Joint-Liability Debt and Fiscal Policies

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Abstract of the Dissertation

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This thesis uses quantitative and computational macroeconomic methods to analyze policies in the presence of financial and economic frictions. The thesis consists two chapters. In particular, the second chapter focuses on finding instruments that mitigate financial crises and stabilize sovereign bond yields, while the third chapter focuses on the optimal capital taxation under the presence of heterogeneity in risk aversion.

In Chapter 2, I assess the consequences of implementing a joint liability debt system in a two-country small open economy model. With joint liability a default of one country makes the other participant liable for its debt. The results highlight a trade-off between the contagion risk, in the sense that this instrument may push some member states to default even though they are individually solvent, and cheaper access to credit on average, since lenders

are at risk only if no participating sovereign is willing to service the debt. The findings suggest that the welfare consequences of this policy proposal hinge critically on the timing of its introduction: Introducing such instruments at the peak of the Eurozone crisis would have helped the Periphery and harm the Core member states, while its adoption during normal times has the potential to make all participants better-off.

In Chapter 3, I introduce risk aversion heterogeneity based on empirical results, in an otherwise standard heterogeneous agents macroeconomic model with incomplete markets, in order to analyze the optimal level of taxation. The heterogeneity in risk aversion affects the precautionary motives on capital and therefore the optimal level of taxation. In the exercise I quantify the welfare implications that occur, because of different tax levels, during the transition period to the long-run equilibrium. The results predict that the optimal capital taxation is increasing when I introduce heterogeneity in risk aversion. This is happening for two reasons, (i) higher precautionary motives compare to the standard case produce higher welfare effects, (ii) agents with lower risk aversion are in favor of a higher capital taxation, since they accumulate less capital.

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Chapter 1

Introduction

In chapter 2, "A Quantitative Analysis of Joint-Liability Debt", I study the effects of introducing Joint Liability Debt instruments (Eurobonds) among the Eurozone Member states, as a potential shield of future financial crises is the implementation of bonds with joint liability. However, with this instrument some member states may have greater intensives to increase their debt accumulation, since they will have easier access to financial markets. On the one hand, this would be problematic since a failure of a country to repay may trigger a contagion effect if the other countries do not have enough resources to absorb the troubled debt. On the other hand, the introduction of joint liability bonds like Eurobonds could provide better access to financial markets especially to those countries under stress. Moreover, it could decrease the incentives for some member countries to abandon the Euro or default, by promoting stability and setting the basis for a prospective fiscal integration, European Commission (2014).

To assess the effects of introducing joint liability debt, I consider two economies with exogenous incomplete markets. In the benchmark case countries can issue only individual sovereign bonds, following Arellano (2008). Then, I study the interactions between two countries that can issue bonds with full joint liability. Following the literature, I study an endowment economy and I abstract from production and input decisions. The endowments follow a stochastic process taken from data on the performance of Core and Periphery countries in the Eurozone. The benchmark model is calibrated for the case of a single country issuing individual bonds. Then, I compare this benchmark to a world where two countries can issue joint liability bonds under two different scenarios: (i) the two countries are subject to different processes of idiosyncratic income risk (asymmetric case), and (ii) the two countries are subject to the same process of idiosyncratic income risk (symmetric case).

The findings show that countries have cheaper access to financial markets in both scenarios, even though the welfare implications differ drastically between both scenarios. In the symmetric case (two core countries issuing joint liability bonds), the model predicts welfare gains for both countries since the cheaper credit effect dominates to the contagion effect. In contrast, in the asymmetric case Periphery countries experiment welfare gains, while Core countries face welfare losses when both countries start with large debt-to-output ratios. If the Periphery countries start with low debt-to-output ratios, then the Core countries could also benefit from the introduction of Eurobonds.

In chapter 3, "Optimal Capital Taxation with Risk Aversion Heterogeneity", I study the optimal level of capital and labor taxation in order to finance the governments budget. The main concern of the policymakers is to decide which group of the population will bear the highest percentage of the tax burden. To give a potential answer to this debate, I developed a DSGE model that nests heterogeneous groups of population, by introducing heterogeneity in risk aversion and income, in order to estimate the desired combination of capital and labor taxes.

The main findings predict that the capital should be taxed heavier than in an economy with homogeneous preferences. The results show that the gap between homogeneous and heterogeneous preferences is caused, firstly because of different precautionary motives in the economies. Second, agents with lower risk aversion are in favor of a higher capital taxation since they tend to accumulate less capital.

In chapter 4, a conclusion to the dissertation is provided. The two main chapters of this thesis show that market frictions have significant influences over the economic and financial performances of sovereigns. A brief discussion of several important directions in the future is provided in offered in this chapter.

Chapter 2

A Quantitative Analysis of Joint-Liability Debt

2.1 Introduction

This paper introduces bonds with joint liability in a model where two small open economies borrow from risk neutral international lenders. Under joint liability a default in one country makes the other country liable for its debt. This feature introduces the potential for contagion of default decisions, while introducing further repayment guarantees for lenders. Hence, the introduction of this instrument generates *contagion* risk, in the sense that this instrument may push some countries to default even though they are individually solvent. On the other hand, it may generate *cheaper credit*, thereby helping enhance financial stability. In this paper I quantify the effects and generate predictions about the welfare implications of introducing joint liability bonds under different underlying fundamental conditions.

The recent Eurozone crisis has highlighted the necessity for the development of financial instruments that mitigate the effects of the financial crisis and stabilize the yields of sovereign bonds. One of the mechanisms that was proposed by the European Commission (2011) as a potential shield of future financial crises is the implementation of bonds with joint liability (Eurobonds)[1]. On the one hand, some member states may increase their debt accumulation with this mechanism, since they will have easier access to financial markets. This would be problematic since a failure of a country to repay may trigger a contagion effect if the other countries do not have enough resources to absorb the troubled debt.[2] On the other hand, the introduction of joint liability bonds like Eurobonds could provide better access to financial markets especially to those countries under stress. Moreover, it could decrease the incentives for some member countries to abandon the Euro or default, by promoting stability and setting the basis for a prospective fiscal integration, European Commission (2014).

I consider two economies with exogenous incomplete markets. In the benchmark case countries can issue only individual sovereign bonds, following Arellano (2008). Then, I study the interactions between two countries that can issue bonds with full joint liability. Following the literature, I study an endowment economy and I abstract from production and input decisions. The endowments follow a stochastic process taken from data on the perfor-

[1]Not to be confused with Eurobond, which are bonds denominated in a currency other than the home currency of the country that issues them.

[2] The European Commission (2011) has tried to asses the feasibility of common issuance of sovereign bonds among Member States of the Eurozone and they have mentioned "moral hazard" as a potential problem. For this reason, all the proposals suggest borrowing limits in order to mitigate this potential problem.

mance of Core and Periphery countries in the Eurozone. For my measurement I use two groups of countries, the first group is wealthier with less income volatile than the other group and represents the Core member states of the Eurozone (Germany, France, Netherlands), while the other group represent the Periphery member states of Eurozone (Portugal, Italy, Greece, Ireland, Spain). In the analysis, the prices of the bonds are endogenously determined and depend on both countries choices, generating a strategic interaction between the two countries. In particular, there exist a two-stage Nash equilibrium. In the first stage countries make their repayment decisions and, conditional on this, they make their borrowing decisions on the second stage. I do not allow for partial default, and the penalty of default is a permanent output loss and exclusion from financial markets.

This paper is related to the novel literature that studies the effects of Eurobonds. Delpla & von Weizsacker (2010) discusses the 'blue and red bond' proposal, in which they propose pooling debt up to 60% of GDP (blue bonds) and using individual bonds issued by each country separately (red bonds) beyond that threshold. Hellwig & Philippon (2011) foresees a mutualization of 10% of GDP for the short term debt. Claessens *et al.* (2012) discusses in depth various proposals of Eurobonds and analyze potential effects in the Eurozone, and Beetsma & Mavromatis (2014) and Tirole (2015) analyze stylized finite-period models of the strategic interactions between two countries that can issue joint liability bonds. They find that Eurobonds might be beneficial under some circumstances. This paper complements that literature by providing quantitative predictions in an infinite horizon general equilibrium model of debt and default.

This paper also builds on the literature on the quantitative implications of debt dynamics and default in incomplete asset markets models: Eaton & Gersovitz (1981), Aguiar & Amador (2013), Aguiar & Gopinath (2006), Cuadra *et al.* (2010), Pouzo & Presno (2014), and Yue (2010).[3] In fact, the benchmark for comparison is Arellano (2008), which accounts for the empirical regularities in emerging markets as an equilibrium outcome of the interaction between risk-neutral creditors and a risk averse borrower that has the option to default.[4] Hatchondo *et al.* (2014) studies the effects of introducing a limited non-defaultable financing option in a small-open economy. However, they abstract from the strategic interactions that might be generated among the participating member states. Their results suggest that access to such an asset for a given country could produce substantial welfare gains and lead to significant reductions in sovereign debt and spreads. Arellano & Bai (2014a), Arellano & Bai (2014b) and Lizarazo (2009) examine the contagion across sovereign defaults through the existence of common lenders. In this paper, I extend this idea and I develop a model that nests common lenders and borrowers. Àbrahàm *et al.* (2015) develop a model of the Financial Stability Fund (FSF) across sovereigns as a long-term partnership with limited ex-post transfers. To the best of my knowledge, none of the papers in the quantitative default literature addresses the impact of the strategic interactions that joint liability bonds might generate.

The benchmark model is calibrated for the case of a single country issuing individual bonds. Then, I compare this benchmark to a world where two

[3]See Aguiar & Amador (2014) or Tomz & Wright (2012) who explore more key issues in this literature.

[4]Alternative models of default focus on rollover risk, such as Cole & Kehoe (2000) and Conesa & Kehoe (2015), but I do not consider this issue in my analysis.

countries can issue joint liability bonds under two different scenarios: (i) the two countries are subject to different processes of idiosyncratic income risk (asymmetric case), and (ii) the two countries are subject to the same process of idiosyncratic income risk (symmetric case). The findings show that countries have cheaper access to financial markets in both scenarios, even though the welfare implications differ drastically between both scenarios. In the symmetric case (two core countries issuing joint liability bonds), the model predicts welfare gains for both countries since the cheaper credit effect dominates to the contagion effect. In contrast, in the asymmetric case Periphery countries experiment welfare gains, while Core countries face welfare losses when both countries start with large debt-to-output ratios. If the Periphery countries start with low debt-to-output ratios, then the Core countries could also benefit from the introduction of Eurobonds.

The paper is structured as follows: Section 2 presents the theoretical models for the benchmark economy and the Eurobonds, Section 3 calibrates the model and assesses the quantitative implications of the model, and Section 4 concludes.

2.2 The Model

I consider two cases of sovereign bonds markets: first the benchmark economy, in which countries issue only individual bonds to the international markets, i.e. no joint liability. Second, both countries are allowed to issue only bonds with joint liability.

I assume that the countries are risk-averse and they cannot affect the world risk free interest rate. The period utility function $u(.) : \mathbb{R}_+ \rightarrow \mathbb{R}$ and is assumed to be strictly increasing, strictly concave and satisfies Inada conditions. The lifetime payoff of each borrowing country i is $E_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t})$, where $\beta \in (0, 1)$ is the discount factor, $i \in \{1, 2\}$ is the index for each set of countries, $c_{i,t}$ denotes each country's level of consumption at period t. Moreover, in each period the countries receive a stochastic endowment of a single perishable consumption good $y_{i,t}$, which is drawn from a compact set $Y = [\underline{y}, \overline{y}]$. These shocks follow a Markov process with transition matrix $\pi_i(y'_i, y_i)$.

In both models, the risk averse countries trade one-period asset with the risk-neutral competitive foreign lenders. The lenders have access to an international credit market where they can trade as much as they need at a constant risk free interest rate r. I assume that the lenders always commit to repay their debt. However, countries have no commitment and each period decide whether to repay their debt or to default.

The lenders have perfect information about the history of endowments and they can observe the demand for next period's assets. Given these two variables they estimate the probability that the countries will be insolvent and they offer an interest rate that compensates for the risk of default. Considering the risk-neutrality and the zero expected profits, the equilibrium prices q are given by,

$$q = \frac{1 - \phi}{1 + r} \tag{2.1}$$

where ϕ is the endogenous derived default probability. The bond price q lies in $[0, \frac{1}{1+r}]$, since, $0 \leq \phi \leq 1$. The probability of default is zero for any positive savings and the sovereign bond price indicates the price of a risk free bond $\frac{1}{1+r}$. When countries have negative savings there might be some positive probability ϕ for the government to default which has a negative effect on the price of the sovereign bond to compensate the international creditors.[5] The sovereign's interest rate is defined as the inverse of the bond price, $r^s = \frac{1}{q} - 1$ and the country's spread is the difference between the interest rate and the risk free interest rate, $s = r^s - r$.

Influenced by the default episodes in various emerging economies, the cost originated by default episodes is two fold: (i) de facto prohibited access to the financial markets because of high interest rates and (ii) a direct output loss due to liquidity problems, outflow of capital, banking problems. If a country chooses to default, I assume that it will remain in permanent financial autarky since the incidence of the insolvency has created bad reputation for the country from the international creditors. The output cost is a function $g(y_i) \leq y_i$ that country has when defaults and is an increasing function respect to y_i, as in Arellano (2008).

2.2.1 Benchmark

This section is the benchmark economy and follows Eaton & Gersovitz (1981) for the theoretical part and Arellano (2008) and Aguiar & Gopinath (2006) for the quantitative part. Define $V(b, y)$ to be the life-time value function for a country that starts the current period with assets b and endowment y. The country chooses to maximize the present value of its welfare by choosing to repay its debt or to default. Therefore, $V(b, y)$ satisfies

[5] Arellano (2008) models the price function in similar method.

$$V(b, y) = \max \ \{W^{def}(y), W^{r}(b, y)\} \tag{2.2}$$

where $W^{def}(y)$ is the value that is associated with the default, while $W^{r}(b, y)$ is the pay-off function associated with repaying:

$$\begin{aligned} W^{r}(b, y) &= \max_{c,b'} u(c) + \beta E_{y'/y} V(b', y) \\ \text{s.t.} \quad c + q(b', y) \cdot b' &= y + b \\ b &> \underline{b} \end{aligned} \tag{2.3}$$

If the country defaults, it faces permanent financial autarky and its consumption equals the endowment, which entails some direct output costs. The value of default, $W^{def}(y)$ is given by the following:

$$W^{def}(y) = u(g(y)) + \beta E_{y'|y} W^{def}(y') \tag{2.4}$$

Let $A(b)$ be the set of y's for which it is optimal for the country to default. The default set of the country, given that it has good credit history is:

$$A(b) = \{y \in Y : W^{r}(b, y) \leq W^{def}(y)\} \tag{2.5}$$

The country may have incentives to default because it had a bad shock in the output combined with a massive debt that is unsustainable. However, the country loses its ability to have an intertemporal consumption smoothing since it has no access to the financial markets. If the country has a bad credit history then the default set is $A(b) = Y$.

The default probability for the country is defined by:

$$\phi(b', y) = \int_{A(b')} d\pi(y'|y) \tag{2.6}$$

When the default set is empty, $A(b) = \emptyset$, then the equilibrium default probability is zero, since it is not optimal for the country to default. When $A(b) = Y$ then the probability to default is equal to one. In general, the probability changes in a positive manner as the assets shift (i.e. if the government debt is high then the probability is higher).

To derive the equilibrium prices I use Eq. 2.1, and we get:

$$q(b', y) = \frac{1 - \phi(b', y)}{1 + r} \tag{2.7}$$

The level of the asset's price depends on the probability that the country will default next period. In the extreme case that the probability is equal to one then the price is equal to zero and the country can not borrow. As the probability decreases, the price gets closer to the price of a risk free bond.

Definition 1: A Recursive Equilibrium for a single country consist of: *(i)* policy functions for borrowing and consumption $\{b'(b, y), c(b, y)\}$ and a value function $\{V(b, y)\}$ *(ii)* the price function for individual bonds $q(b', y)$ st:

1. Given the prices, the policy functions and the value functions of the country solve its maximization problem 2.2 - 2.4.

2. Taking as given country's policy functions and value function, the bond price function satisfies the maximization problem of the foreign lenders 2.7.

2.2.2 Eurobonds

In this part, I lay out the economy in which both countries can issue bonds with joint liability. The vector of endogenous aggregate states consists of the vector of countries' debt holdings, $\{b_i\}_{\forall i}$. Therefore, the economy's state space consists of the endogenous and exogenous states and is denoted by $S = \{b_1, b_2, y_1, y_2\}$. The countries' repayment strategy is denoted by, $\{h_i'\}_{\forall i}$. The repayment strategy is a binary variable, where $h_i' = 0$ stands for good credit, while $h_i' = 1$ stands for bad credit.

In this economy countries interact strategically about their borrowing and repayment decisions simultaneously in two stages as shown in figure 2.1. In the first stage they chose their repayment decision. Conditional on the decision of the first stage, they issue assets on the second stage. Hence, there are three possible scenarios.

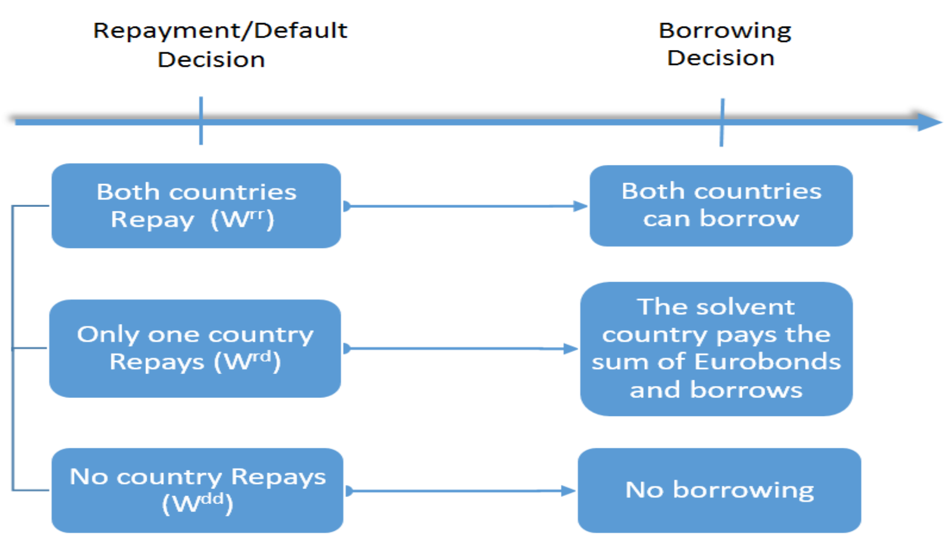


Figure 2.1: Timing of Decision

Scenario I - If both countries choose to repay, the payoff function $W_i^{rr}(S; b'_{i^-})$ of country i, given the arbitrary asset strategy b'_{i^-} for country i^-, solves:

$$
\begin{aligned}
W_i^{rr}(S; b'_{i^-}) = \max_{c_i, b_i{}'} \;\; & u(c_i) + \beta E_{y'_i, y'_{i^-}|S} V_i^E(S') \\
\text{s.t.} \;\; & c_i + q_E(b_i^{'}, b_{i^-}^{'}, y_i, y_{i^-}) \cdot b_i^{'} = y_i + b_i \\
& b_i > \underline{b}
\end{aligned}
\tag{2.8}
$$

Let $V_i^E(S)$ be the associated value function for Eurobonds for each country i, given that both countries have good credit history. It is vital to know the level of debt for country i^-, since it influences the Eurobonds' price $q_E(b_i^{'}, b_{i^-}^{'}, y_i, y_{i^-})$. Next period, since both countries choose to repay, they will be able to borrow again with Eurobonds.

Scenario II/III - If country i chooses to repay while country i^- chooses to default, the payoff function $W_i^{rd}(S)$ solves:

$$
\begin{aligned}
W_i^{rd}(S) = \max_{c_i, b_i^{'}} \;\; & u(c_i) + \beta E_{y'_i|y_i} V_i(b_i^{'}, y'_i) \\
\text{s.t.} \;\; & c_i + q_i(b_i{}', y_i) \cdot b_i{}' = y_i + (b_i + b_{i^-}) \\
& b_i > \underline{b}
\end{aligned}
\tag{2.9}
$$

where, $W_i^{rd}(S)$ is the payoff function when country i chooses to repay while country i^- chooses to default. In this scenario, country i has to pay the sum of all the Eurobonds, while next period it will be in the benchmark case from section 2.2.1. The next period Value function $V_i(b_i, y_i)$ is the same as in the benchmark economy, since next period the country will issue debt without any joint liability. The price $q_i(b_i', y_i)$ that country i receives today is also derived by the benchmark economy, since it reflects the probability that the country to default next period.

Scenario IV - if country i chooses to default, its payoff is:

$$W_i^{dd}(y_i) = W_i^{def}(y_i) \tag{2.10}$$

which is identical to the one in section 2.2.1.

I develop an intra-period game to derive the optimal strategy of repayment and borrowing for each country i, since it internalizes the effects of its strategies and the other's country strategies. The structure of the subgame depends on the aggregate state space S, as well as the repayment and borrowing decisions of both countries. The equilibrium strategies of repayment and borrowing $\{b_i'(S) = b_i^{BR'}(S, b_{i^-}^{BR'}, h_{i^-}^{BR'})\ ,\ h_i'(S) = h_i^{BR'}(S, h_{i^-}^{BR'}, b_{i^-}^{BR'})\}_{\forall i}$ are computed by solving a Nash Equilibrium, thus they reflect the best response of country i given the best response of country i^-.

The best response for the repayment strategy of country i, given the arbitrary current strategies $\{h_{i^-}^{'}, b_{i^-}^{'}\}$ is defined:

$$h_i^{BR'}(S; h'_{i^-}, b'_{i^-}) = \underset{h'_i \in \{0,1\}}{\text{argmax}} \begin{cases} (1-h'_i) \cdot W_i^{rr}(S; b'_{i^-}) + h'_i \cdot W_i^{dd}(y_i) & \text{,if } h'_{i^-} = 0 \\ (1-h'_i) \cdot W_i^{rd}(S) + h'_i \cdot W_i^{dd}(y_i) & \text{,if } h'_{i^-} = 1 \end{cases}$$
(2.11)

The best response for the debt strategy of country i, given the arbitrary current strategies $\{h'_{i^-}, b'_{i^-}\}$ is defined:

$$b_i^{BR'}(S; b'_{i^-}, h'_{i^-}) = \underset{b'_i \in B}{\text{argmax}} \begin{cases} W_i^{rr}(S; b'_{i^-}) & \text{,if } h_i^{BR'} = 0 \ \& \ h'_{i^-} = 0 \\ W_i^{rd}(S) & \text{,if } h_i^{BR'} = 0 \ \& \ h'_{i^-} = 1 \\ 0 & \text{,if } h_i^{BR'} = 1 \end{cases} \quad (2.12)$$

Moreover, $\tilde{V}_i^E(S; \{b'_i, h'_i\}_{\forall i})$ is the payoff function of country i, given the arbitrary current strategies $\{b'_i, h'_i\}_{\forall i}$

$$\tilde{V}_i^E(S; \{b'_i, h'_i\}_{\forall i}) = \begin{cases} W_i^{rr}(S; b'_{i^-}) & \text{,if } h'_i = 0 \ \& \ h'_{i^-} = 0 \\ W_i^{rd}(S) & \text{,if } h'_i = 0 \ \& \ h'_{i^-} = 1 \\ W_i^{dd}(y_i) & \text{,if } h'_i = 1 \end{cases} \quad (2.13)$$

Definition 2: Given the future value functions $\{V_i^E(S'), V_i(b'_i, y'_i)\ , V_i^{def}(y'_i)\}$ and the prices $\{q_i(b'_i, y_i), q_E(b'_1, b'_2, y_1, y_2)\}$, the intra-period Nash Equilibrium consists of the best response strategies for borrowing and repayment $\{b_i^{BR'}(S; b_{i^-}^{BR'}, h_{i^-}^{BR'}), h_i^{BR'}(S; h_{i^-}^{BR'}, b_{i^-}^{BR'})\}_{\forall i}$ s.t.:

1. The best response strategies for repayment and borrowing are the solutions to maximization problem 2.12 and 2.11

2. The equilibrium pay-off value function $V_i^E(S)$ is derived by the equilibrium strategies $\{b_i^{BR'}(S, b_{i^-}^{BR'}, h_{i^-}^{BR'}), h_i^{BR'}(S, h_{i^-}^{BR'}, b_{i^-}^{BR'})\}_{\forall i}$ and equation 2.13 s.t.:
   $V_i^E(S) = \tilde{V}_i^E(S; \{b_i^{BR'}, h_i^{BR'}\}_{\forall i})$

Given the outcome of the intra-period Nash Equilibrium, let $D(b_1, b_2)$ be the set, for which both countries choose to default simultaneously:

$$D(b_1, b_2) = \{y_1 \in Y \;\&\; y_2 \in Y : h_1'(S) \cdot h_2'(S) = 1\} \tag{2.14}$$

To derive the equilibrium prices I use Eq. 2.1 as in the benchmark economy, and I get:

$$q_E(b_1', b_2', y_1, y_2) = \frac{1 - \iint_{D(b_1', b_2')} d\mu_1(y_1'|y_1) d\mu_2(y_2'|y_2)}{1 + r} \tag{2.15}$$

Note that this price reflects the probability that both countries will default simultaneously, and the analysis is similar to the benchmark case.

Definition 3: Given the price function $\{q_i(b_i', y_i)\}_{\forall i}$ and the value function $\{V_i(b_i, y_i), W_i^{def}(y_i)\}_{\forall i}$ from definition 1, a Markov Perfect Equilibrium for this economy consists of: *(i)* policy functions for repayment, borrowing, consumption $\{h_i'(S), b_i'(S)\ , c_i(S)\}_{\forall i}$, value functions $\{V_i^E(S)\}_{\forall i}$ and *(ii)* a price function for bonds $\{q_E(b_i', b_{i-}', y_i, y_{i-})\}$ st:

1. Given the prices $\{q_i(b_i', y_i), q_E(b_i', b_{i-}', y_i, y_{i-})\}$ and the equilibrium value functions from definition 1 and 2, the policy functions and the value

functions are the solution to the maximization problem 2.8 - 2.13 and satisfy definition 3.

2. Taking as given both countries' policies functions and values functions, the bond price function $\{q_E(b'_i, b'_{i-}, y_i, y_{i-})\}$ satisfies the maximization problem of the foreign lenders 2.15.

2.3 Quantitative Analysis

2.3.1 Calibration

Most parameter values of the benchmark economy are set following the literature or exogenously estimated from the data. First, I cluster the Core countries (Germany, France, Netherlands) and the Periphery countries (Portugal, Greece, Italy, Spain, Ireland) of Eurozone. Then I estimate the stochastic processes for the outputs of these groups from their time series. I assume that the stochastic processes of these two groups are independent and follow a log-normal AR(1) process $\log y_t = \rho \log y_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$.[6] The stochastic process is discretized into an independent Markov Chain by using Tauchen & Hussey (1991). Furthermore, the differences between Core and Periphery are not only in their income process, but also the Core is reacher than the Periphery by 20% on average according to Eurostat data. A period in the model refers to a quarter, and the risk free interest rate is set equal to 1.7% as in Arellano (2008). The utility function displays a constant coefficient of relative risk aversion form,

[6] In future work, I will examine the spill-over effects that may be generated by introducing correlation in the endowment processes of the two countries.

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \text{ with } \sigma \neq 1$$

The risk aversion coefficient σ is set to 2, which is a common value used in real business cycle studies. All the Eurobonds proposal had some form of borrowing limit for the member states in order to mitigate moral hazard concerns. For this reason, I set an exogenous borrowing limit, of 66% debt-to-income ratio for the Core and of 83% for the Periphery.[7] As in Arellano (2008), I assume that default entails some direct output cost of the following form:

$$g(y_i) = \begin{cases} \gamma E(y_i) & , \text{ if } y_i > E(y_i) \\ y_i & , \text{ if } y_i \leq E(y_i) \end{cases} \tag{2.16}$$

where γ is the exogenous output cost that I set equal to 0.96, as in Arellano (2008). Finally, I calibrate the discount factors of the benchmark model to match the sovereign spreads of Core and Periphery and I set them equal to 0.89 and 0.88, respectively.

Table 2.2 presents some results on the performance of the benchmark models in comparison with the data. To derive the business cycle statistics, I run many simulations of the model over time until a default occurs and I evaluate the mean statistics of these simulations.

The model matches relatively well the spread for both countries. It predicts that the mean interest rate spread for the Core is 0.5%, while in the

[7] I am in the process of relaxing this assumption. I am solving for the economy that has no exogenous borrowing constraints.

Table 2.1: Calibration

	Values	Target
Risk aversion	$\sigma = 2$	Arellano (2008)
Output cost after default	$\gamma = 0.96$	Arellano (2008)
Risk free interest rate	1.7 %	Arellano (2008)
Core's income process	$\rho = 0.96$, $\sigma_\epsilon = 0.003$	Data
Periphery's income process	$\rho = 0.92$, $\sigma_\epsilon = 0.004$	Data
Output difference	$\bar{y}_c/\bar{y}_p = 1.2$	Data
Core's borrowing limit	66%	Treaty
Periphery's borrowing limit	83%	Treaty
Calibrated parameters		
Core's discount factor	$\beta_c = 0.89$	0.6% spread
Periphery's discount factor	$\beta_p = 0.88$	2.4% spread

data is 0.6%. The model is less successful for the Periphery, since it generates a mean interest rate spread of 1.9%, while in the data is 2.4%. Moreover, the model has an exogenous debt-to-output ratio to match the data.

The model predicts lower volatility than the data. The volatility of the interest rates for the Core is 0.9 % and the Periphery is 2.16% in the data; the model under-predicts the volatility for both countries, since for the Core is 0.019% and the Periphery is 0.042%.

2.3.2 Results

This section first analyzes the policy functions of the benchmark and the Eurobonds models and then examines the quantitative performance of the Eurobonds model in comparison with the benchmark model.

Table 2.2: Business Cycle Statistics: The Benchmark Model and the Data

	Data		Benchmark	
mean(%)	Core	Periphery	Core	Periphery
Debt/Y	66	83	66	79
Spread	0.6	2.4	0.5	1.9
C/Y	77	80	98.8	98.6
std(%)				
Debt/Y	8.10	19	0.16	0.36
Interest rate	0.9	2.16	0.019	0.042
C/Y	1.15	1.9	0.14	0.15

The introduction of joint-liability bonds generates two opposing forces. On the one hand, this instrument generates a contagion effect, in the sense that it may push some countries to default even though they are individually solvent. On the other hand, joint-liability bonds may create cheaper access to credit, since insuring other countries allows for lower rates.

Figure 2.2 shows the effects of introducing Eurobonds and having cheaper access to the financial markets for the Core countries. It compares the spread that is generated by the benchmark and the Eurobonds model. The Eurobonds' price depends also on the Periphery's debt, which is fixed to 55% debt-to-output ratio. When the Core has below 53% debt-to-output ratio, there is no positive externality from the introduction of Eurobonds, since if the Core defaults the Periphery would also be dragged to default with high probability. It would have been very expensive for the Periphery to cover the Core's inherited debt, thus there is no significant effect on the spreads. However, in the region 53%-47%, the Periphery would be willing to cover the inherited debt if the Core defaults, since the Periphery inherits a lower amount of debt. Hence, the Periphery would not default and for this reason the spread decreases and the Core receives a positive externality. In the region above 47%, the Core would not default neither in the benchmark nor in the Eurobonds model, for this reason there is no difference between the two models.

Figure 2.3 shows the effect of introducing Eurobonds and having cheaper access to the financial markets for the Periphery countries. As in figure 2.2, the Eurobonds' price depends also on the Core's debt, which is fixed to 50%, and it compares the spread that is generated by the benchmark and the Eurobonds model. When the Periphery has above 53% debt-to-output ratio, there is no positive externality from the introduction of Eurobonds, since the Periphery would not default neither in the benchmark nor in the Eurobonds model. However, below 53% the Periphery receives lower spreads in the Eurobonds model, because of the insurance mechanism of the Eurobonds. If the Periphery defaults, the Core will cover the inherited debt with high probability, for this reason international lenders are willing to buy bonds at a relatively lower interest rate. As the level of the Core's inherited debt increases the probability, that Core has to payoff the debt, decreases and for this reason the spread increases. It is clear from figure 2.2 and 2.3 that the price effects for the Periphery are bigger than the Core, nonetheless Core also receives some positive externalities by issuing debt with joint-liability.

Figure 2.4 and 2.5 compare the changes on the repayment policy functions for both the Benchmark and the Eurobonds model, given a certain combination of income level of Core and Periphery. The x-axis and y-axis measure the level of asset holding over the average income level for Core and

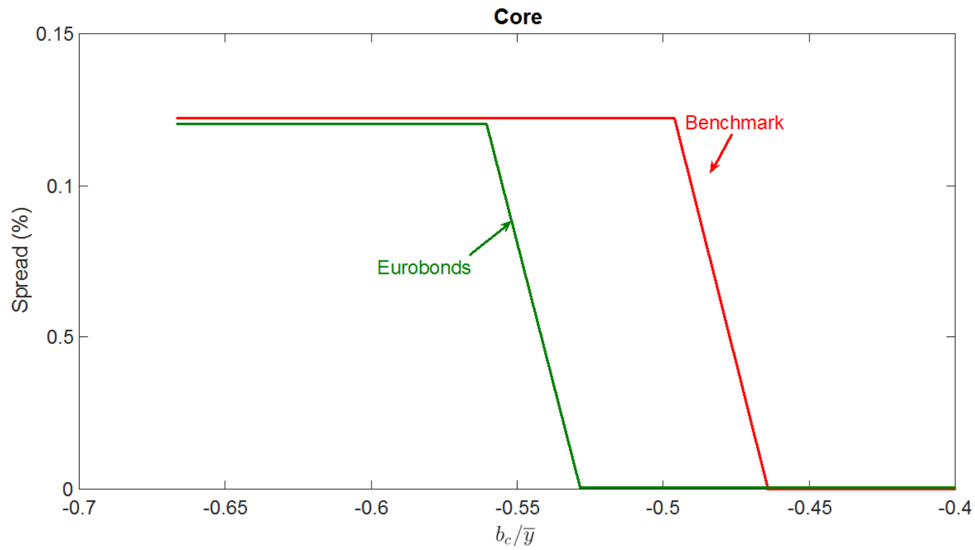


Figure 2.2: Core's spread in the Benchmark and the Eurobonds model, for the same level of debt-to-output ratio and income realization. Periphery's debt-to-output ratio is fixed to 55%.

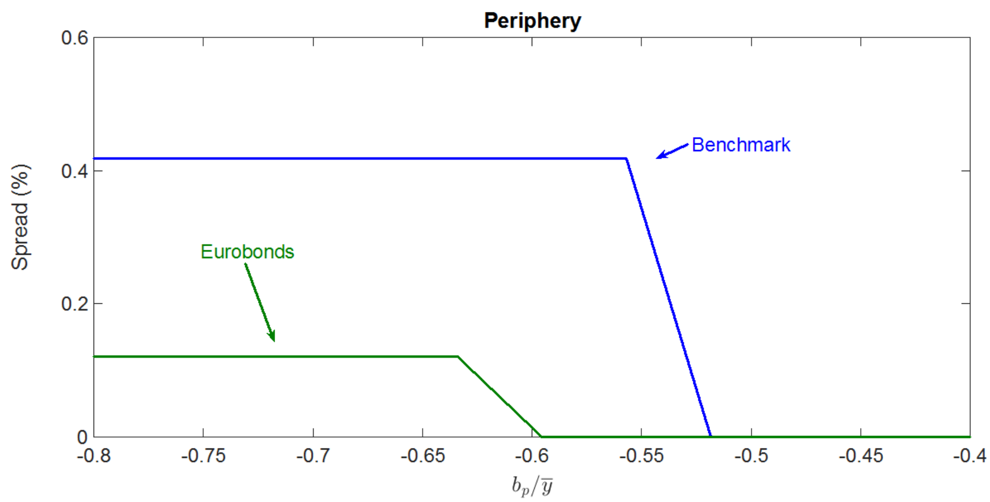


Figure 2.3: Periphery's spread in the Benchmark and the Eurobonds model.

For the same level of debt-to-output ratio and income realization. Core's debt-to-output ratio is fixed to 50%.

Periphery, respectively. The red region represents the combination of the inherited asset levels for which both countries decide to default simultaneously, as in Arellano & Bai (2014a). In the dark green area both choose to repay, while in the light green area I come across with multiple pure strategy Nash equilibrium on the repayment decision and countries choose either to repay or default. In case of multiple equilibrium, I choose by assumption the outcome that yields the highest aggregate welfare, which is the scheme that both economies repay simultaneously. The dark blue region shows the synthesis of asset level for which Core defaults while Periphery repays the sum of Eurobonds and then issues individual bonds. Finally, in the white area no pure strategy Nash Equilibrium exists. For this reason, I solve for the unique mixed strategy Nash Equilibrium of the repayment strategy. The solid yellow lines exhibit the threshold at which countries would default below that level of asset for a specific income realization in the benchmark model.

The analysis of repayment policies explains which of these two opposing effects dominates in the Eurobonds model in comparison with the benchmark model. Figure 2.4 shows the repayment policy functions when the Core and the Periphery face the lowest possible income realization, 4% below the trend of each country. In this figure, the contagion effect dominates, since after the introduction of Eurobonds the region that both countries default simultaneously is growing. On the left panel is the repayment policy function for the benchmark economy, in which there is no form of joint liability or any strategic interaction among the countries. Below the horizontal yellow line the Core countries default while below the vertical yellow line the Periphery countries default. Below the vertical and the horizontal yellow line both

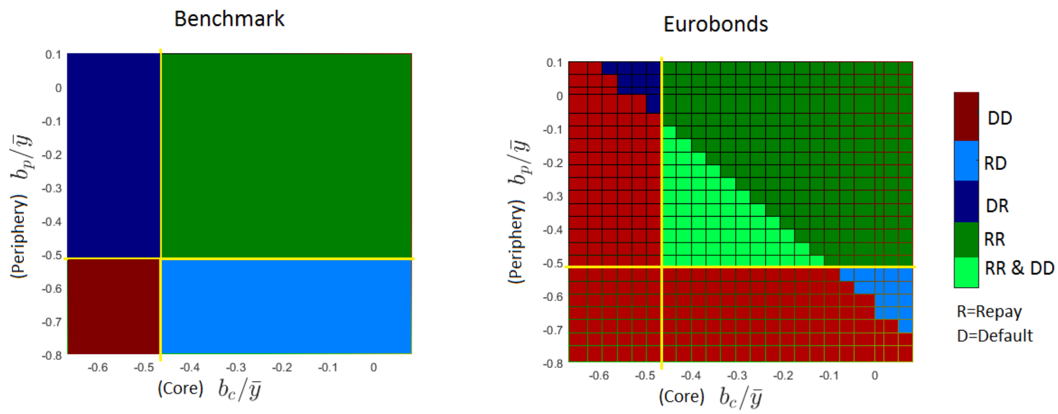


Figure 2.4: Policy function for repayment.

Both countries have a deep recession, 4% below s steady state. DD is when both countries default, RR both countries repay, RD is when Core repays and Periphery defaults, DR is when Periphery repays and Core defaults.

countries choose to default simultaneously while the vertical and the horizontal yellow line above both countries are solvent. On the right panel is the repayment functions for the Eurobonds economy, where countries issue assets with joint liability and they interact strategically on their repayment and borrowing decisions. In this particular case, the contagion effect dominates, since there are regions in which countries choose to default even though they would not in the benchmark economy. The area above the vertical yellow line and below the horizontal yellow line turns from blue in the benchmark to red in the Eurobonds. Here the result is driven by the fact that in this region the Periphery prefers to be insolvent while the Core inherits the sum of Eurobonds. Hence, the Core does not have the means to pay the whole sum of Eurobonds and it is dragged to the default region.

Figure 2.5 presents the repayment policy functions for which the cheaper access to credit effect dominates. In this figure, the cheaper credit effect dominates, since after the introduction of Eurobonds the region that both countries repay simultaneously is growing. The Core faces an income realization of 2% below its trend, while the Periphery has 4% below its trend (same as figure 2.4). On the left panel is the repayment policy function for the benchmark economy and the threshold for the Periphery is the same as in the previous figure although the Core now does not default. Here, the lower interest rate effect dominates since there is a region below the horizontal yellow line that turns from blue in the Benchmark model to dark green in the Eurobonds model. This happens because the Periphery takes advantage of the relatively better income realization of the Core economy and receives a better interest rate. Hence, the Periphery has less incentives to default in this environment with joint liability.

Table 2.3 shows some quantitative predictions of the Eurobonds model. To derive the business cycle statistics, I run many simulations over time and report the mean, until at least one of the countries defaults in the Eurobonds. I use the same parameters as in the benchmark economy, to examine the effects after the introduction of joint liability bonds.

The Eurobonds model predicts that interest rates will decrease significantly in the long run, not only for the Periphery but also for the Core, because of the cheaper credit effect. I conduct two experiments for the Eurobonds model, *(i)* two asymmetric countries (i.e. Core and Periphery) and *(ii)* two symmetric countries (i.e Core and Core). Both experiments predict lower mean spread than the benchmark model. In particular, in both ex-

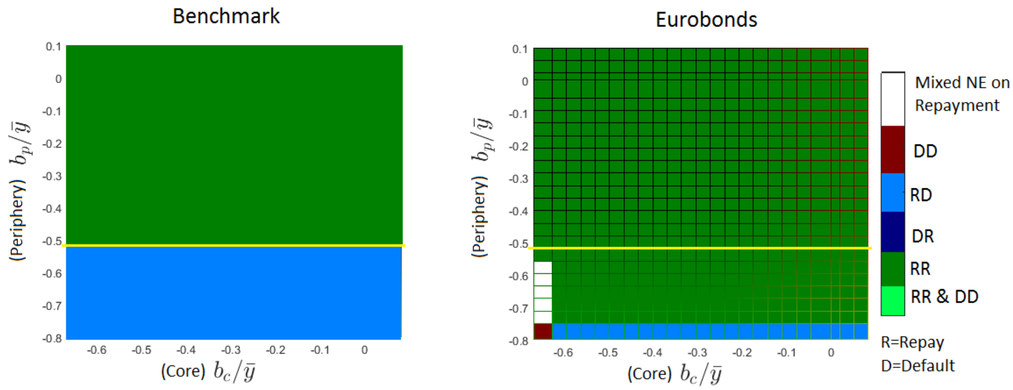


Figure 2.5: Policy function for repayment.

Core has a mild recession and Periphery has a severe recession, 2% and 4% below steady state, respectively. DD is when both countries default, RR both countries repay, RD is when Core repays and Periphery defaults, DR is when Periphery repays and Core defaults.

periments the mean interest rate spread drops to 0.1% for both countries, while in the benchmark economy it was 0.5% for the Core and 1.9% for the Periphery.[8] Moreover, the volatility of interest rates reduces significantly in the Eurobonds model. The volatility of interest rate drops to 0.002% and 0.001% for the asymmetric and symmetric case respectively, while in the benchmark economy it is 0.019% for the Core and 0.042% for the Periphery. It is important to mention that the debt-to-output ratio is the same in the benchmark and Eurobonds models due to the exogenous borrowing limit, therefore there is no need for comparison. However, the goal of this paper is not only to forecast the effects on the spreads per se, but also the potential

[8]Both countries receive the same interest rate, since they issue debt with joint liability. The interest rate in the Eurobonds model reflects the probability that both countries will be insolvent simultaneously.

consequences of the Eurobonds on the countries' welfare, as it will be shown in the next section 2.3.3.

Table 2.3: Business Cycle Statistics: The Eurobonds Model

	Asymmetric		**Symmetric**
mean(%)	Core	Periphery	Core
Debt/Y	66	79	66
Spread	0.1	0.1	0.1
std(%)			
Debt/Y	0.035	0.034	0.029
Interest rate	0.002	0.002	0.001

2.3.3 Welfare Effects of introducing Eurobonds

I first solve for the benchmark economy in which there is no form of joint liability. Then, I measure the welfare effects of an unanticipated announcement explaining that from now on, Core and Periphery will be forced to issue debt with joint liability and interact strategically on their borrowing and repayment decisions. I measure the welfare effects as the proportional changes of consumption that would leave the consumer indifferent between living in the benchmark environment or in the Eurobonds environment, given the stationary ergodic distribution of income. This consumption change is given by

$$\lambda_i = \left(\frac{V_i^E(S)}{V_i(b_i, y_i)} \right)^{\frac{1}{1-\sigma}}$$

where V_i^E and V_i denote the value functions with and without joint liability, respectively.

Figure 2.6 shows the unconditional expected welfare effects for both countries in the asymmetric environment. The Core's debt-to-output is fixed to 50% and countries have welfare gains above the zero line, otherwise they have losses. The Core is getting better off as the Periphery's debt-to-output ratio decreases. This is happening for two reasons, first the Core receives a better price because the Periphery has a lower debt-to-output and therefore a lower probability to be insolvent. Second, the contagion effects decreases, thus the Core has smaller negative externalities if the Periphery defaults. On the other hand, the Periphery is overall better off after the introduction of Eurobonds. More specifically, when the Periphery's debt-to-output ratio is between 80% and 54%, the Periphery faces welfare improvements as the debt level decreases, since the country inherits lower level of debt. However, when the Periphery has a debt-to-output ratio below 55%, the expected welfare effects are getting stagnant, because the Periphery faces negative externalities from the fact that the Core has a relatively high debt-to-output ratio. When the Periphery has a debt-to-output ratio below 38% there is a Pareto improvement, since both countries have welfare gains. The model also predicts that Eurobonds should not have been implemented when they were suggested at the peak of the Eurozone debt crisis. At that time, most of the Eurozone member states, especially the Periphery members, had relatively high debt-to-output ratios. Moreover, as the model predicts, the Periphery member states were in favor of Eurobonds while the Core were not. Nonetheless, the model foresees that when the Periphery has relatively low debt-to-output ratio, all the member states are better off with the introduction of Eurobonds. Hence, if Eurobonds had been introduced before the financial crisis, when

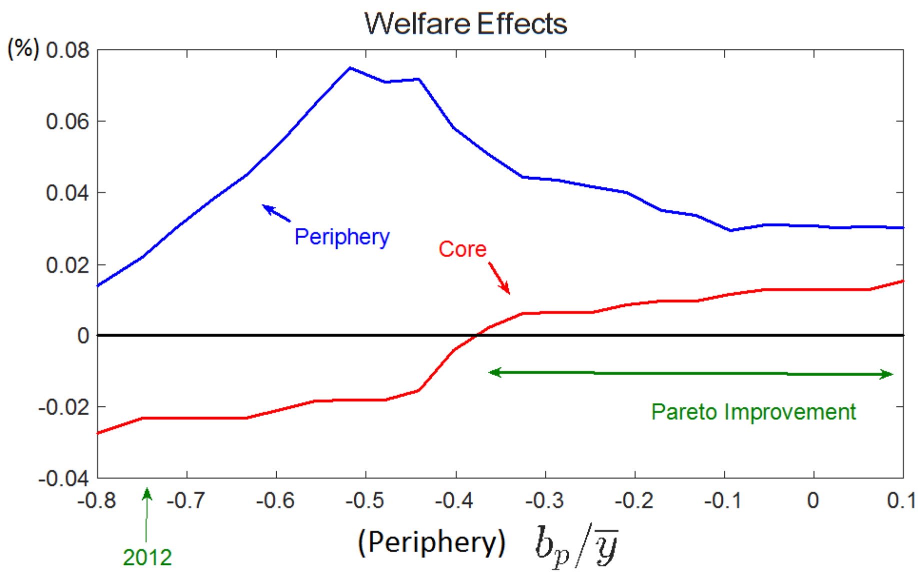


Figure 2.6: Welfare Gains.

Above the zero line countries face welfare gains, while below welfare losses. Core's debt-to-output ratio is fixed to 50%.

almost all the member states had low level of debt, then it would have been beneficial for all the member states.

Figure 2.7 presents the expected Pareto Effects in the asymmetric environment. In contrast with figure 2.6, where the Core has a fixed level of debt, this figure examines the Pareto Effects for all the possible asset combinations. The green region shows all the asset combinations for which there exist a Pareto Improvement. As it is explained in figure 2.6, in order to have Pareto Improvement the Periphery countries should have a relatively low level of debt. Otherwise, there is a Pareto loss, mostly because the Core countries have welfare losses.

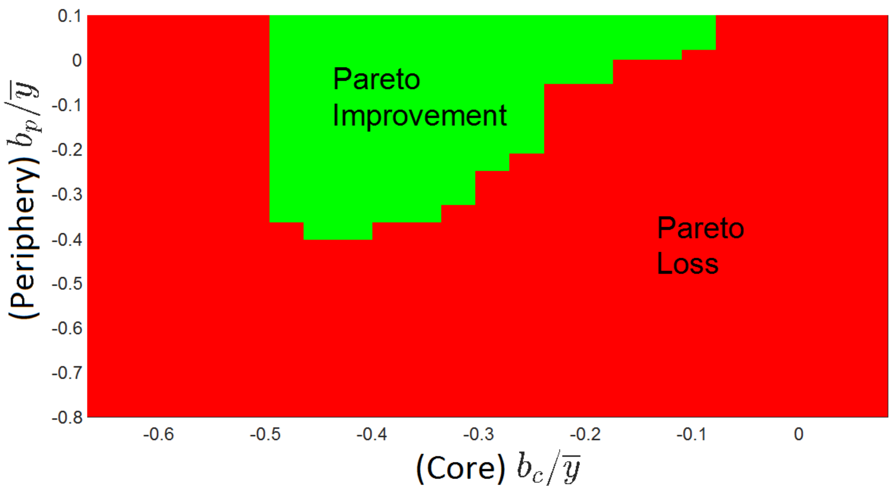


Figure 2.7: Expected Pareto Effects in the Asymmetric environment, for all the possible asset combinations.

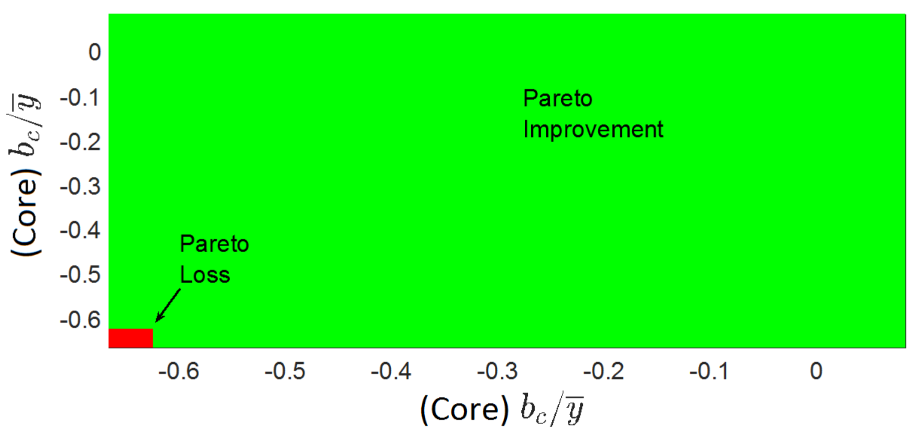


Figure 2.8: Expected Pareto Effects in the Symmetric environment, for all the possible asset combinations

Figure 2.8 shows the expected Pareto effects in the symmetric environment for all the possible asset combinations. As in Tirole (2015), as the countries get more symmetric the welfare improvements are bigger after the introduction of bonds with joint liability. In particular, the Core member states would be better off if they had a Eurobonds agreement with symmetric countries instead of the Periphery countries. For example, Germany would be better off if it had a Eurobonds agreement with France instead of Spain. The main force for this result is the fact that the contagion effect is smaller in comparison with the asymmetric case. Moreover, table 2.3 shows that there is no significant difference on the average spread between the asymmetric and symmetric environment, thus the cheaper credit effect is similar in both environments.

It is likely that this model may underestimate the welfare gains from lowering the sovereign spreads mainly for two reasons. Firstly, lower sovereign spreads lead to better allocation of factors of production and therefore could create significant positive effects as in Mendoza & Yue (2012). Secondly, they decrease the probability of a credit crunch and/or a banking crisis as in Sosa-Padilla (2015) and Bocola (2014). In light of these findings, gains from introducing joint-liability bonds may be larger than the ones I compute.

2.4 Conclusion

Europe faces the dilemma of whether to step forward to a higher degree of unification. This paper develops and analyzes a Eurobonds model where two small open economies issue bonds with full joint liability and interact strategically on their borrowing and repayment decisions. I compare this to the benchmark economy, which builds on a standard default model as in Arellano (2008), under two different scenarios. In the first scenario countries are asymmetric, one country is wealthier and less volatile than the other (i.e Core and Periphery member states of Eurozone), while in the second scenario there are two symmetric countries (i.e Core and Core). The findings show that in both scenarios Eurobonds decrease the yields of sovereign debt for all the member states in the long run. Nonetheless, the welfare consequences in the asymmetric scenario hinge critically on the timing of its introduction. More specifically, introducing such an instrument at the peak of the Eurozone crisis would have brought welfare gains for the Periphery member states and losses for the Core member states. However, adopting Eurobonds in "normal times", when member states have relatively lower debt-to-output ratios, has the potential to make all participants better-off. In the symmetric scenario, the implementation of Eurobonds produces welfare gains for all participants.

A natural extension of the model with the asymmetric scenario would be the analysis of whether or not member states would be willing to take austerity measures to reduce the current high debt-to-output ratios. This would allow the member states to reach the debt-to-output ratio levels at which all participants would be better off with the introduction of Eurobonds. Moreover, it would be newsworthy to explore the option of a joint liability mechanism that allows for bailing-out insolvent participants, as in Azzimonti & Quadrini (2016). In particular, in this environment countries will make transfers in order to decrease the default incentives. This mechanism has the potential to generate not only less default, but also reduce the contagion effect. Finally, it would be interesting to examine the case which countries are not forced to permanent financial autarky and they are permitted to issue debt after a few years of the default incident. We leave these for future research.

2.A Appendix

2.A.1 Additional Results for the Effects of Introducing Eurobonds

Figure 2.9 presents next period's expected inherited debt for Core and Periphery in the asymmetric experiment, when Core's debt-to-output ratio is fixed to 50%. If the model does not allow for default, Core's expected inherited debt would have been an horizontal straight line and the 45 degree line for the Periphery. However, in this model countries are allowed to default and inherit zero debt. For this reason, when Periphery has low debt-to-output ratio the expected inherited debt for Periphery is the 45 degree line and for the Core the expected inherited debt is the horizontal line. Nonetheless, Periphery defaults more frequently as the debt increases, thus below -0.75 Periphery's expected inherited debt line is getting flatter and Core's expected inherited debt is getting larger. Hence, this figure shows the negative externalities that Core countries receive from the fact that inherit higher level of debt.

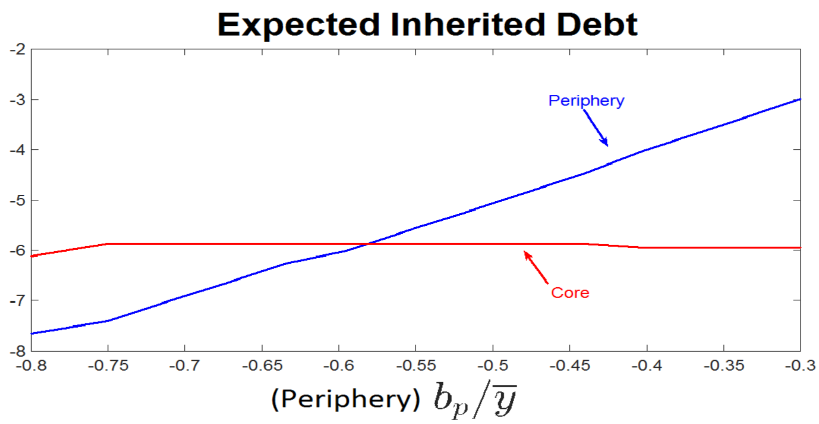


Figure 2.9: The Expected Inherited Debt for Core and Periphery, while Core's debt-to-output ratio is fixed to 50%.

Figure 2.10 shows the unconditional expected welfare effects of introducing joint liability bonds in the asymmetric environment. On the right graph is Periphery's welfare effects after the introduction of Eurobonds. As I discussed previously there are two opposing forces, the effect of cheaper credit and the contagion effect. The introduction of Eurobonds brings welfare gains for Periphery mainly because the cheaper credit effect dominates for all the combination of assets between Core and Periphery.

The left panel of figure 2.10 presents Core's welfare effects after the introduction of Eurobonds. The welfare effects are mixed and they depend on the asset combination of the Core and Periphery. When Core and Periphery have relatively high debt-to-output ratio, the contagion effect dominates and Core is worse off. However, when Periphery has relatively low debt-to-output ratio then Core has welfare gains due to the cheaper credit effect. At this point I would like to mention that the combination of the left and right panel of figure 2.10 generates the Pareto Effects that are shown on figure 2.7.

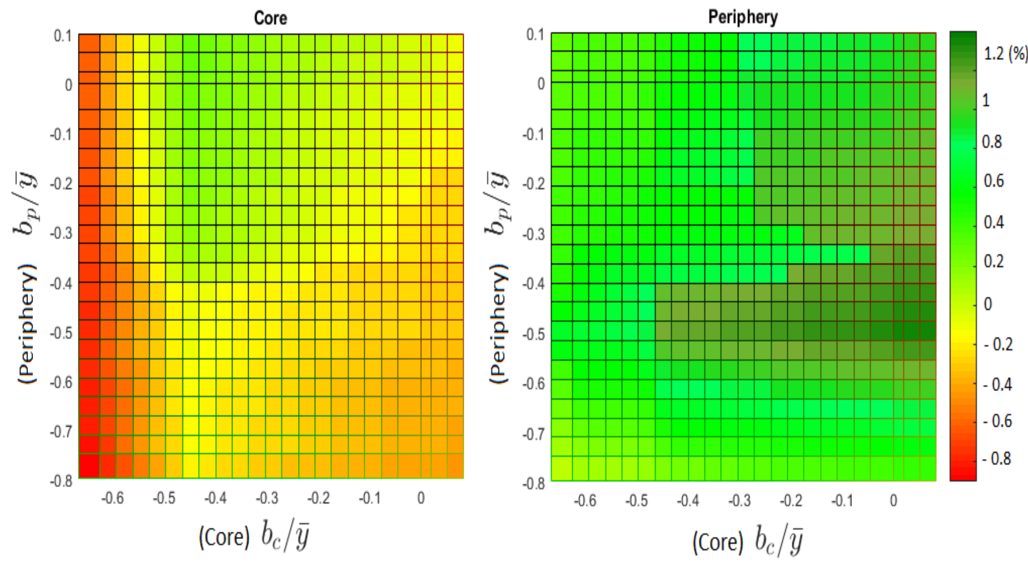


Figure 2.10: Welfare effects after the introduction of Eurobonds (Core & Periphery). Dark green and Red represent the welfare gains and losses, respectively.

Figure 2.11 performs the same experiment as in figure 2.10 for the symmetric environment. In this environment countries are better off because the contagion credit effect is smaller than the asymmetric case, thus countries will be more willing to issue debt with joint liability at any combination of assets. As I explained in the section 2.3.3 the cheaper credit effect is similar in the symmetric and asymmetric environment. The combination of this figure for both participating countries in the symmetric environment derives the Pareto Effects of figure 2.8.

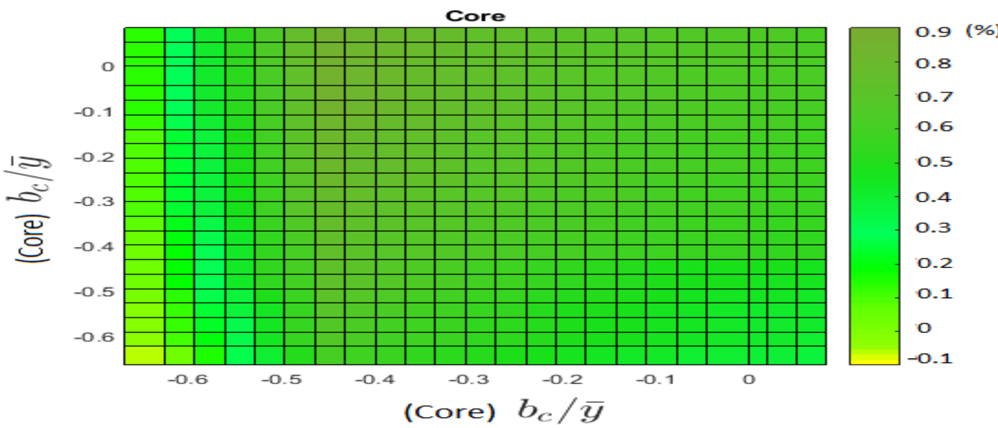


Figure 2.11: Welfare effects after the introduction of Eurobonds (Core & Core).

2.A.2 Computational Algorithm

The following algorithm is used to solve the Benchmark and Eurobonds models:[9]

1. Discretize the state space for assets $b = (b_1; b_2)$ consisting of a grid of 1600 points equally spaced and the endowment space $y = (y_1; y_2)$ into 25 pairs using Tauchen & Hussey (1991) method.

2. Solving the Benchmark model for the two countries separately (following Arellano (2008))

   (a) Start with some guess for the parameters to be calibrated: β_i and γ.

[9]It is important to compute first the Benchmark model separately and then use the equilibrium parameters to compute the Eurobonds model.

(b) Start with a guess for the bond price schedule such that $q_i^0(b_i', y_i) = 1/(1+r)$ for all b_i' and y_i.

(c) Given the bond price schedule, solve the optimal policy functions $c_i(b_i, y_i)$, asset holdings $b'(b_i, y_i)$, repayment sets and default sets $A_i(b_i)$ via value function iteration. I iterate on the value function until convergence for a given q_i^0.

(d) Compute business cycles statistics from 3,000 simulations that each have 3,000 periods. If the model business cycles match the data we stop, otherwise we adjust parameters, and go to step 2.a.

3. Solving the Eurobonds model, given the parameters and the equilibrium outcomes of the Benchmark model:

   (a) Given the price schedules $\{q_i(b_i, y_i)\}_{\forall i}$ and the value functions $\{W_i^{def}(y_i),\, V_i(b_i, y_i)\}_{\forall i}$ from the Benchmark models.

   (b) Derive the pay-off functions $W_i^{rd}(S)$ $\forall i$, this is the scenario that country i repays while country i^- defaults.[10]

   (c) Start with a guess for the eurobonds price schedule such that $q_E^0(S) = 1/(1+r)$ for all the possible combinations of b_i' and y_i.

   (d) Given the price schedules $\{q_E^0(S), q_i(b_i, y_i)\}_{\forall i}$ and the pay-off functions $\{W_i^{def}(y_i),\, V_i(b_i, y_i), W_i^{rd}(S)\}_{\forall i}$. To solve for the value function and the intra-period Nash Equilibrium, for a given price schedule, the following algorithm is being used:

[10] $W_i^{rd}(S)$ is one shot problem since in the first period country i has to pay the sum of Eurobonds and then continues as in the Benchmark model, without issuing assets with joint liability.

i. Assuming that both countries choose to repay, I solve for the pay-off function $\{W_i^{rr}(S; b'_{i^-})\}_{\forall i}$, and the best response of debt policy function $\{b_i^{BR'}(S; 0, b'_{i^-})\}_{\forall i}$, for all the arbitrary next period asset decisions of country i^-, given that the country i^- is solvent.

ii. Given the best response debt policy function for all the arbitrary debt decisions of the other country. I solve for the fixed point that yields the optimal best response of asset and repayment decisions $\{h_i^{BR'}(S; h_{i^-}^{BR'}, b_{i^-}^{BR'}), b_i^{BR'}(S; h_{i^-}^{BR'}, b_{i^-}^{BR'})\}_{\forall i}$ and update the value function for Eurobonds, $\{V_i^E(S) = \tilde{V}_i^E(S; \{h_i^{BR'}, b_i^{BR'}\}_{\forall i})\}_{\forall i}$. I iterate on the the value function for Eurobonds until convergence for a given $q_E^0(S)$.

(e) Given the optimal best response for repayment of both countries I update the price schedule of Eurobonds $\{q_E^0(S)\}$, and go to step 3.d until convergence.

Chapter 3

Optimal Capital Taxation with Risk Aversion Heterogeneity

3.1 Introduction

This paper introduces heterogeneity in risk aversion, in an otherwise standard macroeconomic heterogeneous agent model, with the purpose to examine the effects on the optimal capital taxation. Cozzi (2012) shows that heterogeneity in risk aversion is quantitatively important on the aggregate allocations. Therefore, when I introduce heterogeneity in risk aversion, I should expect different level of optimal capital taxation for two reasons (i) different level of aggregate allocations, and (ii) preferences.

One of the most controversial debates that the policymakers have relates to the level that they should tax capital and labor in order to finance the governments budget. The main concern of the policymakers is to decide which group of the population will bear the highest percentage of the tax burden. To give a potential answer to this debate, I developed a DSGE model that nests heterogeneous groups of population, by introducing heterogeneity in risk aversion and income, in order to estimate the desired combination of capital and labor taxes.

This paper is related to several studies that have focused on optimal taxation. The seminal papers of Judd (1985) and Chamley (1986) estimate that it is optimal to eliminate any level of capital taxation. More recent papers, Atkeson *et al.* (1999) and Chari & Kehoe (1998) relaxed some of the assumptions, of these two seminal papers, and they also conclude that in the long run the optimal capital income taxation should be equal to zero. However, Aiyagari (1995) and Domeij & Heathcote (2004) examine the optimal capital income taxation under a set up with tight borrowing limit and an uninsurable income shocks and they find that taxes should be positive in the long run. Their conclusion is driven by the fact that when I introduce heterogeneous households then they tend to over-accumulate assets in order to secure themselves against idiosyncratic risks and the effect of the wealth redistribution. Àbrahàm & Càrceles-Poveda (2010) introduce endogenous borrowing constraints, while their conclusions are similar to Domeij & Heathcote (2004). However, these papers do not consider that there exist a dispersion on risk aversion among individuals while it affects the aggregate allocations and therefore the desired level of taxes.

This paper is also related to the literature that focuses on heterogeneous preferences. Krusell & Smith (1998) and Coen-Pirani (2004) examine the income wealth inequality by introducing to the model heterogeneous discount factor and risk aversion respectively. Guvenen (2009) tries to solve the equity premium puzzle by considering heterogeneity in risk aversion. However, these papers use an arbitrary preference heterogeneity. Cozzi (2012) is the first paper that adopts a more structural method to estimate the heterogeneity in preferences by using estimates from empirical papers. In this paper, I follow the same approach as in Cozzi (2012) in order to examine the optimal level of capital taxation.

There is a plethora of empirical papers that justify the existence of dispersion in risk aversion among agents. The most notable papers which try to estimate the distribution of risk aversion for US economy, they are using data from Panel Study of Income Dynamics (PSID), Kimball *et al.* (2009) and Health and Retirement Study (HRS), Kimball *et al.* (2008). In addition, there are estimates of risk aversion distribution based on the Italian database Survey of Household Income and Wealth (SHIW), Chiappori & Paiella (2011). Furthermore, Gaudecker *et al.* (2011) estimate the risk preferences in a representative sample from the Dutch with more than 1.400 individuals. In this paper, I use the results from SHIW, Chiappori & Paiella (2011), since these estimations give an average risk aversion equal to 2.23 and the results are much more reasonable than the results of all the other estimations.[1]

The main findings are that the capital should be taxed heavier than in an economy with homogeneous preferences. The results show that the gap

[1] Indicatively, Kimball *et al.* (2009) evaluate an average risk aversion equal to 4.19, which seems to be too high for the model, since the optimal taxation tends to be equal to 100% something that is unrealistic. This insane result is caused due to the reason when I introduce high risk aversion then the precautionary savings are so high that drives agents to save too much even when the taxation is equal to 100%. In addition, the results from the HRS data set are very similar to the result of PSID, while the average risk aversion is 8.2.

between homogeneous and heterogeneous preferences is caused, firstly because of different precautionary motives in the economies. Second, agents with lower risk aversion are in favor of a higher capital taxation since they tend to accumulate less capital.

The rest of the paper is organized as follows. Section 3.2 presents the general equilibrium model. Section 3.3.1 discuss about the calibration that I use for the economy with homogeneous and heterogeneous preferences. Section 3.3.2 analyzes the results and the welfare implication of the reforms for both homogeneous and heterogeneous preferences. Finally, Section 3.4 summarizes and concludes.

3.2 The Model

The economy is populated by a continuum of infinitely lived households ,who are indexed by $i \in I$. In this model, households face an individual productivity shock $\epsilon \in E$ that evolves through the time according to a Markov Chain with a transition matrix Π with dimension $l \times l$, $l < \infty$. Moreover, ex-ante they have a predetermined risk aversion $\gamma \in \Gamma = [\underline{\gamma}, \overline{\gamma}]$, for the rest of their life, and it follows a Gamma distribution according to Chiappori & Paiella (2011).

Let K be the set of all possible values of the household asset wealth. We assume that households are not allowed to borrow therefore $K = \mathcal{R}_+$. Furthermore, I denote X to be the set of all the exogenous aggregate state variables, $X = (\Gamma, E)$, thus X is the set of all possible individual states.

To focus on the effects of the tax changes, I abstract from aggregate productivity shocks or other sources of aggregate risk, and I hold government consumption constant throughout as in Domeij & Heathcote (2004). The time is discrete and indexed by t=0,1,2 . . . .

3.2.1 Household

Agents have heterogeneous additively separable preferences, since they face different risk aversion which is denoted by γ_i and $\gamma_i \in \Gamma$, over a sequence of consumption $c_i = \{c_{it}\}_{t=0}^{\infty}$ in the form of:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_i(c_{it}) \tag{3.1}$$

where $\beta \in (0, 1)$ is the discount factor and the $\mu(.)$ are the transition probabilities that are derived by the transition matrix Π. The period utility function $u(.) : \mathcal{R}_+ \rightarrow \mathcal{R}$ and is assumed to be strictly increasing, strictly concave and satisfies Inada conditions. The momentary utility function is CRRA:

$$U_i(c_{it}) = \frac{c_{it}^{1-\gamma_i}}{1-\gamma_i} \tag{3.2}$$

Each period, households trade in a complete set of assets, however they can not fully insure against idiosyncratic productivity shocks, due to the fact that they face an exogenous borrowing constrain. Let t=0 denote the date of the tax change to the new permanent proportional tax τ^k, where τ^k is the tax rate on asset income and τ^n the tax rate on labor income. Household can use their after tax income to purchase consumption goods or to purchase additional stocks, the real pretax return in period t to one unit of asset purchased at $t-1$ is r_t. The real return to one unit of effective labor at date t is w_t.

Therefore the household budget constrains are given by:

$$c_{it} + k_{it+1} = [1 + (1-\tau^k)r_t]k_{it} + (1-\tau^n)w_t\epsilon_{it}\overline{n} \tag{3.3}$$

$$k_{it+1} \geq 0 \tag{3.4}$$

$\forall\ \epsilon^t \in E^t$ and $\forall t \geq 0$.

Taking as given the sequence for prices $\{r_t, w_t\}_{t=0}^{\infty}$ and the taxes τ^k, τ^n, the solution to the HH's problem is a set of choices k_t,$\forall t$ and $\forall \epsilon^t \in E^t$ such that $k_t \in K = \mathcal{R}_+$ maximizes 3.1 s.t. 3.3 & 3.4.

Aggregate Variables

From date 0 and forward each HH's productivity evolves differently according to the Markov chain. We can normalize the aggregate Labor supply to be equal to 1, while the aggregate assets will be:

$$K = \int_K \sum_{x \in E \times \Gamma} \lambda(k,x) \cdot g_k(k,x)\, dk \tag{3.5}$$

$$N = \int_K \sum_{x \in E \times \Gamma} \lambda(k,x) \cdot \overline{n}\, dk \tag{3.6}$$

where, $\lambda(k, \epsilon, \gamma)$, are the elements of the stationary distribution and $g_k(k,x)$ is the policy function for assets.

3.2.2 Firm

The representative firm hires the aggregate capital stock K_t and labor and combines those two inputs to produce the total output by using a constant return to scale Cobb-Douglas production function

$$Y_t = F(K_t, N) = K_t^{\alpha} N^{1-\alpha} \quad t \geq 0 \tag{3.7}$$

where $\alpha \in [0, 1]$

3.2.3 Government

In each period t government consumes an exogenous constant amount G and it makes no transfers. Government levies taxes from labor and asset holdings. We fix capital taxes and I adjust the labor taxes such that government budget constraint holds, thus I have:

$$\tau^k r_t K_t + t_t^n w_t N = G \quad t \geq 0 \tag{3.8}$$

3.2.4 Welfare Measure

Let c_{ti}^R be the consumption for each individual after the reform took place and c_{ti}^{NR} be the consumption for each individual in the case that there is no-reform. Moreover, I denote with Δ_{x_0} to be the welfare gains for each individual. Therefore, for each individual I will have:

$$\sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t U_i(c_{ti}^R)\pi(e^t) = \sum_{t=0}^{\infty} \sum_{e^t \in E^t} \beta^t U_i((1 + \Delta_i)c_{ti}^{NR})\pi(e^t) \tag{3.9}$$

Using the equation (3.9) I can derive the average welfare gains for the whole economy which is the Δ and solves the following equation:

$$\int_{A,I}\sum_{t=0}^{\infty}\sum_{e^t\in E^t}\beta^t U_i(c_{ti}^R)\pi(e^t)da\,di = \int_{A,I}\sum_{t=0}^{\infty}\sum_{e^t\in E^t}\beta^t U_i((1+\Delta)c_{ti}^{NR})\pi(e^t)da\,di \tag{3.10}$$

In the next step I calculate the hypothetical value for consumption $\hat{c}_t^R$ each individual would get in the case of reform if each agent consumes the same fraction of aggregate consumption as in the case of no reform. Hence, I will have:

$$\hat{c}_{ti}^R = \frac{c_{ti}^{NR}}{C^{NR}}C^R$$

Where C^{NR} & C^R are the levels of aggregate consumption of non-reform and reform respectively. In addition, let Δ_i^a the hypothetical welfare gains for each individual that she/he would have if the individual had the hypothetical consumption.

$$\sum_{t=0}^{\infty}\sum_{e^t\in E^t}\beta^t U_i(\hat{c}_{ti}^R)\pi(e^t) = \sum_{t=0}^{\infty}\sum_{e^t\in E^t}\beta^t U_i((1+\Delta_i^a)c_{ti}^{NR})\pi(e^t) \tag{3.11}$$

Thus the hypothetical average welfare gains will be Δ^a, and I calculate them by the following equation:

$$\int_{A,I}\sum_{t=0}^{\infty}\sum_{e^t\in E^t}\beta^t U_i(\hat{c}_{ti}^{R})\pi(e^t)da\,di = \int_{A,I}\sum_{t=0}^{\infty}\sum_{e^t\in E^t}\beta^t U_i((1+\Delta^a)c_{ti}^{NR})\pi(e^t)da\,di \tag{3.12}$$

Finally I calculate the distributional component, Δ^d, for which it is true that:

$$(1+\Delta) = (1+\Delta^a)(1+\Delta^d)$$

3.3 Quantitative Analysis

3.3.1 Calibration

Initially, I discretize the distribution of relative risk aversion, in order to achieve that I use the estimates of Chiappori & Paiella (2011). The authors of this paper use Italian data from SHIW and they do not specify exactly the type of the distribution, however they have a proxy for the shape of the distribution. Moreover, they calculate that the median of the risk aversion is at 1.7 and 25% of the population has a relative risk aversion larger than 3.

The distributions that fit better to this specifications are beta and gamma, which are very similar to each other and they give almost identical results. Cozzi (2012) follows a beta distribution, while I am using a gamma distribution $\gamma \sim \Gamma(1.4, 1.57)$ that has mean equal to 2.23. For the discretization of the risk aversion I extract 5 point by using the cdf, more analytically I am splitting the cdf into bins and I am taking the middle point of each bin as the risk aversion for the percentage of the population that is inside this bin.

The points that I have for the relative risk aversion are γ = [0.5, 1.5, 2.5, 3.5, 5.65] with probability $prob(\gamma)$= [0.29, 0.27, 0.18, 0.11, 0.15] for each level of risk aversion respectively. At this point I want to mention that if I use the data from PSID or HRS then I get irrational results and the optimal capital taxation is equal to 100%, for both cases the homogeneous and the heterogeneous preferences. About the other parameters I follow similar calibration as in Domeij & Heathcote (2004).

Table 3.1: Parameters Values

Capital's share α	0.36	**Depreciation rate** δ	0.06
Labor's tax τ^n	0.269	**Capital's tax** τ^k	0.397
	Homogeneous Preferences	**Heterogeneous Preferences**	
Risk aversion γ	2.23	$\gamma \sim \Gamma(1.4, 1.57)$	
Discount factor β	0.946	0.942	

The parameters are calibrated in annual terms in Table 3.1. The capital share of the Cobb-Douglas production function α is set to be 0.36 and the depreciation rate δ is equal to 0.06. The discount factor β is set in order to catch the target Capital / Output ratio of 3.31. For the initial tax rate of capital and labor I choose the same as in Domeij & Heathcote (2004) who are using the method of Mendoza *et al.* (1994)

The household productivity process follows the same parameterization as in Domeij & Heathcote (2004) and the idiosyncratic productivity shock E can take the following values $E = \{0.167, 0.839, 5.087\}$. In addition, for the transition productivity matrix will be impossible to jump from the lowest productivity level to the highest and vice versa, each agent has to pass through the middle productivity level. Therefore the transition matrix Π will be:

$$\Pi = \begin{pmatrix} 0.9 & 0.1 & 0 \\ 0.005 & 0.99 & 0.005 \\ 0 & 0.1 & 0.9 \end{pmatrix}$$

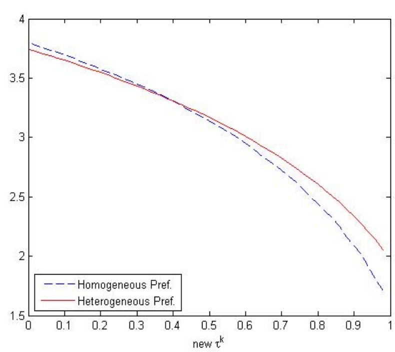
3.3.2 Results

This section analyzes the welfare implications by considering the long run and the short run period. Table 3.2 describes the steady state results of the initial (before any tax change) and the final (after the tax change) for both economies with homogeneous and heterogeneous preferences. Note that this table depicts two final cases, in the first case the new $\tau^k = 0$ for both economies, while in the second case the new τ^k is the optimal taxation for each economy. The two economies have the same initial steady state since they are calibrated to match the capital output (K/Y) ratio of 3.31 and the labor supply is exogenous.

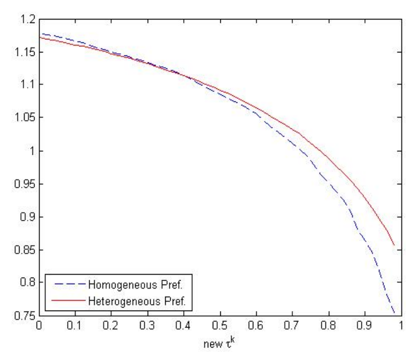
Table 3.2: Aggregate Properties of Initial and Final Steady State

	Homogeneous Preferences			Heterogeneous Preferences		
	Initial	Final	Final	Initial	Final	Final
τ^k	0.397	0.00	0.81 (+104%)	0.397	0.00	0.89 (+124%)
τ^n	0.269	0.34	0.16	0.269	0.34	0.14
G	0.46	0.46	0.46	0.46	0.46	0.46
K/Y	3.31	3.79 (+14%)	2.41 (−27%)	3.31	3.74 (+12%)	2.36 (−28%)
K	6.50	8.05 (+23%)	3.96 (−39%)	6.50	7.87 (+21%)	3.84 (−40%)
r	0.04	0.03 (−25%)	0.08 (+100%)	0.04	0.03 (−25%)	0.09 (+125%)

As it is expected in both economies when I reduce the capital taxation then the aggregate capital is increasing and vise versa. However, the K/Y for homogeneous preference is steeper than the K/Y of heterogeneous preferences at their steady states as I change the capital taxation, see Figure 3.1a. The responses in the heterogeneous preferences are smoother because in this economy there are agents with various risk aversions as a consequence they respond differently to any change of taxation and that makes the outcome flatter. The same explanation holds for the aggregate consumption, see Figure 3.1b.



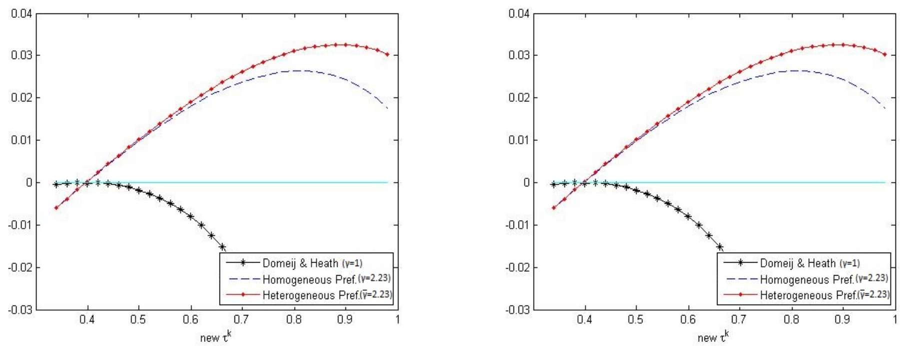
(a) K/Y



(b) **Aggregate Consumption**

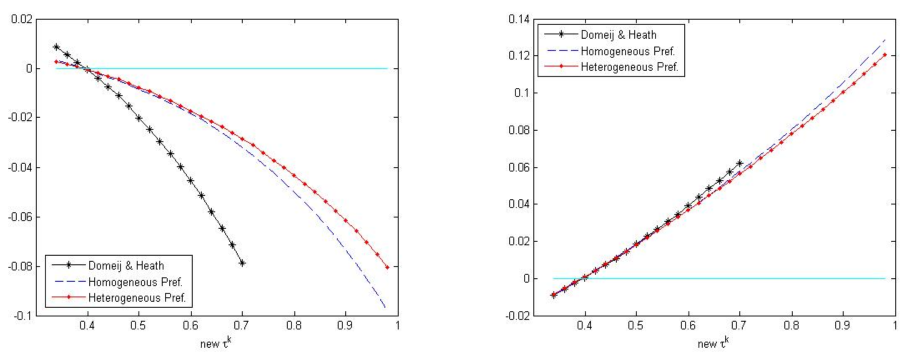
Figure 3.1: Steady State at different taxes

In what follows, I analyze the average welfare gains for both economies, firstly when I change the capital taxation immediately, see Figure 3.2a. Secondly, I examine the average welfare gains under the assumption that there is a delay of one year for the government to impose the new capital taxation, see Figure 3.2b. In the second experiment there is a slight drop of the welfare gains compared to the first experiment. However, the optimal taxation is not affected and in both experiments the results are 0.81 for homogeneous



(a) **Immediate Tax reform** (b) **Delayed Tax reform**

Figure 3.2: Average Welfare Gains



(a) Aggregate Component-Immediate Tax Reform (b) **Distributional Component-Immediate Tax Reform**

Figure 3.3: Welfare effects-Immediate Tax Reform

preferences and 0.89 for heterogeneous preferences. The welfare gains after the immediate reform at the optimal capital taxation is 2.64 percent and 3.25 percent for the homogeneous and the heterogeneous preferences respectively. The first reason for the difference between heterogeneous and homogeneous preferences is caused due to the fact that in the economy with heterogeneous preferences there is higher precautionary savings since there is a non-linearity between precautionary savings and risk aversion as I mentioned in **??**. Similar intuition underlies in Aiyagari (1995) & Domeij & Heathcote (2004). Although, more work needs to be done in order to be able to specify at which level this results is caused by precautionary savings and by how much is affected by the distributional effect. For this reason, I have to solve the no-income risk case at which there is not precautionary savings but only the redistributional effect.

It is clear for Figure 3.2 & 3.3 that the difference between the homogeneous and heterogeneous preferences is getting smaller as the new taxation is closer to the initial tax rate 0.397. The reason for this is because the welfare gains should be equal to zero when the new taxation comes close to the initial taxation.

Moreover, I replicate the results of the homogeneous preferences with risk aversion equal to 1 and the optimal taxation is 0.397, as in Domeij & Heathcote (2004).[2] We can observe that in an economy with homogeneous preferences there is an increase of the optimal capital taxation if I increase the risk aversion from 1 to 2.23 since it boosts from 0.397 to 0.81. Therefore, this result is problematic because the households respond too much with the precautionary savings as I change the value of the risk aversion.

The consumption equivalent depends significantly on the value of the risk aversion, as we notice in Figure 3.4. Nonetheless, not only in the case with low risk aversion but also in the case with high risk aversion, households with lower assets support any reform that increases the capital taxation. This is

[2]This experiment is almost the same as in Domeij & Heathcote (2004) the only difference is that in my model I do not consider any government bond. However, the results are almost the identical as in their paper.

because their dominant source of their income is from their labor supply, therefore they benefit from any reform that reduces the labor taxes. When the risk aversion increases, the households are less willing to increase the taxation due to the reason that households with higher risk aversion tend to have higher precautionary motives, therefore they will be in favor of a lower level of capital taxation. Furthermore, it is important to mention that at the lower levels of risk aversion, Figure 3.4a & 3.4b the households with high productivity level and low assets are benefited more by increasing the capital taxation, than the households who have the same level of asset and productivity shock, since they tend to save less due to lower risk aversion. Note that if all the households in the economy have the same low risk aversion, for example equal to 0.5, then the optimal capital taxation would be bellow 0.397. This is happening because in this hypothetical economy the households do not accumulate so much capital, thus increasing the capital taxation is not favored. However, in the economy with heterogeneous preferences this is not a problem any more since the capital accumulation is high enough, thus imposing such a high taxation is becoming more preferable.

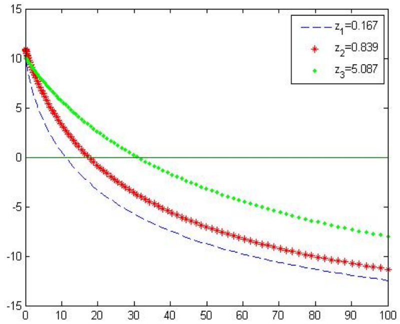
The distribution of the risk aversion is a key element in order to derive the optimal capital taxation. In fact, if we increase the proportion of the population that have lower risk aversion then the optimal taxation will be even greater and vice versa. In Figure 3.5a, I have the weighted average of all the consumptions equivalents of Figure 3.4, while in the Figure 3.5b is the consumption equivalent of the economy with the homogeneous preferences at its optimal taxation which is $\tau^k = 0.81$.

To conclude, the results illustrate that the introduction of heterogeneity in risk aversion alters the optimal capital taxation. The gap between homogeneous and heterogeneous preferences is created for two reasons, first because of different precautionary savings between the two economies and second because, the households with lower risk aversion benefit more since they tend to accumulate less capital and for this reason they prefer higher capital taxation.

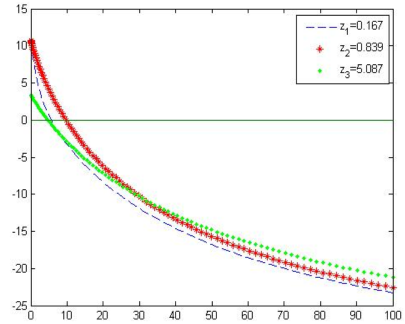
3.4 Conclusions

The present work studies the optimal capital taxation in an economy with exogenous incomplete markets, capital accumulation and heterogeneous preferences. The main findings show that the capital should be taxed heavier than in an economy with homogeneous preferences. The gap between homogeneous and heterogeneous preferences is caused, firstly because of different precautionary motives in the economies. Secondly, in the economy with heterogeneous preferences agents with lower risk aversion are in favor of a higher capital taxation.

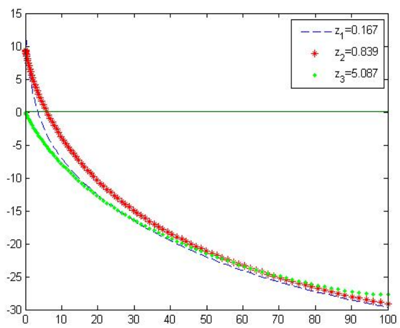
Further research should focus on the examination of the homogeneous case, why the optimal taxation changes so significantly when we shift the risk aversion from 1 to a higher risk aversion. In addition, it would be newsworthy to study optimal capital taxation in a model with occupational choices (entrepreneurs and workers) similar to Cagetti & De Nardi (2006). This might be crucial because by introducing to the model the heterogeneity in risk aversion, the results contradict the conclusions of Kimball *et al.* (2008), who support a negative relation between risk aversion and wealth in stocks.



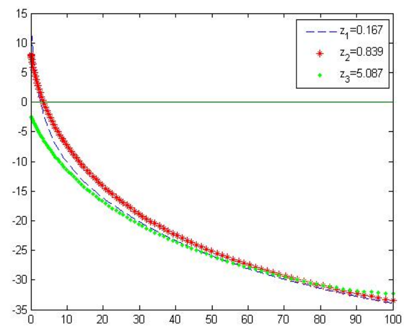
(a) $\gamma = 0.5$



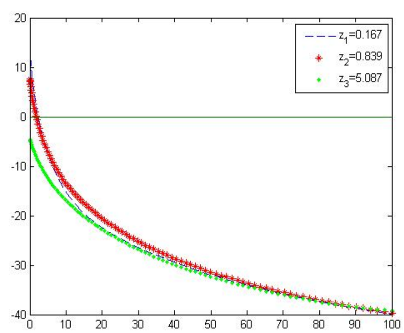
(b) $\gamma = 1.5$



(c) $\gamma = 2.5$

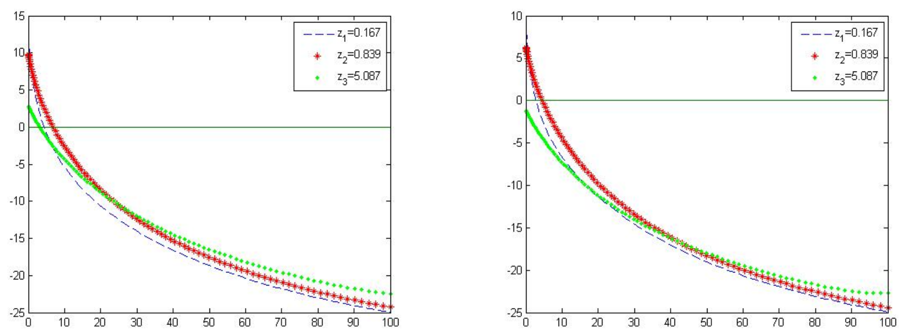


(d) $\gamma = 3.5$



(e) $\gamma = 5.65$

Figure 3.4: Welfare Gains (Consumption Equivalent) for Heterogeneous Preferences: $\tau^k = 0.89$



(a) $\gamma_{mean} = 2.23$, $(\tau^k = 0.89)$ (b) $\gamma = 2.23$, $(\tau^k = 0.81)$

Figure 3.5: Aggregate Welfare Gains for Heterogeneous and Homogeneous Preferences at each Economy's Optimal Taxation

3.A Appendices

3.A.1 Definition of Equilibrium

A stationary equilibrium consists a pair of policy functions $g_k(k,x)$ and $g_c(k,x)$, a probability distribution $\lambda(k,\epsilon,\gamma)$, factor prices (w,r), and a vector of aggregate assets and labor (K,N) such that:

1. The factor prices satisfies the conditions for profit maximization.

$$\pi = F(K_t, N) - r_t K_t - w_t N$$

2. The household solves the following problem given the factor prices.

$$\{g_k(k,x), g_c(k,x)\} = argmax\, E_0 \sum_{t=0}^{\infty} \beta^t U_i(c_{it})$$

$$\begin{aligned} s.t.\ \ c_{it} + k_{it+1} &\leq [1 + (1-\tau^k) r_t] k_{it} + (1-\tau^n) w_t \epsilon_{it} \overline{n} \\ k_{it+1} &\geq 0 \end{aligned}$$

3. The probability distribution $\lambda(k,\epsilon,\gamma)$ is the stationary distribution associated with $g_k(k,x)$, Π and $Prob(\gamma)$.

4. Market clearing conditions

$$
\begin{aligned}
C_t + G + K_{t+1} - (1-\delta)K_t &= F(K_t, N) \\
K_{t+1} &= \int_K \sum_{x \in E \times \Gamma} \lambda(k,x) \cdot g_k(k,x)\, dk \\
N &= \int_K \sum_{x \in E \times \Gamma} \lambda(k,x) \cdot \overline{n}\, dk \\
C_t &= \int_K \sum_{x \in E \times \Gamma} \lambda(k,x) \cdot g_c(k,x)\, dk
\end{aligned}
$$

5. The government budget constraint is satisfied.

$$\tau^k r_t K_t + t^n w_t N = G \quad t \geq 0$$

3.A.2 Computational Algorithm

1. Discretize the grid space for $\gamma \in [\gamma_{min}, ..., \gamma_{max}]$.

2. Discretize the grid space for $a \in [0, ..., a_{max}]$.

3. solve the problem for the initial τ^k, by this algorithm:

   - Given the τ^k and the τ^n.
   - Guess r_0.
   - Calculate the policy functions.
   - Get the stationary distribution.
   - Check the capital demand and capital supply.
   - Adjust the interest rate r until convergence.

- Calculate the G.

4. Choose the new τ^k and assume that this has been announced before households make decision in the first period.

5. Assume that the economy converges in the new steady state in period T.

6. Guess a sequence $K_2, ..., K_{T-1}$ for Aggregate Asset during the transition.

7. Given the new τ^k I solve the problem as in step 1 and I can find the final stationary distribution, the K_T and the τ^n that solves for the initial G.

8. Solve for the transition: Given K_T, K_{T-1}, and c_T and the distribution we can calculate the c_{T-1}. Following the same idea we can calculate c_i where $i = 1, .., T-2$.

9. Now we have to move forward and for given the consumption path, that I just calculate in the previous step, I will find the new aggregate assets.

10. This will continue until the path of the aggregate assets is not changing significantly. If the new aggregate assets is different than the old then set that to the new aggregate asset path and go to step 5.

11. Now check if the transition time T is enough by increasing T, if the transition path is not changing then I have found the optimal path.

Chapter 4

Conclusion

Europe faces the dilemma of whether to step forward to a higher degree of unification. Chapter 2 develops and analyzes a Eurobonds model where two small open economies issue bonds with full joint liability and interact strategically on their borrowing and repayment decisions. The finding show that introducing such an instrument at the peak of the Eurozone crisis would have brought welfare gains for the Periphery member states and losses for the Core member states. Chapter 3 studies the optimal capital taxation in an economy with exogenous incomplete markets, capital accumulation and heterogeneous preferences. The main findings show that the capital should be taxed heavier than in an economy with homogeneous preferences. I believe it is shown that continued research efforts should be devoted to identifying and understanding market frictions which widely exist over the sovereigns. I provide here several extensions for future research.

A natural extension of Chapter 2 would be the analysis of whether or not member states would be willing to take austerity measures to reduce the current high debt-to-output ratios. Moreover, it would be interesting to examine the case which countries are not forced to permanent financial autarky and they are permitted to issue debt after a few years of the default incident.

Regarding Chapter 3, it would be newsworthy to study optimal capital taxation in a model with occupational choices (entrepreneurs and workers) similar to Cagetti & De Nardi (2006). This might be crucial because by introducing to the model the heterogeneity in risk aversion, the results contradict the conclusions of Kimball *et al.* (2008), who support a negative relation between risk aversion and wealth in stocks.

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