three holding patterns (shown as blue cirular disks) along the thick path (shown in green).

Emergency holding pattern

Problem Definition: Given a polygonal domain P with obstacles (polygons within P), we want to find a thick (non negligible width) path (avoiding the obstacles) from source s to sink t (s and t are edges of P), such that we are able to construct holding patterns (circular disks of radius r) before (distance $< \ell$) any corridor (area blocked on both sides by constraints) of length $> k$ and width $< w$.

These corridors are undesirable regions, the flights are traveling between two bad weather conditions, the path would be completely blocked if the weather moved a little and therefore we desire an holding pattern just before as it will allow the flights some wait item in case of unwanted events.

Again the holding pattern must avoid obstacles and can be of different shapes.

Refer to Figure 6, we see an undesirable corridor and therefore we have a holding pattern (shown as blue circular disk) just before that.

It is open weather these problems are hard or polynomially solvable.

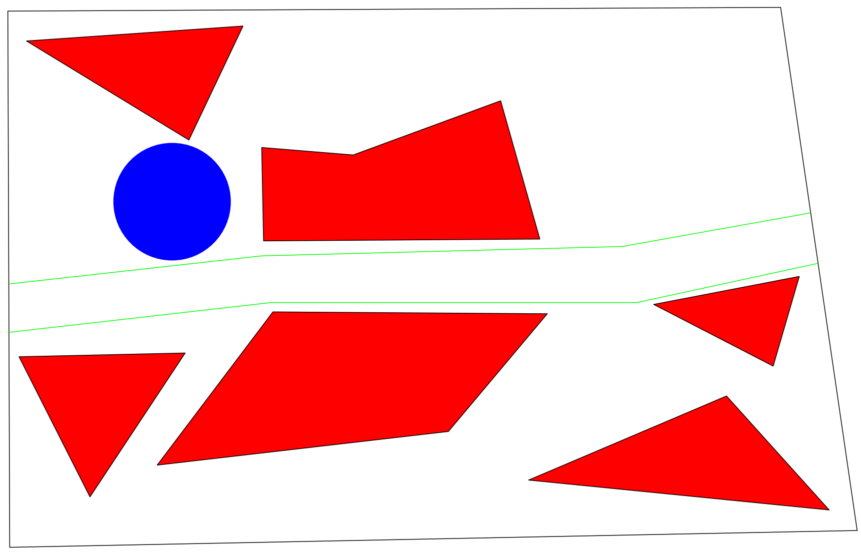

Figure 6: Holding pattern shown in blue just before a long skinny corridor.

4.3 Maximize Throughput

In the problems before we were usually trying to get a single high probable path through a domain with uncertain weather. There are weather conditions, say region with light to moderate turbulence(which we will call "soft" constraints as opposed to "hard" constraints which a flight cannot pass through), through which a flight can pass but with low throughput, i.e because these areas are not completely clear we want more distance between the flights.

Problem Definition: We are given a polygonal domain P with hard and soft constraints (polygons within P). The paths in P have to avoid the hard constraints. Say the number of planes that can traverse on a path in P is n_1 if it avoids the soft constraints and n_2 if it passed through a soft constraint. We have to get thick (non negligible width) paths from source s to sink t (s and t are edges of P), in order to maximize the throughput (number of flights per unit time) of the region.

In Figure 7, the hard constraints are shown in red and the soft constraint in blue. We have drawn a path (shown in yellow) from source to sink passing through a soft constraint, which can take only 10 planes an hour and another (shown in green) which can take 20 planes an hour.

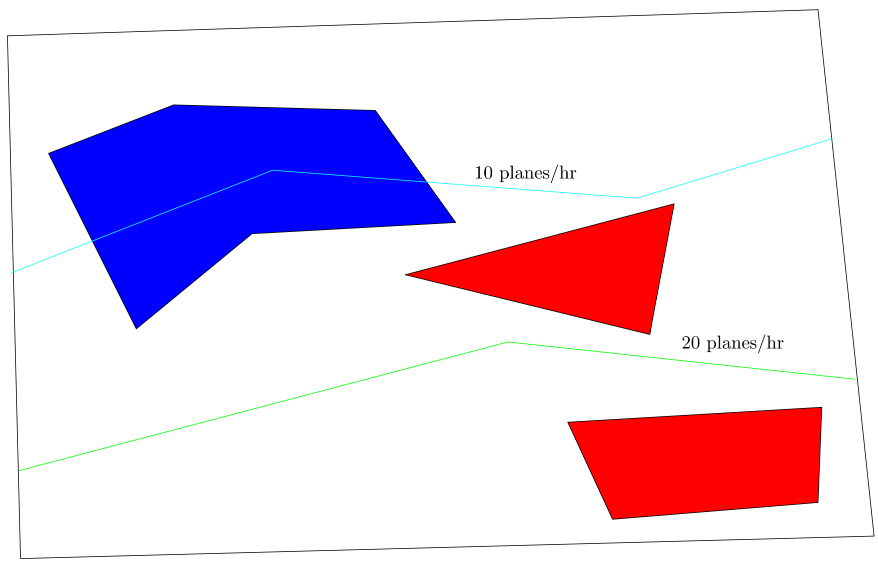

Figure 7: Lanes with different throughput.

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