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Coupling Spin Resonances In RHIC And AGS

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The polarization of proton beam during the acceleration process in a particle accelerator is affected by the existence of spin resonances. Coupling spin resonances can be excited in the presence of the betatron coupling introduced by rolled quadrupoles and solenoids, as well as in when the stable direction of the spin deviates from the vertical (for instance, by partial Snakes or spin rotators). In this study, We extended the ASPIRRIN code to account for the effect of solenoidal magnets and related betatron coupling on the spin resonances. The examples of the coupling spin resonance for RHIC are given. In addition, an analysis is presented for spin coupling resonances produced in the AGS due to the partial Snakes.

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Chapter 1

1 Introduction to spin beam dynamics

High energy polarized beams are as a fundamental necessities for research in high energy and nuclear physics sciences using accelerators facilities. Spin is an intrinsic feature of all of the accelerated particle species. Most lepton-proton scattering experiments have shown that quark spin accounts for only a portion of the proton spin. Studying polarized proton-proton collisions also is a practical approach to examining the spin content of protons.

The missing portion of the proton spin may come from gluons and from the movement of quarks and gluons relative to each other. A particular quark polarization can be measured through the parity violation during different collisions. Examples are W and Z bosons that are produced during the collisions.

The spin program at RHIC is intended to probe the internal structure of the proton. RHIC was the first machine in the world able to collide protons with both longitudinal and transverse polarization up to a beam energy of 250 GeV. Although other proton accelerators have been accelerating proton beams up to hundreds of GeVs, the acceleration of polarized protons to these high energies has to deal with possible beam depolarization as a result of depolarizing resonances. These resonances are produced by the fields required for accelerating and focusing the protons and occur at special energies.

The particle's spin is affected by its own motion. The particles traverse magnetic field that results in the precessing of their spins around different axes is defined by the field direction. The spin of a particle moving on closed orbit would precess around the vertical axis in perfect circular accelerator with vertical guiding magnetic fields; spin tune ν_s is defined as the rate of spin precession in a circular accelerator.

The linear field errors caused by the presence of misaligned magnets and by the betatron motion can add up coherently to tip the spin motion away from the vertical direction. The action of perturbed fields would be enhanced when the tune of perturbing magnetic fields equals the spin tune; this enhancement results in the depolarization of the beam. These resonances fall into three categories: coupled spin resonances, intrinsic resonances and imperfection resonances.

Imperfection resonances are caused by the misalignment of the magnets.

The spin would precess on the distorted closed orbit and the strength of the resonance is proportional to the vertical distortion of the closed orbit so produced. The only condition for that resonance to occur when $G\gamma$ equals an integer, is where $G\gamma$ is the spin tune.

The intrinsic spin resonances are encountered in nature as betatron motion is an intrinsic feature of particle motion in synchrotrons, even for the ideal case of perfectly aligned machine. They are enhanced when the spin tune equals the harmonic of vertical betatron tune.

In general, vertical and horizontal betatron oscillations are not coupled. Such coupling of motion can be introduced by skew quadrupoles and the rotational misalignment of regular quadrupole and solenoids. In the case of fouling resonances the horizontal betatron motion would drive additional spin perturbation that can cause the deviation of the spin from the vertical.

Several techniques were considered and employed to compensate for depolarizing resonances. The most powerful one was proposed by Derbenev and Kondratenko in 1978 using a powerful spin rotator called Siberian snake to rotate the spin vector around a horizontal axis by 180^o .

In the presence of one snake or several of them, the spin tune ν_s would be energy independent at fixed value, generally 1/2. As a major consequence, all first-order resonance conditions can be avoided. Hence, the spin tune would never cross any imperfection or intrinsic depolarizing resonances during the acceleration up to arbitrary high energy values. Two Siberian snakes are installed in each of RHIC proton rings[3].

Each snake consists of four super-conducting helical dipoles. Using 2 full snakes in each ring overcame both the intrinsic and imperfection resonances. By installing full snakes,the RHIC delivered high polarization up to 250 GeV, A partial snake also was used to preserve polarization; partial implies that the angle of spin rotation is less than 180^o.

A superconducting helical-field partial snake was installed at the AGS. With a strength of 5% the spin vector was rotated by 9^o ,i.e, sufficient to correct for all imperfection resonances in the AGS.

For calculating depolarizing resonances the DEPOL code generally has been used. However, DEPOL assumes that direction of the stable spin is vertical everywhere on the closed orbit and does not include the effect of snakes or other spin rotators. For RHIC it was necessary to do the assessment with full configuration of snakes and spin rotators, and also to consider the specific case of betatron coupling. The general algorithm used for calculating the spin resonances with arbitrary orientation of periodical spin was added to the ASPIRRIN code (Analysis of Spin Resonances in Rings). In course of this my work ASPIRRIN abilities were extended to calculate the strength of coupled spin resonances. The newly modified ASPIRRIN code allowed us to calculate both the horizontal and vertical resonance harmonics for the case of strong betatron coupling.

This thesis begins with an overview of the basic fundamental concepts for the spin dynamics in chapter 2. In Chapter 3 the formulations of contributions to spin resonances harmonics for rotated quadrupole ,solenoidal magnets and combined-magnets function are derived . In chapter 4 the newly modified code ASPIRRIN will be used for examining the major coupling sources of the depolarization in both AGS and RHIC. In Chapter 5 conclusions will be presented.

2 SPIN BEAM DYNAMICS IN SYNCHROTRON

In this chapter we introduce first the analytic basics of the spin motion. Then we discuss the spin depolarization resonances conditions.

Additionally the use of Siberian snakes in mitigating the spin depolarization resonances is covered in details, followed by introducing a general formula for spin resonance harmonics.

2.1 Basic Spin Motion

The dynamics of polarized beam particles is governed by the interaction of magnetic moment with the surrounding external electromagnetic fields. Since our main concern is the acceleration of polarized protons, it is more convenient to discuss the spin dynamics,particularly of spin 1/2 particles. For a beam of 1/2 spin particles, the degree of polarization is given by

$$P = \frac{N_+ - N_-}{N_+ + N_-} \tag{1}$$

$N_\pm$ are the populations in two spin states $|1/2, \pm 1/2 >$. The beam is 100% totally polarized when all the spin vectors are pointing to the same point. The polarization **P** of a beam is defined as the absolute value of the average spin taken over all N particles of the beam.[5]

$$P = |\frac{1}{N}\sum_{j=1}^{N} S_j| \tag{2}$$

Polarization is a collective property of the entire beam ; it cannot be understood by the behavior of individual particle , nor a specific point in phase space.

For a moving particles with magnetic and electric fields, the evolution of the spin motion in the laboratory frame using Lorentz transformation is described by the spin precession motion [3].

$$\frac{d\mathbf{S}}{dt} = \frac{e}{m\gamma} \times [(1+G\gamma)\mathbf{B}_\perp + (\mathbf{1}+\mathbf{G})\mathbf{B}_\parallel + (\mathbf{G}\gamma + \frac{\gamma}{\gamma+\mathbf{1}})\frac{\mathbf{E}\times\beta}{\mathbf{c}}] \tag{3}$$

where **S** is the spin vector in the rest frame, $\mathbf{B}_\perp$, $\mathbf{B}_\parallel$ are the magnetic field components, which are parallel and magnetic to the particle's momentum,e and γ are the charge and the relativistic Lorentz factor. For simplification, the electric field is neglected. Converting the Thomas-BMT equation to the accelerator reference system $(\hat{e}_x, \hat{e}_s, \hat{e}_y)$, related with a reference design closed orbit, we get the spin precession equation:

$$\frac{\mathbf{dS}}{d\theta} = (\mathbf{W_0} + \mathbf{w}) \times \mathbf{S} \tag{4}$$

where $\mathbf{W_0}$ is the spin precession vector on the reference orbit and $\mathbf{w}$ describes the precession due to orbit errors and betatron oscillations. The components of W are defined as by the following Ref. [2]

$$\mathbf{W_{0x}} = \nu_0 K_x \qquad \mathbf{W_{0y}} = (1+G)K_y \qquad \mathbf{W_{0z}} = \nu_0 K_z \tag{5}$$

While the components of $\mathbf{w}$ are

$$\mathbf{w_x} = (\nu_0 + 1)z^{"} + (\nu_0 + \frac{G}{\gamma_0})K_x P_\sigma + (1+G)K_y x^{'} \tag{6}$$

$$\mathbf{w_y} = (1+G)(K_x^{'} x + K_z^{'} z + \delta K_y - K_y p_\sigma) - (\nu_0 - G)(K_x p_x + K_z p_z) \tag{7}$$

$$\mathbf{w_z} = (\nu_0 + 1)x^{"} + (\nu_0 + \frac{G}{\gamma_0})K_z p_\sigma + (1+G)K_y z^{'} \tag{8}$$

As p_σ is the relative momentum and is defined as $\frac{\triangle p}{p_0}$ where the longitudinal motion is parametrized by a dimensionless time lag and is defined as $\sigma = \theta - w_0 t$ where w_0 is the circulation frequency along the reference orbit. $K_{x,y,z}$ are defined as the dimensionless scaled magnetic fields on the reference orbit and are given by $K_{x,y,z} = \frac{B_{x,y,z}}{B_0}$

For ideal case of a particle in the reference orbit the spin vector in one direction comes back to the same direction after one revolution. This vector is called the periodical stable spin vector $\mathbf{n_0}$. In most cases, $\mathbf{n_0}$ is oriented vertically. For vertical bending fields the spin vector will precess around the vertical direction faster than the orbital motion by values of $G\gamma$ per one revolution.

2.2 Spin Depolarization Resonances

With a vertically stable spin vector,$\mathbf{n_0}$, depolarization happens if the particle's spins deviated from the vertical direction because of the distorted orbit errors or betatron oscillations. Based on the main sources of spin resonances, they can be broken down into three categories; Intrinsic-,imperfection- and coupled-spin resonances.

Imperfection resonance can be caused by vertical closed orbit errors due to faults in aligning of the magnets. The resonances become more enhanced with time, become quite strong; may flip the direction of polarization. The effects of the misalignment are considered significant when the spin tune crosses an integer and that would lead to the domination of the horizontal fields that produce spin precessions away from the vertical direction. Careful corrections are needed for the distorted closed orbit to limit the delivered horizontal field components when the imperfection resonances are crossed. Intrinsic resonances are driven by the natural vertical focusing oscillations. The condition for the occurrence of standard intrinsic resonances is when the spin precessions build up coherently due to the vertical betatron oscillations deflections. The condition can be written as

$$G\gamma = KP \pm \nu_y \tag{9}$$

where K is an integer and P is accelerator lattice super-periodicity. In the presence of the coupling elements, such as the rotated quadrupole ,rotational misalignment of the regular quadrupole and solenoids in the lattice would cause the horizontal tune to come into play. The resonance condition is $G\gamma = N \pm \nu_x$ where ν_x is the horizontal betatron tune and N represents an integer. All of the depolarizing resonances are strong when N is a multiple of the lattice's super-periodicity. Nevertheless, a large fraction of the depolarizing resonances could be eliminated by a lattice of higher periodicity compared to a low super-periodicity lattice.

2.3 Siberian Snakes and spin rotators

The concept of Siberian Snakes has been imposed for correcting spin resonance to enable the acceleration of spin polarized beams up to high energies without crossing the depolarizing resonances. By using the Siberian snakes, the spin vector would be rotated by an angle of less than or equal to 180^o around an axis in the horizontal plane. Full Siberian snakes are composed of sequence of magnets so that the overall spin rotations would add up to 180^o. With a single full snake in a ring or with a properly placed pair of snakes the spin tune will be fixed, usually to 0.5, so that no first order resonances have to be crossed. The value of the spin tune is independent of the beam energy. Two snakes are used in RHIC rings to maintain a reasonable amount of polarization as the beam is accelerated to high energies up to $E = 250$ GeV. Two full snakes installed on opposite sides in RHIC to assure the vertically stable spin direction as shown in the following figure 1

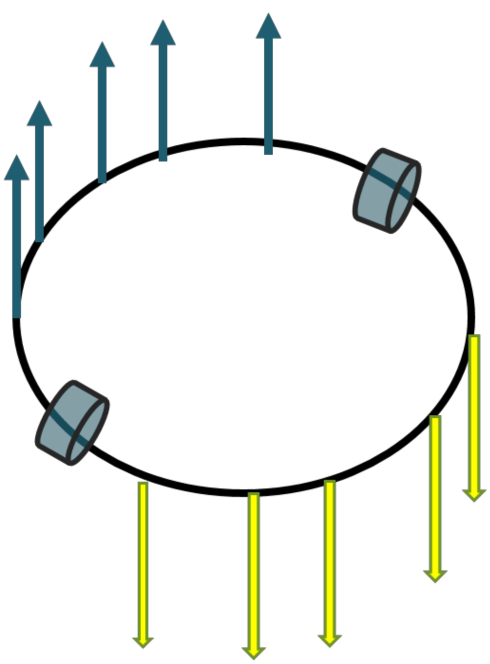

Figure 1: Schematic of spin vector in the ring with two spin

Another polarization control device used in accelerators is the spin rotator. They are used to rotate the stable spin direction from the vertical to the horizontal plane, and back again, as most particle physics experiments with polarized beams require longitudinally polarized beams. The spin rotators concept did exist as the spin and the orbit rotate through different angles when passing through a magnet. As with the snakes the amount of rotation essentially is independent of energy. However, in RHIC there is an energy-dependent precessions due to the net horizontal bend between the rotator and the interaction region. These precessions must be compensated to maintain the required longitudinal polarization. Furthermore,in RHIC,four rotators orient the spin axially at two collisions points. A partial snake , that rotates the spin by less than 180^o, is very effective in counteracting imperfection resonances and was used successfully in the AGS. Partial snakes represent a tool suitable for use in low-to-medium energy synchrotrons.

2.4 General formula for spin resonance harmonics

The stable spin direction on the design closed orbit may deviate from the vertical even when various kinds of spin rotating devices are applied. Since rings are frequently designed so that n_o is not vertical everywhere, a new algorithm was extended to allow the identification of the spin-resonance harmonics with this complex configuration of the magnetic field in the design orbit. We implemented the algorithm of the intrinsic resonance harmonic calculation in a program code that was added to the existing code, ASPIRRIN.

A 2π periodical coordinate system would be used to define the spin motion so that the spin would rotate on the design orbit with a constant angle $2\pi\nu_{spo}$. This coordinate system is defined by the right-handed orthonormal triad $(\mathbf{l}, \mathbf{m}, \mathbf{n_o})$, where $\mathbf{n_o}$ is the stable direction of spin on the design orbit, and the vectors $\mathbf{l}, \mathbf{m}$ can be derived using the eigen-solutions of the one turn spin map on the design orbit. We also will use the complex vector $\mathbf{k} = \mathbf{l} + i\mathbf{m}$. In fact, as the spin tune on the design orbit can be defined up to the integer number, the choice of special periodic system also is unlimited. In the accelerator with the snakes, we will use the periodical coordinate system in which the spin tune is 1/2 (or other constant value between 0 and 1). To describe the invariant spin field related with the orbital oscillations, we can parametrize by a complex variable $\alpha$and the spin precession presentation would be described as:

$$\mathbf{n} = \sqrt{1 - |\alpha|^2}\mathbf{n_o} + \Re(\alpha \mathbf{k}^*) \tag{10}$$

The spin motion equation can be written as

$$\frac{d\alpha}{d\theta} = i\nu\alpha - i\mathbf{w} \cdot \mathbf{k}\sqrt{\mathbf{1} - |\alpha|^{\mathbf{2}}} \tag{11}$$

where $\mathbf{w}$ describes the spin's perturbation by the orbital oscillations. We consider the spin perturbation as the linear form in the orbit variables: $\mathbf{w_j} =$

$\mathbf{T_{jq}x_q}$. Also, the linear orbital motion can be presented as: $x_q = F_{qr}A_r$, where F_{qr} is the matrix compiled from the eigen-vectors of the orbital motion, and A_r is the vector of the orbital amplitude. We note that currently we have implemented the algorithm for the transverse betatron motion, and therefore the q and r indexes run from 1 to 4.

The first-order spin resonance harmonics are defined as the coefficients of the Fourier decomposition of the spin perturbation term:

$$\mathbf{w} \cdot \mathbf{k} = \sum_{\mathbf{r},\mathbf{p}} \mathbf{w_{rp}} \mathbf{e^{i\nu_{rp}\theta}} \tag{12}$$

where $\nu_{rp} = p + Q_r$. The Q_r are the components of the betatron tune set: $(Q_x, -Q_x, Q_y, -Q_y)$. For strong betatron coupling, the tunes of the betatron modes would be more convenient for the calculation. Since we are only considering linear fields and linear orbital motion, the resonance harmonics are called first-order in the sense of both the spin motion and the orbital motion.

Keeping only the terms corresponding to the betatron motion:

$$\mathbf{w} \cdot \mathbf{k} = \sum_{j,q,r} T_{jq} F_{qr} A_r k_j = \sum_r A_r e^{iQ_r\theta} \tilde{V}_r \tag{13}$$

Here, the index j runs from 1 to 3, while the indices q and r run from 1 to 4. Futher, we introduced a four-component vector $\tilde{V}$, that is the periodical function of the azimuth θ and, therefore, can be expanded into the Fourier series:

$$\tilde{V}_r = [e^{-iQ_r\theta} \sum_{j,q} T_{jq} F_{qj} k_j] = \sum_p \tilde{v}_{rp} e^{ip\theta} \tag{14}$$

Comparing the expressions (13) and (14) with (12) we see that $\tilde{v}_{rp}$ presents the spin resonance harmonic v_{rp} normalized by the corresponding betatron amplitude A_r. To calculate $\tilde{v}_{rp}$ we need to take the integral over one turn:

$$\tilde{v}_{rp} = \frac{1}{2\pi} \int_0^{2\pi} \tilde{V}_r e^{-ip\theta} d\theta = \frac{1}{2\pi} \int_0^{2\pi} [e^{i(\nu - \nu_{rp})\theta} \sum_{j,q} T_{jq} F_{qr} k_{0j}] d\theta \tag{15}$$

In the program the integral over one turn is taken via element-by-element integration.

$$|w_{l1m}| = |w_1| + |w_{-1}|; \qquad |w_{l1s}| = ||w_1| - |w_{-1}|| \tag{16}$$

Here w_{l1m} presents main linear harmonic, which amplitude is large. And w_{l1s} denotes secondary linear harmonic, which is small and can be considered as a perturbation.

In the similar way the program can calculate the intrinsic resonance harmonics, which are located further away from the spin tune (w_2,w_{-2},w_3,w_{-3}, and so on) and the corresponding linear harmonics. Calculating the harmonics of the resonances related with the horizontal betatron motion which is considered the new modification for the ASPIRRIN code, and in the case when this motion has a non negligible effect on spin dynamics (betatron coupling).

3 TECHNIQUES FOR CALCULATING OF THE RESONANCE STRENGTH

The transverse motion can be treated separately in the horizontal and vertical plane. Proper selection and alignment of the magnets are the main means to achieve a better standing for the concept of transverse motion. The perturbation caused by the rotated quadrupole would create a linear betatron coupling for both the vertical and horizontal betatron oscillations. The most general sources that would introduce coupling to the beam motion are the rotated quadrupole, the skew quadrupoles and the solenoid fields. To compensate for the coupling in controlled way, its own dynamics must be understood in more detail. Initially betatron coupling was considered as unpreventable effect and unfortunately it has become as an intrinsic feature for many accelerator proposals.

The general equations of motion in presence of solenoid and rotated quaduroples magnets are Ref.[6]

$$x^{"} + gx = -y\kappa_c + y^{'}K_s + y\frac{1}{2}K_s^{'} \qquad y^{"} - gy = -x\kappa_c - x^{'}K_s + x\frac{1}{2}K_s^{'} \quad (17)$$

where $\kappa_c = g \cdot sin(2\phi)$ are defined by the quadrupole rotation angle ϕ and the quadrupole's field strength g. K_s is the normalized solenoid field and is defined as

$$K_s(y) = \frac{eB_s(y)}{p} \quad (18)$$

And $B_s(y)$ is the longitudinal field component.

For (17) the analytic solution cannot be determined because the distribution of solenoid and an arbitrary rotated quadrupole is random and could change many times depending on the different parameters to take into account. Consequently, we discuss the solutions for individual magnets only.

3.1 Coupled Spin Resonance For Rotated Quadrupole

In order to obtain $\tilde{v}_{rp}$ for rotated quadrupole we start with considering the following equations of particle motion of particle in arbitrarily rotated quadrupole:

$$x^{''} = -g_c x - \kappa_c y \tag{19}$$

$$y^{''} = g_c y - \kappa_c x \tag{20}$$

where $g_c = g \cdot cos(2\phi)$

From (20) we can get the expressions for the orbital motion eigen-vectors:

$$F_{3r} = -AF_{3r}^{''} + BF_{1r}^{''} \tag{21}$$

$$F_{1r} = AF_{1r}^{''} + BF_{3r}^{''} \tag{22}$$

Here:

$$A = -cos(2\phi)/g \tag{23}$$

$$B = -sin(2\phi)/g \tag{24}$$

The spin response formalism treats transverse coupling with full or partial snakes and spin rotators; however, the main assumption is that the orbital and spin motion are enhanced by the presence of the magnetic fields only.The components of the horizontal and vertical components of the spin perturbations in a quadrupole are given by the following:

$$w_x = (1 + \nu_0) y^{''} \tag{25}$$

$$w_y = -(1 + \nu_0) x^{''} \tag{26}$$

where $\nu_0 = G\gamma$, with magnetic moment anomaly G and relativistic factor γ. From (12)and (26) the contribution to a spin resonance harmonic produced by a betatron mode oscillating with the tune Q_r from a quadrupole can be formulated as the following:

$$\tilde{v}_{rp}^{quad} = (1 + \nu_0)(k_{0x} I_3 - k_{0y} I_1) \tag{27}$$

where I_1 and I_2 are the integrals defined as

$$I_1 = \int_{\theta_1}^{\theta_2} F_{1r}^{''} e^{i\delta_{rp}\theta} d\theta \tag{28}$$

$$I_3 = \int_{\theta_1}^{\theta_2} F_{3r}^{''} e^{i\delta_{rp}\theta} d\theta \tag{29}$$

and $\delta_{rp} = \nu - (p + Q_r)$ describes the detuning of the spin tune from a resonance tune. And θ_1 and θ_2 correspond to the entrance and the exit of the quadrupole.

To find the integrals (28) and(29) one can apply twice the integration by parts and the expressions (22) to get the set of linear equations for I_1 and I_3. Resolving the equations one generates the solution in the matrix form:

$$\begin{pmatrix} I_1 \\ I_3 \end{pmatrix} = M^{-1} \begin{pmatrix} C_1 \\ C_3 \end{pmatrix} \tag{30}$$

where M, C_1 and C_3 are expressed as

$$M = \begin{pmatrix} 1 + A\delta_{rp}^2 & B\delta_{rp}^2 \\ B\delta_{rp}^2 & 1 - A\delta_{rp}^2 \end{pmatrix} \tag{31}$$

$$C_3 = [(F_{3r}^{'} - i\delta_{rp} F_{3r}) e^{i\delta_{rp}\theta}]_{\theta_1}^{\theta_2} \tag{32}$$

$$C_1 = [(F_{1r}^{'} - i\delta_{rp} F_{1r}) e^{i\delta_{rp}\theta}]_{\theta_1}^{\theta_2} \tag{33}$$

Then the final formula which defines the contribution from an individual rotated quadrupole to the spin resonance harmonic is:

$$\begin{aligned} \tilde{v}_{rp}^{quad} = & Y[k_{0x}(\delta_{rp}^2 \sin(2\phi) C_1 + (g - \delta_{rp}^2 \cos(2\phi)) C_3) \\ & -k_{0y}((g + \delta_{rp}^2 \cos(2\phi)) C_1 + \delta_{rp}^2 \sin(2\phi) C_3)] \end{aligned} \tag{34}$$

where

$$Y = g(1 + \nu_0)/(g^2 - \delta_{rp}^4) \tag{35}$$

It is clear from (34) that spin resonance harmonics is a function of the rotation angle. The same formula would be applicable for the the skew quadrupole if we assign a value for the rotation angle round its longitudinal axis as 45^o. Summing these contributions from all quadrupoles yields the full spin resonance harmonic defined by r and p.

3.2 Coupled Spin Resonance For Solenoid

The strength of this coupled spin resonances will be a function of the coupling coefficient, which itself is a function of the strength of the distributed coupling elements in the lattice and the distance between the horizontal and vertical tunes. So, As we go through the derivation of the formula for the resonance strength produced by the quadrupole, it also was crucial to go through the same one for the other element which contributes to a huge fraction of producing the polarization loss.

To obtain $\tilde{v}_{rp}$ for the solenoid, we start by considering the following equations of motion of particle in solenoidal field as in [7]

$$x^{"} = y^{'} + \frac{1}{2}K_s y \qquad y^{"} = x^{'} - \frac{1}{2}K_s x \tag{36}$$

Using floquet Transformations the equation of motion can be written as the following

$$F_1^{"} = F_2 + \frac{1}{2}K_s F_3 \qquad F_2^{"} = F_4 - \frac{1}{2}K_s F_1 \tag{37}$$

where K_s is the normalized field of the solenoid

The components of longitudinal and vertical components of the spin perturbations for the body of the solenoid are given by:

$$w_x = (G - \nu_0)K_s x^{'} - \frac{1}{2}(1 + \nu_0)K_s x \tag{38}$$

$$w_y = (G - \nu_0)K_s y^{'} - \frac{1}{2}(1 + \nu_0)K_s y \tag{39}$$

And the spin perturbations for produced at edges are

$$w_x = -\frac{1}{2}(1 + \nu_0)K_s x[\delta(\theta - \theta_1) - \delta(\theta - \theta_2)] \tag{40}$$

$$w_y = \frac{1}{2}(1 + \nu_0)K_s y[\delta(\theta - \theta_1) - \delta(\theta - \theta_2)] \tag{41}$$

Using the same technique as previously mentioned for the quadrupole, the contribution of spin resonance harmonics from a solenoid was derived as:

$$
\begin{aligned}
\tilde{v}_{rp}^{sol} =& \frac{(G-\nu_0)K_s}{\delta^2-(GK_s)^2}(-i\delta_{rp}[e^{i\delta_{rp}\theta}((F_{2r}+\frac{1}{2}K_sF_{3r})k_{0x} \\
&+(F_{4r}-\frac{1}{2}K_sF_{1r})k_{0y})]_{\theta_1}^{\theta_2}+GK_s[e^{i\delta_{rp}\theta}((F_{2r}+\frac{1}{2}K_sF_{3r})k_{0y} \\
&-(F_{4r}-\frac{1}{2}K_sF_{1r})k_{0x})]_{\theta_1}^{\theta_2}) \\
&+\frac{(1+\nu_0)K_s}{2}[e^{i\delta_{rp}\theta}(F_{1r}k_{0x}+F_{3r}k_{0y})]_{\theta_1}^{\theta_2}
\end{aligned}
\tag{42}
$$

3.3 Spin Resonance For The Combined Magnet Function

A combination of bending and focusing magnets is required for specific applications. Sometimes functions like bending/focusing are combined into a single element. In general, combined magnets are used to help reducing the length (and, therefore , the cost) of a beamline. The combined function also can help to improve the dynamical properties of the lattice.

The general expression for the equation of motion can be stated as follows:

$$y^{"}-gy=0 \qquad x^{"}+(g+k_y^2)x=0 \tag{43}$$

where the focusing effects from the quadrupole magnet and that from a bending dipole may be combined into one parameter as stated in the previous equation

$$\mathcal{K}=g+K_y^2 \tag{44}$$

To conclude the general spin perturbations, $K_y \neq$ zero and the gradient has a non zero value too.The spin perturbations are simply described by

$$w_x=(1+\nu_0)y^{"} \qquad w_y=-(1+\nu_0)x^{"} \qquad w_z=0 \tag{45}$$

$$\tilde{v}_{rp}^{c.f}=\int_{\theta_1}^{\theta_2}k_xw_xe^{-i\delta_{rp}\theta}d\theta+\int_{\theta_1}^{\theta_2}k_yw_ye^{-i\delta_{rp}\theta}d\theta \tag{46}$$

Where k_x and θ are defined as

$$k_x = k_{0x} \cos \nu_0 \psi + k_{0y} \sin \nu_0 \psi \quad , \theta = \frac{\rho \psi}{R} \tag{47}$$

And where ρ is the local bend radius in the horizontal plane and R is the ring's average radius .

The horizontal spin perturbation then can be expanded as

$$\int_{\theta_1}^{\theta_2} k_x w_x e^{-i\delta_{rp}\theta} d\theta = k_{0x} \int_{\theta_1}^{\theta_2} W_x \cos(\nu_0 \psi) e^{-i\delta_{rp}\theta} d\theta + k_{0y} \int_{\theta_1}^{\theta_2} W_y \sin(\nu_0 \psi) e^{-i\delta_{rp}\theta} d\theta \tag{48}$$

The general formula for the combined magnet function is derived from the previous equations

$$\begin{aligned} \tilde{\upsilon}_{rp}^{c.f} = & \frac{K_{0x}}{2} M_1 [(F_{4r} - i(\delta_{rp} + b) F_{3r}) e^{i(\delta_{rp}+b)\theta}]_{\theta_1}^{\theta_2} + \\ & \frac{K_{0x}}{2} M_1 [(F_{4r} - i(\delta_{rp} - b) F_{3r}) e^{i(\delta_{rp}-b)\theta}]_{\theta_1}^{\theta_2} - \\ & i \frac{K_{0y}}{2} M_2 [(F_{4r} - i(\delta_{rp} + b) F_{3r}) e^{i(\delta_{rp}+b)\theta}]_{\theta_1}^{\theta_2} - \\ & i \frac{K_{0y}}{2} M_2 [(F_{4r} - i(\delta_{rp} - b) F_{3r}) e^{i(\delta_{rp}-b)\theta}]_{\theta_1}^{\theta_2} + \\ & K_z M_3 [(F_{2r} - i\delta_{rp} F_{1r}) e^{i\delta_{rp}\theta}]_{\theta_1}^{\theta_2} \end{aligned} \tag{49}$$

where M_1 and M_2 and M_3 are defined as

$$M_1 = (1 + \nu_0)(\frac{k + k_y^2}{k + k_y^2 + (\delta_{rp} - b)}) \tag{50}$$

$$M_2 = (1 + \nu_0)(\frac{k + k_y^2}{k + k_y^2 + (\delta_{rp} + b)}) \tag{51}$$

$$M_3 = (1 + \nu_0)(\frac{k}{k + \delta_{rp}^2}) \tag{52}$$

4 STUDY OF THE BETATRON COUPLING AT RHIC and AGS

In this section we show calculations which were done with ASPIRRIN after modifying it with the new spin coupling calculation algorithm.

The main motivation for our calculation is to test the code's ability for calculating the resonance strength amplitude in the general case for the betatron oscillation coupling. The calculations were done for the RHIC lattice with two Siberian Snakes, which had uncoupled tunes $Q_x = 27.685$, $Q_y = 29.673$ with betatron amplitudes of $10\mu m\cdot$ rad in both planes.

The spin tune in RHIC with the Siberian Snakes is equal to 0.5, and because of that in all plots in this section we show the result for resonance harmonics that are closest to 0.5 value (that is with the resonance tunes in $[0-1]$ range). Moreover, we add the amplitudes of the two spin resonance harmonics, one below and another above 0.5, corresponding to the same betatron tune (Q_I or Q_{II}). This gives us the resonance amplitude of the linear resonance harmonic, which is more natural when considering an accelerator with the Siberian Snake [1].

First, we started to test the new modified code by applying a small random error roll for one of the quadrupoles in the IR, and observed by how much the sensitivity of the measurement of resonance strength varied after applying the error. In figure 2 rolling the quadruple by 2 mrad introduced high values for vertical resonance strength by a maximum value of almost 0.4 for G_γ of value 425 with minimum betatron tune split ΔQ_{min} of 0.017.

In figure 3 we noticed that even with the small value of the error roll of 2 mrad , it is still strong enough to cause a modest excitation of the horizontal strength resonance.

A more flexible approach is used to observe the variation in the behavior of the resonance strength by examining another case for a rolled Quadrupole by 45^o.

By randomly varying the gradient strength assigned for each skew quadrupole corrector located at the IR for the range of $10^{-3}\ m^{-2}$ the vertical polarization loss can be reduced by turning the skew quadrupole correctors for betatron amplitude =10 μm as is declared in figure 4.

A typical pattern of behavior appeared for the horizontal resonance strength, consistent with the previous case. Thus, for rolled quads, the horizontal resonance amplitudes increase up to value of 0.2 above the zero values that

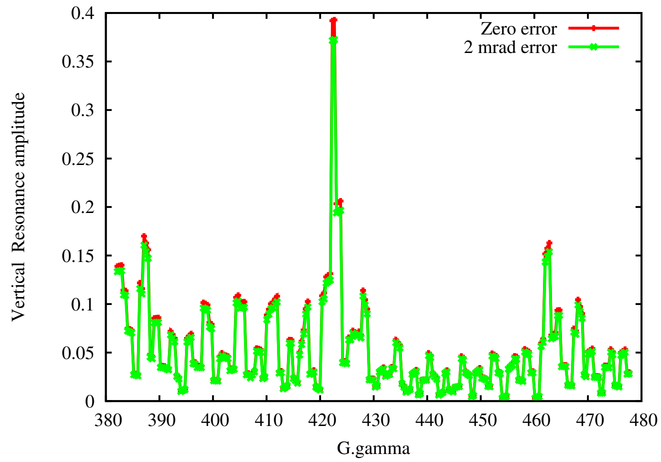

Figure 2: calculated vertical resonance strength from the artificial quadruple roll errors.

are produced when the Skew Quads are off as shown in figure 5. A better understanding of the behavior of these resonances is much needed so that an effective remedy can be implemented.

If we start to vary the field strength of different IR skew quadrupoles correctors, which are used to compensate for coupling caused by the rolled quadrupoles, we can observe how the spin-resonance harmonics depend on the betatron coupling, which is characterized by a minimum betatron tune split ΔQ_{min}. Figure 6 depicts the typical behavior of the vertical and horizontal resonance harmonics caused by varying the gradient strength of skew-quadrupole (SQ06C2B); thus, the horizontal resonance amplitude increases non linearly, while the vertical amplitude behaves in the opposite way.

Figure 7 demonstrates another typical feature of the resonance harmonic dependence observed during the skew quadrupole variation studies. While the vertical and horizontal harmonics change the sum of their squares remains approximately constant: $\tilde{v}_{hor}^2 + \tilde{v}_{ver}^2 \approx const.$ This indicates a rotational type transformation between the horizontal and vertical resonance harmonics when betatron coupling is introduced.

Next, we considered the actual coupling errors present in RHIC, where the strong sources of the the betatron coupling are due to the rotation of the IR quadrupoles. These quad rolls are well known from the beam-based and magnetic measurements. Thus, we applied the known rolls and well as

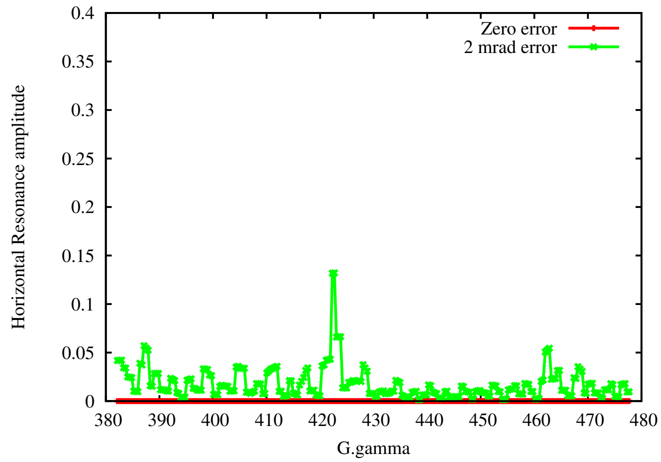

Figure 3: Excited Horizontal resonance strength generated by the artificial quadruple roll errors.

the local skew quadrupole corrector strengths used at RHIC operation [4]. Figure 8 shows the calculated values of the vertical resonance amplitudes versus $G\gamma$ in a region of strong resonances in RHIC. The figure reveals some reduction of the values of vertical harmonics amplitude after applying the quad rolls and the skew quad correction.

Figure 9 shows the calculated values of the the horizontal resonance amplitude versus $G\gamma$; the amplitudes are excited up to 0.15 when the actual quadrupole rolls and skew quad corrector strengths are used (in this case $\delta Q_{min} \approx 0.01$). Thereafter, we used optimized values for certain local skew quadrupole gradients, which reduced the δQ_{min} coupling parameter to 0.001. The horizontal resonance harmonics clearly are reduced in this case.

The plot also shows that the horizontal resonance amplitudes are suppressed to zero when the skew quadrupoles are turned off and the quadrupole rolls are 0.

The horizontal and longitudinal solenoids can act in a coherent fashion to depolarize the beam. Strong betatron oscillation coupling is introduced into RHIC enhancing the coupled resonances. For a given range of G_γ the range of the corresponding solenoidal filed strength would vary in a similarly to that presented for the skew quadrupole. As expected, with stronger solenoidal field comes stronger resonances as shown by the vertical resonance amplitude as a function of G_γ in Fig 10.

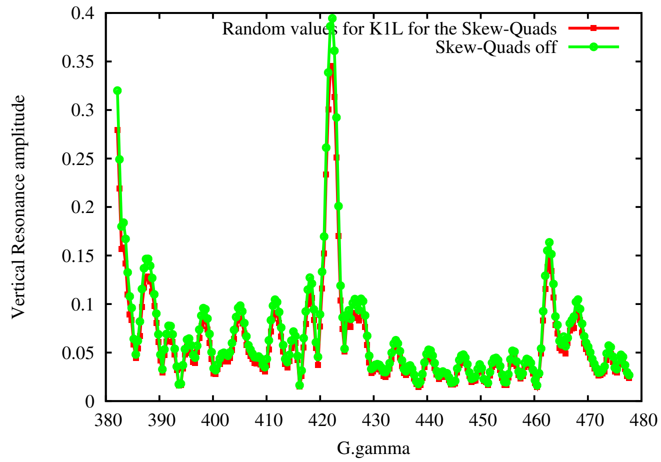

Figure 4: calculated vertical polarization loss as a function of G.gamma.

We found that the behavior of strength of the horizontal resonances matched fairly well with previous predictions of the exciting of the Resonance strength with increasing the energy for the case when the solenoidal magnets are on, as shown in fig11.

The same conservation law for the resonance strength that was predicted for the skew quadrupole is applicable for the case of solenoid as illustrated in fig 12

After modifying ASPIRRIN with the combined magnet resonance strength calculations, we were able to better compare our findings against DEPOL's predictions. For a vertical emittance of 10 mrad and a zero horizontal one, the assessments from ASPIRRIN and DEPOL calculations agreed well as shown in fig 13.

We undertook another study using the modified ASPIRRIN code for comparing the resonance strength of AGS lattice with DEPOL data; in this case, the agreement with DEPOL is not good and we plan to investigate this further.

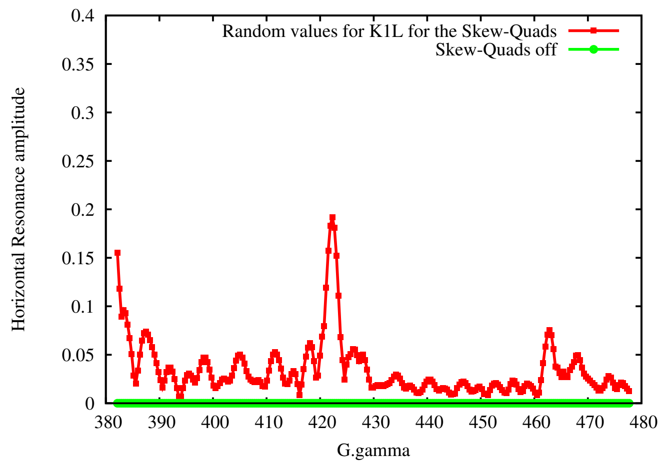

Figure 5: calculated Horizontal polarization loss as a function of G.gamma.

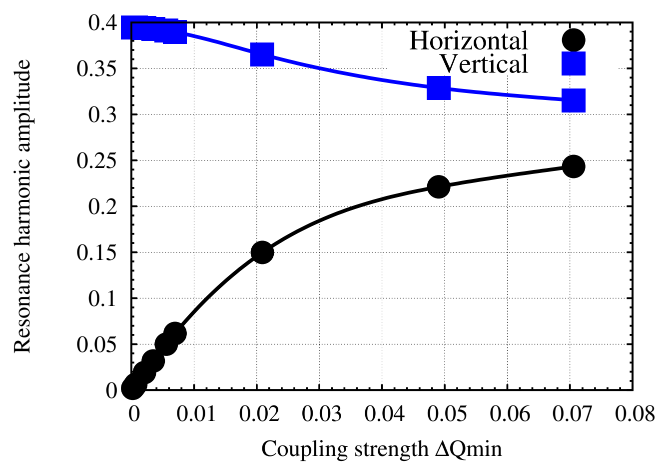

Figure 6: Dependence of the vertical and horizontal resonance harmonic amplitudes on the coupling strength when varying a skew quadrupole corrector. $G\gamma = 422.3$.

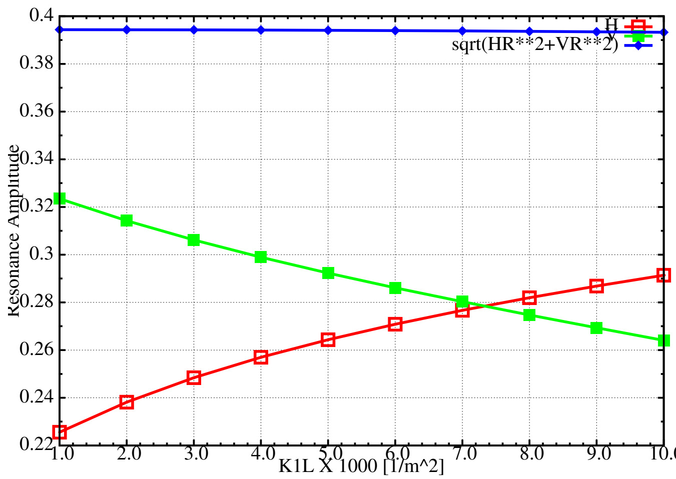

Figure 7: Conservation of the sum of squares of the harmonic amplitudes of the horizontal and vertical resonance when varying a skew quadrupole corrector(SQ08C2B). $G\gamma = 422.3$.

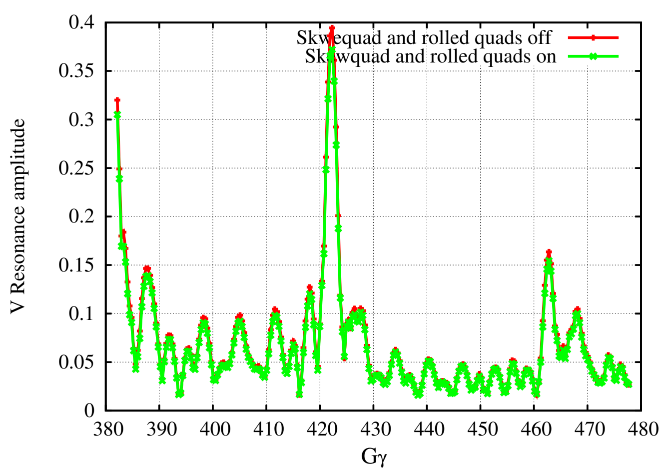

Figure 8: Calculated vertical spin resonance harmonics with and without the actual RHIC IR quadrupole rolls and local IR skew corrections.

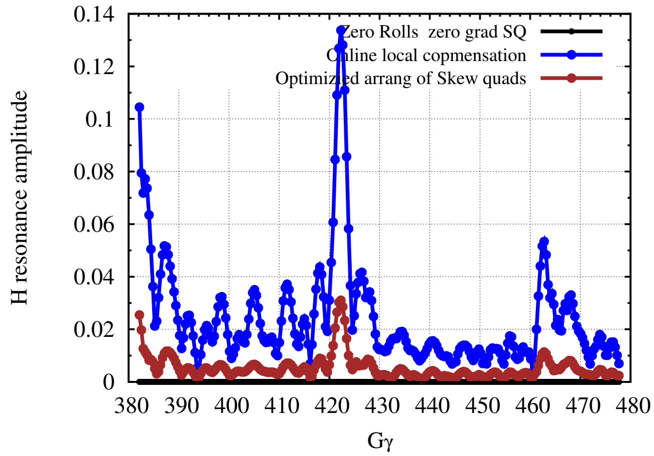

Figure 9: Calculated horizontal spin resonance harmonics with and without the actual RHIC IR quadrupole rolls and local IR skew corrections. The result for optimized corrections is also shown.

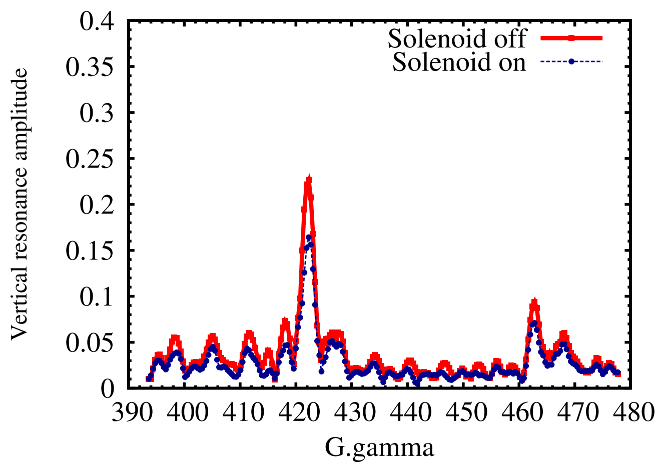

Figure 10: Calculated vertical spin resonance harmonics for zero solenoidal fields and when the solenoidal magnets are turned on versus G_γ.

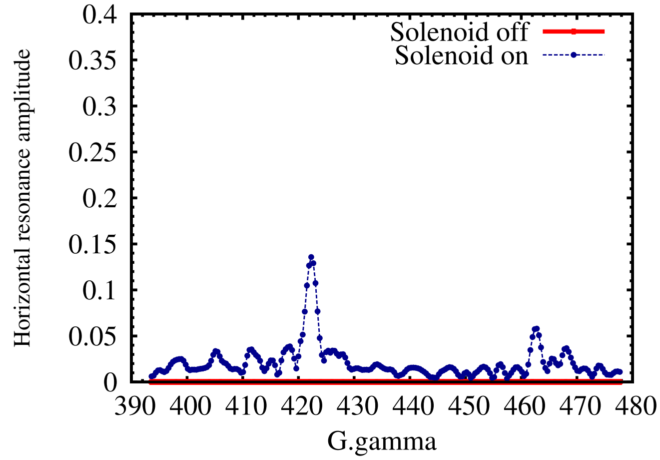

Figure 11: Calculated horizontal spin resonance harmonics for zero solenoidal fields and when the solenoidal magnets are turned on versus G_γ.

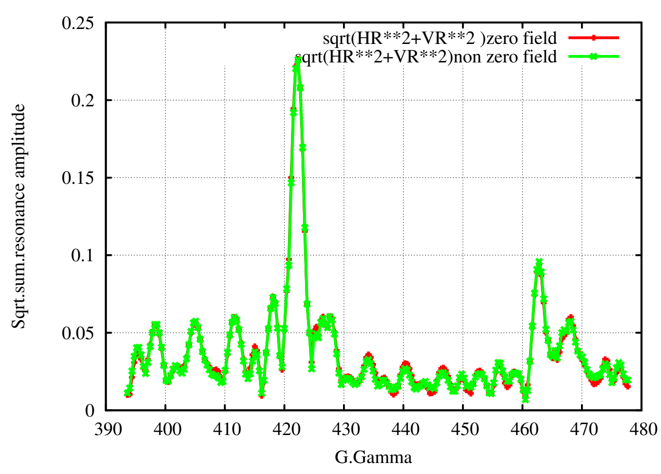

Figure 12: Conservation of the sum of squares of the horizontal and vertical resonance harmonic amplitudes while varying the solenoidal field strength.

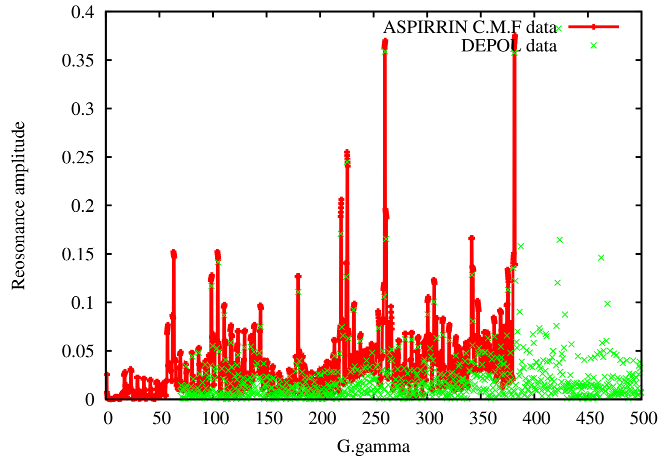

Figure 13: Comparison of the modified ASPIRRIN by combined magnet calculation For RHIC lattice with DEPOL Results.

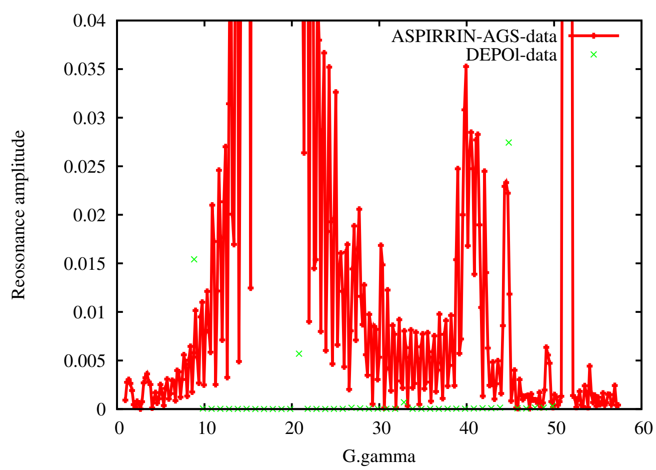

Figure 14: Comparison of the modified ASPIRRIN by combined magnet calculation For AGS lattice with DEPOL Results.

5 Conclusions

The calculation of spin resonance harmonics with coupled transverse betatron motion was implemented in the ASPIRRIN code.The modified code was used to study coupled spin resonances in RHIC.

We noticed a rotational-type transformation of the resonance strength of vertical and horizontal harmonics when introducing the betatron coupling and the conversation for resonance amplitude was proved for the two cases, which are Quadrupole and Solenoid.

The local coupling correction for actual values of quad rolls and interaction region skew quad correctors at 6 Interaction regions in RHIC excites the horizontal resonance strength up to 0.15 at G of the value of 422.3.

Optimizing the arrangement of the skew quadrupoles to better compensate for the local coupling helps to reduce the horizontal spin resonance amplitudes even before applying the global coupling correction. Modifying the ASPRRIN code with the combined magnet resonance strength calculations allowed us to observe the significant agreement between ASPIRRIN and DEPOL calculations for the vertical Resonance strength.

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