A Distributed MAC Protocol for Efficient Channelization in Wireless Networks

A Thesis Presented

by

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to

The Graduate School

in Partial Fulfillment of the

Requirements

for the Degree of

Master of Science

in

Computer Science

Stony Brook University

December 2013

Stony Brook University
The Graduate School

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Abstract of the Thesis

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Abstract— There is a significant interest in designing new wireless multiple access protocols that split a wide frequency channel into multiple sub-channels and assign these sub-channels to competing transmissions. Doing this adaptively depending on the number of competing transmissions has a tremendous potential both in high-speed and white-space networks. While such protocols have been developed, they suffer from limitations such as considerable protocol overheads, dependence on a centralized controller, and assumptions about the network being static, etc. In this work, we develop a new multiple access protocol, Ez-Channel, that adaptively and efficiently channelizes the spectrum for improving throughput, without encountering the limitations of the past solutions. Ez-Channel performs efficient channelization and assignment of sub-channels to links by resourcefully utilizing the OFDM sub-carriers. In addition to circumventing hidden and exposed terminal problems, Ez-Channel adapts channel assignments whenever the topology or direction of links change in the network. In order to eliminate the need for a centralized controller and to avoid an overwhelming amount of information exchange, the protocol takes advantage of randomization techniques that facilitate localized decision making at nodes. Our extensive analytical and simulation studies show that Ez-Channel yields significant throughput improvements as compared with the state-of-the-art protocols in various settings.

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1 Introduction

Splitting the channel resources in time and/or frequency domain have been widely considered in wireless networks to accommodate multiple competing transmissions. Straightforward analysis shows that splitting across frequency domain (i.e., use of multiple orthogonal channels and FDM) achieves a greater performance relative to that across time domain (TDM scheduling) under the maximum transmit power constraint (the typical practical constraint wireless networks operate under) [1, 2]. Also, as the network speed increases (say, over 1 Gb/s) the scheduling-based approaches increasingly face higher normalized overheads. This is because the per-packet overheads are largely independent of channel bit rate, but the useful time spent on the channel on a per-packet basis reduces with channel bit rate. This has been explained in [3] and is even experienced in relatively slower networks, e.g., 802.11n. This problem is directly addressed by using concurrent packet transmissions on multiple channels.

The advantage of using multiple channels is not restricted to high-speed networks alone. In networks where a large amount of spectrum (possibly non-contiguous) is used – white space networks [4] being good examples – appropriate radio front ends to exploit very large bandwidths may not always be cost effective. Here, use of multiple smaller channels becomes a natural choice.

While traditionally multichannel systems have used a pre-defined and fixed channel split, recent work has focused on 'channelization,' i.e., adaptively determining how the channel is to be split and then assigning the individual sub-channels to competing transmissions [5]. Since the number of competing transmissions change dynamically in any network, such channelization approaches can show much superior performance relative to the use of fixed channels. Several such (adaptive) channelization approaches have appeared in recent literature (e.g., [6, 7, 8, 9]).

Concurrently with the above development, the advent of OFDM technology has given rise to the prospect of fast exchange of control information using OFDM sub-carriers. For example, [10, 11], and [12] use OFDM sub-carriers to carry out frequency-domain contention that is much more efficient than time-domain contention schemes, especially for high data rates.

Motivated by the ever-growing benefits of channelization, we demonstrate that rich potentials of OFDM sub-carriers can be employed to design efficient channelization. We present *Ez-Channel*, a distributed MAC protocol to efficiently channelize the spectrum and assign the resulting sub-channels (a set of contiguous OFDM sub-carriers), to the communicating links. The sizes of the sub-channels are dynamically determined by the number of active links and their interference relationships. A randomization technique is used to eliminate the need of any central controller to determine the sub-channels. The protocol operates seamlessly in either an ad hoc network or a wireless LAN scenario.

After reviewing the related work (Section 2), the paper presents the Ez-Channel protocol (Section 3) and a proposed synchronization protocol that makes various protocol stages synchronous (Section 5). Ez-Channel's performance is evaluated analytically (Section 4) as well as via simulations against a suite of multichannel/channelization protocols and protocols that perform frequency-domain contention (Section 6). We show that while Ez-Channel performs at par with the state-of-the-art in some of the simpler scenarios (e.g., all links interfere with one another), it provides a far superior performance in more complex interference scenarios.

2 Related Work

The idea of considering the spectrum as a set of multiple sub-channels has been investigated for a long time. Earlier work was focused on assigning a fixed set of sub-channels to network nodes and ensure that the transmitter and the receiver of each link will operate on the same sub-channel (see SSCH [13], MMAC [14], DCA [15], xRDT [16], and HMCP [17] for instance). As opposed to these static protocols, Ez-Channel is a dynamic channelization scheme in which the sub-channels are determined based on the current requirements of the network. Taking another perspective, centralized channelization techniques (e.g., [18, 19, 6, 20]) rely on a centralized entity in the network for channelization. In contrast, Ez-Channel is a distributed protocol.

We also briefly review some major techniques developed for dynamic channel access. WiFi-NC [21] utilizes a single radio that is capable of simultaneously operating on multiple channels. The complexity and cost of the WiFi-NC hardware coupled with the well-studied inefficiencies of 802.11 DCF (i.e., wasting time on back-offs and shortcomings in fairness) on each sub-channel confines the scheme's performance gain. B-Smart [7] requires a dedicated control channel and a separate radio on each node for exchanging control information. The protocol relies on estimating the number of nodes in the network in order to allocate time and spectrum to the links in an efficient manner and prevent the control channel from turning into a bottleneck. White-Fi [22] is a scheme for Access Point (AP)-Client communication over TV white spaces. If there are multiple APs and clients while no primary users are present, nodes will access the entire wide spectrum using 802.11 DCF that can perform poorly. In the work presented in [23], white space spectrum is distributed amongst clients and APs by maintaining and updating a conflict graph at each node as well as using a dedicated control channel. Jello [8] and Papyrus [9] are two distributed platforms for finding free spaces in the spectrum, but they are not designed for packet switched networks. Additionally, a node may sense and capture any portion of the spectrum for as long as it desires that exacerbates fairness.

There are a few MAC protocols that improve throughput in high data rate WLANs. WiFi-Nano [24] improves throughput of 802.11 DCF by reducing the slot size at the cost of using a complex radio that is capable of self-interference cancellation and prolonging preamble times. However, the underlying protocol is 802.11 DCF, which has major efficiency drawbacks. Recently, two related schemes have been exploited to significantly reduce the overhead of wireless MAC protocols: frequency-domain contention (see [10, 11], and [12]) and sending acknowledgments via OFDM symbols [12]. Back2F [10] and REPICK [12], however, do not take advantage of splitting the channel amongst links. Even though in FICA [11] the wide channel is divided into multiple sub-channels, fixing the widths of sub-channels limits the performance gain yielded by channelization. Ez-Channel, in contrast, divides the channel at the granularity of OFDM sub-carriers and based on the number of current active links. Furthermore, frequency-domain contention and acknowledgments are integral parts of Ez-Channel.

3 Ez-Channel

Ez-Channel splits the channel (i.e., available bandwidth) such that interfering transmissions use separate sub-channels as to avoid interference. In this section, we first provide an overview of its channelization results via three examples. The protocol is then formally defined, and its different aspects are discussed.

3.1 Overview

The key idea of Ez-Channel is to split the channel into as many sub-channels of equal sizes as required at the current time to ensure interference-free transmissions. Figure 1 provides an insight into what Ez-Channel achieves via given sample scenarios shown on the left-hand side of the figure. The right-hand side of the figure demonstrates the sub-channel(s) assigned to each link as yielded by Ez-Channel in distinct colors. The enclosing rectangle in each case represents the channel that is split into the colored sections that indicate the sub-channels. Note that a transmitter causes interference at the receiver of another simultaneously active link if they are both in the same collision domain[1] and operate on the same frequency.

Let us consider three examples of channelizations resulted by Ez-Channel. Receivers prevent the hidden terminal problem [25] by relaying transmission requests so that transmitters will use separate sub-channels (Figure 1-a). Likewise, the exposed terminal [26] problem is prevented, as in Ez-Channel, channelization decisions are based on receivers' view of the network (Figure 1-b). Figure 1-c depicts a network with four links all of which are located within the same collision domain. In this case, the interfering links simultaneously operate over non-overlapping sub-channels. The mechanism through which these channelizations are achieved becomes clear in the next sub-section.

3.2 The Protocol

In Ez-Channel, transmitters and receivers exchange *tones* (i.e., short signals on OFDM channel sub-carriers) systematically to learn how many links they may interfere with. This information is then used to split the channel into non-interfering sub-channels in a decentralized fashion. Since the total number of sub-carriers is limited and also nodes local information to determine sub-channels may not be sufficient, there can be inefficiencies in the protocol that are discussed later.

Ez-Channel is executed in *rounds*. In each round, active nodes will first split the channel among themselves such that links in the same collision domain use separate frequency ranges of the channel. The links then use the corresponding sub-channels to transfer data. Finally, the receivers acknowledge successful transmissions.

[1]If two links are located in the same *collision domain*, they will interfere with each other if their frequencies overlap.

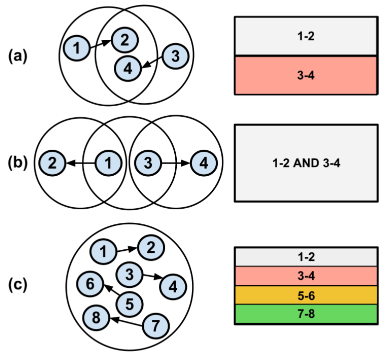


Figure 1: Example networks (on the left) along with channelization results of Ez-Channel (on the right) (Nodes within a circle will interfere with one another if they operate on the same frequencies). The hidden and exposed terminal problems are circumvented (a and b, respectively). Further, the available channel is split into as many sub-channels as needed depending on the number of links that would interfere otherwise (c).

Each round is composed of five *stages* as numbered in Figure 2. Note that the time periods in the figure are not to scale, and stages 1, 2, 3, and 5 (each of length T_{sub}) as well two SIFS periods (each of length T_{SIFS}) are very short compared with data transmission time in stage 4 (T_{data}). Sub-channels are determined and assigned to the network links in the first three stages, and the last two stages are designated to transmitting data and acknowledgements. Except for stage 4, all other stages merely involve transmission of tones whose short transmission periods substantially reduce the channelization and acknowledgement time.

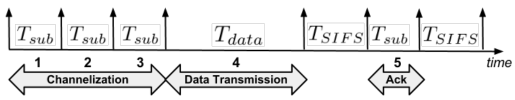


Figure 2: Five stages of a round in Ez-Channel.

In Ez-Channel, the channel, which is composed of N_s sub-carriers (indexed 1 through N_s), is divided into *clusters* of equal sizes each of which includes C contiguous sub-carriers. Clusters, as discussed shortly, provide a useful correspondence between the transmitter and the receiver of a link. Suppose there are n nodes in the network. Let i and j denote the indices of a typical transmitter and receiver, respectively; and Id_j represent a unique identifier of node j such as its MAC address. Let $\%$, $\lfloor\ \rfloor$, and $\lceil\ \rceil$ denote the remainder (modulo 10), floor, and ceiling operators, respectively. Then $Cluster_j = C \times (Id_j \ \% \ \lfloor \frac{N_s}{C} \rfloor) + 1$ is the index of the first sub-carrier in node j's cluster.

In what follows, the five stages of each round of Ez-Channel are described in the context of data transmission from source node i to destination node j:

Stage 1 (Contention): Node i transmits a tone on a randomly chosen sub-carrier that belongs to node j's cluster. Formally, the sub-carrier index chosen by node i is a random number $u_{i,j}$ computed as follows:

$$u_{i,j} = Cluster_j + Rand_{0,C-1} \tag{1}$$

where $Rand_{0,C-1}$ is an integer chosen at random from the set $\{0, 1, \ldots, C-1\}$. Node j stores the indices of all sub-carriers it hears tones on as 1s in $S_{j,1}$, a binary, zero-initialized array of size N_s. (Subscript 1 in $S_{j,1}$ indicates stage 1 of the protocol.)

Stage 2 (Contention resolution and channelization on transmitter side): The goal of this stage is to determine the winners of the contention and split the channel among them. If $S_{j,1}$ includes at least one element with a value of 1 located in j's cluster (i.e., located between indices $Cluster_j = C \times (Id_j \ \% \ \lfloor \frac{N_s}{C} \rfloor) + 1$ and $Cluster_j = C \times (Id_j \ \% \ \lfloor \frac{N_s}{C} \rfloor) + C$ of $S_{j,1}$), it means that some other node(s) are trying to send data to node j. In this case, node j selects a winner among the requests. The receiver arbitrates the requests based on a pre-defined rule. Without loss of generality, suppose the rule mandates that the request with the smallest index wins the contention.[2] Node j populates a zero-initialized array, $S_{j,2}$, based on $S_{j,1}$ as follows: in each cluster of $S_{j,1}$, if there is more than one element with a value of 1, only the first one is copied over into $S_{j,2}$; the other elements of $S_{j,2}$ are set to zero. Node j then transmits tones on sub-carriers that correspond to the elements of $S_{j,2}$ that have a value of 1.

Now consider node i. It stores 1s corresponding to the sub-carriers on which it hears tones in stage 2 in a zero-initialized binary array, $S_{i,2}$. By scanning $S_{i,2}$, node i can determine if $u_{i,j}$ is the first non-zero element in node j's cluster in $S_{i,2}$. If so, node i concludes that its request to transmit to node j has been approved. Furthermore, node i can determine the total number of approved transmission requests in the neighborhood simply by counting the elements with a value of 1 in $S_{i,2}$. Finally, node i can infer its sub-channel which is the set of sub-carriers that will be used by node i for transmission to node j as follows.

Once $S_{i,2}$ is populated, the transmitters that have won, can determine their sub-channles. Suppose node i is the winner among the nodes contending to transmit to node j, and element $u_{i,j}$ of $S_{i,2}$ is the r_ith non-zero element among a total of R non-zero elements in $S_{i,2}$, then node i splits the channel into R sub-channels of almost equal sizes and assigns the r_ith sub-channel to itself. Let $X = \lfloor \frac{N_s}{R} \rfloor$ and $Y = (N_s \ mod \ R)$, node i's sub-channel start at sub-carrier $Start_i$ and ends at sub-carrier End_i where:

$$Start_i = \begin{cases} r_i \times (X+1) - X & \text{if } r_i \leq Y \\ Y + X \times (r_i - 1) + 1 & \text{Otherwise} \end{cases} \tag{2}$$

$$End_i = \begin{cases} Start_i + X & \text{if } r_i \leq Y \\ Start_i + X - 1 & \text{Otherwise} \end{cases} \tag{3}$$

The goal of (2) and (3) is to ensure that the channel is split and assigned to the winners in a systematic way (i.e., without overlap between sub-channels and without leaving any part of the channel unassigned).

Stage 3 (Channelization on receiver side): In this stage, each receiver determines the sub-channel it has to listen to in Stage 4 (data transmission). To this end, each approved transmitter i sends a tone corresponding to each element of $S_{i,2}$ with a value of 1. By hearing these symbols, the receivers can determine the sub-channels they will be listening to during the following stage (data transmission)

[2] If node j has sent a transmission request in stage 1, it needs to do arbitration in this stage only if it has decided to proceed as a receiver. Otherwise, it will ignore all requests.

in the exact same way that the transmitters identified their sub-channels in stage 2. Upon completion of stages 2 and 3, the transmitter and receiver of each link will have identical sub-channels.

Stage 4 (Data transmission): Node i sends data to node j over the corresponding sub-channel.

Stage 5 (Acknowledgement): If the transmission from node i has been successful, node j sends an acknowledgement using a tone on sub-carrier $u_{i,j}$.

The SIFS period between stages 4 and 5 ensures a receiver has enough time to determine whether data transmission has been successful before sending acknowledgement. The second SIFS (after stage 5 and before the next round) separates consecutive rounds.

3.3 Practical Considerations

There are several aspects of a system implementation of Ez-Channel that are worth discussing here. First, as is the case in protocols that perform frequency-domain contention (e.g, [10, 12]), Ez-Channel requires that the same stage of the protocol be running by all active nodes at any given time. This topic is discussed in Section 5 where we design a synchronization method. Second, in order for concurrent transmissions over neighboring sub-channels not to interfere with each other, tones on different sub-channels must be maintained aligned in time. FICA [11] presents a distributed method to overcome symbol misalignment using the cyclic-prefix (CP) mechanism that can be directly adopted by Ez-Channel. Third, in Ez-Channel, every node needs to be equipped with transmit/receive antennas to simultaneously listen to and transmit tones on sub-carriers. This is shown to be feasible on commodity software radio platforms (see [10]). Another practical aspect is setting the value of data transmission time (T_{data}): if it is too large, the channel will be left unutilized for a substantial amount of time under sporadic packet arrival patterns. Conversely, if data time is too short, the time overhead of the protocol grows. Thus, the expected traffic pattern, coming from network measurement logs, is the main factor in choosing an appropriate value for T_{data}. Finally, the feasibility of data transmission on a subset of sub-carriers has been demonstrated in previous related work (e.g., [8] and [11]).

3.4 Protocol Features and Use Cases

Here, we summarize the advantages of Ez-Channel. First, it dynamically adapts to network topology changes, as channelization depends on current link interferences. Thus, the protocol is efficient with respect to spectrum usage in that no part of the channel may be wasted due to the use of pre-defined, fixed sub-channels. For instance, if the network in Figure 1-a transforms into the network in Figure 1-b, the nodes sub-channels will be adapted accordingly in the following round. Second, the protocol prevents hidden and exposed terminals (Figure 1-a, b). Third, each node only needs local information for channelization. However, access to global information will enhance the performance (see Section 3.5).

As mentioned in the introduction, Ez-Channel is well-suited to high-speed wireless as well as white space networks. Since the protocol, as described, can be directly used in high-speed wireless networks, here we concisely discuss how to extend it to accommodate white space networking. First, active nodes must stop operating as soon as they detect a primary user. Second, while the description of Ez-Channel assumes contiguity of channel sub-carriers, in white space networks, the available bandwidth may be composed of non-contiguous chunks of sub-carriers, as some arbitrary parts of the channel may be occupied by primary users. Suppose the channel is composed of sub-carriers indexed 1 through N_s, as before. At a new stage inserted before stage 1 of the base protocol (Section 3.2), each node senses the channel and finds the set of free sub-carriers represented by $S = S_1 \cup \cdots \cup S_F$, where: *(i)* S_p is a set of contiguous and free sub-carriers, *(ii)* there is a gap of at least one occupied sub-carrier between the sub-carriers in S_p and those in S_q, and *(iii)* all sub-carrier indices in S_p are smaller than the corresponding values in S_q ($\forall p, q \in \{1, \ldots, F\}$ and $p < q$). Let $N'_s = \sum_{p=1}^{F} |S_p|$ (Note that $N'_s \leq N_s$.). By relabeling the indices of sub-carriers in S consecutively from 1 to N'_s starting with S_1 and ending with S_F in order, Ez-Channel is applicable to this setting.

3.5 Imperfect Channelization

A perfect channelization protocol must ensure no overlap between sub-channels of interfering links. This, however, requires information on the global topology of the network. It can be seen from the protocol description that Ez-Channel performs perfect channelization only if the interference between any two links is bidirectional;[3] otherwise, the resulting information asymmetry between the links may cause interference as highlighted next via an example.

Figure 3 depicts an example network topology in which Ez-Channel will yield imperfect channelization. The middle link ($3 \to 4$) will use one third of the channel, while the other two links ($1 \to 2$ and $5 \to 6$) will each use one half of the channel. The fundamental problem is that links $1 \to 2$ and $5 \to 6$ are not aware of each other, but link $3 \to 4$ is aware of both of them. Even worse, if also $u_{1,2} < u_{3,4} < u_{5,6}$, link $3 \to 4$ will use the middle third of the channel which results in interference on all links.

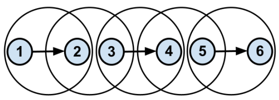


Figure 3: An example topology to highlight imperfect channelization.

Our simulations involving this situation (not presented here due to space limitations), reveal, however, that this problem does not significantly hinder the performance of Ez-Channel. Nonetheless, the severity of the problem can be reduced by providing each node a wider view of the network such that all neighbors of the neighbors of any node are aware of its existence This can be achieved by repeating

[3]A network whose nodes are all located within a single collision domain is an example of this case.

stages 1 and 2 of the protocol in order to propagate the transmission requests one hop further in the neighborhood. We omit such details for brevity.

3.6 Resolving Collisions

Transmissions may collide under certain circumstances (see Sections 3.5 and 4). To deal with collisions, the colliding transmitters should back-off. One such back-off mechanism is as follows: each transmitter i maintains an aggressiveness parameter, p_i, which is the probability that the transmitter participates in the next round of the protocol, and is initialized to 1. If the transmitter fails in a transmission attempt, it halves p_i. Otherwise, it will update p_i to the new value of $min(2p_i, 1)$. In the evaluations (Section 6), we simulate the base protocol only as described in Section 3.2 without this collision resolution mechanism.

4 Analysis of Ez-Channel

This section presents the analytical formulations pertaining to the performance of Ez-Channel when all network nodes are located in a single collision domain. Since the total number of channel sub-carriers N_s is finite, there are two possible sources of failure in the protocol. A *sub-carrier collision* occurs when multiple transmitters that are contending for transmitting to the same receiver win the contention. A *cluster collision*, on the other hand, refers to the event that the same cluster is assigned to multiple receivers. In this section, we probabilistically analyze the effect of these two types of collisions and formulate the efficiency of the protocol. Subsequently, we show how to choose the best value for cluster size.

4.1 Sub-Carrier Collision

A sub-carrier collision signifies that more than one transmitter among the contenders that are trying to transmit to a certain receiver win the contention, as all of these winners have chosen the sub-carrier with the smallest index among all the chosen sub-carriers within the receiver's cluster. Suppose the set of active nodes in a given round of Ez-Channel is composed of n_r receivers, and n_t transmitters contend for transmitting to each receiver. In practice, n_t may reflect the average or maximum number of transmitters that simultaneously contend for any receiver if we are to analyze the protocol performance in an average or worst case sense, respectively.

Let us focus on receiver j, where $j \in \{1, \ldots, n_r\}$. A sub-carrier collision signifies that more than one of the n_t sub-carriers randomly chosen by the transmitters in stage 1 (all of which are within the receiver's cluster by construction) rank first. Let A denote such an event. If A_i represents the event that the ith element of the cluster is the first non-zero element of node j's cluster and is chosen by more than one contending transmitter, where $i \in \{1, 2, ..., C\}$, the probability of sub-carrier collision will be:

$$P(A) = \sum_{i=1}^{C} P(A_i) \tag{4}$$

In order to calculate $P(A_i)$, suppose A_{i1} is the event that the ith element of the cluster is chosen by more than one contending transmitter, and A_{i2} is the event that it is ranked first among all non-zero elements of the cluster; then:

$$P(A_i) = P(A_{i1} \cap A_{i2}) = P(A_{i1}|A_{i2})P(A_{i2}) \tag{5}$$

And it can be proven that:

$$P(A_{i1}|A_{i2}) = 1 - n_t \frac{1}{C-i+1}(1 - \frac{1}{C-i+1})^{(n_t-1)} - (1 - \frac{1}{C-i+1})^{n_t} \tag{6}$$

$$P(A_{i2}) = (1 - (1 - \frac{1}{C-i+1})^{n_t})(1 - \frac{i-1}{C})^{n_t} \tag{7}$$

The proof is presented in Appendix A.

4.2 Cluster Collision

A cluster collision means a given cluster is assigned to more than one receiver. Formally, a cluster collision signifies there exist at least two receivers j and k that $Cluster_j = Cluster_k$ and receivers j and k are located such that their transmissions would interfere with one another. Let random variable X_c denote the number of receivers to which cluster c is allocated. We define indicator random variable X_{cj} as follows:

$$X_{cj} = \begin{cases} 1 & \text{If cluster } c \text{ is assigned to receiver } j \\ 0 & \text{Otherwise} \end{cases} \tag{8}$$

where $c \in \{1, \ldots, N_{cluster}\}$[4] and $j \in \{1, \ldots, n_r\}$. Since $P(X_{cj}) = \frac{1}{N_{cluster}}$, by applying the definition of expected value, it can be observed that:

$$E[X_c] = \frac{n_r}{N_{cluster}} \tag{9}$$

See Appendix B for details.

4.3 Aggregate Collision Probability

Having the sub-carrier and cluster collisions defined, the *aggregate collision probability*, shown by $P(B)$, reflects the combined effects of sub-carrier and cluster collisions. To combine the effects of these two types of collisions, parameter n_t in (6) and (7) must be replaced with $\lceil E[X_c] \rceil \times n_t$ because, effectively, at most $\lceil E[X_c] \rceil \times n_t$ transmitters contend within any given cluster. Therefore, the aggregate collision probability $P(B)$ is defined exactly as $P(A)$ was defined, with the difference that in calculating $P(A_{i1}|A_{i2})$ and $P(A_{i2})$, n_t must be substituted by $\lceil \frac{n_r}{N_{cluster}} \rceil \times n_t$.

[4] $N_{cluster}$ denotes the total number of clusters given by $N_{cluster} = \lfloor \frac{N_s}{C} \rfloor$.

4.4 Channel Use Efficiency

What fraction of the time does the channel involve data transmission using Ez-Channel? To answer this question, we need to account for the overhead associated with aggregate collision probability as well as the overhead of an Ez-Channel round (i.e., the entire duration of a round except for stage 4 – see Figure 2). It is easy to verify that the expected number of successful links (i.e., non-interfering transmitter-receiver pairs) in each round is $min\,(N_{cluster}, n_r) \times (1 - P(B))$. Furthermore, the overhead of the protocol in each round, which is the time taken by stages 1, 2, 3, and 5 and the two SIFS intervals, is equal to $4T_{sub} + 2T_{SIFS}$. Therefore, the efficiency of Ez-Channel in a single collision domain is given by:

$$E_{Ez-Channel} = \frac{min(N_{Cluster}, n_r) \times (1 - P(B)) \times T_{data}}{4T_{sub} + 2T_{SIFS} + min(N_{Cluster}, n_r) \times T_{data}} \tag{10}$$

To gain some intuition about $E_{Ez-Channel}$, we have also formulated the efficiency of REPICK [12] taking into account collision probability in a similar manner to the analysis presented in this section (see Appendix C). By plugging values for the parameters of (10) based on IEEE 802.11n and setting the cluster size to its optimal (see Section 4.5), we observed that Ez-Channel outperforms REPICK by 35% in average. Low collision probabilities due to the notion of clusters, small protocol overhead in each round, and using multiple sub-channels are the key reasons behind the observed performance gap. As an example, the aggregate collision probability using Ez-Channel in a network with 128 nodes, where $N_s = 104$, was only 13%; the corresponding value in REPICK approaches 100%.

4.5 Setting Cluster Size

A practical question is "What value of the cluster size C will maximize $E_{Ez-Channel}$?". We consider two cases that will be referred to as *downlink setting* and *uplink setting.* Downlink setting is characterized by $n_t = 1$ that implies exactly one transmitter contends for each receiver. In uplink setting, however, $n_t > 1$. These two scenarios are reminiscent of the downlink and uplink traffic in a wireless LAN, respectively.

We have numerically studied values of $E_{Ez-Channel}$ using an extensive set of real-world values of parameters n_t, n_r, N_s, T_{sub}, T_{SIFS}, and T_{data}, while varying C (we do not show the corresponding plots due to space limitations). We have found the following heuristic for finding the optimal value of cluster size, shown by C^* (i.e., the cluster size that maximizes $E_{Ez-Channel}$): *In downlink setting, if number of receivers is smaller than or equal to the total number of sub-carriers,* $C^* = 1$*; otherwise,* $C^* = N_s$*. In uplink setting* $C^* = N_s$. For instance, in a wireless LAN that is not extremely dense (i.e., $n_r \leq N_s$) client and APs, as receivers, should be assigned cluster sizes of 1 and N_s, respectively.

5 Round Synchronization

Similar to existing frequency-domain contention protocols, in Ez-Channel, all active nodes across the network that may interfere with each other are required to be executing the same stage of the protocol at any given time. Such synchronization is attainable by using either out-of-band or in-band solutions. Out-of-band solutions, such as equipping each node with a GPS [14, 27], would incur no synchronization time overhead to Ez-Channel. If all nodes are located within the same collision domain, the in-band synchronization method of the work in [10] can be used. For the general case of multiple collision domains, we propose an in-band solution for achieving stage synchronization that can also be applied to the existing frequency-domain contention protocols that lack any in-band synchronization technique [10, 12]. Our synchronization method has two main components as described in what follows.

5.1 Synchronizing Nodes at ACK Stage

A certain sub-carrier of the channel, called ACK-SYN, is dedicated to denoting stage 5 of Ez-Channel. Each node sends a tone on ACK-SYN during stage 5 regardless of whether it has been active (transmitting/receiving) in the current round. Any new node joining the network may not start operating until it first overhears a tone on ACK-SYN. In such an event, the node will set its current stage to stage 5 and becomes synchronized with other nodes. This simple solution, while expected to be sufficient in many scenarios, has a shortcoming that is discussed and addressed next.

5.2 Network-wide Synchronization Protocol

Suppose the network is composed of multiple isolated islands of nodes such that the nodes within an island are synchronized, but nodes across islands may not be synchronized. If an existing node moves to, or a new node arrives at a position at which it can hear nodes from two isolated and unsynchronized islands, it will bridge the two otherwise isolated groups of nodes. We call this situation the *connected islands problem* in which case the mechanism based on ACK-SYN will not suffice for achieving synchronization across the nodes in the connected islands. The second component of our synchronization protocol tackles this issue.

The main idea is simple. If the connected islands problem occurs, all nodes across all connected islands must stop their operations so that they can become synchronized together. We augment the ACK-SYN method with a halt mechanism. A predetermined sub-carrier, denoted by STOP, which is different from ACK-SYN, is used to cause a domino effect that will stop all activities across the connected islands as follows. [5] If node u: *(i)* hears a tone on ACK-SYN, *(ii)* does not hear any tone on STOP, and *(iii)* its current stage is not stage 5, the connected islands problem has occurred. In such an event, node u will continuously send tones on STOP. Any node that receives this tone, must immediately stop its activities, and constantly send tones on STOP. Therefore, the STOP tones will reach any node for which there is a path to/from u. Node u keeps sending tones on STOP for a duration of t since initiating it. Parameter t, which has a predefined value, should be large enough to ensure that STOP tones can reach all nodes reachable from u within t. The next operations allow the nodes determine when they must stop sending tones

[5] Sub-carriers ACK-SYN and STOP are not used for any operations other than what they are reserved for.

on STOP and resume executing Ez-Channel rounds (while all nodes have become synchronized).

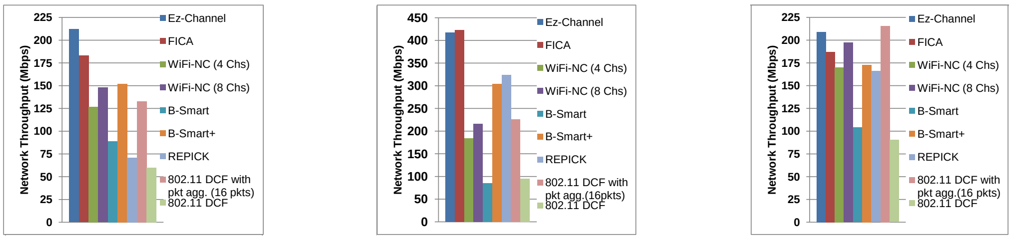
Let h denote a number that is larger than or equal to the diameter of the network in terms of number of hops and $h \leq N_s$. Starting at node u as the origin, each hop k is identified by a unique sub-carrier s_k. After a time period of t from the time node u initiated the STOP tones, it will transmit a *hop tone* on s_1 (i.e., the first of N_s sub-carriers) for a time period of T_{sub}. Any node that receives a hop tone on s_k, will transmit a hop tone on s_{k+1} (for a duration of T_{sub}). This hop relaying process will continue until the last hop. Note that during this process nodes are still constantly sending tones on STOP. The node that receives a hop tone on s_h will not relay any hop tone on s_{h+1}. All nodes are aware of the duration of the entire hop relaying process ($h \times T_{sub}$). Consider node w ($w \neq u$). By knowing its hop distance from u (i.e., the index of the sub-carrier it has first heard a hop tone on), once node w hears a hop tone, it knows the amount of time δ_w it has to wait before the entire hop relaying process ends. After an elapsed time of δ_w, node w will stop sending tones on STOP and start the T_{SIFS} period that follows stage 5 of Ez-Channel. Thus, all nodes located in the connected islands become synchronized.

In the rare case that multiple nodes initiate tones on STOP (i.e., more than two unsynchronized islands of nodes emerge at the same time) there may exist a node w that hears a hop tone on s_k originated from node x, and shortly after that, another hop tone on s_l originated from node y. In this case, w will calculate a new waiting time as follows. If the new waiting time is greater than or equal to what w calculated before, then w will ignore the more recent tone. Otherwise, w will update δ_w accordingly and will relay tones on s_{l+1}. Finally, if w hears two hop tones, s_k and s_l, at the same time, then w will ignore one of the tones based upon same calculations as above, and will update its δ_w in a similar manner.

The proposed method ensures synchronization across any connected region of the network. While it can take a considerable amount of time, it is not expected to be triggered too often in the networks with low to medium mobility.

6 Simulations

In this section, we conduct extensive simulations in order to evaluate the performance of Ez-Channel with respect to network throughput and fairness. Ez-Channel's performance is compared with seven other protocols (all reviewed in section 2): FICA [11], WiFi-NC [21], REPICK [12], plain 802.11 DCF [28], 802.11 DCF with packet aggregation [29], B-Smart [7], as well as B-Smart+. In B-smart, a dedicated control channel is carved out from the given channel for exchanging the protocol information. To further improve the performance of B-smart, we develop B-smart+ in which the bandwidth and collision overheads associated with the control channel are ignored. We have developed custom simulators for the above eight protocols. We do not consider Back2F [10] as a comparison point because its basic idea has been shown to be substantially outperformed by REPICK. The simulations are carried out over a wide variety of network topologies and show that while Ez-Channel performs at par with the state-of-the-art in some of the simpler scenarios (e.g., all links interfere with one another), it provides a far superior performance in more



(a) Results for scenario of Figure 1-a.

(b) Results for scenario of Figure 1-b.

(c) Results for scenario of Figure 1-c.

Figure 4: Network throughput for sample scenarios of Figure 1.

complex interference scenarios in terms of both network throughput and fairness.

6.1 Simulation Methodology

The simulators capture important aspects of both PHY and MAC layers as described next.

6.1.1 Physical layer models

The OFDM PHY is simulated and the SINR model is used to determine packet reception at receivers. The channel is 160 MHz.[6] The nodes operate on the 5 GHz band. The transmit power of each node is 100 mW, the noise-level is -91 dBm for the 160 MHz channel, and the carrier-sense threshold is 5 dB. The free-space path loss model is used to model signal propagation. The modulation is QPSK and bit error rate (BER) at receiver is calculated based on the Q-function [2]. BER values determine the probability of correct reception of incoming packets. The PHY data rate in this setting is 256 Mbps.

Since FICA proposes new PHY and MAC schemes for high data rate WLANs, we closely follow the specifications of its PHY as presented in [11]. The rest of the protocols conform to the 802.11 PHY specifications [30] based on which the 160 MHz channel is composed of 512 sub-carriers. We observed a slight mismatch between the PHY data rates supported by FICA and the other protocols; however, the difference is negligible. The transmit power per active sub-carrier across all nodes is constant, so the transmission range of nodes is independent of the fraction of the channel they use.

6.1.2 MAC layer models

The important aspects of the MAC layer in the simulations are presented here. T_{SIFS} and T_{sub} (slot time) are 16 μsec and 9 μsec, respectively. These values are the same for both the FICA MAC and the other protocols, and are taken from the 802.11 standard.

The number of clusters in Ez-Channel is set based upon the analysis in Section 4.5. T_{data} is set to be enough for sending eight 1500-Byte packets if the entire

[6]It will be supported in the forthcoming 802.11ac standard for high data rate WLANs [30].

channel is used, so $T_{data} = 40$ time slots. We found this value to be appropriate via experiments. Based on the sub-channel width of each winning transmitter, the transmitter determines the number of back-to-back packets that it can send. For instance, if the transmitter is one of a total of four winning transmitters in the current round of Ez-Channel, it can send at most two 1500-Byte packets.

In order to make conditions favorable for the FICA MAC protocol, we have used FICA's $AIMD$ back-off scheme, which is shown to be better than the FICA's $Rmax$ back-off scheme [11]. We have examined the 802.11 DCF that operates on the entire wide channel, both with and without RTS/CTS. While both cases perform much worse than Ez-Channel in different scenarios, we only present the results for the case with RTS/CTS off, as it is the default option at high data rates due to higher expected throughput [31]. For REPICK, besides following the details provided in [12], we also assume that nodes are always synchronized in terms of rounds without any synchronization overhead. For 802.11 DCF with packet aggregation, a maximum of 16 packets can be sent back-to-back. The control channel in B-Smart is 16 MHz, and nodes can only use discrete channel sizes of 16, 32, 64, and 128 MHz.

The aforementioned settings hold throughout the simulations unless otherwise noted. All results are averaged over 10 simulation runs.

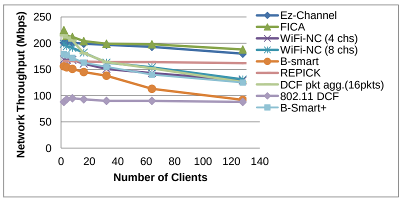
6.2 Results for Three Sample Scenarios

As the first step, we evaluate the protocols in the three sample scenarios of Figure 1. Figure 4-a shows the network throughput for the hidden terminal case (Figure 1-a). It can be observed that Ez-Channel achieves a high channel utilization (about 82%). There are three reasons for this. First, the channelization process is based on the receivers' view of the network which results in preventing the hidden terminal problem. Second, the protocol overhead of Ez-Channel is small (roughly $4Tsub + 2T_{SIFS}$ in this case). Finally, the channel is divided into two non-overlapping sub-channels each being assigned to one of the links (Figure 1-a).

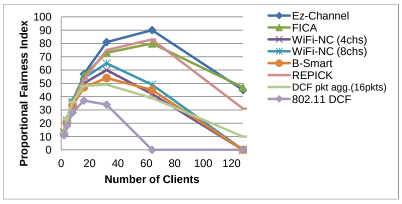
In this scenario, FICA also attempts to improve performance by splitting the wide channel between the two competing transmissions. However, we have observed that, in doing so, FICA occasionally causes collisions on sub-channels, and thus, wastes portions of the spectrum. Moreover, FICA may leave some sub-channels idle, so in this scenario it achieves a channel utilization of 73%. Nodes are made synchronized in FICA at no overhead cost. If the two senders in FICA are not synchronized, then its performance would drastically drop.

In WiFi-NC with k sub-channels, the 160 MHz channel is divided into k sub-channels of equal sizes. Concurrent transmissions on sub-channels independently execute the 802.11 DCF. As the results show, while WiFi-NC surely provides better throughput than single-channel 802.11 DCF, it still performs significantly worse than Ez-Channel. WiFi-NC suffers from collisions as well as the channel remaining idle due to time-domain back-offs. While not shown here, we observed similar results for WiFi-NC with 16 sub-channels.

For similar reasons as the case of WiFi-NC, 802.11 DCF with packet aggregation achieves a significantly lower network throughput compared with Ez-Channel. Note that, this is despite the large number of aggregated packets (16 1500-byte packets). If fewer number of packets were aggregated by 802.11 DCF, then even lower throughput



(a) Network throughput in random single collision domain networks.



(b) Fairness in random single collision domain networks.

Figure 5: Evaluations in single collision domain.

would be resulted by this scheme. For comparison purposes, we also show the throughput of 802.11 DCF, where each transmitter sends only one packet every time it gains access to the channel.

While REPICK also adopts a frequency-domain contention scheme, which shortens the contention and acknowledgement periods (each becoming equal to $Tsub$), it performs poorly in this scenario. In fact, it performs close to the plain 802.11 DCF, and Ez-Channel yields a 2.8 times improvement over REPICK. In REPICK, the transmitter contends for the first packet in a sequence of packets. Therefore, the hidden terminal problems cannot always be avoided. The hidden terminal occurs in B-Smart, as it uses 802.11 DCF over its control channel. Finally, B-Smart+, while naturally performs better than B-Smart, faces degraded performance because of using sub-channels of predetermined sizes.

Figure 4-b shows the network throughput for the exposed terminal examples of Figure 1-b. Ez-Channel performs well in this scenario too and achieves a network throughput of around 400 Mbps. This is another example of where Ez-Channel's *adaptive* channelization proves helpful in enhancing network throughput. Note that in the prior case (i.e., hidden terminal), Ez-Channel was able to split the channel into two sub-channels, which was the ideal choice for that scenario. Here, Ez-Channel identifies, in a distributed fashion, that both transmissions should be provided with the entire channel as their sub-channels (Figure 1-b). It can be seen that WiFi-NC and 802.11 with packet aggregation attain much lower throughput values than Ez-Channel because at most one of the two transmitters can transmit at a given time on the same portion of the channel despite the fact that it would harmless if both links were simultaneously active. Note that REPICK's reverse contention mechanism can sometimes cause only one of the senders to transmit at a time which reduces the network throughput. As expected, 802.11 provides the poorest performance in this scenario. Conversely, FICA performs well. However, if the two senders get desynchronized then FICA's performance could be degraded to half of its current value.

Figure 4-c demonstrates the network throughput for the single collision domain scenario of Figure 1-c. Similar to the previous examples, Ez-Channel performs well in this scenario too. Ez-Channel splits the entire channel into four sub-channels and assigns a separate sub-channel to each link. Even though 802.11 with packet aggregation and WiFi-NC perform close to Ez-Channel, it should be noted that only 4 links are contending in the network. As we will see shortly, the performance of these protocols deteriorate in networks with a larger number of nodes because they suffer from collisions and back-off overhead.

It is clear from the right-hand side of Figure 1 that Ez-Channel leads to fair utilization of the channel, as the contending nodes gain equal shares of the channel in these three examples. As we will see in §6.3, this characteristic of the protocol holds to a great extent in more sophisticated scenarios too.

6.3 Results for Random Scenarios

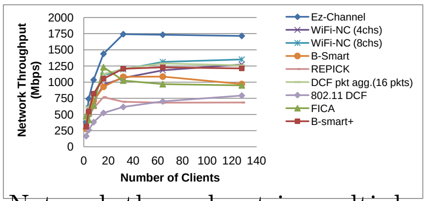
We also evaluate the protocols in random network topologies where we consider both networks with a single and multiple collision domain(s). The following formula is used for evaluating the level of proportional fairness P in the network [32]: $P = \log_2(\prod_{l=1}^{L} r_l)$, where L is the number of links, and r_l is the total throughput observed for both downlink and uplink flows of link l. A larger value of P indicates a better level of fairness for a given number of links. Note that B-Smart+ is excluded from the fairness evaluations, as its channelization decisions are assumed to be given by an oracle.

In the single collision domain setting, three APs are randomly placed in the network. The number of randomly located clients varies between 2 and 128. Figure 5-a shows the network throughput values. Ez-Channel performs at par with FICA, and both of them perform better than the other protocols. Figure 5-b shows that the high network throughput of Ez-Channel is not an artifact of reduced fairness.

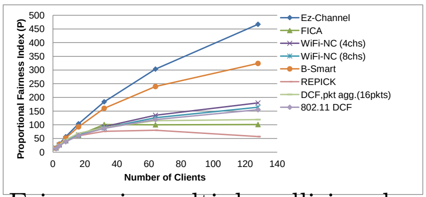
For the multiple collision domain settings, 20 APs are randomly placed within a $250m \times 250m$ area, and the number of clients in the network is varied from 2 to 128. Figure 6 demonstrates the network throughput and proportional fairness. Ez-Channel significantly outperforms all other protocols with respect to both metrics. It is interesting that FICA's throughput reduces significantly in multiple collision domains with a larger number of nodes in the network. This is because nodes may encounter constant collisions, unnecessary retransmissions, and starvation in FICA [33]. The comparison points other than FICA face the limitations that are mentioned §6.2, namely, collisions, back-off time, the hidden and exposed terminals, and fixed sub-channels. It is noteworthy that Ez-Channel successfully handles a large number of nodes because of the low probability of collisions and effective channelization.

7 Conclusions

In this paper, we have introduced Ez-Channel, a MAC protocol for channelization in wireless networks. It is distributed, and adaptive to changes in the network. Ez-Channel utilizes OFDM sub-carriers to parsimoniously exchange the information



(a) Network throughput in multiple collision domains.



(b) Fairness in multiple collision domains.

Figure 6: Evaluations in multiple collision domains.

needed by network nodes to make channelization decisions locally. Mathematical analysis as well as simulation studies show that Ez-Channel outperforms the state-of-the-art MAC protocols in realistic settings of high-speed networks. Moreover, the in-band round synchronization mechanism we proposed as a part of Ez-Channel is independently applicable to existing frequency-domain protocols.

Two separate directions can be considered in the future work. The first direction is studying the extent to which the protocol can be made centralized (in order to minimize imperfect channelization) while scalable. System implementation and evaluation of Ez-Channel is another possible extension to this work.

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8 Appendix A

Sub-carrier collisions: As defined in 4.1, $P(A_{i1}|A_{i2})$ is the probability of the event that the ith element of a cluster is chosen by more than one contending transmitter given the ith element is the first non-zero element of the cluster. The complement of event $A_{i1}|A_{i2}$ includes two events E_1 and E_2:
E_1: The ith element of a cluster is chosen by exactly one contending transmitters.
E_2: The ith element of a cluster is chosen by none of the contending transmitters.
where both events E_1 and E_2 are conditioned on the event that the ith element is the first non-zero element of the cluster. It can be seen that the probabilities of events E_1 and E_2 are given by $n_t \frac{1}{C-i+1}(1-\frac{1}{C-i+1})^{(n_t-1)}$ and $(1-\frac{1}{C-i+1})^{n_t}$, respectively. Hence:

$P(A_{i1}|A_{i2}) = 1 - n_t \frac{1}{C-i+1}(1 - \frac{1}{C-i+1})^{(n_t-1)} - (1 - \frac{1}{C-i+1})^{n_t}$

On the other hand, $P(A_{i2})$ is the probability of the event that the ith element of the cluster is the first non-zero element of the cluster. Event A_{i2} can be thought of as the intersection of the following two events:

E_3: Element i of the cluster is non-zero.

E_4: The first $i-1$ elements of the cluster are all zero.

Therefore, $P(A_{i2}) = P(E_3 \cap E_4) = P(E_3|E_4)P(E_4)$ where $P(E_3|E_4)$ and $P(E_4)$ are given as follows:

$P(E_3|E_4) = (1 - (1 - \frac{1}{C-i+1})^{n_t})$

$P(E_4) = (1 - \frac{i-1}{C})^{n_t}$

Thus, $P(A_{i2})$ can be expressed as:

$P(A_{i2}) = (1 - (1 - \frac{1}{C-i+1})^{n_t})(1 - \frac{i-1}{C})^{n_t}$

9 Appendix B

Expected number of receivers assigned to cluster c: Since X_{cj}, defined in 4.2, is an indicator random variable, using the definition of expected value we have:

$E[X_c] = \sum_{j=1}^{n_r} 1 \times P(X_{cj}) + 0 \times P(\bar{X}_{cj}) = \sum_{j=1}^{n_r} P(X_{cj}) = \frac{n_r}{N_{cluster}}$

where $c \in \{1, \ldots, N_{cluster}\}$, $j \in \{1, \ldots, n_r\}$, and $\bar{X}_{cj}$ denotes the complement of X_{cj}.

10 Appendix C

Probability of collision in REPICK: The value of this probability can be calculated based on probability of sub-carrier collision in Ez-Channel by making the following observations:

1- In REPICK, each contending transmitter chooses a random sub-carrier from a set of N_C sub-carriers where $N_C = N_s - (n_t \times n_r + n_r)$ (see Section 4 and [12]).

2- All $n_t \times n_r$ transmitters contend with each other irrespective of their receivers. In Ez-Channel's terminology, REPICK uses only one cluster, which is the entire channel.

The rest of the calculations is identical to computing $P(A)$ (the probability of sub-carrier collision in Ez-Channel in Section 4.1) as shown below:

$P(D) = \sum_{i=1}^{N_C} P(D_i)$

Defining D_{i1} and D_{i2} analogous to A_{i1} and A_{i2} respectively, the probability of D_i is given by:

$P(D_i) = P(D_{i1} \cap D_{i2}) = P(D_{i1}|D_{i2})P(D_{i2})$

$P(D_{i1}|D_{i2}) = 1 - n_t \times n_r \frac{1}{N_C-i+1}(1 - \frac{1}{N_C-i+1})^{(n_t \times n_r - 1)} - (1 - \frac{1}{N_C-i+1})^{n_t \times n_r}$

$P(D_{i2}) = (1 - (1 - \frac{1}{N_C-i+1})^{n_t \times n_r})(1 - \frac{i-1}{N_C})^{n_t \times n_r}$