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Abstract of the Dissertation

A quantitative policy analysis of Bronx County public high school students' mathematics course completion

by

Ramón Emilio Fernández

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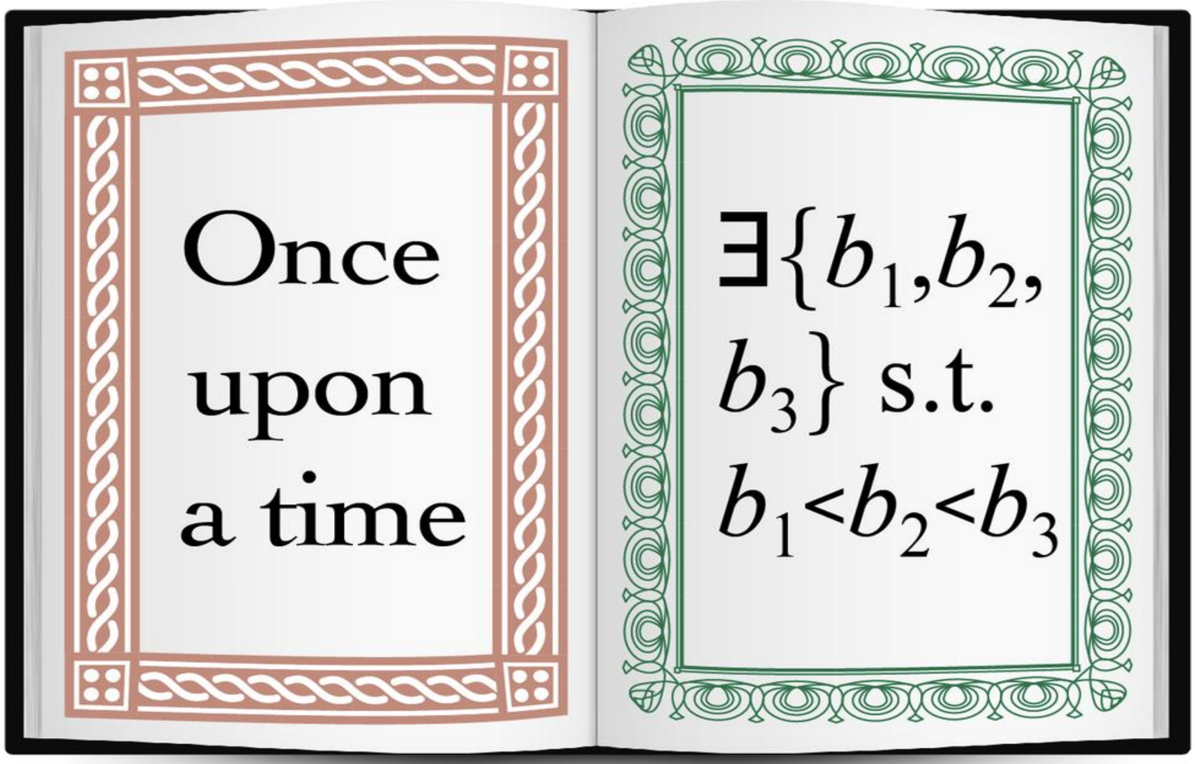
Education reform remains a work in progress. Research, government, and policy reports emphasize that a high-quality high school mathematics education is key to the future of the nation. Technical and research work in science, technology, engineering, mathematics (STEM), social sciences, and life sciences have been identified as essential requirements to bolster the economic growth, national competitiveness, and security of the United States. The academic preparedness, college aspiration, persistence, graduation, as well as earnings of high school graduates who complete college preparatory and advanced math courses are all positively, strongly, and significantly correlated to their math course completion. Despite various efforts from the federal government, such as the No Child Left Behind Act, warnings, and recommendations from national reports such as A Nation at Risk, and the documented importance of high school mathematics in preparing highly skilled and critically thinking workers, ethnic minorities and economically disadvantaged students remain severely underrepresented in STEM disciplines. High schools located in inner-city, high-minority and economically disadvantaged neighborhoods offer college preparatory and advanced math courses at rates that are significantly lower than schools located in suburban or rural neighborhoods. White, Asian, or economically advantaged students surpass Black and Hispanic students in the completion of standard through advanced level math courses. However, Black and Hispanic students surpass White and Asian students in Basic through Preformal math courses. We developed and answered the following research questions within the aforementioned context: 1. How are the key socio-demographic and performance indicators of New York City public high schools distributed? 2. How comparable are key demographic and academic performance indicators of Bronx public high schools with a reported 4-year mathematics requirement to those of other Bronx public high schools? 3. How comparable are the high school mathematics course completion rates of Bronx County public high school students attending schools with a reported 4-year mathematics requirement to other Bronx public high school students?

Dedication Page

This work is dedicated to the many ethnically underrepresented or economically disadvantaged students, like myself, who would benefit from receiving positive examples, successful role models, as well as positive media coverage from the Black and Hispanic communities around the globe. Stories and statistics about defeat, disgrace, or negativity abound, but they are not the only ones among our people. I have no doubt that, if as a collective, we provide you with the opportunities and tools that help people succeed in life and accomplish their life goals, you will rise to your greatest potential, if that is something you choose to do.

Frontispiece

As a mathematician, humanist, and engineer I believe that math is a learnable/translatable art.



Mathematics is a language. We should teach it that way. To solve the problems of math education, we should study how language education solves the same problems.

I don’t know who wrote the above statement for am unable to properly acknowledge the author; whether mathematics is a language or not is irrelevant to me. What is relevant to me is that to understand mathematics, one has to understand the language through which mathematics is taught. Mathematics allows us to use our language in the most abstract way, but before we can abstract our languages, we need to understand the basic foundations of our languages: grammar. We need to provide our students a stronger foundation of syntax and morphology. We can acquire vocabulary by means of social and cultural interactions. But understanding the grammar is more complex. Mathematics should, I propose, be accessible to us in such a way that translating it into real life becomes second nature to us given that math is always around us. I read somewhere, that *Mathematics may not teach us how to add love or subtract hate. However, it can give us many reasons to hope that every problem has a solution.* Freedom, I believe, is the ability to think for ourselves, so that we can comprehend the ramifications of our decisions. Freedom, I believe, can be more easily achieved, if we can abstract and simultaneously consider more than one choice; math is a great tool to help us do just that; to help us find solutions…

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Preface

Writing this thesis was very fulfilling. Nonetheless, settling into it, developing it, selling it, accepting it, was not. While this work was completed in the year of 2016, it probably started to form, within my thoughts, by the year of 2004. It was during that year that the terms “minority”, “underrepresented”, “underachievement”, among other, started to be implanted deeply within my universal grammar, and became part it. Moreover, it was during the years of 2005 through 2009, after successfully completing Engineering and Math internships, completing dual degrees in Pure Math and Hispanic Languages and Literature, and realizing that I was the only Black and Latino male in each one of my math classes, that I started to make more sense of what it really means to be Hispanic or Latino, Black, and economically disadvantaged within the academic circles of higher education. Based on my experiences and feedback as an Engineering student, I think that I would have become a fine professional engineer. After all, I loved that work very much. But, I felt that as an engineer, I was not going to have much access to the social aspects of my work. I then decided that becoming a mathematician and humanist may do the trick. I love both, pure math and the humanities, but I also found both to be too isolationist for my very own good. Then, it clicked in my head that a multidisciplinary program, where I could devote time to my math and engineering love, my interest and passion for social welfare and policy, and my innate calling for teaching, was a good place to continue my academic journey. It was indeed a good place to start, but it was also an uneasy place to be. My work was not easy for it touches on sensitive topics at all levels: racially, politically, socially, professionally, personally. But these challenges were what made undertaking this work most fulfilling.

Outside of its academic relevance, the main purpose of this thesis is to serve as a platform, with me as an agent of change. The aim is then to show the world, primarily to underrepresented students at the doctoral and professoriate level, particularly those that are economically disadvantaged, that fundamentally, all humans, irrespective of race, color, or economic status, are capable of greatness, when they are provided with the tools to achieve greatness. I learned so many things through this journey. Primarily that education disparity is a big problem that gets far less attention than it should from the people who are better equipped to work with it. This work definitely changed me in many ways; specifically, my journey through the Ph.D. and getting into the professoriate helped me understand that one has to learn which battles not to fight. It has also taught me that, a great idea whose time has not come, to remember the great Maya Angelou, should remain silent but active until its time comes about. Great ideas will always come about.

People say that it is difficult for ethnically underrepresented or economically disadvantaged students to make it through academia, but having survived the streets as a poor black man taught me many things, particularly that academia is a safe-haven for most. I challenge those of you who are out there struggling, to make a plea for sacrifice; not one that offers immediate rewards, but one that can bring about safety and fulfillment through the longevity of your life. You may not overcome been underrepresented or poor; but your education, your human capital, will always be yours and fully under your control. That, my dear friend, cannot be taken away from you.

Acknowledgments

The work printed in this thesis belongs to many individuals; so many, that a single page cannot do justice to them. My adviser Dr. Lisa Berger has been my greatest supporter in this process. I have no doubt that I have the best adviser any student can have and if I go through this again, I will choose the same advisers. Lisa, your genuine interest in my academic development, my ideas, and my becoming the best professional I can become was always, and continues to be evidenced, by your investment and willingness to help me acquire the tools I needed, and by your openness to accepting my opinions and my grounds. The example that you have set while directing my thesis is the example I aspire to abide by, should I become an adviser to any student. Lisa, I very much thank you for having taken the responsibility of directing my work and for owning that responsibility to the best of your abilities. My co-adviser and longtime mentor, Dr. David Ferguson. Dave, our history supersedes your advice in my thesis. You have served as an inspiration to me since I became a S-STEM recipient as an undergraduate student at Stony Brook University. To date, I have not met a person who is so personally, professionally, and willingly committed to issues of diversity and outreach in academia. I can only hope that the fire that fuels my passion for those same interests lasts as long as yours continues to last. Your multifaceted support, ranging from our one-on-one research time, our research collaborations, our trying to understand how to make academia, and by extension the world, a better place for all, to your ensuring that I had financial support throughout my Ph.D. tenure, are just a few of the quantifiable goods that you have consistently provided to me. My desires to become a scholar that will serve students and society at large have always been fueled by the example of both of my advisers, the best adviser in the world. Dr. Keith Sheppard, my thesis defense committee chair, has provided me so much in this process. You provided me many ideas about data, including data acquisition and management. After our first meeting about data gathering I felt so lost, but every time I came to your office, I left with a sense of hope that I could do this, and I did it. Dr. Angela Kelly, who kindly served as the outside reader of the thesis provided me with invaluable comments, suggestions, and reassurances about my work. I am indebted to you for your strong commitment to your profession. Every single comment and suggestion you made to me has been invaluable, particularly in the areas of methodology and writing style. The Center for Inclusive Education, under the direction of Ms. Nina Maung Gaona has been most valuable to me. The support the Center provided me over my Ph.D. tenure is simply unquantifiable. The Department of Technology and Society has been a supporter of my career since before I became a Stony brook student. Mr. Paul Siegel was the first, and continuous to be a, supporter I had at Stony Brook. From ensuring that I was receiving the CSEMS scholarship to finding ways to provide me with a summer tutor for my survival of advanced undergraduate and master level math courses in my last semester as an undergraduate, to showing up to events to support me and my fellow ethnic minority colleagues in STEM, Paul has been my champion. A great team of ladies that I admire: Rita Reagan-Redko, Marypat Taveras, and Joyce Flynn. You have been so great to me in so many ways that there is no space here to being describing how grateful I am for your continuous support. Rita always made sure that I had some kind of work to help me support my summer work, and ensured that I was compensated fairly. Marypat has provided some of the most reassuring and comforting conversations I have had and has always ensured that my academic paperwork was ready and sound. Joyce has never failed to get any of my paperwork prepared, signed, or submitted. She always ensured that my appointments were booked or re-booked on a timely fashion. The STEP and CSTEP programs, of which I have participated in every single capacity imaginable, have always inspired me to keep doing what I do and to love it. The many ethnically underrepresented or economically disadvantaged students from grades fourth through college that I have taught have been, and continue to be, so inspirational and important to my work, and my believing in it. Professor Malcolm Read introduced me to the great world of "Ideology". Malcolm, while this doctoral thesis is not devoted to ideology, the notions and theories I worked with you have shaped this work dramatically. Dr. Henry Frisz, my first math instructor in the United States was, I have no doubt, my first academic champion and supporter in academia. Most of this work was inspired by you and your passion for teaching mathematics to students whose social circumstances made them become part of the statistics of math underperformance. Dr. Madelaine Bates, I will always be thankful to you for your support, and inspired by your commitment and love for teaching and helping students like myself.

My family and friends have been the greatest to me. My late grandfather, Antonio Nicanor Núñez Arias, always reminded me that an education is something that no one can take away from me. I aspire to one day be as wise a man as you were, and continue to be within myself. My grandmother and mother have both been very supportive. They never questioned my decision to be a student well beyond what is expected in my culture and they are always proud me. Even when she had no clue what I was doing, my mother found a way to help finance my studies during my teenage years. My siblings, you mean the world to me and part of this work belongs to you. My friends, who at this point are my family: Miguel López and Mohammed Osman: You have seen me struggle and celebrate, and your continuous support has shaped the person that I am today. Thank you both for always been there for me. Yvan Arnó, while you don't talk much but feelings, I know you are secretly very proud of me. Maria Célleri and Ana Mirón: you are both such great and inspiring women in my life. You both have shown me what it means to be true to the self and that been a feminist, in principle, does not have to take away been feminine and realistic. I love you both very much. Nina Maung Gaona, you have become a great supporter and friend of mine. I cannot imagine what this journey would have been, had I not counted with your support in so many crucial times. Ryan Clements, thank you for all the ways in which you helped me realize that most of the answers I searched for, exist inside me. Dr. Lucienne Buannic, whose name is Lucy. I am in no doubt that you are the person who has more closely understood me and supported me through this lengthy path I chose. For the lack of time and space, I just want to tell you Thank you so much. A lot of the hard and unquantifiable work of this thesis was overcome thanks to you.

Many people have touched my work in the most positive ways,

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Those who tried to push me down, and did not succeed:

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Chapter 1

1. Introduction

This chapter provides a general overview of the research literature that highlights the need for a detailed analysis of high school mathematics course completion and advanced mathematics course access in highly populated, ethnically and economically diverse and large public school districts such as New York City. This study is grounded on ongoing dialogues regarding high school graduates' mathematics preparedness, college access and success in science, technology, engineering, and mathematics (STEM), mathematics and science education in urban settings, and the call from the government and the private sector to increase the mathematics and science literacy of high school graduates and the STEM workforce.

Despite continued efforts to increase high school students' mathematics course completion in the United States, evidence from research articles and reports has indicated that the proportion of high school graduates who are not ready for college or careers right after graduation is too high (National Science Board, 2004; Hill, 2006; Anderson & Chang, 2011). In the last two decades, there has been a research surge in mathematics education, and the general public continues to gain a better understanding of the need for, and the importance of, high school mathematics curricula of high intensity and quality (National Research Council, 2013). Nonetheless, the research literature treating large, high minority, and economically depressed school districts such as New York City is scarce. The scarcity of both comprehensive and systematically researched information makes it difficult for parents, teachers, principals, district administrators, scholars, and the general public to understand the high school mathematics education that students of large and diverse urban school districts, such as New York City, receive. There is a need, therefore, for an updated and systematic review of the high school mathematics course completion literature to provide guidance to students, teachers, high school and district administrators, faculty, higher education administrators, government and private researchers, and policy makers in regard to the critical importance of high school mathematics course completion and the policies that affect this course completion in Bronx County public high schools. It is particularly important to ensure that new research in mathematics education contextualizes students' mathematics course completion based on the number and type of courses completed. It is imperative for researchers to distinguish and document the vertical and horizontal (Tyson & Roksa, 2016) dimensions of mathematics course completion.

This thesis revisits the national high school mathematics course completion literature to understand and model the high school mathematics course completion of Bronx County public high school students. We frame our work around the high school mathematics course completion policies implemented by the New York State Board of Regents in the period covering the academic years of 2010 to 2013. These academic years are especially relevant in the policy arena. The Integrated Algebra, Geometry, and Algebra2/Trigonometry Regents exams[1] were being offered in their entirety since the year of 2008; but up until the year of 2010, these exams were given in conjunction with the Math A and Math B Regents exams. Hence, the academic years of 2010-2013 were the years in which only the Integrated Algebra, Geometry, and Agebra2/Trigonometry

[1] Appendix B provides a concise description of the Regents exams, their development and scoring mechanisms.

Regents exams were available to students in New York State. Therefore, the only academic years that provide us with true representative samples of the 2008 Regents exam-taking policies are the years of 2010-2013 because in the academic year of 2014, the Integrated Algebra, Geometry, and Algebra2/Trigonometry Regents exams were administered in conjunction with the exams aligned with the Common Core State Standards for Mathematics (EngageNY, 2013). Therefore, this study will serve as a baseline for future analyses of the Common Core State Standards which were fully implemented in the academic year of 2014-15 and which changed the high school mathematics course completion policy for all New York State public high schools.

Chapter 2 presents the reader with a systematic and concise review of the literature on students' high school mathematics course taking and completion at the national level; and we highlight the past and current efforts from government, public and private sectors to create policies that advance the mathematics preparedness of high school graduates. Our review starts by describing many of the benefits that students gain from completing high school mathematics courses and ends with the literature on the predictive power of students' high school mathematics course completion. Chapter 3 is a detailed discussion of the methodology we use to explore and answer our research questions and test our hypotheses. In it, we present our methodological approach, a discussion of how to implement the statistical tools of choice and a concise justification about why we chose the statistical tools employed throughout this thesis. This section starts with a global descriptive analysis of New York City public high schools; we present the reader with both qualitative and quantitative analyses of New York City public high schools and we end by conducting a quasi-experiment that allows us to determine how the mathematics course completion of Bronx county public high school students is distributed. The results of the study are presented in Chapter 4. We use the results of statistical models to both, answer our research questions, and make inferential deductions about Bronx students' high school preparedness and its impact on their subsequent life, particularly their higher education aspirations. Chapter 5 is a concise summary of the study's results that contextualizes the results connecting them with the literature review; it also presents a summary of this thesis' contribution alongside the significance of this research and new research avenues opened by this research.

Chapter 2. High School Mathematics Course Taking: A Review

2.0. Introduction

The following chapter is presented in two parts concerning students' high school mathematics course completion. The chapter begins this with a descriptive analysis of United States high school students' mathematics course completion or course taking. This overview includes a diverse set of scholarly work including national reports, journal articles, and doctoral theses. The chapter ends by highlighting how students' high school mathematics course completion, or course taking, is a strong predictor of their success, whether they decide to move on to higher education or to go directly into the workforce. Most nationally representative research studies on the topic of high school math course completion focus on the importance of the course completion in lessening or potentially eradicating the educational achievement gap found among ethnically underrepresented or economically disadvantaged students in various STEM-related fields. Among STEM disciplines, mathematics is of primary importance for various reasons. First, at the higher education level, most science, technology, and engineering courses require an understanding of mathematics that goes beyond algebraic manipulations. These topics rely heavily on mathematical knowledge at or above the level of matrix algebra and vectors, which are usually covered in advanced high school mathematics courses. Mathematics literacy is important even for students who are not intending to pursue STEM degrees. Many social science programs, including but not limited to, Sociology, Psychology, and Higher Education Administration require one to three quantitative methodology courses, all of which require mathematical knowledge at the level of a probability and statistics courses. These courses are most commonly offered as advanced electives in high school. We selected studies that included nationally and demographically representative samples when analyzing students' high school math course completion. Incidentally, most of these studies highlighted the issue of high school math course completion among ethnically underrepresented and economically disadvantaged students. This fact speaks to the importance that high school mathematics plays in the democracy and well-being of nations as conceptualized in Ball & Bass, 2008; Stemhagen & Smith, 2008; and Steen & Madison, 2011.

2.1. Mathematics high school course completion: a descriptive review of the literature

Students' high school math course completion remains a topic of utmost importance in mathematics education. Various factors, where the most notable are socioeconomic status, school geographic location, and ethnicity, have been found to affect students' mathematics course completion in high school (Adelman, 1999; Tyson et al., 2007; Wilensky, 2007). Many studies conclude that the academic preparedness, college aspirations, persistence, and graduation, as well as earnings of high school graduates who complete college preparatory mathematics curricula, particularly curricula that include advanced courses, are all positively, strongly, and significantly correlated to their math course completion (Achieve Inc., 2008; Adelman, 1999, 2006; Bozick & Lauff, 2007; Jones, Davenport Jr., Bryson, Bekhuis, & Zwick, 1986; Ma & Wilkins, 2007; National Research Council, 2013; National Science Board, 2004, 2010; Nord et al., 2011; Oakes, 1990). Education in STEM is a topic of importance and of concern in the United States. Policy makers have realized that STEM education is a priority at various levels. The main priorities include but are not limited to: national security, the rise, expansion and sustainability of global markets, and the re-establishment of the United States into the top ranks in international benchmark tests such as the Programme for International Student Assessment (PISA) (National Science Board, 2010; President's Council of Advisors on Science and Technology, 2010).

The policy needs of the United States, coupled with the rapid growth of scientific and technological innovations, put a premium on education, requiring that high school graduates have a robust education to be competitive in college, especially if they intend to pursue STEM careers. There is now significant conclusive evidence from academic, government, and private research studies supporting the thesis that students' high school mathematics course completion is the single best predictor of their academic success across various academic subjects, especially in STEM (Achieve Inc., 2008; Adelman, 1999; 2006; Bozick & Lauff, 2007; Bryson, Bekhuis, & Zwick, 1986; Jones, Davenport Jr., National Research Council, 2013; Ma & Wilkins, 2007; National Science Board, 2004, 2010; Nord, et al., 2011; Oakes, 1990). Moreover, students' intentions of studying mathematics and science in college are strongly and positively related to the mathematics courses they complete in high school. For example, the results of Burkam and Lee's (2003) analysis of the National Educational Longitudinal Study of 1988 (NELS:88) suggested that, as students progress through their high school math courses, their intentions of studying mathematics and science in college increase.

Studies quantifying high school math course completion provide valuable insight into the various ways in which students' math course completion relates to their success during high school and their access to, and their persistence and success in higher education (Hill, 2006; Tyson et al., 2007; Wilensky, 2007). When high school students take advanced math and science courses, their likelihood of earning higher scores on key performance assessment indicators, pursuing higher education degrees in STEM, and completing baccalaureate degrees increases substantially (Adelman, 1999: 2006; Bozick & Lauff, 2007; Chen, 2009; National Center for Education Statistics, 2010; Nord et al., 2011). The results and conclusions of research, government, and policy reports suggest that students should graduate from high school having completed advanced courses in mathematics and science, or academically intense four-year math and science curricula (Achieve Inc., 2008; Chen, 2009; Finkelstein & Fong, 2008; Hill, 2006; National Science Board, 2004).

Since the 1980s, education reform has remained a work in progress. Research, government, and policy reports since the 1980s have continuously emphasized that a robust high school mathematics education plays an important role in shaping the future of public school students and the nation (Achieve Inc., 2008; Adelman, 1999; Burkam & Lee, 2003; Frank, Muller, Riegle-Crumb, Mueller, Crosnoe, & Pearson, 2008; Ma & Wilkins, 2007; National Commission on Excellence in Education, 1983; National Research Council, 2013). High school mathematics course completion has been shown to be a regulator of students' success at various levels (Frank et al., 2008; Chen, 2009). These regulating effects prove to be positive across students' ethnic and social backgrounds. Various studies are conclusive in finding that when students from historically underperforming groups in higher education, particularly in STEM disciplines, complete four-year math sequences, or complete academically challenging math courses in high school, their high school and college performance are close to, or the same as, that of their counterparts (Adelman, 1999; Burkam & Lee, 2003; Tyson, Lee, Borman, & Hanson, 2007; Nord et al., 2011). Despite strong evidence about how high school math course completion positively affects students, the access to advanced high school math courses varies significantly across public high schools of the United States; for example, research has shown that, on average, high schools located in inner-city, high-minority, and economically disadvantaged neighborhoods failed to provide their students high-quality, competitive, or academically challenging mathematics curricula (Anderson & Chang, 2011; Finkelstein & Fong, 2008; Fernández, 2015a, 2015b; Graham, 2009).

The NSF report *Science and Engineering Indicators* (2016) indicated that, on average, the mathematics and science teachers from high-poverty and high-minority schools were not as highly qualified as the teachers from wealthier and low-minority schools. Also, when compared to students coming from families with incomes at or above 200% of the Federal Poverty Level, students from families with incomes below this level continue to underperform in mathematics as early as kindergarten (National Science Board, 2016). As an example, for the academic year of 2011-2012, the average free or reduced price lunch[2] eligibility of Bronx County public high school students was 81.1%. These students displayed high academic deficiencies in mathematics as measured by their advanced Regents exams completion and passing rates (Fernández, 2015a). Research has indicated, however, that irrespective of their ethnic or socioeconomic backgrounds, when students completed an International Baccalaureate program, an intense 9th grade Advanced math or science course, or a rigorous secondary curriculum that includes three to four years of math courses, on average, they scored at the proficiency level on the math sections of the National Assessment of Educational Progress (Nord et al., 2011). Our results showed that when Bronx students were held accountable to 4-years of mathematics, all of their key academic performance indicators increased significantly (Fernández, 2015a). For example, Bronx high school students graduating from schools that required 4-years of mathematics earned Advanced Regents diplomas at ten times the rate of Bronx students from other high schools.

The National Research Council report of the National Science Foundation, *The Mathematical Sciences in 2025,* asserted that mathematical knowledge is rapidly becoming essential for people working in many social and life sciences disciplines. Moreover, technical and research work in these areas have been identified as essential requirements to bolster the economic growth, national competitiveness and security of the United States (National Research Council, 2013). Despite the importance of high school mathematics in preparing highly skilled and critically thinking workers, ethnic and racial minorities who are a representative component of the population of the United States, such as Blacks and Hispanics, are severely underrepresented in the mathematical sciences (National Research Council, 2013). Davenport, E. C. et al. (1998) used 1990 National Educational Progress (NAEP) transcript data to analyze students' high school math course taking by gender and ethnicity. The study's results indicated that about 54% of the mathematics Carnegie[3] units earned by all students "were earned in the Standard Sequence: Algebra 1, Geometry, and Algebra 2" (504) and only 10% of the net Carnegie units were earned in any one of the advanced sequences, "Algebra 3; Trigonometry; Analytic Geometry; Trigonometry and Solid Geometry; Algebra and Analytic Geometry; Analysis, Introduction to

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[2] At the time the data of this study was released, free lunch was available for students whose families earned from $0 to $30,615 a year for a family of four. Students whose families earned from $30,616 to $43, 568 were eligible for reduced price lunch (United States Department of Agriculture, 2013).

[3] "A Carnegie Unit Carnegie Units were proposed in 1906 as a basis for measuring school work. A unit would represent a single subject taught for one classroom period for five days a week for a semester. Fractional units would be awarded for subjects taught less frequently" (Department of Education's International Affairs, 2008).

Calculus, and Analytic Geometry; Calculus; Advanced Placement Calculus" (501).. In terms of course entry, results indicated that White and Asian students surpassed Black and Hispanic students at both the *standard* and *advanced* levels based on the net number of high school mathematics Carnegie units completed. The course entry rate of Black and Hispanic students was about half that of White students and about one-third that of Asian students. However, Black and Hispanic students surpassed White and Asian students in the total mean Carnegie units attempted in *Basic* and *Preformal* math courses. The researchers believe that observed differences in the net Carnegie unit completion rates of Black and Hispanic students in the basic and preformal categories could be attributed to both the differing entry rates into the preformal and basic categories, as well as to net differences in the number of courses taken by students entering in different course categories. For example, the proportions of Asian and White students entering into the standard categories were higher than those of Black and Hispanic students: White 0.78**, Black 0.71**, Hispanic, 0.74**, Asian 0.87**. There is indeed an entry gap, when the course category of reference is the standard math sequence, where Black and Hispanic students fall behind in comparison to White and Asian students. To make matters worse, the advanced math course entry gap of Black and Hispanic students, when compared to White and Asian students, was even greater than the standard math course entry gap: White 0.28**, Black 0.14**, Hispanic 0.16**, Asian 0.48**. These results indicate that the mathematics gaps begin with course entry or enrollment, where Hispanic and Black students had higher entry into math courses below the standard sequences and lower entries into math courses at or above the standard sequences.

Finkelstein and Fong (2008) investigated California high school minority students' course-taking patterns, focusing on whether the courses that students took met university entrance requirements. The researchers drew samples from the Transcript Evaluation Service (TES) for the academic year of 2004-05 and school-level data reported by the California Department of Education; both data sets were representative of the student population of the state of California. The results of the study suggested that the completion of academically demanding or challenging math courses can mitigate the persistent college completion gaps between ethnically underrepresented or economically disadvantaged students and White, Asian, or wealthier students. Nonetheless, these students completed advanced or academically demanding math courses at very low rates. For example, the researchers found that completing courses such as one-year college-preparatory math or English in 9th grade is a big challenge for many students. In their sample, over forty percent of the students failed to complete a one-year sequence of college preparatory mathematics. The researchers' results suggest that many ethnically underrepresented or economically disadvantaged students failed to meet the A-G[4] CSU and UC requirements because their enrollment rate in college preparatory courses, particularly math, was very low. Black students, for example, registered in these courses at a rate of 40% in 9th grade. These results were aligned with those of (Oakes, 1990), who found that Black and Hispanic students completed four-year math and science course sequences or college preparatory math and science courses at very low rates. The researcher's results also indicate that socioeconomic status is a regulating variable of the math course taking patterns of high school students, as indicated in the table below:

[4] The CSU and UC system categorizes their course requirements as A-G; math courses fall into the C category, and the minimum requirement for math is three years or six units.

Table 1
Students Exhibiting Academic Course completion Patterns by Socioeconomic Status and Race (percentages of students exhibiting pattern)

	SES					
	High	Middle	Low	White	Black	Hispanic
Academic math	69.1	45.7	25.1	51.5	28.1	28.9
Academic science	58.3	36.9	19.6	40.7	26.1	23.8
Computer science	17.4	12.4	8.4	13.8	10.5	8.0

Source. National Center for Educational Statistics (1985a). Taken from Oakes, (1990).

In spite of the results depicted in Table 1, when students completed Algebra I or college prep courses in 9th grade, they created paths of positive academic trajectories. College preparation starts with 9th-grade courses, and graduating college ready when not completing college preparatory courses in 9th grades has been very difficult for students (Finkelstein & Fong, 2008). Students who completed math college preparatory courses in 9th grade were more likely to attend a 4-year CSU or UC institution than a two-year community college. Roughly 42% of California high school students were, by the end of 9th grade, at risk of not meeting CSU or UC institutions' entrance requirements because they did not complete 2 semesters of Algebra I or higher with a grade of C or better; by the end of 10th and 11th grade, this number increased to 53% and 62%, respectively. However, results showed that this number decreased to 52% by the end of 12th grade (Finkelstein & Fong, 2008). The researchers suspect that it is because CSU and UC institutions required only 3 years of high school mathematics that the percentage of students at risk of not meeting these institutions' math requirements decreased as they moved from grade 11 into grade 12. The net percentage of students meeting the minimum 3 math units[5] required by CSU or UC institutions increased by one percentage point as students reached 12th grade; nonetheless, when disaggregating the data by students' ethnicity, results indicate that ethnic minorities' math requirements completion rates were lower than those of their counterparts. By the end of 12th grade, students in each ethnic category completed the minimum 3 math units required by CSU or UC institutions at the following rates: Asian 64.5%, Hispanic 41.7%, Black 34.9%, and White 58.7%. From these numbers, it is clear that the math academic gap mentioned in many of the studies we cite in this thesis still persists among Black and Hispanic students. If we consider that these students register at very low rates in college preparatory courses, then one could argue that we need to set stronger mathematics course completion policies. For example, various studies, including *Rising Above the Gathering Storm* and *The Mathematical Sciences in 2025*, draw on nationally representative data to make the case that students should be completing at least 3 years of mathematics up to an advanced math course, such as Precalculus or Statistics. *The Mathematical Sciences in 2025*, as well as The City University of New York and the University of California and California State University, recommend at least four years of high school mathematics. We need policies that hold students accountable to higher mathematics course completion standards, irrespective of their socio-demographic characteristics and all students should complete four years of high school mathematics, including advanced courses. A number of empirical studies support the latter claim. For example, California public high school students increased their likelihood of

[5] A unit is equivalent to a full year course.

been accepted into a four-year CSU or UC institutions by taking math courses in 12th grade (Finkelstein & Fong, 2008); Bronx County students graduating from schools that reported a 4-year math requirement graduated college ready at ten times the rate of the other Bronx public high school students (Fernández, 2015a); besides, there are many advantages of completing advanced math courses: a greater likelihood of completing Bachelor's degrees (Adelman, 1999), mediation of students background characteristics (Frank, et al., 2008), positive academic outcomes throughout students' high school and post-secondary education, and the workforce (National Science Board, 2004). Despite efforts from the federal government and research-oriented policy recommendations, students of underrepresented ethnic backgrounds or low socioeconomic status remain underrepresented in STEM disciplines and, when compared to other groups, their chances of having access to challenging math and science courses are minimal (Graham, 2009; Kelly S., 2009; Oakes, 1990). Many African American and Hispanic students begin high school underprepared in mathematics, which truncates their upward mobility within the mathematics course ladder (Adelman, 1999; Riegle-Crumb, 2006). Research has suggested that students attending urban or poor high schools tend to take fewer advanced math courses than students from suburban, rural or wealthier high schools (Lippman, Burns, McArthur, Burton, Smith, & Kaufman, 1996; Graham, 2009). For example, a study by Lippman et al. (1996) showed that graduates from poor high schools took Geometry at a rate of 60%, and graduates from wealthier high schools took this course at a rate of 74%. It becomes apparent that gaining a comprehensive understanding of the high school mathematics course completion of students from large urban public school districts that serve highly diverse student populations such as New York City, is timely and relevant.

2.2. Mathematics high school course completion: a review of the predictive literature

Empirical evidence allows us to conclude that the number and types of high school courses students complete play a pivotal role in their high school academic preparedness, as well as in their subsequent professional and academic interest. In an analysis of the 1980 high school sophomore cohort from the *High School and Beyond* data set, Jones, Davenport Jr., Bryson, Bekhuis, and Zwick, (1986) found positive and statistically significant correlations between students' senior year mathematics achievement test scores and the high school math courses they completed at the level of Algebra I or higher. When students completed the course sequence Algebra I and II, Geometry, Trigonometry, and Calculus, they achieved higher scores on their senior year mathematics achievement tests. Grounded on their findings, Jones et al. (1986) concluded that students' average math test scores were directly proportional to the number of advanced high school math courses they completed in high school. Similarly, the results of Adelman's (1999) report, *Answers in the toolbox: Academic intensity, attendance patterns, and bachelor's degree attainment* indicated that the highest level of mathematics that students studied in high school had the strongest positive correlation to their bachelor's degree completion. For example, students entering higher education having completed high school math courses beyond Algebra2 –for example, Trigonometry or Pre-Calculus- more than doubled their chances of completing bachelor's degrees. Moreover, the academic intensity, rigor, or quality of Black and Hispanic students' high school curriculum was the strongest contributing factor to their academic preparedness. The impact of intense or high quality math courses was greater for Black and Hispanic students than it was for White and Asian students, as indicated in Table 2 below. The variable Curriculum, which Adelman refers to as Academic Intensity, was manipulated for quality by gradations. For example, for advanced math courses, a gradation level of 0, 1, or >1 was recorded, depending on the level of the course. In regards to regular high school mathematics, a +1 was added for courses in Geometry or higher, a 0 was added for Algebra 2, and a -1 was added for courses at levels lower than Algebra 2. When any math course was remedial, subtractions were made accordingly. This operational definition resulted in an enhanced curriculum indicator with 40 gradations. The gradations were equally spaced in intervals of 2.5, ranging from 2.5 to 100. As an example, students reaching the highest curricular indicator, 100 points, would have completed 3.77 or more Carnegie units in high school mathematics, with no remedial courses. Adelman made it clear that the enhanced curriculum is a criterion variable and therefore, it is not a relative measure of performance like test scores or class rank scales. The variable Test Scores comes from the senior year test that was taken by 92.7% of the students from the *High School and Beyond* dataset. The variable Class Rank/GPA refers to either the rankings that schools make on their students, i.e., to 10%, to 15% and students' actual high school GPA. The author had to combine both measures because of a lack of measures among students. This was done because these variables were correlated at 0.841 (Adelman, 1999). As summarized in Table 2 below, based on the results of his study, Adelman made the claim that, to improve bachelor degree completion rates, particularly among Latinos, one ought to concentrate on students' high school curriculum, particularly, the quality and intensity of curricula.

Table 2 Comparative improvement in bachelor's degree attainment rates by moving into the top 40 percent on each component measure of the ACRES index, by race, High School & Beyond/Sophomore cohort, 1982-1993

Components	White	Black	Latino	All
Curriculum	+10.4%	+27.5%	+18.5%	+12.7%
Test Scores	+5.1	+22.0	+58	+78
Class Rank/GPA	+3.5	+13.7	-3.8	+5.0

SOURCE: National Center for Education Statistics: High School & Beyond /Sophomore Cohort, NCES CD#98-135. -Taken from Adelman, (1999)-.

Richard O. Hill studied the correlations between the senior-year high school mathematics courses and the entry-level mathematics courses of Michigan State University students. The study's results suggest that both Advanced[6] and non-Advanced calculus courses provided students with a successful preparation for entry-level mathematics at Michigan State University. Descriptive statistics indicated that the overall college preparation of students who did not take mathematics in their senior year was very poor. For example, 75% of those students placed in remedial mathematics at Michigan State University displayed the following high school mathematics preparation: during their senior year, about 35% did not take mathematics, about 16% took only one math course, about 18% took low-level or non-algebraically demanding math courses, and about 6% took a demanding course but received grades lower than C (Hill, 2006). Table 3 below shows the background of students based on the course tier on which they were

[6] The Advanced Placement (AP) program is administered by College Board, which is "a mission-driven not-for-profit organization that connects students to college success and opportunity" (The College Board, 2015). "The AP is a rigorous academic program built on the commitment, passion, and hard work of students and educators from both secondary schools and higher education" (College Board, 2003).

placed at Michigan State University. Tier 5 is the lowest level math course indicating remedial mathematics and Tier 1 is the highest course indicating advanced level mathematics: Tier 5 – Remedial Math; Tier 4 – College Algebra and Finite Math; Tier 3 – Precalculus; Tier – 2: Five different courses: (a) business/biological science calculus, (b) trigonometry, (c) mathematics for elementary education students, (d) any statistics course, and (e) a (relatively new) liberal arts math course."; Tier - 1: Technical calculus or a higher-level course (Hill, 2006).

Table 3
Summary of results of students by Tier.

Tier	*n*	%[a]	Average Grade[b]	HS Background Comments
5	550	18%	2.3	75% from a weak background
4	689	24%	2.2	60% from a weak background; 30% had ≥ C in calc. or precalc.
3	435	15%	2.3	38% had ≥ C in calc. or precalc; most had ≥ C.[c]
2	410	14%	2.8	50% had ≥ C in calc. or precalc; most had ≥ C.[c]
1	604	21%	2.8	79% had calc. or precalc, ≥ B.[c]

[a] Of all students in the study. [b]In the MSU course. [c]In whatever senior-level math course they had in high school. Taken from (Hill, 2006).

The positive correlations between advanced high school math courses and students' proficiency on their mathematics exams, as demonstrated by Hill's (2005) work and the work of others (Fernández, 2015a; Jones, Davenport Jr., Bryson, Bekhuis, & Zwick, 1986) are more important than they may seem a priori; exams are an increasingly important key performance indicator considered by policy makers (Adelman, 1999). In New York State, for example, public schools are required to provide information on students' Regents exam scores. A 1986 study by Jones and others found that the number of advanced high school math courses completed by students, both self-reported and transcript extracted, respectively, explained 53% and 46% of students' test-scores variance. The inverse relationship also held true: as students took fewer advanced courses, their math exams' responses were at or near the chance level; i.e., their response where similar to that of a student who would choose his or her answers randomly. Frank et al. (2008) analyzed course transcript data from the 1995-2002 *Adolescent Health and Academic Achievement Study* (AHAA). The study focused on the academic years 1994-95 and 1995-96. One of the study's conclusions was that social norms, including those established by parents and peers, contributed to students' fundamental social dynamics; the latter mediated students' educational motivation and success. The study's findings supported the contention that adolescents have a "considerable agency" (1680) in mapping their social contexts based on the math courses they choose to take. We contend, however, that limited or restricted mathematics course access in high school nulls such agency. In his 1999 study, Shawn Kelly used data from the National Education Longitudinal Study of 1988 (NELS: 88) student, school administrator, and parent surveys. He supplemented this data with the 1990 student and school administrator surveys and the high school transcript file. The study focused on students' high school math course completion by grade 10th. The study's results suggest that starting high school with a history of low-intensity math courses was detrimental to many Black students due to the sequential nature of math courses; hence, if high school students had not moved upward in the hierarchical sequence of math courses by their sophomore year, they had great difficulties catching up and completing college preparatory math sequences. Finkelstein & Fong (2008) arrived at the same conclusion. Courses such as Pre-Algebra, Algebra, I, Geometry and Trigonometry make significant contributions to students' mathematical growth (Ma & Wilkins, 2007), and these same courses have been found to be positively and strongly correlated to students' senior-year test scores (Jones, Davenport Jr., Bryson, Bekhuis, & Zwick, 1986).

Studies also have documented that professionals with various accomplishments in STEM, such as scholarly publications, Ph.D. completion, academic tenure, and patents, were involved in many advanced and strong STEM high school courses and activities such as advanced math and science courses (National Research Council, 2013). In a representative study of Florida's public four year college graduates in STEM, Tyson et al. (2007) concluded that Black and Hispanic students generally completed fewer advanced high school math courses than Asian and White students; however, when they compared Black and Hispanic students who took advanced math courses at rates above average for these students, their overall academic achievement was comparable to that of Asian and White students, and in some cases higher than that of White students. Another nationally representative report indicated that a high proportion of students entering STEM fields completed Trigonometry, Precalculus, and Calculus courses in in high school; these results established strong and positive correlations among students' advanced high school math course completion and their access to STEM fields in higher education. Correlations showed that as high school students advanced through math courses, their likelihood of entering a STEM career increased. Advanced high school math course completion was also strongly related to higher degree completion rates in STEM. Students completing STEM degrees completed Trigonometry and Precalculus at rates of 37.5% and 58.6%, respectively. Students who left STEM fields completed the same courses at the respective rates of 23.4% and 20.2% (Chen, 2009). The United States was for a long time at the forefront of technological and engineering development. However, evidence suggests that public high schools in the U.S. are not offering math curricula with the same focus and coherence of other developed foreign countries (Schmidt, 2003). Most of the studies we cited in the section above suggest that the mathematics expectations for high school students have been inconsistent; these expectations varied and correlated significantly to students' racial and socioeconomic backgrounds. This thesis begins to address some of these issues by taking a policy evaluation perspective.

Chapter 3. Research Methodology

3.0. Introduction

In this chapter, we present and discuss the theoretical framework guiding this thesis and the methodologies we employ in analyzing and answering the research questions of the study. This chapter is divided into several sections. Section 1 presents the research questions of the study; section 2 presents the research design; section 3 provides a description of the New York State Regents examinations; section 4 covers the population from which we draw our samples; section 5 discusses the collection of the data; section 6 details the methodological considerations based on the type of data used in the study; section 7 discusses how the results are presented; section 8 explains the statistical measures we present; section 9 provides a detailed description of the research questions.

3.1. Research Questions

Social capital and human capital tend to be distinctive features of the educated, middle, and upper classes. Social capital, in particular, plays significant roles in the creation of human capital (Coleman, 1998). High school mathematics, in particular, is an important base for the creation of human capital, given its potential, as a gateway or gatekeeper, to allow students access to other advanced math, science courses, and college (Frank, 2008). Social capital, according to Bourdieu, (1986), is an individual's actual or potential management and acquisition of resources that come about by their networking with acquaintances and by their recognition of being members of recognized institutions such as schools, colleges, universities, or clubs (Bourdieu, 1986). Hence, academic institutions are an important part of the machinery that builds social capital, particularly those that touch humans during their developmental stages, such as high schools. Human capital, as defined by Coleman (1998), is more abstract. While individuals' social capital develops by means of dynamic changes in the relations among those who facilitate social action or networking, Coleman argued, human capital manifests through the skills and knowledge that individuals acquire over time (Coleman, 1998). Hence, social capital appears to be a necessary condition to the development of human capital. Coleman coined a term that, I argue, combines both the networking and social recognition that individuals gain: "Information Channels". Information channels are, in essence, the means by which individuals are able to capitalize from and build up social capital. High schools are an important type of information channels. High schools provide individuals, at least in theory, with a prerequisite knowledge that can help them either enter into the workforce or into higher education. It is now well established that a minimum of three years of high school mathematics is essential to gain access to four-year higher education institutions. Therefore, high schools play at least a double role in the socio-cultural development of their students: they are a strong platform for the development and expansion of students' social networking; and, they have the potential to help students gain and develop mathematical skills that build upon and augment their human capital. For example, it is estimated that by the year of 2025, the economy of the United States will need an additional 140,000 to 190,000 workers with high-level mathematical and analytical skills to help the nation move forward not only in its scientific and technological development but also in the administration and management of such employees. Corporate decision making, in particular, will need individuals with an understanding of analytical concepts, particularly in the areas of statistics and machine learning (National Research Council, 2013). These additional workers will require high levels of both cultural and human capital; and an important type of human capital would be a sound high school mathematics education.

A main purpose of this study is to help the research community, students, parents, policy makers, and the general public to better understand an important aspect of high school mathematics education in New York City public high schools: the high school mathematics course completion distribution of New York City public schools. The high school mathematics course completion or course taking review of the literature, chapter 2, provides the theoretical framework through which we model the high school mathematics course completion distribution of New York City public high schools. We take Bronx County as the baseline county of comparison throughout the study. We contextualize our model by using math course completion as a proxy that measures students' human capital. By regulating mathematics course offerings, schools have the potential to limit or augment the amount of human capital that their students can acquire, hence either limiting or expanding their graduates' opportunities post high school. I developed the research questions of this thesis within this framework.

1. **How are key socio-demographic and performance indicators of New York City public high schools distributed?**
    a. How do the socioeconomic and ethnic distributions of Bronx County public high schools compare to those of New York City public high schools?
    b. How does the average reported mathematics Regents exams passing and mastery rates of Bronx County public high schools compare to those of New York City?
    c. How do Bronx and New York City public high school graduates' diploma attainment and post-secondary aspiration rates compare to one another?

2. **How comparable are key demographic and academic performance indicators of Bronx public high schools with a reported 4-year mathematics requirement to those of other Bronx public high schools?**
    a. How do the socioeconomic and ethnic distributions of Bronx County public high school students attending schools with a reported 4-year of mathematics requirement compare to those of Bronx County public high school students from other schools?
    b. How do the average mathematics Regents exams passing and mastery rates of Bronx County public high school students attending schools with a reported 4-year mathematics requirement compare to those of other Bronx County public high school students?
    c. How do the high school diploma conferment and post-graduation aspiration rates of students from Bronx schools with a reported 4-year mathematics requirement compare to those of other Bronx public high school students?

3. **How comparable are the high school mathematics course completion rates of Bronx County public high school students from schools with a reported 4-year mathematics requirement and other Bronx public high school students?**

3.2. Research Design

This thesis is a descriptive quantitative quasi-experimental study; a study in which subjects are not randomly assigned to specific conditions (Shadish, Cook, & Campbell, 2001). Ideally, researchers would be developing their research by means of experimental designs. When an experimental research study has been properly designed, the results of the research have a high degree of internal validity. This is due to the researcher's full control of the independent variable, as the researcher can if needed or required, manipulate it. Also, the researcher generally has full control of treatment assignment, where cause/effect conclusions have stronger validity, as most confounding variables can be controlled (Cook & Campbell, 1979).

In quasi-experimental designs, random assignments to treatment conditions are not feasible for various, primarily ethical, reasons. However, in our particular case, a quasi-experimental design is the best tool that we have to study our target population: public high schools and their students. Researchers have many tools to set up quasi-experimental designs that can lead to strong conclusive evidence. To optimize the external validity of our quasi-experiment, we pick comparison groups with similar baseline characteristics (White & Sabarwal, 2014). Because of the practical methodological constraints of our research questions, a quasi-experimental design is the most optimal tool to help us undertake our study. In a quasi-experimental design, researchers do not have control over treatment conditions. However, in well-designed quasi-experiments, we do have control of over the directionality problem because we fully control independent variables, which can be manipulated prior to measuring dependent variables (Price, 2012). While we do not have control over treatment assignment, we did control for many confounding variables and the conditions under which we compare our groups are as homogeneous as we could have made them be. We accomplished this by creating stratified[7] samples from which we later randomly selected representative samples; specifically, in addressing research question three, we drew two representative samples from each of our strata: Bronx public high schools that reported a 4-year math requirement and other Bronx public high schools. We drew fifteen schools from each sample, each of which met pre-determined selection criteria: school size, socioeconomic, and ethnic distributions. That is, in each sample, we have that each school is comparable to every other school in the aforementioned variables.

Through the thesis, we draw conclusions that apply to both schools and students. However, we are careful when drawing conclusions to avoid confusing the reader. Our sample selection criteria were always determined at the school-level. That is, we looked at schools as the population from which we drew our samples. Research questions one and two ask specifically about schools' characteristics. However, the analyses carried out to answer these questions uses student-level data. When interpreting our results, we drew our conclusions in relation to the students, not the schools. Nonetheless, the results of research questions two and three could be interpreted in relation to schools or to their students. At the core, research question three is concerned with schools' profiles. However, like research question one, this question is answered using student-level data. Nonetheless, because we are asking about schools' profiles, we interpreted the results at the school level. Hence, at the core, research question two provides answers about schools, not

[7] When groups are not homogeneous, the use of a procedure called *stratified random sampling* is appropriate. This procedure allows us to select strata within our sample of interest in such a way that no stratum is over or under represented (Walpole R. E., Myers, Myers, & Ye, 2011).

students[8]. Throughout this thesis, we carry out our various analyses based on mutually exclusive or inclusive samples. To answer research question 1, we used two mutually exclusive representative random samples: a sample of 94 public high schools from Bronx County[9], and a randomly selected representative sample of 91 New York City secondary schools. The samples used to answer research question one consist of 50,649 students from Bronx public high schools and 93,490 students from New York City public high schools. We answered research question two by selecting two samples from Bronx public high schools: a sample of fifteen schools that report a 4-year mathematics requirement in the New York City High School Directory, and a sample of eighty one public high schools of Bronx County, which are all public high schools that did not report a 4-year mathematics requirement. These high schools abided by the mathematics requirements set forth by the New York State Board of Regents: 6 credits of mathematics[10] (New York City Department of Education, 2013b). The samples used to answer research question two consisted of 6,859 students attending Bronx public high schools that reported a 4-year of mathematics requirement and 42,203 students attending Bronx public high schools without this reported requirement. We answer research question three using a stratified random sample from each, the group of Bronx public schools reporting 4-year math requirements, and the other Bronx public schools. Note, however, that the data used in our statistical tests is student-level data. The samples used to answer research question three consisted of 2,898 students from schools reporting a 4-years of mathematics requirement and 2,523 students from schools not reporting this requirement. We let the reader know that, while a small subset of Bronx public high schools reported a 4-year math requirement, only 45% of the students we sampled in the academic year of 2012-13 had completed mathematics courses during that academic year. In other Bronx public high schools, this figure was also 45%.

3.3. The New York State Regents Examinations

a. Implementation and Evolution

Developed and managed by The New York State Board of Regents[11], the Regents exams are the oldest academic assessment exams in the United States. As early as 1865, the State of New York was assessing eighth-grade students via entry exams. In 1790, the State of New York authorized the Board of Regents to take over the administration of some state lands and to use the lands' revenues to fund the Literature fund, which became the main source of aid for colleges and academies (University of the State of New York, 1905). Propelled by the rapid increase of the

[8] While we interpret our data based on the way in which our research question was formulated, our results could also be used to draw conclusions not only about schools, but also students.

[9] This sample is in fact the population because these are all of the schools that fit our selection criteria.

[10] Neither set of schools specifies the type of courses required, nor how the courses are distributed by year.

11 In May 1, 1784, a Corporation conformed by the Governor, the mayors of New York and Albany, plus twenty four appointed persons was created by statue. This is what we know as The Board of Regents of the University of the State of New York (USNY) (Folts, 1996). Since its creation, the Board of Regents has been responsible for the general supervision of all educational activities within the State, presiding over the University of the State of New York which is the most comprehensive and unified education system in the United States, consisting of all elementary, secondary, and post-secondary public and private institutions, including public libraries. As of today, "The Board comprises 17 members elected by the State legislature for 5 year terms. (…) Regents are unsalaried and are reimbursed only for travel and related expenses in connection with their official duties" (New York State Education Department, 2010).

academies and the boom of high schools during the mid-nineteenth century, in 1865 the Board of Regents established the Regents examinations (Office of State Assessment, 1987) to determine, according to the law, which scholars were entitled to receive aid from the Literature fund (Murray, 1881), cited in (Lott, 1986). Ever since their implementation, the Regents exams have undergone various changes, mainly to keep up to date with evolving academic demands and expectations. The Board of Regents established the local diploma in 1906 to serve as a pathway to graduation for students who either did not take the Regents exams or did not achieve the minimum passing score on the required Regents Exams (Folts, 1996).

In the early 1990s, a national trend of raising the academic standards of K-12 education emerged. Its intense academic standards in K-12 have always positioned New York State as a leader in educational reform. In 1996, the Board of Regents started to undertake new educational reforms to improve the educational performance of New York State students. The core of the reform was centered on accountability and higher standards for all: schools, school administrators, teachers, parents, and students. The Board of Regents dictated that beginning in 2006 all students would take and pass at least five Regents exams in order to graduate from high school. Some immediate results of this mandate included the phasing out of the local diploma, the implementation of a higher minimum Regents exam passing score, and the renaming of the old Regents diploma now called Regents Diploma with Advanced Designation or Advanced Regents Diploma (McCall, 2000). In low-performing and high-minority[12] school districts, the Regents competency tests had been the primary pathway to students' graduation; the 1996 policy was set to eliminate this pathway because it was considered to be of low rigor when compared to the Regents exams. The new education reform rested on the hypothesis that holding students accountable to higher academic standards would boost their academic success (McCall, 2000; Knecht, 2007). After a decade, in 2008, the Board of Regents fully implemented the Regents exams policies put in place in 1996. As of today, with exceptions that apply primarily to special education students, all high school students in New York State must pass at least five Regents exams with a minimum scaled score of 65 points to be granted a diploma. It is worth noting that students can take the Regents exams as many times as they wish until they achieve passing or self-satisfying scores (Willens, 2013). With their respective sub-fields, the five required Regents exams provide students with eleven options to choose from:

12 The State Education Department defines high-minority schools as those with minority populations at or higher than 80% of its student population (McCall, 2000).

Table 4
Regents exams options for New York State Students

Subject	Courses
Mathematics	Integrated Algebra Geometry Algebra2/Trigonometry
Science/Biology	The Living Environment
Science/The Physical Settings	Earth Science Chemistry Physics
Social Studies	Global History and Geography US History and Government
English Language and Arts	Comprehensive English
Foreign Language	Courses vary by schools

As of 2013, New York State public high school students may qualify for three different Regents diplomas. A Regents diploma is granted to students who pass at least five subject-specific Regents exams (Comprehensive English, Any Math, Any Science, Global History and Geography, and US History and Government) with a minimum score of 65 scaled points. However, graduating with a Regents diploma does not automatically mean that a student is college ready. We measure academic success and college readiness based on the guidelines set forth by the City University of New York, CUNY and the recommendations of the New York City Department of Education. According to these institutions, a student is college ready, i.e. places out of remedial courses, when he or she has completed a Regents diploma with Advanced or Honors designation, or has earned a score of 75 or higher on the Comprehensive English Regents exam and of 80 or higher on at least one of the applicable Mathematics Regents exams. Students are granted a Regents diploma with Honors Designation when they achieve a minimum score average of 90 on the five required Regents exams. A Regents Diploma with Advanced designation is given to students who pass nine or more Regents exams with the following distribution: at least one Foreign language, the Comprehensive English, two Social Studies, all three Mathematics, and at least two Science Regents exams (New York City Department of Education, 2007; NYSED Commissioner of Education, 2010; Willens, 2013).

b. Curriculum-Based External Exit Examinations

A curriculum-based external exit exam stands out from other exam systems because it possesses the following six traits:

- It produces signals of student accomplishment that have real consequences for the student;
- It defines achievement relative to an external standard, not relative to other students in the classroom or school;
- It is organized by discipline and keyed to the content of specific course sequences;
- It signals multiple levels of achievement in the subject;
- It covers almost all secondary school students. The school system as a whole must be made to accept responsibility for how students perform on the exams, although it is not essential that a single exam is taken by all;

- It assesses a major portion of what students studying a subject are expected to know or be able to do (Bishop, 1998).

The New York State Regents exams are the oldest academic assessment exams in the United States; they are also the oldest curriculum-based external exit exams, and the only of their type until the early 1990s (Bishop, 2000). Centered on the traits of Curriculum-Based External Exist Exams Systems (CBEEES), researchers have argued that, in their absence, schools' reputations are determined by the social class of their student population and standardized exam scores. Because CBEEES are tied to a secondary school's curriculum, and because they are designed by agencies external to the school, they minimize the influence that the social class of the school's student body exerts on the school's reputation. CBEEES also diminish the leverage that school administrators and teachers have in determining the intensity or weakness of the academic standards in their schools to either maintain or boost their schools' reputation. CBEEES are assumed to objectively and strongly assess whether students have adequate and equitable high school preparation to be granted a high school diploma, by holding them accountable to identical and rigorous educational standards (Knecht, 2007; Bishop, 2000).

c. Development and Scoring

The State of New York has a Commissioner's Technical Advisory Committee[13] that helps develop challenging, consistent and reliable exams (New York State Education Department, 2013; Office of State Assessment, 2013). The Regents exams are developed using IRT Rasch and Partial Credit Models. Since the 1980s, Item Response Theory (IRT) has been the most common alternative to Classical Test Theory (CTT). While there are many forms of CTT, they all rely on the same basic underlying assumption: the actual (X) or raw score of a test taker is comprised of a true component (T) and a random error (E) component; the true score is expressed by the equation $X = T + E$. This actual score is assumed to be the score of an individual, were they able to take the same exam an infinite number of times. Because of the impossibility of an individual to perform any task an infinite number of times, the value T is hypothetical. Domain sampling is the most commonly used form of CTT for practical purposes. In Domain sampling it is assumed that in any particular test, the items presented to the examinee are samples of items that come from an infinite domain of potential items (Kline, 2005).

Contextualizing the CTT and its assumptions, if a student takes the same test many times, the mean score of all his or her completed tests regresses toward this student's true test score. Item Response Theory is a psychometric alternative approach to CTT. Formally speaking, IRT is a psychometric approach highlighting that an individual's response to a particular test is affected by qualities of both the individual and the test (Kline, 2005). For example, an individual's ability to correctly answer the question "In the right triangle ABC, side a measures 2cm and side b measures 3cm. What is the length of side c?" depends on two factors. The first factor is the individual's academic traits in relation to the question; in this case, the Pythagorean Theorem and the algebraic properties of the real numbers. If the individual's knowledge of mathematics is at or above the level of the question, his or her probability of answering correctly is at the higher end. The second

13 Some of the members of this committee are: Robert Brennan, University of Iowa, Andrew Ho, Harvard Graduate School (NYSED, Questions regarding data, 2013).

factor is the item's difficulty, which is directly correlated to factor 1. If the item is of great difficulty and the individual's academic traits do not reach the level of the question, this individual's probability of answering correctly lies on the lower end. Summarizing, IRT states that an individual's likelihood of answering a test item correctly depends on his or her academic traits and the item's level of difficulty (Kline, 2005).

3.4. Description of the Population

New York State takes pride in having a rich history of publically or state funded education that started in 1795 with a statewide system of support for public schools. The first school districts of the state were small, and as the state's population grew, so did the school districts. To accommodate the needs of various geographical areas, various types of school districts emerged. Below we describe the type of school district from which we drew our population of interest. We then provide a succinct description of the population of interest.

New York City Public High Schools

The New York City public school district is the largest and most diverse public school district in the United States. There were 423 public high schools during the year of 2012-2013 in New York City. In the academic year of 2013-2014, there were 1,031,000 K-12 students registered in New York City public schools. The distribution of students by sex was: 51% males and 49% females. The distribution of students' ethnicity was: American Indian or Alaska Native, 1%; Black or African American, 28%; Hispanic or Latino 40%; Asian or Native Hawaiian/Other Pacific Islander, 15%; White, 14%; Multiracial, 1%. It is noteworthy that 69% of the student body served by the New York City public school district was of ethnically underrepresented backgrounds and that 74% of the students were deemed Economically Disadvantaged.

Most of New York City public high schools were structured under a comprehensive system; this is the common system in all of New York State. Under this system, school assignment is primarily tied to its neighborhood's population. This is what is called the Comprehensive school system, which makes up a common school culture that reflects the strengths of the belief in the existence of one best system (Farmer, 2000). Under the comprehensive school structure of New York City, students' school enrollment is based on the following admission priorities:

1. Continuing Eighth Graders Priority: Priority to continuing eighth graders who apply to their current school for ninth grade. Continuing students have a guaranteed match to their school regardless of the Admissions Method(s) and available seats in their school.
2. Feeder School Priority: Priority to applicants who attend identified middle schools.
3. Geographic Priority: Priority to applicants who live in a specific district, borough, or geographic area.
4. Limited Unscreened Priority: Priority to students who attend a school's information session(s) or open house event(s), or visit the school's table at any one of the High School Fairs.
5. Screened for Language Priority: Priority to applicants based on English Language Learner status and home language.
6. Single Gender Priority: Single gender schools are open to only male or only female students.

7. Zoned Priority: Priority – or in some cases, a guaranteed match – to applicants who live in the zoned area (New York City Department of Education, 2014).

The above selection algorithm applies to most public schools in New York City; there are nine public high schools that have different admission selection criteria. Also, some charter schools may have their own selection criteria, generally aligned with the schools' missions and visions.

b. New York City Specialized High Schools

New York City has nine specialized public high schools. Many of these high schools continuously rank in the top 20% in the list of the best high schools in the United States and within New York State. In eight of these schools, the admission criterion is the score earned by students on the Specialized High School Admissions Test (SHSAT) (New York City Department of Education, 2014). One school, Fiorello H. LaGuardia High School of Music & Art and Performing Arts, has as admission criteria, a portfolio, a performance, or both. These schools are the specialized high schools of New York City. These high schools were established under New York State Law 2590 – Section G, and they are:

- The Bronx High School of Science
- The Brooklyn Latin School
- Brooklyn Technical High School
- High School for Mathematics, Science and Engineering at the City College of New York
- High School of American Studies at Lehman College
- Queens High School for the Sciences at York College
- State Island Technical High School
- Stuyvesant High School
- Fiorello H. LaGuardia High School of Music & Art and Performing Arts (New York City Department of Education, 2014a).

Two of these schools are located in Bronx County: The Bronx High School of Science and the High School of American Studies at Lehman College. However, while these schools are located in Bronx County, their demographics are not comparable to those of the County or the County's public high schools. An important component of each of the specialized schools is their comprehensive core curricula, which include a minimum of three years of mathematics and science coursework at or above the Regents level. At Bronx Science, students must successfully complete three years of Regents level mathematics and are recommended a total of 4 years of mathematics. Students must take four years of laboratory sciences including Regents physics (The Bronx High School of Science, 2015).

The goal of the High School for American Studies at Lehman College is to prepare students to be admitted into highly competitive colleges and majors ranging from careers in the arts to STEM (New York City Department of Education, 2014a). They set a four-year program so that their students have an opportunity to complete the necessary number of credits to earn a specialized high school diploma endorsed by the High School of American Studies, which consists of the following:

Social Studies – 12 credits[14], including:

1. A.P. U.S. History I, II, III, IV, V, and VI – 6 credits
2. Global History – 4 credits, including:
   - Global History I and II – 2 credits
   - A.P. World History I and II – 2 credits
- Government – 1 credit
- Economics – 1 credit

English – 8 credits, including:

1. English I and II – 2 credits
2. English III and IV – 2 credits
3. English V and VI or A.P. English Language I and II – 2 credits
4. English VII and VIII or A.P. English Literature I and II – 2 credits

Mathematics – 8 credits, among:

- Algebra – 2 credits
- Geometry – 2 credits
- Algebra II and Trigonometry – 2 credits
- Pre-calculus, A.P. Calculus, or approved elective – 2 credits

Science – 6 credits, including:

- Biology – 2 credits
- Chemistry – 2 credits
- Physics – 2 credits

Foreign Language – 6 credits
Music – 1 credit
Art – 1 credit
Health – 1 credit
Physical Education – 7 semesters (4 credits) (HSFAS, 2013).

Stuyvesant students are required to successfully complete four years of mathematics, including all Regents level courses and the completion of Pre-calculus or an Advanced Algebra course, four years of science, including Living Environment, Chemistry and Physics at or above the Regents level, and a year of Science Electives (Stuyvesant High School, 2015). At The Bronx High School of Science, students must successfully complete three years of Regents level mathematics and are recommended a total of 4 years of mathematics. Students must take four years of laboratory sciences including Regents physics (The Bronx High School of Science, 2015). The Brooklyn Latin High School gives students the option to graduate with an International Baccalaureate (IB)[15] diploma. All students must complete four years of mathematics and science coursework (New York City Department of Education, 2014a). Brooklyn Technical High School's (BTHS) diploma is recognized by the National Consortium of Specialized Secondary Schools for Math, Science & Technology. In order to qualify for this diploma, students must satisfy the following four requirements:

[14] In New York State public schools, 1 credit is equivalent to one semester of instruction. For example, if an Algebra course consists of 2 credits, this course spans over two semesters.

[15] An IB programs challenge students to excel both academically and personally; "IB learners strive to become inquirers, knowledgeable, thinkers, communicators, principled, open minded, caring, risk-takers, balanced and reflective" (International Baccalaureate Organization, 2013)

1. Earn an Advanced Regents Diploma.
2. Complete the required Community Service.
3. Pass Freshmen Design & Drafting for Production, Sophomore Digital Electronics, Chemistry & Physics.
4. Complete all courses required for the students' major (Brooklyn Technical High School, 2015).

Students who do not pursue a BTHS diploma can graduate with a Regents or and Advanced Regents diploma. The High School for Mathematics, Science, and Engineering at the City College of New York seeks to challenge gifted and talented students to expand their intellect. They achieve this mission by instilling in their students the habits of "inquiry, written, and verbal expressions and critical thinking focusing on math, science, and engineering" (New York City Department of Education, 2014a).

The goal of the High School for American Studies at Lehman College is to prepare students to be admitted to highly competitive colleges and majors ranging from careers in the arts to STEM. They set a four-year program so that their students have an opportunity to fulfill the necessary number of credits to earn a specialized high school diploma endorsed by the High School of American Studies, which consists of the following:

Social Studies – 12 credits, including:
- A.P. U.S. History I, II, III, IV, V, and VI – 6 credits
- Global History – 4 credits, including:
  - Global History I and II – 2 credits
  - A.P. World History I and II – 2 credits
- Government – 1 credit
- Economics – 1 credit

English – 8 credits, including:
- English I and II – 2 credits
- English III and IV – 2 credits
- English V and VI or A.P. English Language I and II – 2 credits
- English VII and VIII or A.P. English Literature I and II – 2 credits

Mathematics – 8 credits, among:
- Algebra – 2 credits
- Geometry – 2 credits
- Algebra II and Trigonometry – 2 credits
- Pre-calculus, A.P. Calculus, or approved elective – 2 credits

Science – 6 credits, including:
- Biology – 2 credits
- Chemistry – 2 credits
- Physics – 2 credits

Foreign Language – 6 credits
Music – 1 credit
Art – 1 credit
Health – 1 credit
Physical Education – 7 semesters (4 credits) (High School of American Studies at Lehman College, 2014)

Queens High School for the Sciences at York College is dedicated to providing a rigorous curriculum whose bedrock is science and mathematics. Most students complete the New York State Regents requirements by the 11^{th} grade, and by then they are taking college level classes at York College (New York City Department of Education, 2013-2014 Specialized High School Student Handbook, 2013a). At State Island Technical High School, students have the opportunity to take Advanced level courses in mathematics, science, social and behavioral sciences, computers, engineering, humanities and the performing arts as well as robotics (New York City Department of Education, 2014a). Staten Island Tech offers a Career & Technology Education (CTE) certificate[16]. The vision of its pre-engineering program is to graduate students capable of handling the challenges of college and career readiness by means of robust STEM curricula (Staten Island Technical High School, 2015). Staten Island Tech expects all of its students to successfully complete the pre-engineering curriculum by grade 11^{th}. At this point, all students would have completed two terms of physics, electronics, and pre-calculus. Fiorello H. LaGuardia High School of Music & Art and Performing Arts prides itself on its international reputation for being the "first and foremost high school dedicated to nurturing students gifted in the arts" (New York City Department of Education, 2014a, p. 8). Whilst this school specializes in the arts, it has a strong STEM curriculum; for its non-academically accelerated students, there is a minimum of three mathematics Regents courses; accelerated students complete a minimum of two math Regents, an AP calculus, and an AP statistic course. In science, all students must complete a four-year sequence that includes: Living Environment, Chemistry or Earth Science, Physics and at least an AP in either of the latter courses, hence all students graduate with at least a Regents diploma with advanced designation (Fiorello H. LaGuardia High School of Music & Art , 2015).

Adelman (1999) described curricula, such as those of the specialized high schools, as intense and of high quality. Indeed, specialized schools are continuously ranked among the best high schools in the United States. For example, on the 2014 best high schools ranking of the US News & World Report, specialized –also called magnet- schools, are described as follows: "Magnet schools are public high schools that attract the most talented students in a region using an application process that typically involves test scores and grade-point averages" (US News Ranking, 2014). In this report, The High School of American Studies at Lehman College and The Bronx High School of Science were ranked 11^{th} and 13^{th}, respectively. Under the US World & News Report's section of best STEM high schools, Stuyvesant High School was ranked #11 (US News Report, 2014). In a Business Insider issue, Staten Island Technical High School and Stuyvesant High School were ranked respectively as # 5 and # 4 among the 25 best public high schools in the United States (Business Insider, 2014).

c. *Charter Schools*

Charter schools are a particularly interesting case to study. Charters fall under the category of public schools because they are publicly financed; however, their per-pupil expenditures are not the same as those of traditional public schools (Davis, 2013). Charters have to be approved by the

[16] These programs were established to raise the quality and rigor of courses than can prepare students for careers in post-secondary education and the labor market upon high school graduation. It is a program of study that involves a multiyear course sequence integrating core-academic with technical and occupation knowledge. Some high schools confer an Associate's degree upon the completion of a CTE program (New York State Education Department, 2013b).

New York State Department of Education, but unlike other public schools in New York State, which are managed by the University of the State of New York with the Board of Regents as their governing board, charters are usually developed and managed by non-governmental groups such as parents, community, or educational management organizations (EMO), and they do not follow the selection algorithm of New York City public schools. Nonetheless, since they are public schools, local or state governing boards have the power to sanction or close charter schools if they do not make adequate academic progress. It has been suggested that while charters are on the rise, there is a paucity of research on their formation (Renzulli, 2005). Some studies have documented that teachers in charter schools feel a higher sense of autonomy. However, evidence by Torres (2014) may suggest that this may be changing. The study challenges the notion of teachers' perceived freedom and documents, by means of case studies, that teachers in charters managed by CMOs did not feel high levels of independence (Torres, 2014). There have been arguments, primarily from market theorists, supporting the notion that the growing pace of charters will infuse a competition into the education market that has the potential to force public schools to upgrade their structure as to provide students with a better education. Nonetheless, in his study about the competition of charters, Davis (2013) presents little to no evidence supporting this claim.

d) Private high schools

The New York City school district has 306 private high schools serving a total of 132,926 students. Most of these high schools are of some kind of religious denomination. The main religious affiliations of private high school are Roman Catholic followed by Jewish. They are distributed as follows: New York, 73 high schools serving 30,310 students; Queens, 44 high schools serving 18,405 students; Brooklyn, 153 high schools serving 65,503 students; Staten Island, 14 high schools serving 5,570 students; Bronx 22 high schools serving 13,138 students (Private School Review, 2015).

e) Consortium Schools

The consortium schools arose from a grassroots movement. A group of education leaders who believed in active student learning, exemplary professional development and innovative curriculum and teaching strategies for 21st-century students came together and decided to provide an alternative to standardized education. The consortium schools do not follow the same rubrics and standards of the Board of Regents. Consortium schools are only required to administer the Comprehensive English Regents exam. Therefore, students in these schools do not take any of the mathematics Regents exams. Consortium schools devised a system of assessments consisting of eight components aligned with state standards, professional development, and external review, formative and summative data. The mission of Consortium schools is to help their students to gain high levels of analytical thinking, reading comprehension, research writing skills, the application of mathematical and computation and problem-solving skills, computer technology skills, and more. Students' evaluations or assessments are conducted by teachers' and experts external to the schools such as universities and the business sector. The performance-based assessment of the schools is managed by The Performance Assessment Review Board, Inc. This board is comprised of educators, test experts, researchers and members of the legal and business world (Consortium, 2003).

3.5. Data Collection and Instruments

We gathered studies relevant to our literature review via digital sources using the keywords "Mathematics course taking, course completion[17], high school mathematics course taking, mathematics course access, high school mathematics, mathematics courses in New York City, and mathematics course pipelines". We performed searches in *Google Scholar, Educational Resources Information Center (ERIC), Journal Storage (JSTOR), Stony Brook University Library's various databases, and Google.* The author also acquired a number of studies from the syllabi of the various courses completed during his doctoral coursework. We also included major national reports about K-16 education in the United States. To answer our research questions we gathered school samples from the New York State Report Cards data and from data provided to us by the New York City Department of Education. To obtain more detailed data, which includes, among other statistics, students' course completion with grades, we submitted a research proposal to the New York City Department of Education's Institutional Review Board. This process took seventeen months. The New York State Report Cards are available to the general public and are published yearly. We collected a sample of 192 eligible schools for this study, for a total sample of about 45%. Taking into consideration that not all 423 New York City public high schools fit our selection criteria, our sample is close to 50%. The size of our sample ensures a high degree of internal validity. The individual samples, New York City and the Bronx, both consist of 96 representative secondary public schools. Bronx County has 106 public schools, hence, our Bronx sample size of 96 make up a sample of about 91%. However, since these 96 schools are all of the schools that fit our selection criteria, this is the population, not a sample of Bronx County public high schools. The population of New York City public high schools, excluding the Bronx, is 317; hence, our New York City sample size is a little over 30%. The average New York City public high school enrollment over three years, before separating the Bronx from New York City, varied from 123, the smallest, to 5300, the largest. The largest of all schools was Brooklyn Technical high school, one of the nine specialized schools in New York City. The school size was distributed as follows: about 55% were small, about 31% were medium, and about 14% were large. The school size distribution for the New York City sample was: about 41% small, about 34% medium, and about 25% large. The school size distribution for Bronx County was: about 68% small, about 29% medium, and about 3% large.

[17] We also used the compound words course taking and course completion.

3.6. Methodological Considerations: Data Type

The statistical and probabilistic analyses that will be carried out in this thesis rely heavily on various assumptions about our data. These assumptions rest on the Central Limit Theorem:

Central Limit Theorem: If $\overline{X}$ is the mean of a random sample of size n taken from a population with mean μ and finite variance σ^2, then the limiting form of the distribution

of $\dfrac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$ as $n \to \infty$, is the standard normal distribution $n(z; 0, 1)$.

The proof of this theorem is beyond of the scope of this thesis. However, we shall be relying upon this theorem when employing statistical tests and making inferences. Also, note that the theorem can be easily extended to two or more sample cases. Researchers have different set guidelines as to what constitutes a large enough sample size in regards to whether or not the Central Limit Theorem Applies. These guidelines vary significantly from the "hard" to the social sciences, and they also depend on the types of tests been applied. In probability theory, $\bar{x}$ will be approximately normal given an $n \geq 30$ (Walpole R. E., Myers R., Myers S., & Ye K., 2012). Throughout the thesis, the statistical tool most appropriate, and hence the only one used, is a *t-test* of proportion difference. The goal of this test is to determine whether a difference between two groups exists. However, if the samples used in the test are extremely small, the tests may not detect a significant difference when one in fact exists, resulting in a Type II error. In the study of proportion differences, however, the convention is that if $m\hat{p}$, *and* $n\hat{q}$, where m and n represent sample proportion sizes, and $\hat{p}$ and $\hat{q}$, represent the probability of success drawn from samples m and n, respectively, are both greater than or equal to 10, then we can safely assume the condition of normality (Devore, 2012). At the school level, our smallest sample consists of fifteen schools with a reported 4-year mathematics requirements. Moreover, the summary statistics that we compare from group to group are all means in the form of proportions and the Central Limit Theorem implies that, irrespective of the population distribution from which our samples are drawn, our samples are normality distributed. Therefore, we safely put the normality assumption to rest. Except for Bronx County, where we work from the population, based on inclusion criteria, every sample of schools and students were chosen at random from their respective school districts and high schools. We chose our samples in the following manner: 1. We entered the name of each New York City public high school into an excel spreadsheet. 2. We used the *rand*() function on excel to draw a sample of 60%. 3. After the sample was created, we used our selection criteria to eliminate schools. 4. Our final random representative sample size was about 45%.

3.7. Presentation of Results

In our results tables, we report the pooled proportion of success , and failure , the pooled standard error (SE), the *z*-value and the *p*-value of any tested proportion differences, and the Confidence Interval (CI) of the calculated effect sizes. These statistical values allow us to determine whether any observed differences among sample proportions are statistically significant, or likely due to chance. Social science education researchers are usually concerned with the effect size or power of statistical models. In light of this, we remind the reader that our effect size is the proportion difference we acquired by performing a *t-test*. In the language of social science education research, this effect size can be presented in the following three forms:

1. The difference in sample proportions = $\hat{p}_1 - \hat{p}_2$. (This value is sometimes called the risk difference.
2. The Relative Risk or Risk Ratio (RR) = $\frac{p_1}{p_2}$ or $\frac{p_2}{p_1}$
3. The Odds Ratio (OR)

We represent the effect size by means of the difference in sample proportions.

3.8. Statistical Measures and Inferences

We answered research question one by compiling descriptive statistics of the schools' key socio-demographic and performance indicators. All variables measured here were school-level variables. We summarize the means of variables of interest and then determine whether these means are statistically significantly different by using t-tests. Since our interest was to determine whether schools were different, irrespective of how they were different, we employed two-tailed t-tests. Following convention, we considered test results to be statistically significant when a *p*-value is less than or equal to 0.05. This statistic tells how probable it is that any observed differences were due to chance. With an alpha or significance level of 0.05, the probability that observed differences in our t-tests are due to chance, is less than or equal to 1/20. We also generated a confidence interval, which together with the *p* statistic helped determine whether we should reject the null hypothesis of any test. A confidence interval provides us with a range of values within which, if all our probabilistic assumptions were met, our statistics would be contained. The confidence interval is bounded from the left and the right in the number line. Following convention, we set the confidence interval to 95%. The interval tells us that our calculated statistics should fall within our calculated range 95% of the time. It is important to understand the *p*-values within a context. By themselves, *p*-values are not as valuable. Summary statistics such as means, medians, standard deviations, standard errors and correlation coefficients (r_S) allow us to understand data in specific contexts. Within our statistical context, our data consists of actual values. That is, for the most part, we do not perform data analysis on aspirational data: a typical measure of people's plans or desires, simulated data: data acquired when situations don't allow for actual data collection, such as hurricane data, or survey data: self-reported data. In each of the previous examples of data type, the researcher is in control of the data generation technique but the data is almost completely out of his or her control. In our case, we have actual data that is almost within our control. That is not to say that our data if perfect. In fact, our data is subject to many limitations, where the fact that we did not collect it is the main one. Nonetheless, given the exactitude of our data, the effect sizes we calculated were fairly strong by themselves. Let us recall that a *p*-value allows us to make statements about a null hypothesis; the more plausible a null hypothesis is, the more confident we can feel about accepting or rejecting it (Nuzzo, 2014). Our decision to reject or fail to reject a null hypothesis was informed by at least two explicit factors: the parameters of the statistical tests; as stated before, the effect size should be very strong given the data type and samples sizes we employed. Secondly, results aligned with our review of the mathematics course completion literature were expected, for our decisions took into consideration the overall conclusions of the literature review. We inform the reader, however, that for research question two, we display summary statistics within the *t*-test for convention purposes only, since this question is answered using not a sample, but the population of interest.

After we determined the distributions of the mathematics course completion of Bronx County's public school students, we categorized them according to our adaptation of Burkam and Lee's (2003) mathematics course completion pipeline measures[18]. To ensure homogeneity in our samples, we only included high schools that had been operating long enough so that any eligible students had an opportunity to complete all three mathematics Regents exams. This is important because *Mathematics Regents Exams* is a primary independent variable in our study. To be included in this study, a high school would have had to enroll students from grades 9-12 only. It must have been offering the Regents exams since 2006. We decided to exclude schools that only offer grades 9, 10-11, schools that had been offering the Regents exams only after 2006 and schools that are part of the Consortium[19]. We exclude the Consortium schools because they were exempted from all but the English Regents exam. We also excluded schools offering grades 6-12, 7-12 and 8-12. We did this to be able to properly account for secondary school students key demographic variables. This process ensures that students have similar baseline characteristics, which is essential in any quasi-experimental study (Agresti & Finlay, 2009; Shadish , Cook , & Campbell , 2001). We place schools into one of three categories, depending on their enrollment capacity: small, medium, or large.

We created multi-year databases to answer research questions one and two. Each database contained information that was averaged over three academic years: 2010-11, 2011-12, 2012-13. By taking an average over three academic years, we minimized the possibility of inflation or

[18] One should note that the values to the left have been filled in by us, using the course categories of New York City public high schools.

1. Non-Academic	Other Math (general, basic, and consumer)
2. Low-Academic	Integrated Algebra over three or more semesters
3. Middle-academic I	Integrated Algebra, Geometry
3. Middle-Academic II	Algebra 2/Trigonometry
4. Advanced I	Probability/Statistics
5. Advanced II	Pre-calculus
6. Advanced III	Calculus, AP Calculus,

[19] For a detailed account of the Consortium see: Knecht, D., (2007). The Consortium and the Commissioner: A Grass Roots Tale of Fighting High Stakes Graduation Testing in New York. The Urban Review, 39, 1, 45-65.

deflation on each of the variables we analyzed. For example, since many of Bronx public high schools have been undergoing various changes over the last fifteen to twenty years, particularly in size, we minimized the possibility of categorizing a school as small when this school was in fact of medium and not small size. This technique would also allow us to draw inferential conclusions about students' probabilities of passing or mastering a particular mathematics Regents exam within a four-year period, which is the expected time to complete high school. We wanted to do the same for research question three; however, as stated in the New York City Department of Education's document entitled o*rganization of course and grades datasets*:

> As of the 2012-13 school year, schools were instructed to begin using a standardized course code system. As a result course codes for this school year are much more reliable than in previous years... Prior to the 2012-13 school year, there was no standardized course code system making it more difficult to identify courses using the course_code variable. As an alternative, use the course_title or course_description variables to identify courses (New York City Department of Education, 2015d).

Since the year of 2000, with the enactment of the No Child Left Behind Act, the New York City Department of Education has made significant innovations in their data collection procedures. We agree with the New York City Department of Education in that, any data collected prior to the academic year of 2012-13 would pose a number of challenges. The primary challenge we faced was the heterogeneity of course descriptions. We used only the academic year of 2012-13 to determine students' high school mathematics course completion rates.

3.9. Variables

Dependent Variables: Each research question has its own set of independent variables. Research question one is descriptive in nature for we do not use the dependent/independent variable classification when addressing it. Research questions two and three are concerned with proportion differences and any inferences made are made about the populations. For this reason, our study is bi-correctional in nature, and each variable is looked at independently of any other. However, the samples of research questions two and three have a fundamental difference, namely, one set of schools has a self-reported 4-year mathematics requirement and another set does not. Therefore, conceptually, the reported 4-year mathematics requirement is the main independent variable in research questions two and three. We contextualize the 4-year mathematics requirement as students' mathematics course completion. There is now sufficient evidence to make the claim that this variable is the most important when determining students' academic success (Adelman, 1999; Fernández , 2015a; Finkelstein & Fong, 2008; Hill, 2006; Jones, Davenport Jr., Bryson, Bekhuis, & Zwick, 1986).

Independent Variable:

Research suggests that math courses provide cross-disciplinary benefits to first-year college students (Sadler & Tai, 2007). Thus math courses are reported to be gatekeepers of advanced math and science courses. While the primary independent variable of this thesis is *students' math course completion,* each research question, with its sub-questions, has its own independent variable. We refer to these independent variables as *school-level variables;* this is because they allow us to draw conclusions about, not only school students, but schools themselves. Nonetheless, when we perform tests of comparison proportion, the population that we use on our calculation belongs to the universe of students, not schools. Even though the data for the Regents exams is aggregated, we do have the total count of students taking the exams.

School Size:

This variable measures the total number of students enrolled in a school. Most of the schools selected in the study offered grades 9-12 only. However, a number of schools, mostly in Bronx County, offered grades 6-12. Because we did not have demographical data by grade levels, the enrollment of schools offering grades 6-12 is taken in its entirety. By design, this variable is categorical. Our three categories are adaptations of those of (Patrice, Schwartz, Stiefel, & Chellman, January 01, 2008)[20]: small (0, 500], medium [501, 1200], and Large ≥ 1201.

Ethnicity:

This variable defines students' ethnic backgrounds and by design it is categorical. The New York State Report Cards classify students' ethnic backgrounds as American Indian or Alaska Native, Black or African American, Hispanic or Latino, Asian or Native Hawaiian/Other Pacific Islander, White and Multiracial. Because across every sample we use the ethnic category 'multiracial' constituted less than half a percent, we did not use it in this study.

Free or Reduced Lunch (FRL)

This variable provides an assessment of the social position of students' families in relation to one another and it is also categorical by design. However, for research question three, this variable is presented as binomial, consisting of the category eligible for free and reduced-price lunch and not eligible for free or reduced-price lunch. Free meals are available for students whose families annual income ranges up to but no higher than, 100% of the federal poverty line, which is, $0 to $ 23,850 a year for a family of four. Reduced-price meals are available to students whose family's annual income ranges from 101% up to 185% of the federal poverty line, which is $23,851 to $ 44,123 for a family of four (United States Department of Agriculture, 2013). Please note that this variable has an inverse relation to social class, as students' free lunch eligibility increase, their families' socioeconomic status decreases. The dollar cut-offs of this variable's categories apply to the academic year of 2014-2015 which spans from July 1, 2014, through June 30, 2015. While not significantly, these figures tend to change from year to year.

Regents Passing and Mastery Percentages[21]

This variable represents the percentage of students who passed or mastered either of the mathematics Regents exams: Integra Algebra, Geometry, Algebra2/Trigonometry. The passing and mastery scores on all exams are, respectively, 65 and 85 points. Please note that the category mastery is a subset of the passing category.

[20] They used five categories: very small (0, 300], small [301, 500], medium [501, 1200], large [1201, 2000], and very large ≥2001.

[21] Please note that these scores are scaled. For a detail description of the scaling system, please refer to Appendix A.

Course completion percentage

This variable represents the percentage of students who completed, irrespective of the marking grade, any of the following mathematics courses: Calculus, Precalculus, Probability/Statistics, Algebra2/Trigonometry, Geometry, Integrated Algebra, or Other[22] Math courses.

Students' Mathematics Expectations

This variable is binary by design and it refers to the reported 4-year mathematics requirements of some New York City Public high schools, as published in the New York City public high schools handbook (New York City Department of Education, 2013). Please note that this 4-year of mathematics requirement is reported by the schools in the New York City public high school handbook, which is available to parents and students every year.

Regents Diploma

This variable represents the percentage of student who in a given school year graduates with a Regents Diploma.

Advanced Regents Diplomas (ARD)

It represents the total number of students who graduate with Regents or Advanced Regents Diplomas at the end of a given school year. Students graduating with ARDs are potentially graduating college ready.

Total Graduates

This variable consists of the total number of students who graduate in a particular school year. With this variable, we were able to compute the variables Regents Diploma (RD) and Advanced Regents Diploma (ARD) Conferment rates. While the variable ADR is contained within the variable RD, we treat both variables as mutually exclusive. We did this to make it easier for the reader to figure out the total percentage of graduates who do not achieve college readiness. The category RD is, numerically, less than the variable Total Graduates. This is because up until the academic year of 2012-2013, the Report Card did not include the category "Local Diploma", which, numerically, together with the variable RD add up to the variable Total Graduates.

Individualized Education Program (IEP) Diploma

This diploma recognizes an individual student's achievement. Unlike students receiving RD and ARD, the New York State Education Department does not consider students receiving IEP as graduates.

Post-secondary Plans of Completers

This variable measures students' aspirations upon graduation. In the New York State Report Card, this variable is expressed categorically consisting of the following categories: Attending 4 Year College in State, Attending 4 Year College Out of State, Attending 2

[22] Please refer to Appendix B for the titles of the Other Math courses.

Year College in State, Attending 2 Year College Out of State, Other Post Graduation Plans in State, Other Postsecondary Plans out of State, To the Military, To Employment, To Adult Services, Other Plans, and Plans Unknown. For analytical purposes, we combined the categories expressed as in State and Out of State as a single category. We combined the categories 2 Year College and Other Postsecondary Plans as 2 Year College. We combined the categories To Employment and To Adult Services. We combined the Categories Other Plans and Plans Unknown. In our analysis, this variable consists of the following categories: To Senior College (SC), To Community College (CC), To the Military, To Employment, (TE), and Plans Unknown (PU).

Students' Dropout Rate

According to the New York State report Cards, this variable represents the percentage of students in grades 9 through 12 whom, for any reason, other than death, left their school prior to graduation and did not transfer into another school or high school equivalency program, or at least their school was not notified of the students' enrollment status. The variable is calculated by adding the total number of students who left the high school and then dividing it by the sum of all students registered in grades 9 through 12 during a given school year. This decimal is then multiplied by 100 and is expressed as a percentage (The University of the State of New York, 2014; Council on Children & Families, 2003-2016).

Percentage of Students with Limited English Proficiency

The Student Information Repository System Manual defined this variable represents as the percentage of students who have limited proficiency in English (The University of the State of New York, 2014). On Education Law 3204 section 2, students with Limited English Proficiency are defined as, "pupils who, by reason of foreign birth or ancestry have limited English proficiency" (New York State Education Department, 2011).

Chapter 4. Results

4.0. Introduction

As discussed in Chapter 3, The New York City public school district is the largest and most diverse school district in the United States. There were 423 public high schools during the academic year of 2012-2013 in New York City. We collected a sample of 192[23] eligible schools for this study, for a total sample size, if we consider our selection criteria, of about 50%. Since our sample size is sufficiently large, we are confident that the results of our statistical tests are robust against sampling errors and design fragility. Individually, the New York City and the Bronx samples each consist of 96 representative secondary public schools. Bronx County has a total of 106 public schools, and ten of these did not meet our inclusion criteria. Therefore, we are working with the population and not a sample of Bronx County public high schools.

4.1. Research question I: How are key socio-demographic and performance indicators of New York City public high schools distributed?

a. How do the socioeconomic and ethnic distributions of Bronx County public high schools compare to those of New York City public high schools?

By answering section (a) of research question one, we determined whether the socioeconomic and ethnic distributions of New York City public school students are comparable to those of Bronx County public schools, a subset of New York City public high schools[24].

Table 5
Social and ethnic distributions of New York City and Bronx Borough (2014)

Variable	NYC	Bronx	Difference	SE	99% CI
Hispanic (of any race)	24%	54%	-30%***	.0004494	[-30.1%, - 29.9%]
White	37%	10%	27%***	.0003118	[26.9%, 27.1%]
Black	21%	30%	-9%***	.0004153	[-9.1%, -8.9%]
American Indian	0.18%	0.22%	-.0004%***	.0000398	[-0.05%, -0.03%]
Asian alone	15%	3.5%	11.5%***	.0003109	[11.45%, 11.55%]
Native Hawaiian	0.03%	0.02%	0.01%***	.0000155	[0.007%, 0.014%]
Two or more races	2.67%	1.59%	1.08%***	.0001437	[1.05%, 1.11%]
Median Household Income	$61,329	$34,388	$26941%***	.0009228	[$26,941, $26,941]

Note: * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$ for a two-tailed test. The first and second numbers of the CI are, respectively, the lower bound and upper bounds of the interval. NYC (*N*) = 6,941,323; Bronx (*N*) = 1,413,566.

The results of Table 1 show significant differences across all of the demographic indicators of New York City and The Bronx. Particularly, the median household income of New York City is almost twice that of Bronx County. This is not a surprise since Bronx County has continuously been reported as one of the poorest counties in the United States (Ink, 2011). As of 2010, for example, South Bronx was the poorest district in the United States with 30% of its population living below the poverty line (Daily News Washington Bureau, 2010). Most of the literature in

[23] This includes the six specialized schools.
[24] For the purpose of this study, New York City excludes the borough "The Bronx".

STEM education, particularly as it relates to mathematics high school course completion, asserts that there is a wide attainment gap between ethnically represented and ethnically unrepresented students in STEM disciplines (National Center for Education Statistics, 2007; National Commission on Excellence in Education, 1983; Oakes, 1990; Tyson, Lee, Borman, & Hanson, 2007). To help the reader contextualize the meaning of "ethnically", we conceptualize the variable "Ethnicity" as binary: ethnically represented and ethnically underrepresented students. The category ethnically represented is composed of White and Asian students. The category ethnically underrepresented is composed of Black or African American, American Indian and Alaskan Native, Native Hawaiian or Other Pacific Islander, and Hispanic students. We decided not to include the category two or multiple races or ethnicities because this category made up barely 0.13% of the total population. In light of the suggestive evidence of our literature review, and the statistics of Table 1 above, when we refer to ethnic minorities, we adopt the terminology "ethnically underrepresented" or "economically disadvantaged" students. Most of the studies in our literature review show that ethnically underrepresented students in STEM, particularly in relation to high school mathematics course completion, are also economically disadvantaged students. The results of Table 1 also indicate that the population of Bronx County is significantly poorer than that of New York City. Our literature review and the results of Table 1 led us to develop the following hypothesis to help us answer research question 1a: "How do the socioeconomic status and ethnic distributions of Bronx County public high schools compare to those of New York City public high schools?"

H_0 := There are no statistically significant differences between average socioeconomic and limited English proficiency distributions of New York City and Bronx public high school students.
H_{1a} := The average socioeconomic and limited English proficiency distributions Bronx public high school students are statistically different from those of New York City public high school students.

Table 6
Average Schools' socioeconomic distribution (2010-13)

Variables	Bronx	New York City	Difference	SE	95% CI
Free Lunch	75%	60%	15%***	.0026268	[14.5%, 15.5%]
Reduced Price Lunch	5%	7%	-2%***	.0013402	[-2.3%, -1.8%]
Limited English Proficiency	18%	13%	5%***	.0019568	[4.6%, 5.4%]

Note: * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$ for a two-tailed test. The first and second numbers of the CI are, respectively, the lower bound and upper bounds of the interval. Bronx (*N*= 50,649); New York City (*N*= 93,490).

The parameters of the test of proportion difference indicate that, on average, the free lunch and reduced-price lunch eligibility, and limited English proficiency rates of the Bronx and New York City public high school students were statistically different. The results of the test of proportion difference, together with our literature review, let us confidentially reject the null hypothesis and conclude that, on average, the Bronx public high school students' free and reduced price lunch eligibilities are higher than those of New York City public high school students. Similarly, on average, the Bronx public high school students have higher limited English proficiency rates than New York City public high school students. We want to state that while the reduced price lunch eligibility income range for a student from a family of four lies between 101% and 185% of the federal poverty line, the high end cut off of the income is well below the middle-class range cut off. According to the Pew Research Center, based on income alone, a family of four needs to earn between $48,347 to $145,041 to make it into the middle class[25] of the United States (PewResearchCenter, 2015).

Table 7 below helps us answer the part of research question 1a that is concerned with students' ethnic or racial distribution. We warn the reader that the category Hispanic has to be interpreted carefully. Hispanic is not a racial but an ethnic[26] category; therefore, as recognized by the census of 2014, Hispanics can be of any race. We took care to disaggregate each of the races presented in Table 1 by race alone. For example, those classified as White and Black were individuals who did not select the category of Hispanic. Since the ethnic categories presented in the New York State Report Cards add up to 100%, we suspect that New York State schools treat the variable Hispanic as mutually exclusive of the racial categories Black and White. However, this is an educated guess and we do not have empirical evidence to justify why the Report Card's ethnic categories add up to 100%.

Table 7
Schools' ethnic distributions (2010-13)

Variables	Bronx	New York City	Difference	SE	95% CI
American Indian	0.53%	0.56%	-.03%**	.0012572	[-.11%, .05%]
Black	32%	33%	-1%***	.0025872	[-1.5%, -.5%]
Hispanic	61%	34%	27%***	.0027351	[26%, 28%]
Asian	3%	19%	-16%***	.0018781	[-16.3%, -15.7%]
White	3.6%	13%	-9.4%***	.0016326	[-9.7%, -9.1%]

Note: * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$ for a two-tailed test. The first and second numbers of the CI are, respectively, the lower bound and upper bounds of the interval.

The results of Table 7 indicate that each ethnic variable differs from one school set to the next. These results allow us to reject the null hypothesis and we conclude that, on average, Bronx public high schools enrolled American Indian, Asian, and White students at rates that are lower than those of New York City public high schools. Conversely, Bronx high schools enroll Black and Hispanic students at rates that are higher than those of New York City public high schools. The variables Hispanic and Asian, as distributed in the Census data from Table 1, have distributions similar to those of the schools' data. This may indicate that the enrollment distribution difference between Hispanics and Asians is to a certain degree expected. However, while the White population of Bronx County is 10%, the White student enrollment in Bronx County schools is just 3.6%. Because the White population of New York City is 37% and the White student enrollment at New York City public high schools is 19%, we see that there is a similar relationship between the geographic region race distributions and the school ethnic enrollment distributions. It then becomes apparent that enrollment distributions of New York City and Bronx public high schools, when the variables of reference are race or ethnicity, are comparable to the racial and ethnic distributions of the Bronx and New York City.

[25] This definition serves as point of reference and it ought to be interpreted cautiously.
[26] The concept of Ethnicity is beyond the scope of this thesis. For specific details refer to the works of: Jenkins, 2014: 2008, & Eriksen, (2012).

If we recall the census data, then, the observed racial and economic disparities between Bronx County and New York City public high schools are expected. The United States government, the private sector, public and private policy makers, all agree that education is paramount for the country to set itself back at the forefront of science and technological development (National Research Council, 2013; National Science Board, Science and engineering indicators 2012, 2012). Nonetheless, despite various efforts, such as the No Child Left Behind Act, warnings, and recommendations from national reports such as “A Nation at Risk”, racial and ethnic minorities as well as economically disadvantaged students such as Blacks and Hispanics, continue to attend high-poverty high schools at rates much higher than other students. Data from the National Center for Education Statistics show that in 2005, Black and Hispanic students had greater probabilities to attend high-poverty high schools than their white or Asian counterparts (National Center for Education Statistics, 2007). Our results today are not that different from of 2005. It is within this framework that we ask research question 1b.

4.2. Research question Ib: How do the average reported mathematics Regents exams passing and mastery rates of Bronx County public high schools compare to those of New York City public high schools?

We measured the average aggregate mathematics Regents exam performance of the Bronx and New York City public high schools and compared them to determine whether they were statistically significantly different. The measures of mathematical performance that we used are their reported student passing and mastery rates on the Integrated Algebra, Geometry, and Algebra2/Trigonometry Regents[27] exams. To help the reader contextualize the students’ academic performance in each of the exams, it is helpful to understand the intensity of each of them. Table 7 indicates that Integrated Algebra and Geometry both fall under the Middle-Academic I category.

Table 7
Burkam & Lee’s high school course classification (2003)

Intensity	Course Description
1. Non-Academic	Nonacademic courses (general, basic, and consumer)
2. Low-Academic	Pre-algebra, Algebra IA, or Algebra 1B
3. Middle-academic I	Algebra I, Geometry
3. Middle-Academic II	Algebra II/Trigonometry
4. Advanced I	Trigonometry, Analytical Geometry, Probability/Statistics
5. Advanced II	Pre-calculus
6. Advanced II	Calculus, AP Calculus,

Adapted from (Burkam & Lee, 2003).

Given the low intensity of these courses, in principle, one would expect students to be passing and mastering these courses, as determined by their exit examinations, the Regents exams, at relatively high rates.

[27] Refer to Appendix B for a detailed explanation of the exams’ scaling system.

H_0 = There are no statistically significant differences between the average passing and mastery rates of the Bronx and New York City public high schools on the Integrated Algebra, Geometry, and Algebra2/Trigonometry Regents exams.
H_{1b} = Bronx public high schools' average passing and mastery rates on the Integrated Algebra, Geometry, and Algebra2/Trigonometry Regents exams are statistically significantly different from those of New York City public high schools.

Table 8
Students' passing mathematics Regents scores (2010-12)

Variable	Bronx	New York City	Difference	SE	95% CI
Integrated Algebra	50%	61%	-11%***	.0045214	[-12%, -10%]
Geometry	46%	62%	-16%***	.0070617	[-17%, -14.6%]
Algebra2/Trigonometry	32%	52%	-20%***	.0099543	[-21.8%, -18%]

Note: * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$ for a two-tailed test. The first and second numbers of the CI are, respectively, the lower bound and upper bounds of the interval. Integrated Algebra: Bronx, N= 19,118; New York City, N= 32,203. Geometry: Bronx N= 6,547; New York City, N= 19,271. Agebra2/Trigonometry: Bronx, N= 3,335; New York City, N = 10,219.

Let us start by highlighting that on average, as the intensity of the exams increases, so does the average passing rate differences between the Bronx and New York City public high schools. Grounded on the results of our literature review, and the *p*-value of the test, we reject the null hypothesis and conclude that on average, the Integrated Algebra passing rate of the Bronx public high school students is eleven percentage points lower than that of New York City public high school students. The New York City Department of Education considers the Geometry and Algebra2/Trigonometry courses as advanced[28] math courses. From the results of the Geometry and Algebra2/Trigonometry test of proportion difference, we observe that, on average, as the intensity of the exams increases, the performance gap of Bronx public high school students also increases. The results of the test for the Geometry exam show that the Bronx public high school students pass this exam at a rate that is less than two-thirds that of New York City public high school students. Similarly, the passing rate of Bronx public high school students in the Algebra2/Trigonometry exam is over one and a half times lower than that of their counterparts in New York City public high schools.

Table 9
Schools' mastery mathematics Regents scores (2010-12)

Variable	Bronx	New York City	Difference	SE	95% CI
Integrated Algebra	2%	5%	-3%***	.0017637	[-3.3%, -2.7%]
Geometry	4%	14%	-10%***	.0045574	[-10.6%, -9.3%]
Algebra2/Trigonometry	5%	18%	-.13%***	.0070819	[-14%, -11.9%]

Note: * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$ for a two-tailed test. The first and second numbers of the CI are, respectively, the lower and upper bounds of the interval. Integrated Algebra: Bronx, N= 19,118; New York City, N= 32,203. Geometry: Bronx N= 6,547; New York City, N= 19,271. Agebra2/Trigonometry: Bronx, N= 3,335; New York City, N = 10,219.

[28] In an unpublished systematic review of the high school mathematics course taking literature, the preliminary results of Fernández (In Progress) indicate that, on average, most researchers conclude that Geometry and Algebra II are Intermediate and not advanced courses.

The comparison test results for the mastery level are similar to but more compelling than, the passing scores for Bronx and New York City public high schools. Unlike the passing rate, which certifies that students satisfy the minimum graduation requirement, mastery indicates that students have mastered a course; moreover, a mastery score also indicates that a student is college ready[29] in the mastered discipline (New York City Department of Education, 2013; City University of New York, 2016); There is a pattern similar to, but acuter than, that that which we observed in the exams' passing rates: as the intensity of the exams increases, the achievement gap of Bronx public high school students also increases. Our test provides sufficient evidence to confidently reject the null hypothesis and conclude that, in all mathematics Regents exams, Bronx public high school students have lower mastery rates than their counterparts from New York City public high schools. This is particularly the case for the more advanced exams, Geometry and Algebra2/Trigonometry, in which Bronx students achieved mastery at rates that were, respectively, over three and a half times lower than those of New York City students.

The correlation is clear: as the intensity and stakes of the mathematics Regents exams increases, Bronx public high school students' achievement in these exams, measured by their average passing and mastery rates, decreases. Let us recall from our literature review that in Adelman's (1999) study about Bachelor's degree completion rates, high school graduates who earned Bachelor degrees, completed as the highest level course Algebra 1 at a rate of 7%, Geometry at a rate of 28.5%, Algebra 2 at a rate of 44.4%, and Trigonometry at a rate of 62.2% (Adelman, 1999).

4.3. Research question Ic: How do Bronx and New York City public high school graduates' diploma attainment and post-secondary aspiration rates compare to one another?

Research question 1c is arguably the most important question within research question 1. As the educational demands of careers and professions increase, people realize the importance of a proper education in the areas of mathematics and science. For example, the High School for Health Professions and Human Services, a public high school located in Manhattan, states the following about the Regents diplomas:

> Passing the required Regents tells everyone that you at least have a basic high school education. Most colleges require a minimum of a Regents diploma to even consider an applicant for acceptance. It is more rigorous to achieve an Advance Regents Diploma and is recommended for students who plan to attend college after high school. The Advance Regents Diploma shows you did more than the minimum to graduate high school. An Advance Regents Diploma shows you had a sound high school education. Commission of Education David Steiner said the Advance Regents Diploma should really be the standard (The High School for Health Professions , n.d.).

This school has a clear understanding of what the high school mathematics standards ought to be, and it is not alone on this issue. Take as an example, the position of The Intersegmental Committee of Academic Senates (ICAS), which is composed of the California Community

[29] CUNY measures college readiness in mathematics as follows: the achievement of a score of 80 or higher in any exam and the successful completion of an advanced math course, i.e. Algebra2/Trigonometry or higher (City University of New York, 2016).

Colleges, The California State University, and The University of California. The ICAS states that the following mathematical topics are *essential areas of focus for all entering college students*:

1. Variables, Equations, and Algebraic Expressions
2. Families of Functions and Their Graphs
3. Geometric Concepts, Probability
4. Data Analysis and Statistics
5. Argumentation and Proof

The ICAS understood that students are best served when their high schools provide them with deep mathematical experiences and that the aforementioned areas of focus serve as a brief compilation of truly essential topics. It also identifies *desirable areas of focus for all entering college students.* While the ICAS understood that no curriculum could include study in all the areas unless it is at the expense of depth; it understands that these areas provide an excellent context for the development of mathematical learning and that any successful high school mathematics program would include some of the following topics:

- Discrete Mathematics: Topics such as set theory, graph theory, coding theory, voting systems, game theory, and decision theory.
- Sequences and Series: Geometric and arithmetic sequences and series; the Fibonacci sequence; recursion relations.
- Geometry: Right triangle trigonometry; transformational geometry including dilations; tessellations; solid geometry; three-dimensional coordinate geometry, including lines and planes.
- Number Theory: Prime numbers; prime factorization; rational and irrational numbers; triangular numbers; Pascal's triangle; Pythagorean triples (The Intersegmental Committee of Academic Senates, 2010).

The University of California, a member of the ICAS, and a highly regarded public higher education system, also understood the importance of high school mathematics and therefore, has the following minimum mathematics entrance requirements for their aspiring undergraduates:

Three units (equivalent to three years or six semesters) of college-preparatory mathematics are required (four units are strongly recommended) including or integrating topics covered in:

- Elementary algebra
- Advanced algebra
- Two- and three-dimensional geometry

Also acceptable are courses that address the above content areas, and include or integrate:

- Trigonometry
- Statistics (Regents of the University of California, 2015).

The City University of New York has very similar entrance requirements, a "Score of 80 or higher in Integrated Algebra, Geometry or Algebra 2/Trigonometry and the successful completion of the Algebra 2/Trigonometry or a higher-level course" (City University of New York, 2016). Taking the University of California and the City University of New York higher education systems as a reference, it becomes clear that a Regents Diploma[30] does not prepare students to face the mathematics requirements of college. The Advanced Regents Diploma, which requires the completion of a minimum of three years of mathematics, has greater potential to guarantee college readiness. We say it has the potential because students are required to pass, not master, the exams to be granted the Advanced Diploma. However, should a student earn an 80 or higher in any exam, then he or she would be college ready. In a 2010 study by the Board of Regents, only 28.3% of students scoring below 80 on their math Regents exams achieved scores higher than 500 on their SAT. Meanwhile, 81.2% of the students who achieved scores of 80 or higher on their math Regents exams achieved scores of over 500 in their SAT (Board of Regents, 2010). Parts a and b of research question one, together with the literature review, allowed us to create the following hypothesis to contextualize the answer to research question one part c.

H_0 = There are no statistically significant differences between the average Regents diploma conferment rates of Bronx and New York City public high school graduates.
H_{1c} = The average Regents Diploma conferment rates of Bronx public high school students are statistically significantly different from those of New York City public high school graduates.

Table 10
High school graduates' diplomas (2010-12)

Variable	Bronx	New York City	Difference	SE	95% CI
Regents Diploma	80%	87%	-7%***	.0047522	[-8%, 6%]
Advanced Regents Diploma	9%	23%	-14%***	.0051672	[-15%, -13%]
Individualized Ed Diploma	4%	3%	1%***	.0023702	[.51%, 1.5%]

Note: * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$ for a one-tailed test. The first and second numbers of the CI are, respectively, the lower bound and upper bounds of the interval. Bronx, *N*= 8,221; New York City, *N*= 18, 512.

The difference of -7% points indicates that, on average, Bronx County public high schools awarded fewer Regents diplomas than New York City public schools. Essentially, Bronx public high schools graduated fewer students than New York City public high schools. The -14% point difference indicated that, on average, Bronx public high school students were less than half as likely as New York City public high school students to graduate with Advanced Regents diplomas. This difference is significant not only in magnitude but also in terms of access. As we previously discussed, an Advanced Regents Diploma is a credential that can potentially show that a student is college ready. These results were expected given the low rates at which Bronx public school students passed and mastered the Geometry and Algebra2/Trigonometry exams.

[30] The academic years for which we conduct our study precedes the credit and examination requirements set forth with the implementation of the New York State Common Core Learning Standards for mathematics. After the full implementation of these Learning Standards, the Regents Diploma specifies that the six required credits in mathematics must include at least two credits of "Advanced math", i.e. Geometry or Algebra II (New York City Department of Education, 2013b). Hence, while these requirements still fall short of the minimum requirements of CUNY and the Unified California system, it is significantly better than what we had prior to their inception.

H_0 = There are no statistically significant differences between average post-graduation plans of Bronx and New York City public high school graduates.
H_{1c} = The average 4-year college, 2-year college, employment, and unknown aspirations of Bronx public high school graduates are different from those of New York City public high school graduates.

Table 11
Graduates' post-graduation plans (2010-13)

Variable	Bronx	New York City	Difference	SE	95% CI
To Attend 4 YR Colleges	32%	44%	-12%***	.0064265	[-13.2%, -10.8%]
To Attend 2 YR Colleges	39%	29%	10%***	.006126	[8.8%, 11.2%]
To Employment	4%	4%	0%	.002569	[-.5%, .5%]
Plans Unknown	25%	22%	3%***	.005514	[1.9%, 4.1%]

Note: * $p < 0.05$, ** $p < 0.01$, and *** $p < 0.001$ for a two-tailed test. The first and second numbers of the CI are, respectively, the lower bound and upper bounds of the confidence interval. Bronx, *N*= 8,568; New York City, *N*= 18,130.

Each variable tested, except for employment aspirations, was significantly different. Altogether, Bronx and New York City public high school graduates aspired to higher education at a similar rate, if we think of college aspiration as a single category, 71%, and 73%, respectively. There is extensive research about the time to degree completion differences of students who begin their higher education at 2-year colleges in comparison to those who being their higher education at 4-year colleges (Dougherty, 1994; Karen & Dougherty, 2005). Research indicates that high school graduates' higher education institution of choice matters a lot. While the depth of this research is beyond the scope of this thesis, we like to highlight that students who start their higher education at community colleges tend to complete fewer degrees, particularly four-year degrees, than students who start their higher education at four-year institutions, by large margins (Dougherty, 1994; Karen & Dougherty, 2005). Bronx public high school graduates aspired to four-year colleges at a rate that is almost one third that of their counterparts from New York City public high schools. Conversely, they aspired to attend two-year institutions at a rate that is almost one-third higher than that of New York City public high school graduates. To some extent, New York City high school graduates seem to have had more definite post-graduation plans than Bronx graduates.

Summarizing research question one: The families of Bronx county public high school students were statistically significantly less wealthy than the families of New York City public high school students. Their likelihood of being from ethnically underrepresented groups was also higher. On average, Bronx public high school students showed lower academic performance, measured by their passing and mastery scores in the mathematics Regents exams and their Regents and Advanced Regents Diploma attainment rates, than New York City public high school students. The higher education aspirations of Bronx public high school graduates were different from those of New York City public high school graduates. Specifically, Bronx graduates were less likely to aspire to attend 4-year colleges than New York City graduates.

4.4. Research question II. How comparable are key demographic and academic performance indicators of Bronx public high schools with a reported 4-year mathematics requirement and those of other Bronx public high schools?

a. How do the socioeconomic and ethnic distributions of Bronx County public high school students attending schools with a reported 4-year mathematics requirement compare to those of Bronx County public high school students from other schools?

Research question two emanates directly from research question one. In research question one we found a number correlations, all of which indicated that, on average, Bronx public high schools enrolled more ethnically underrepresented and economically disadvantaged students, reported lower passing and mastery rates in the mathematics Regents exams, and graduated students with lower aspirations to attend four year colleges than New York City public high schools. Based on these results, a number of questions can be asked. For example: Why are Bronx high school students underperforming in the mathematics Regents exams, why do they achieve Regents and Advanced Regents diplomas at lower rates, and why do they aspire to attend 4-year colleges at rates lower than New York City public high school students? These are questions of causation and are very difficult to answer with complete certainty. The researcher is well aware of the limitations of quasi-experimental designs[31], especially of the issue of internal validity[32]. While we have full control of the independent variables, “the reported 4-year mathematics requirement” and “students’ high school mathematics course completion”, we did not manipulate them. Under research question two, we were interested to know whether the reported 4-year mathematics requirement has direct, significant, and causal effects on students’ academic performance and higher education aspirations. Since we use the populations, and not samples of interest to answer research question two, *p*-values and other summary statistics are provided to keep consistency, but they are unnecessary. However, we present these summary statistics for the purpose of consistency.

Table 12
Average Schools demographic distributions (2010-13)

Variables	Bronx	Bronx+4	Difference	SE	95% CI
Free or Reduced Lunch	78%	84%	-6%***	.0052972	[-7%, -5%]
Limited English P.	19%	10%	9%***	.0049583	[8.2%, 9.8%]
American Indian	0.5%	0.5%	0%	.000914	[-.18%, 18%]
Black	31%	35%	-4%***	.0060207	[-5.2%, -2.7%]
Hispanic	63%	58%	5%***	.0062787	[3.7%, 6.3%]
Asian	3%	3%	0%	.0007087	[-.1%, 1%]
White	3%	4%	-1%	.000724	[-.3%, .01%]

Note: Since each group under the comparison represents the population and not a population sample, we present the level of significance, standard error, and confidence interval of the tests just to be consistent. Bronx, N = 45,203; Bronx+4, N = 6,859.

[31] In this type of design, a researcher has no control of the subjects and their assignation to control or treatment group. In our case, participants were pre-assigned, i.e., a group of students enrolled in schools with a reported 4-year mathematics requirement and another group of students did not.

[32] Recall the quasi-experimental design discussion in the methodology section.

The test of proportion differences, as indicated by the results of Table 12, indicate that these two sets of schools were identical across three ethnic variables: the enrollment percentages of American Indian, Asian and White students. We conclude, based on the test of proportion difference that, on average, Bronx public high schools reporting a 4-year mathematics requirement had a higher enrollment of economically disadvantaged, limited English proficiency, and Black students. These schools had a lower enrollment of Hispanic students. A relevant question that arises in quasi-experimental studies concerned with schools with better academic standards is whether there is a self-selection bias among students. That is, it is legitimate to ask whether stronger students are more academically driven therefore self-selecting into high schools with higher academic standards. While this question is indeed legitimate, we remind the reader that in New York State public high schools, including New York City, school assignment follows a stringent selection algorithm[33]. Therefore, while there may exist a self-selection factor among students attending schools with a reported 4-year mathematics requirement, such self-selection factor should account, in practice, based on the school placement algorithm criteria, for a small percentage of students. The results of socioeconomic status are rather surprising, given that they contradict the long standing research evidence that poverty is negatively associated with students' achievement in mathematics. However, these results are contradicted by the data we use to answer research question three.

4.5. Research question IIb: How do the average mathematics Regents exams passing and mastery rates of Bronx County public high school students attending schools with a reported 4-year mathematics requirement compare to those of other Bronx County public high school students?

Table 13
Bronx Students' passing rates (2010-13)

Variable	Bronx	Bronx+4	Difference	SE	95% CI
Integrated Algebra	49%	62%	-13%***	.0105982	[-15%, -11%]
Geometry	44%	53%	-9%***	.0152686	[-12%, -6%]
Algebra2/Trigonometry	31%	34%	-3%***	.0183003	[-6.6%, -0.6%]

Note: Since each group under the comparison represents the population and not a sample, summary statistics are presented for consistency. Integrated Algebra: Bronx, *N*= 16,976; Bronx+4, *N*= 2,561. Geometry: Bronx *N*= 5,371; Bronx+, *N*= 1,328. Agebra2/Trigonometry: Bronx, *N*= 2,544; Bronx+, *N* =868.

Let us recall that within the years covered in this study, 2010-2013, the minimum mathematics requirement for students to earn a Regents Diploma was a passing score on the Integrated Algebra exam (New York City Department of Education, 2013b). The results of Table 13 indicate that Bronx students attending schools with a reported 4-year mathematics requirement, on average, had better academic performance, as indicated by their passing rates in the mathematics Regents exams. However, contrary to the results of the comparison between Bronx and New York City public high schools, when comparing Bronx schools with a reported 4-year mathematics requirement to other Bronx schools, as the intensity of the mathematics Regents exams increases, the performance gap between students from schools requiring 4-years of math

[33] Please refer to page 20 where we include the school assignment algorithm.

and other Bronx students decreases. Since research supports the notion that students who complete four years of mathematics perform significantly better than their counterparts, we were expecting that, as the intensity of the exams increased, the performance gap between students attending schools with a reported 4-year mathematics requirement and other Bronx students would increase. That said, with a three, nine, and thirteen percentage point advantage in the Algebra2/Trigonometry, Geometry, and Integrated Algebra exams, respectively, Bronx students from schools with a reported 4-year mathematics requirement outperformed the other Bronx public high school students. Our results indicate a strong and positive correlation between the reported 4-year mathematics requirement and students' passing scores in their mathematics Regents exams.

Table 14
Bronx Students' mastery rates (2010-13)

Variable	Bronx	Bronx+4	Difference	SE	95% CI
Integrated Algebra	1%	4%	-3%***	.0024847	[-3.7%, -2.2%]
Geometry	3%	7%	-4%***	.0058542	[-5.4%, 2.5%]
Algebra2/Trigonometry	4%	6%	-2%***	.0081564	[-3.7%, -.2%]

Note: Since each group under the comparison represents the population and not a sample, summary statistics are presented for consistency. Integrated Algebra: Bronx, N= 16,976; Bronx+4, N= 2,561. Geometry: Bronx N= 5,371; Bronx+, N= 1,328. Agebra2/Trigonometry: Bronx, N= 2,544; Bronx+, N =868.

The results of Table 14 indicate expected magnitudes and directions, for the test of proportion differences. The negative signs indicate that, on average, students attending schools with a reported 4-year mathematics requirement outperform the other Bronx public high school students in their mastery of each mathematics Regents exam. However, as one would expect, the mastery performance gap between students from schools with the 4-year mathematics requirement and other Bronx students is significantly higher than the one passing performance gap. Their mastery rates on the Integrated Algebra, Geometry, and Algebra2/Trigonometry exams are, respectively, four, over two, and over one third times higher than those of their counterparts. Nonetheless, when thinking globally, Bronx County students' mathematics Regents exams mastery is rather low.

4.6. Research question IIc. How do the high school diploma conferment and post-graduation aspiration rates of students from Bronx schools with a reported 4-year mathematics requirement compare to those of the other Bronx public high school students?

Table 15
Bronx graduates diploma type (2010-13)

Variable	Bronx	Bronx+4	Difference	SE	95% CI
Regents Diploma	79%	90%	-11%***	.0127715	[-13%, -9%]
Advanced Regents Diploma	8%	18%	-10%***	.0093721	[-12.3%, -7.7%]
Individualized Ed Diploma	5%	1%	4%***	.0066543	[3.2%, 4.7%]

Note: Since each group under the comparison represents the population and not a sample, we present summary statistics for consistency. Bronx (*N*) = 7,218; Bronx+4 (*N*) = 1,112.

The results of Table 15 are in accord with our literature review: students held to higher mathematics standards, on average, outperform other students in their high school academic performance, measured by graduation rates and diploma types. The -11% difference in Regents Diplomas indicates that, on average, students graduating from schools with a reported 4-year mathematics requirement graduated at a rate that is higher than the other Bronx public high school students. A more significant result is the rate at which these students completed Advanced Regents diplomas. On average, students attending schools requiring 4-years of mathematics earned Advanced diplomas at over twice the rate of their counterparts. This could translate as on average, students graduating from Bronx high schools requiring 4-years of mathematics were more than two times as likely to graduate college ready as their counterparts. These students were four times less likely to graduate with an Individualized high school diploma than the other Bronx high school graduates.

Table 16
Bronx graduates post-graduation plans (2010-13)

Variable	Bronx	Bronx+4	Difference	SE	95% CI
To Attend 4 YR Colleges	29%	46%	-17%***	.0188829	[-20.6%, -13.4%]
To Attend 2 YR Colleges	39%	40%	-1%	.0190488	[-4.7%, 2.7%]
To Employment	4%	2%	2%***	.0066224	[.7%, 3.3%]
Plans Unknown	27%	11%	16%***	.0152295	[13%, 19%]

Note: Since each group under the comparison represents the population and not a population sample, we present summary statistics for consistency. Bronx (*N*) = 1288; Bronx+4 (*N*) = 1348.

Up to now, our results indicate that, on average, a reported 4-year mathematics requirement had the strongest impact on students' post-graduation plans. This was particularly true for students' aspirating to attend four year colleges. If we consider the variable to college as a single category, on average, 96% of these students aspired to higher education, versus 68% of students attending other Bronx public high schools. This finding is significant and re-enforces the results of our literature review. More importantly, students from schools with a reported 4-year mathematics requirement aspired to attend 4-year colleges at a rate that is over one and a half times higher than that of the other Bronx public high school students. This result translates as: based on the community college research, students from schools with a reported 4-year mathematics requirement will have greater chances to complete bachelor degrees than their counterparts (Adelman, 1999; Dougherty, 1994; Karen & Dougherty, 2005). Also, these students were less likely to have unknown post-graduation plans and aspired to join the workforce at a rate that was half that of counterparts.

4.7. Research question III. How comparable are the high school mathematics course completion rates of Bronx County public high school students from schools with a reported 4-year mathematics requirement and the other Bronx public high school students?

Before presenting the results of research question three, we remind the reader how we treated the data we used to answer it. As stated earlier, we did not manipulate the independent variable, the "reported 4-year mathematics requirement"; we created the strata from which we selected our samples by controlling for the variables: schools' aggregate socioeconomic status, size, and geographic district location. These selection criteria ensured the creation of homogeneous strata, which allowed us to assimilate a controlled experimental environment, therefore increasing the internal validity of our results. Once the strata were completed, we assigned schools to each of our comparison groups by means of stratified random sampling. As a result, our comparison groups were very similar across all demographic variables, as displayed in Table 3.1. Please note that the data used to answer research question three differs from those of research question two. The data from research question two were extracted from the New York State Report Cards and those used on research question three were provided to us by the New York City Department of Education, as a result of our submission of a research proposal. Also, unlike research question two, wherein we used the populations of interest, for research question three we have representative samples. Since in theory, on average, students attending schools with a reported 4-year mathematics requirement would enroll in math courses every year, we expected that our comparison group "Bronx+4" would consist of the population and not a sample; however, our data analysis indicate that only 44% of students that were enrolled in schools with a 4-year mathematics requirement during the academic year of 2012-2013 completed math courses during that year.

Table 16
Average student demographic distribution by schools (2012-13)

Variables	Bronx	Bronx+4	Difference	SE	95% CI
Free or Reduced Lunch	86%	86%	0%	.0135202	[-2.6%, 2.6%]
Limited English P.	14%	8%	-6%***	.0121584	[3.6%, 8.3%]
American Indian	0.5%	0.7%	-2%**	.0092289	[-4%, -2%]
Black	38%	32%	6***	.0185565	[2.4%, 9.6%]
Hispanic	58%	59%	-1%	.0191985	[-5%, 3%]
Asian	1.6%	3.8%	-2.2%***	0063439	[-3.4%, -1%}
White	1.6%	3.7%	-2.1%***	.0062857	[-3.3%, -1%]

Note: *p < 0.10, **p < 0.05, and ***p < 0.01 for a two-tailed test. In the 95% Confidence Interval (CI), the first and second numbers are, respectively, the lower and upper bounds of the interval. Bronx (*N*) = 1288; Bronx+4 (*N*) = 1348.

The results of Table 16 show that the families of Bronx students attending schools with a reported 4-year mathematics requirement were as wealthy as the families of other Bronx students. Schools with a reported 4-year mathematics requirement had, respectively, lower enrollments of students that are, Black, and are limited in their English proficiency. They also had a higher enrollment of White and Asian students. Their enrollment of White and Asian students was about two and a third times higher than the enrollment of White and Asian students in other Bronx public high schools. Table 17 below shows the mathematics course completion distributions of the students of each group of schools. On research question two we determined that, on average, students from Bronx County public high schools reporting a 4-year mathematics requirement had better academic performance than the other Bronx public high school students; therefore, we expected that these students would have high school mathematics course completion rates that are significantly higher than their counterparts across all but the least demanding or non-academic mathematics courses: Integrated Algebra, and Other Math courses.

Table 17
Average mathematics course completion distribution by schools (2012-13)

Variables	Bronx	Bronx+4	Difference	SE	95% CI
Calculus	0.08%	1.1%	-1.02%***	.0021464	[-1.4%, -0.62%]
Precalculus	1.9%	1.6%	0.3%	.0011346	[-0.4%, 1%]
Statistics	1.7%	1%	.7%**	.0031141	[.08%, 1.3%]
Algebra2/Trigonometry	15.1%	18.9%	-3.8%%***	.0102587	[-5.8%, -1.8%]
Geometry	22%	34.2%	-12.2%***	.0122936	[-14.6%, -9.8%]
Integrated Algebra	49.7%	37.3%	12.4%***	.013482	[9.8%, 15%]
Other Math	9.6%	5.8%	3.8%***	.0072012	[2.4%, 5.2%]

Note: *p < 0.10, **p < 0.05, and ***p < 0.01 for a two-tailed test. In the 95% Confidence Interval (CI), the first and second numbers are, respectively, the lower and upper bounds of the interval. Bronx (*N*) = 2523; Bronx+4 (*N*) = 2899.

The students' mathematics course completion rates were only the same for the Precalculus courses. Within the Advanced category, the biggest gap was observed in the Calculus courses, with students from schools with a reported 4-year mathematics requirement completing Calculus at a rate that is about fifteen times that of the other Bronx public high school students. In statistics, the difference was less pronounced. While students in schools with a reported 4-year mathematics requirement outperformed the other Bronx public high school students in the advanced mathematics course completion category, their advanced math course completion distribution was rather low, if we considered that these students had a 4-year mathematics requirement. The trend was the same for the results of the Middle-Academic II category course, Algebra2/Trigonometry. Although students from schools with a reported 4-year mathematics requirement outperformed other students, their Algebra2/Trigonometry completion rate was smaller than we anticipated.

In Geometry, however, the difference was more pronounced with students from schools with the 4-year mathematics requirement completing this course at over one and a half times the rate of other Bronx students. As expected, these students completed the Low-Academic and Non-Academic category courses, Integrated Algebra, and Other Math courses, respectively, at rates that were lower than those of their counterparts. Their Integrated Algebra and Other Math completion rates were just three quarters and two thirds, respectively, that of their counterparts from other Bronx public high schools.

Chapter 5. Conclusion

5.0. Concluding Remarks

The aim of this thesis was to determine and quantify an important measure of mathematics literacy, the high school mathematics course completion of Bronx public high school students. We found that when compared to public high schools from other New York City Boroughs, on average, Bronx public high schools enrolled higher percentages of poor, Black, Hispanic, and limited English proficiency students. Conversely, Bronx County public high schools enrolled lower percentages of White and Asian students. With 80% of their enrolled students eligible for free or reduced price lunch and a 94.5% minority enrollment, Bronx public high schools were both high minority and high-poverty schools as defined in (New York State Office of the Comptroller, 2000). Academically, students from Bronx public high schools were outperformed by other New York City public high school students. Bronx students' passing and mastery rates in all mathematics Regents exams were significantly lower than those of other New York City public high school students. Similarly, their Regents conferment rates, particularly their Advanced Regents diploma conferment rates, were statistically significantly lower than those of their counterparts in other New York City public high schools.

A significant finding of this thesis is the rate at which Bronx students aspired to attend 2-year colleges. Starting college at a two-year institution has long lasting repercussions on students, one of which is a very low probability of completing a bachelor's degree. Bronx public high school students aspired to attend two-year colleges at a rate that was over one and a third times as high as that of other New York City public high school students. Conversely, they aspired to attend 4-year colleges at a rate that was less than three quarters that of other New York City public high school students. Our investigation of research question two, which was answered using aggregate school-level data, allowed us to determine that the demographics of students from Bronx schools with a reported 4-year mathematics requirement differed from those of other Bronx students, but not across all of the variables we measured. Schools with a reported 4-year mathematics requirement were, on average, less wealthy and enrolled more Hispanic and English proficient, and fewer Black students, than other Bronx high schools. However, students from schools with a 4-year of mathematics requirement had better academic performance than other Bronx students. We highlight, however, that in the mathematics Regents exams, as the intensity of the exams increases, the performance gap between students from schools with a reported 4-year mathematics requirement and other Bronx schools decreased. For example, in the Integrated Algebra exam, the least intensive of the exams, students from schools reporting a 4-year math requirement had a passing rate that was about one and a fifth, and a mastery rate that was about four times those of other Bronx students. However, in the Algebra2/Trigonometry exam, the most academically intensive of the three exams, students from schools with a reported 4-year mathematics requirement had a passing rate that was one and one eleventh times, and a mastery rate that was one and a third times as high as those of the other Bronx public high school students. Research question three, which was answered using student-level data provided by the New York City Department of Education, and not the New York State report Cards, showed that the demographic differences between students from schools requiring 4-years of mathematics differed from those determined in research question two. Specifically, the data from research question three indicated no economic differences between Bronx schools with a reported 4-year mathematics requirement and other Bronx schools. Also, schools with a reported 4-year mathematics requirement enrolled Asian and White students at more than twice the rate of other Bronx schools. Similar to the results of research question two, the results of research question three showed that as the intensity of the mathematics Regents courses increased, the mathematics course completion gap between students from schools requiring 4-years of mathematics and students from the other Bronx high school students decreased. For example, Bronx students complete the Middle-Academic II category, Algebra2/Trigonometry courses, at a rate that was close to 80%, and the Middle-Academic I category, Geometry courses, at a rate that was just over 64%, that of Bronx students from schools with a reported 4-year math requirement. Conversely, their completion rate in In the Middle Academic Category, Integrated Algebra courses, was about 133% that of students from Bronx schools with a reported 4-year mathematics requirement. Their completion rate in the Non-Academic category, Other Math courses, was about 165% that of students from Bronx schools with a reported 4-year mathematics requirement. Only in the Advanced III course category, Calculus, did students have a really large achievement gap difference. Students from schools requiring 4-years of mathematics complete these courses at a rate that is almost fifteen times that of their counterparts. Nonetheless, this completion rates is rather low at 1.1%. We can then deduce that, on average, there was a correlation between a reported 4-year mathematics requirements and students' academic performance. Specifically, students from Bronx schools with a reported 4-years of mathematics requirement had higher passing and mastery rates in all mathematics Regents exams, complete more advanced and intermediate math courses, aspired to attend 4-year colleges, and had Regents and Advanced Regents diploma attainment rates that were, on average, higher than those of students from other Bronx schools.

The results of this thesis validate key findings of past and current research that investigate the unintended consequences of attending public schools that serve high-minority or poor students (Graham, 2009; Kelly S., 2009; Oakes, 1990). We found that, on average, students from Bronx county public high schools had greater aspirations to attend community colleges as opposed to senior colleges. When grouping their aspirations to attend two- and four-year institutions, Bronx County public high school students had very low aspirations to higher education, had higher dropout rates, higher tendencies to go to high school equivalency programs, and higher rates of unknown plans than students from other New York City public high schools.

5.1. Policy Implications

This thesis has many implications that ought to be highlighted. First, there are clear correlations between students' geographic location, socioeconomic status and their mathematics expectations and preparedness. Students from Bronx public high schools display, on average, lower academic performance, lower college aspirations; particularly, these students completed fewer advanced and intermediate math courses than students from other New York City public high schools.

This thesis uses data from the academic years of 2010, 2011, and 2012, which were the only years in which, the Integrated Algebra, Geometry, and Algebra2/Trigonometry exams were administered by themselves. Starting in the academic year of 2013, New York City public high schools started to introduce the Regents exams that were aligned with the Common Core State Standards for Mathematics. Therefore, this thesis serves as a point of reference for comparative analysis of the new curriculum and exams, which are aligned with the Common Core State Standards.

5.2. Policy Recommendations

Our results indicate that, on average, Bronx public high school students from schools with a reported 4-year mathematics requirement displayed better academic performance than the other Bronx public high school students, despite similar demographics among these students. Therefore, we recommend the following policies for consideration. The New York City Department of Education, in conjunction with the New York State Board of Regents, should:

- Hold all students accountable to higher mathematics standards by requiring them, regardless of race or socioeconomic status, to complete four years of mathematics.
- Ensure that mathematics curricula lead to college readiness in mathematics. This could be accomplished by having college preparatory math curricula which preferably includes advanced math courses.

In the area of policy, the implementation stage is paramount. We want to highlight that it is important to understand the meaning of four years of mathematics. Tyson & Roksa, (2016) provide a very good framework that complements the recommendations of the University of California and the City University of New York. Four year math curricula should always consider both the vertical (whether students begin 9th grade with Algebra 1) and horizontal (the different ways in which Algebra 1 can be taught: over one year, over one and a half years, over two years, accelerated, honors, advanced) components of curricula (Tyson & Roksa, 2016). This is an important, yet under studied difference in curriculum implementation.

5.3. Limitations

We discussed most of the limitations of this study in the methodology section; in summary, this study is a quasi-experiment, for we did not have full control of condition assignment. We controlled for all possible extraneous variables we found, but we cannot say that we controlled all of them. However, our design and the data we used allowed us to design experiments that were robust against model bias, or internal validity problems. To us, the primary limitation of this study was the data type and the samples of choice. We worked with secondary data and we trust that the data we were provided are valid and accurate. Nonetheless, these data are not exempt from human error in data entry. The selection of samples was very limited. For example, few schools reported a "4-year math requirement" for we were unable to do random sampling. A major limitation of this study lies in the reported "4-year math requirement". When analyzing the proportion of schools with a reported "4-year math requirement" whose students completed math courses, data analyses showed that only 44% of students completing math courses, during that same academic year, in schools without a reported 4-year math requirement was 45%. Of course, we were able to analyze only one academic year and not four.

5.4. Suggestions for Future Research

Our descriptive analyses validate past and current research focusing on the effect of advanced mathematics courses on students' aspirations and access to, and success, in higher education (Anderson & Chang, 2011; Finkelstein & Fong, 2008; Graham, 2009; Hoyt & Sorensen, 2001). Our results have open potential research routes, among which we highlight the following: a more detailed analysis of the schools with a reported 4-year mathematics requirement. Correlational studies that can build statistical models to determine which of the variables we used in this study are the best predictors of students' success, and an analogous study, focusing on students' science courses.

Bibliography

Achieve Inc. (2008). *The building blocks of success: Higher-level math for all students.* Washington, D.C.: Achieve Policy Brief.

Adelman, C. (1999). *Answers in the tool box: Academic intensity, attendance patterns, and bachelor's degree attainment.* Washington, DC: U.S. Department of Education, Office of Educational Research and Improvement.

Adelman, C. (2006). *The toolbox revisited: Paths to degree completion from high school through college.* Washington, D.C.: Office of Vocational and Adult Education, U.S. Dept. of Education.

Agresti, A., & Finlay, B. (2009). *Statistical methods for the social sciences.* Upper Saddle River, NJ: Pearson Prentice Hall.

Anderson, R., & Chang, B. (2011). Mathematics course-taking in rural high school. *Journal of Research in Rural Education 26(1)*, 1-10.

Ball, D. L., & Bass, H. (2008). Chapter 12: The role of mathematics education for democracy. *Yearbook of the National Society for the Study of Education, 107, 1*, 171-184.

Bishop, J. H. (1998). *Do Curriculum-Based External Exit Exam Systems Enhance Student Achievement?* Philadelphia: Consortium for Policy Research in Education University of Pennsylvania Graduate School of Education .

Bishop, J. H. (2000). Curriculum-Based External Exit Exam Systems: Do students learn more? How? *Psychology, Public Policy, and Law*, 199-215.

Board of Regents. (2010). *A new standard for proficiency: College readiness.* Albany, NY: Board of Regents of the University of the State of New York. Retrieved February 24, 2016, from http://usny.nysed.gov/A_New_Proficiency_Public_Version07_22.pdf

Bogatta, E. F., & Jackson, D. J. (1980). Aggregate data analysis: An overview. In E. F. Bogatta, & D. J. Jackson (Eds.), *Aggregate data: Analysis and interpretation* (pp. 7-12). Beverly Hills: SAGE Publications, Inc.

Bourdieu, P. (1986). The forms of capital. In J. G. Richardson, *Handbook of theory and research for the sociology of education.* Greenwood.

Bozick, R., & Lauff, E. (2007). *Education longitudinal study of 2002 (ELS:2002): A first look at the initial postsecondary experiences of the sophomore class of 2002 (NCES 2008-308).* Washington, DC.: National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education.

Brooklyn Technical High School. (2015). Retrieved October 1, 2015, from NCSSSMST Endorsed Diploma (BTHS Diploma): http://www.bths.edu/GUIDANCE/NCSSSMST_DIPLOMA.jsp

Burkam, D. T., & Lee, V. E. (2003). *Mathematics, foreign language, and science course taking and the NELS:88 transcript data.* Washington, D.C.: U.S. Department of Education, National Center for Education Statistics.

Business Insider. (2014). *The 25 Best Public High Schools In The US*. Retrieved February 5, 2015, from Business Insider: http://www.businessinsider.com/best-public-high-schools-in-the-us-2014-11#1-high-technology-high-school-lincroft-nj-25

Chen, X. (2009). *Students who study science, technology, engineering, and mathematics (STEM) in postsecondary education.* Washington, DC: National Center for Education Statistics.

City University of New York. (2016). *Admissions requirements*. Retrieved February 23, 2016, from CUNY assessment tests-testing: http://www.cuny.edu/academics/testing/cuny-assessment-tests/admissions-requirements.html

Coleman, J. S. (1988). Social capital and the formation of human capital. *The American Journal of Sociology*, S95-S120.

College Board. (2003). *College Board*. Retrieved September 26, 2015, from A Brief History of the Advanced Placement Program: http://www.collegeboard.com/prod_downloads/about/news_info/ap/ap_history_english.pdf

Consortium. (2003). *Alternatives to High-Stakes Testing*. Retrieved February 6, 2015, from Performance assessment: http://performanceassessment.org/index.html

Cook, T. D., & Campbell, D. T. (1979). *Quasi-experimentation: Design & analysis issues for field settings.* Boston, MA: Houghton Mifflin.

Council on Children & Families. (2003-2016). *Annual Dropouts-Public Schools*. Retrieved from Council on children & families: Kid's well-being Indicators clearinghouse: http://www.nyskwic.org/get_data/indicator_narrative_details.cfm?numIndicatorID=15

Daily News Washington Bureau. (2010, September 28). *South Bronx is the poorest district in the nation, U.S. Census Bureau finds: 38% live below poverty line*. Retrieved from Daily News: http://www.nydailynews.com/new-york/south-bronx-poorest-district-nation-u-s-census-bureau-finds-38-live-poverty-line-article-1.438344

Davis, T. M. (2013). Charter school competition, organization, and achievement in traditional public schools. *Education Policy Analysis Archives, 21*(88).

Department of Education's International Affairs. (2008, February). *Structure of the U.S. education system: Credit System.* Washington, D.C.: International Affairs Office: U.S. Department of Education.

Devore, J. L. (2012). Inferences concerning a difference between population proportions. In J. L. Devore, *Probability and statistics for engineering and the sciences* (pp. 375-382). Boston, MA: Brooks/Cole.

Dougherty, K. J. (1994). *The contradictory college: The conflicting origins, impacts, and futures of the community college.* Albany, NY: State University of New York Press.

EngageNY. (2013, November 18). *Common Core Implementation Timeline*. Retrieved from EngageNY: https://www.engageny.org/resource/common-core-implementation-timeline?au=network-teamsntes

Eriksen, T. H. (2012). Ethnicity. *The Wiley-Blackwell Encyclopedia of Globalization*.

Fernández, R. E. (In Progress). Academic success and its relations high school mathematics course-taking: A systematic review of the literature.

Fernández, R. E. (2015a). An effective policy for the high school to college transition: The case of Bronx County public high schools. *EDULEARN15* (pp. 349-357). Barcelona: The International Academy of Technology, Education and Development (IATED).

Finkelstein, N. D., & Fong, A. B. (2008). *Course-taking patterns and preparation for postsecondary education in California's public university systems among minority youth.* Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory West.

Fiorello H. LaGuardia High School of Music & Art . (2015). *Fiorello H. LaGuardia High School of Music & Art and Performing Arts*. Retrieved October 1, 2015, from Academic and Studio Programs: http://laguardiahs.org/programs/

Folts, J. D. (1996). *History of the University of the State of New York and the State Education Department 1784-1996.* Albany: New York State Education Dept., Publications Sales Desk.

Frank, K. A., Muller, C., Riegle-Crumb, C., Mueller, A. S., Crosnoe, R., & Pearson, J. (2008). The social dynamics of mathematics course taking in high school. *American Journal of Sociology 113, 6*, 1645-1696.

Freedman, D. A. (1999). *Ecological Inference and the Ecological Fallacy.* Berkeley: Department of Statistics: University of California at Berkley.

High School of American Studies at Lehman College. (2014). *High School of American Studies at Lehman College Course Catalog*. Retrieved October 11, 2015, from Graduation, Diploma, and Examination Requirements: http://www.hsas-lehman.org/academics/CourseCatalogue.pdf

Hill, R. O. (2006). *On the transition in mathematics from high school to Michigan state university.* East Lansing: math.msu.edu. Retrieved October 17, 2013, from http://www.math.msu.edu/~hill/Transition.pdf

House, J. S., & Williams, D. R. (2003). Understanding and reducing socioeconomic and racial/ethnic disparities in health. In R. Hofrichter (Ed.), *Health and social justice: Politics, ideology, and inequity in the distribution of disease* (pp. 89-131). San Francisco, CA: Jossey-Bass.

Ink, T. B. (2011, September 22). *Bronx County remains the poorest county in the country, New York Times*. Retrieved from The Bronx Ink: http://bronxink.org/2011/09/22/16425-bronx-county-remains-the-poorest-county-in-the-country-reports-the-new-york-times/

International Baccalaureate Organization. (2013, August). *What is an IB education?* Retrieved October 1, 2015, from http://www.ibo.org/globalassets/digital-tookit/brochures/what-is-an-ib-education-en.pdf

Jenkins, R. (2008). *Rethinking ethnicity.* Thousand Oaks, CA: SAGE Publications.

Jenkins, R. (2014). *Social identity.* New York, NY: Routledge, Taylor & Francis Group.

Jones, L. V., Davenport Jr., E. C., Bryson, A., Bekhuis, T., & Zwick, R. (1986). Mathematics and Science Test Scores as Related to Courses Taken in high school and Other Factors. *Journal of Educational Measurement*, 197-208.

Karen, D., & Dougherty, K. J. (2005). Necessary but not sufficient: Higher education as a strategy of social mobility. In G. Orfield, P. Marín, & C. L. Horn (Eds.), *Higher education and the color line: College access, racial equity, and social change* (pp. 33-57). Cambridge, Mass: Harvard Education Press.

Kilpatrick, J. (2011). *Review of math performance in global perspective: How well does each state do at producing high-achieving students?".* Boulder, CO:: National Education Policy Center.

Kline, T. J. (2005). Classical Test Theory: Assumptions, Equations, and Item Analyses and Item Response Theory and Rasch Models. In *Psychological Testing: A Practical Approach to Design and Evaluation* (pp. 91-106). Thousand Oaks, CA: SAGE Publications, Inc.

Knecht, D. (2007). A grass roots tale of fighting high school stakes graduation testing in New York. *The Urban Review, 39*, 1, 45-65. doi:10.1007/s11256-007-0043-0

Ma, X., & Wilkins, J. L. (2007). Mathematics Coursework Regulates Growth in Mathematics Achievement. *Journal for Research in Mathematics Education*, 230-257.

National Center for Education Statistics. (2007, September). *Status and change in the education of racial/ethnic minorities*. Retrieved from IES Institute of Education and Science: http://nces.ed.gov/pubs2007/minoritytrends/

National Center for Education Statistics. (2010). *The nation's report card: Grade 12 reading and mathematics 2009 national and pilot state results (NCES 2011–455).* Washington, D.C.: Institute of Education Sciences, U.S. Department of Education.

National Center for Education Statistics. (2014, February). *High School Course taking* . Retrieved February 9, 2015, from The Condition of Education: http://nces.ed.gov/programs/coe/indicator_cod.asp

National Commission on Excellence in Education. (1983). *A Nation at risk:* Washington, D.C.: United States Department of Education.

National Research Council. (2013). *The Mathematical Sciences in 2025.* Washington, D.C.: The National Academies Press.

National Science Board. (2004). *Science and engineering indicators 2004.* Arlington, VA: National Science Foundation (volume 1, NSB 04-1).

National Science Board. (2012). *Science and engineering indicators 2012.* Arlington, VA: National Science Foundation.

National Science Board. (2016). *Science and engineering indicators.* Arlington, VA: National Science Foundation (NSB-2016-1).

New York City Department of Education. (2007). *New York City Department of Education Office of School & Youth Development*. Retrieved from Credit Requirements: http://schools.nyc.gov/NR/rdonlyres/215FF06B-DCA3-442B-89DF-18E674DC867E/0/Acpolicygened.pdf

New York City Department of Education. (2013). *2012-2013 Publications: Directory of New York City Public High Schools*. Retrieved October 11, 2013, from 2013-2014 High School Directory: Introduction: http://schools.nyc.gov/NR/rdonlyres/6C650E9E-C2B6-4B78-9D4F-927DF94E4012/0/Introduction.pdf

New York City Department of Education. (2013a). *2013-2014 Specialized High School Student Handbook*. Retrieved October 11, 2013, from http://schools.nyc.gov/NR/rdonlyres/D6C63FDF-7A92-41DE-B42C-719634D9172C/0/SHSAT_StHndbk_20132014.pdf

New York City Department of Education. (2013b, May). *Graduation requirements for students entering 9th grade in 2008, 2009, 2010, and 2012*. Retrieved February 1, 2014, from Credit Requirements: http://schools.nyc.gov/NR/rdonlyres/215FF06B-DCA3-442B-89DF-18E674DC867E/0/Acpolicygened.pdf

New York City Department of Education. (2014). *2014-2015 Directory of New York City Public High Schools*. Retrieved October 15, 2015, from http://schools.nyc.gov/NR/rdonlyres/30E4141F-89AC-484A-906B-E26E10452AAA/0/20142015HSDIntroduction.pdf

New York City Department of Education. (2014a). *2014-2015 Specialized Schools Student Handbook*. Retrieved October 1, 2015, from http://schools.nyc.gov/NR/rdonlyres/1FDB8183-E675-42D9-A17C-237C18E4C255/0/SHSAT_StHdbk_201415.pdf

New York City Department of Education. (2015d, August 22). Organization of courses and grades datasets. Stony Brook, New York, United States: New York City Department of Education. Retrieved March 31, 2016

New York State Education Department. (2010, December 23). *New York State Education Department*. Retrieved September 29, 2013, from About the Board of Regents: http://www.regents.nysed.gov/about/

New York State Education Department. (2011, August 17). *Section 3204*. Retrieved from Student support services: http://www.p12.nysed.gov/sss/lawsregs/3204.html

New York State Education Department. (2013, October 10). Questions regarding data. (D. L. Ferguson, Interviewer)

New York State Education Department. (2013b, May 10). *Career & Technical Education*. Retrieved October 19, 2013, from Program Approval Process: http://www.p12.nysed.gov/cte/ctepolicy/

New York State Education Department. (2015a, August 26). *Information and Reporting Services (IRS)*. Retrieved September 29, 2015, from Directory of Public and Non-Public Schools and Administrators in New York State: 2014-15 School Directory and General Information: http://www.p12.nysed.gov/irs/schoolDirectory/

New York State Education Department. (2015b, May 25). *School District Organization*. Retrieved September 28, 2015, from Guide to the Reorganization of School Districts in New York State: http://www.p12.nysed.gov/mgtserv/sch_dist_org/GuideToReorganizationOfSchoolDistricts.htm

New York State Education Department, N. (2013, October Friday 26). *Engage New York.* Retrieved September Monday 2, 2013, from New York State P-12 Common Core Learning Standards for Mathematics: http://www.engageny.org/resource/new-york-state-p-12-common-core-learning-standards-for-mathematics

New York State Office of the Comptroller. (2000). *Higher standards for all: New York State's educational performance gap and necessary reforms.* Albany, NY: Office of the State Comptroller, Public Information Office, Governor Smith State Office Building. Retrieved March 27, 2016, from http://www.osc.state.ny.us/reports/schools/2000/perfgap.pdf

New York State School Boards Association. (2012, December 17). *Types of school districts in New York State*. Retrieved October 1, 2015, from Better School Boards Lead to Better Student Performance: http://www.nyssba.org/news/2012/12/14/on-board-online-december-17-2012/types-of-school-districts-in-new-york-state/

Nord, C., Roey, S., Perkins, R., Lyson, M., Lemanski, N., Brown, J., & Schuknecht, J. (2011). *The Nation's Report Card: America's High School Graduates.* Washington, DC: U.S. Government Printing Office: U.S. Department of Education, National Center for Education Statistics.

Nuzzo, R. (2014). Scientific method: Statistical errors. *Nature, 506*, 7487, 150-152. doi:http://doi.org/10.1038/506150a

NYC Council Committees on Education and Higher Education. (2012). *College and Career Readiness in NYC.* New York City: New York City Department of Education.

NYSED Commissioner of Education. (2010, August). *Part 100 Regulations*. Retrieved January 22, 2014, from 100.5 Diploma Requirements: http://www.p12.nysed.gov/part100/pages/1005.html#a

Oakes, J. (1990). Opportunities, achievement, and choice: Women and minority students in science and mathematics. *Review of Research in Education*, 153-222.

Office of State Assessment. (1987, November 24). *History of Regents Examinations: 1865 to 1987*. Retrieved from NYSED.gov: the Office of State Assessment: http://www.p12.nysed.gov/assessment/hsgen/archive/rehistory.htm

Office of State Assessment, N. (2013). *P-12: Prekindergarten through Grade 12 Education.* Retrieved January 22, 2014, from 2013 Edition School Administrator's Manual Secondary Level Examinations: http://www.p12.nysed.gov/assessment/sam/secondary/hssam-13rev.pdf

Patrice, I., Schwartz, A. E., Stiefel, L., & Chellman, C. C. (January 01, 2008). Small schools, large districts: Small-school reform and New York City's students. *Teachers College Record, 110*, 9, 1837-1878.

PewResearchCenter. (2015, December 9). *The American Middle Class Is losing ground: No longer the majority and falling behind financially*. Retrieved from Pew Research Center social and demographic trends: http://www.pewsocialtrends.org/2015/12/09/the-american-middle-class-is-losing-ground/

President's Council of Advisors on Science and Tec. (2010). *Prepare and inspire: K–12 education in science, technology, engineering, and math (STEM) for America's future.*

Washington, D.C.: Office of Science and Technology Policy, Executive Office of the President.

Price, P. (2012). *Research methods in psychology: Core concepts and skills.* Unidentified: Unidentified.

Private School Review. (2015). *Bronx County Private High Schools*. Retrieved February 5, 2015, from Private School Review: http://www.privateschoolreview.com/county_private_schools/stateid/NY/county/36005/high

Private School Review. (2015). *New York City Private Schools*. Retrieved October 1, 2015, from Private School Review: http://www.privateschoolreview.com/new-york

Regents of the University of California. (2015). *Mathematics ("c")*. Retrieved February 24, 2016, from A-G Subject requirements: http://www.ucop.edu/agguide/a-g-requirements/c-mathematics/index.html

Renzulli, L. A. (2005). Organizational environments and the emergence of charter schools in the United States. *Sociology of Education, 78*, 1-26.

Schmidt, W. (March 01, 2003). Too little too late: American high schools in an international context. *Brookings Papers on Education Policy, 2003, 1*, 253-308.

Shadish , W. R., Cook , T. D., & Campbell , D. T. (2001). *Experimental and quasi-experimental designs for generalized causal inference.* New York, NY: Wadsworth Publishing.

Staten Island Technical High School. (2015). Retrieved October 1, 2015, from Career & Technology Education (CTE): http://www.siths.org/apps/pages/index.jsp?uREC_ID=203749&type=d&termREC_ID=&pREC_ID=404295

Steen, L. A., & Madison, B. L. (2011). Reflections on the tenth anniversary of mathematics and democracy. *Numeracy, 4, 1*, 1-6. doi: http://dx.doi.org/10.5038/1936-4660.4.1.1.

Stemhagen, K., & Smith, J. W. (January 01, 2008). Dewy, democracy, and mathematics education: Reconceptualizing the last bastion of curricular certainty. *Education and Culture Purdue University Press-, 24, 2*, 25-40.

Stuyvesant High School. (2015). Retrieved October 1, 2015, from Graduation Requirements: http://stuy.enschool.org/academics/grad_require1.jsp

Tam Cho, W. K., & Manski, C. F. (2008). Cross-Level/Ecological Inference. In J. M. Box-Steffensmeier, H. E. Brady, & D. Collier, *The Oxford handbook of political methodology* (pp. 1-23). Oxford: Oxford University Press.

The Bronx High School of Science. (2015). Retrieved October 1, 2015, from Graduation Requirements: http://www.bxscience.edu/apps/pages/index.jsp?uREC_ID=41056&type=d&termREC_ID=&id=3&hideMenu=1

The College Board. (2015). *College Board*. Retrieved September 26, 2015, from About us: https://www.collegeboard.org/about

The High School for Health Professions . (n.d.). *Benefits of Regents Diploma*. Retrieved February 24, 2016, from The High School for Health Professions and Human Services: http://www.hphsnyc.org/apps/pages/index.jsp?type=d&uREC_ID=246434&pREC_ID=568534

The Intersegmental Committee of Academic Senates. (2010, April 26). *Statement on competencies in mathematics expected of entering college students*. Retrieved February 24, 2016, from Intersegmental Committee of the Academic Senates of the California Community Colleges, The California State University, and The University of California: http://icas-ca.org/Websites/icasca/Images/ICAS-Math-competency.pdf

The University of the State of New York, I. (2014). *New York State Student Information Repository System (SIRS) Manual.* Albany: The University of the State of New York. Retrieved February 3, 2014, from http://www.p12.nysed.gov/irs/sirs/2011-12/2011-12SIRSGuidance/NEWER/2013-14SIRSManual9-1_20131107.pdf

Torres, A. (2014). "Are we architects or construction workers?" Re-examining teacher autonomy and turnover in charter schools. *Education Policy Analysis Archives, 22*(124), 1-24.

Tyson, W., & Roksa, J. (2016). How schools structure opportunity: The role of curriculum and placement in math attainment. *Research in Social Stratification and Mobility, 44*, 124–135.

Tyson, W., Lee, R., Borman, K., & Hanson, M. A. (2007). Science, technology, engineering, and mathematics (STEM) pathways: High school science and math coursework and postsecondary degree attainment. *Journal of Education for Students Placed at Risk*, 243-270.

U.S. Census Bureau. (2015, December 2). *State and County QuickFacts*. Retrieved from United States Census Bureau: http://quickfacts.census.gov/qfd/states/36/36085.html

U.S. DOE. (2011, May 5). *Elementary and Secondary Education Act (ESEA).* Retrieved September 10, 2014, from No Child Left Behind: http://www2.ed.gov/policy/elsec/leg/blueprint/publication_pg3.html#part3

United States Department of Agriculture. (2013, August 8). *School meals income eligibility guidelines: Corrections*. Retrieved February 10, 2015, from United States Department of Agriculture: http://www.fns.usda.gov/sites/default/files/IEG_Table-032913.pdf

University of the State of New York, (. (1905). *New York State Education Department Bulletin.* Albany: New York State Education Department.

US News Ranking. (2014). *Best High Schools National Ranking*. Retrieved February 5, 2015, from US News & World Report Education: http://www.usnews.com/education/best-high-schools/national-rankings/magnet-school-rankings?int=966408

US News Report. (2014). *STEM Schools Best High Schools*. Retrieved February 5, 2015, from US News & World Report: http://www.usnews.com/education/best-high-schools/national-rankings/stem

USNEI. (2008, February). *Structure of the U.S. education system: Credit System.* Washington, D.C.: International Affairs Office: U.S. Department of Education.

Walpole, R. E., Myers, R. H., Myers, S. L., & Ye, K. (2012). *Probability & statistics for engineers & scientists.* Boston, MA: Prentice Hall.

West, J., Miller, W., & Diodata, L. (1985). *An analysis of course offerings and enrollments as related to school characteristics.* Washington, D.C.: National Center for Education Statistics.

White, H., & Sabarwal, S. (2014). *Quasi-experimental design and methods.* Florence, Italy: United Nations Children's Fund (UNICEF).

Wilensky, R. (2007). *High schools have got it bad for higher ed-and that ain't good.* Phi Delta Kappan.

Appendix A. Regents Examinations' Scaling System

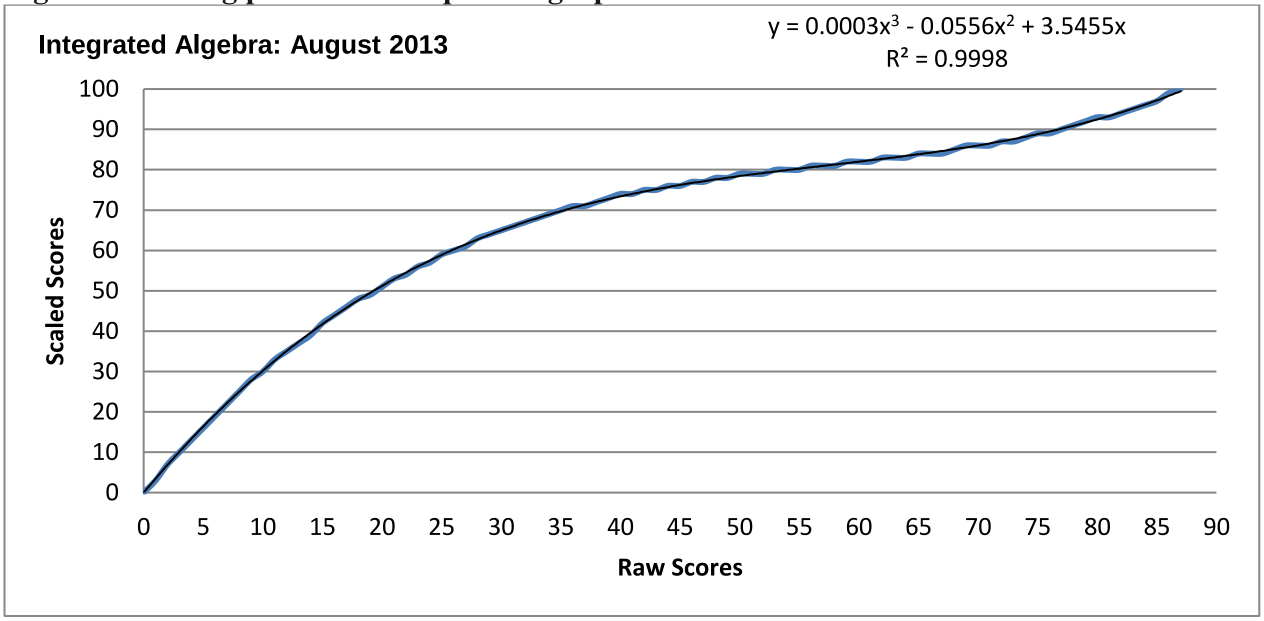
In order to have a better understanding of the Regents exams and their development, we communicated with the New York State Education department via e-mail. They provided us a copy of Section IV: Scaling of Operational Test Forms (NYSED, 2013) of their technical report. The Board of Regents and the New York States Commissioner's Technical Advisory Committee employ IRT Rasch and Partial Credit Modeling to develop their tests. The development of the test is tied to its scoring. In developing the Regents exams, the Board of Regents and the Technical Advisory Committee first determine a baseline raw score which, for exams we analyze, was determined to be June 2008. This baseline will remain until the parameters are re-visited. Because the Regents Exams are developed by means of psychometric tools, their scores are scaled to reflect the traits of both the test items and the test takers (NYSED, Questions regarding data, 2013). As a concrete example, the Integrated Algebra tests from 2008 up to August 2013 have a total raw score of 87 points. Raw scores are determined by test developers and their knowledge of the field. Raw scores are then mapped to scaled scores. The lowest and highest raw scores are mapped to their equivalent scaled scores using the polynomial equation: $p(x) = m_3x^3 + m_2x^2 + m_1x + m_0$. The scaled scores 65 and 85 are fixed parameters. In the case of Integrated Algebra exam, these values are mapped onto 30 and 65 or 31 and 67, respectively (NYSED, Questions regarding data, 2013). Using six conversion charts for the Algebra exam, we solved for the parameters (m) of the polynomial equation $p(x) = m_3x^3 + m_2x^2 + m_1x + m_0$, and obtained the value of the constants m_i and each of the estimated equations' R^2.

Conversion Equations

August 2008	$y = 0.0003x^3 - 0.0564x^2 + 3.5879x$	$R^2 = 0.9991$
June 2009	$y = 0.0003x^3 - 0.0558x^2 + 3.5563x$	$R^2 = 0.9997$
January 2010	$y = 0.0003x^3 - 0.0488x^2 + 3.3623x$	$R^2 = 0.9992$
June 2011	$y = 0.0003x^3 - 0.0563x^2 + 3.5446x$	$R^2 = 0.9991$
January 2012	$y = 0.0004x^3 - 0.0633x^2 + 3.7591x$	$R^2 = 0.9959$
August 2013	$y = 0.0003x^3 - 0.0556x^2 + 3.5455x$	$R^2 = 0.9998$

With the exception of the January 2012 test, the parameters of the cubic polynomials are virtually identical from test to test. Since the Regents exams are curriculum-based, this homogeneity in the values of (m) is expected. Because Regents exams are curriculum-based, the content difficulty of test items should be homogenous from test to test within the same content blocks. Also, since the exams are tied to the academic curriculum, it is expected that students are being taught the same topics with a somewhat similar academic intensity; hence, on average, students should acquire similar academic preparedness. Figure 2 below provides a visual of how the Regents exams' raw scores are mapped onto scaled scores by means of IRT Rasch modeling.

Figure 1: Scaling parameters' equation graph



Based on the pre-determined raw to scaled score values of 65 and 85, figure two suggests that the New York State Education Department, NYSED expects that the majority of its students have mathematical traits at or near the 65 scaled points threshold, which is 30 raw points. It is also expected that students achieving mastery scores [85, 100] are those with the highest academic traits in Algebra, for their raw scores have lower weights when mapped onto scaled scores. The Integrated Algebra exam is comprised of four parts, each containing the following raw score distribution: part 1, thirty items, each worth 2 points for a total of 60 points; part 2, three items, each worth 2 points for a total of 6 points; part 3, three items, each worth 2 points for a total of 6 points and part four three items, each worth 4 points for a total of 12 points. If a student is able to correctly answer half of part 1, he or she will have a passing scaled score of 65. That is, with a raw score of 30 out of 87, students pass the test. If a student is able to correctly achieve a raw score of 68 out of 87 points he or she will earn a scaled score of 85, which shows high proficiency.

Appendix B. Other Mathematics Courses Category

MQF11S	**Numeracy Development**
MQF82	**MATHEMATICS ELECTIVE**
MQF84	**MATHEMATICS ELECTIVE**
MQN11S	**Numeracy**
MQS11	**Financial Algebra**
MQS11	**Problem Solving**
MQS11C1	**Thinking and Reasoning**
MQS11QJA	**ELECTIVE ALGEBRA**
MQS11QMF	**MATHEMATICS OF FORENSICS**
MQS11QS	**Math Independent Study**
MQS11QX	**Consumer Math**
MQS11QX	**Liberal Arts Math**
MQS11S	**Algebra Skills**
MQS21	**BUSINESS MATH**
MQS21	**MATH ELECTIVE**
MQS21QM	**MATHEMATICS ELECTIVE**
MQS21QWT	**BUSINESS MATH TERM 1**
MQS21QX	**MATH ELECTIVE 1 OF 2 EXTENDED DAY**
MQS22	**Algebra 2**
MQS22	**BUSINESS MATH**
MQS22	**MATH ELECTIVE**
MQS41QW	**BUSINESS MATH TERM 1**
MQS81QM	**MATHEMATICS ELECTIVE**
MQS82	**MATH ELECTIVE 9**
MQS82QM	**MATH ELECTIVE 9 EXCELSIOR**
MQS84H	**MATH ELECTIVE HONOR**
MQS84QM	**MATH ELECTIVE EXCELSIOR**
MQS85	**MATH ELECTIVE**
MQS85H	**MATHEMATICS ELECTIVE HONOR**
MQS86	**MATH ELECTIVE**
MQS86H	**MATH ELECTIVE HONOR**
MQS87	**MATH ELECTIVE**
MQS87H	**MATH ELECTIVE HONOR**
MQT41	**FOUND IN GEOM1**
MQT42	**FOUND IN GEOM2**
MQT43	**FOUND IN GEOM3**
MQT44	**FOUND IN GEOM4**
MQT66	**GEOMETRY SKILLS**
MQF11	**Algebra Skills**
MQN11	**Applied Math**
MQN11	**GEOMETRY ENRICHMENT**

MQN11QA	**Engineering Design Elective**
MQN11QB	**Business Math**
MQN11QB	**Business Elective**
MQN11QPF	**Personal Finance**
MQN11QS	**Applications of Mathematics IS**
MQS11	**PERSONAL FINANCE**
MQS11	**MATHEMATICS FOR THE REAL WORLD**
MQS11QC	**CONSUMER MATH**
MQS11QP	**EXPLORATORY MATH**
MQS11QT	**PRACTICAL MATHEMATICS**
MQS11QT	**PRACTICAL MATH**
MQS21	**APPLIED MATHEMATICS 1 OF 2**
MQS21	**CUNY MATH T1**
MQS21	**COLLEGE ALGEBRA 1**
MQS21C	**College Math 1**
MQS21QC	**Finite Math**
MQS21QCM	**Consumer Math**
MQS21QCU	**CUNY MATH 1 of 2**
MQS21QMP	**MATH PROBLEM SOLVING 1 OF 2**
MQS21QSA	**SAT Math Wed**
MQS22	**CUNY MATH T2**
MQS22	**COLLEGE ALGEBRA 2**
MQS22QC	**Finite Math Term 2**
MQS22QCU	**CUNY Math 2 of 2**
MQS22QCW	**CUNY MATH WED**
MQS22QMP	**MATH PROBLEM SOLVING 2 OF 2**
MQS22QSA	**SAT Math Wednesday**
MQSA9	**Math Proficiency 1**
MQSAA	**Math Proficiency 2**
MQT11	**Math**
MQT11	**Math Elective**