Sequential and Non-Sequential Licensing of Innovation with Potential Entrants

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Abstract of the Dissertation

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This dissertation consists of two essays that study the economic impact of innovations when the buyers are not symmetric. Namely, when potential licensees involves both entrants and incumbent firms. In Chapter 2, a non-sequential licensing approach is analyzed. Licenses in this chapter are sold simultaneously by auction aiming to maximize the revenue of the innovator. The post innovation market structure, the diffusion of the innovation and the incentive to innovate are analyzed and compared with the case where licenses are sold only to incumbent firms and not to entrants. In Chapter 3, a sequential licensing approach is analyzed in a specific industry with one incumbent firm. An outside innovator holds a patent that allows him to bring in entry and the incumbent firm is willing to buy the ownership of the patent either to use it himself or to limit further entry. The innovator sells licenses (or patent right) to entrants (or incumbent firm) sequentially. It is shown, quite surprisingly, that before bargaining with the incumbent regarding the sale of the patent right, the innovator may benefit from selling a few licenses to new entrants. Such action reduces the total industry profit to be allocated but enables a better credible threat on the incumbent firm and hence may increase the innovator's payoff. As a result, the bargaining outcome is not ex-ante Pareto-efficient.

Contents

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Chapter 1

1 Introduction

The analysis of optimal licensing strategies of an innovator, the post innovation market structure as well as the incentive to innovate has been extensively studied in the literature, starting with Katz and Shapiro (1985), Katz (1986), Kamien and Tauman (1984), Kamien and Tauman (1986), Kamien, Oren, and Tauman (1992). A review of the first decade results on this topic is Kamien (1992). The literature on the optimal market structure which provides the highest incentive to innovate starts with Arrow (1962) showing that the revenue of an innovator who sells licenses by means of a per unit royalty is maximized in a competitive market. Kamien and Tauman (1986) and the extended analysis in Sen and Tauman (2007) show that the revenue of an innovator who sells licenses by either an upfront fee determined by an auction or by a per unit royalty (or by a combination of the two) is maximized in an oligopoly market of a size which depends on the magnitude of innovation, demand intensity and the marginal cost of production. In these papers as well as most other papers on optimal licensing of new innovations it is assumed that incumbent firms are the only potential licensees. This dissertation analyzes the economic impact of innovations when the buyers are not symmetric. Namely, when potential licensees involves both entrants and incumbent firms.

In Chapter 2, a non-sequential licensing approach is analyzed. Licenses in this chapter are sold simultaneously by auction aiming to maximize the revenue of the innovator when firms are compete à la Cournot. It is shown, quite surprisingly, that opening the market to entrant licensees, the incentive to innovate is maximized in a monopoly market rather than oligopoly or competitive markets and this is true for drastic as well as non-drastic innovations. This result is consistent with the observation in Schumpeter (1942) that monopolistic industries, those in which individual firms have a measure of control over their products price, provide a more hospitable atmosphere for innovation than purely competitive ones. We doubt however that Schumpeter visioned an outside innovator who may benefit from creating a competition by selling licenses for the use of his invention to new entrants, in addition to the monopolist incumbent. But as we show this option makes a monopolist market more attractive for the innovator than any oligopoly or competitive market.

For innovations of significant magnitude the innovator chooses to sell licenses only to incumbent firms and not to entrants and the diffusion of the innovation is the same as in the case where entry is excluded. Although entrants are willing to pay for a license typically more than incumbent firms, the competition effect on the revenue of the innovator dominates the revenue he can obtain from additional entrant. For less significant innovations the innovator sells licenses to some entrants and to all incumbent firms. Furthermore, we show that the post-innovation market size is larger the smaller is the magnitude of the innovation. Namely smaller innovations diffuse more.

As expected, opening the market to entrant licensees yields the innovator a higher revenue compared with the case where entry is excluded. The marginal effect of the entry market on the innovator's revenue is higher, the smaller is the magnitude of the innovation and the smaller is the pre-innovation market size. In particular, a monopoly not only provides the highest incentive to innovate but also maximizes the incremental incentive to innovate due to entry. It is also shown that opening the market to new entrants has positive effect on social welfare, which is decreasing in the magnitude of the innovation.

In Chapter 3, a sequential licensing approach is analyzed. We consider a specific industry with one incumbent and many potential entrants. It is assumed that initially the high entry cost does not enable a profitable entry and the incumbent is a monopoly. Suppose that an outside innovator obtains a patent on a new technology that eliminates the entry cost but has a marginal cost which is different from the current one. The innovator can sell his intellectual property (IP) to the incumbent through bargaining. Even though the technology itself maybe useless for the incumbent, he may purchase the IP to limit or exclude further entry. However, before approaching the incumbent, the innovator may sell a few licenses to new entrants. A licensing contract with an entrant specifies the license fee together with the maximum number of licenses that can be sold. The contracts are signed sequentially and they are bound by previous commitments.

Selling licenses before bargaining (with the incumbent) reduces the total industry profit to be allocated but enables a better credible threat on the incumbent firm and hence may increase the innovator's payoff. As a result, the bargaining outcome is not ex-ante Pareto-efficient. We show that such inefficiency occurs when the bargaining time takes a large proportion of the patent right life; or when there is a constraint on the number of times the two bargainers can meet. Furthermore, we show that the inefficiency is less significant when the innovator has a higher bargaining power; when the new technology is less efficient; or when the patent lasts for a longer period.

Chapter 2

2 Non-Sequential Licensing of Innovation

2.1 Introduction

Licenses in our model are sold by auction aiming to maximize the revenue of the innovator when firms are compete à la Cournot. The post innovation market structure, the diffusion of the innovation and the incentive to innovate are compared with the case where licenses are sold only to incumbent firms and not to entrants.

It is shown, quite surprisingly, that opening the market to entrant licensees, the incentive to innovate is maximized in a monopoly market rather than oligopoly or competitive markets and this is true for drastic as well as non-drastic innovations. This is because the total number of licensees is no longer constrained by the number of incumbent firms and each licensee's willingness to pay for a license is higher the smaller is the pre-innovation market size. This result is consistent with the observation in Schumpeter (1942) that monopolistic industries, those in which individual firms have a measure of control over their products price, provide a more hospitable atmosphere for innovation than purely competitive ones. We doubt however that Schumpeter visioned an outside innovator who may benefit from creating a competition by selling licenses for the use of his invention to new entrants, in addition to the monopolist incumbent. But as we show this option makes a monopolist market more attractive for the innovator than any oligopoly or competitive market.

We show that for innovations of significant magnitude the innovator chooses to sell licenses only to incumbent firms and not to entrants and the diffusion of the innovation is the same as in the case where entry is excluded. Although entrants are willing to pay for a license typically more than incumbent firms, the competition effect on the revenue of the innovator dominates the revenue he can obtain from additional entrant. For less significant innovations the innovator sells licenses to some entrants and to all incumbent firms. Furthermore, we show that the post-innovation market size is larger the smaller is the magnitude of the innovation. Namely smaller innovations diffuse more. To clarify this point notice that adding an entrant licensee on one hand reduces the willingness to pay for a license of each licensee, but on the other hand the innovator adds to his payoff the entire profit of this entrant. For small number of entrants the latter effect exceeds the former effect. The net effect (which depends on the original market size) while positive in the beginning, it decreases with the number of entrant licensees. For the innovator, the optimal number of entrant licensees is the one for which this net effect vanishes. In the Cournot model this net effect is larger the smaller is the magnitude of innovation. Hence, the optimal number of entrant licensees is larger the less significant is the innovation.

As expected, opening the market to entrant licensees yields the innovator a higher revenue compared with the case where entry is excluded. The marginal effect of the entry market on the innovator's revenue is higher, the smaller is the magnitude of the innovation and the smaller is the pre-innovation market size. In particular, a monopoly not only provides the highest incentive to innovate but also maximizes the incremental incentive to innovate due to entry. It is also shown that opening the market to new entrants has positive effect on social welfare, which is decreasing in the magnitude of the innovation.

We are aware of only few papers which deals with the licensing of innovations to both incumbent firms and potential entrants. The one closest to this paper is Hoppe, Jehiel, and Moldovanu (2006), (HJM here after). In HJM the innovator sells licenses through a uniform auction (UA). The innovator in UA decides on the number k of licenses to sell. The auction welcomes bids from both incumbent firms and entrants. Each one of the k highest bidders (whether incumbent firm or entrant) obtains a license and all licensees pay the same amount which is the $(k+1)$th highest bid. Externality plays an important role in UA. Incumbent firms and entrants have different willingness to pay for a license and both of them depend not only on the number of licensees but also on the distribution of entrants and incumbent licensees. The problem however with UA is that it has multiple equilibrium points and multiple equilibrium payoffs of the innovator even if bidders do not use dominated strategies. This paper extends the analysis of HJM, who deals with a general setup and focuses on whether entrants can be winners of licenses in an auction which is open to both incumbent firms and entrants. Their study mostly deals with the sale of an exclusive license as well as some special cases involving multiple licenses. Our paper provides a general analysis of the optimal licensing strategy of the innovation, the post-innovation market structure and the incentive to innovate but in a specific set-up: Cournot oligopoly market, linear demand and a constant per-unit cost. We show that for any number of licenses, $k \geq 1$, every partition (k_1, k_2) of k $(k_1+k_2 = k)$ can be supported as an equilibrium outcome where k_1 is the number of incumbent licensees and k_2 is the number of entrant licensees. While the innovator controls k he has no control over the partition of k to incumbent and entrant licensees, making it difficult to predict the outcome of the UA. In particular, it is not clear what should be the innovator's choice of k. To have some UA benchmark we compute the highest equilibrium payoff of the innovator in UA and compare it with the equilibrium outcomes of the alternative auctions offered in this paper.

In attempt to escape the multiplicity problem we offer two alternative non-uniform auctions, NUA and SUA (semi-uniform auction). The innovator in both auctions chooses in addition to k the exact partition (k_1, k_2) of k. The winners of the auctions are the k_1 highest incumbent bidders and the k_2 highest entrant bidders (ties are resolved at random). The license fee is the same across licensees in SUA and to ensure the participation of incumbent firms in SUA the license fee is set to be the k-th highest bid[1]. The license fee is not uniform across licensees in NUA. Each incumbent licensee in NUA pays the $(k_1 + 1)$th highest bid among the incumbents' bids while each entrant licensee pays the $(k_2 + 1)$th highest bid among the entrants' bids[2]. Externalities do not play a role in both NUA and SUA since the post innovation market structure is determined by (k_1, k_2) regardless of the bids. Given (k_1, k_2), the equilibrium outcome in undominated strategies is unique, for both NUA and SUA.

We first analyze NUA. In the first glance it seems that the ability to choose (k_1, k_2) and differentiating the license fee of entrants from incumbent firms should always yield the innovator a higher payoff than his payoff in UA. But this may not be the case. Indeed every entrant licensee pays in NUA her entire profit (assuming zero entrant's opportunity cost) while in UA it is (like any incumbent licensee) only the incremental profit of an incumbent licensee. However, an incumbent firm may be willing to pay more in UA if he can limit entry. While every incumbent licensee in NUA takes the place of some other incumbent firm and thus does not change the number of active firms, in UA an incumbent firm may take the place of an entrant and thus reduces the number of active firms by 1. In this case his willingness to pay for a license in UA maybe higher. We show that the most optimistic innovator (who expects to obtain his highest equilibrium payoff in UA) prefers NUA on UA if and only if the magnitude of the innovation is relatively small. We provide general analyses for NUA and SUA and use them to study the impact of the innovation on the market structure and the incentive to innovate when entry is allowed. The multiplicity of equilibrium payoffs of the innovator in UA makes it basically impossible to predict which equilibrium will emerge. On the other hand each of the NUA and SUA has a unique equilibrium outcome in undominated strategies and as such provide sharp predictions.

[1]If (as in UA) every licensee pays the highest losing bid, it may happen that the bids of all incumbent licensees fall below the $(k+1)$th highest bid. This is the case if the $(k+1)$th highest bid is submitted by an entrant who bids her entire industry profit. Such bid exceeds the willingness to pay of incumbent firms and incumbents are best off not participating in the auction.

[2]We will obtain the same equilibrium outcome if the license fee incumbents (entrants) pay is the k_1-th (k_2-th) highest bid among the incumbent (entrant) bids. Setting the license fee to be the (k_1+1)th highest bid among incumbent bids guarantee that bidding truthfully (the true willingness to pay) is a weakly dominant strategy of every incumbent firm (similar for entrants).

The analysis of the semi-uniform auction (SUA) shows that irrespective of the market size and the magnitude of innovation the innovator extracts in NUA at least as much as in SUA. Like in NUA, a monopoly market provides the highest incentive to innovate in SUA. Moreover, like in NUA, for relatively significant innovation the total number of licenses the innovator sells in SUA is decreasing in the magnitude of the innovation and if the innovator sells some licenses to entrants, it is only if he also sells licenses to all (but 1) incumbent firms. In contrast to NUA, for relatively small innovation the innovator in SUA sells licenses only to new entrants and not to incumbent firms. The reason is the ability of the innovator to extract the entire industry profit of every entrant licensee, as opposed to the case where he sells some licenses also to incumbent firms. In the latter case the license fee an entrant pays is equal to the willingness to pay of an incumbent licensee which decreases to zero as the magnitude of innovation decreases to zero. In contrast, the innovator in NUA can discriminate the entrant licensees and can extract their entire industry profit whether or not he sells licenses to incumbent firms. Therefore in NUA the innovator sells licenses to both new entrants and incumbent firms, even for less significant innovation and the diffusion of the innovation is higher in NUA than in SUA. We conclude that the ability to charge entrant licensees different fee than the fee incumbent licensees pay has positive effect on welfare for less significant innovations: it induces higher diffusion of innovation as well as lower post innovation market price.

Another related paper studying the innovator's optimal licensing strategy in the presence of potential entrants is Tauman, Weiss, and Zhao (2016). It deals with a pre-innovation monopoly industry where the innovator sells his intellectual property to the incumbent through bargaining. The paper shows that before approaching the incumbent, the innovator may sell a few licenses to new entrants. This on one hand reduces the total industry profit but enables a better credible threath on the incumbent and hence may increase the innovator's payoff.

2.2 The Model

Consider an industry with a set $N = \{1, ..., n\}$ of incumbent firms who produce one product with marginal cost $c > 0$. Potential entrants are unable to enter the market either because of high fiexed cost or since the current technology is protected by patent. An outside innovator (Inn) comes along with an innovation which eliminates the fixed cost and reduces the constant per unit cost from c to $c - \epsilon$, $0 < \epsilon \leq c$[3].

[3]In principle even $\epsilon \leq 0$ may be valuable, and entrants may be willing to pay for inefficient technology if it allows them a profitable entry. We confine in this paper to $\epsilon > 0$.

The number of potential entrants is assumed to be sufficiently large and it exceeds the optimal number of licenses sold by Inn.

The inverse demand function is linear, $p = \max(a - Q, 0)$. Denote by $\pi_1(m_0, m_1)$ and $\pi_0(m_0, m_1)$ the Cournot profit of a licensee and a non-licensee, respectively, when there are m_0 firms producing at a unit cost c and m_1 firms producing at a unit cost $c - \epsilon$. It can be verified that

$$\pi_0(m_0, m_1) = \begin{cases} \left(\frac{(a-c)-\epsilon m_1}{m_0+m_1+1}\right)^2 & \text{if } m_1 \leq \frac{a-c}{\epsilon} \\ 0 & \text{if } m_1 > \frac{a-c}{\epsilon} \end{cases} \tag{1}$$

$$\pi_1(m_0, m_1) = \begin{cases} \left(\frac{(a-c)+(m_0+1)\epsilon}{m_0+m_1+1}\right)^2 & \text{if } m_1 \leq \frac{a-c}{\epsilon} \\ \left(\frac{(a-c)+\epsilon}{m_1+1}\right)^2 & \text{if } m_1 > \frac{a-c}{\epsilon} \end{cases}$$

Without loss of generality we normalize $a - c$, the quantity demanded at the price c, to be 1. We make the following assumption throughout the paper.

Assumption 1. *Training and installing the new technology is costly and paid only by the innovator. This cost is sufficiently small for any licensee and has no effect on the optimal number of licenses the innovator sells. The training cost is smaller for incumbent licensees than entrant licensees.*

This assumption simplifies the tie breaking rule. If a tie involves both entrants and incumbent firms, the innovator prefers to sell a license to an incumbent firm. If a tie involves only one type of bidders, the tie is resolved at random.

In UA the players are engaged in a three-stage game, G_u. In the first stage Inn chooses and announces the number k of licenses to be auctioned off to both incumbent firms and new entrants. In the second stage the licenses are allocated to the winners of a uniform auction where each one of the k highest bidders obtains a license and pays the $(k+1)$th highest bid. In the third and last stage the firms (incumbents and entrant licensees) compete à la Cournot. Let $G_u(k)$ be the subgame of G_u which starts in the second stage.

Let G_{nu} be the game associated with NUA. In the first stage Inn chooses and announces (k_1, k_2), where $0 \leq k_1 \leq n-1$ and $k_2 \geq 0$ are the number of licenses he auctions off to incumbent firms and entrants, respectively. Let $G_{nu}(k_1, k_2)$ be the subgame of G_{nu} which starts in the second stage of G_{nu}. In $G_{nu}(k_1, k_2)$, licenses are sold through a non-uniform auction. Let E be the set of entrant bidders. Each of the k_1 highest incumbent bidders obtains a license and pays the bid of (k_1+1)th highest bidder in N. Similarly, each of the k_2 highest entrant bidders obtains a license and pays the bid of the (k_2+1)th highest bidder in E. In the third stage the firms in the industry (the licensees and the non-licensees) engage in Cournot competition. Note that the auction is not well define for $k_1 = n$. Thus we limit k_1 to $n-1$.

In $G_{nu}(k_1, k_2)$, the value of a license is uniquely determined for each bidder. This is not the case in UA where the value of a license depends on the distribution of incumbent and entrant licensees. One exception is when the number of licenses the innovator sells is sufficiently large to drive down the Cournot price below the pre-innovation marginal cost. As a result all non-licensee firms are driven out of the market and the value of the license for each bidder is uniquely determined even in UA. Finally note that bidders do not usually have dominant strategies in UA.

Proposition 1. *Suppose bidders do not use dominated strategies. (i) If the innovator auctions off a total of $\frac{1}{\epsilon}$ licenses (using either UA or NUA), then the Cournot price is c, the pre-innovation marginal cost, and every non-licensee firm is driven out of the market. Each licensee pays his entire profit and the innovator obtains the total industry profit. (ii) It is never optimal for the innovator in both UA and NUA to auction off more than $\frac{1}{\epsilon}$ licenses.*

It will be shown (see Proposition 3 and Proposition 5, below) that for $\epsilon > \frac{2}{n+1}$ the optimal number of licenses for the innovator is $k = \frac{1}{\epsilon}$ in both UA and NUA .

Proof. Part (i) is a straight forward consequence of (1). Part (i) asserts that when $k = \frac{1}{\epsilon}$ only licensees are active firms in the market, and this is obviously true for all $k \geq \frac{1}{\epsilon}$. Since the total industry profit is decreasing in k for $k \geq \frac{1}{\epsilon}$, part (ii) follows. $\square$

By Proposition 1, without loss of generality we only consider the case where in both UA and NUA the total number k of licenses does not exceed $\frac{1}{\epsilon}$. In case $\epsilon \geq 1$ (drastic innovation) even if the innovator sells an exclusive license, every non-licensee firm is driven out of the market and the innovator extracts the monopoly profit under the new technology. It is left to analyze only the non-drastic innovation case, namely $\epsilon < 1$.

2.2.1 Uniform Auction

Consider the subgame $G_u(k)$ of G_u, for $1 \leq k \leq \frac{1}{\epsilon}$. Suppose (k_1, k_2) is an equilibrium outcome of $G_u(k)$, where k_1, $0 \leq k_1 \leq n$, is the number of incumbent licensees and $k_2 = k - k_1$ is the number of entrant licensees. Let $b_{(i)}$ be the ith highest bid in UA ($b_{(i)} = b_{(i+1)}$ if more than one bidder bids $b_{(i)}$).

The willingness to pay of an incumbent firm for a license is the difference between his profit $\pi_1(n-k_1, k)$ as a licensee and his profit as a non-licensee. The latter depends on which type of licensee replaces him if he drops out, an entrant or an incumbent firm. If it is an entrant, the total number of firms increases by 1 and his willingness to pay is

$$w_{ih}^k(k_1) = \pi_1(n - k_1, k) - \pi_0(n - k_1 + 1, k). \tag{2}$$

If it is an incumbent firm, his willingness to pay is

$$w_{il}^k(k_1) = \pi_1(n - k_1, k) - \pi_0(n - k_1, k). \tag{3}$$

Note that w_{ih}^k can be regarded as an incumbent's willingness to pay for both, limiting entry and using the superior technology.

The willingness to pay of an entrant for a license is simply her Cournot profit,

$$w_e^k(k_1) = \pi_1(n - k_1, k). \tag{4}$$

By Proposition 1, for any k_1, $0 \leq k_1 \leq \min(k, n)$, if $k \geq \frac{1}{\epsilon}$ $w_e^k(k_1) = w_{ih}^k(k_1) = w_{il}^k(k_1)$. If $k < \frac{1}{\epsilon}$ then $\pi_0(n - k_1, k) > \pi_0(n - k_1 + 1, k) > 0$ and $w_e^k(k_1) > w_{ih}^k(k_1) > w_{il}^k(k_1)$.

For $k < \frac{1}{\epsilon}$ any entrant licensee is willing to pay for a license more than any incumbent licensee. Nevertheless it is possible that in equilibrium some incumbent firm wins a license. To clarify this point observe that an entrant with no license who outbids an incumbent licensee not only increases the number of active firms by 1 but also increases the license fee from $b_{(k+1)}$ to $b_{(k)}$. This may reduce the profit of each licensee to a level below the new license fee, causing the deviant entrant a loss.

By Proposition 1, when the innovator chooses $k = \frac{1}{\epsilon}$ (or actually $k \geq \frac{1}{\epsilon}$), the willingness to pay of each bidder is independent of the distribution of licensees between entrants and incumbent firms and each bidder's willingness to pay in $G_u(k)$ is his Cournot profit. If, however, $k < \frac{1}{\epsilon}$, each bidder's willingness to pay depends in addition to k on the distribution of winners. We next analyze the innovator's equilibrium payoff in this case.

Proposition 2. *Let $1 \leq k < \frac{1}{\epsilon}$. Then (i) any (k_1, k_2), $0 \leq k_1 \leq n$ and $k_2 \geq 0$ s.t. $k_1 + k_2 = k$, is an equilibrium outcome of $G_u(k)$. (ii) For $k_1 = 0$, π is an equilibrium payoff of the innovator in $G_u(k)$ if and only if $\pi \in [0, kw_e^k(0)]$. (iii) For $1 \leq k_1 \leq n$, π is an equilibrium payoff of the innovator in $G_u(k)$ if and only if $\pi \in [0, kw_{ih}^k(k_1)]$.*

Proof. (i) Let $k \geq 1$ and let (k_1, k_2) s.t. $0 \leq k_1 \leq n - 1$ and $k_2 = k - k_1$ (the case where $k_1 = n$ will be dealt separately). Let us show that (k_1, k_2) is an equilibrium outcome of $G_u(k)$. Denote $b = \pi_1(n - k_1 - 1, k)$ (b is well defined since $k_1 \leq n - 1$) and $\underline{b} = \pi_1(n - k_1, k) - \pi_0(n - k_1 + 1, k)$. Suppose that exactly k_1 incumbent firms and k_2 entrants bid b and only one entrant bids $\underline{b}$. All other incumbents or entrants bid below $\underline{b}$. Clearly $b_{(1)} = ... = b_{(k)} = b$, $b_{(k+1)} = \underline{b}$ and $\underline{b} \leq b$. We claim that these bid profile constitutes an equilibrium of $G_u(k)$. Any incumbent licensee, i, obtains

$$\pi_1(n - k_1, k) - b_{(k+1)} = \pi_0(n - k_1 + 1, k).$$

If i lowers his bid below $\underline{b}$ the entrant who bids $\underline{b}$ will become a licensee and will replace i. As a result there will be $n - k_1 + 1$ firms producing with the inferior technology and i will obtain $\pi_0(n - k_1 + 1, k)$, the same as his payoff as a licensee. Since the opportunity cost of any entrant is zero, an entrant licensee (when $k_2 \geq 1$) has no incentive to lower her bid. Next let us show that a non-licensee (incumbent or entrant) can not benefit from outbidding a licensee. Suppose j (incumbent or entrant) outbids a licensee i. Then he/she will increase the license fee from $\underline{b}$ to b. We claim that the industry profit of j is at most b and hence he has no incentive to become a licensee. Indeed, if both j and i are incumbent firms the industry profit of j as a licensee will be $\pi_1(n - k_1, k)$ which is smaller than $b = \pi_1(n - k_1 - 1, k)$. If j is an incumbent firm and i is an entrant, the number of firms using the inferior technology will reduce to $n - k_1 - 1$. The gross profit of j as a licensee will be b and his payoff, net of the new license fee, is zero. If j is an entrant, j will obtain an industry profit of $\pi_1(n - k_1 + 1, k) < b$ if i is an incumbent firm and $\pi_1(n - k_1, k) < b$ if i is an entrant. In both cases j's net payoff is negative. To complete the proof of part (i) suppose that $k_1 = n$ and hence $k_2 = k - n$. Suppose every incumbent firm and exactly k_2 entrants bid $b = \pi_1(0, k)$, one entrant only bids $\underline{b} = \pi_1(0, k) - \pi_0(1, k)$ and every other bidder bids below $\underline{b}$. The license fee is $\underline{b}$ and it is easy to verify that these bids constitute an equilibrium of $G_u(k)$.

(ii) Let $k_1 = 0$, $\tilde{b} \in [0, \pi_1(n, k)]$ and $b = \pi_1(n - 1, k)$. Suppose exactly k entrants bid b, only one entrant bids $\tilde{b}$ and every other bidder bids below $\tilde{b}$. The license fee is $b_{(k+1)} = \tilde{b}$. Since $\pi_1(n, k) - \tilde{b} \geq 0$, no (entrant) licensee benefits from lowering his bid below $\tilde{b}$. Suppose next that a non-licensee j (incumbent or entrant), bids above b. Then the new license fee will increase to $b = \pi_1(n - 1, k)$ and j's industry profit is $\pi_1(n - 1, k)$ if j is an incumbent firm, or $\pi_1(n, k)$ if j is an entrant. In both cases the industry profit does not exceed the license fee. Finally, there is no equilibrium of $G_u(k)$ with $k_1 = 0$ and s.t. $b_{(k+1)} > \pi_1(n, k)$. Otherwise, the industry profit of a licensee does not cover the license fee.

(iii) Suppose $1 \leq k_1 \leq n - 1$ and let $\tilde{b} \in [0, w^k_{ih}(k_1)]$, where $w^k_{ih} = \pi_1(n - k_1, k) - \pi_0(n - k_1 + 1, k)$. Denote $b = \pi_1(n - k_1 - 1, k)$. Suppose exactly k_1 incumbent firms and k_2 entrants bid b, only one entrant bids $\tilde{b}$ and every other bidder bids below $\tilde{b}$.

Then $b_{(k+1)} = \tilde{b}$ is the license fee. An incumbent licensee obtains

$$\pi_1(n - k_1, k) - \tilde{b} \geq \pi_0(n - k_1 + 1, k). \tag{5}$$

If he lowers his bid below $\tilde{b}$ he will obtain $\pi_0(n-k_1+1, k)$. By (5) this will not benefit him. A non-licensee j (incumbent or entrant) who outbids a licensee i (incumbent or entrant) will increase the license fee from $\tilde{b}$ to $b = \pi_1(n - k_1 - 1, k)$. It is easy to verify that independently of the identity of j and i, j's industry profit will not exceed $\pi_1(n - k_1 - 1, k)$.

Next suppose $k_1 = n$. Let $\tilde{b} \in [0, \pi_1(0, k) - \pi_0(1, k)]$ and let $b = \pi_1(0, k)$. Suppose every incumbent firm and exactly $k_2 = k - n$ entrants bid b. Suppose also that only one entrant bids $\tilde{b}$ and all other bidders bid below $\tilde{b}$. Then the license fee is $b_{(k+1)} = \tilde{b}$. A licensee obtains

$$\pi_1(0.k) - \tilde{b} \geq \pi_0(1, k) \geq 0. \tag{6}$$

If an incumbent licensee lowers his bid below $\tilde{b}$ he will obtain $\pi_0(1, k)$ and by (6) he does not improve his payoff. If a non-licensee entrant j outbids a licensee i the new license fee will be $b = \pi_1(0, k)$ and again, independent of the identity of i, the industry profit of j will not exceed $\pi_1(0, k)$.

Finally, for $k \geq 1$ the willingness of an incumbent firm to pay for a license is at most $w_{ih}^k(k_1)$. Thus there is no equilibrium b^* of $G_u(k)$ s.t. $1 \leq k_1 \leq n$ and $b^*_{(k+1)} > w_{ih}^k(k_1)$.

□

Proposition 2 asserts that there are multiple equilibrium points in $G_u(k)$ $(1 \leq k < \frac{1}{\epsilon})$. There are two types of multiplicity. First, any (k_1, k_2) s.t. $k_1 + k_2 = k$ is an equilibrium outcome of $G_u(k)$. Second, every (k_1, k_2) generates continuum of equilibrium payoffs of the innovator. In fact for every equilibrium outcome $\left(b^*_{(1)}, ..., b^*_{(k)}, b^*_{(k+1)}\right)$ and for every b, $0 \leq b \leq b^*_{(k+1)}$, $\left(b^*_{(1)}, ..., b^*_{(k)}, b_{(k+1)} = b\right)$ is also an equilibrium outcome. By parts (ii) and (iii) of Proposition 2 for any (k_1, k_2), the innovator's equilibrium payoff can be as low as zero, and as high as $kw_e^k(0) = k\pi_1(n, k)$ if $k_1 = 0$ and as high as $kw_{ih}^k(k_1) = k[\pi_1(n - k_1, k) - \pi_0(n - k_1 + 1, k)]$ if $1 \leq k_1 \leq n$.

The multiplicity of equilibrium outcomes is a problem even if dominated strategies are eliminated. HJM dealt with the game G_u with this restriction. The equilibrium analysis of G_u then is very complicated. HJM analyzed only the case $k = 1$ and some other special cases. They too found multiple equilibrium points. The conclusion is that there is no obvious way to predict the outcome of G_u nor the choice k of the innovator. To provide some comparison between the innovator's payoff in UA and in NUA, we focus here on a specific equilibrium of UA, the one where for any k the innovator is lucky to obtain his highest equilibrium payoff in $G_u(k)$. Namely, we focus in UA on the payoff of the "luckiest" innovator. We next analyze the optimal number of licenses of the "luckiest" innovator in G_u.

Lemma 1. *For any $1 \leq k \leq \frac{1}{\epsilon}$, the innovator's highest equilibrium payoff in $G_u(k)$ is obtained when either $k_1 = 0$ or $k_1 = \min(k, n)$.*

Proof. By Proposition 2, given an arbitrary $1 \leq k \leq \frac{1}{\epsilon}$, any $0 \leq k_1 \leq \min(k, n)$ can emerge as an equilibrium outcome. In addition the highest payoff of the innovator is $k\pi_1(n, k)$ if $k_1 = 0$ and $kw_{ih}^k(k_1)$ if $1 \leq k_1 \leq n$. It is shown in the Appendix (see A.1.2) that $w_{ih}^k(k_1)$ is increasing in k_1. Thus the innovator obtains the highest payoff for $k_1 = \min(k, n)$. □

Proposition 3. *Suppose the innovator obtains for every k the highest equilibrium payoff in $G_u(k)$. (i) The corresponding equilibrium number of licensees in G_u is*

$$k_u^*(n, \epsilon) = \begin{cases} n+1 & \text{if } 0 < \epsilon < g(n) \\ n & \text{if } g(n) \leq \epsilon \leq f(n) \\ \tilde{k}(n, \epsilon) & \text{if } f(n) < \epsilon < \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} \leq \epsilon < 1. \end{cases}$$

(ii) If $0 < \epsilon < g(n)$, all licensees are entrants and if $g(n) < \epsilon < 1$, all licensees are incumbent firms.

The formulas of $f(n)$, $g(n)$ and $\tilde{k}(n, \epsilon)$ are quite complicated and not revealing. This is the reason they all appear in the Appendix (A.1.1).

Remark: Let $\frac{2}{n+1} \leq \epsilon < 1$ and suppose bidders do not use dominated strategies.. Then the unique optimal strategy of the innovator is to auction off $k = \frac{1}{\epsilon}$ licenses. In this case he sells at most $\frac{n+1}{2}$ licenses, the Cournot price reduces to the pre-innovation marginal cost c, and every non-licensee firm is driven out of the market. Consequently, the multiplicity of equilibrium points of UA occurs only when $\epsilon < \frac{2}{n+1}$.

Proof. See A.1.3 of the Appendix. □

Proposition 3 shows that for relatively small innovation the innovator obtains the highest equilibrium payoff when all licensees are entrants. Indeed an entrant licensee increases the number of active firms by 1 causing the Cournot profit of each firm to shrink. However when selling licenses only to entrants each entrant licensee is willing to pay all her profit for a license as opposed to the case where the innovator sells some licenses to incumbent firms (in the latter case the license fee is only the incremental profit of an incumbent licensee). When the innovation is relatively small the effect of the additional license fee from entrant licensees exceeds the higher competition on the revenue of the innovator and innovator's highest equilibrium payoff is obtained when all licensees are entrants.

Corollary 1. *The highest equilibrium payoff of the innovator in G_u is*

$$\pi_u^*(n,\epsilon) = \begin{cases} (n+1)\pi_1(n,n+1) & \text{if } 0<\epsilon<g(n) \\ n\big(\pi_1(0,n)-\pi_0(1,n)\big) & \text{if } g(n)\le\epsilon\le f(n) \\ \tilde{k}\Big(\pi_1\big(n-\tilde{k},\tilde{k}\big)-\pi_0\big(n-\tilde{k}+1,\tilde{k}\big)\Big) & \text{if } f(n)<\epsilon<\frac{2}{n+1} \\ \epsilon & \text{if } \frac{2}{n+1}\le\epsilon<1. \end{cases}$$

where $\tilde{k}=\tilde{k}(n,\epsilon)$.

Proof. Follows immediately from Proposition 3. □

2.2.2 Non-Uniform Auction

In this section we allow the innovator to choose and announce both the number of licenses to be sold to incumbent firms ($0\le k_1\le n-1$) and the number of licenses to be sold to potential entrants ($k_2\ge 0$). Each incumbent licensee pays the (k_1+1)th highest bid among the incumbents' bids. Each entrant licensee pays the (k_2+1)th highest bid among the entrants' bids. In $G_{nu}(k_1,k_2)$ the willingness to pay of each incumbent firm is $\pi_1(n-k_1,k_1+k_2)-\pi_0(n-k_1,k_1+k_2)$ and the willingness to pay of each entrant is $\pi_1(n-k_1,k_1+k_2)$. Since bidding the true valuation is a (weakly) dominant strategy for each bidder, it is assumed that bidders bid truthfully in NUA. The innovator's equilibrium payoff in $G_{nu}(k_1,k_2)$ is uniquely determined and it is given by

$$\pi_{nu}(k_1,k_2)=k_1\big(\pi_1(n-k_1,k_1+k_2)-\pi_0(n-k_1,k_1+k_2)\big)+k_2\pi_1(n-k_1,k_1+k_2). \quad (7)$$

The analysis of the highest incentive to innovate does not require the characterization of the equilibrium licensing strategy of the innovator in NUA.

Proposition 4. *A monopoly industry maximizes the revenue of the innovator if he sells licenses by NUA.*

Proposition 4 asserts that a monopoly industry provides the highest incentive to innovate if licenses are sold by NUA. The proof does not make use of the linear structure of our demand and it applies to any demand function.

Proof. Suppose there are n, $n \geq 2$ incumbent firms. Denote by (k_1^*, k_2^*) the optimal licensing strategy in G_{nu}. Let $K_{nu}^* = k_1^* + k_2^*$. The innovator's highest payoff is

$$\alpha \equiv k_1^*\big(\pi_1(n - k_1^*, K_{nu}^*) - \pi_0(n - k_1^*, K_{nu}^*)\big) + k_2^*\pi_1(n - k_1^*, K_{nu}^*). \tag{8}$$

Suppose one of the incumbent firms drops out and only $(n-1)$ incumbent firms remain.

Case 1. $k_1^* \geq 1$. Using the licensing strategy (k_1^*-1, k_2^*+1), the innovator obtains

$$\beta \equiv (k_1^* - 1)\big(\pi_1(n - k_1^*, K_{nu}^*) - \pi_0(n - k_1^*, K_{nu}^*)\big) + (k_2^* + 1)\pi_1(n - k_1^*, K_{nu}^*). \tag{9}$$

Clearly for $K_{nu}^* = \frac{1}{\epsilon}$, $\pi_0(n - k_1^*, K_{nu}^*) = 0$ and $\alpha = \beta$. For $K_{nu}^* < \frac{1}{\epsilon}$, $\beta > \alpha$. The innovator obtains more in $G_{nu}(n-1, \epsilon)$ compare with $G_{nu}(n, \epsilon)$.

Case 2. Suppose $k_1^* = 0$. Using the licensing strategy $(0, k_2^*)$, the innovator obtains

$$\gamma \equiv k_2^*\pi_1(n-1, k_2^*) \geq k_2^*\pi_1(n, k_2^*) \equiv \alpha. \tag{10}$$

Again for $k_2^* = \frac{1}{\epsilon}$, $\gamma = \alpha$. For $k_2^* < \frac{1}{\epsilon}$, $\gamma > \alpha$

Combining Cases 1 and 2, if $K_{nu}^* < \frac{1}{\epsilon}$ the innovator extracts strictly higher revenue with $n-1$ than with n incumbent firms. For $K_{nu}^* = \frac{1}{\epsilon}$, when the market size is $n-1$ the innovator obtains a payoff which is at least as high as in case where the market size is n. Since this is true for all $n \geq 2$, the proof is complete. $\square$

We next characterize the equilibrium of G_{nu}.

Proposition 5. *Consider the game G_{nu}. The unique equilibrium licensing strategy of the innovator is*

(i) For $n \geq 3$

$$k_1^{n*}(n, \epsilon) = \begin{cases} n-1 & \text{if } 0 < \epsilon \leq \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} \leq \epsilon < 1, \end{cases}$$

$$k_2^{n*}(n, \epsilon) = \begin{cases} \frac{2(n+2\epsilon)}{2n\epsilon+1} - (n-1) & \text{if } 0 < \epsilon \leq \frac{1}{2n-4} \\ 0 & \text{if } \frac{1}{2n-4} \leq \epsilon < 1. \end{cases}$$

(ii) For $n = 2$

$$k_1^{n*}(2, \epsilon) = 1,$$

$$k_2^{n*}(2, \epsilon) = \begin{cases} \frac{3}{4\epsilon+1} & \text{if } 0 < \epsilon \leq \frac{1}{2} \\ \frac{1}{\epsilon} - 1 & \text{if } \frac{1}{2} \leq \epsilon < 1. \end{cases}$$

Proof. See A.1.4 of the Appendix. □

By Proposition 5 for $n \geq 3$ the total number of licenses the innovator sells is decreasing in ϵ. For small ϵ he sells $2n$ licenses ($n-1$ licenses to all (but 1) incumbent firms and $n+1$ licenses to new entrants). As ϵ grows the number of licenses decreases continuously to 1, as the innovation becomes closer to a drastic innovation ($\epsilon \to 1$). The innovator sells at least one license to incumbent firm and for $\epsilon > \frac{1}{2n-4}$ he sells no licenses to new entrants. Let $K^*_{nu} = k_1^{n*} + k_2^{n*}$.

Corollary 2. *For $n \geq 2$ the number of licenses decreases continuously in ϵ with a maximum of $2n$ for small magnitude of innovation and minimum of one license when the innovation becomes drastic. In particular*

For $n \geq 3$

$$K^*_{nu}(n,\epsilon) = \begin{cases} \frac{2(n+2\epsilon)}{2n\epsilon+1} & \text{if } 0 < \epsilon \leq \frac{1}{2n-4} \\ n-1 & \text{if } \frac{1}{2n-4} \leq \epsilon \leq \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} \leq \epsilon < 1, \end{cases}$$

For $n = 2$

$$K^*_{nu}(n,\epsilon) = \begin{cases} \frac{3}{4\epsilon+1} + 1 & \text{if } 0 < \epsilon \leq \frac{1}{2} \\ \frac{1}{\epsilon} & \text{if } \frac{1}{2} \leq \epsilon < 1 \end{cases}$$

Proof. It follows immediately by Proposition 5. □

Remark 1: The NUA where $k_1 = n$ is not defined (the formal definition of NUA assigns the incumbent winners license for the highest losing bid among all incumbent bidders). We could extend our definition to the case $k_1 = n$ if we allow the innovator to charge a minimum reservation price. For $n = 1$ the minimum reservation price should be $\pi_1(0,k_2) - \pi_0(1,k_2)$. With this definition it is easy to verify that the optimal (k_1,k_2) in case $n = 1$ is $k_1 = 1$ and $k_2 = 0$.

Corollary 3. *In equilibrium of G_{nu} (i) the innovator sells licenses to entrant only if he also sells licenses to all (but one) incumbent firms. (ii) The post-innovation market size is larger the smaller is the magnitude of innovation. The post-innovation market size is at most double the size of the pre-innovation market. (iii) For a significant innovation ($\epsilon \geq \frac{1}{2}$), regardless of the pre-innovation market size, the optimal number of licenses for the innovator is the minimum number needed to drive any non-licensee firm out of the market.*

Proof. (i) For $n \geq 3$, the claims follow from Proposition 5 part (i), the inequality $\frac{1}{2n-4} \leq \frac{2}{3n-5}$, and from $k_1^*(n,\epsilon)$ being decreasing in ϵ. If $n = 1$ or $n = 2$ the claim is an immediate consequence of part (ii) of Proposition 5. Parts (ii) and (iii) are straightforward again from Proposition 5. □

Part (i) of Corollary 3 asserts that if $K^*_{nu} > n-1$ Inn sells $n-1$ licenses to incumbent firms and the remaining $K^*_{nu}-(n-1)$ licenses he sells to entrants. If $K^*_{nu} \leq n-1$ he sells all K^*_{nu} licenses just to incumbent firms.. On one hand each entrant is willing to pay all her profit for a license (while each incumbent is willing to pay only his incremental profit), but on the other hand an entrant licensee increases the number of active firms by 1 causing the Cournot profit of each firm to shrink. The effect of a smaller competition on the revenue of the innovator is larger and the innovator prefers incumbent firms on entrants. Part (ii) asserts that the diffusion of technology is higher for smaller magnitude of innovation. The (negative) competition effect of additional licensee on the innovator's revenue is more significant the higher is the magnitude of the innovation and as a result the innovator is more reluctant to issue a large number of licenses.

Proposition 6. *Consider the game G_{nu}. (i) the innovator's equilibrium payoff is:*
For $n \geq 3$

$$\pi^*_{nu}(n,\epsilon) = \begin{cases} \frac{4\epsilon^2+4n\epsilon+1}{4(n+1)} & \text{if } 0 < \epsilon \leq \frac{1}{2n-4} \\ \frac{(n-1)\left(-(n-3)\epsilon^2+2\epsilon\right)}{n+1} & \text{if } \frac{1}{2n-4} < \epsilon \leq \frac{2}{3n-5} \\ \frac{(n\epsilon+\epsilon+2)^2}{8(n+1)} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \epsilon & \text{if } \frac{2}{n+1} \leq \epsilon < 1. \end{cases}$$

For $n = 2$

$$\pi^*_{nu}(n,\epsilon) = \begin{cases} \frac{4\epsilon^2+4n\epsilon+1}{4(n+1)} & \text{if } 0 < \epsilon \leq \frac{1}{2} \\ \epsilon & \text{if } \frac{1}{2} \leq \epsilon < 1. \end{cases}$$

(ii) The post innovation market price is:
For $n \geq 3$

$$p^*_{nu}(n,\epsilon) = \begin{cases} c + \frac{1-2\epsilon}{2(n+1)} & \text{if } 0 < \epsilon \leq \frac{1}{2n-4} \\ c + \frac{1-(n-1)\epsilon}{n+1} & \text{if } \frac{1}{2n-4} \leq \epsilon \leq \frac{2}{3n-5} \\ c + \frac{2-(n+1)\epsilon}{4(n+1)} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ c & \text{if } \frac{2}{n+1} \leq \epsilon < 1, \end{cases} \tag{11}$$

For $n = 2$

$$p^*_{nu}(n,\epsilon) = \begin{cases} c + \frac{1-2\epsilon}{2(n+1)} & \text{if } 0 < \epsilon \leq \frac{1}{2} \\ c & \text{if } \frac{1}{2} \leq \epsilon < 1 \end{cases}$$

Proof. See A.1.5 of the Appendix. □

By Proposition 6 $p^*_{nu}(n,\epsilon) \geq c$ for $0 < \epsilon < 1$. For significant innovations ($\frac{2}{n+1} \leq \epsilon < 1$ for $n \geq 3$ and $\frac{1}{2} \leq \epsilon < 1$ for $n = 2$) the post innovation market price is c, the pre-innovation marginal cost. If the innovation is drastic ($\epsilon > 1$) then the market price, $c - \frac{\epsilon - 1}{2}$, is the monopoly price under the new technology and it is smaller than c. The innovator sells in this case an exclusive license to an incumbent firm and all other incumbent firms or entrants are out of the market.

Corollary 1 and Corollary 6 allow us to compare the payoff of the luckiest innovator in UA with the payoff of the innovator in NUA.

Proposition 7. *Let $n \geq 3$. Then $\pi^*_{nu}(n,\epsilon) > \pi^*_u(n,\epsilon)$ iff $\epsilon < h(n)$.*

Here $h(n) \geq g(n) \geq 0$. The formula of $h(n)$ is given in A.1.1 of the Appendix.

Proof. See A.1.6 of the Appendix. □

When the innovation is less significant, the innovator in both UA and NUA sells relatively large number of licenses (propositions 3 and 5). In particular, in NUA the innovator sells large number of licenses to entrants, in addition to all (but one) incumbent firms (Proposition 5). In this case (i) the level of competition in UA is already high and the additional willingness to pay of every incumbent licensee for further entry prevention is relatively small and (ii) the number of entrant licensees in NUA is relatively large and the innovator's gain from collecting the entire profit of entrant licensees (as opposed to UA where he can extract only a portion of this profit) is large. The net effect of less significant innovations on the revenue of the innovator is in favor of NUA. This net effect is opposite for significant innovations, in which case the innovator in both UA and NUA sells relatively small number of licenses (in NUA he sells fewer, sometimes even 0, licenses to entrants). The additional willingness to pay of every incumbent licensee in UA for further entry prevention is high compare with the innovator's gain in NUA from collecting the entire profit of entrant licensees, especially since their number is relatively small.

If however the innovator is not too lucky, UA could emerge with an equilibrium outcome where all winners are entrant firms. In this case if $0 < \epsilon < \frac{2}{n+1}$, the innovator obtains in UA strictly less than in NUA. Note that the innovator obtains the same payoff in both auctions if $\frac{2}{n+1} \leq \epsilon < 1$. A less optimistic innovator therefore may prefer the non-uniform auction irrespective of the magnitude of innovation. We illustrate the above in the following example.

Example: Suppose $\epsilon = 0.2$ and $n = 5$. In NUA, the innovator's unique equilibrium payoff is $4\big(\pi_1(1,4) - \pi_0(1,4)\big) = 0.213$ which is obtained when he auctions off 4 licenses only to incumbent firms. In UA the innovator's highest equilibrium payoff is $4\big(\pi_1(1,4)-\pi_0(2,4)\big)=0.214$. It is obtained when he auctions off 4 licenses and all winners happen to also be incumbent firms. Interestingly enough, to support this equilibrium in UA the 5th highest bid of 0.214 has to be submitted by entrants only. In this case each of the 4 incumbent licensees pays more in UA than in NUA in attempt to limit entry. However, there are other equilibrium points in UA which yields the innovator a much lower payoff. For instance there is an equilibrium in which all of the 4 winners are entrants (this follows by Proposition 2). In this case the innovator obtains only $4\pi_1(5,4)=0.194$[4].

2.2.3 Semi-Uniform auction

We introduce another auction mechanism, a semi-uniform auction (SUA), with a weaker asymmetry requirement than the non uniform auction. In this auction the innovator chooses (k_1,k_2), $1\le k_1\le n-1$ and $k_2\ge 0$. The k_1 highest incumbent bidders and the k_2 highest entrant bidders win the auction and all of them pay the same license fee which is the lowest winning bid[5]. Note that the willingness to pay of an incumbent is 0 if $k_1=n$. This is the reason we restrict our analysis to $k_1\le n-1$. In SUA, like in NUA, the innovator controls the number of incumbent and the number of entrant licensees, but unlike NUA the innovator charges the same amount to every licensee.

Let G_{su} be the game associated with SUA. In the subgame $G_{su}(k_1,k_2)$ of G_{su} each incumbent is willing to pay

$$w_l(k_1,k_2)=\pi_1(n-k_1,k_1+k_2)-\pi_0(n-k_1,k_1+k_2).$$

Each entrant is willing to pay

$$w_e(k_1,k_2)=\pi_1(n-k_1,k_1+k_2).$$

It is easy to verify that the innovator's equilibrium payoff in $G_{su}(k_1,k_2)$ is $(k_1+k_2)w_l(k_1,k_2)$ for $k_1>0$ and $k_2w_e(0,k_2)$ for $k_1=0$.

Remark: Notice that some entrants that bid above the SUA license fee do not obtain license. Yet in equilibrium the innovator has no incentive to increase k_2 since

[4] This is the highest equilibrium payoff when 4 winners are all entrants. There are other equilibrium in which the innovator ends up with zero equilibrium payoff.

[5] Note that in SUA the highest losing bid, if submitted by an entrant, may be higher than the willingness to pay of an incumbent winner. To avoid this problem we define the license fee as the lowest winning bid.

it will increase competition and will lower his total revenue.

Let $\pi^*_{su}(n,\epsilon)$ be the innovator's equilibrium payoff in G_{su}.

$$\pi^*_{su}(n,\epsilon) = \max\left(\pi^0_{su}(n,\epsilon), \hat{\pi}_{su}(n,\epsilon)\right) \tag{12}$$

where

$$\pi^0_{su}(n,\epsilon) = \max_{k_2 \geq 1} k_2 w_e(0,k_2)$$

and

$$\hat{\pi}_{su}(n,\epsilon) = \max_{\substack{1\leq k_1 \leq n-1\\ 0\leq k_2}} (k_1+k_2) w_l(k_1,k_2). \tag{13}$$

Note that when $k_1 = 0$ each entrant licensee pays her entire profit for a license. But when $k_1 > 0$ each entrant licensee pays less, only the willingness to pay of an incumbent licensee.

Proposition 8. *(i) $\pi^*_{su}(n,\epsilon) \leq \pi^*_u(n,\epsilon)$ and (ii) $\pi^*_{su}(n,\epsilon) \leq \pi^*_{nu}(n,\epsilon)$.*

Proof. (i) In UA the highest equilibrium payoff of the innovator is

$$\pi^*_u(n,\epsilon) = \max\left(\pi^0_u(n,\epsilon), \hat{\pi}_u(n,\epsilon)\right)$$

where $\pi^0_u(n,\epsilon) = \max_{k\geq 1} k w_e(0,k)$ and $\hat{\pi}_u(n,\epsilon) = \max_{\substack{1\leq k_1 \leq n-1\\ 0\leq k_2}} (k_1+k_2) w_h(k_1,k_2)$. Part (i) follows from $w_h(k_1,k_2) \geq w_l(k_1,k_2)$ for any (k_1,k_2).

(ii) Follows from the fact that for any (k_1,k_2), NUA yields the innovator a higher payoff than SUA. □

The innovator in UA can choose only k while in SUA he can choose in addition the partition of k. In the first glance the innovator should always obtain a higher payoff in SUA than in UA. But this is not necessarily the case. There are cases in which UA yields the innovator a higher payoff than SUA since an incumbent licensee is willing to pay more in UA for further entry prevention. As for the comparison between SUA and NUA, note that the innovator can charge the entrant and incumbent licensees different fee, therefore for any (k_1,k_2), $k_2 > 0$, NUA yields the innovator a higher payoff than SUA.

Like in NUA also in SUA the innovator obtains the highest payoff in a monopoly market. This is stated in the next proposition.

Proposition 9. *Suppose the innovator sells licenses by SUA. Then monopoly industry provides the innovator with the highest incentive to innovate.*

Proof. The proof is similar to that of Proposition 4, and hence omitted. □

We next analyze for any industry size n the optimal licensing strategy of the innovator in SUA, as a function of ϵ. Unlike NUA this strategy is discontinuous for one value of ϵ. The equilibrium revenue of the innovator is however, continuous for all $\epsilon > 0$. We will discuss this point after we state our next proposition.

Proposition 10. *Consider the equilibrium of G_{su}. For $n \geq 3$ there exists $r(n)$, $r(n) > 0$, such that (i) if $r(n) < \epsilon \leq 1$ then the innovator sells positive number of licenses to entrants only if he sells $n-1$ licenses to all (but 1) incumbent firms. In this region the total number of licensees is larger, the smaller is the magnitude of innovation. (ii) At $\epsilon = r(n)$ the innovator has two optimal licensing strategies: either selling $n-1$ licenses to incumbent firms and some licenses to entrants, or selling $n+1$ licenses to only entrants. (iii) if $0 < \epsilon < r(n)$, the innovator sells $n+1$ licenses to only entrants.*

Proof. See A.1.10 of the Appendix. □

The maximizer of $\pi^*_{su}(n, \epsilon)$ is given by

$$k_1^{s*}(n,\epsilon) = \begin{cases} 0 & \text{if } 0 < \epsilon < r(n) \\ \hat{k}_1(n,\epsilon) & \text{if } r(n) \leq \epsilon < 1 \end{cases}$$

and

$$k_2^{s*}(n,\epsilon) = \begin{cases} n+1 & \text{if } 0 < \epsilon < r(n) \\ \hat{k}_2(n,\epsilon) & \text{if } r(n) \leq \epsilon < 1. \end{cases}$$

Where for $n \geq 3$

$$\hat{k}_1(n,\epsilon) = \begin{cases} n-1 & \text{if } 0 < \epsilon \leq \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} \leq \epsilon < 1 \end{cases}$$

$$\hat{k}_2(n,\epsilon) = \begin{cases} 2\sqrt{2+\frac{1}{\epsilon}} - (n+1) & \text{if } 0 < \epsilon \leq \frac{4}{n^2+2n-7} \\ 0 & \text{if } \frac{4}{n^2+2n-7} \leq \epsilon < 1. \end{cases}$$

For $n = 2$

$$\hat{k}_1(2,\epsilon) = 1$$

$$\hat{k}_2(2,\epsilon) = \begin{cases} 2\sqrt{2+\frac{1}{\epsilon}} - 3 & \text{if } 0 < \epsilon \leq \frac{1}{2} \\ \frac{1}{\epsilon} - 1 & \text{if } \frac{1}{2} \leq \epsilon < 1. \end{cases}$$

It is shown in the Appendix that for $3 \leq n \leq 16$, $0 < r(n) < \frac{4}{n^2+2n-7}$. Therefore for $3 \leq n \leq 16$

$$k_1^{s*}(n,\epsilon)=\begin{cases} 0 & \text{if } 0<\epsilon\leq r(n) \\ n-1 & \text{if } r(n)<\epsilon\leq \frac{2}{3n-5} \\ \frac{n+1}{4}+\frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5}\leq\epsilon\leq\frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1}\leq\epsilon<1 \end{cases}$$

and

$$k_2^{s*}(n,\epsilon)=\begin{cases} n+1 & \text{if } 0<\epsilon\leq r(n) \\ 2\sqrt{2+\frac{1}{\epsilon}}-(n+1) & \text{if } r(n)<\epsilon\leq\frac{4}{n^2+2n-7} \\ 0 & \text{if } \frac{4}{n^2+2n-7}\leq\epsilon<1. \end{cases}$$

For $3\leq\epsilon\leq 16$, when $\epsilon=1$ the innovator sells an exclusive license to incumbent firm and he increases the number of licenses he sells to incumbent firms as ϵ decreases as long as $\epsilon\geq\frac{4}{n^2+2n-7}$. At this point he sells $n-1$ licenses, all of them to incumbent firms. When $r(n)<\epsilon<\frac{4}{n^2+2n-7}$, the innovator sells $2\sqrt{2+\frac{1}{\epsilon}}-(n+1)$ to new entrants in addition to the licenses he sells to incumbent firms. When $0<\epsilon<r(n)$ he sells $n+1$ licenses, all of them to entrants.

The reason for selling licenses only to entrant in SUA for less significant innovation is the ability of the innovator to extract the entire industry profit of every entrant licensee, as opposed to the case where he sells some licenses also to incumbent firms. In the latter case the license fee an entrant pays is equal to the willingness to pay of an incumbent licensee which decreases to zero as $\epsilon\to 0$. To illustrate this point suppose that $\epsilon=0$[6]. In this case if the innovator sells some licenses to incumbent firms, every licensee in SUA will pay zero license fee to the innovator. If instead, the innovator sells licenses only to entrants, he obtains the entire industry profit of all new entrant licensees (entrants would not be able to enter the market otherwise). If there are n incumbent firms the linear demand assumption implies that the innovator maximizes his revenue if the number of entrant licensees is $n+1$.

Let us compare the outcome of SUA with the outcome of NUA. First observe that for $\epsilon>r(n)$ in both SUA and NUA the total number of licenses the innovator sells is decreasing in the magnitude of the innovation and the innovator may sell licenses to entrants, only if he also sells licenses to all (but 1) incumbent firms. The main difference between SUA and NUA is when $\epsilon<r(n)$. In this case, unlike NUA, the innovator in SUA sells licenses only to new entrants and not to incumbent firms. This shift in the innovator's optimal strategy generates a discontinuity in the number of licenses at $\epsilon=r(n)$. In contrast, the innovator in NUA can discriminate

[6] This is the case where the innovator provides no improvement in cost but his technology allows free entry.

the entrant licensees and can extract their entire industry profit whether or not he sells licenses to incumbent firms. Therefore in NUA the innovator sells licenses to both new entrants and incumbent firms, even for small ϵ.

Let K^*_{su} and K^*_{nu} be the total number of licenses the innovator sells in SUA and NUA, respectively.

Proposition 11. *Suppose $n \geq 2$. There exists $l(n)$, $0 < l(n) < 1$, such that if $0 < \epsilon \leq l(n)$, $K^*_{nu}(n, \epsilon) > K^*_{su}(n, \epsilon)$.*

Proof. See A.1.13 of the Appendix. □

Proposition 11 shows that comparing with SUA, for relatively small innovation, NUA results in a higher diffusion of technology. As shown in Proposition 10 for less significant innovation, the innovator in SUA does not sell licenses to incumbent firm while in NUA he sells licenses to entrants in addition to all (but 1) incumbent firms. Therefore the ability to charge new entrant licensees different than incumbent licensees has positive effect for less significant innovation as it induces higher diffusion of innovation. This also implies lower post innovation market price and higher social welfare in NUA as compare to SUA[7].

2.3 Entry Vs. No Entry

Most of the literature on optimal licensing of process innovations ignore possible entry. Our next goal is to compare our results with the existing literature on optimal licensing where entry is excluded. As shown in the previous section, UA has continuum of equilibrium points and there is no obvious way to predict which equilibrium will emerge. SUA and NUA both have unique equilibrium outcome but SUA always yields a lower payoff of the innovator compared to NUA (Proposition 8). Therefore we base our study on the comparison between G_0 and G_{nu}, where G_0 is the game defined similarly to G_{nu}, but where entry is excluded.

Suppose bidders do not use dominated strategies. The willingness to pay of each bidder in $G_0(k)$, $k \geq 1$, is uniquely determined and so is the innovator's equilibrium payoff. The next proposition characterizes the innovator's optimal licensing strategy in G_0.

Proposition 12. *The unique equilibrium licensing strategy of the innovator in G_0 is:*

[7]When $\epsilon \leq l(n)$ the market price in SUA is $c + \frac{1-(n+1)\epsilon}{2(n+1)}$ (easy to verify) while the market price in NUA is $c + \frac{1-2\epsilon}{2(n+1)}$ (Proposition 6).

(i) For $n \geq 3$

$$k_0^*(n,\epsilon) = \begin{cases} n-1 & \textit{if } 0 < \epsilon < \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \textit{if } \frac{2}{3n-5} \leq \epsilon < \frac{2}{n+1} \\ \frac{1}{\epsilon} & \textit{if } \frac{2}{n+1} \leq \epsilon < 1. \end{cases} \tag{14}$$

(ii) For $n \leq 2$

$$k_0^*(n,\epsilon) = n-1.$$

For a proof see Kamien, Oren, and Tauman (1992).

Observe that by Proposition 12, $k_0^*(n,\epsilon) = k_1^*(n,\epsilon)$ where $k_1^*(n,\epsilon)$, the optimal number of incumbent licensees in G_{nu}, is given in Proposition 5. This is not very surprising in light of part (i) of Corollary 3.

Corollary 4. *Allowing entry will not change the innovator's revenue nor the social welfare if either (i) $\epsilon > 0$ and n is sufficiently large, or (ii) $n \geq 3$ and ϵ is sufficiently large.*

Proof. By propositions 5 and 12 (i) $k_0^*(n,\epsilon) = k_1^*(n,\epsilon)$ for any n and ϵ and (ii) $k_2^*(n,\epsilon) = 0$ for $n \geq 3$ and $\frac{1}{2n-4} \leq \epsilon < 1$. □

By propositions 5 and 12 for less significant innovations ($0 < \epsilon \leq \frac{1}{2n-4}$), $k_0^*(n,\epsilon) = k_1^*(n,\epsilon)$ and $k_2^* > 0$. In this case G_{nu} results in a higher diffusion of technology and larger post-innovation market size. The difference in market size is larger for smaller magnitude of innovation.

The next proposition characterizes the innovator's revenue and the post-innovation market price in G_0.

Proposition 13. *Consider the game G_0. (i) the innovator's equilibrium payoff is For $n \geq 3$*

$$\pi_0^*(n,\epsilon) = \begin{cases} \frac{(n-1)\left(-(n-3)\epsilon^2+2\epsilon\right)}{n+1} & \textit{if } 0 < \epsilon \leq \frac{2}{3n-5} \\ \frac{(n\epsilon+\epsilon+2)^2}{8(n+1)} & \textit{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \epsilon & \textit{if } \frac{2}{n+1} \leq \epsilon < 1. \end{cases}$$

For $n \leq 2$

$$\pi_0^*(n,\epsilon) = \frac{(n-1)\big(-(n-3)\epsilon^2+2\epsilon\big)}{n+1}.$$

(ii) The post-innovation market price is

For $n \geq 3$

$$p_0^*(n,\epsilon) = \begin{cases} c + \frac{1-(n-1)\epsilon}{n+1} & \text{if } 0 \leq \epsilon \leq \frac{2}{3n-5} \\ c + \frac{2-(n+1)\epsilon}{4(n+1)} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ c & \text{if } \frac{2}{n+1} \leq \epsilon < 1. \end{cases}$$

For $n \leq 2$

$$p_0^*(n,\epsilon) = c + \frac{1-(n-1)\epsilon}{n+1}.$$

Proof. Follows from Proposition 12 □

Corollary 5. *Suppose* $n \geq 3$. $\pi_{nu}^*(n,\epsilon) - \pi_0^*(n,\epsilon)$ *and* $p_0^*(n,\epsilon) - p_{nu}^*(n,\epsilon)$ *are both decreasing to* 0 *if either* n *is increasing indefinitely or* ϵ *is increasing to* $\frac{1}{2}$.

Proof. See A.1.7 of the Appendix. □

Corollary 5 asserts that for any $\epsilon > 0$ the difference in price and in the innovator's payoff between G_{nu} and G_0 are decreasing in n. The innovator sells licenses to entrants only if he sells licenses to all (but one) incumbent firms (Corollary 3). Therefore the larger is the pre-innovation market size the more reluctant is the innovator to sell licenses to entrants and the smaller is the difference between G_{nu} and G_0. Corollary 5 also asserts that for any $n \geq 3$ this difference is decreasing in ϵ, when the magnitude of innovation is relatively large, the innovator is best off selling smaller number of licenses (Proposition 5) and entrants are less likely to become licensees. The difference between G_{nu} and G_0 is larger for relatively less significant innovation.

Next we characterize the market structure that provides the highest incentive to innovate in G_0.

Proposition 14. *An oligopoly industry with size* $n = \max\left(3, 2\sqrt{2+\frac{1}{\epsilon}} - 1\right)$ *maximizes the revenue of the innovator in* G_0.

Proof. See A.1.8 of the Appendix. □

When entry is excluded, the incentive to innovate is maximized when the market is oligopoly (with at least 3 firms). This is not the case where the market is open to entry. In this case the incentive to innovate is maximized in a monopoly market (Proposition 4).

Chapter 3

3 Sequential Licensing of Innovation

3.1 Introduction

Keurig holds the patent for the packaging lines used to manufacture k-cups. Green Mountain is the first and the only licensee of this patent until 2000. In 2000 and 2001, Keurig issued four additional licenses[8]. Soon after this, Green Mountain increased its percentage ownership of Keurig (by 2003 it had a 43% ownership) and finished the full acquisition in 2006. In principle, the total industry profit is maximized under a monopolistic market compared to oligopoly since more firms induces more competition which damages the total revenue. Therefore it is puzzling why did Keurig took an action which damages the total industry profit to be shared (selling four additional licenses) before bargaining with the current incumbent, Green Mountain. It worth notice that in addition to this example, the action of an innovator to sell licenses to entrants in a pre-innovation monopolistic market is not rarely observed. This paper provides a novel explanation to the inefficient outcome in bargaining under the patent licensing context. The main idea is that the option of licensing an innovation can be used by innovators strategically to raise their bargaining power with incumbent firms. The action of selling licenses to entrants on one hand reduces the total industry profit to be shared, while on the other hand makes a more severe threat on the incumbent credible.

We consider a specific industry with one incumbent and many potential entrants. It is assumed that initially the high entry cost does not enable a profitable entry and the incumbent is a monopoly. Suppose that an outside innovator obtains a patent on a new technology that eliminates the entry cost but has a marginal cost which is different from the current one. The innovator can sell his intellectual property (IP) to the incumbent through bargaining. Even though the technology itself maybe useless for the incumbent, he may purchase the IP to limit or exclude further entry. However, before approaching the incumbent, the innovator may sell a few licenses to new entrants. A licensing contract with an entrant specifies the license fee together with the maximum number of licenses that can be sold. The contracts are signed sequentially and they are bound by previous commitments.

Selling licenses before bargaining (with the incumbent) reduces the total industry profit to be allocated but enables a better credible threat on the incumbent firm and

[8] To Tully's coffee, Timothy's World Coffee, Diedrich Coffee and Van Houttee

hence may increase the innovator's payoff. As a result, the bargaining outcome is not ex-ante Pareto-efficient. We show that such inefficiency occurs when the bargaining time takes a large proportion of the patent right life; or when there is a constraint on the number of times the two bargainers can meet. Furthermore, we show that the inefficiency is less significant when the innovator has a higher bargaining power; when the new technology is less efficient; or when the patent lasts for a longer period.

3.2 Model

We consider an economy with one good that is initially produced by a single monopoly at marginal cost c. Total demand is fixed at $p = a - q$, $a > c$. There are many potential entrants who are currently unable to enter the market due to the high current fixed entry cost. An innovator holds a patent that eliminates the entry cost but with a different marginal cost $c + \epsilon$. Here ϵ can be either positive or negative. The patent right expires after T periods. Denote $x = \frac{\epsilon}{a-c}$. Throughout this paper, we assume $a - c$ to be fixed and normalize it to be one.

The innovator (Inn) may sell his intellectual property (IP) to the incumbent (Inc) through bargaining but may sell licenses to new entrants before approaching Inc. If licenses are sold to new agents Cournot competition arises at the end of each period. Selling licenses to entrants takes no time while the bargaining between Inn and Inc takes one period. In the appendix we relax this assumption and allow an agreement to be reached earlier if both bargainers agree to do so. It turns out that Inc is always best off delaying making the agreement and the bargaining always takes the full period.

After the bargaining stage, the owner of the IP chooses the number of additional licenses to sell, and exclusively collects all corresponding license fees. Namely if the an agreement between Inn and Inc was reached, the new owner of the IP, Inc, chooses the number of additional entrants to bring in. If, however, previous bargaining fails, Inn chooses the number of additional licenses to sell.

The licensing contract given to entrants and the bargaining procedure between Inn and Inc will be specified later on. Note that here we assume Inc has some power on the bargaining with Inn, while the entrants have none. The reason follows from the assumption that there are many potential entrants, so that each one of them has negligible bargaining power related to the innovator, which enables the innovator to give a take-it-or-leave-it offer when selling them the license. The monopoly incumbent, on the other hand, stands in a better bargaining position.

The goal of Inn is to divide the total industry profit with Inc, through bargaining. In the main part of the paper, we assume the bargaining between Inn and Inc can take place only once. Such assumption fits the scenario in which either Inc can credibly commit not to meet with Inn again if the first bargaining fails, or the bargaining takes a large proportion of the patent right life so that it's not profitable for Inn to approach Inc for a second time. One of the interesting outcome of our model is that Inn may benefit from selling licenses to new entrants before approaching Inc. Such action, although brings more competition to the market and thus reduces the total industry profit, increases the threatening position of Inn in the subsequent bargaining game.

We then relax the assumption that Inn and Inc can bargain only once and allow Inn to approach Inc even before he carries out the action that causes the inefficiency. It is shown that even if the bargainers can meet twice and anticipate the inefficient outcome, there is no credible way to avoid it. This is because under some parameters, it is best off for Inn to bring in some entrants even before the first meeting.

It worth notice that the model described in this section is of a hybrid one, which combines the strategic choice of Innovator together with the cooperative concept *generalized Nash bargaining solution*. The hybrid model can be transformed into a pure non-cooperative game by introducing a mediator who makes offers to Inc on behalf Inn and collects a fee if a deal is reached. This non-cooperative approach is shown in the appendix for the special case $T = 2$ and $\epsilon \geq 0$. The result remains the same despite the different set-up.

3.3 $T = 2$

First we analyze the special case $T = 2$ and show that inefficient outcome may arise as a result of Inn bringing in entrants before bargain with Inc. We show that Inn is less willing to bring in entrants before bargaining if (i) Inn has a higher bargaining power; (ii) the innovation is less efficient or (iii) the patent lasts longer. It is then shown that if there is a probability of the patent being obsolete at the end of each period, then Inn is more willing to bring in entrants before meeting with Inc. It is shown later that these results hold also for $T > 2$.

3.3.1 $\epsilon \geq 0$

It is assumed that the patent right expires after two periods and that Cournot competition takes place at the end of each period. The technology held by Inn has a higher marginal cost compared to that held by Inc thus the technology itself is useless for Inc, though he may purchase the IP to limit or exclude further entry.

The innovator first decides on the number of entrants to bring into the market (denoted by t), as well as the contract that is offered to each one of them. Each contract is of the form (α, δ), where α is an upfront license fee and δ is a commitment of Inn to sell no more than δ licenses in total.

Then Inn and Inc engage into a bargaining in which the Pareto frontier is defined as the subsequent optimal industry profit; while the disagreement payoffs are defined as Inn's (or Inc's) payoff in case the bargaining fails (to be specified below). The generalized Nash bargaining solution is adopted as the outcome of this bargaining problem.

(i) The constrained optimal total industry profit to be shared in the bargaining game is defined as

$$v_2 = m(x,t)\pi_e(m(x,t)+t) + \pi_0(m(x,t)+t) \tag{15}$$

Here

$$m(x,t) = \underset{m \leq \delta, m \in \mathbb{N}_0}{\operatorname{argmax}} \left[m\pi_e(m+t) + \pi_0(m+t)\right] \tag{16}$$

represents the number of additional licenses Inc will sell in order to maximize his subsequent profit if he obtains the IP. This maximized subsequent profit of Inc is in turn defined as the size of the "cake" to be shared in the bargaining process.

(ii) The disagreement payoffs are defined as

$$d_{inn} = n(t)\pi_e(t+n(t)) \tag{17}$$

and

$$d_{inc} = \pi_0(t+n(t)) \tag{18}$$

Here

$$n(t) = \underset{n \leq \delta, n \in \mathbb{N}_0}{\operatorname{argmax}} \, n\pi_e(t+n) \tag{19}$$

represents the number of additional licenses Inn will sell in order to maximize his subsequent profit if bargaining fails. In turn, Inn's disagreement payoff is defined as the Cournot profit of all these additional licensees, and Inc's disagreement payoff is defined as his Cournot profit facing $n(t)$ additional competitors. Note that although a second bargaining between Inn and Inc is allowed, Inn cannot benefit from such action in the case $T = 2$. This is because the value of the patent goes to zero at the end of the second bargaining.

To summarize, given any choice of t, a bargaining game is defined by $v_2(t)$ and $(d_{inn}(t), d_{inc}(t))$. The generalized Nash bargaining solution is adopted to this bargaining game. Namely, let

$$s = v_2 - (d_{inn} + d_{inc}) \tag{20}$$

be the surplus. If $s < 0$, Inn and Inc each obtains d_{inn} and d_{inc} from the bargaining process. If $s \geq 0$, from the bargaining process, Inn and Inc each obtains

$$b_{inn} = \beta s + d_{inn} \tag{21}$$

$$b_{inc} = (1-\beta)s + d_{inc} \tag{22}$$

Here β is an exogenously given parameter which captures the relative bargaining power of the innovator. When $\beta = 0.5$, their payoffs coincide with that of the Nash bargaining solution.

It is assumed that for each of the t entrants that are brought into the market before the bargaining stage, he is willing to pay up to $\pi_e(t)+\pi_e(m(x,t)+t)$ if $s(t) \geq 0$, and $\pi_e(t) + \pi_e(n(t)+t)$ if $s(t) < 0$. Namely each entrant is willing to pay up to his total Cournot profit for the license.

It is easy to verify that $s(t) \geq 0$ for all t (under any $x \in [0, 0.5]$ and $\beta \in [0, 1]$). Thus, each of the first t entrants is willing to pay $\alpha = \pi_e(t) + \pi_e(m(x,t)+t)$ for the license. The total payoffs of Inn and Inc are decided by the summation of their respective bargaining payoff and the fee or profit collected prior to the bargaining stage.

$$\pi_{inn} = t\big[\pi_e(t) + \pi_e(m(x,t)+t)\big] + b_{inn} \tag{23}$$

$$\pi_{inc} = \pi_0(t) + b_{inc}$$

The innovator then maximizes over t his payoff π_{inn} where

$$\begin{aligned} \pi_{inn}(x,\beta,t) =& \beta[t\pi_e(t) + \pi_0(t) \\ &+ (t+m(x,t))\pi_e(t+m(x,t)) + \pi_0(t+m(x,t)) \\ &- t(\pi_e(t) + \pi_e(m(x,t)+t)) - n(t)\pi_e(t+n(t)) \\ &- \pi_0(t) - \pi_0(t+n(t))] \\ &+ t(\pi_e(t) + \pi_e(t+m(x,t))) + n(t)\pi_e(t+n(t)) \end{aligned} \tag{24}$$

After simplification

$$\begin{aligned} \pi_{inn}(x,\beta,t) = & \overbrace{\beta[(m(x,t)+t)\pi_e(m(x,t)+t) + \pi_0(m(x,t)+t)] + (1-\beta)t\pi_e(m(x,t)+t)}^{\text{part 1}} \\ & + \underbrace{(1-\beta)n(t)\pi_e(t+n(t)) - \beta\pi_0(t+n(t))}_{\text{part 2}} + t\pi_e(t) \end{aligned} \tag{25}$$

Note that by definition, the value of $m(x,t)$ and $n(t)$ depends on the quantity commitment δ. Let $\tilde{m}(x,t)$ (or $\tilde{n}(t)$) be the number of additional licenses Inn (or Inc) will choose if there are no constraint on the quantity of total licensees. It is shown in the appendix that $\delta < t+\tilde{n}(t)$ can never be an optimal choice. First note that if Inn sets $t+\tilde{m}(x,t) \leq \delta < t+\tilde{n}(t)$, then the constraint only reduces the "threat" Inn can impose on Inc during bargaining while keep the size of the total "cake" to be shared the same - which is not beneficial for Inn. If instead, Inn sets $\delta < t+\tilde{m}(x,t)$, then the same number of additional licenses will be sold in the second period, irregardless of the ownership of the IP. In this case Inn can get nothing from bargaining with Inc so the problem reduced to the one in which Inn chooses the number of entrants to maximizes the two periods' license fee, which is shown to be less profitable for Inn compare to relaxing such quantity constraint. As a result, Inn always commits on some $\delta \geq t+\tilde{n}(t)$ to each of the first t entrants. In other words, in the contract signed with the first t entrants, the quantity commitment δ is not binding. Which $\delta \geq t+\tilde{n}(t)$ Inn chooses doesn't affect our result below.

The optimal choice of t is summarized in Figure 1. Note that when $x \geq 0.5$, the technology held by Inn is so inefficient that its licensee can make no profit, which case is trivial to analyze. Thus we concentrate our analysis on $x \in [0, 0.5]$.

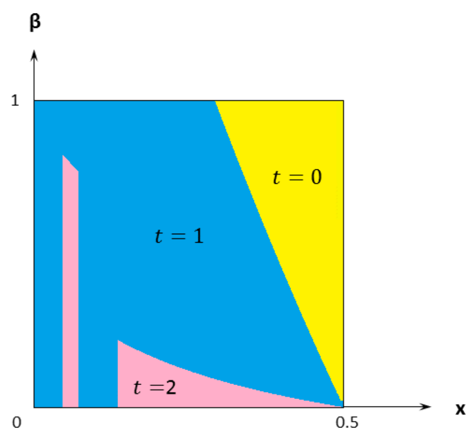


Figure 1

The result is a obtained simply by comparing the value of $\pi_{inn}(x,\beta,0)$, $\pi_{inn}(x,\beta,1)$ and $\pi_{inn}(x,\beta,2)$. It is left to be shown that $t \geq 3$ can not be an optimal choice of Inn, which is degraded into the appendix.

It worth notice that the disconnection of the pink area in Figure 1 results from the restriction that $m(x,t)$ being an integer. If such restriction is removed, namely if Inc is allowed to sell a fraction of licenses once obtaining the IP, Figure 1 transforms into Figure 2. The detailed argument is given in the appendix.

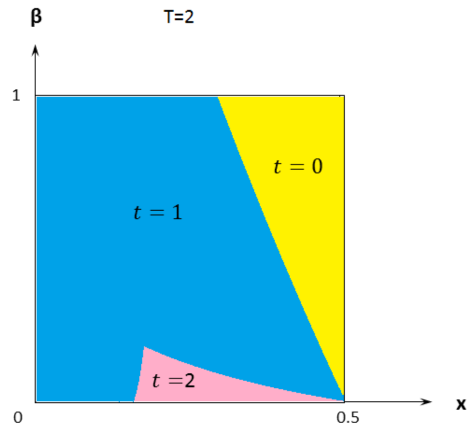


Figure 2

Clearly, when fixing x and increasing β, Inn is best off bringing in less entrants before bargaining with Inc. This is because when Inn obtains a bigger share of the total "cake", his benefit from introducing more competition (which increases the threat on the incumbent) does not compensate for the loss of shrinking the size of the "cake". As a result, when Inn has a bigger bargaining power, he is best off selling fewer licenses prior to the bargaining stage.

Next, assume β is fixed. Note first that if we assume Inc will sell no additional licenses after obtaining the IP (which is true for $t = 0, 1, 2$ under $\frac{1}{6} < x \leq \frac{1}{2}$), Inn is best off bringing in more entrants before bargaining when for smaller x. This is because when the innovation is more efficient, by bringing in one additional entrant, the "damage" imposed on the size of the total "cake" increases, and the increment on the *severity of the threat*[9] on Inc also increases. The damage of sharing a smaller size of the "cake" is divided by both bargainers, while the increment on the severity of the threat benefits Inn exclusively. As a result, when x decreases, the benefit for Inn to bring in more competition exceeds the loss on the size of the "cake", thus he is more willing to bring in more entrants before bargaining with Inc.

[9]The *severity of the threat on Inc* is defined as the negation of the interception of the line containing the disagreement point and the bargaining solution, which formal definition is given in the appendix.

When we take the effect of x on $m(x,t)$ into account, recall that

$$m^*(x,0) = 0$$

$$m^*(x,1) = \begin{cases} -8x+1 & 0 \leq x \leq \frac{1}{8} \\ 0 & \frac{1}{8} < x \leq \frac{1}{2} \end{cases}$$

$$m^*(x,2) = \begin{cases} -12x+2 & 0 \leq x \leq \frac{1}{6} \\ 0 & \frac{1}{6} < x \leq \frac{1}{2} \end{cases}$$

The above argument of Inn's trade off still holds for $\frac{1}{6} < x \leq \frac{1}{2}$, while it is inverted for $0 \leq x \leq \frac{1}{6}$. This is because for big enough innovation (small enough x), after obtaining the IP, Inc who maximizes his subsequent profit sells more additional licenses if there are more entrants already in the market. Clearly, the more licenses he sells the less is the cake to be divided ex-ante since each entrants brings in additional competition. After taking this into account, Inn sells less licenses than he would have.

As an example, Figure 3 shows Inn's total payoff for $t = 0, 1$ and 2 when $\beta = 0.15$. The green, red and black line represent Inn's payoff $\pi_{inn}|_{t=1}$, $\pi_{inn}|_{t=2}$ and $\pi_{inn}|_{t=0}$ respectively.

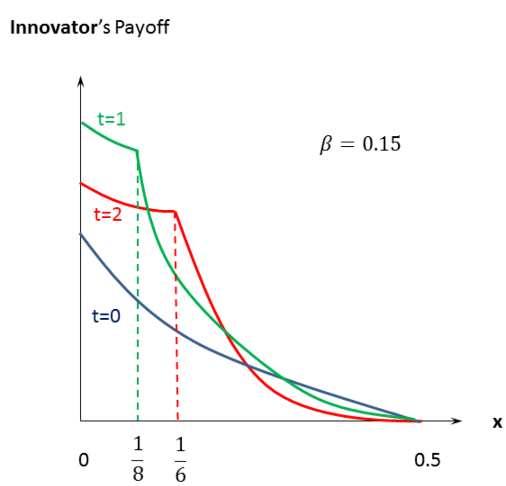


Figure 3

Extension - Nash Variable Threat In the previous section when Inn and Inc bargain, no one has the power to commit on an action in case bargaining fails. In case bargaining fails, the number of additional licenses to sell is defined as the one that maximizes the patent holder's subsequent payoff; while the quantity to produce is defined as the one that maximizes the firm's Cournot profit. We show that Inn can make the disagreement action more harmful to Inc by strategically bringing in entrants prior to the bargaining stage. In this section we assume that each player can commit to a disagreement strategy which will be carried out in case bargaining fails. Inn commits to the number n of entrants to bring in; Inc commits to the quantity q_I to produce.

Let c_i and c_e be the marginal cost of the incumbent and the new entrants, respectively. It is assumed that $a \geq c_i$, $a \geq c_e$ and $c_e = c_i + \epsilon$ with $\epsilon \geq 0$. It can be easily verified that

(i) if $\frac{a-c_i}{2} \geq a - c_e$, the marginal cost of any entrant is so high that producing any positive amount is not profitable. Inc commits to the quantity $q_I = \frac{a-c_i}{2}$, which is the quantity that maximizes his monopoly profit; while Inc commits to any $n \geq 0$. The disagreement payoff of Inn is 0, the disagreement payoff of Inc is $(\frac{a-c_i}{2})^2$. In this case, Inc obtains all the monopoly profit.

(ii) if $\frac{a-c_i}{2} < a - c_e$, Inc commits to the quantity $a - c_e$. This quantity drives the price of the product below c_e, so that entrants are best off producing nothing; while Inn commits to any $n \geq \frac{a-c_e}{c_e-c_i} - 1$. The disagreement payoff of Inn is 0, the disagreement payoff of Inc is $(a-c_e)(c_e-c_i)$. The final payoffs of the two bargainers are

$$\pi_{inn}^{nvt} = \frac{1}{8}(a - c_i - 2\epsilon)^2$$

$$\pi_{inc}^{nvt} = \frac{1}{4}(a - c_i)^2 - \frac{1}{8}(a - c_i - 2\epsilon)^2$$

Clearly $\pi_{inn}^{nvt} < \pi_{inc}^{nvt}$

Next we compare this solution with the one obtained in the previous section, in which case commitment on the disagreement action is not allowed. Assume $\beta = 0.5$. If $\frac{a-c_i}{2} > a - c_e$, again, since there is no profitable entry, Inc obtains all the monopoly profit. If $\frac{a-c_i}{2} < a - c_e$, for the bargaining game in which Inn approaches Inc without bringing in any entrant in advance

$$\pi_{inn}^{0} = \frac{1}{32}(a - c_i)^2 \Big(\frac{2\epsilon}{a - c_i}\Big)\Big(\frac{2\epsilon}{a - c_i} - 1\Big)$$

It's easy to verify that $\pi_{inn}^{nvt} < \pi_{inn}^{0}$ for all ϵ satisfying $\frac{a-c_i}{2} < a - c_e$. Thus, Inn is worse off when commitment on the disagreement action is allowed for both bargainers.

3.3.2 $\epsilon < 0$

In this section we analyze the game for $\epsilon < 0$. That is, the new technology does not only eliminate the entry cost but also reduce the marginal cost.

In this case, upon obtaining the IP, Inc will use the technology himself instead of putting it on the shelf. Denote the number of new entrant firms by b. Let $k \in \{0, 1\}$ be an indicator. It is 1 if the incumbent uses the new technology and 0 otherwise. Let $\pi_0(k, b)$ and $\pi_e(k, b)$ be the Cournot profit of the incumbent and each entrant, respectively. Note that $\pi_e(1, b)$ represents the entrant's payoff when there are in total $b + 1$ firms producing at cost $c - \epsilon$.

Denote $x = \frac{\epsilon}{a-c}$. Assume $a - c$ to be fixed and normalize it to be one. Whenever $x < -1/b$, the incumbent is driven out of the market if he does not have an access to the new technology. It is easy to verify that the Cournot profit of the incumbent and the entrant are:

$$\pi_0(1, b) = \pi_e(1, b) = (a - c)^2 (\frac{1 - x}{b + 2})^2$$

$$\pi_e(0, b) = \begin{cases} (a-c)^2(\frac{1-x}{b+1})^2 & \text{if } b \geq -\frac{1}{x} \\ (a-c)^2(\frac{1-2x}{b+2})^2 & \text{if } b < -\frac{1}{x} \end{cases}$$

$$\pi_0(0, b) = \begin{cases} 0 & \text{if } b \geq -\frac{1}{x} \\ (a-c)^2(\frac{1+bx}{b+2})^2 & \text{if } b < -\frac{1}{x} \end{cases}$$

Given the t entrants which are already in the market and the quantity commitment δ Inn made to them, in the bargaining stage Inn and Inc bargain on the surplus which is the difference between (i) constrained optimal total industry profit in case an agreement is reached and (ii) the total payoffs of the two bargainers in case the bargaining fails.

(i) the constrained optimal total industry profit to be shared in the bargaining game is defined as

$$v_2 = m(t)\pi_e(1, m(t) + t) + \pi_0(1, m(t) + t) \tag{26}$$

Here

$$m(t) = \underset{m \leq \delta, m \in \mathbb{N}_0}{\operatorname{argmax}} [m\pi_e(1, m + t) + \pi_0(1, m + t)] \tag{27}$$

represents the number of additional licenses Inc will sell in order to maximizes his subsequent profit if he obtains the IP. It can be easily verified that $m(t) = t$.

(ii) The disagreement payoffs are defined as

$$d_{inn} = n(t,x)\pi_e(0, t + n(t,x)) \tag{28}$$

and

$$d_{inc} = \pi_0(0, t + n(t,x)) \tag{29}$$

Here

$$n(t,x) = \underset{n \le \delta, n \in \mathbb{N}_0}{\operatorname{argmax}} \, n\pi_e(0, t + n) \tag{30}$$

represents the number of additional licenses Inn will sell in order to maximize his subsequent profit if bargaining fails (provided that no previous contract is violated). Again, for the case $T = 2$, Inn has no incentive to engage into a second bargaining since at the end of this bargaining the value of the patent goes to zero.

Let

$$s = v_2 - (d_{inn} + d_{inc}) \tag{31}$$

It is assumed that if $s \ge 0$ an agreement will be reached where the innovator and the incumbent each obtains a proportion of the surplus s together with their disagreement payoff. Suppose that the relative bargaining power of the innovator and the incumbent are β and $1 - \beta$ respectively where $0 \le \beta \le 1$. Their final payoffs in case $s \ge 0$ are

$$\pi_{inn} = t\alpha + \big[\beta s + d_1\big] \tag{32}$$

$$\pi_{inc} = \pi_0(0,t) + \big[(1-\beta)s + d_2\big] \tag{33}$$

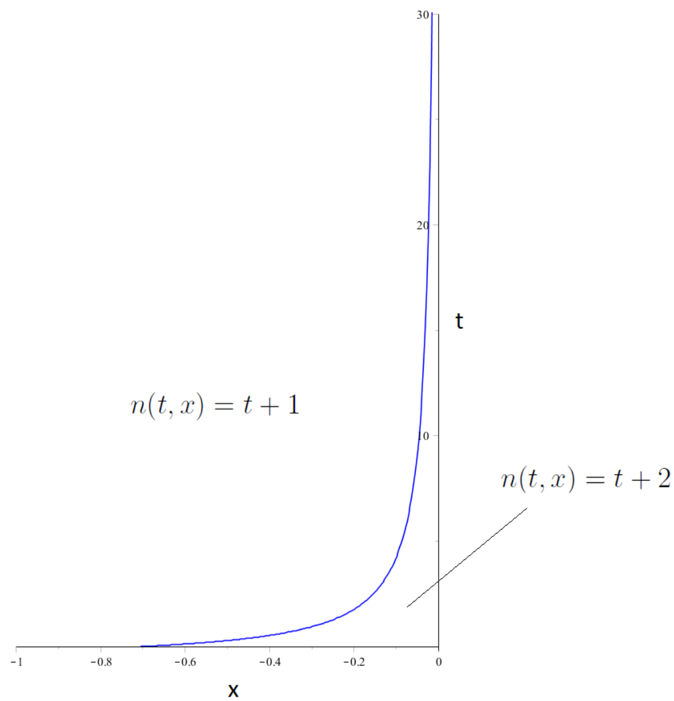
It is assumed that $\alpha = \pi_e(0,t) + \pi_e\big(1, t + m(t)\big)$ if $s \ge 0$, while $\alpha = \pi_e(0,t) + \pi_e\big(0, t + n(t,x)\big)$ if $s < 0$. Namely each entrant is willing to pay up to his total Cournot profit for the license.

Lemma 2. *Given t entrants are already in the market and the bargaining between Inn and Inc fails, if there is no constraint on the number of total licenses Inn can sell, it's best off for him to sell additional $n(t,x)$ licenses, which satisfies*

$$n(t,x) = \begin{cases} t+1 & \textit{if } t \ge f(x) \\ t+2 & \textit{if } t < f(x) \end{cases}$$

with $f(x) = -\frac{2x^2-1}{x(3x-2)}$.

The graphical interpretation of Lemma 1 is shown below. The blue curve represents function $f(x)$. The proof is given in the appendix.



First we analyze Inn's optimal choice of t under the constraint that $\delta \geq t + n(t)$. Namely when the quantity commitment made to the first t entrants are not binding.

Lemma 3. *Once Inc obtains the IP, he uses the technology himself.*

This is not a trivial question since when using the technology himself Inc increases his own Cournot profit but in the meanwhile brings in more severe competition which damages the Cournot payoff of the additional licenses he brings in. The proof is shown in the appendix.

Lemma 4. *Once engaging into bargaining, an agreement between Inn and Inc is always reached.*

Proof. From 3, once obtaining the IP, Inc is best off using it himself in addition to licensing to others. Thus

$$\overbrace{\max_k \Big(k\pi_e(0, t+k) + \pi_0(0, t+k)\Big)}^{\text{Inc not use tech}} \leq \overbrace{\max_m \Big(m\pi_e(1, t+m) + \pi_0(1, t+m)\Big)}^{\text{Inc use tech}} \qquad (34)$$

Using this result we have

$$
\begin{aligned}
d_{inn} + d_{inc} &= n(t,x)\pi_e(0, t + n(t,x)) + \pi_0(0, t + n(t,x)) \\
&\leq \max_k \Big(k\pi_e(0, t+k) + \pi_0(0, t+k) \Big) \\
&\leq \max_m \Big(m\pi_e(1, t+m) + \pi_0(1, t+m) \Big) = v_2
\end{aligned}
\tag{35}
$$

Thus once engaging into a bargaining, an agreement between Inn and Inc is always reached. □

Lemma 5. *If the quantity commitment Inn made to the first t entrants are not binding, namely $\delta \geq t + n(t)$, Inn is best off bringing in one entrant and bargain with Inc for all (x, β) with $x \leq 0$.*

Proof. The action of bringing in one additional entrant before approaching Inc has three effects on Inn's payoff. It (i) changes the amount of license fee Inn collects prior to bargaining. This effect is positive when bringing in the first entrant; it is negative thereafter. (ii) decreases the size of the "cake" to be shared and (iii) decreases the severity of the threat on the incumbent.

Clearly $t \geq 2$ cannot be an optimal choice, since all three effects are negative in such cases. To find the optimal choice of t, we need only to compare $t = 0$ and $t = 1$. It turns out that when bringing in the first entrant, the magnitude of the first effect exceeds that of the later two, thus it is always best off for Inn to bringing in one entrant before bargaining. It worth notice that when $x < -1/b$, Inc is driven out of the market so that Inn can charge the only entrant his monopoly profit.

□

Next we analyze the case when Inn making the quantity commitment $\delta < t + n(t)$.

Lemma 6. *If the quantity commitment Inn made to the first t entrants are binding, namely $\delta < t + n(t)$, Inn is best off committing on $\delta = t$ in which case he obtains only the licensing fee from entrants and does not bargain with Inc. In particular, he sells one license for $x < -\frac{\sqrt{2}}{2}$ and sells two licenses for $x \geq -\frac{\sqrt{2}}{2}$.*

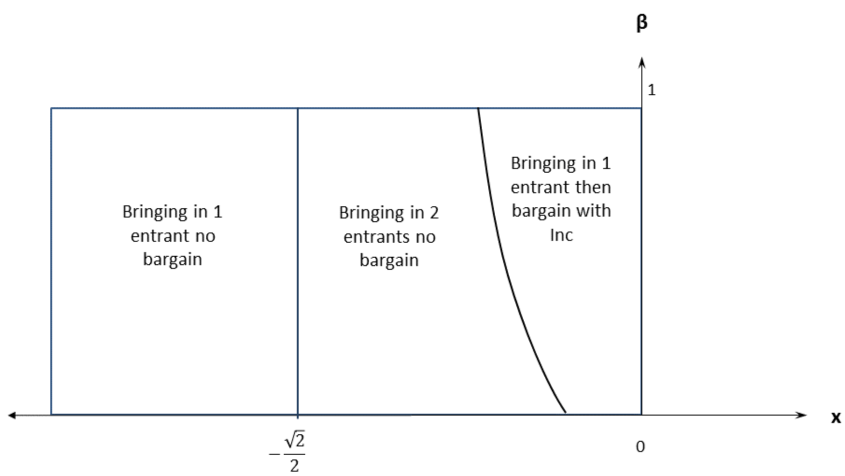
Proof. It's easy to verify that $m(t) < n(t,x)$. Denote $b = \delta - t$. If $m(t) \leq b < n(t,x)$ then the quantity constraint only decreases the severity of the threat on Inc while keep the size of the "cake" to be shared intact. Clearly making the commitment $t + m(t) \leq \delta < t + n(t,x)$ is not beneficial for Inn.

Now consider $b < m(t)$. In this case the same number of licenses will be sold irregardless of the bargaining result (this is shown in the previous section), thus Inn cannot benefit from bargaining with Inc. The problem is then degraded into the one that Inn chooses the number of licenses to maximize the total licensing fee. In other words, if Inn chooses to make a binding quantity commitment to the first t entrants ($\delta < t + n(t)$), then he is best off committing on $\delta = t$ and chooses t that maximizes the Cournot profit of all entrants. By Lemma 2, Inn's payoff is maximized with one entrant for $x < -\frac{\sqrt{2}}{2}$ and with two entrants for $x \geq -\frac{\sqrt{2}}{2}$. Note that for $x < -\frac{1}{2}$ Inc is driven out of the market with two entrants and for $x < -1$ Inc is driven out of the market with one entrant.

□

Combining the above two Lemmas and comparing Inn's payoff for $\delta \geq t + n(t)$ and $\delta < t + n(t)$, Inn's optimal licensing strategy is summarized in the following proposition.

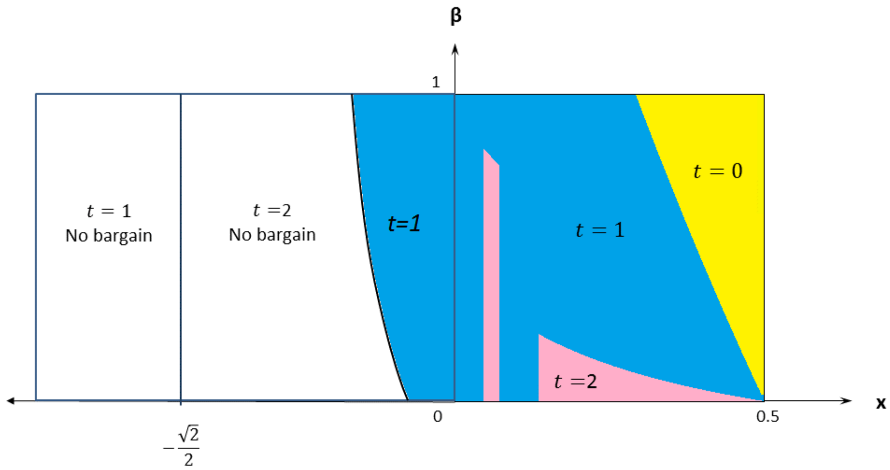
Proposition 15. *Inn's optimal licensing strategy for $T = 2$ and $\epsilon < 0$ is shown in the following graph.*



The following graph summarizes Inn's optimal licensing strategy for both positive and negative ϵ. Note that as long as Inn and Inc engage into bargaining, the quantity commitment Inn made to the first t entrants are not binding.

3.4 $T \geq 3$

In this section, under the assumption of Inn and Inc can meet only once, the effect of the duration of the patent on Inn's behavior is analyzed. We show that



Inn is less willing to bring in entrants before bargaining with Inc when k increases. This is because the licensing fee collects during the bargaining stage accounts for a smaller proportion in Inn's total payoff, which decreases Inn's willingness to bring in entrants prior to the bargaining with Inc. However, it still increases the severity of the threat on Inc thus the range of parameters under which Inn brings in entrants prior to bargaining does not vanish even when $k \to \infty$.

Formally, if the bargaining takes place at the first period and it fails, Inn chooses $n(t)$ that

$$n(t) = \underset{n \leq \delta, n \in \mathbb{N}_0}{\operatorname{argmax}} (k-1) n \pi_e(t+n) \tag{36}$$

Clearly the optimal choice of n remains the same irregardless of k. The disagreement payoffs are

$$d_{inn} = (k-1) n(t) \pi_e(t + n(t)) \tag{37}$$

and

$$d_{inc} = (k-1) \pi_0(t + n(t)) \tag{38}$$

If an agreement is reached, Inc chooses the number $m(x,t)$ of additional entrants to bring in, which satisfies

$$m(x,t) = \underset{m \le \delta, m \in \mathbb{N}_0}{\operatorname{argmax}} (k-1)[m\pi_e(m+t) + \pi_0(m+t)] \tag{39}$$

Clearly the choice of $m(x,t)$ doesn't depend on k. The total "cake" to be shared in the bargaining game is given as

$$v_2 = (k-1)\big[m(x,t)\pi_e(m(x,t)+t) + \pi_0(m(x,t)+t)\big] \tag{40}$$

in which

Given any choice of t, a bargaining game is defined by $v_2(t)$ and $(d_{inn}(t), d_{inc}(t))$. The generalized Nash bargaining solution (to be specified later) is adopted to this bargaining game. To be precise, let

$$s = v_2 - (d_{inn} + d_{inc}) \tag{41}$$

be the surplus. If $s < 0$, Inn and Inc each obtains d_{inn} and d_{inc} from the bargaining process. If $s \geq 0$, from the bargaining process, Inn and Inc each obtains

$$b_{inn} = \beta s + d_{inn} \tag{42}$$

$$b_{inc} = (1-\beta)s + d_{inc} \tag{43}$$

It is assumed that for each of the t entrants that are brought into the market before the bargaining stage, he is willing to pay up to $\pi_e(t) + (k-1)\pi_e(m(x,t)+t)$ if $s(t) \geq 0$, and $\pi_e(t) + (k-1)\pi_e(n(t)+t)$ if $s(t) < 0$.

It is easy to verify that $s(t) \geq 0$ for all t (under any $x \in [0, 0.5]$ and $\beta \in [0,1]$). Thus, each of the first t entrants is willing to pay $\alpha = \pi_e(t) + (k-1)\pi_e(m(x,t)+t)$ for the license. The total payoffs of Inn and Inc are decided by the summation of their respective bargaining payoff and the fee or profit collected prior to the bargaining stage.

$$\pi_{inn} = t\big[\pi_e(t) + (k-1)\pi_e(m(x,t)+t)\big] + b_{inn} \tag{44}$$

$$\pi_{inc} = \pi_0(t) + b_{inc}$$

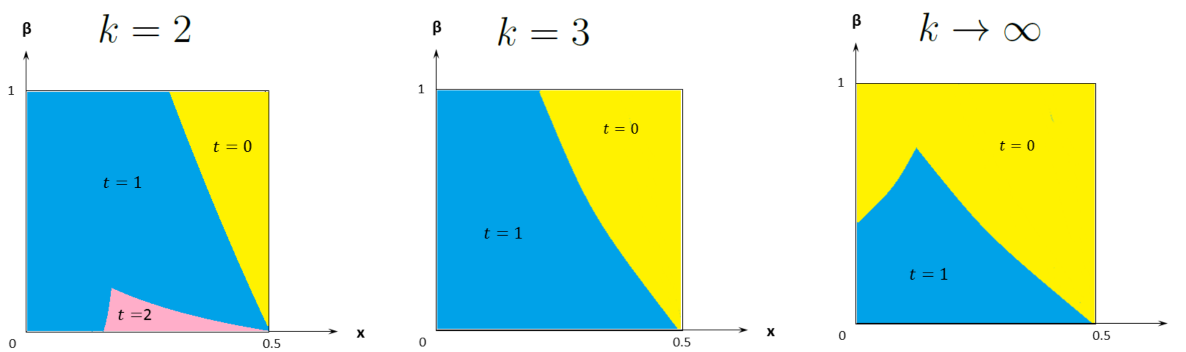
The innovator then maximizes over t his payoff π_{inn} where

$$\begin{aligned}\pi_{inn}(x,\beta,t) =& (k-1)\Big[\beta\big[m(x,t)\pi_e\big(t+m(x,t)\big)+\pi_0\big(t+m(x,t)\big)\\ &-n(t)\pi_e\big(t+n(t)\big)-\pi_0\big(t+n(t)\big)\big]\\ &+t\pi_e\big(t+m(x,t)\big)+n(t)\pi_e\big(t+n(t)\big)\Big]+t\pi_e(t)\end{aligned} \tag{45}$$

When k increases, for any given (x,β), the optimal choice of t decreases. If k is very large, the effect of $t\pi_e(t)$ becomes negligibly small. Then Inn chooses t that

$$\begin{aligned}\max_t\Big[&\beta\big[m(x,t)\pi_e\big(t+m(x,t)\big)+\pi_0\big(t+m(x,t)\big)\\ &-n(t)\pi_e\big(t+n(t)\big)-\pi_0\big(t+n(t)\big)\big]\\ &+t\pi_e\big(t+m(x,t)\big)+n(t)\pi_e\big(t+n(t)\big)\Big]\end{aligned} \tag{46}$$

Relaxing the assumption that $m(x,t)$ has to be an integer The following graph shows the optimal choice of t when $k=2$, $k=3$ and $k\to\infty$.



A Appendix

A.1 Appendix for Chapter 2

A.1.1 Formula

$$f(n) = \frac{n^3 + n^2 + 2n + 4 + \sqrt{n^6 + 8n^5 + 30n^4 + 56n^3 + 50n^2 + 20n + 4}}{3n^4 + 8n^3 + 10n^2 + 4n - 4}$$

$$\tilde{k}(n,\epsilon) = \frac{2n^3\epsilon+10n^2\epsilon+16n\epsilon+4n+8\epsilon+6-\sqrt{4n^6\epsilon^2+34n^5\epsilon^2+119n^4\epsilon^2+4n^4\epsilon+220n^3\epsilon^2+26n^3\epsilon+227n^2\epsilon^2+62n^2\epsilon+124n\epsilon^2+4n^2+64n\epsilon+28\epsilon^2+12n+24\epsilon+9}}{3(2n+3)\epsilon}$$

$$g(n) = \max\left(0, \frac{3n^4 + 6n^3 + 7n^2 + 4n - 4 - 2\sqrt{n^8 + 9n^7 + 31n^6 + 49n^5 + 29n^4 - 9n^3 - 16n^2 - 4n}}{5n^5 + 15n^4 + 19n^3 + 9n^2 + 4}\right)$$

$$h(n) = \max\left(0, \frac{n^4 + n^3 + 2n^2 + 4n - \sqrt{3n^7 + 14n^6 + 18n^5 + 7n^4 + 24n^3 + 40n^2 - 16}}{n^5 + 2n^4 + n^3 + n^2 + 4n + 4}\right)$$

$$r(n) = \begin{cases} e_1(n) & \text{if } n \geq 17 \\ f_1^{-1}(n) & \text{if } 2 \leq n \leq 16 \end{cases}$$

where

$$e_1(n) = \frac{3\,n - 5 - 2\,\sqrt{n^2 - 4\,n + 3}}{5\,n^2 - 14\,n + 13}$$

and

$$f_1(\epsilon) = -\frac{8\,\sqrt{1+2\,\epsilon}\epsilon^{3/2}-11\,\epsilon^2+2\,\sqrt{-48\,\sqrt{1+2\,\epsilon}\epsilon^{7/2}-12\,\sqrt{1+2\,\epsilon}\epsilon^{5/2}+68\,\epsilon^4+34\,\epsilon^3+2\,\epsilon^2}-3\,\epsilon}{\epsilon^2}$$

Figure 4 shows that the inverse function of $f_1(\epsilon)$ exists for $0 < \epsilon < \frac{1}{2}$.

A.1.2 Let us prove that $\pi_1(n-k_1, k) - \pi_0(n-k_1+1, k)$ is increasing in k_1

Let $m = n - k_1$, we will show that $\pi_1(m,k) - \pi_0(m+1,k)$ is decreasing in m.

$$\pi_1(m,k) - \pi_0(m+1,k) = \frac{\big(1+(m+1)\epsilon\big)^2}{(m+k+1)^2} - \frac{(1-k\epsilon)^2}{(m+k+2)^2}$$

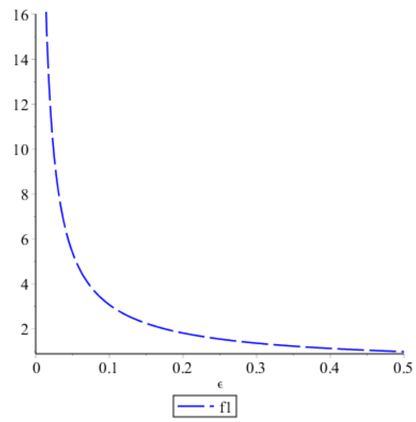


Figure 4: The value of $f_1(\epsilon)$

The first order condition (using Maple) is

$$\frac{\partial\big(\pi_1(m,k)-\pi_0(m+1,k)\big)}{\partial m} = G\epsilon^2 + H\epsilon + I$$

where

$$G = \frac{2\,k^5+8\,k^4m+12\,k^3m^2+8\,k^2m^3+2\,km^4+8\,k^4+30\,k^3m+36\,k^2m^2+14\,km^3+18\,k^3+54\,k^2m+36\,km^2+26\,k^2+40\,mk+16\,k}{(m+k+1)^3(m+2+k)^3} > 0$$

$$H = \frac{-2\,k^4-8\,k^3m-12\,k^2m^2-8\,km^3-2\,m^4-2\,k^3-18\,k^2m-30\,km^2-14\,m^3-36\,mk-36\,m^2-12\,k-40\,m-16}{(m+k+1)^3(m+2+k)^3}$$

and

$$I = \frac{-6\,k^2-12\,mk-6\,m^2-18\,k-18\,m-14}{(m+k+1)^3(m+2+k)^3}$$

Therefore $\frac{\partial\left(\pi_1(m,k)-\pi_0(m+1,k)\right)}{\partial m}$ is in quadratic in ϵ with $G > 0$.

The equation $\frac{\partial\left(\pi_1(m,k)-\pi_0(m+1,k)\right)}{\partial m} = 0$ has two solutions in ϵ.

$$\epsilon_1 = -\frac{3\,k^2+6\,mk+3\,m^2+9\,k+9\,m+7}{k^4+4\,k^3m+6\,k^2m^2+4\,km^3+m^4+4\,k^3+15\,k^2m+18\,km^2+7\,m^3+9\,k^2+27\,mk+18\,m^2+13\,k+20\,m+8} < 0$$

and

$$\epsilon_2 = \frac{1}{k} > 0$$

Therefore for $0 < \epsilon \le \frac{1}{k}$, $\frac{\partial\left(\pi_1(m,k)-\pi_0(m+1,k)\right)}{\partial m} < 0$ and $\pi_1(m,k) - \pi_0(m+1,k)$ is decreasing in m. Since $m = n - k_1$, $\pi_1(n - k_1, k) - \pi_0(n - k_1 + 1, k)$ is increasing in k_1.

A.1.3 Proof of Proposition 3

By Lemma 1 the highest equilibrium payoff of the innovator in G_u is

$$\pi_u^*(n, \epsilon) = \max\big(\pi^0(n, \epsilon), \hat{\pi}(n, \epsilon)\big)$$

where $\pi^0(n, \epsilon) = \max_{k \geq 1} k\pi_1(n, k)$ and $\hat{\pi}(n, \epsilon) = \max_{k \geq 1} k\Big(\pi_1\big(n - \min(k, n), k\big) - \pi_0\big(n - \min(k, n) + 1, k\big)\Big)$.

Let $k_u^*(n, \epsilon)$, $k^0(n, \epsilon)$ and $\hat{k}(n, \epsilon)$ be maximizers of $\pi_u^*(n, \epsilon)$, $\pi^0(n, \epsilon)$ and $\hat{\pi}(n, \epsilon)$, respectively. Clearly either $k^0(n, \epsilon)$ or $\hat{k}(n, \epsilon)$ is a maximizer of $\pi_u^*(n, \epsilon)$.

Lemma 7. *$k(\pi_1(0, k) - \pi_0(1, k))$ is decreasing in k.*

Proof. Let

$$J = k\big(\pi_1(0, k) - \pi_0(1, k)\big) = k\Big(\big(\frac{1 + \epsilon}{1 + k}\big)^2 - \big(\frac{1 - k\epsilon}{2 + k}\big)^2\Big)$$

Then

$$\frac{\partial J}{\partial k} = A\epsilon^2 + B\epsilon + C$$

where A, B and C are functions of k. In particular,

$$A = -\frac{k^6 + 9\,k^5 + 22\,k^4 + 24\,k^3 + 12\,k^2 - 4\,k - 8}{(1 + k)^3\,(2 + k)^3}$$

$$B = -\frac{-6\,k^4 - 14\,k^3 - 12\,k^2 - 16\,k - 16}{(1 + k)^3\,(2 + k)^3}$$

$$C = -\frac{4\,k^3 + 9\,k^2 + k - 6}{(1 + k)^3\,(2 + k)^3}$$

and

$$B^2 - 4AC = \frac{-16(k^3 + k^2 - 2k - 1)}{(k + 2)^4(1 + k)^2}$$

Clearly $A < 0$ for $k \geq 1$ and $B^2 - 4AC < 0$ for $k \geq 2$. Therefore $\frac{\partial J}{\partial k} < 0$ for $k \geq 2$ and J is maximized either at $k = 1$ or at $k = 2$.

Since $J|_{k=1} = \frac{1}{36}(\epsilon+5)(5\epsilon+1)$ and $J|_{k=2} = -\frac{1}{72}(10\epsilon+1)(2\epsilon-7)$, for every ϵ

$$J|_{k=1} - J|_{k=2} = \frac{5}{12}\epsilon^2 - \frac{2}{9}\epsilon + \frac{1}{24} > 0$$

Thus $J = k\big(\pi_1(0,k) - \pi_0(1,k)\big)$ is decreasing in k for $k \geq 1$. □

Lemma 8. *(i)* $k^0(n,\epsilon) \leq n+1$ *and (ii)* $\hat{k}(n,\epsilon) \leq n$.

Proof. (i) By (1) and Proposition 1

$$\pi^0(n,\epsilon) = \max_{1 \leq k \leq \frac{1}{\epsilon}} \frac{k\big(1+(n+1)\epsilon\big)^2}{(n+k+1)^2}$$

and it is maximized at $k = \min(n+1, \frac{1}{\epsilon})$. Hence $k^0(n,\epsilon) \leq n+1$.

(ii) Let

$$\hat{\pi}_1(n,\epsilon) = \max_{1 \leq k < n} k\big(\pi_1(n-k,k) - \pi_0(n-k+1,k)\big)$$

and

$$\hat{\pi}_2(n,\epsilon) = \max_{k \geq n} k\big(\pi_1(0,k) - \pi_0(1,k)\big)$$

Then $\hat{\pi}(n,\epsilon) = \max\big(\hat{\pi}_1(n,\epsilon), \hat{\pi}_2(n,\epsilon)\big)$. But $k\big(\pi_1(0,k) - \pi_0(1,k)\big)$ is decreasing in k (Lemma 7). This implies $\hat{k}(n,\epsilon) \leq n$. □

The next lemma characterizes both $k^0(n,\epsilon)$ and $\hat{k}(n,\epsilon)$.

Lemma 9.

$$k^0(n,\epsilon) = \begin{cases} n+1 & \textit{if } 0 < \epsilon < \frac{1}{n+1} \\ \frac{1}{\epsilon} & \textit{if } \frac{1}{n+1} \leq \epsilon < 1 \end{cases} \tag{47}$$

and

$$\hat{k}(n,\epsilon) = \begin{cases} n & \textit{if } 0 < \epsilon < f(n) \\ \tilde{k}(n,\epsilon) & \textit{if } f(n) \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \textit{if } \frac{2}{n+1} < \epsilon < 1 \end{cases} \tag{48}$$

where

$$f(n) = \frac{n^3+n^2+2n+4+\sqrt{n^6+8n^5+30n^4+56n^3+50n^2+20n+4}}{3n^4+8n^3+10n^2+4n-4},$$

$$\tilde{k}(n,\epsilon) = \frac{2n^3\epsilon+10n^2\epsilon+16n\epsilon+4n+8\epsilon+6-\sqrt{4n^6\epsilon^2+34n^5\epsilon^2+119n^4\epsilon^2+4n^4\epsilon+220n^3\epsilon^2+26n^3\epsilon+227n^2\epsilon^2+62n^2\epsilon+124n\epsilon^2+4n^2+64n\epsilon+28\epsilon^2+12n+24\epsilon+9}}{3(2n+3)\epsilon}$$

and $\frac{1}{\epsilon} \leq \tilde{k}(n,\epsilon) \leq n$ for $f(n) \leq \epsilon \leq \frac{2}{n+1}$.

Proof. (47) follows from the proof of part (i) of Lemma 8. We next analyze $\hat{k}(n,\epsilon)$.

By part (ii) of Lemma 8, $\hat{k}(n,\epsilon) \leq n$. Then

$$\begin{aligned}\hat{\pi}(n,\epsilon) &= \max_{1\leq k\leq n} k\Big(\pi_1(n-k,k) - \pi_0(n-k+1,k)\Big)\\ &= k\Big(\frac{(1+(n-k+1)\epsilon)^2}{(n+1)^2} - \frac{(1-k\epsilon)^2}{(n+2)^2}\Big)\end{aligned}$$

The first order condition is

$$\frac{\partial\hat{\pi}(n,\epsilon)}{\partial k} = Dk^2 + Ek + F \tag{49}$$

where

$$D = \frac{6\,n\epsilon^2 + 9\,\epsilon^2}{(n+1)^2\,(n+2)^2} > 0,$$

$$E = \frac{-4\,n^3\epsilon^2 - 20\,n^2\epsilon^2 - 32\,n\epsilon^2 - 8\,n\epsilon - 16\,\epsilon^2 - 12\,\epsilon}{(n+1)^2\,(n+2)^2},$$

$$F = \frac{n^4\epsilon^2 + 6\,n^3\epsilon^2 + 2\,n^3\epsilon + 13\,n^2\epsilon^2 + 10\,n^2\epsilon + 12\,n\epsilon^2 + 16\,n\epsilon + 4\,\epsilon^2 + 2\,n + 8\,\epsilon + 3}{(n+1)^2\,(n+2)^2}$$

and

$$E^2 - 4DF = 4\,\frac{\epsilon^2\big(4\,n^6\epsilon^2+34\,n^5\epsilon^2+119\,n^4\epsilon^2+4\,n^4\epsilon+220\,n^3\epsilon^2+26\,n^3\epsilon+227\,n^2\epsilon^2+62\,n^2\epsilon+124\,n\epsilon^2+4\,n^2+64\,n\epsilon+28\,\epsilon^2+12\,n+24\,\epsilon+9\big)}{(n+1)^4(n+2)^4} > 0$$

Let c_1 and c_2 be the solution in k of the quadratic function $\frac{\partial\hat{\pi}(n,\epsilon)}{\partial k} = 0$. Then

$$c_1 = \frac{2n^3\epsilon+10n^2\epsilon+16n\epsilon+4n+8\epsilon+6-\sqrt{4n^6\epsilon^2+34n^5\epsilon^2+119n^4\epsilon^2+4n^4\epsilon+220n^3\epsilon^2+26n^3\epsilon+227n^2\epsilon^2+62n^2\epsilon+124n\epsilon^2+4n^2+64n\epsilon+28\epsilon^2+12n+24\epsilon+9}}{3(2n+3)\epsilon}$$

$$c_2 = \frac{2n^3\epsilon+10n^2\epsilon+16n\epsilon+4n+8\epsilon+6+\sqrt{4n^6\epsilon^2+34n^5\epsilon^2+119n^4\epsilon^2+4n^4\epsilon+220n^3\epsilon^2+26n^3\epsilon+227n^2\epsilon^2+62n^2\epsilon+124n\epsilon^2+4n^2+64n\epsilon+28\epsilon^2+12n+24\epsilon+9}}{3(2n+3)\epsilon}$$

It can be easily verified that when $\epsilon \geq 0$ and $n \geq 1$, $c_1 > 0$. Next we compare c_1 with $\frac{1}{\epsilon}$.

$$\frac{1}{\epsilon} - c_1 = \frac{s(n,\epsilon) - t(n,\epsilon)}{3(2n+3)\epsilon} \tag{50}$$

where

$$s(n,\epsilon) = \sqrt{4n^6\epsilon^2 + 34n^5\epsilon^2 + 119n^4\epsilon^2 + 4n^4\epsilon + 220n^3\epsilon^2 + 26n^3\epsilon + 227n^2\epsilon^2 + 62n^2\epsilon + 124n\epsilon^2 + 4n^2 + 64n\epsilon + 28\epsilon^2 + 12n + 24\epsilon + 9}$$

and

$$t(n,\epsilon) = 2n^3\epsilon + 10n^2\epsilon + 16n\epsilon + 8\epsilon - 2n - 3 \tag{51}$$

For $\epsilon \geq 0$, $s(n,\epsilon) > 0$ and it can be easily verified that $t(n,\epsilon) \leq 0$ iff $\epsilon \leq \frac{2n+3}{2(n+1)(n+2)^2}$. By (50) for $\epsilon \leq \frac{2n+3}{2(n+1)(n+2)^2}$, $c_1 \leq \frac{1}{\epsilon}$. If, however, $\epsilon > \frac{2n+3}{2(n+1)(n+2)^2}$ by (51) $t(n,\epsilon) > 0$. It can be easily verified that

$$\big(s(n,\epsilon)\big)^2 \geq \big(t(n,\epsilon)\big)^2 \quad \text{iff} \quad 0 \leq \epsilon \leq \frac{2}{n+1} \tag{52}$$

and for all $n \geq 1$, in which case, again, $c_1 \leq \frac{1}{\epsilon}$. It can also be verified that $\frac{2}{n+1} > \frac{2n+3}{2(n+1)(n+2)^2}$ for $n \geq 1$. Therefore $c_1 > \frac{1}{\epsilon}$ iff $\epsilon > \frac{2}{n+1}$. Since the optimal k is bounded above by $\frac{1}{\epsilon}$ (Proposition 1), for $\epsilon > \frac{2}{n+1}$, $\hat{k}(n,\epsilon) = \frac{1}{\epsilon}$. Next we analyze the case $0 \leq \epsilon \leq \frac{2}{n+1}$ (or equivalently $c_1 \leq \frac{1}{\epsilon}$). We first compare the value of $\frac{1}{\epsilon}$ and c_2.

$$\frac{1}{\epsilon} - c_2 = \frac{-s(n,\epsilon) - t(n,\epsilon)}{3(2n+3)\epsilon}$$

as shown above, $t(n,\epsilon) \geq 0$ iff $\epsilon \geq \frac{2n+3}{2(n+1)(n+2)^2}$. For $\frac{2n+3}{2(n+1)(n+2)^2} \leq \epsilon \leq \frac{2}{n+1}$, $c_2 \geq \frac{1}{\epsilon}$. Since $\big(s(n,\epsilon)\big)^2 \geq \big(t(n,\epsilon)\big)^2$ for $0 \leq \epsilon < \frac{2n+3}{2(n+1)(n+2)^2}$, again $c_2 \geq \frac{1}{\epsilon}$. Thus for any $\epsilon \leq \frac{2}{n+1}$, $c_2 \geq \frac{1}{\epsilon}$. This together with (49) imply that $\hat{\pi}(n,\epsilon)$ is maximized at $k = c_1$.

Finally we compare the value of c_1 with n.

$$n - c_1 = \frac{s(n,\epsilon) - (2n^3\epsilon + 4n^2\epsilon + 7\epsilon n + 4n + 8\epsilon + 6)}{3(2n+3)\epsilon}$$

It can be easily verified that

$$\big(s(n,\epsilon)\big)^2 - (2n^3\epsilon + 4n^2\epsilon + 7\epsilon n + 4n + 8\epsilon + 6)^2 =$$
$$(18n^5 + 75n^4 + 132n^3 + 114n^2 + 12n - 36)\epsilon^2 - (12n^4 + 30n^3 + 42n^2 + 84n + 72)\epsilon - (12n^2 + 36n + 27)$$

Thus $c_1 \geq n$ iff the last term ≤ 0. The solution of this quadratic inequality is

$$\frac{n^3+n^2+2n+4-\sqrt{n^6+8n^5+30n^4+56n^3+50n^2+20n+4}}{3n^4+8n^3+10n^2+4n-4} \leq \epsilon \leq \frac{n^3+n^2+2n+4+\sqrt{n^6+8n^5+30n^4+56n^3+50n^2+20n+4}}{3n^4+8n^3+10n^2+4n-4} \equiv f(n)$$

It can be easily verified that $\frac{n^3+n^2+2n+4-\sqrt{n^6+8n^5+30n^4+56n^3+50n^2+20n+4}}{3n^4+8n^3+10n^2+4n-4} < 0$ for $n \geq 1$. It can also be verified that $f(n) \leq \frac{2}{n+1}$ for $n \geq 1$.

Consequently, for $0 \leq \epsilon \leq f(n)$, $n \leq c_1 \leq \frac{1}{\epsilon}$ and $\hat{k}(n,\epsilon) = n$; for $f(n) < \epsilon \leq \frac{2}{n+1}$,

$c_1 < n$, $c_1 < \frac{1}{\epsilon}$ and $\hat{k}(n,\epsilon) = c_1$; for $\frac{2}{n+1} \leq \epsilon \leq 1$, $\frac{1}{\epsilon} < c_1 < n$ and $\hat{k}(n,\epsilon) = \frac{1}{\epsilon}$, and the proof of Lemma 9 is complete. $\square$

We are now ready to characterize the equilibrium number of licensees in G_u, for the "lucky" innovator.

Case 1: Suppose $0 \leq \epsilon \leq \min\left(\frac{1}{n+1}, f(n)\right)$, then $k^0(n,\epsilon) = n+1$ and $\hat{k}(n,\epsilon) = n$.

$$\begin{aligned}\pi^0(n,\epsilon) - \hat{\pi}(n,\epsilon) &= (n+1)\pi_1(n,n+1) - n\big(\pi_1(0,k) - \pi_0(1,k)\big) \\ &= \frac{(5n^5+15n^4+19n^3+9n^2+4)\epsilon^2 - (6n^4+12n^3+14n^2+8n-8)\epsilon + n^3 - 3n^2 - 4n + 4}{4(n+1)^2(n+2)^2}\end{aligned} \tag{53}$$

It is easy to verify that $\pi^0(n,\epsilon) \leq \hat{\pi}(n,\epsilon)$ iff

$$\frac{3n^4+6n^3+7n^2+4n-4-2\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n}}{5n^5+15n^4+19n^3+9n^2+4} \leq \epsilon \leq \frac{3n^4+6n^3+7n^2+4n-4+2\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n}}{5n^5+15n^4+19n^3+9n^2+4}$$

Let $d_1 = \frac{3n^4+6n^3+7n^2+4n-4-2\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n}}{5n^5+15n^4+19n^3+9n^2+4}$ and $d_2 = \frac{3n^4+6n^3+7n^2+4n-4+2\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n}}{5n^5+15n^4+19n^3+9n^2+4}$. We next show that $d_1 < \min\left(\frac{1}{n+1}, f(n)\right) < d_2$. First observe that

$$d_2 - \frac{1}{n+1} = \frac{(n+1)\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n} - (n^5+3n^4+3n^3+4-n^2)}{\frac{1}{2}(5n^5+15n^4+19n^3+9n^2+4)(n+1)}$$

It can be easily verified that $d_2 > \frac{1}{n+1}$ for $n \geq 1$. Thus $d_2 \geq \min\left(\frac{1}{n+1}, f(n)\right)$. Next observe that

$$\frac{1}{n+1} - d_1 = \frac{(n+1)\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n} + (n^5+3n^4+3n^3+4-n^2)}{\frac{1}{2}(5n^5+15n^4+19n^3+9n^2+4)(n+1)} > 0$$

thus $d_1 < \frac{1}{n+1}$. The analytical comparison between the value of d_1 and $f(n)$ is complicated. The numerical comparison is shown in Figure 6. Form the figure, d_1 (blue) is less than $f(n)$ for $1 \leq n \leq 100$.

Since $d_1 < \min\left(\frac{1}{n+1}, f(n)\right) < d_2$, for $0 \leq \epsilon < d_1$, $\pi^0(n,\epsilon) \geq \hat{\pi}(n,\epsilon)$ and $k_2^*(n,\epsilon) = k^0(n,\epsilon) = n+1$. For $d_1 \leq \epsilon \leq \min\left(\frac{1}{n+1}, f(n)\right)$, $\pi^0(n,\epsilon) < \hat{\pi}(n,\epsilon)$ and $k_2^*(n,\epsilon) = \hat{k}(n,\epsilon) = n$.

Case 2: Suppose $\frac{2}{n+1} \leq \epsilon < 1$, then $k^0(n,\epsilon) = \hat{k}(n,\epsilon) = \frac{1}{\epsilon}$ and $\pi^0(n,\epsilon) = \hat{\pi}(n,\epsilon) = \epsilon$. Clearly $k_2^*(n,\epsilon) = \frac{1}{\epsilon}$.

Case 3: Suppose $\min\left(\frac{1}{n+1}, f(n)\right) < \epsilon < \frac{2}{n+1}$. Consider first the case $\frac{1}{n+1} \leq f(n)$. By Lemma 9 $\hat{k}(n,\epsilon) < \frac{1}{\epsilon}$ thus $\hat{\pi}(n,\epsilon) > \epsilon$. Since $k^0(n,\epsilon) = \frac{1}{\epsilon}$ and $\pi^0(n,\epsilon) = \epsilon$, $\hat{\pi}(n,\epsilon) > \pi^0(n,\epsilon)$.

Consider next the case $\frac{1}{n+1} > f(n)$. (i) Suppose $\frac{1}{n+1} \leq \epsilon < \frac{2}{n+1}$, then the previous

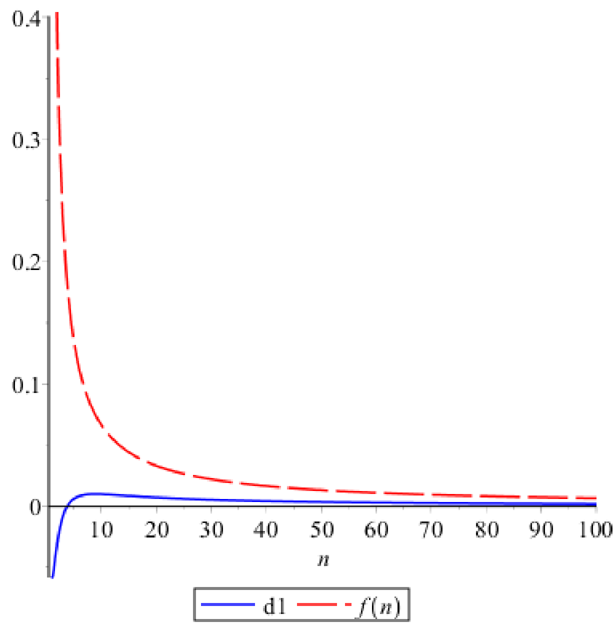


Figure 5: Comparison between d_1 and $f(n)$

argument applies and $\hat{\pi}(n,\epsilon) > \pi^0(n,\epsilon)$. (ii) Suppose $f(n) < \epsilon < \frac{1}{n+1}$, $k^0(n,\epsilon) = n+1$ and $\hat{k}(n,\epsilon) = \tilde{k}(n,\epsilon)$. We next compare $\pi^0(n,\epsilon)$ and $\hat{\pi}(n,\epsilon)$ in this case. First observe that $\tilde{k}(n,\epsilon) < n$ for $f(n) < \epsilon < \frac{1}{n+1}$, thus $\hat{\pi}(n,\epsilon) > n\big(\pi_1(0,n) - \pi_0(1,n)\big)$. If we can show that

$$n\big(\pi_1(0,n) - \pi_0(1,n)\big) > (n+1)\pi_1(n,n+1) \tag{54}$$

then the proof is complete. This is indeed true since (54) holds iff $d_1 \leq \epsilon \leq d_2$ and we have shown that $d_1 \leq f(n)$ and $d_2 \geq \frac{1}{n+1}$. Denote $g(n) = \max(d_1, 0)$, Proposition 3 is complete.

A.1.4 Proof of Proposition 5

By Proposition 1, we focus only on the case where $k_1 + k_2 \leq \epsilon$. The innovator solves

$$\max_{k_1,k_2} \overbrace{k_1\big(\pi_1(n-k_1,k_1+k_2) - \pi_0(n-k_1,k_1+k_2)\big) + k_2\pi_1(n-k_1,k_1+k_2)}^{\pi_D}$$

s.t.

$$
\begin{aligned}
&0 \leq k_1 \leq n-1 \\
&0 \leq k_2 \\
&k_1 + k_2 \leq \frac{1}{\epsilon}
\end{aligned}
\tag{55}
$$

$$\pi_D = -\frac{(k_2\epsilon^2+2n\epsilon^2+2\epsilon^2)k_1^2}{(n+k_2+1)^2} - \frac{(k_2^2\epsilon^2+2k_2n\epsilon^2-n^2\epsilon^2+2k_2\epsilon^2-2n\epsilon^2-2n\epsilon-\epsilon^2-2\epsilon)k_1}{(n+k_2+1)^2} + \frac{k_2n^2\epsilon^2+2k_2n\epsilon^2+2k_2n\epsilon+k_2\epsilon^2+2k_2\epsilon+k_2}{(n+k_2+1)^2}$$

Note first that π_D is continuous on k_1 and k_2. Moreover, for any k_2, π_D is quadratic in k_1. Denote (k_1^*, k_2^*) the optimal choice of the innovator. Given any k_2, let $k_1(k_2)$ be the maximizer of π_D. Then

$$k_1(k_2) = \min\{\underbrace{-\frac{k_2^2\epsilon + (2n\epsilon+2\epsilon)k_2 - n^2\epsilon - 2n\epsilon - 2n - \epsilon - 2}{2\epsilon(k_2+2n+2)}}_{k_1^s}, n-1, \frac{1}{\epsilon} - k_2\}$$

It can be easily verified that

$$k_1^s < n-1 \quad \text{iff} \quad k_2 > \underbrace{\frac{-2n\epsilon + \sqrt{\epsilon(n^2\epsilon+2n\epsilon+2n+5\epsilon+2)}}{\epsilon}}_{c_1}$$

and

$$k_1^s < \frac{1}{\epsilon} - k_2 \quad \text{iff} \quad k_2 < \underbrace{\frac{2-n\epsilon-\epsilon}{\epsilon}}_{c_2}$$

It can also be verified that $c_1 \leq c_2$ iff $\epsilon \leq \frac{1}{2}$. We first analyze the case $0 < \epsilon \leq \frac{1}{2}$.

Case 1. $0 < \epsilon < \frac{1}{2}$ *Subcase 1.1:* Suppose $k_2 \leq c_1$, then $n-1 < k_1^s < \frac{1}{\epsilon} - k_1$ and $k_1(k_2) = n-1$. Substituting k_1 in π_D with $n-1$,

$$\pi_D^1 = -\frac{(n-1)\epsilon^2k_2^2 + (2n^2\epsilon^2 - 4n\epsilon^2 - 2n\epsilon - 2\epsilon^2 - 2\epsilon - 1)k_2 + \epsilon(n-1)(n+1)(n\epsilon - 3\epsilon - 2)}{(n+k_2+1)^2}$$

It can be easily verified that $\frac{\partial \pi_D^1}{\partial k_2}$ is decreasing in k_2. Let $\tilde{k}_2$ be the solution of $\frac{\partial \pi_D^1}{\partial k_2} = 0$. Then

$$\tilde{k}_2 = \frac{2n\epsilon + n + 4\epsilon + 1 - 2n^2\epsilon}{2n\epsilon + 1}$$

It can be verified that $\tilde{k}_2 \leq c_1$ iff $\epsilon \leq \frac{1}{2}$ thus π_D is maximized at $k_2 = \tilde{k}_2$ for $k_2 \leq c_1$.

Subcase 1.2: Suppose $c_1 \leq k_2 \leq c_2$, then $k_1^s < n-1$, $k_1^s < \frac{1}{\epsilon} - k_2$ and $k_1(k_2) = k_1^s$. Substituting k_1 in π_D with k_1^s,

$$\pi_D^2 = \frac{k_2^2\epsilon^2 + (2n\epsilon^2 + 2\epsilon^2)k_2 + n^2\epsilon^2 + 2n\epsilon^2 + 4n\epsilon + \epsilon^2 + 4\epsilon + 4}{4(k_2 + 2n + 2)}$$

It can be verified that π_D^2 is decreasing in k_2 for $0 \leq k_2 \leq c_2$, thus π_D is maximized at $k_2 = c_1$ for $c_1 \leq k_2 \leq c_2$.

Subcase 1.3: Suppose $c_2 \leq k_2$, then $\frac{1}{\epsilon} - k_2 < k_1^s < n-1$ and $k_1(k_2) = \frac{1}{\epsilon} - k_2$. Since Inn's payoff is the same for all (k_1, k_2) s.t. $k_1 + k_2 = \frac{1}{\epsilon}$, for any $k_2 \geq c_2$ Inn obtains the same payoff. By Assumption 1 in this case π_D is maximized at $k_2 = c_2$ in the region $k_2 \geq c_2$.

To summarize, for $k_2 \leq c_1$, π_D is maximized at $(k_1 = n-1, k_2 = \tilde{k}_2)$; for $k_2 \in [c_1, c_2]$, π_D is maximized at $(k_1 = k_1^s, k_2 = c_1)$; for $k_2 \geq c_2$, π_D is maximized at $(k_1 = \frac{1}{\epsilon} - c_2, k_2 = c_2)$. Since π_D is continuous in k_2, π_D is maximized at $k_2 = \max(\tilde{k}_2, 0)$.

For $n \geq 3$, it can be easily verified that $\tilde{k}_2 \geq 0$ iff $\epsilon \leq \frac{1}{2n-4}$. Then

$$k_2^*(n,\epsilon) = \begin{cases} \frac{2n\epsilon+n+4\epsilon+1-2n^2\epsilon}{2n\epsilon+1} & \text{if } 0 < \epsilon < \frac{1}{2n-4} \\ 0 & \text{if } \frac{1}{2n-4} \leq \epsilon \end{cases}$$

Next we analyze k_1^*. Suppose $0 < \epsilon < \frac{1}{2n-4}$. Since $k_2^* = \tilde{k}_2$ and $\tilde{k}_2 < c_1$ (for $0 \leq \epsilon < \frac{1}{2}$), $k_2^* < c_1$. Following the analysis in subcase 1.1, $k_1(k_2^*) = n-1$. Suppose next $\frac{1}{2n-4} \leq \epsilon$, $k_1(k_2^*)$ then depends on the relation between 0, c_1 and c_2. (i) If $0 \leq c_1$ ($\frac{1}{2n-4} \leq \epsilon \leq \frac{2}{3n-5}$), following the analysis of subcase 1.1, $k_1(k_2^*) = n-1$. (ii) If $c_1 < 0 \leq c_2$ ($\frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1}$), following the analysis of subcase 1.2, $k_1(k_2^*) = k_1^s|_{k_2=0} = \frac{n+1}{4} + \frac{1}{2\epsilon}$. (iii) If $c_2 < 0$ ($\frac{2}{n+1} < \epsilon$), following the analysis of subcase 1.3, $k_1(k_2^*) = \frac{1}{\epsilon}$. Thus for $n \geq 3$

$$k_1^*(n,\epsilon) = \begin{cases} n-1 & \text{if } 0 < \epsilon < \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} < \epsilon < \frac{1}{2} \end{cases}$$

$$k_2^*(n,\epsilon) = \begin{cases} \frac{2n\epsilon+n+4\epsilon+1-2n^2\epsilon}{2n\epsilon+1} & \text{if } 0 < \epsilon < \frac{1}{2n-4} \\ 0 & \text{if } \frac{1}{2n-4} \leq \epsilon < \frac{1}{2} \end{cases}$$

Consider next $n = 2$. It can be easily verified that $\tilde{k}_2 > 0$. Therefore $k_2^*(2,\epsilon) = \tilde{k}_2$ and $k_2^*(2,\epsilon) < c_1$ (since $\tilde{k}_2 < c_1$ for $0 \leq \epsilon < \frac{1}{2}$). Following subcase 1.1 $k_1^*(2,\epsilon) = 1$. The innovator's optimal payoff is obtained for $k_1^* = 1$ and $k_2^* = \tilde{k}_2|_{n=2} = \frac{3}{4\epsilon+1}$.

Finally consider $n = 1$. Clearly $k_1^* = 0$. It is easy to verify that $k_2^* = 2$.

Case 2. $\frac{1}{2} \leq \epsilon < 1$ In this case $c_1 > c_2$, $\tilde{k}_2 > \frac{1}{2}$ and $\tilde{k}_2 > c_1$.

Subcase 2.1: Suppose $k_2 \leq c_2$, then $n-1 < k_1^s \leq \frac{1}{\epsilon} - k_2$ and $k_1(k_2) = n-1$. Following similar argument as in Subcase 1.1, π_D is maximized at $\min(\tilde{k}_2, c_2)$. Since $\tilde{k}_2 > c_1$ and $c_1 > c_2$, $\tilde{k}_2 > c_2$. Therefore π_D is maximized at $k_1 = n-1$ and $k_2 = c_2$.

Subcase 2.2: Suppose $c_2 \leq k_2 \leq c_1$, then $k_1^s \geq n-1$, $k_1^s \geq \frac{1}{\epsilon} - k_2$ and $k_1(k_2) = \min(n-1, \frac{1}{\epsilon} - k_2)$. It can be easily verified that $c_2 \leq \frac{1}{\epsilon} - n + 1 \leq c_1$ for $\epsilon \geq \frac{1}{2}$. (i) Suppose first $c_2 \leq k_2 \leq \frac{1}{\epsilon} - n + 1$ (or equivalently $n-1 \leq \frac{1}{\epsilon} - k_2$). Then $k_1(k_2) = n-1$. Following similar argument as in Subcase 1.1, π_D is maximized at $\min(\tilde{k}_2, \frac{1}{\epsilon} - n + 1)$. Since $\tilde{k}_2 > c_1 \geq \frac{1}{\epsilon} - n + 1$, π_D is maximized at $(k_1 = n-1, k_2 = \frac{1}{\epsilon} - n + 1)$. (ii) Suppose next $\frac{1}{\epsilon} - n + 1 \leq k_2 \leq c_1$ (or equivalently $n-1 \geq \frac{1}{\epsilon} - k_2$) then $k_1(k_2) = \frac{1}{\epsilon} - k_2$. The innovator's payoff is maximized at $k_1 + k_2 = \frac{1}{\epsilon}$.

Subcase 2.3: Suppose $k_2 \geq c_1$, then $\frac{1}{\epsilon} - k_2 < k_1^s < n-1$ and $k_1(k_2) = \frac{1}{\epsilon} - k_2$. The innovator's payoff is maximized again at $k_1 + k_2 = \frac{1}{\epsilon}$.

Consider first $n \geq 3$. Since $\epsilon \geq \frac{1}{2}$, $n - 1 \geq \frac{1}{\epsilon}$ holds. Therefore Subcase 2.1 and part (i) of Subcase 2.2 are irrelevant. In this case π_D is maximized at $k_1 + k_2 = \frac{1}{\epsilon}$. By Assumption 1, $k_1^* = \frac{1}{\epsilon}$ and $k_2^* = 0$.

Consider next $n \leq 2$. Since $\frac{1}{2} \leq \epsilon \leq 1$, $n-1 \leq \frac{1}{\epsilon}$ holds. Therefore Part (ii) of Subcase 2.2 and Subcase 2.3 are irrelevant. Since π_D is continuous on k_2, combining Subcases 2.1 and Part (i) of Subcase 2.2 π_D is maximized at $(k_1 = n-1, k_2 = \frac{1}{\epsilon} - n + 1)$. Proposition 5 follows.

A.1.5 Proof of Proposition 6

(i) Follows from (7) and Proposition 5.

(ii) Let $q_1(m_0, m_1)$ and $q_0(m_0, m_1)$ be the equilibrium quantity produced by a licensee and a non-licensee, respectively, when there are m_0 firms producing at a unit cost c and m_1 firms producing at a unit cost $c - \epsilon$. It can be verified that

$$q_0(m_0, m_1) = \begin{cases} \frac{1-\epsilon m_1}{m_0+m_1+1} & \text{if } m_1 \leq \frac{a-c}{\epsilon} \\ 0 & \text{if } m_1 > \frac{a-c}{\epsilon} \end{cases} \tag{56}$$

$$q_1(m_0,m_1)=\begin{cases}\frac{1+(m_0+1)\epsilon}{m_0+m_1+1} & \text{if } m_1\le\frac{a-c}{\epsilon}\\ \frac{1+\epsilon}{m_1+1} & \text{if } m_1>\frac{a-c}{\epsilon}\end{cases} \tag{57}$$

Since $p^*_{nu}(n,\epsilon)=(c+1)-\Big((n-k^*_1)q_0(n-k^*_1,k^*_1+k^*_2)+(k^*_1+k^*_2)q_1(n-k^*_1,k^*_1+k^*_2)\Big)$, by (56), (57) and Proposition 5, part (ii) of Proposition 6 follows.

A.1.6 Proof of Proposition 7

Lemma 10. *Consider the case $n\ge 3$. (i) If $\epsilon\le g(n)$, $\pi^*_{nu}(n,\epsilon)>\pi^*_u(n,\epsilon)$. (ii) If $\frac{1}{2n-4}\le\epsilon<\frac{2}{n+1}$, $\pi^*_{nu}(n,\epsilon)<\pi^*_u(n,\epsilon)$. (iii) If $\frac{2}{n+1}\le\epsilon<1$, $\pi^*_{nu}(n,\epsilon)=\pi^*_u(n,\epsilon)$.*

Proof. (i) If $\epsilon<g(n)$, in UA the innovator's highest payoff is $\hat{\pi}=(n+1)\pi_1(n,n+1)$ which is obtained when he auctions off $n+1$ licenses and all winners are entrants. In NUA if the innovator chooses $k_1=0$ and $k_2=n+1$ he obtains $\hat{\pi}$. But he can obtain more by choosing other combinations of (k_1,k_2). It can be shown that $g(n)<\frac{1}{2n-4}$ for $n\ge 3$ (the analytic proof is difficult, see Figure 6 for a numerical comparison). Thus by Proposition 5 when $\epsilon<g(n)$, $k^*_1(n,\epsilon)>0$ and $\pi^*_{nu}>\hat{\pi}$.

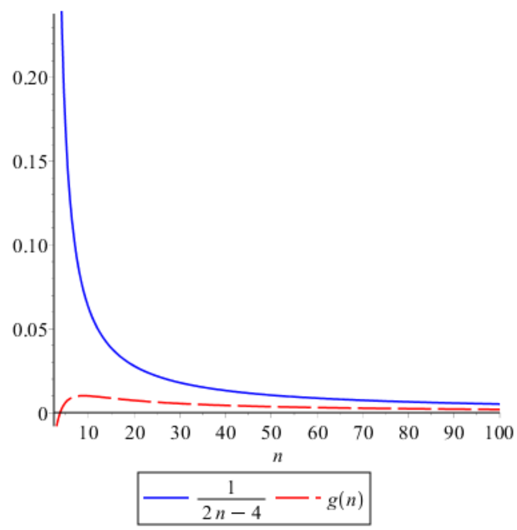


Figure 6: Comparison between $\frac{1}{2n-4}$ and $g(n)$

(ii) If $\frac{1}{2n-4}\le\epsilon<\frac{2}{n+1}$ then in NUA $k^*_2(n,\epsilon)=0$. Since the advantage of NUA on UA lies only on the innovator's ability to charge for a license a higher price to entrants than to incumbent firms, this advantage disappears when $k^*_2(n,\epsilon)=0$. Moreover, for any (k_1, k_2) an incumbent's willingness to pay in NUA is $\pi_1(n-k_1, k_1+k_2) - \pi_0(n-k_1, k_1+k_2)$ while it can be as high as $\pi_1(n-k_1, k_1+k_2) - \pi_0(n-k_1+1, k_1+k_2)$ in UA (incumbent may be willing to pay more to limit entry). Thus $\pi^*_{nu}(n,\epsilon) \leq \pi^*_u(n,\epsilon)$. Since $K^*_{nu} < \frac{1}{\epsilon}$, $\pi^*_{nu}(n,\epsilon) < \pi^*_u(n,\epsilon)$.

(iii) If $\frac{2}{n+1} \leq \epsilon < 1$ the innovator auctions off in total $\frac{1}{\epsilon}$ licenses in both UA and NUA. By Proposition 1, the innovator obtains the same payoff which is the total industry profit ϵ in both auctions. □

Next we focus on the analysis of $g(n) < \epsilon < \frac{1}{2n-4}$. Clearly $\frac{1}{2n-4} < \frac{2}{3n-5} < \frac{2}{n+1}$ for $n \geq 3$. Thus in G_{nu}, $k^*_1(n,\epsilon) = n-1$ and $k^*_2(n,\epsilon) = \frac{2n\epsilon+n+4\epsilon+1-2n^2\epsilon}{2n\epsilon+1}$. In G_u, $\pi^*_u(n,\epsilon)$ depends on whether $\epsilon \leq f(n)$ or $\epsilon > f(n)$.

Case 1: Suppose $f(n) \leq \frac{1}{2n-4}$ (this inequality holds for $n \leq 8$). Then

$$\pi^*_u(n,\epsilon) = \begin{cases} n\Big(\pi_1(0,n) - \pi_0(1,n)\Big) & \text{if } g(n) \leq \epsilon \leq f(n) \\ \tilde{k}\Big(\pi_1\big(n-\tilde{k},\tilde{k}\big) - \pi_0\big(n-\tilde{k}+1,\tilde{k}\big)\Big) & \text{if } f(n) < \epsilon < \frac{1}{2n-4} \end{cases}$$

We first analyze $g(n) \leq \epsilon \leq f(n)$.

$$\pi^*_{nu} - \pi^*_u = \frac{(4n^5+8n^4+4n^3+4n^2+16n+16)\epsilon^2}{4(n+1)^2(n+2)^2} - \frac{(4n^4+4n^3+8n^2+16n)\epsilon}{4(n+1)^2(n+2)^2} + \frac{n^3-3n^2-4n+4}{4(n+1)^2(n+2)^2}$$

It can be easily verified that $\pi^*_{nu} < \pi^*_u$ iff

$$\frac{3n^4+6n^3+7n^2+4n-4-2\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n}}{5n^5+15n^4+19n^3+9n^2+4} < \epsilon < \frac{3n^4+6n^3+7n^2+4n-4+2\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n}}{5n^5+15n^4+19n^3+9n^2+4}$$

Let $e_1 = \frac{3n^4+6n^3+7n^2+4n-4-2\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n}}{5n^5+15n^4+19n^3+9n^2+4}$ and $e_2 = \frac{3n^4+6n^3+7n^2+4n-4+2\sqrt{n^8+9n^7+31n^6+49n^5+29n^4-9n^3-16n^2-4n}}{5n^5+15n^4+19n^3+9n^2+4}$. Figure 7 compare the value of $f(n)$, $g(n)$, e_1 and e_2. Note that $n \in [3, 8]$ since in this section we deal with $n \geq 3$ and $f(n) \leq \frac{1}{2n-4}$.

It can be easily verified that $g(n)$ and e_1 intersect at $g(n) = e_1 = 0$. Thus $\pi^*_{nu} > \pi^*_u$ for $g(n) \leq \epsilon < e_1$ and $\pi^*_{nu} \leq \pi^*_u$ for $e_1 \leq \epsilon \leq f(n)$.

Next consider the case $f(n) < \epsilon < \frac{1}{2n-4}$. Again, the analytic comparison between π^*_u and π^*_{nu} is difficult and Figure 8 shows that $\pi^*_u(n,\epsilon) - \pi^*_{nu}(n,\epsilon) \geq 0$ numerically.

To summarize, in case $f(n) \leq \frac{1}{2n-4}$, $\pi^*_{nu} > \pi^*_u$ for $g(n) \leq \epsilon < e_1$ and $\pi^*_{nu} \leq \pi^*_u$ for $e_1 \leq \epsilon \leq \frac{1}{2n-4}$.

Case 2: Suppose $f(n) > \frac{1}{2n-4}$ (this inequality holds for $n \geq 9$). Clearly for $g(n) < \epsilon < \frac{1}{2n-4}$, $\pi^*_u(n,\epsilon) = n\Big(\pi_1(0,n) - \pi_0(1,n)\Big)$. Again $\pi^*_{nu} < \pi^*_u$ iff $e_1 < \epsilon < e_2$. Figure 9 shows that $e_2 > \frac{1}{2n-4} > e_1 > g(n)$ numerically. Clearly $\pi^*_{nu} > \pi^*_u$ for

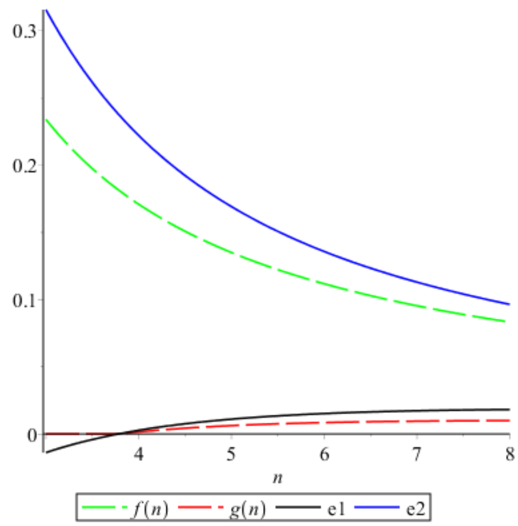


Figure 7: Comparison between $f(n)$, $g(n)$, e_1 and e_2

$g(n) \leq \epsilon < e_1$ and $\pi^*_{nu} \leq \pi^*_u$ for $e_1 \leq \epsilon < \frac{1}{2n-4}$. Let $h(n) = \max(0, e_1)$, Proposition 7 follows.

A.1.7 Proof of Corollary 5

(i) For $n \geq 3$

$$\pi^*_{nu}(n,\epsilon) - \pi^*(n,\epsilon) = \begin{cases} \frac{\left(1-(2n-4)\epsilon\right)^2}{4(n+1)} & \text{if } 0 \leq \epsilon \leq \frac{1}{2n-4} \\ 0 & \text{if } \frac{1}{2n-4} \leq \epsilon < 1, \end{cases} \tag{58}$$

Let $E = \frac{\left(1-(2n-4)\epsilon\right)^2}{4(n+1)}$.

$$\frac{\partial E}{\partial \epsilon} = \frac{\big((2n-4)\epsilon - 1\big)(n-2)}{n+1}$$

Observe that (58) is continuous in ϵ and $\frac{\partial E}{\partial \epsilon} \leq 0$ for $0 < \epsilon \leq \frac{1}{2n-4}$. Thus for any $n \geq 3$, $\pi^*_{nu} - \pi^*$ is non-increasing in ϵ for $0 < \epsilon < 1$.

Next observe that for $0 < \epsilon < 1$

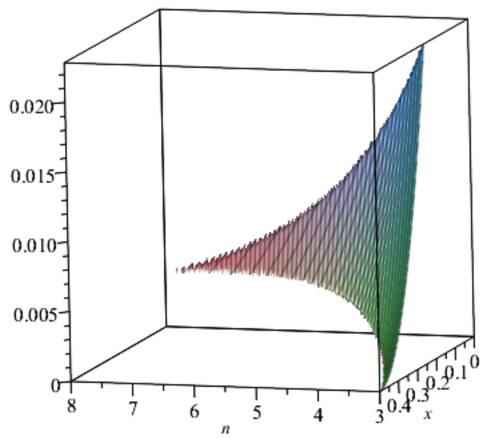


Figure 8: The value of $\pi^*_u(n,\epsilon) - \pi^*_{nu}(n,\epsilon)$

$$\pi^*_{nu}(n,\epsilon) - \pi^*(n,\epsilon) = \begin{cases} \frac{\left(1-(2n-4)\epsilon\right)^2}{4(n+1)} & \text{if } 3 \le n \le \frac{1}{2\epsilon} + 2 \\ 0 & \text{if } \frac{1}{2\epsilon} + 2 \le n, \end{cases} \tag{59}$$

and (59) is continuous in n. Since

$$\frac{\partial E}{\partial n} = \frac{\left((2n-4)\epsilon - 1\right)(2n\epsilon + 8\epsilon + 1)}{4(n+1)^2},$$

$\pi^*_{nu} - \pi^*$ is non-increasing in n for $n \ge 3$.

(ii) For $n \ge 3$

$$p^*(n,\epsilon) - p^*_{nu}(n,\epsilon) = \begin{cases} \frac{1-(2n-4)\epsilon}{2(n+1)} & \text{if } 0 \le \epsilon \le \frac{1}{2n-4} \\ 0 & \text{if } \frac{1}{2n-4} \le \epsilon < 1, \end{cases} \tag{60}$$

Clearly for any $n \ge 3$, $p^*(n,\epsilon) - p^*_{nu}(n,\epsilon)$ is non-increasing in ϵ.

For $0 < \epsilon < 1$,

$$p^*(n,\epsilon) - p^*_{nu}(n,\epsilon) = \begin{cases} \frac{1-(2n-4)\epsilon}{2(n+1)} & \text{if } 3 \le n \le \frac{1}{2\epsilon+2} \\ 0 & \text{if } \frac{1}{2\epsilon} + 2 \le n, \end{cases} \tag{61}$$

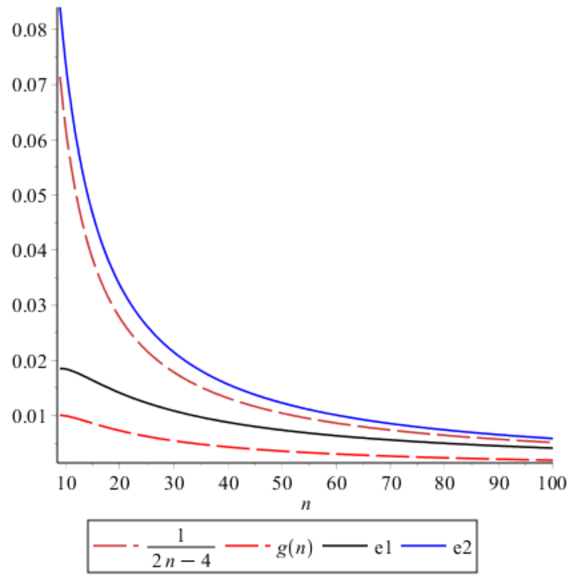


Figure 9: Comparison between $\frac{1}{2n-4}$, $g(n)$, e_1 and e_2

Let $F = \frac{1-(2n-4)\epsilon}{2(n+1)}$ It can be easily verified that

$$\frac{\partial F}{\partial n} = -\frac{6\epsilon + 1}{(n+1)^2}$$

Since (61) is continuous in n and $\frac{\partial F}{\partial n} < 0$, (61) is non-increasing in n for $n \geq 3$.

A.1.8 Proof of Proposition 14

By Proposition 13, for $n \geq 3$

$$\pi_0^*(n, \epsilon) = \begin{cases} \frac{(n-1)\left(-(n-3)\epsilon^2+2\epsilon\right)}{n+1} & \text{if } 0 < \epsilon \leq \frac{2}{3n-5} \\ \frac{(n\epsilon+\epsilon+2)^2}{8(n+1)} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \epsilon & \text{if } \frac{2}{n+1} \leq \epsilon < 1. \end{cases}$$

For $n \leq 2$

$$\pi_0^*(n, \epsilon) = \frac{(n-1)\big(-(n-3)\epsilon^2 + 2\epsilon\big)}{n+1}. \tag{62}$$

Consider first $n \geq 3$. First note that for any $\epsilon \geq \frac{1}{2}$, $\pi_0^* = \epsilon$ regardless of the value of n. We next focus on $0 < \epsilon < \frac{1}{2}$.

Subcase 1: Suppose $n < \frac{2}{3\epsilon} + \frac{5}{3}$ (or equivalently $0 < \epsilon < \frac{2}{3n-5}$). Denote

$$\pi_0^{*1} = \frac{\epsilon(n-1)(3\epsilon + 2 - n\epsilon)}{n+1}.$$

It can be easily verified that

$$\frac{\partial \pi_0^{*1}}{\partial n} = \frac{\epsilon(-n^2\epsilon - 2n\epsilon + 7\epsilon + 4)}{(n+1)^2},$$

$$\frac{\partial \pi_0^{*1}}{\partial n} > 0 \quad \text{if} \quad 0 \leq n < 2\sqrt{2 + \frac{1}{\epsilon}} - 1$$

and

$$\frac{\partial \pi_D^{*1}}{\partial n} \leq 0 \quad \text{if} \quad n \geq 2\sqrt{2 + \frac{1}{\epsilon}} - 1$$

Denote $n^{*1} = 2\sqrt{2 + \frac{1}{\epsilon}} - 1$. It can be easily verified that $3 < n^{*1} < \frac{2}{3\epsilon} + \frac{5}{3}$ if $0 < \epsilon < \frac{1}{2}$. Therefore $3 < n^{*1} < \frac{2}{3\epsilon} + \frac{5}{3}$ for $0 < \epsilon \leq \frac{1}{3n-5}$ and n^{*1} is the maximizer of π_0^* for $3 < n < \frac{2}{3\epsilon} + \frac{5}{3}$.

Subcase 2: Suppose $\frac{2}{3\epsilon} + \frac{5}{3} \leq n \leq \frac{2}{\epsilon} - 1$ (or equivalently $\frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1}$). Deonte

$$\pi_0^{*2} = \frac{(n\epsilon + \epsilon + 2)^2}{8(n+1)}.$$

It can be easily verified that

$$\frac{\partial \pi_0^{*2}}{\partial n} = \frac{\epsilon^2 n^2 + 2\epsilon^2 n + \epsilon^2 - 4}{8(n+1)^2}$$

and

$$\frac{\partial \pi_0^{*2}}{\partial n} < 0 \quad \text{for} \quad 0 \leq n < \frac{2}{\epsilon} - 1.$$

Therefore $n^{*2} = \frac{2}{3\epsilon} + \frac{5}{3}$ is the maximizer of π_0^{*2} for $\frac{2}{3\epsilon} + \frac{5}{3} \leq n \leq \frac{2}{\epsilon} - 1$.

Subcase 3: Suppose $\frac{2}{\epsilon} - 1 \leq n$ ($\frac{2}{n+1} \leq \epsilon \leq \frac{1}{2}$). Then $\pi_0^*(n, \epsilon) = \epsilon$ and the innovator's payoff is the same for any $\frac{2}{\epsilon} - 1 \leq n$.

Combining subcases 1-3, for $n \geq 3$, since π_0^* is continuous in n, $n^* = 2\sqrt{2 + \frac{1}{\epsilon}} - 1$

is the maximizer of π_0^*. Let π_D^* be the innovator's equilibrium payoff when $n = n^*$.

$$\pi_D^* = \begin{cases} \frac{2\epsilon\left(2\epsilon+1-\sqrt{\epsilon(2\epsilon+1)}\right)\left(\sqrt{\epsilon(2\epsilon+1)}-\epsilon\right)}{\sqrt{\epsilon(2\epsilon+1)}} & \text{if } 0 < \epsilon < \frac{1}{2} \\ \epsilon & \text{if } \frac{1}{2} \leq \epsilon < 1 \end{cases}$$

Consider next $n = 2$. By (62), $\pi_0^*|_{n=2} = \frac{1}{3}\epsilon^2 + \frac{2}{3}\epsilon$. Finally consider the case $n = 1$. By (62) the innovator obtains 0 since we restrict $k \leq n-1$. To provide a more reasonable comparison we assume in this case that the innovator sells the license to the incumbent firm by fixed fee. The innovator's payoff is then $\pi_0^{*'}|_{n=1} = \pi_1(0,1) - \pi_1(1,0) = \frac{1}{4}\epsilon^2 + \frac{1}{2}\epsilon$.

Figure 10 provides the comparison of the innovator's payoff when $n^* = 2\sqrt{2+\frac{1}{\epsilon}} - 1$, $n = 2$ and $n = 1$. Clearly in G_0 the innovator obtains the highest payoff in an oligopoly market with $n^* = 2\sqrt{2+\frac{1}{\epsilon}} - 1$ firms.

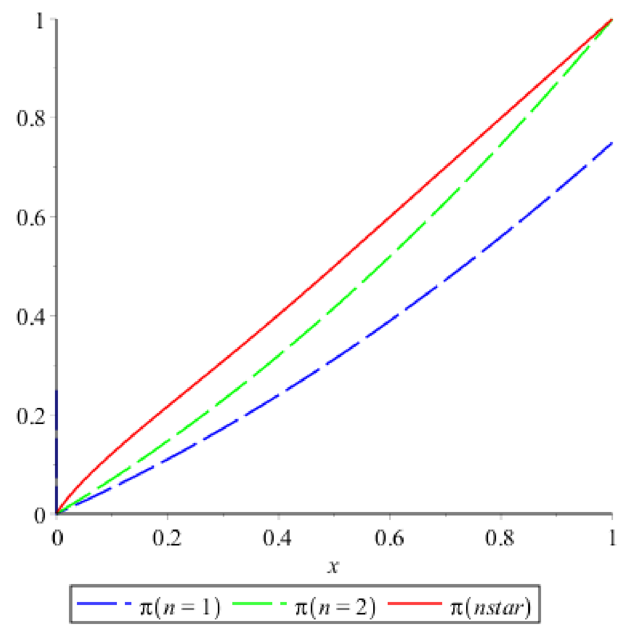


Figure 10: The innovator's payoff under different n

A.1.9 The maximizer of $\pi_{su}^*(n, \epsilon)$

Let $(k_1^{s*}(n,\epsilon), k_2^{s*}(n,\epsilon))$ be the maximizer of $\pi_{su}^*(n,\epsilon)$.

For $n \geq 3$

$$k_1^{s*}(n,\epsilon) = \begin{cases} 0 & \text{if } 0 < \epsilon < r(n) \\ n-1 & \text{if } r(n) \leq \epsilon \leq \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} \leq \epsilon < 1 \end{cases}$$

and

$$k_2^{s*}(n,\epsilon) = \begin{cases} n+1 & \text{if } 0 < \epsilon < r(n) \\ 2\sqrt{2+\frac{1}{\epsilon}} - (n+1) & \text{if } r(n) \leq \epsilon \leq \frac{4}{n^2+2n-7} \\ 0 & \text{if } \frac{4}{n^2+2n-7} \leq \epsilon < 1. \end{cases}$$

For $n = 2$

$$k_1^{s*}(2,\epsilon) = \begin{cases} 0 & \text{if } 0 < \epsilon < r(2) \\ 1 & \text{if } r(2) \leq \epsilon \leq 1 \end{cases}$$

and

$$k_2^{s*}(2,\epsilon) = \begin{cases} 3 & \text{if } 0 < \epsilon < r(2) \\ 2\sqrt{2+\frac{1}{\epsilon}} - 3 & \text{if } r(2) \leq \epsilon \leq \frac{1}{2} \\ \frac{1}{\epsilon} - 1 & \text{if } \frac{1}{2} \leq \epsilon < 1. \end{cases}$$

For $n = 1$, $k_1^{s*}(1,\epsilon) = 0$ and $k_2^{s*}(1,\epsilon) = 2$.[10]

A.1.10 Proof of Proposition 10

Let $(0, k_2^0)$ and $(\hat{k}_1, \hat{k}_2)$ be maximizers of $\pi_{su}^0(n,\epsilon)$ and $\hat{\pi}_{su}(n,\epsilon)$, respectively. Clearly either $(0, k_2^0)$ or $(\hat{k}_1, \hat{k}_2)$ is a maximizer of $\pi_{su}^*(n,\epsilon)$.

Proposition 16.

$$k_2^0(n,\epsilon) = \begin{cases} n+1 & \textit{if } 0 < \epsilon < \frac{1}{n+1} \\ \frac{1}{\epsilon} & \textit{if } \frac{1}{n+1} \leq \epsilon < 1 \end{cases}$$

Proof. Easy to verify. □

Next we focus on the analysis of $\hat{\pi}_{su}(n,\epsilon)$. Note that $n = 1$ does not apply here since $k_1 = 0$ in this case.

[10] In G_{su} we restrict $k_1 \leq n-1$ therefore $k_1^{s*}(1,\epsilon) = 0$. If, instead, an auction with minimum reservation price is conducted to the monopoly incumbent, there are parameters under which the innovator sells licenses to the incumbent firm in addition to entrants.

Proposition 17. *(i) For* $n \geq 3$

$$\hat{k}_1(n,\epsilon) = \begin{cases} n-1 & \textit{if } 0 < \epsilon \leq \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \textit{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \textit{if } \frac{2}{n+1} \leq \epsilon < 1 \end{cases}$$

$$\hat{k}_2(n,\epsilon) = \begin{cases} 2\sqrt{2+\frac{1}{\epsilon}} - (n+1) & \textit{if } 0 < \epsilon \leq \frac{4}{n^2+2n-7} \\ 0 & \textit{if } \frac{4}{n^2+2n-7} \leq \epsilon < 1 \end{cases}$$

(ii) For $n = 2$

$$\hat{k}_1(2,\epsilon) = n-1$$

$$\hat{k}_2(2,\epsilon) = \begin{cases} 2\sqrt{2+\frac{1}{\epsilon}} - (n+1) & \textit{if } 0 < \epsilon \leq \frac{1}{2} \\ \frac{1}{\epsilon} - (n-1) & \textit{if } \frac{1}{2} \leq \epsilon < 1 \end{cases}$$

Note that $\frac{4}{n^2+2n-7} \leq \frac{2}{3n-5}$ for $n \geq 3$.

Proof. See A.1.11 of the Appendix. □

To find the optimal licensing strategy of the innovator, we next compare $\pi^0_{su}(n,\epsilon)$ and $\hat{\pi}_{su}(n,\epsilon)$.

Lemma 11. *For* $n \geq 2$, $\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ *iff* $\epsilon < r(n)$.

The formula of $r(n)$ is quite complicated and it appears in the Appendix (A.1.1).

Proof. See A.1.12 of the Appendix. □

We are now ready to characterize the optimal licensing strategy of the innovator.

Proposition 18. *For* $n \geq 2$

$$k_1^*(n,\epsilon) = \begin{cases} 0 & \textit{if } 0 < \epsilon < r(n) \\ \hat{k}_1(n,\epsilon) & \textit{if } r(n) \leq \epsilon < 1 \end{cases}$$

and

$$k_2^*(n,\epsilon) = \begin{cases} n+1 & \textit{if } 0 < \epsilon < r(n) \\ \hat{k}_2(n,\epsilon) & \textit{if } r(n) \leq \epsilon < 1. \end{cases}$$

Proof. Follows immediately from propositions 16, 17 and Lemma 11. □

A.1.11 Proof of Proposition 17

We first shows that the innovator in SUA sells licenses to entrants iff he sells licenses to all (but one) incumbent firms.

Lemma 12. *For any $n \geq 2$ and $0 < \epsilon < 1$, $\hat{k}_2(n,\epsilon) > 0$ iff $\hat{k}_1(n,\epsilon) = n-1$.*

Proof. Denote $k = k_1 + k_2$. Suppose first $k = \frac{1}{\epsilon}$ the innovator obtains the entire industry profit and by Assumption 1 he sells licenses to incumbent firms and only when he exhausts all (but 1) incumbents will he sell licenses to entrants. Suppose next $1 \leq k < \frac{1}{\epsilon}$,

$$\frac{\partial w_l(k_1, k-k_1)}{\partial k_1} = -2\,\frac{\epsilon\ (k\epsilon - 1)}{(n - k1 + k + 1)^2} > 0. \tag{63}$$

For any k, $1 \leq k < \frac{1}{\epsilon}$, the license fee paid by each licensee is increasing in the number of incumbent licensees in k. Therefore the innovator in this case also sells licenses to incumbents first. Lemma 12 follows. $\square$

By Lemma 12, if $k \leq n-1$, $k_1 = k$ and $k_2 = 0$. If, however, $k > n-1$, $k_1 = n-1$ and $k_2 = k - (n-1)$. Therefore

$$\hat{\pi}_{su}(n,\epsilon) = \max\left(\max_{1\leq k\leq n-1} k w_l(k,0), \max_{k_2}\big((n-1+k_2)w_l(n-1,k_2)\big)\right) \tag{64}$$

Suppose first $n \geq 3$. It can be verified that the maximizer of $\max_{1\leq k\leq n-1} k w_l(k,0)$ is

$$\tilde{k}_1(n,\epsilon) = \begin{cases} n-1 & \text{if } 0 < \epsilon \leq \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} \leq \epsilon < 1 \end{cases} \tag{65}$$

and the maximizer of $\max_{k_2}\big((n-1+k_2)w_l(n-1,k_2)\big)$ is

$$\bar{k}_2(n,\epsilon) = \begin{cases} 2\sqrt{2+\frac{1}{\epsilon}} - (n+1) & \text{if } 0 < \epsilon \leq \frac{4}{n^2+2n-7} \\ 0 & \text{if } \frac{4}{n^2+2n-7} \leq \epsilon < 1 \end{cases} \tag{66}$$

(66) states that for $\frac{4}{n^2+2n-7} \leq \epsilon < 1$, the innovator is best off selling 0 licenses to entrants even if he sells $n-1$ licenses to incumbent firms. By Lemma 12 $\hat{k}_1(n,\epsilon) = \tilde{k}_1(n,\epsilon)$ and $\hat{k}_2(n,\epsilon) = 0$ in this case. As for $0 < \epsilon \leq \frac{4}{n^2+2n-7}$, the innovator is best off selling positive number of licenses to entrants if he sells $n-1$ licenses to incumbent

firms. Since $\frac{4}{n^2+2n-7} \leq \frac{2}{3n-5}$ by (65) the innovator in this case is best off selling $n-1$ licenses to incumbent firms even if $k_2 = 0$. Therefore $\hat{k}_1(n,\epsilon) = n-1$ and $\hat{k}_1(n,\epsilon) = \hat{k}_2(n,\epsilon)$ in this case. Part (i) of Lemma 17 follows.

Suppose next $n = 2$. By Lemma 12, $\hat{k}_1(2,\epsilon) = 1$. It can be easily verified that $\hat{k}_1(2,\epsilon) = \min(2\sqrt{2+\frac{1}{\epsilon}} - 3, \frac{1}{\epsilon} - 1)$.

A.1.12 Proof of Lemma 11

By Proposition 16 it is easy to verify that

$$\pi^0_{su}(n,\epsilon) = \begin{cases} \frac{(\epsilon(n+1)+1)^2}{4(n+1)} & \text{if } 0 < \epsilon < \frac{1}{n+1} \\ \epsilon & \text{if } \frac{1}{n+1} \leq \epsilon < 1 \end{cases} \tag{67}$$

By Proposition 17 it is easy to verify that For $n \geq 3$

$$\hat{\pi}_{su}(n,\epsilon) = \begin{cases} 2\,\epsilon\,\left(\sqrt{1+2\,\epsilon} - \sqrt{\epsilon}\right)^2 & \text{if } 0 < \epsilon \leq \frac{4}{n^2+2n-7} \\ \frac{(n-1)\left(-(n-3)\epsilon^2+2\epsilon\right)}{n+1} & \text{if } \frac{4}{n^2+2n-7} < \epsilon \leq \frac{2}{3n-5} \\ \frac{(n\epsilon+\epsilon+2)^2}{8(n+1)} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \epsilon & \text{if } \frac{2}{n+1} \leq \epsilon < 1 \end{cases}$$

For $n = 2$

$$\hat{\pi}_{su}(n,\epsilon) = \begin{cases} 2\,\epsilon\,\left(\sqrt{1+2\,\epsilon} - \sqrt{\epsilon}\right)^2 & \text{if } 0 < \epsilon \leq \frac{1}{2} \\ \epsilon & \text{if } \frac{1}{2} \leq \epsilon < 1 \end{cases}$$

Suppose $\epsilon \geq \frac{1}{n+1}$, then $\pi^0_{su}(n,\epsilon) = \epsilon$ and $\hat{\pi}_{su}(n,\epsilon) \geq \epsilon$. In this case $\hat{\pi}_{su}(n,\epsilon) \geq \pi^0_{su}(n,\epsilon)$. We next focus on the case $0 < \epsilon \leq \frac{1}{n+1}$.

Case 1: Consider first $n \geq 7$. In this case $\frac{1}{n+1} \geq \frac{2}{3n-5}$.

Subcase 1.1: Suppose $\frac{2}{3n-5} \leq \epsilon \leq \frac{1}{n+1}$.

$$\hat{\pi}_{su}(n,\epsilon) - \pi^0_{su}(n,\epsilon) = -\frac{(n+1)^2\epsilon^2 - 2}{8(n+1)}$$

where $\hat{\pi}_{su}(n,\epsilon) \geq \pi^0_{su}(n,\epsilon)$ iff $-\frac{\sqrt{2}}{n+1} \leq \epsilon \leq \frac{\sqrt{2}}{n+1}$. Therefore $\hat{\pi}_{su}(n,\epsilon) \geq \pi^0_{su}(n,\epsilon)$ holds for $\frac{2}{3n-5} \leq \epsilon \leq \frac{1}{n+1}$.

Subcase 1.2: Suppose $\frac{4}{n^2+2n-7} \le \epsilon \le \frac{2}{3n-5}$.

$$\pi_{su}^0(n,\epsilon) - \hat{\pi}_{su}(n,\epsilon) = \frac{(5\,n^2 - 14\,n + 13)\,\epsilon^2}{4(n+1)} + \frac{(-6\,n+10)\,\epsilon}{4(n+1)} + (4\,n+4)^{-1}$$

Note that $\frac{5n^2-14n+13}{4(n+1)} > 0$ for $n \ge 7$. It can be easily verified that $\pi_{su}^0(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff $\epsilon < \frac{3\,n-5-2\sqrt{n^2-4\,n+3}}{5\,n^2-14\,n+13}$ or $\epsilon > \frac{3\,n-5+2\sqrt{n^2-4\,n+3}}{5\,n^2-14\,n+13}$.

Denote $e_1 = \frac{3\,n-5-2\sqrt{n^2-4\,n+3}}{5\,n^2-14\,n+13}$ and $e_2 = \frac{3\,n-5+2\sqrt{n^2-4\,n+3}}{5\,n^2-14\,n+13}$. Figure 11 compares the value of e_1, e_2, $\frac{4}{n^2+2n-7}$ and $\frac{2}{3n-5}$ numerically. Note that e_1 and $\frac{4}{n^2+2n-7}$ intersects at $n = 16.19$. Thus in case $\frac{4}{n^2+2n-7} \le \epsilon \le \frac{2}{3n-5}$, for $7 \le n \le 16.19$, $\hat{\pi}_{su}(n,\epsilon) > \pi_{su}^0(n,\epsilon)$ holds. For $n > 16.19$, $\pi_{su}^0(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff $\frac{4}{n^2+2n-7} \le \epsilon < e_1$.

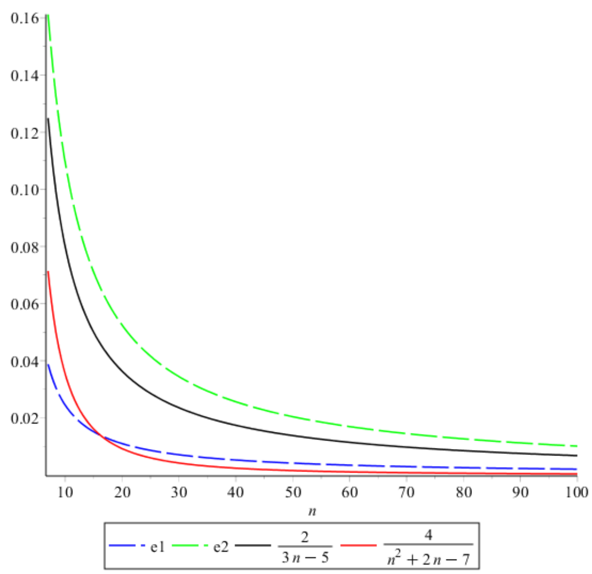


Figure 11: Comparison between e_1, e_2, $\frac{4}{n^2+2n-7}$ and $\frac{2}{3n-5}$

Subcase 1.3: Suppose $0 < \epsilon \le \frac{4}{n^2+2n-7}$.

$$\pi_{su}^0(n,\epsilon) - \hat{\pi}_{su}(n,\epsilon) = \frac{n^2\epsilon^2 + \left(16\sqrt{1+2\,\epsilon}\epsilon^{3/2} - 22\,\epsilon^2 - 6\,\epsilon\right)n + 16\sqrt{1+2\,\epsilon}\epsilon^{3/2} - 23\,\epsilon^2 - 6\,\epsilon + 1}{4\,n+4}$$

It can be easily verified that $\pi_{su}^0(n,\epsilon) \le \hat{\pi}_{su}(n,\epsilon)$ iff

$$-\frac{8\sqrt{1+2\,\epsilon}\epsilon^{3/2} - 11\,\epsilon^2 + 2\sqrt{-48\sqrt{1+2\,\epsilon}\epsilon^{7/2} - 12\sqrt{1+2\,\epsilon}\epsilon^{5/2} + 68\,\epsilon^4 + 34\,\epsilon^3 + 2\,\epsilon^2} - 3\,\epsilon}{\epsilon^2} \le n \le \frac{-8\sqrt{1+2\,\epsilon}\epsilon^{3/2} + 11\,\epsilon^2 + 2\sqrt{-48\sqrt{1+2\,\epsilon}\epsilon^{7/2} - 12\sqrt{1+2\,\epsilon}\epsilon^{5/2} + 68\,\epsilon^4 + 34\,\epsilon^3 + 2\,\epsilon^2} + 3\,\epsilon}{\epsilon^2}$$

Denote $f_1 = -\frac{8\sqrt{1+2\epsilon}\epsilon^{3/2}-11\epsilon^2+2\sqrt{-48\sqrt{1+2\epsilon}\epsilon^{7/2}-12\sqrt{1+2\epsilon}\epsilon^{5/2}+68\epsilon^4+34\epsilon^3+2\epsilon^2}-3\epsilon}{\epsilon^2}$

and $f_2 = \frac{-8\sqrt{1+2\epsilon}\epsilon^{3/2}+11\epsilon^2+2\sqrt{-48\sqrt{1+2\epsilon}\epsilon^{7/2}-12\sqrt{1+2\epsilon}\epsilon^{5/2}+68\epsilon^4+34\epsilon^3+2\epsilon^2}+3\epsilon}{\epsilon^2}$.

Note that for $n \geq 7$, $0 < \epsilon \leq \frac{4}{n^2+2n-7}$ iff $n \leq 2\sqrt{2+\frac{1}{\epsilon}}-1$. Figure 12 shows that $f_2 > 2\sqrt{2+\frac{1}{\epsilon}}-1$ always holds. Note that ϵ is constraint to $\frac{1}{14}$ since we are dealing in this subcase $\epsilon \leq \frac{4}{n^2+2n-7}$ and $n \geq 7$.

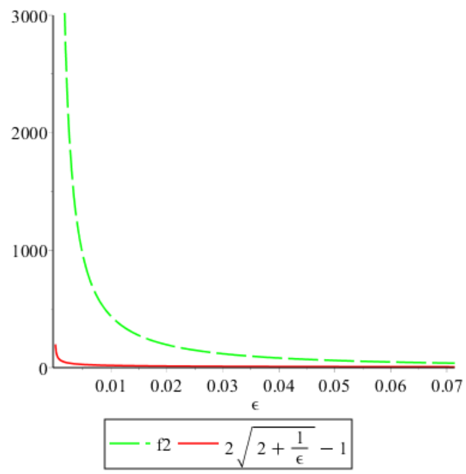


Figure 12: Comparison between f_2 and $2\sqrt{2+\frac{1}{\epsilon}}-1$

Figure 13 compares the value of f_1 and $2\sqrt{2+\frac{1}{\epsilon}}-1$. Note that f_1 and $2\sqrt{2+\frac{1}{\epsilon}}-1$ intersects at $\epsilon = 0.0139$ and $n = 16.19$. By Figure 13, $\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff either $\epsilon < 0.0139$ or $\epsilon > 0.0139$ and $n < f_1(\epsilon)$. Or equivalently, when $0 < \epsilon \leq \frac{4}{n^2+2n-7}$, $\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff either $n > 16.19$ or $n \leq 16.19$ and $\epsilon < f_1^{-1}(n)$ (the existence of $f_1^{-1}(n)$ is shown in Figure 4).

Combining subcases 1.1-1.3, for $n \geq 17$, $\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff $0 < \epsilon < e_1$. For $7 \leq n \leq 16$, $\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff $0 < \epsilon < f_1^{-1}(n)$.

Case 2: Consider next $1+2\sqrt{3} \leq n < 7$. In this case $\frac{4}{n^2+2n-7} \leq \frac{1}{n+1} < \frac{2}{3n-5}$.

Subcase 2.1: Suppose $\frac{4}{n^2+2n-7} \leq \epsilon \leq \frac{1}{n+1}$.

$$\pi^0_{su}(n,\epsilon) - \hat{\pi}_{su}(n,\epsilon) = \frac{(5n^2-14n+13)\epsilon^2}{4(n+1)} + \frac{(-6n+10)\epsilon}{4(n+1)} + (4n+4)^{-1}$$

where $5n^2-14n+13 > 0$ for $1+2\sqrt{3} \leq n < 7$. By the same argument as in Subcase

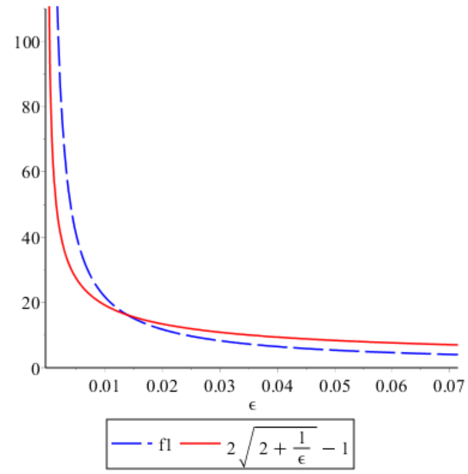


Figure 13: Comparison between f_1 and $2\sqrt{2+\frac{1}{\epsilon}}-1$

1.2, $\pi^0_{su}(n,\epsilon) \leq \hat{\pi}_{su}(n,\epsilon)$ iff $e_1 \leq \epsilon \leq e_2$. Figure 14 shows that $\pi^0_{su}(n,\epsilon) \leq \hat{\pi}_{su}(n,\epsilon)$ for $\frac{4}{n^2+2n-7} \leq \epsilon \leq \frac{1}{n+1}$.

Subcase 2.2: Suppose $0 < \epsilon \leq \frac{4}{n^2+2n-7}$, or equivalently $1+2\sqrt{3} \leq n \leq \min\left(7, \frac{-\epsilon+2\sqrt{2\epsilon^2+\epsilon}}{\epsilon}\right)$. Clearly $\epsilon \leq \frac{1}{2(1+\sqrt{3})}$. By the same argument as in Subcase 1.3, $\pi^0_{su}(n,\epsilon) \leq \hat{\pi}_{su}(n,\epsilon)$ iff $f_1 \leq n \leq f_2$. It can be easily verified that $f_2 > 7$ for $0 < \epsilon \leq \frac{1}{2(1+\sqrt{3})}$. Figure 15 compares the value of f_1 and $\frac{-\epsilon+2\sqrt{2\epsilon^2+\epsilon}}{\epsilon}$. Therefore for $1+2\sqrt{3} \leq n \leq \min\left(7, \frac{-\epsilon+2\sqrt{2\epsilon^2+\epsilon}}{\epsilon}\right)$, $\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff either $0 < \epsilon < f_1^{-1}(7)$ or $f_1^{-1}(7) \leq \epsilon$ and $n < f_1$. Or equivalently,$\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff $\epsilon < f_1^{-1}(n)$.

Combining subcases 2.1-2.2, for $1+2\sqrt{3} \leq n < 7$, $\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff $\epsilon < f_1^{-1}(n)$.

Case 3: Suppose $3 \leq n \leq 1+2\sqrt{3}$. In this case $\frac{1}{n+1} \leq \frac{4}{n^2+2n-7}$. Consider $0 < \epsilon \leq \frac{1}{n+1}$ (or equivalently, $3 \leq n \leq \min\left(1+2\sqrt{3}, \frac{1}{\epsilon}-1\right)$). Clearly $\epsilon \leq \frac{1}{4}$. By the same argument as in Subcase 1.3, $\pi^0_{su}(n,\epsilon) \leq \hat{\pi}_{su}(n,\epsilon)$ iff $f_1 \leq n \leq f_2$. Figure 16 compares the value of f_1, f_2 and $\frac{1}{\epsilon}-1$.

Figure 16 shows that for $3 \leq n \leq 1+2\sqrt{3}$, $\pi^0_{su}(n,\epsilon) > \hat{\pi}_{su}(n,\epsilon)$ iff $\epsilon < f_1^{-1}(n)$.

Finally suppose $n=2$. Clearly for $\frac{1}{3} \leq \epsilon < 1$, $\pi^0_{su}(2,\epsilon) \leq \hat{\pi}_{su}(2,\epsilon)$ since $\pi^0_{su}(2,\epsilon) = \epsilon$ and $\hat{\pi}_{su}(2,\epsilon) \geq \epsilon$. For $0 < \epsilon \leq \frac{1}{3}$, $\pi^0_{su}(2,\epsilon) > \hat{\pi}_{su}(2,\epsilon)$ iff either $f_1(\epsilon) > 2$ or $f_2(\epsilon) < 2$. It can be easily verified that $f_2(\epsilon) > 2$ for any $0 < \epsilon \leq \frac{1}{3}$. Therefore $\pi^0_{su}(2,\epsilon) > \hat{\pi}_{su}(2,\epsilon)$ iff either $f_1(\epsilon) > 2$. Or equivalently, $\pi^0_{su}(2,\epsilon) > \hat{\pi}_{su}(2,\epsilon)$ iff $\epsilon < f_1^{-1}(2)$.

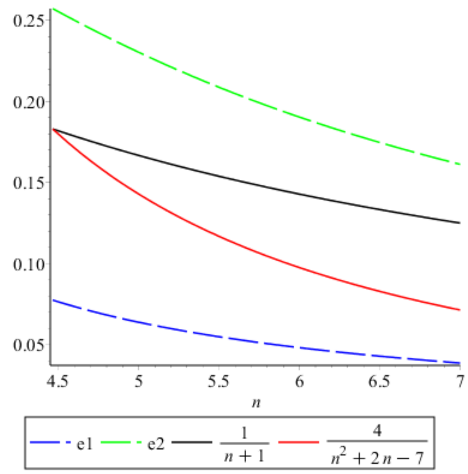


Figure 14: Comparison between e_1, e_2, $\frac{4}{n^2+2n-7}$ and $\frac{1}{n+1}$

To summarize, for any $n \geq 2$, $\pi^0_{su}(n,\epsilon) \leq \hat{\pi}_{su}(n,\epsilon)$ iff $0 < \epsilon < r(n)$ where $r(n) = e_1$ for $n \geq 16.19$ and $r(n) = f_1^{-1}(n)$ for $1 \leq n < 16.19$.

A.1.13 Proof of Proposition 11

For $n \geq 3$,

$$K^*_{su}(n,\epsilon) = \begin{cases} n+1 & \text{if } 0 < \epsilon < r(n) \\ 2\sqrt{1+\frac{1}{\epsilon}} - 2 & \text{if } r(n) \leq \epsilon \leq \frac{4}{n^2+2n-7} \\ n-1 & \text{if } \frac{4}{n^2+2n-7} \leq \epsilon \leq \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} \leq \epsilon < 1 \end{cases}$$

$$K^*_{nu}(n,\epsilon) = \begin{cases} \frac{2(n+2\epsilon)}{2n\epsilon+1} & \text{if } 0 < \epsilon \leq \frac{1}{2n-4} \\ n-1 & \text{if } \frac{1}{2n-4} \leq \epsilon \leq \frac{2}{3n-5} \\ \frac{n+1}{4} + \frac{1}{2\epsilon} & \text{if } \frac{2}{3n-5} \leq \epsilon \leq \frac{2}{n+1} \\ \frac{1}{\epsilon} & \text{if } \frac{2}{n+1} \leq \epsilon < 1 \end{cases}$$

Observe that $K^*_{nu}(n,\epsilon) \xrightarrow{\epsilon \to 0} 2n > n+1$. Since $K^*_{nu}(n,\epsilon)$ is continuous on ϵ and $K^*_{su}(n,\epsilon) = n+1$ for $0 < \epsilon < r(n)$, Proposition 11 follows. The case $n = 2$ is easy to verify following the same argument.

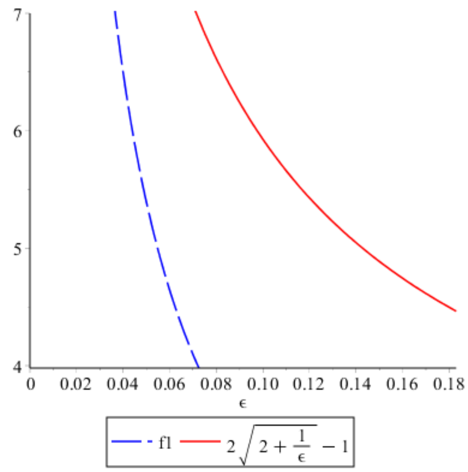


Figure 15: Comparison between f_1, f_2 and $\frac{-\epsilon+2\sqrt{2\epsilon^2+\epsilon}}{\epsilon}$

A.2 Appendix for Chapter 3

A.2.1 A non-cooperative approach

Here we study the case $T = 2$ and $\epsilon \geq 0$ under a non-cooperative set up.

It is assumed that the patent right expires after two periods and that Cournot competition takes place at the end of each period. At the beginning of the first period, the innovator chooses the number $t_1 \geq 0$ of licenses to sell to new entrants as well as the contract that is offered to each one of them. Each contract is of the form (α, δ), where α is an upfront license fee and δ is a commitment of Inn to sell no more than δ licenses in total. Selling licenses to entrants takes no time.

Next, the innovator brings in a Mediator (M), who offers the incumbent (Inc) to purchase the IP from the innovator (Inn) with price y. That is, M offers Inn and Inc to share the future "cake". Note that M can choose the offer contingent on the number t_1. Inn and Inc simultaneously announce whether they accepts M's offer or not. The deal is signed if and only if both accept. Otherwise the deal fails. The process of M making an offer, Inn and Inc deciding on to accept or to reject takes one period.

At the beginning of the second period, the owner of the IP (Inn if the deal fails and Inc if the deal is signed) decides on the number of additional licenses to sell. Denote t_2^{inn} and t_2^{inc} as the number of additional licenses chosen by Inn and Inc respectively.

The payoffs depend on whether or not a deal was reached. If the deal fails, Inn

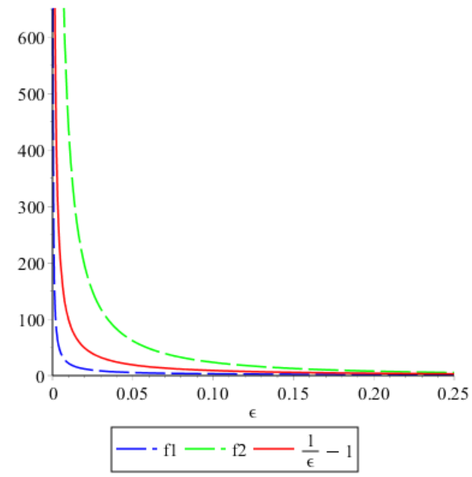


Figure 16: Comparison between f_1, f_2 and $\frac{1}{\epsilon} - 1$

obtains the total license fees he extracts from the $t_1 + t_2^{inn}$ entrants. Inc obtains his two periods' Cournot profit where in the first period he competes with t_1 entrants and in the second period with $t_1 + t_2^{inn}$ entrants. M obtains nothing.

If a deal is signed, Inn obtains the license fees from the first t_1 licensees as well as the payment y made by Inc. Inc obtains his two periods' Cournot profit in addition to the Cournot profits of the t_2^{inc} future new entrant licensees he brings to the market at the second period, subtracts the payment y for purchasing the IP. As for M, he obtains a payoff $u(t_1, y)$.

If $u(t_1, y) = y$, M fully represents Inn and his best offer is equivalent to a take-it-or-leave-it offer to Inc by Inn. If $u(t_1, y) = 1/y$ the best offer of M is equivalent to a take-it-or-leave-it offer of Inc to Inn. If $u(t_1, y) = 1$ then M is only interested in making a deal rather than in the terms of the deal.

Payoffs If M's offer was rejected, in the second period the innovator is best off bringing in additional t_2^{inn} entrants that satisfies

$$t_2^{inn} = \underset{t_2 \in \mathbb{N}_0}{\operatorname{argmax}} \left[t_2 \pi_e(t_1 + t_2) \right]$$

In which case the second period payoffs of the innovator and the incumbent are

$$d_2^{inn} = t_2^{inn} \pi_e(t_1 + t_2^{inn})$$

$$d_2^{inc} = \pi_0(t_1 + t_2^{inn})$$

Clearly, the innovator rejects any offer

$$y < d_2^{inn}$$

If an offer was singed, in the second period it is best off for the incumbent, who becomes the new owner of the IP, to choose t_2^{inc} new entrants to bring in which satisfies

$$t_2^{inc} = \operatorname*{argmax}_{t_2 \in \mathbb{N}_0} \left[t_2 \pi_e(t_1 + t_2) + \pi_0(t_1 + t_2) \right]$$

By paying y for the IP, in the second period the incumbent instead of obtaining d_2^{inc}, obtains

$$v_2 = t_2^{inc} \pi_e(t_1 + t_2^{inc}) + \pi_0(t_1 + t_2^{inc})$$

Thus the incumbent rejects any payment y which exceeds his *additional benefit* from obtaining the IP. Namely Inc rejects any

$$y > v_2 - d_2^{inc}$$

To summarize, under equilibrium, the offer y will be accepted by both Inn and Inc if and only if

$$d_2^{inn} \leq y \leq v_2 - d_2^{inc}$$

Such offer exists if and only if

$$d_2^{inn} + d_2^{inc} \leq v_2 \tag{68}$$

By the definition of t_2^{inc}, inequality (68) always holds.

It is easy to show that irrespective of the value of x

$$t_2^{inn}|_{t_1} = t_1 + 2$$

On the contrary, the optimal choice of t_2^{inc} depends on both t_1 and x. It is easy to verify that

$$t_2^{inc}|_{t_1=0} = 0$$

and

$$t_2^{inc}|_{t_1=1} = \begin{cases} 1 & 0 \leq x \leq \frac{1}{14} \\ 0 & \frac{1}{14} < x \leq \frac{1}{2} \end{cases}$$

$$t_2^{inc}|_{t_1=2} = \begin{cases} 2 & 0 \leq x \leq \frac{1}{22} \\ 1 & \frac{1}{22} < x \leq \frac{7}{54} \\ 0 & \frac{7}{54} < x \leq \frac{1}{2} \end{cases}$$

As shown in the appendix, in the following two sections, $t_1 \geq 3$ can never be in equilibrium. Thus we do not need the formula for $t_2^{inc}|_{t_1}$ when $t_1 \geq 3$ for now.

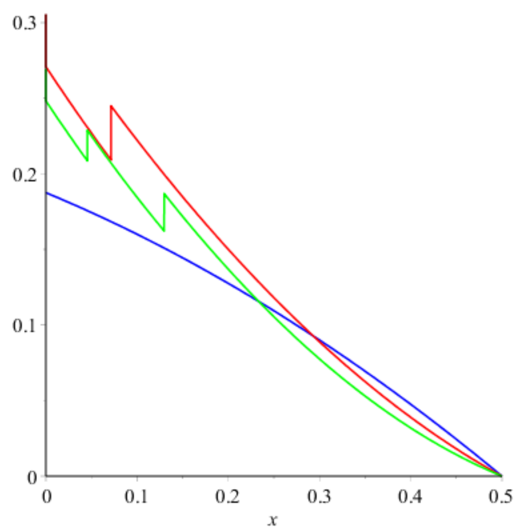
Case $u(y) = y$ In this case, the mediator offers $y = v_2 - d_2^{inc}$. It is equivalent to a take-it-or-leave-it offer made by the innovator to the incumbent. It is shown in the previous section that both players accept the offer under equilibrium. The total payoff of the innovator is

$$\pi^{inn} = t_1\big(\overbrace{\pi_e(t_1) + \pi_e(t_1 + t_2^{inc}|_{t_1})}^{\alpha_1}\big) + y \tag{69}$$

Here α_1 is the license fee charged to each of the t_1 entrants under equilibrium. Rewrite

$$\begin{aligned}\pi^{inn} =& t_1\big(\pi_e(t_1) + \pi_e(t_1 + t_2^{inc}|_{t_1})\big) \\ &+ t_2^{inc}\pi_e(t_1 + t_2^{inc}|_{t_1}) + \pi_0(t_1 + t_2^{inc}|_{t_1}) - \pi_0(t_1 + t_2^{inn}|_{t_1})\end{aligned} \tag{70}$$

The next graph shows the total payoff of the innovator under different choices of t_1. The magnitude of the technology x is plotted on the horizontal axis, with π^{inn} on the vertical axis. The blue, red and green line represents the case for $t_1 = 0$, $t_1 = 1$ and $t_1 = 2$ respectively. The discontinuity of the red and the green line results from the discontinuity of $t_2^{inc}|_{t_1=1}$ and $t_2^{inc}|_{t_1=2}$.



As shown in the graph, for $0.29 < x < 1/2$, it is best off for the innovator to approach the mediator directly (without bringing in an entrant first). For $0 \leq x \leq$

0.29, it is best off for the innovator to first bring in one entrant then to approach the mediator, even though in this section the mediator fully represents the innovator (the offer made by M equivalents to a take-it-or-leave-it offer made by Inn to Inc).

The action of bringing in one entrant before approaching the mediator introduces competition to the market thus reduces the total industry profit to be allocated. But on the other hand it has two positive effects on the innovator's total payoff. (i) it enables the innovator to collect profit through out the bargaining process. (ii) it changes the optimal subsequent choice of $t_2^{inn}|_{t_1}$, which affects the disagreement payoffs of the two bargainers. To be more specific on the second effect, if the innovator approaches the mediator directly (without bringing in any entrants first), the disagreement payoff of the incumbent is his Cournot profit when facing two competitors ($t_2^{inn}|_{t_1=0} = 2$). However, if the innovator brings in one entrant before approaching the mediator, the disagreement payoff of the incumbent is his Cournot profit when facing four competitors (one entrant is already in the market, plus $t_2^{inn}|_{t_1=1} = 3$ subsequent entrants). Clearly the incumbent faces a more severe threat under the second case, thus he is willing to give up a larger size of the cake to prevent the failing of reaching an agreement.

Case $u(y) = 1/y$ In this case, the mediator is best of offering the smallest acceptable y. It is equivalent to a take-it-or-leave-it offer made by Inc to Inn. The mediator offers

$$y = d_2^{inn}$$

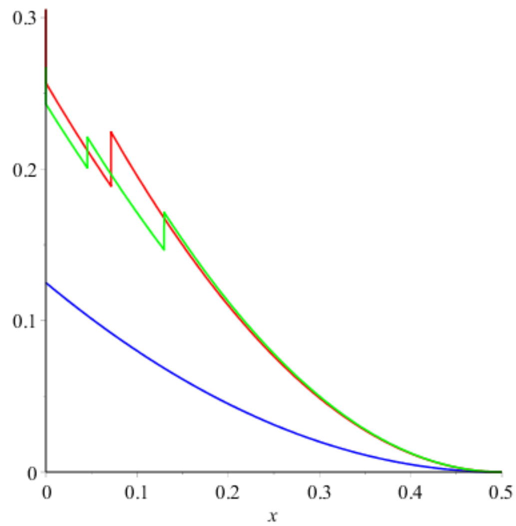
which is accepted by both Inn and Inc under equilibrium. The total payoff of the innovator is

$$\begin{aligned}\pi^{inn} =& t_1\alpha_1 + y \\ =& t_1\big(\pi_e(t_1) + \pi_e(t_1 + t_2^{inc}|_{t_1})\big) + t_2^{inn}|_{t_1}\pi_e(t_1 + t_2^{inn}|_{t_1})\end{aligned} \tag{71}$$

The next graph shows the total payoff of the innovator under different choices of t_1. The magnitude of technology x is plotted on the horizontal axis, with π^{inn} on the vertical axis. The blue, red and green line represents the case $t_1 = 0$, $t_1 = 1$ and $t_1 = 2$ respectively.

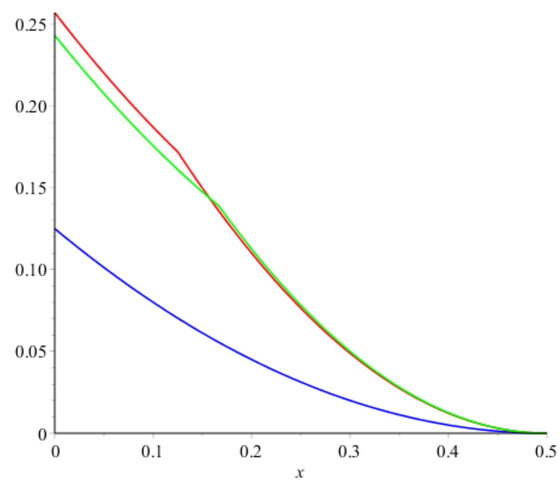
For $0 < x < 1/22$ and $1/14 < x < 7/54$, it is best off for Inn to bring in one entrant before approaching the mediator. For $1/22 < x < 1/14$ and $7/54 < x < 1/2$, it is best off for Inn to bring in two entrants before approaching the mediator.

Combining the results for both cases, note that no matter which bargainer has the full bargaining power, for $0 \le x \le 0.29$ it is best off for the innovator to bring in some entrants before approaching the mediator. In other words, when the technology



is relatively efficient, the inefficient outcome appears no matter whom the mediator represents.

It worth notice that in the above graph, the range of x under which $t_1 = 2$ is the optimal choice is $(\frac{1}{22}, \frac{1}{14}) \cup (\frac{7}{54}, \frac{1}{2})$, which is disconnected. This disconnection results from the restriction that $t_2^{inc}|_{t_1}$ has to be an integer. If we relax such assumption, the disconnection disappears and above graph transforms into the following one



u(y)=1 In this case the mediator only interested in striking a deal rather than the term of the deal. He is indifferent making the offer y as long as it is accepted, namely

$$d_2^{inn} \leq y \leq v_2 - d_2^{inc}$$

which can be written as

$$t_2^{inn}\pi_e(t_1 + t_2^{inn}) \leq y \leq t_2^{inc}\pi_e(t_1 + t_2^{inc}) + \pi_0(t_1 + t_2^{inc}) - \pi_0(t_1 + t_2^{inn}) \qquad (72)$$

Since the Cournot profit of each entrant decreases with the total number of entrants in the market, the left hand side decreases with t_1. It is shown in the appendix that the right hand side also decreases with t_1. As a result, the range of acceptable offers shifts to the left when t_1 increases.

A.2.2 Allow Bargainers to Reach an Agreement Immediately

Let's assume for the bargaining between Inn and Inc, an agreement can be reached immediately if both of them want to do so. However, if either one wants to waste time and delay reaching an agreement, he can delay it for a maximum of one period.

Given the number of entrants already in the market, Inn intends to share part of the cake originally belongs to the incumbent through bargaining. First note that Inn has no incentive delay reaching the agreement since whatever he can obtain prior to bargaining, he can guarantee in the bargaining process (otherwise he is best off not engaged in bargaining at all). Since the size of the cake to be shared remains the same, and Inn benefit from the bargaining procedure, Inc will end up worse off. Namely, Inc has to cut part of his "cake" to Inn to set an agreement. As a result, Inc is always best off delaying reaching the agreement. Thus, even though we allow bargaining to end immediately, it will always take the full period.

A.2.3 Optimal δ for $T = 2, \epsilon \geq 0$

In principle we could allow Inn signing different contracts with the first t entrants specifying different number of total licensees to be sold. But the commitment that really binding is the minimum number specified. Thus there is no point introducing such complication.

First, we show how does δ affects the choice of $n(t)$ and $m(x, t)$. Denote $b = \delta - t$, which represents the constraint on the number of licenses can be sold in addition to the t licensees already in the market. Recall that in the case negotiation fails, Inn chooses n that maximizes the profit of the subsequent licensees

$$n\pi_e(t+n) \tag{73}$$

Since

$$\frac{\partial[n\pi_e(t+n)]}{\partial n} = \frac{(a-c)^2(2x-1)^2(t+2-n)}{(t+n+2)^3}$$

If there are no constraint on n, (73) is maximized when $n = t+2$. Denote $\tilde{n}(t) = t+2$. If, however, $b < t+2$, then (73) is maximized at $n = b$. Thus $n(t) = \min\{b, \tilde{n}(t)\}$.

Next, given the first t licenses and assuming an agreement has been reached, the incumbent chooses to sell additional m licenses where m is the maximizer of

$$M(m,t) = m\pi_e(m+t) + \pi_0(m+t) \tag{74}$$

Since

$$\frac{\partial[m\pi_e(m+t) + \pi_0(m+t)]}{\partial m} = \frac{(a-c)^2[m(2x-1) + 8tx^2 - 6tx + 8x^2 + t - 4x]}{(m+t+2)^3} \tag{75}$$

If there are no constraint on m, (73) is maximized when

$$\frac{\partial[m\pi_e(m+t) + \pi_0(m+t)]}{\partial m} = 0$$

since the right-hand side of (75) is decreasing in m. It is easy to verify that the solution is $t(1-4x) - 4x$. Denote $s(x,t) = t(1-4x) - 4x$ and note that it may not be a non-negative integer. Denote $\tilde{m}(x,t)$ as the non-negative integer that maximizes (75) when there are no constraint on m. It can be easily verified that

$$\tilde{m}(x,0) = 0 \quad \text{for} \quad 0 \leq x \leq 1/2$$

and

$$\tilde{m}(x,1) = \begin{cases} 1 & 0 < x < 1/14 \\ 0 & 1/14 < x < 1/2 \end{cases}$$

etc. In case there is a constraint on the number of total licensees, since (75) is bell shape, we have

$$m(x,t) = \min\{b, \tilde{m}(x,t)\}$$

Next we prove by contradiction that the optimal δ has to satisfy $\delta \geq t + \tilde{n}(t)$. The case $t = 0$ is not relevant since no contract is signed prior to bargaining. For $t \geq 1$, first note that if $\delta \geq t+\tilde{n}(t)$ (which imply $\delta \geq t+\tilde{m}(x,t)$ since clearly $\tilde{n}(t) \geq \tilde{m}(x,t)$), rewrite (25), Inn's total profit becomes

$$\pi_{inn}(x,\beta,t) = \overbrace{\beta[(\tilde{m}(x,t)+t)\pi_e(\tilde{m}(x,t)+t)+\pi_0(\tilde{m}(x,t)+t)] + (1-\beta)t\pi_e(\tilde{m}(x,t)+t)}^{\text{part 3}} \\ + \underbrace{(1-\beta)\tilde{n}(t)\pi_e(\tilde{n}(t)+t) - \beta\pi_0(\tilde{n}(t)+t)}_{\text{part 4}} + t\pi_e(t) \tag{76}$$

Case 1: suppose $t+\tilde{m}(x,t) \leq \delta < t+\tilde{n}(t)$. Since $b = \delta - t$ we have

$$\tilde{m}(x,t) \leq b < \tilde{n}(t)$$

By definition

$$n(t) = b$$

$$m(x,t) = \tilde{m}(x,t)$$

Rewrite (25), the total profit of Inn is

$$\pi^1_{inn}(x,\beta,t,b) = \overbrace{\beta[(\tilde{m}(x,t)+t)\pi_e(\tilde{m}(x,t)+t)+\pi_0(\tilde{m}(x,t)+t)] + (1-\beta)t\pi_e(\tilde{m}(x,t)+t)}^{\text{part 5}} \\ + \underbrace{(1-\beta)b\pi_e(t+b) - \beta\pi_0(t+b)}_{\text{part 6}} + t\pi_e(t) \tag{77}$$

Note that part 5 of (77) coincides with part 3 of (76). But part 6 is smaller than part 4. To see this recall that $k\pi_e(k+t)$ is maximized for $k = \tilde{n}(t)$ and $\pi_0(k+t)$ is decreasing in k. Thus, if Inn restricts the number of licenses he can sell in a manner such that (i) the optimal number of additional licenses to sell if bargaining fails cannot be met, but (ii) the number if bargaining succeeds is intact, then such restriction cannot benefit Inn because it moves the disagreement point to the direction that is disadvantageous to him while keep the size of the "cake" to be shared the same. Thus $m(t) \leq b < n(t)$ cannot be an optimal choice.

Case 2: suppose $\delta \leq t+\tilde{m}(x,t)$. Namely

$$b \leq \tilde{m}(x,t)$$

By definition

$$n(t) = b$$

$$m(x,t) = b$$

Here the restriction on the number of licenses imposed by Inn on one hand decreases the severity of the threat on Inc, on the other hand limits the number of additional competitors can be brought in by Inc if he obtains the IP. In other words, as the result of this restriction, the disagreement point moves to the direction that is not beneficial for Inn while the size of the "cake" to be shared increase. We'll show below that as a combination of these two effects, Inn is worse off.

If an agreement is reached, the surplus is

$$\begin{aligned} v_2 - [d_{inn} + d_{inc}] &= [b\pi_e(t+b) + \pi_0(t+b)] - [b\pi_e(t+b) + \pi_0(t+b)] \\ &= 0 \end{aligned} \tag{78}$$

Inn's total payoff is

$$\begin{aligned} \pi^2_{inn}(x,\beta,t,b) &= t\big[\pi_e(t) + \pi_e(t+b)\big] + b\pi_e(t+b) \\ &= t\pi_e(t) + (t+b)\pi_e(t+b) \end{aligned} \tag{79}$$

Which is maximized when $b = 0$ and $t = 2$. Intuitively, since no matter who is the new owner of the IP in the second period, the same number of additional licenses will be sold, Inn can get nothing from the bargaining process. Now the maximization problem faced by Inn is equivalent to the one assuming he doesn't approach Inc at all and chooses the number of entrants to maximizes the license fee from the two periods. Next we compare the maximum payoff Inn can obtain in the case with the payoff if he doesn't impose a tight constraint on the total number of licensees.

$$\pi_{inn}(x,\beta,1) - \pi^2_{inn}(x,\beta,2,0) = \begin{cases} \frac{1}{36}x^2 - \frac{1}{36}x + \frac{1}{144} - \frac{5}{18}\beta x^2 + \frac{1}{9}\beta x + \frac{1}{72}\beta & 0 \le x \le \frac{1}{14} \\ \frac{2}{9}x^2 - \frac{2}{9}x + \frac{1}{18} - \frac{2}{3}\beta x^2 + \frac{1}{3}\beta x & \frac{1}{14} < x \le \frac{1}{2} \end{cases} \tag{80}$$

It can be easily verified that (80) is non-negative for $1 \le x \le \frac{1}{2}$. Thus Inn is worse off setting the constraint $b < \tilde{m}(x,t)$. As a summary of these two cases, we conclude that $\delta < t + \tilde{n}(t)$ can never be an optimal choice of Inn. In other words, Inn always chooses $\delta \ge t + \tilde{n}(t)$, in which case by definition

$$m(x,t) = \tilde{m}(x,t)$$

$$n(t) = \tilde{n}(t)$$

A.2.4 Relaxing the assumption of $m(x,t)$ being an integer

The "disconnection" of the pink region in Figure 1 disappears if we relax the assumption of $m(x,t)$ being an integer. Recall that

$$m(x,t) = \underset{m\le\delta, m\in\mathbb{N}_0}{\operatorname{argmax}} \left[m\pi_e(m+t) + \pi_0(m+t)\right] \tag{81}$$

Since δ is not binding, as is proved in the previous section, we have

$$m(x,0) = 0$$

$$m(x,1) = \begin{cases} 1 & 0 \le x \le \frac{1}{14} \\ 0 & \frac{1}{14} < x \le \frac{1}{2} \end{cases}$$

$$m(x,2) = \begin{cases} 2 & 0 \le x \le \frac{1}{22} \\ 1 & \frac{1}{22} < x \le \frac{7}{54} \\ 0 & \frac{7}{54} < x \le \frac{1}{2} \end{cases}$$

If we replace the assumption of $m \in \mathbb{N}_0$ by $m \in \mathbb{R}_+$. Let

$$m^*(x,t) = \underset{m\le\delta, m\in\mathbb{R}_+}{\operatorname{argmax}} \left[m\pi_e(m+t) + \pi_0(m+t)\right] \tag{82}$$

It is easy to verify that

$$m^*(x,0) = 0$$

$$m^*(x,1) = \begin{cases} -8x+1 & 0 \le x \le \frac{1}{8} \\ 0 & \frac{1}{8} < x \le \frac{1}{2} \end{cases}$$

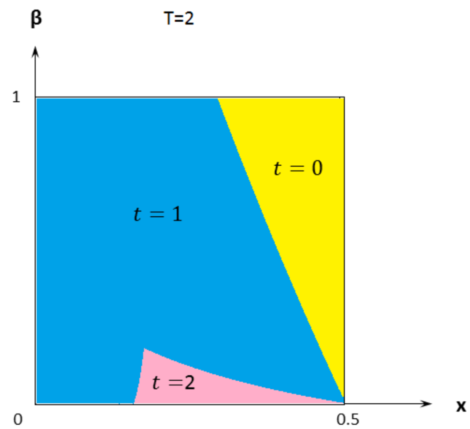
$$m^*(x,2) = \begin{cases} -12x+2 & 0 \le x \le \frac{1}{6} \\ 0 & \frac{1}{6} < x \le \frac{1}{2} \end{cases}$$

Once replacing the $m(x,t)$ in (24) with $m^*(x,t)$, the optimal choice of t is summarized in the following graph.

A.2.5 Severity of the Threat on Inc

Suppose the relative bargaining power of Inn and Inc is fixed. Recall that

$$\begin{aligned}\pi_{inn} =& t\big[\pi_e(t) + \pi_e\big(m(x,t)+t\big)\big] + \beta(v_2 - d_{inn} - d_{inc}) + d_{inn} \\ =& \underbrace{t\big[\pi_e(t) + \pi_e\big(m(x,t)+t\big)\big]}_{\text{license fee collected prior to bargaining}} + \beta(\underbrace{v_2}_{\text{"cake"}} + \underbrace{\frac{1-\beta}{\beta}d_{inn} - d_{inc})}_{\text{severity of threat on Inc}}\end{aligned} \tag{83}$$



The *severity of threat on Inc* given any β is defined as

$$-\Big[d_{inc} - \frac{1-\beta}{\beta} d_{inn}\Big]$$

which is the negation of the interception of the line containing the disagreement point and the bargaining solution. It is shown next that fixing the Pareto frontier and Inn's relative bargaining power, any point lies on the same line yields the same bargaining outcome. Moreover points lies on the line that has a smaller interception yields Inn a higher payoff (or equivalently yields Inc a lower payoff).

In Figure 17, the Pareto frontier is determined by the size of the "cake" v_2. The line containing $[A, B]$ can be written as

$$\{(x_1, x_2)|x_1 + x_2 = v_2)\}$$

Given the relative bargaining power of the innovator, β, let $[C, S]$ be on the line

$$\{(x_1, x_2)|x_2 - \frac{1-\beta}{\beta} x_1 = \hat{k}\}$$

for some $\hat{k}$. It's easy to check that any disagreement point D that lies on $[C, S]$ yields the same bargaining solution. Thus, Inn is better off (and Inc is worse off) when the disagreement point $D' = (d_{inn}, d_{inc})$ lies on a line, parallel to $[C, S]$, but with a smaller interception $\hat{k}'$. Namely, the threat on the incumbent is more severe when $d_{inc} - \frac{1-\beta}{\beta} d_{inn}$ is smaller. Or equivalently, when $-\Big[d_{inc} - \frac{1-\beta}{\beta} d_{inn}\Big]$ is bigger.

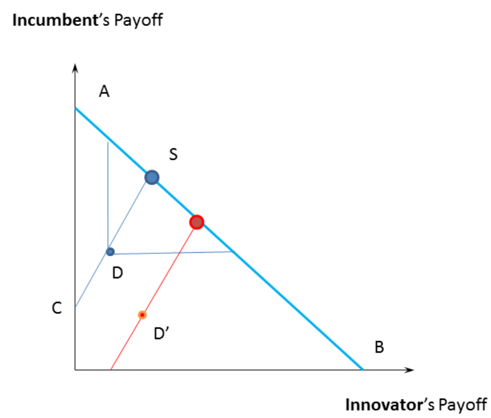


Figure 17

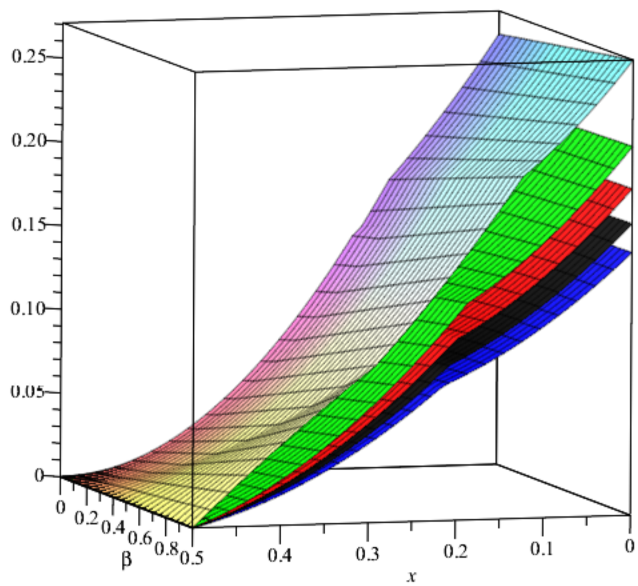
A.2.6 $t \geq 3$ can not be an Optimal Choice of Inn

Denote

$$\bar{\pi}_{inn} = \max\{\pi_{inn}(x, \beta, 0), \pi_{inn}(x, \beta, 1), \pi_{inn}(x, \beta, 2)\}$$

The following graph plots Inn's total payoff under different choices of t. The multicolored plane corresponds to the value of $\bar{\pi}_{inn}$; while the green, red, black, blue plane represents the value of $\pi_{inn}(x, \beta, 3)$, $\pi_{inn}(x, \beta, 4)$, $\pi_{inn}(x, \beta, 5)$ and $\pi_{inn}(x, \beta, 6)$ respectively.

A clear trend can be observed, namely Inn's payoff decreases in t for $t \geq 3$. Recall that bringing in entrants before bargaining has two effects on Inn's payoff. It on one hand enables Inn to collect license fee through the long bargaining process (the number t of entrants that yields Inn the optimal total licensing fee is $t = 2$); on the other hand changes the bargaining game. In the bargaining game, although a larger number of established licenses enables a more severe credible threat on Inc, but it also damages more the total "cake" to be shared. As a result Inn will not bring in more than 2 entrants prior to the bargaining. This being said, a formal proof is still needed.



A.2.7 Proof for Lemma 2

Let $g(n,t,x) = n\pi_e(0,t+n)$. Given t and x, the innovator chooses n that maximizes $g(n,t,x)$

$$g(n,t,x) = \begin{cases} \frac{n}{(t+n+2)^2}(a-c)^2(1-2x)^2 & \text{if } n < -\frac{1}{x} - t \\ \frac{n}{(t+n+1)^2}(a-c)^2(1-x)^2 & \text{if } n \geq -\frac{1}{x} - t \end{cases}$$

It's easy to verify that $\frac{n}{(t+n+2)^2}(a-c)^2(1-2x)^2$ is maximized with $n = t+2$, while $\frac{n}{(t+n+1)^2}(a-c)^2(1-x)^2$ is maximized with $n = t+1$. Note that with $n = -\frac{1}{x} - t$,

$$\frac{n}{(t+n+2)^2}(a-c)^2(1-2x)^2 = \frac{n}{(t+n+1)^2}(a-c)^2(1-x)^2 = (-\frac{1}{x} - t)x^2$$

Thus $g(n,t,x)$ is continues with respect to n. The relation between n, t and x can be divided into the following three cases

Case 1: Suppose $t+2 \leq -\frac{1}{x} - t$ (or equivalently $t \leq -\frac{1}{2x} - 1$), then $n(t,x) = t+2$.
Case 2: Suppose $t+1 \geq -\frac{1}{x} - t$ (or equivalently $t \geq -\frac{1}{2x} - \frac{1}{2}$), then $n(t,x) = t+1$.
Case 3: Suppose $t+1 < -\frac{1}{x} - t < t+2$ (or equivalently $-\frac{1}{2x} - 1 < t < -\frac{1}{2x} - \frac{1}{2}$), then $g(n,t,x)$ is maximized with $n = -\frac{1}{x} - t$. Since $n(t,x)$ has to be a non-negative integer[11], we need to compare $g(t+1,t,x)$ and $g(t+2,t,x)$.

$$g(t+1,t,x) - g(t+2,t,x) = \frac{tx(3x-2) + 2x^2 - 1}{(2t+3)^2} \tag{84}$$

Denote $f(x) = -\frac{2x^2-1}{x(3x-2)}$. It's easy to check that (84) is positive when $t \geq f(x)$, and it is negative otherwise. Thus, $n(t,x) = t+1$ for $t \geq f(x)$, and $n(t,x) = t+2$ for $t < f(x)$. Last note that $-\frac{1}{2x} - 1 < f(x) < -\frac{1}{2x} - \frac{1}{2}$ for $x < 0$. The result in Lemma 1 follows immediately.

A.2.8 Whether Inc Uses the New Technology Once Obtaining the IP

It is not clear after obtaining the IP, if Inc should use the technology himself or not[12]. Although Inc obtains less Cournot profit when using the inferior technology, but he can charge a higher license fee to each of the additional entrants. Note that this requires Inc to be able to commit on using a technology with a higher marginal cost and such action is verifiable to the others. In most cases such requirement is not appropriate and it is convincing to assume Inc always uses the superior technology once obtaining it. In this section we show that in our model even if Inc can commit to use the inferior technology, he is better off use the superior one.

First, if Inc uses the new technology himself, he then chooses the number m of additional licenses to sell which satisfies

$$m(t) = \underset{m \in \mathbb{N}_0}{\operatorname{argmax}} [m\pi_e(1, m+t) + \pi_0(1, m+t)] \tag{85}$$

It can be easily verified that $m(t) = t$. Inc's payoff is

$$\begin{aligned} v_2^{new} &= m(t)\pi_e(1, m(t)+t) + \pi_0(1, m(t)+t) \\ &= (a-c)^2 \frac{(1-x)^2}{4(t+1)} \end{aligned} \tag{86}$$

If instead, Inc commits on not using the new technology himself (whether such

[11]It's easy to verify that n(t,x) can be either $t+1$ or $t+2$

[12]Similarly for case $\epsilon \geq 0$ it is not clear if Inc should use his old technology or use the new but inferior one

commitment is reliable and verifiable is another question we do not address here), he then chooses m to solve the following optimization problem

$$v_2^{old} = \max_m [\overbrace{m\pi_e(0, m+t) + \pi_0(0, m+t)}^{\text{Part A}}] \tag{87}$$

Note that

$$\text{Part A} = \begin{cases} (a-c)^2 m\left(\frac{1-x}{t+1+m}\right)^2 & \text{if } t+m \geq -\frac{1}{x} \\ (a-c)^2\left[m\left(\frac{1-2x}{t+m+2}\right)^2 + \left(\frac{1+(t+m)x}{t+m+2}\right)\right] & \text{if } t+m < -\frac{1}{x} \end{cases}$$

It's easy to verify that $(a-c)^2 m\left(\frac{1-x}{t+1+m}\right)^2$ is maximized when $m = t+1$ and $(a-c)^2\left[m\left(\frac{1-2x}{t+m+2}\right)^2 + \left(\frac{1+(t+m)x}{t+m+2}\right)\right]$ is maximized when $m = -4tx + t - 4x$. In addition, $(a-c)^2 m\left(\frac{1-x}{t+1+m}\right)^2 = (a-c)^2\left[m\left(\frac{1-2x}{t+m+2}\right)^2 + \left(\frac{1+(t+m)x}{t+m+2}\right)\right]$ when $m = -\frac{1}{x} - t$.

Thus, if Inc chooses $m < -\frac{1}{x} - t$, Part A is maximized at $m = \min\{-\frac{1}{x} - t, t - 4x - 4tx\}$. If instead, Inc chooses $m \geq -\frac{1}{x} - t$, Part A is maximized at $m = \max\{-\frac{1}{x} - t, t+1\}$.

It's easy to verify that

$$-\frac{1}{x} - t > t - 4x - 4tx \quad \text{iff} \quad x > -\frac{2}{4t+4}$$

$$-\frac{1}{x} - t > t + 1 \quad \text{iff} \quad x > -\frac{2}{4t+3}$$

(i) for $x > -\frac{2}{4t+4}$, $-\frac{1}{x} - t > t - 4x - 4tx$ and $-\frac{1}{x} - t > t+1$. The optimal m under the restriction of $m < -\frac{1}{x} - t$ is $m = \min\{-\frac{1}{x} - t, t - 4x - 4tx\} = t - 4x - 4tx$; the one under the restriction of $m \geq -\frac{1}{x} - t$ is $m = \max\{-\frac{1}{x} - t, t+1\} = -\frac{1}{x} - t$. Since Part A is continues, it is maximized when $m = t - 4x - 4tx$.

$$v_2^{new} - v_2^{old} = \frac{-x(4tx + 3x + 2)}{4(t+1)}$$

This difference is non-negative when $-\frac{2}{3+4t} \leq x \leq 0$, which is automatically satisfied in this case. Thus here Inc is better off using the new technology.

(ii) for $-\frac{2}{3+4t} < x < -\frac{2}{4t+4}$, $-\frac{1}{x} - t < t - 4x - 4tx$ and $-\frac{1}{x} - t > t+1$. The optimal

m under the restriction of $m < -\frac{1}{x} - t$ is $m = \min\{-\frac{1}{x} - t, t - 4x - 4tx\} = -\frac{1}{x} - t$; the one under the restriction of $m \geq -\frac{1}{x} - t$ is $m = \max\{-\frac{1}{x} - t, t+1\} = -\frac{1}{x} - t$. When Inc chooses $m = -\frac{1}{x} - t$.

$$v_2^{old} = (a-c)^2(-\frac{1}{x} - t)\Big(\frac{1-x}{t+1+(-\frac{1}{x}-t)}\Big)^2$$

$$v_2^{new} - v_2^{old} = \frac{(2tx+x+1)^2}{4(t+1)} \geq 0$$

Thus Inc is better off using the new technology.

(iii) for $x < -\frac{2}{3+4t}$, $-\frac{1}{x} - t < t - 4x - 4tx$ and $-\frac{1}{x} - t < t+1$. The optimal m under the restriction of $m < -\frac{1}{x} - t$ is $m = \min\{-\frac{1}{x} - t, t - 4x - 4tx\} = -\frac{1}{x} - t$; the one under the restriction of $m \geq -\frac{1}{x} - t$ is $m = \max\{-\frac{1}{x} - t, t+1\} = t+1$. Again, since Part A is continues, Inc chooses $m = t+1$. Since

$$t + m = 2t + 1 > -\frac{1}{x}$$

with the old technology the Incumbent firm is driven out of the market. Here $v_2^{new} = v_2^{old}$. Namely Inc is indifferent between using the new technology himself or not.

As a conclusion, in our model, after obtaining the IP for the new technology, in addition to issue additional licenses, Inc is always better off using it himself.

A.2.9 The r.h.s. of (72) Decreases with t_1

Since $t_2^{inn}|_{t_1} = t_1 + 2$, we only need to prove that

$$\begin{aligned} &t_2^{inc}|_{t_1}\pi_e(t_1 + t_2^{inc}|_{t_1}) + \pi_0(t_1 + t_2^{inc}|_{t_1}) - \pi_0(2t_1+2) \\ \geq &t_2^{inc}|_{t_1+1}\pi_e(t_1 + 1 + t_2^{inc}|_{t_1+1}) + \pi_0(t_1 + 1 + t_2^{inc}|_{t_1+1}) - \pi_0(2t_1+4) \end{aligned} \tag{88}$$

It is equivalent to prove

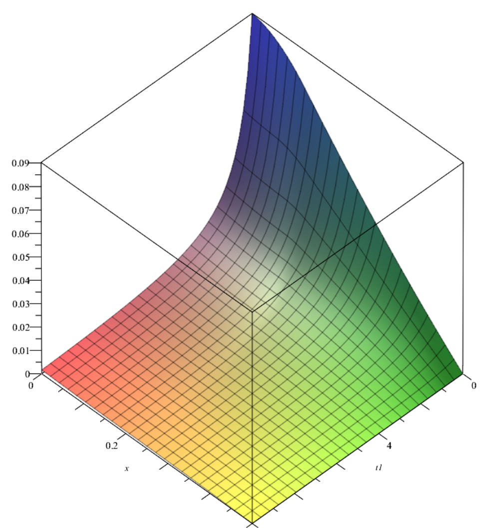
$$\begin{aligned} &\pi_0(2t_1+2) - \pi_0(2t_1+4) \leq \\ &\underbrace{\big[t_2^{inc}|_{t_1}\pi_e(t_1 + t_2^{inc}|_{t_1}) + \pi_0(t_1 + t_2^{inc}|_{t_1})\big]}_{\text{Part C}} - \underbrace{\big[t_2^{inc}|_{t_1+1}\pi_e(t_1+1+t_2^{inc}|_{t_1+1}) + \pi_0(t_1+1+t_2^{inc}|_{t_1+1})\big]}_{\text{Part D}} \end{aligned} \tag{89}$$

First, the following 3d graph plotted the value

$$\overbrace{\left[t_2^*|_{t_1}\pi_e(t_1+t_2^*|_{t_1})+\pi_0(t_1+t_2^*|_{t_1})\right]}^{\text{Part A}}-\overbrace{\left[t_2^*|_{t_1+1}\pi_e(t_1+1+t_2^*|_{t_1+1})+\pi_0(t_1+1+t_2^*|_{t_1+1})\right]}^{\text{Part B}}$$
$$-\left[\pi_0(2t_1+2)-\pi_0(2t_1+4)\right] \tag{90}$$

With

$$\begin{aligned} t_2^* &= \operatorname*{argmax}_{t_2\geq 0}\left[t_2\pi_e(t_1+t_2)+\pi_0(t_1+t_2)\right] \\ &= \max\{0, t_1(1-4x)-4x\} \end{aligned} \tag{91}$$



Clearly, the following inequality holds.

$$\pi_0(2t_1+2)-\pi_0(2t_1+4)\leq$$
$$\underbrace{\left[t_2^*|_{t_1}\pi_e(t_1+t_2^*|_{t_1})+\pi_0(t_1+t_2^*|_{t_1})\right]}_{\text{Part A}}-\underbrace{\left[t_2^*|_{t_1+1}\pi_e(t_1+1+t_2^*|_{t_1+1})+\pi_0(t_1+1+t_2^*|_{t_1+1})\right]}_{\text{Part B}} \tag{92}$$

Next, substitute the $t_2^{inc}|_{t_1}$ in Part C with $t_2^*|_{t_1}+1$ and denote it as Part E; substitute the $t_2^{inc}|_{t_1+1}$ in Part D with $t_2^*|_{t_1+1}+1$ and denote it as Part F. By the definition of t_2^{inc} and t_2^*, clearly

$$E \leq C$$

$$F \leq D$$

We already shown that

$$A - B \geq \pi_0(2t_1 + 2) - \pi_0(2t_1 + 4)$$

If we can show that

$$|(C - D) - (A - B)| \leq (A - B) - \Big[\pi_0(2t_1 + 2) - \pi_0(2t_1 + 4)\Big] \tag{93}$$

Then it follows easily

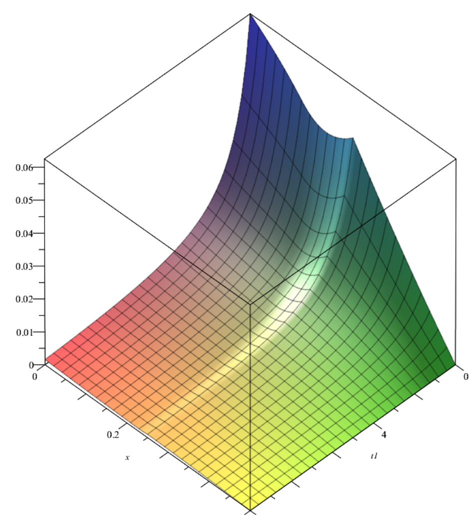
$$C - D \geq \pi_0(2t_1 + 2) - \pi_0(2t_1 + 4)$$

To show (93), observe that

$$|(C - D) - (A - B)| = |(A - C) + (D - B)| \leq |A - C| + |B - D| \leq (A - E) + (B - F)$$

The following graph plotted the value

$$\Big[(A - B) - \big[\pi_0(2t_1 + 2) - \pi_0(2t_1 + 4)\big]\Big] - \Big[(A - E) + (B - F)\Big] \tag{94}$$



Clearly, (94) is non-negative. Thus inequality (93) holds true. The prove is thus finished.

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