Essays in Financial Econometrics and Game Theory

A dissertation presented

by

Abhinav Anand

to

The Graduate School

in partial fulfillment of the

requirements

for the degree of

Doctor of Philosophy

in

Economics

Stony Brook University

August 2014

Stony Brook University

The Graduate School

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Abstract of the Dissertation

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My thesis analyzes two different topics: the estimation of the equity risk posed by the "too-big-to-fail" banks during the period encompassing The Great Recession; and a comparative analysis of the welfare effects of two different classes of affirmative action mechanisms. Both essays have been inspired by a desire to analyze currently enforced policies and to attempt to improve upon them by using arguments from Financial Econometrics and Game Theory respectively.

The first essay is titled "*The Empirical Foster-Hart Risk of the Global Banking Stock Market*" and measures how much equity risk the too-big-to-fail banks posed on the common public during the recent financial crisis. In this essay I use an "ARMA(1,1)-GARCH(1,1)-Normal Tempered Stable" statistical model to capture the skewed and leptokurtotic nature of stock returns; and employ the "Foster-Hart risk measure" to better capture equity risk. This union of sophisticated risk modeling

with fat-tailed statistical modeling bears fruit, as the paper is able to measure the equity risk during the Great Recession much more accurately than is possible with current techniques.

The second essay is titled "Quotas versus Handicaps: A Game Theoretic Analysis of Affirmative Action Policies in India". In this essay, I analyze and compare the Quota Policy — in which preference is given to the disadvantaged section of the populace by reserving a certain fraction of jobs for them; and a hypothetical "Handicap Policy" — in which the performance index of the disadvantaged is given an added boost, by means of an additive handicap. After modeling this situation as a game, I am able to conclude that on many important metrics of performance, Quotas and Handicaps can be shown to be equivalent to each other.

Keywords: ARMA-GARCH model, Normal Tempered Stable Distribution, Foster-Hart risk, Value-at-Risk (VaR), Average Value-at-Risk (AVaR), Too-big-to-Fail, Systemically Important Financial Institutions (SIFI), Affirmative Action, Quotas, Handicaps

JEL classification: C13, C22, C58, C65, C72, G32, I24, I28

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Acknowledgements

"All this happened, more or less", thus begins Kurt Vonnegut's Slaughterhouse-Five, the opening lines echoing the hazy, ambiguous, somewhat amorphous mix of sentiments the culmination of my PhD evokes in me. It has been an eventful journey and I daresay I have learned much, though a sense of humility, engendered by the depth of my ignorance; and a feeling of excitement of learning *so* much more, remain the stronger impressions.

While I will be credited as the sole author of this thesis, it is definitely a collaborative effort only made possible by the support of so many. My deepest thanks go to my dissertation advisor Professor Sandro Brusco whose razor-sharp mind, awe-inspiring erudition and insistence on independent thinking have shaped my development in important ways. I thank Professor Yair Tauman for his unconditional support, good humor and ever-useful advice all through my PhD. I also wish to thank Professor Pradeep Dubey for first suggesting the problem that kick-started my PhD, for helping me shift to the Economics department from Applied Mathematics; and for inculcating in me a love for writing which is clear, precise, even beautiful.

I have been incredibly lucky to have been mentored by Professor Svetlozar Rachev whose lectures in Mathematical Finance in general, and those on fat-tailed distributions in particular, taught me whatever little I do know in Finance. A large part of my research builds upon the pioneering work done by him over the years and the wonderful textbooks he has written on the subject. I also thank Professor Young Shin Kim for his constant guidance and for supervising my research work in Finance which owes a great debt to a sequence of papers written by him recently. It is indeed an honor to co-author a paper with him and it was my great fortune to work with him in the College of Business in my final stages of PhD. I also want to applaud Professor Hugo Benitez-Silva and his helmsmanship of the Economics department.

His adept handling of all difficulties — academic, financial or otherwise — helped me stay on course and complete my PhD.

Among fellow graduate students and friends, I must first thank Bruno for teaching me almost everything I know in Economics; Sama and Soyol for always being there when it mattered; Tiantian and Kurosaki san for being *so* patient during our collaboration; Monstha and Vatsa for all the years of companionship; Vatsa, Vicky and Abhra for their "Fellowship for Impecunious Graduate Students"; Jacques for five years of uninterrupted comedy; Tamara and Anju for making commuting a fun adventure; Xin and Kaya for being inspirations; Deniz for her invaluable friendship; Michael, Lala and Sumit babu for being my wonderful little faux-Karamazovian family at 213 Main Street; and *of course* to Shishir for being a friend indeed.

Thanks also to all my family whose total trust and support I'm lucky enough to have enjoyed all through my life — in particular, to Mitu bhaiya and Abha bhabhi for being the iron pillars of support I could rely on. I end by thanking my late grandfather Pt. Dharnidhar Dwivedi in whose memory this thesis is dedicated.

Chapter 1

The Empirical Foster-Hart Risk of the Global Banking Stock Market

Abstract

The measurement of financial risk relies on two factors: determination of riskiness by use of an appropriate risk measure; and the distribution according to which returns are governed. Wrong estimates of either, severely compromise the accuracy of computed risk. We analyze the equity risk of an equally weighted portfolio of "Global Systemically Important Financial Institutions" by means of the "Foster-Hart risk measure" — a new, reserve based measure of risk, extremely sensitive to tail events. We model banks' stock returns as an ARMA(1,1)-GARCH(1,1) process with "Normal Tempered Stable" innovations, to capture the skewed and leptokurtotic nature of stock returns. Our union of sophisticated risk modeling with fat-tailed statistical modeling bears fruit, as we are able to measure the equity risk during the Great Recession much more accurately than is possible with current techniques.

Keywords: ARMA-GARCH model, Normal Tempered Stable Distribution, Foster-Hart risk, Value-at-Risk (VaR), Average Value-at-Risk (AVaR), Too-big-to-Fail, Systemically Important Financial Institutions (SIFI)

JEL classification: C13, C22, C58, C65, C72, G32

1.1 Introduction

While accurate measurement of financial risk is an important theoretical problem in its own right, the Great Recession, its attendant financial turmoil and the Eurozone crisis have catapulted the subject into public limelight and highlighted its centrality in framing economic policy. Following the Lehman Brothers' bankruptcy in September 2008, the Basel Committee for Banking Supervision (BCBS) formulated a successor to the then Basel II regulatory framework. The new framework, Basel III, insisted on additional capital requirements for *Global Systemically Important Financial Institutions* (G-SIFIs) whose failure or distress, it claimed, could trigger instability in the global financial markets.[1] In turn, the Financial Stability Board (FSB) was set up in April 2009 which drew up an initial list of 29 G-SIFIs in November 2011.[2]

The Basel Committee for Banking Supervision is not alone in its calls for additional capital requirements. Admati and Hellwig (2013) make equally strident calls for increase in capital requirements for banks. Similar concerns have been echoed by Lord Adair Turner, the Chairman of the Financial Stability Authority (FSA) of the UK in numerous speeches and press briefings.[3] All of these commentators make the point that benefits accruing from the long term financial stability of the systemically important institutions far exceed the modest risk of slow GDP growth in the near future. Moreover, the moral hazard generated by governments' intervention by means of taxpayer funded bailouts has detrimental consequences for corporate incentives and public finances. The recent nationalization and subsequent breakup and restructuring of Dexia, a G-SIFI, highlights the importance of adequate capitalization, especially for those institutions that are exposed to positions whose riskiness becomes more pronounced during systemic market downturns.

Motivated by such concerns, we aim to measure the equity risk posed by the G-SIFI constituents accurately. To this end, we form an equally weighted portfolio of G-SIFIs and track its normalized equity risk on the basis of banks' stock market returns from January 4th, 2000 to February 28th, 2014. This is done by means of three different measures of risk — the currently in use Value at Risk (VaR) and Average Value at Risk (AVaR) — and the recently discovered *Foster-Hart risk*. Foster and Hart (2009) have proposed a new measure of risk which computes the minimal wealth needed to avoid bankruptcy for an agent who faces an unknown sequence of risky gambles. In addition, it enjoys many other highly desirable properties that experts in the field deem "coherent" (Artzner *et al.*, 1998).

[1] In Basel II, capital requirements for banks were uniform and not dependent on their systemic importance.

[2] For the latest list of G-SIFIs, please visit `http://www.financialstabilityboard.org/publications/r_131111.pdf`

[3] The FSA was dissolved from April 1st, 2013 and its duties were split between the Prudential Regulation Authority, the Financial Conduct Authority and the Bank of England.

We postulate an underlying standard *Multivariate Normal Tempered Stable* distribution for the returns of banking stocks and model their temporal dependence by means of an ARMA(1,1)-GARCH(1,1) stochastic process. In order to establish our skewed and fat-tailed distributional hypothesis's contribution to statistical modeling, we compare it with the standard specification of stock returns being distributed as Multivariate Normal random variables.

We emphasize that we are not measuring the equity risk *of* G-SIFIs but in fact measuring the equity risk *posed by* them on a hypothetical investor who holds the equally weighted G-SIFI portfolio. We interpret this hypothetical investor to be in fact, the common public, whose financial fortunes are closely tied with those of the systemically important banks. A rise in the levels of portfolio risk implies a rise in the financial risk faced by ordinary citizens. Although they hold the portfolio only symbolically, they face the quite real consequences of savings lost due to financial panic, which in turn is often fueled by risky G-SIFI behavior. In this sense, the analysis in our paper is helpful in measuring the extent to which the common public is exposed to equity risks. We also note that in order to measure the equity risk *of* banks, one must have access to banks' internal portfolios. However, our methodology is flexible enough for banks' in-house risk management teams to adapt and thus, our approach can be used to compute equity risks *of* banks as well.

The rest of the paper is organized as follows. In section 2 we introduce the theory behind risk measurement and statistical modeling. In section 3 we describe the data used while the 4^{th} section outlines the methodology employed to estimate the statistical model and the portfolio risks. Section 5 describes the results and interprets them. We outline our conclusions in section 6 and end the paper by including additional results and tables in the Appendices.

1.2 Theory

Our research work is primarily based on two parallel strands of literature — one from the theory of risk measurement and the other from the theory of time series econometrics.

In the following sections, we focus on some relevant aspects of both theories to properly ground our work in the broader context of related literature and delineate its main features.

1.2.1 Risk Measurement

An agent undertaking a gamble can win as well as lose. From a mathematical point of view, the gamble being undertaken is just a random variable whose outcomes are governed by a certain probability distribution. In case of a win, it stands to reason that the agent's wealth increases when compared to its pre-gamble level and decreases in case of a loss.

Since gambles may be viewed as random variables, can we ascribe their riskiness as stemming from the measures of dispersion (such as variance) associated with their distributions? After all, economic agents, usually modeled as risk averse, prefer less uncertainty to more uncertainty; and measures of dispersion quantify uncertainty. Such was the reasoning behind Markowitz (1952) in which he identified standard deviation as a proxy for risk of a gamble.

Prima facie, it may seem to be a good assumption. However, on closer inspection, we can detect problems with this approach. Standard deviation, like other symmetric measures of dispersion, penalizes *equally*, deviations from the mean — whether in the positive direction (big gains) or in the negative direction (big losses). Risk, however, has traditionally been more concerned with losses than with gains. Arguably, if an agent is proposed a gamble in which there are two equally probable outcomes — a gain of \$50 or a gain of \$150, while the standard deviation of this gamble would be positive, its *risk* posed to the agent is exactly 0! It was observations such as these that led Markowitz to propose *semi standard deviation* as a proxy for risk (Markowitz, 1959).

While semi standard deviation — in which only losses are taken into account — may seem to be an acceptable solution, it turns out that it is still not sophisticated enough to capture the notion of risk. In fact, it may be proved that *all* dispersion measures will fail to be true risk measures! (Rachev *et al.*, 2008, 180–181)

Having underscored the essential difference between measures of dispersion and those of risk, we introduce some popular and well studied measures of risk in the following sections.

Value at Risk

Value at Risk is defined as the minimum level of loss at some specified, sufficiently high confidence level (say 99%) for a given time horizon (say one day). As an example, if a gamble's (or equivalently, portfolio's) one day 99% VaR is $1 million, it means that the gamble (portfolio) may lead to a loss of an amount more than a million over a period of one day with probability 1%.[4]

Mathematically:

$$\mathrm{VaR}_\alpha(g) := -\inf\{g : G(g) \geq \alpha\} = -G^{-1}(\alpha)$$

where g is the gamble, $G(\cdot)$ is its payoff distribution (profit and loss distribution) and $(1 - \alpha) * 100$ is the (percentage) confidence level.[5]

Even though VaR has been embraced by financial institutions and regulators alike, it suffers from some serious deficiencies as outlined here:

Deficiencies

1. *Uninformativeness about losses beyond the VaR level*: As an example, if an agent has two portfolios — one in which 5% VaR is $1 million and another one in which 5% VaR is $1 million *and* 4% VaR is $10 million, the risk of both portfolios at 95% confidence level will be reported by VaR to be at the same level of $1 million. This is clearly misleading as the risk posed by holding the second portfolio is much higher than the first. In other words, VaR is silent about losses posed beyond the VaR level.

[4] VaR was popularized by JP Morgan in the late 1980s and remains the most widely used risk measure in the industry. In the mid 1990s, it was endorsed by the Basel Committee of Banking Supervision (BCBS) to be used to compute capital reserves of financial institutions.

[5] The second right hand side follows if we assume that G is continuous.

2. *Failure to capture "Diversification Effects"*: While it is a well understood phenomenon in Finance that holding different assets in a portfolio leads to an overall reduction in risk (often dubbed the "diversification effect"), one may easily come across examples in which portfolio VaR risk *exceeds* that of the sum of its constituents. That is,

$$\mathrm{VaR}_\alpha(X + Y) > \mathrm{VaR}_\alpha(X) + \mathrm{VaR}_\alpha(Y)$$

Due to these crippling defects, researchers have argued that VaR should in fact be abandoned as a measure of risk altogether (Rachev *et al.*, 2008, 185).

Average Value at Risk

A more sophisticated measure of risk is Average Value at Risk, which as the name suggests, averages the different VaR values beyond the confidence level. Mathematically, it is defined to be the following:

$$\mathrm{AVaR}_\alpha(g) := \frac{\int_0^\alpha \mathrm{VaR}_p(g)dp}{\alpha}$$

The Average Value at Risk is much more informative about the losses beyond the stipulated confidence level and enjoys highly desirable properties of "coherence" (Artzner *et al.*, 1998) that VaR lacks.

This definition solves *both* the problems described above with VaR — not only is AVaR much more informative about levels of losses beyond the VaR level — it is also able to capture the diversification effect!

While AVaR is a significant improvement over VaR, it still suffers from some deficiencies that one cannot overlook.

Deficiencies

1. *Dependence on arbitrary confidence levels*: AVaR's dependence on arbitrary, ad hoc confidence levels is a serious shortcoming especially since prima facie it

is not clear what ought to be “the best” confidence level.

2. *Problems with backtesting*: Due to the definition of AVaR, checking whether reported AVaR values are reliable, requires complete knowledge of the structure of the tails of the distribution according to which portfolio returns are assumed to be distributed. This is often very difficult to assess and therefore reported AVaR numbers may not be easily verified for accuracy — thus posing a significant challenge to regulators.

Foster-Hart Risk

Foster and Hart (2009) proposed a new measure that was based on considerations of how much wealth an agent should possess, irrespective of her utility function, to render a gamble “not risky”. They describe “gamble” to be any *bounded* random variable with a positive expectation and a positive probability of losses:

$$\mathbb{E}(g) > 0 \text{ and } \mathbb{P}(g < 0) > 0$$

In their setup, a sequence of gambles is offered to an agent with an arbitrary utility function where the *only* assumption made about her is that she prefers non bankruptcy to bankruptcy. The agent rejects the gamble if she deems it “too risky”, otherwise, accepts it.

To further motivate the setup, we offer an example taken from Foster and Hart (2009). Suppose the gamble being offered gives gains of \$120 and losses of \$100 with equal probability:

$$g = \begin{cases} 120 \text{ with probability } 1/2 \\ -100 \text{ with probability } 1/2 \end{cases}$$

Foster and Hart (2009) ask some important questions about the riskiness of this gamble. Since the analysis is free of any knowledge of the agent's utility, their questions probe the extent to which the wealth of the agent, and her preference of non-bankruptcy over bankruptcy influences her perception of risk inherent in the gamble. For example, if the agent possesses wealth of \$100 will she deem the gamble to be “too risky”? What if, her wealth is \$1 million?

It is clear that in the first case, the agent’s probability of bankruptcy is 50% thus making the gamble risky for her to play, while in the second instance, bankruptcy probability is 0 and so the gamble is not risky at all! The central concern of Foster and Hart (2009) is: *“Is there a critical level of wealth below which playing the gamble is risky but beyond which it is not so?”*. The answer they provide is “Yes!” and the critical wealth level is proven to be unique as well!

Foster-Hart Setup:

The agent’s wealth in time periods 1, 2, ... is assumed to be $W_1, W_2, \ldots$ and it is assumed that she is being offered, in each time period, gambles $g_1, g_2, \ldots$

In each time period the agent may choose to either accept (play) the gamble or reject (not play) the gamble. If she accepts, her wealth next period becomes $W_{t+1} = W_t + g$ — where g is the realization of the gamble g_t, while if she rejects the gamble, her new wealth equals her old wealth, i.e., $W_{t+1} = W_t$. The sequence of gambles is assumed to be arbitrary.

For each offered gamble g_t, Foster and Hart (2009) define a “*Critical Wealth Function*” which computes a wealth level below which gambles are rejected and above which, they are accepted. The use of such a function induces a “simple strategy” that instructs the agent to accept a gamble *only* if her wealth is more than that computed by the critical wealth function.

It is assumed that such critical wealth functions $Q(g)$ satisfy two important criteria:

1. *Homogeneity*: $\forall\ \lambda > 0,\ Q(\lambda g) = \lambda Q(g)$ — doubling gambles doubles wealth requirement.

2. *Distribution*: The critical wealth function $Q(\cdot)$ is objective and depends *only* on the distribution of the gamble.

Bankruptcy is defined to be the situation in which, $\exists\ t < \infty$: $W_t = 0$ or if $\lim\limits_{t\to\infty} W_t = 0$. Likewise, non bankruptcy is satisfied if $\Pr\left(\lim\limits_{t\to\infty} W_t = 0\right) = 0$.

A simple strategy s_Q, based on a critical wealth function $Q(\cdot)$ *guarantees* no bankruptcy (almost surely) if

1. For each starting wealth W_1, and

2. For each sequence of gambles $g_1, g_2, \ldots$

$$\Pr\left(\lim_{t\to\infty} W_t = 0\right) = 0$$

The main result of Foster and Hart (2009) is that for each gamble g, there is a function $R(g)$ which such that a simple strategy s_Q guarantees no bankruptcy iff $Q(g) \geq R(g)$. Moreover, $R(g)$ is the unique positive solution to the following equation:

$$\mathbb{E}\left(\log\left[1+\frac{g}{R(g)}\right]\right) = 0$$

This function $R(g)$ — the Foster-Hart risk measure — may be interpreted as the *minimal reserve* needed to play the gamble g.

The Foster-Hart risk is more sophisticated than VaR and AVaR since it is free from arbitrary confidence levels and time horizons; and because it guarantees non-bankruptcy in the face of infinite, unknown sequences of arbitrary gambles. (In other words, the confidence level is 100%, the time horizon is infinite and the risk measure is coherent.)

In addition, the Foster-Hart risk enjoys many other highly desirable properties:

Selected Properties:

1. *Homogeneity*: $\forall\ \lambda > 0,\ R(\lambda g) = \lambda R(g)$ — doubling gambles doubles wealth requirement.

2. *Subadditivity*: $R(g+h) \leq R(g) + R(h)$ — a diversified portfolio has lesser risk than that of a sum of its components.

3. *First Order Monotonicity*: If gamble g first order stochastically dominates gamble h, $R(g) < R(h)$.

4. *Second Order Monotonicity*: If gamble g second order stochastically dominates gamble h, $R(g) < R(h)$.

5. *Black Swan Proofness*:[6] For each gamble, Foster-Hart riskiness is at least as high as the *maximum loss* L of that gamble.

6. *Continuity*: If a sequence of gambles converges and their maximum losses converge, their Foster-Hart risks converge.

Foster-Hart Risk for General Gambles

In the Foster-Hart setup, gambles faced by the agent are assumed to have discrete probability mass functions. As shown in Riedel and Hellmann (2014), the defining equation for the Foster-Hart riskiness has no solution for many common continuous distributions. However, the Foster-Hart risk may be consistently extended to such continuous random variables, in which case it coincides with *maximum loss* L, i.e.,

$$R(g) = \begin{cases} r^* : \mathbb{E}\left(\log\left[1 + \frac{g}{r^*}\right]\right) = 0 & \text{if } \mathbb{E}\left(\log\left[1 + \frac{g}{L}\right]\right) < 0 \\ L & \text{if } \mathbb{E}\left(\log\left[1 + \frac{g}{L}\right]\right) \geq 0 \end{cases} \tag{1.1}$$

1.2.2 Statistical Modeling

While in principle, it is very simple to compute the Foster-Hart risk — after all it is the unique positive solution to (1.1) — one presupposes that we have the "correct" distribution according to which gambles are governed. After many decades of investigation, researchers now agree that return distributions of various financial instruments are *skewed and leptokurtotic* — more peaked around the mean, with fat, Pareto-type tails.[7] Hence, using recent findings from Kim *et al.* (2012) we choose the "Normal Tempered Stable" distribution to model stock returns.

[6] Taleb (2007) has popularized the idea of "Black Swan Events" — extremely rare events which nonetheless have tremendous, often incalculable impacts on society.

[7] For a comprehensive review, see (Haas and Pigorsch, 2009).

A simple way to find out the distribution according to which the payoffs or returns of a gamble are distributed is to simply assume that the observations are the realizations of an unobservable underlying distribution — say Normal with some mean and standard deviation — and use a standard statistical method (say Maximum Likelihood) to choose from the class of Normal distributions, the one whose mean and standard deviation produce the best "fit" to the data (are most likely).

An obvious shortcoming of this approach is that it is not clear how to choose this underlying distribution. For example there is a substantially large body of literature, starting from Alexander (1961), Mandelbrot (1963a), Mandelbrot (1963b), Fama (1963), Fama (1965), Mandelbrot (1967) etc. that overwhelmingly rejects the normal family as candidates for underlying data generating distributions. (For a comprehensive literature review, see Haas and Pigorsch (2009).)

Subsequent work done after Mandelbrot and Fama confirmed and extended their findings (Haas and Pigorsch, 2009).

In fact, as is discussed at length in Rachev and Mittnik (2000), a promising way to model return distributions is through the family of *Stable Distributions*. Also, Rachev *et al.* (2005b) shows that Stable Distributions provide better fits to real world stock returns than the Normal Distribution.

Stable Distributions: A Short Primer

Normal distributions and Cauchy distributions are special cases of Stable distributions. In fact, just as Normal distributions emerge as limiting distributions as a consequence of the classical Central Limit Theorem, Stable distributions emerge as limiting distributions as a consequence of the *Generalized Central Limit Theorem*.

The Central Limit Theorem provides the conditions under which an appropriately centered and normalized sum of iid random variables with finite variances converges in limit to a Normal distribution. If however, the assumption of finiteness of variances is relaxed, as a consequence of the Generalized Central Limit Theorem, the summands converge in limit to a non-Normal, *Stable Distribution*. (Any properly centered and normalized iid sum will converge to a Stable Distribution — when the variances of the random variables are finite, a subclass of Stable Distributions — the Normal Distribution — is the limiting distribution.)

Other than the Cauchy and Normal distributions, however, the members of the Stable family do not have closed forms for their density or distributions; and hence researchers have to rely on their *characteristic functions* ($\equiv \mathbb{E}(e^{iuX})$).

Parametrization:

Just as the Normal distribution is parametrized by its mean μ and standard deviation σ, the Stable family may be parametrized by the following four parameters: (Rachev *et al.*, 2005a)

1. $\alpha \in (0, 2]$: The tail parameter. The closer α is to 0, the fatter and more "non-Normal" the Stable distribution becomes.
   (a) At $\alpha = 2$, the distribution is Normal.
   (b) For $\alpha \in [1, 2)$, the variance is infinite.
   (c) For $\alpha \in (0, 1)$, both mean and variance are infinite.
2. $\beta \in [-1, 1]$: The skewness parameter. Positive values imply that the distribution is positively skewed.
3. $\sigma > 0$: The scale parameter. If finite, coincides with the standard deviation.
4. $\mu \in \mathbb{R}$: The location parameter. If finite, coincides with mean.

Stable distributions have *polynomially decaying tails* — much "fatter" than the exponentially decaying tails of the Normal distribution:

$$\lim_{x\to\infty} \Pr(|X| > x) \propto x^{-\alpha}$$

However, when it comes to modeling stock return distributions, the infinite variance property of the Stable Distribution becomes undesirable since although returns have a very large variance, it is not quite infinite (Rachev *et al.*, 2008, 123). In order to solve this issue, the use of "*Tempered Stable Distributions*" is encouraged, for which, all moments exist and are finite. It provides better fits to asset return distributions than those provided by the Stable Distributions while retaining the existence of moments of all orders (Bianchi *et al.*, 2010).

Normal Tempered Stable Distribution:

The Normal Tempered Stable distribution is built upon the *Classical Tempered Stable* (CTS) distribution with parameters $\alpha \in (0, 2)$ and $\theta > 0$ with characteristic function:

$$\phi_T(u) = \exp\left(-\frac{2\theta^{1-\frac{\alpha}{2}}}{\alpha}((\theta - iu)^{\frac{\alpha}{2}} - \theta^{\frac{\alpha}{2}})\right)$$

The Normal Tempered Stable Distribution is then defined as:

$$X = \mu + \beta(T - 1) + \gamma\sqrt{T}\epsilon$$

where $\mu, \beta \in \mathbb{R}$, $\gamma > 0$, $\epsilon \sim \mathcal{N}(0, 1)$ and T is the Classical Tempered Stable subordinator defined above with parameters α and θ. The CTS subordinator is independent of the standard normal $\epsilon \sim \mathcal{N}(0, 1)$. The Normal Tempered Stable (NTS) random variable so defined is denoted as $X \sim \text{NTS}(\alpha, \theta, \beta, \gamma, \mu)$.

Standardization of the NTS random variable involves setting $\mu = 0$, $\gamma = \sqrt{1 - \beta^2 \left(\frac{2-\alpha}{2\theta}\right)}$ with $|\beta| < \sqrt{\frac{2\theta}{2-\alpha}}$. This yields a standard NTS random variable with mean 0 and variance 1 and is denoted as $X \sim \text{std NTS}(\alpha, \theta, \beta)$. We use the Multivariate Standard Normal Tempered Stable (MNTS) Distribution to model G-SIFI returns (see Kim *et al.* (2012) for more details).

Modeling Temporal Dependencies

Real life asset return data exhibit autocorrelation and volatility clustering. Postulating an underlying return distribution, no matter how sophisticated, cannot capture the dependence effect owing to the implicit assumption of observed returns being iid.

Hence the common assumption in the literature is the use of ARMA(1,1)-GARCH(1,1) stochastic process:

$$r_{t+1} = \mu_{t+1} + \sigma_{t+1}\epsilon_{t+1}$$

where r_{t+1} denotes asset return at time $t+1$, μ_{t+1} is the conditional mean, while σ_{t+1} is the conditional standard deviation. ϵ_{t+1} is the "*innovation*" and is a standard random variable with 0 mean and unit standard deviation.

The conditional mean and the conditional standard deviation themselves are modeled as autoregressive processes, giving rise to the nomenclature of ARMA (Autoregressive Moving Average) and GARCH (Generalized Autoregressive Conditional Heteroskedasticity).

The ARMA(1,1) process for the conditional mean is:

$$\mu_{t+1} = c + ar_t + b\sigma_t\epsilon_t$$

The GARCH(1,1) process for the conditional standard deviation is:

$$\sigma_{t+1}^2 = d + f(\sigma_t\epsilon_t)^2 + g\sigma_t^2$$

We assume the innovations to be standard Normal Tempered Stable, i.e.,

$$\epsilon_t \sim \text{std NTS}(\alpha, \theta, \beta)$$

1.2.3 The Monte Carlo Method

The Monte Carlo Method is a powerful computational technique that does not rely on any closed form expressions and yields accurate results. Our specification of the statistical model that generates asset returns enables us to put this method to use. We achieve this in the following steps:

1. *Specification of a Statistical Model for Asset Returns*: We choose an ARMA(1,1)-GARCH(1,1) with standard NTS innovations as the underlying data generating process whose realizations are the observed values of asset returns.

2. *Estimation of Parameters of the Statistical Model*: Choosing a set of observed asset return data, we estimate all the parameters of the model using the Maximum Likelihood Estimation technique.

3. *Generation of Scenarios from the Fitted Model*: Once the underlying data generating process has been estimated, we generate a large number of independent simulations from it. These are possible realizations of asset returns in which the specifications of the (already estimated) statistical model are embedded.

4. *Computation of Risk*: On the basis of generated scenarios, we compute risk as specified by its definition.

1.3 Data

We construct an equally weighted portfolio of the "*Global Systemically Important Financial Institutions*" (G-SIFIs) — a list of 29 banks compiled by the Financial Stability Board (FSB) in November 2013.[8] Our data set comprises daily log returns of 28 of these 29 banks from January 4$^{\text{th}}$, 2000 to February 28$^{\text{th}}$, 2014. The only bank among the G-SIFIs that we exclude is Groupe BPCE since it is unlisted. We use the S&P 1200 financial sector index (SGFS) to represent the global banking stock market.

We exclude American non-business days from the data set, which gives us 3694 observations for each bank's stocks. The two Chinese banks: Bank of China and Industrial and Commercial Bank of China, the three Japanese banks: Mitsubishi UFJ FG, Mizuho FG and Sumitomo Mitsui FG; and the French bank Group Crédit Agricole do not have sufficient historical data to cover the entire sample period. For all of these G-SIFIs, we backfill historical data using "*Cognity*" — a risk management software developed by FinAnalytica Inc. using the one-factor bootstrapped method. We note that our approach in dealing with missing data via backfilling and substitution is consistent with recent literature (Kurosaki and Kim, 2013).

[8]The list of Global Systemically Important Financial Institutions as it stood in November 2013, is presented in the Appendices.

1.4 Methodology

Our methodology may be considered to be built up largely of two steps — the first one being the estimation of the ARMA-GARCH and innovation process's parameters — and the second one being the generation of scenarios from the fitted model and subsequent computation of risk by the Monte Carlo method.

1.4.1 Estimation Methodology

We model the log returns of banking stocks as an ARMA(1,1)-GARCH(1,1) stochastic process:

$$r_{t+1}^i = \mu_{t+1}^i + \sigma_{t+1}^i \epsilon_{t+1}^i$$

$$\mu_{t+1}^i = c_i + a_i r_t^i + b_i \sigma_t^i \epsilon_t^i$$

$$(\sigma_{t+1}^i)^2 = d_i + f_i (\sigma_t^i)^2 + g_i (\sigma_t^i \epsilon_t^i)^2$$

The index $i \in \{1, 2, \ldots, 28\}$ runs through the list of banks in the G-SIFI list.

$\epsilon_t = [\epsilon_t^1, \epsilon_t^2, \ldots, \epsilon_t^{28}]^T$ is the multivariate innovation. We choose the "*Multivariate Normal Tempered Stable*" (MNTS) distribution to model the innovations' process since as demonstrated in Kim *et al.* (2012), it captures the skewed, interdependent and leptokurtotic nature of real life stock returns.

The NTS distribution has two tail parameters α and θ and one skewness parameter β. Following the example of Kurosaki and Kim (2013), we represent G-SIFI stocks by the S&P 1200 financial index (SGFS) and assume common tail parameters α and θ for G-SIFIs' NTS marginals and estimate them from the SGFS index. This leaves the skewness parameters $\{\beta_i\}_{i=1}^{28}$ to be calibrated for each bank in the portfolio. We follow Kim *et al.* (2012) in joining NTS marginals by means of their covariance matrix into MNTS and note that such a method is computationally feasible even in very high dimensional settings (Kurosaki and Kim, 2013). For comparative purposes, we also estimate an ARMA(1,1)-GARCH(1,1) model with multivariate Normal innovations. Computation of the covariance matrix for both MNTS and Multivariate

Normal innovations is done on the basis of the most recent 250 days of daily returns.[9]

The reason for explicitly modeling the dependence between various banking stocks' innovations by means of their covariance matrix is because at each time period it is unreasonable to assume that movements in the stock price of one banking stock are unrelated to those of another G-SIFI. Hence while there is no temporal correlation between innovations (they remain an iid process with 0 mean and unit variance), they do depend on each other by means of a covariance matrix.

For more details on the estimation of parameters of the MNTS distribution, we refer the reader to Kim *et al.* (2012).

1.4.2 Scenario Generation and Computation of Risk

Using the above methodology, we can estimate the parameters of both statistical models — ARMA(1,1)-GARCH(1,1) with MNTS (AGMNTS) and Multivariate Normal (AGMNormal) innovations respectively. We then employ the estimated models to compute the time series of risk.

We use a 1250 days' forward moving time window to generate such a series.[10] Our first time window starts at January 4$^{\text{th}}$, 2000 and ends at October 18$^{\text{th}}$, 2004. For this time window, we estimate all parameters of the AGMNTS and AGMNormal model, thereby automatically estimating all parameters of the G-SIFI portfolio which in turn, is distributed as a one dimensional standard Normal Tempered Stable; and standard Normal random variable respectively (Kim *et al.*, 2012). Once all parameters are estimated, risk is computed corresponding to that particular time window and the time window is then shifted one period ahead. Using such a moving time window entails estimation of model parameters for 2445 different time windows — each window shifted one trading day to the right of its preceding one.

We then generate a large number, N ($= 10^6$, in our paper) of scenarios for the one dimensional portfolio log return one period ahead: $\{r_{t+1}^{\text{port},1}, \ldots, r_{t+1}^{\text{port},N}\}$ for

[9] There are about 250 trading days in a typical year. Hence our choice of a 250-day window corresponds to the most recent one year of return data for computing the covariance matrix.

[10] 1250 daily returns correspond to about five years of financial data, since each year has about 250 daily returns.

both AGMNTS and AGMNormal statistical models.[11] These N simulations are the realizations of the one dimensional return distribution of the G-SIFI portfolio which is distributed as Normal Tempered Stable and Normal random variable respectively. We note that the distribution of the portfolio remains a one dimensional NTS and Normal respectively for *any* linear combination of individual stock returns.

On the basis of the above generated scenarios, we compute three different measures of risk for each specification of a statistical model (AGMNTS and AGMNormal): Value at Risk (VaR), Average Value at Risk (AVaR) and Foster-Hart Risk (FH). The time horizon for both VaR and AVaR is taken to be one day.

Finally, we update the conditional means and conditional standard deviations of the two statistical models for the next period and move the time window one period forward after which we re-estimate all parameters of AGMNTS and AGMNormal models, generate scenarios, compute risks and again move the time window one period ahead.

1.5 Results

We divide our description of results in three broad categories:

1. Tests of the validity of the Statistical Model (AGMNTS versus AGMNormal).

2. Analysis of the time series of portfolio risk (FH risk versus VaR and AVaR).

3. Backtesting of the computed VaR and AVaR estimates.

1.5.1 Tests of Statistical Model Validity

In order to test which statistical model provides a better fit to the observed data, we rely on the Kolmogorov-Smirnov (KS) test, which is a goodness of fit based test that compares empirical distributions to a specific, hypothesized distribution. We test the

[11] As is proved in Kim *et al.* (2012) if each stock return follows the NTS and Normal distribution respectively, the portfolio return — a linear combination of each stock return — also follows the NTS and Normal distribution respectively.

standardized innovations of each banking stock against distributional hypotheses of standard Normal Tempered Stable (NTS) and standard Normal random variables.

We first carry out this exercise on the S&P Global 1200 (SGFS) index which we have assumed, represents the global banking stock market.

SGFS Index: NTS versus Normal

We fit the SGFS log return series to our standard ARMA-GARCH with Normal and NTS innovations time series. The empirical densities of the residuals of the ARMA(1,1)-GARCH(1,1)-Normal and ARMA(1,1)-GARCH(1,1)-NTS stochastic processes are then compared with those of standard Normal and standard NTS densities respectively.

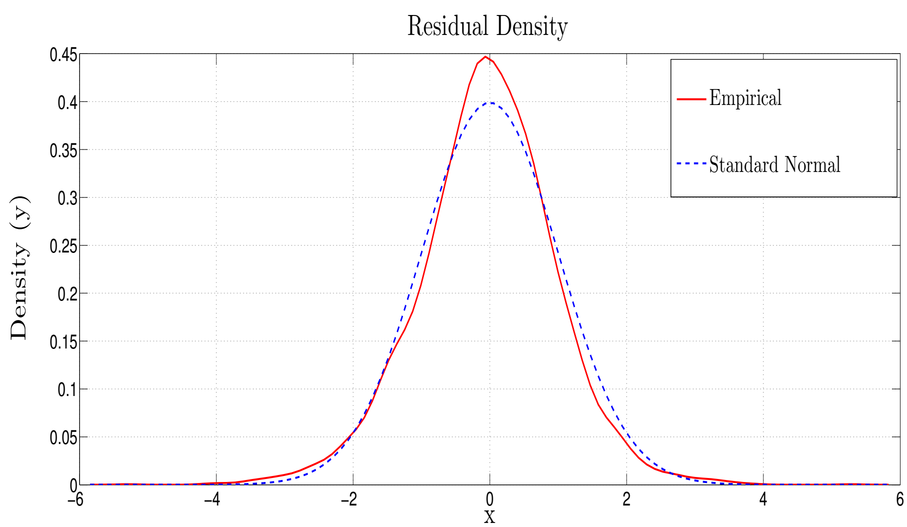

Figure 1.1: Comparing the empirical density of the residual to that of standard normal

The SGFS log return series is leptokurtotic and hence the standard normal distributional hypothesis is not adequate. We can check this further by plotting the quantiles of the empirical distribution of residuals and comparing it to the quantiles of standard Normal and standard NTS distributions respectively.

If the QQ plot is (almost) a 45° straight line, it implies that the quantiles of the empirical distribution are (almost) the same as that of the hypothesized distribution.

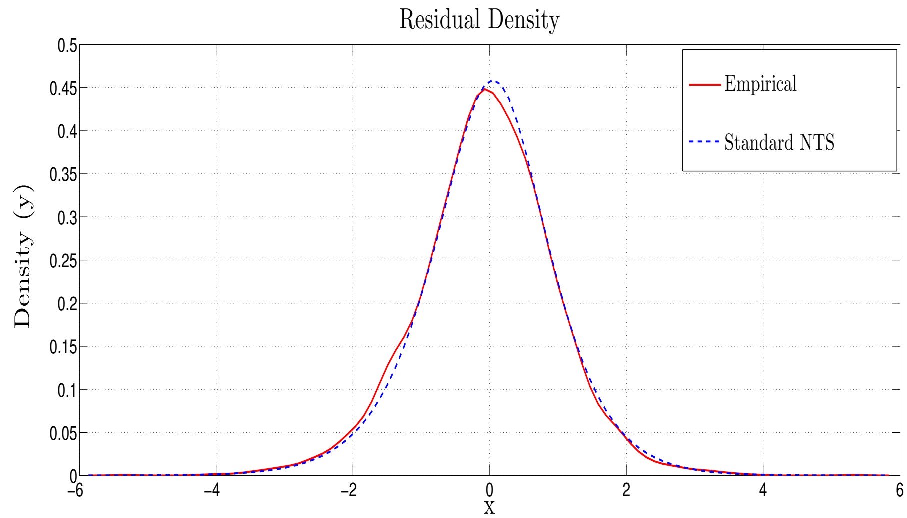

Figure 1.2: Comparing the empirical density of the residual to that of standard NTS

Again, due to leptokurtosis, Normal quantiles are a bad fit — especially in the tails.

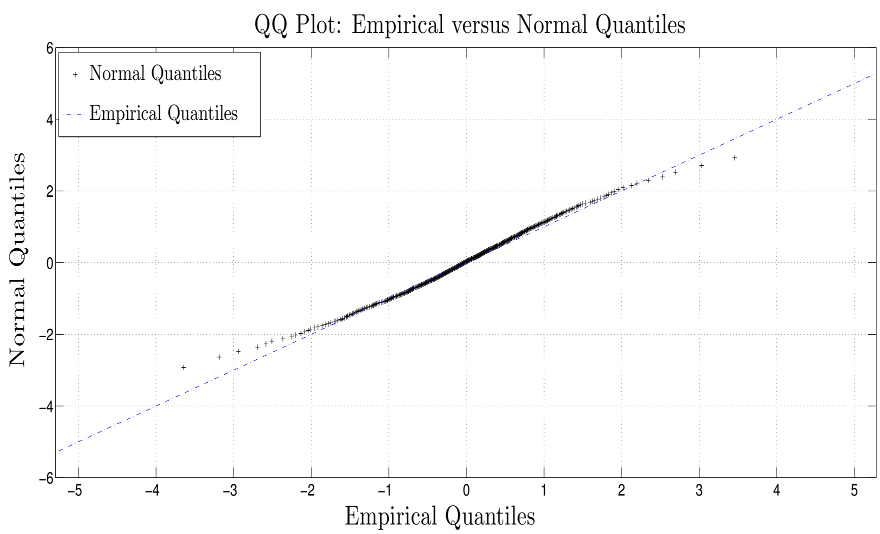

Figure 1.3: QQ plots of empirical and standard Normal distribution

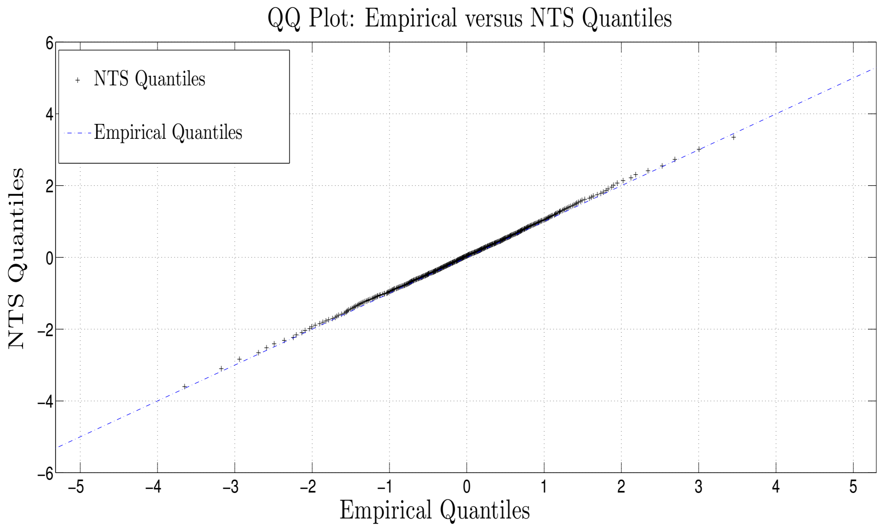

Figure 1.4: QQ plots of empirical and standard NTS distribution

The Multivariate Case: AGMNTS versus AGMNormal

Based on 2445 daily estimations of the AGMNTS and the AGMNormal models, we apply the Kolmogorov-Smirnov test 2445 times for each banking stock. Table 1.1 reports the number of days on which NTS and Normal assumptions for each banking stock are rejected at 0.5, 1, 5 and 10% significance levels respectively.

For *all* of the 28 banks in the G-SIFI list, and for *all* of the above significance levels, the NTS hypothesis outperforms the Normal hypothesis. This is true not just for the higher significance levels of 10% and 5% but also for the more conservative significance levels of 1% and 0.5%. As Table 1.1 points out, not only does the NTS hypothesis outperform the Normal, it does so quite overwhelmingly. Taking a few examples, even at the relatively high significance level of 10%, for the banks Barclays, BNP Paribas, Group Crédit Agricole, HSBC, Unicredit Group, Standard Chartered and Mitsubishi UFG, the NTS hypothesis, from a total of 2445 estimations, is rejected only 65, 13, 3, 11, 90, 12 and 5 times respectively. The corresponding figures for the Normal distribution are 1967, 1922, 1624, 2421, 1616, 1635 and 1976 respectively. At the opposite end of spectrum, with a very small significance level of 0.5%, the banks mentioned above, under the NTS hypothesis, are rejected only 3, 0, 1, 0, 6, 1 and 1 times (out of a possible 2445 estimations) respectively. Again, the Normal hypothesis performs much worse comparatively, with corresponding rejection figures being 8, 598, 126, 303, 308, 477 and 118 respectively.

For the other banks also, the NTS hypothesis is rejected far fewer times than the Normal hypothesis at *all* significance levels. The overall result is that the Normal Tempered Stable distributional hypothesis provides much better fits than the Normal distribution. Such overwhelming evidence in our opinion, provides strong support in favor of the AGMNTS model over the AGMNormal for stock returns for G-SIFIs.

1.5.2 Portfolio Risk Analysis

Before analyzing the time series of the portfolio equity risk for the G-SIFIs, we briefly discuss the behavior of the (log) return time series of the Standard and Poor's Global 1200 Financial Sector (SGFS) index.

We plot the (log) return time series of the index and superimpose on it, an HP filtered time series of the index returns with $\lambda = 20000$. As is clear from the SGFS time series graph, there are two sharp downturns in the index returns — one during the first quarter of 2008 — corresponding to the collapse and eventual sale of Bear Stearns to JP Morgan Chase; and a second, even more pronounced fall in the final quarter of 2008 — corresponding to the bankruptcy of Lehman Brothers in September 2008. A second, though less severe, phase of large negative returns occurred in the second quarter of 2010, with the onset of the Greek crisis, Standard and Poor's downgrade of Greek debt ratings to junk bond status (April 27$^{\text{th}}$) and the subsequent nationwide protests against austerity policies in Greece. Finally there is another heavy downslide in the third and fourth quarters of 2011, coinciding with Standard and Poor's downgrading of US federal government in August 2011; the bankruptcy of Dexia, an ex-G-SIFI, in October 2011; and the controversial Greek announcement of a referendum on the Eurozone debt deal in November 2011.

A qualitatively similar, though not identical behavior is exhibited by the constituent banks of the index. For ease of comparison, we have included the log-return time series of four of the 29 such banks. These are: Barclays (UK) and Credit Suisse

Table 1.1: Number of rejections of distributional assumptions for each banking stock on the basis of KS test (out of a total of 2445 estimations)

	Significance Level 10%		Significance Level 5%		Significance Level 1%		Significance Level 0.5%	
	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS
BAC	2443	1522	2443	1220	2406	400	2362	96
BK	2445	1889	2445	1583	2436	931	2431	679
C	2391	1787	2278	1419	1742	523	1493	432
GS	2331	899	2030	580	1203	1	835	0
JPM	2443	1487	2400	846	1787	92	1375	28
MS	2249	1725	1938	1129	1457	370	1259	154
STT	2444	1403	2444	1268	2443	946	2440	708
WFC	1878	742	1718	606	1540	418	1370	280
BARC	1967	65	827	7	9	5	8	3
BNP	1922	13	1692	4	1024	1	598	0
CSGN	2135	329	1705	53	1042	2	834	0
DBK	2087	332	1637	74	869	8	790	8
ACA	1624	3	1132	1	362	1	126	1
HSBA	2421	11	2193	0	609	0	303	0
INGA	1998	1048	1522	772	655	15	245	6
NDA	2442	668	2348	574	1778	346	1395	109
RBS	2351	959	2184	796	1613	348	1448	227
SAN	2354	1726	2164	1439	1293	768	1159	455
GLE	2419	663	2212	290	871	7	493	2
UBSN	1664	531	1136	312	528	75	354	39
UCG	1616	90	1307	15	709	7	308	6
BBVA	2405	941	2294	428	1832	27	1593	3
STAN	1635	12	1383	2	677	2	477	1
BOC	1991	1455	1737	1420	1445	1279	1383	1172
MUFG	1976	5	1341	2	319	1	118	1
MHFG	2445	1461	2442	1403	2041	900	1586	613
SMFG	2435	979	2433	180	2359	5	2069	3
ICBC	1936	1190	1475	1065	1210	946	1103	871

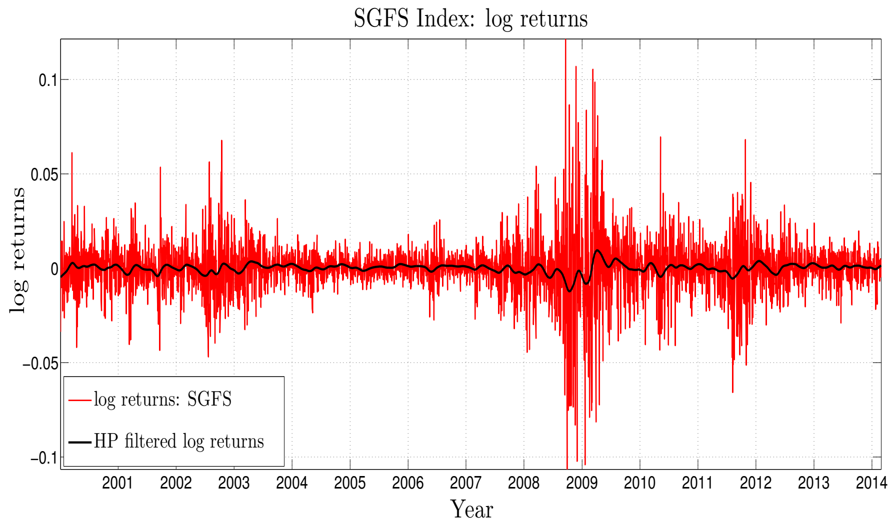

Figure 1.5: Log-returns of the S&P Global 1200 Financial (SGFS) index ($\lambda = 20000$ for the HP filter)

(Switzerland) from the Euro area; JP Morgan from the US; and Mitsubishi UFG from Asia. While all of these banks show sharp downturns in the last quarter of 2008 with the collapse of Lehman Brothers, the two European banks show much heavier losses from the Eurozone crisis than their American and Asian counterparts, as should be expected.

Clearly, such sharp downturns amplified the equity risk posed by banks on their shareholders dramatically. Our portfolio of G-SIFI stocks also exhibits these observations. In light of this information, we discuss the time series of our G-SIFI portfolio equity risk below.

In order to track the risk profile of our equally weighted portfolio of G-SIFI stocks, we plot the time series of portfolio risk according to three different risk measures — VaR, AVaR and FH — assuming an underlying AGMNTS model. For VaR and AVaR, we compute risk for 90%, 95% 99% and 99.5% confidence levels.

By definition, the risk according to VaR and AVaR increases with increase in confidence levels.[12] Again, in keeping with the definition, AVaR for a given confidence

[12]Graphs corresponding to variation of VaR and AVaR with confidence levels is presented in the Appendices.

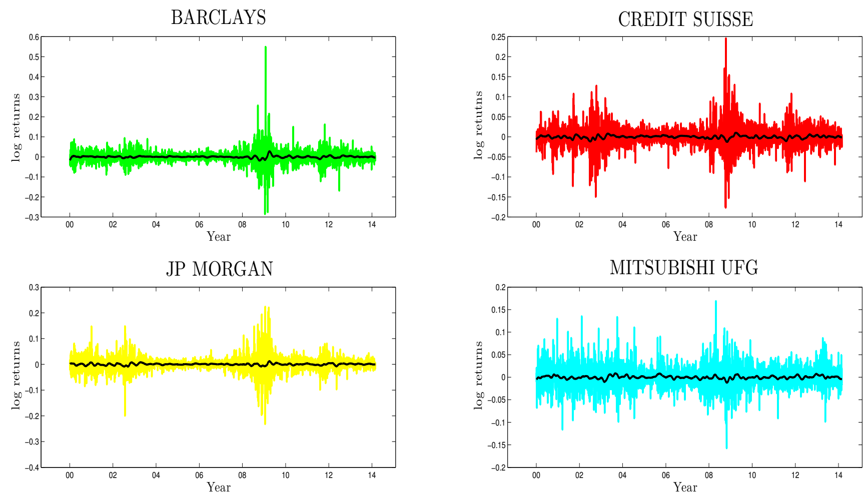

Figure 1.6: Log-returns of some G-SIFIs ($\lambda = 20000$ for the HP filter)

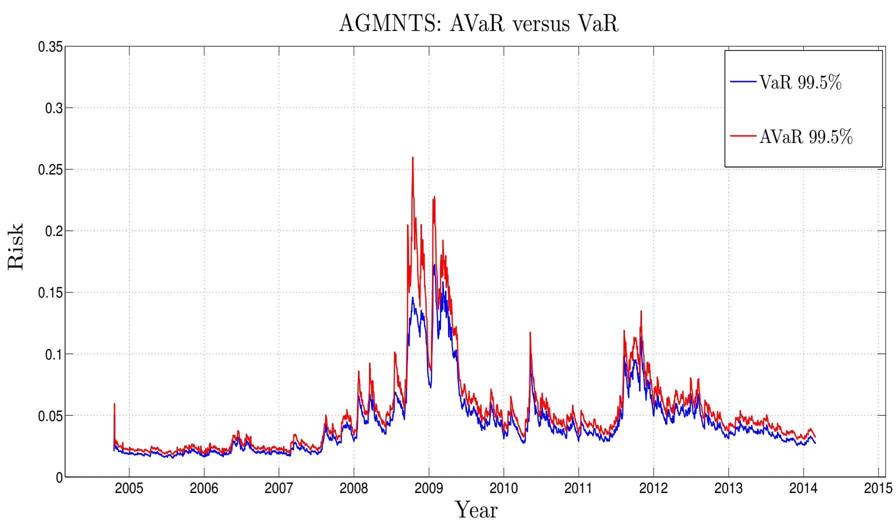

Figure 1.7: AVaR versus VaR at 99.5% confidence level under the AGMNTS model

level is more than the corresponding VaR.

Theory suggests that the Foster-Hart risk should be more than both VaR and AVaR at any confidence level. This is so because the risk as measured by Foster-Hart is Black Swan proof — i.e., even if the maximum loss possible were to occur (with howsoever small a probability) wealth levels computed by means of the Foster-Hart riskiness will always prove adequate since it is always at least as large as the maximum loss. Such a property is not possessed by either VaR or AVaR. Indeed even for very high confidence levels, the probability that a large loss outstrips wealth levels computed via VaR or AVaR — an event that results in bankruptcy — is positive.

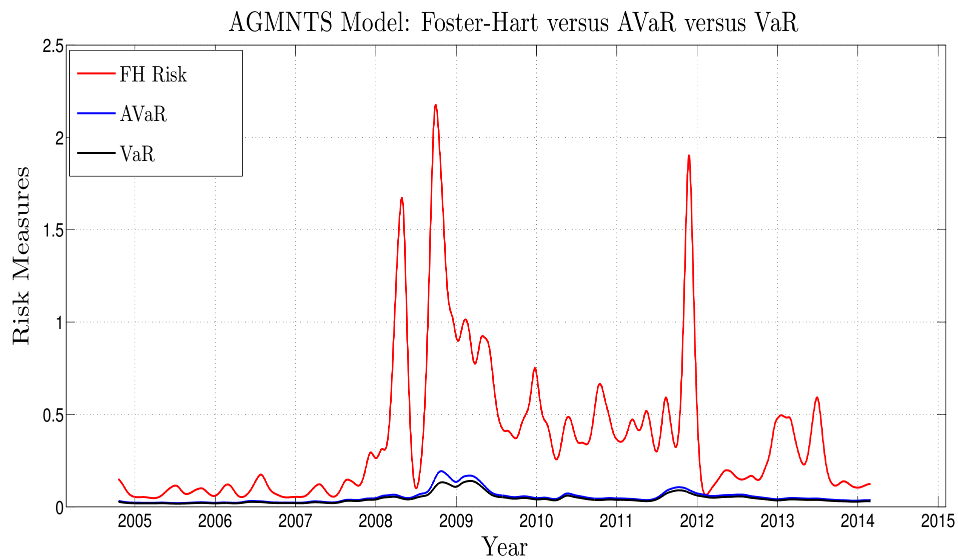

Figure 1.8: FH versus AVaR and VaR at 99.5% confidence level. All series have been smoothed by using the Hodrick-Prescott filter with $\lambda = 20,000$.

This is exactly what we observe: a comparative study of risk profiles as measured by VaR, AVaR and FH risk shows that not only is the FH risk always more than that according to VaR and AVaR but in fact, the degree of divergence between the two is much higher during times of crises. While it is true that during the worst of the financial crisis (March 2008 – Bear Stearns's sale to JP Morgan and September 2008 – Lehman Brothers' bankruptcy) VaR and AVaR are able to pinpoint the equity risk posed by, and reflected in, abysmal stock market performance; and exhibit local maxima at the aforementioned events, the corresponding FH risk puts the equity risk value to be much higher.

Such a result goes on to show that equity risk, as computed by even the most sophisticated of statistical models (ARMA(1,1)-GARCH(1,1)-MNTS process) and the most widely used risk measures in the industry today (VaR and AVaR) severely underestimated the equity risk posed during the financial crisis! In fact, it also implies that if the banks' in-house risk management groups, based on the knowledge of their own internal portfolios, were to use VaR and AVaR as risk measures to compute critical reserves, their computations would have proved far from adequate.

However, without much difficulty, the internal risk management teams of banks can adopt our methodology and on the basis of their bank's portfolio, come up with capital reserve computations that are much more accurate, robust, reliable and sensitive to information embedded in the stock market.

1.5.3 Backtesting

In order to evaluate the accuracy of forecasted VaR and AVaR according to the AGMNTS and AGMNormal statistical models, we perform backtesting using the Christofferson Likelihood Ratio (CLR) test (Christofferson, 1998) and the Berkowitz Likelihood Ratio (BLR) test (Berkowitz, 2001). We check the marginal VaR of the 28 G-SIFIs by reporting the p-values of the CLR test; and check the VaR of the equally weighted portfolio by both the CLR and the BLR tests. Finally, we analyze our backtests further using a multi-period framework.

Marginal Risk Backtesting

The CLR test has three parts: the CLR unconditional coverage test ('uc'),[13] the CLR independence test ('ind'); and the CLR joint test of coverage and independence ('cc'). We report the p-values of 28 marginal CLR tests in Table 1.2, and report how many of them are below 5% and 1% respectively.

[13]This test is also called the "proportion of failures" test in Kupiec (1995).

Note that a p-value smaller than 5% (1%) means being rejected by the corresponding 95% (99%) confidence level CLR test. We conclude the following: while both univariate ARMA(1,1)-GARCH(1,1)-Normal and ARMA(1,1)-GARCH(1,1)-NTS models perform well for VaR backtesting at the 95% level, the non-Gaussian model overwhelmingly outperforms the Gaussian model for both 99% and 99.5% levels of VaR backtesting — especially in CLR uc and cc tests. For the Gaussian model, more than 24 out of the 28 marginal 99.5% VaRs; and more than 21 out of 28 marginal 99% VaRs do not pass the 95% confidence level CLR uc/cc test, which strongly indicates that the univariate ARMA(1,1)-GARCH(1,1)-Normal model should not be used to measure daily tail risk. For the ARMA(1,1)-GARCH(1,1)-NTS model, the number of rejections for the 95% confidence level CLR tests is less than 6 for the 99.5% VaR; and less than 5 for the 99% VaR.

Therefore, we conclude that on the basis of all three CLR tests, there is strong evidence in favor of the VaRs computed under the non-Gaussian statistical model.

We note that our marginal VaR and AVaR backtesting results are broadly consistent with those obtained in recent literature (Kurosaki and Kim, 2013).

Portfolio Risk Backtesting

We construct an equally weighted portfolio of the 28 G-SIFI stocks and apply to it, the same CLR tests to check if our estimations of portfolio VaR are reasonable. Since the portfolio risk estimation is based on the 28 dimensional joint distribution model, this test actually examines the reliability of the AGMNormal and AGMNTS models. This is in contrast to the previous marginal VaR backtests for each G-SIFI constituent which relied only on univariate Normal and NTS distributions.

Table 1.3 reports the p-values of CLR tests, based on which we conclude that the AGMNTS model is much more reliable than the AGMNormal model in the CLR uc/cc tests; and while its p-values are smaller than that of the AGMNormal model in 95% and 99% VaR independence backtests, the AGMNTS model is still able to pass the CLR independence test. We note that this result is consistent with the backtests of the individual G-SIFI constituents.

Table 1.2: p-values of CLR test for marginal VaR

	99.5% VaR						99% VaR						95% VaR					
	uc		ind		cc		uc		ind		cc		uc		ind		cc	
	Normal	NTS	Normal	NTS	Normal	NTS	Normal	NTS	Normal	NTS	Normal	NTS	Normal	NTS	Normal	NTS	Normal	NTS
BAC	0.0000	0.0701	0.4029	0.5845	0.0002	0.1665	0.0000	0.1372	0.0745	0.3557	0.0000	0.2156	0.5266	0.3584	0.0629	0.0450	0.1456	0.0884
BK	0.0000	0.1910	0.3262	0.6248	0.0000	0.3766	0.0006	0.0061	0.7889	0.2596	0.0027	0.0122	0.9443	0.7975	0.3593	0.6173	0.6555	0.8551
C	0.0000	0.0397	0.0136	0.1561	0.0000	0.0440	0.0000	0.0637	0.0835	0.0114	0.0000	0.0073	0.2620	0.3024	0.8813	0.9150	0.5244	0.5813
GS	0.0701	0.6086	0.5845	0.6873	0.1665	0.8082	0.2668	0.7814	0.3846	0.5076	0.3690	0.7730	0.9815	0.6567	0.9600	0.8698	0.9984	0.8957
JPM	0.0113	0.0701	0.5263	0.5845	0.0328	0.1665	0.0267	0.8951	0.5613	0.2584	0.0723	0.5233	0.4666	0.4104	0.7690	0.4434	0.7373	0.5333
MS	0.0006	0.0397	0.2818	0.5648	0.0015	0.1019	0.0035	0.4678	0.6896	0.4194	0.0130	0.5537	0.9815	0.4104	0.4422	0.1157	0.7440	0.2075
STT	0.0001	0.1910	0.3315	0.6248	0.0003	0.3766	0.0003	0.5958	0.0517	0.3062	0.0002	0.5141	0.0371	0.0172	0.6413	0.9273	0.1032	0.0589
WFC	0.0000	0.0056	0.3557	0.5076	0.0000	0.0174	0.0003	0.0418	0.8224	0.3121	0.0015	0.0752	0.6277	0.7635	0.2481	0.2820	0.4552	0.5350
BARC	0.0000	0.0397	0.0632	0.1561	0.0000	0.0440	0.0020	0.0946	0.0047	0.0095	0.0002	0.0085	0.7635	0.6277	0.0360	0.0985	0.1060	0.2266
BNP	0.0006	0.7314	0.2818	0.0387	0.0015	0.1114	0.0035	0.5958	0.0300	0.3062	0.0013	0.5141	0.3468	0.6277	0.2869	0.1954	0.3627	0.3837
CSGN	0.0006	0.6086	0.4536	0.6873	0.0020	0.8082	0.0267	0.9430	0.2984	0.4892	0.0498	0.7854	0.9443	0.5901	0.4159	0.2984	0.7168	0.5049
DBK	0.0027	0.7314	0.4892	0.7519	0.0088	0.8973	0.0035	0.9430	0.2475	0.4892	0.0072	0.7854	0.0286	0.0805	0.2672	0.9491	0.0487	0.2148
ACA	0.0056	0.1183	0.2144	0.1225	0.0100	0.0894	0.0102	0.7396	0.6247	0.2818	0.0325	0.5299	0.7975	0.8704	0.0951	0.0025	0.2406	0.0102
HSBA	0.0001	0.0216	0.4194	0.5454	0.0004	0.0593	0.0006	0.1938	0.0454	0.3710	0.0004	0.2873	0.6943	0.7975	0.0405	0.0169	0.1134	0.0557
INGA	0.0000	0.1183	0.0043	0.1225	0.0000	0.0894	0.0061	0.5958	0.0259	0.0027	0.0019	0.0097	0.0805	0.1930	0.0468	0.0568	0.0298	0.0695
NDA	0.0013	0.6086	0.4712	0.6873	0.0042	0.8082	0.0061	0.4678	0.2596	0.4194	0.0122	0.5537	0.7260	0.6943	0.3426	0.8462	0.6005	0.9067
RBS	0.0000	0.0027	0.1897	0.2359	0.0000	0.0055	0.0000	0.0102	0.4646	0.6247	0.0000	0.0325	0.6943	0.6567	0.0065	0.0045	0.0228	0.0162
SAN	0.0000	0.2946	0.3557	0.6453	0.0000	0.5185	0.0000	0.8951	0.9561	0.4712	0.0001	0.7645	0.0971	0.3468	0.7285	0.7446	0.2359	0.6065
GLE	0.0013	0.6086	0.4712	0.0683	0.0042	0.1664	0.0020	0.1938	0.2358	0.0632	0.0042	0.0764	0.6277	0.6277	0.3542	0.1954	0.5775	0.3837
UBSN	0.0006	0.7314	0.2818	0.7519	0.0015	0.8973	0.0061	0.7814	0.1570	0.0180	0.0085	0.0586	0.9076	0.4104	0.0655	0.0505	0.1820	0.1056
UCG	0.0000	0.4336	0.0043	0.0803	0.0000	0.1592	0.0003	0.7396	0.0517	0.0304	0.0002	0.0907	0.0012	0.0017	0.5704	0.5404	0.0046	0.0058
BBVA	0.0000	0.1910	0.4700	0.6248	0.0000	0.3766	0.0000	0.1372	0.4369	0.4408	0.0000	0.2454	0.0542	0.0356	0.8812	0.6307	0.1536	0.0969
STAN	0.0002	0.1910	0.4363	0.6248	0.0009	0.3766	0.0035	0.1372	0.6896	0.4408	0.0130	0.2454	0.7975	0.4104	0.3662	0.1157	0.6440	0.2075
BOC	0.0397	0.8146	0.5648	0.7086	0.1019	0.9070	0.2668	0.3606	0.3846	0.5648	0.3690	0.5588	0.1111	0.0736	0.4904	0.4359	0.2231	0.1502
MUFG	0.4336	0.8146	0.0803	0.0574	0.1592	0.1599	0.5958	0.1134	0.3062	0.1074	0.5141	0.0784	0.0471	0.0736	0.5235	0.8551	0.1147	0.2002
MHFG	0.6086	0.7314	0.0683	0.0387	0.1664	0.1114	0.7396	0.2571	0.2818	0.1388	0.5299	0.1763	0.0028	0.0290	0.0256	0.0244	0.0010	0.0074
SMFG	0.0113	0.1910	0.1940	0.6248	0.0173	0.3766	0.4678	0.9430	0.3315	0.2359	0.4787	0.4942	0.1111	0.0908	0.0504	0.1100	0.0418	0.0673
ICBC	0.0013	0.9598	0.2584	0.0476	0.0029	0.1403	0.0418	0.8951	0.5303	0.2584	0.1030	0.5233	0.1618	0.1618	0.2993	0.5480	0.2207	0.3157
less than 5%	25	6	3	3	24	4	23	3	4	5	21	5	5	4	5	5	6	4
less than 1%	22	2	2	0	21	1	19	1	1	2	16	3	2	1	1	2	2	2

Table 1.3: p-values of CLR test for portfolio VaR

99.5% VaR					
uc		ind		cc	
AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS
0.0000	0.6076	0.3868	0.6873	0.0000	0.8082
99% VaR					
uc		ind		cc	
AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS
0.0006	0.1931	0.7889	0.3710	0.0027	0.2873
95% VaR					
uc		ind		cc	
AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS
0.0178	0.0961	0.3086	0.2941	0.0360	0.1445

Table 1.4: BLR tail test for the equally weighted G-SIFI portfolio

Tail probability	0.5%		1%		5%	
Model	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS
Statistics value	27.7675	0.497	27.0884	1.6883	26.3461	2.9099
p-value	0.0000	0.7800	0.0000	0.4299	0.0000	0.2334

In addition to the CLR tests, we also perform the BLR tail test to examine the reliability of the estimated AVaR. In Table 1.4, we backtest for three levels of tail probability: $\{0.5\%, 1\%, 5\%\}$. As expected, the AGMNormal model cannot pass any of BLR tail tests; while the AGMNTS model generally exhibits satisfying p-values, thereby indicating the validity of AVaR estimations from the non-Gaussian model.

As a final exercise in backtesting, we divide the entire time series into five equally spaced periods as follows:

1. 2004–2006: from 11/02/2004 to 09/12/2006

2. 2006–2008: from 09/13/2006 to 07/23/2008

3. 2008–2010: from 07/24/2008 to 06/03/2010

4. 2010–2012: from 06/04/2010 to 04/13/2012

5. 2012–2014: from 04/16/2012 to 02/24/2014

Table 1.5: Multi-period p-values of CLR test for portfolio 99% VaR

	uc		ind		cc	
	AGMNormal	AGMNTS	AGMNormal	AGMNTS	AGMNormal	AGMNTS
2004-2006	0.6161	0.6859	0.6982	0.7965	0.8180	0.8913
2006-2008	0.0021	0.0915	0.3497	0.5596	0.0058	0.2032
2008-2010	0.0061	0.1903	0.4352	0.6044	0.0172	0.3710
2010-2012	0.3600	0.6161	0.6507	0.6982	0.5937	0.8180
2012-2014	0.9493	0.6859	0.7469	0.7965	0.9473	0.8913

We then repeat the CLR test on the 99% VaR period by period; and illustrate the results in Table 1.5. We observe that the AGMNormal model completely fails the CLR uc/cc tests in the second and third periods: from September 2006 to June 2010 — roughly the pre-crisis and crisis periods. During the same periods, while the AGMNTS model has relatively small p-values, it definitely outperforms the Gaussian model. For example, in the CLR cc test, the AGMNTS model has a p-value of 0.2032 in the pre-crisis period, and 0.3710 in the crisis period; while the AGMNormal model has corresponding p-values as 0.0058 and 0.0172 respectively. Hence, based on the multi-period CLR backtesting results, risk computations under the AGMNormal model are reliable only in periods when the market is relative peaceful; and cannot be trusted during periods of high volatility. On the other hand, due to its capacity to capture extreme tail risk, the AGMNTS model can provide accurate risk estimations, even in volatile periods.

We do not perform backtesting for the Foster-Hart risk measure because conventional backtesting techniques cannot be applied to the FH risk estimates. Since the Foster-Hart risk estimates are at least as high as the maximum realized loss of the empirical portfolio distribution, there are no “exceedances” — instances in which the observed losses are larger than the risk estimates. We note that devising backtesting procedures for the Foster-Hart risk is very much an open problem.

1.6 Conclusions

In this paper, we compute the equity risks posed by the Global Systemically Important Financial Institutions (G-SIFIs) which constitute a very important part of the global financial system and are its most critical component from the point of view of maintenance of overall stability of the global financial markets.

In order to illustrate that current methods of risk assessment underestimate the real equity risk posed by the G-SIFIs, we employ the Foster-Hart risk measure. The Foster-Hart risk measure is a recent breakthrough in the field of risk measurement which is based on the observation that irrespective of utility functions, the risk posed to each agent depends critically on the amount of wealth she needs to possess in order to stave off bankruptcy.

In order to facilitate this exercise, we assume that stock returns of G-SIFIs are distributed according to the ARMA(1,1)-GARCH(1,1) stochastic process whose innovations are distributed according to the Multivariate Normal Tempered Stable distribution — from a family of tempered α-Stable distributions with finite moments. Use of such a sophisticated distribution for the innovations process leads to much better statistical fits as compared to those that rely on standard Normal based techniques. Specifically, the superiority of the MNTS to the Multivariate Normal statistical model has been demonstrated for our dataset by both in-sample (the KS test) and out-of-sample tests (the CLR and BLR test).

We show that during the Great Recession, equity risks posed by the G-SIFIs were much higher than that suggested by conventional measures of risk such as VaR and AVaR. The difference between the FH risk and that computed by VaR and AVaR is not static — it evolves with time and is the most pronounced during times of high volatility and during times of crises.

Our findings have important policy implications. Regulators looking to measure equity risk posed by banks on the financial system as a whole might end up underestimating it if they rely on VaR and AVaR. To accurately assess the risks posed, our approach, based on the Foster-Hart measure, is an improvement over current methodologies. Another application of our paper is in the area of critical reserve computations for banks. Current methods endorsed by regulators for computation of reserve requirements are not necessarily adequate and may be improved upon by employing the Foster-Hart measure of risk.

This paper should prove useful to the banking industry as well, since the banks' in-house risk management teams can adapt our methodology relatively easily and based on the knowledge of their internal portfolio composition, may track their own equity risks much more accurately.

Chapter 2

Quotas versus Handicaps: A Game Theoretic Analysis of Affirmative Action Policies in India

Abstract

We analyze the affirmative action policy implemented in India, the Quota Policy, in which preference is given to the disadvantaged section of the populace by reserving a certain fraction of positions for them. We compare it to a hypothetical policy called the "Handicap Policy" in which the performance index of the disadvantaged is given an artificial boost instead, by means of an additive handicap.

We conclude that if the degree of asymmetry between the disadvantaged and the rest is not too high, on many important metrics of performance of affirmative action policies, Quotas and Handicaps can be shown to be equivalent to each other.

Keywords: Affirmative Action, Quotas, Handicaps
JEL classification: I24, I28

2.1 Introduction

Affirmative Action policies are a means of reduction of socio-economic inequalities in a society by favoring historically disadvantaged, or discriminated against groups over the rest of the population by granting them special opportunities to help them compete on an equal footing with the general populace. Governments the world over implement many varieties of affirmative action techniques in order to combat social inequalities. There are however, two main ways in which affirmative action policies are implemented — the "Quota" policy — in which a certain fraction of positions are reserved for those hailing from disadvantaged backgrounds; and the "Handicap" policy, in which the performance indices of the disadvantaged are given "handicaps" or additional points to help them compete with the rest of the population. For example, India enforces the Quota policy to ensure that the backward castes are guaranteed representation in government jobs, college admissions etc.[1] The People's Republic of China, on the other hand, favors the "handicap" policy by giving

[1] Among other nations, Romania and Pakistan also use quotas to ensure Roma people's and women and non-Muslim citizens' representation in government jobs and universities respectively.

grade boosts ("bonus points") to minority students who take the National Higher Education Entrance Examination (gaokao).

Supporters of affirmative action policies claim that it is unfair to assume that those who have been victims of centuries of systematic discrimination can compete for scarce resources in the economy on an equal footing; and that it is the moral obligation of modern governments to help by leveling the playing field. However, such opinions are not shared by all. Detractors point out the negative effects on incentives that such policies may have on those belonging to the general population, claiming that it is in fact, unfair for the progeny of the advantaged to be penalized for the excesses of their forefathers' era.

Moral and philosophical arguments aside, our task in this paper is to probe the extent to which the *economic* decision making of agents — both disadvantaged and otherwise — is affected by different implementations of affirmative action policies. Indeed, the question that we are most interested in asking is "Is there a *most efficient* way in which we could implement affirmative actions?", or, in absence of a clear answer to the previous question, the following, more modest inquiry: "Among some affirmative action policies, can we compare them in terms of efficiency and rank them from most to least efficient?" While we define what we mean by "efficient" more precisely in a later section, we use these natural question to delineate the scope and limits of our study.

We use a game theoretic model for studying this problem. We envisage a situation in which a large number of students take a competitive exam. The top few rankers of this exam will be awarded highly coveted jobs. However, there are two sections of students who differ in their "test-taking skills". The government can choose to intervene and help those with lower test-taking skills by implementing some forms of affirmative action; or it may choose to refrain from affecting the outcome of the competition. The students may choose to put in full effort to prepare for the examination or may decide to not study at all if they feel that they have no chance of winning the job. The reason why we use a game theoretic model is because a student's decision to study hard or slack off may influence *all* other students' decisions — imparting the problem a classic game theoretic flavor. As an easy illustration consider the case of only two students — one of them being disadvantaged. If the disadvantaged student works hard, she gets a score between 30 and 60 out of a 100; while in case of slacking off, she gets a score between 0 and 30. Imagine that the other student obtains between 65 and 80 in case she shirks; and gets between 81 and 100 in case she studies hard. Having access to this information will compel the disadvantaged student to slack off, since she can never win the job no matter what, which decision in turn will make life easy for the advantaged student who will then shirk too. The equilibrium of the game will entail both students shirking — the disadvantaged getting a score between 0 and 30 — while the other student scoring between 60 and 80 and winning the job. Clearly, this is a "bad equilibrium" but can we institute changes, namely by some forms of affirmative action so as to nudge the players' behavior towards a "better" equilibrium? This is the type of analysis we carry out in our paper.

The rest of the paper is organized as follows: section 2 discusses related work done in this area; the third section describes the basic model while section 4 formally introduces affirmative action policies. In section 5, we lay out general procedures that governments may use to "shop" for affirmative action policies they prefer most and formally compare different policies on the basis on some natural criteria. Section 6 concludes the paper.

2.2 Related Work

While there have been many attempts to understand the trade-offs involved between efforts and incentives in the presence of affirmative action policies that favor a particular group of people, the paper that comes the closest in spirit to our work is Brent R. Hickman (2010) in which explicit comparisons between the different affirmative action mechanisms like Quota, Handicap and No Intervention are carried out.[2] Like our model, the author studies the game induced by the imposition of the particular affirmative action policy under question and studies equilibrium allocational

[2] In Brent R. Hickman (2010), the No Intervention policy is referred to as the Color Blind policy, and the rule for the Quota system is a little different from the one that we use.

outcomes.

However, our work is significantly different from the one in Brent R. Hickman (2010). This is because Brent R. Hickman (2010) studies affirmative action mechanisms in context of American college admissions. Hence the objects for which the students (players) compete are heterogeneous. The Quota policy considered in that paper is that of "simple" quotas, in which a certain section of positions are reserved for the backward section of society and they compete among themselves for *only* those reserved positions. Also, there is no scarcity in the model — *all* players in the game are guaranteed some position at some college.

All these features are different for the problem we are trying to tackle. In our model students compete for a set of jobs that are *identical*, though highly valued by all of them. The rules of the Quota game are also different. While there is a certain fraction of jobs reserved for those coming from the backward sections of the society, for the rest of the unreserved jobs, there is open competition between *all* players, regardless of which section of society they hail from. This is how the quota rule is currently enforced in India. Also, there is acute scarcity in our model in that the measure of available jobs is much smaller than the measure of those competing for them.

All these differences, in our opinion, radically change the game that is induced and hence the behavior of players in equilibrium.

2.3 The Basic Model

Our model reflects the situation in which there are a large number of students taking a competitive examination. The top rankers in this examination will be awarded highly coveted jobs, the measure of which is very low compared to the measure of students taking the exam.

We first analyze the basic game in which there is only one group of students and then introduce affirmative action policies within two groups of disparately skilled population of students by means of quotas and handicaps.

2.3.1 The Game

We first model the basic situation in which there is only one group of students, all of whom may have different "test taking abilities". We postulate the existence of a continuum of players located uniformly in the interval [0,1]. The location $z \in [0, 1]$ of a player completely specifies her test-taking abilities. Hence a player situated close to 1 has a higher test-taking skill than that of a player located close to 0.

Each player is equipped with two possible actions denoted by $e(z) \in \{0, 1\}$, with '0' standing for "shirking" or putting no effort in preparing for the examination, while '1' stands for "working", or exerting full effort. We also assume that exerting full effort is costly for the student and she incurs a constant disutility $\delta > 0$ when $e(z) = 1$. On the other hand, not exerting any effort is assumed costless and there is no disutility associated with shirking.

We assume that when students (players) exert full effort, their scores in the exam are higher than if they were to exert no effort. Each player receives a final score on the competitive test which is dependent on whether she shirked or worked. If she shirked, her score is governed by a random variable $s(z, 0)$ while in case of working, the score is given by $s(z, 1)$. We assume that the score functions' structure is as follows:

$$s(z, 0) \sim \mathbb{U}[f(z), f(z) + \eta]$$

$$s(z, 1) \sim \mathbb{U}[g(z), g(z) + \eta]$$

where $\mathbb{U}(\cdot)$ is the uniform distribution, η is an exogenous positive constant and $f(\cdot)$ and $g(\cdot)$ are increasing functions of z.

This situation may also be represented pictorially in Fig. 2.1.

The students compete for a scarce set of jobs of measure $J < 1$ all of which are homogeneous and are valued sufficiently highly by the players at $v > 0$. The jobs are to be allocated to measure J top rankers in the competitive examination. We refer to the set of winners as W.

Based on the description of the game above, the payoffs of the players may be

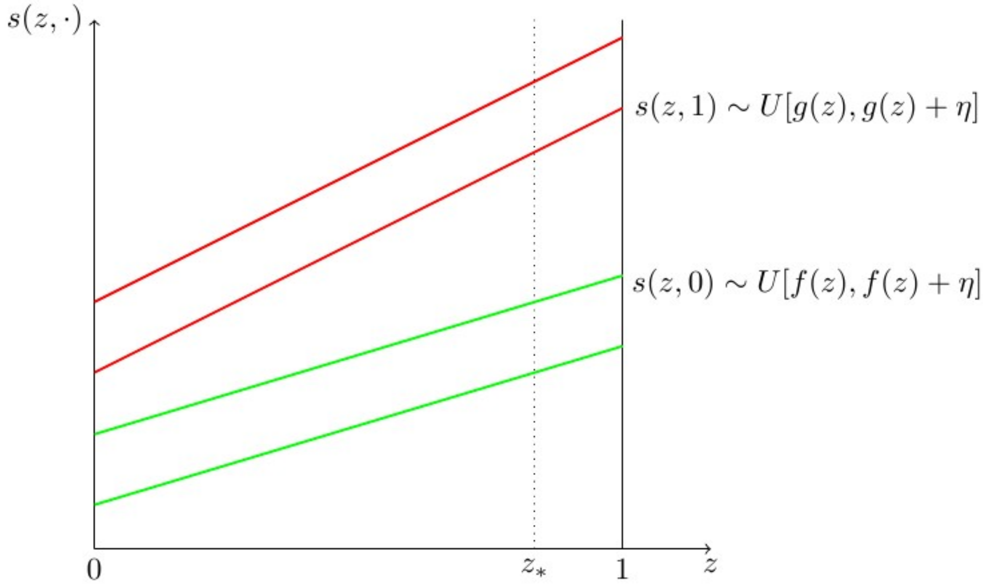

Figure 2.1: The Basic Game

written as:

$$u_z(e(z), e(-z)) = \begin{cases} u_z(0,\cdot) = v\,\Pr(z \in W) \\ u_z(1,\cdot) = v\,\Pr(z \in W) - \delta \end{cases}$$

Here, in spirit of the notation s_{-i}, we use $e(-z)$ to denote the action profile of all players other than z.

We can further understand the term $\Pr(z \in W)$ by describing it in terms of the score function of the student z:

$$\Pr(z \in W) = \Pr\{s(z,\cdot) \geq \underline{s}\}$$

where $\underline{s}$ is the cutoff score — the lowest score at which one gets a job.

Nash Equilibrium: The Nash Equilibrium of the above game is a threshold equilibrium in which players beyond a certain cutoff level of skill exert full effort and win

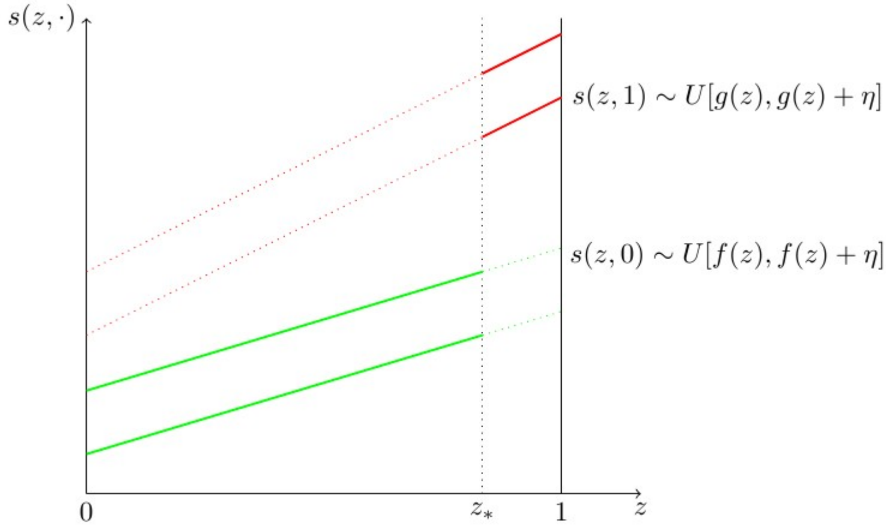

Figure 2.2: Nash Equilibrium of the Basic Game

from the set of jobs J; while the rest shirk:

$$e(z) = \begin{cases} 0 & \forall\, z < z_* \\ 1 & \forall\, z \geq z_* \end{cases}$$

This may be represented by Fig. 2.2.

It is easy to check that this threshold strategy is indeed a Nash Equilibrium by observing that those beyond the critical threshold have a positive expected payoff when exerting full effort and 0 payoff when shirking; while those with $z < z_*$ have a negative expected payoff while exerting full effort and 0 payoff when shirking. The student at $z = z_*$ is the one who is indifferent between shirking and working.

The Nash Equilibrium may be characterized by two different equations — the critical player z_*'s payoff equation and the second condition of the measure of winners being equal to the measure of jobs.

The Critical Player's Condition:

$$u_{z_*}(1, e(-z_*)) = v \Pr(z_* \in W) - \delta = 0$$

$$\Rightarrow \Pr(s(z_*, 1) \geq \underline{s}) = \frac{\delta}{v}$$

The equation stems from the fact that the critical player must be indifferent between working and shirking in equilibrium.

Measure of Winners Equals the Measure of Jobs

$$\int_{z_*}^{1} \Pr(s(z, 1) \geq \underline{s}) dz = J$$

We note that essentially, the above condition is the Law of Large Numbers.

2.4 Affirmative Action Policies

Now we introduce two different groups of players — the αs and the βs who differ from each other in terms of their test-taking skills. The βs are assumed to be hailing from the disadvantaged section of the society which fact is reflected in their relatively lower test-taking skills.

We assume that the source of difference between the two sections of society is captured by an exogenous "degree of asymmetry" $\Delta > 0$.

While we assume that the players have the same score functions, valuations over jobs and the same disutility from exerting full effort, we posit that their test-taking skills are dependent on Δ as follows:

$$z_\beta \in [0, 1]$$

while

$$z_\alpha \in [\Delta, 1 + \Delta]$$

We represent this pictorially in Fig. 2.3.

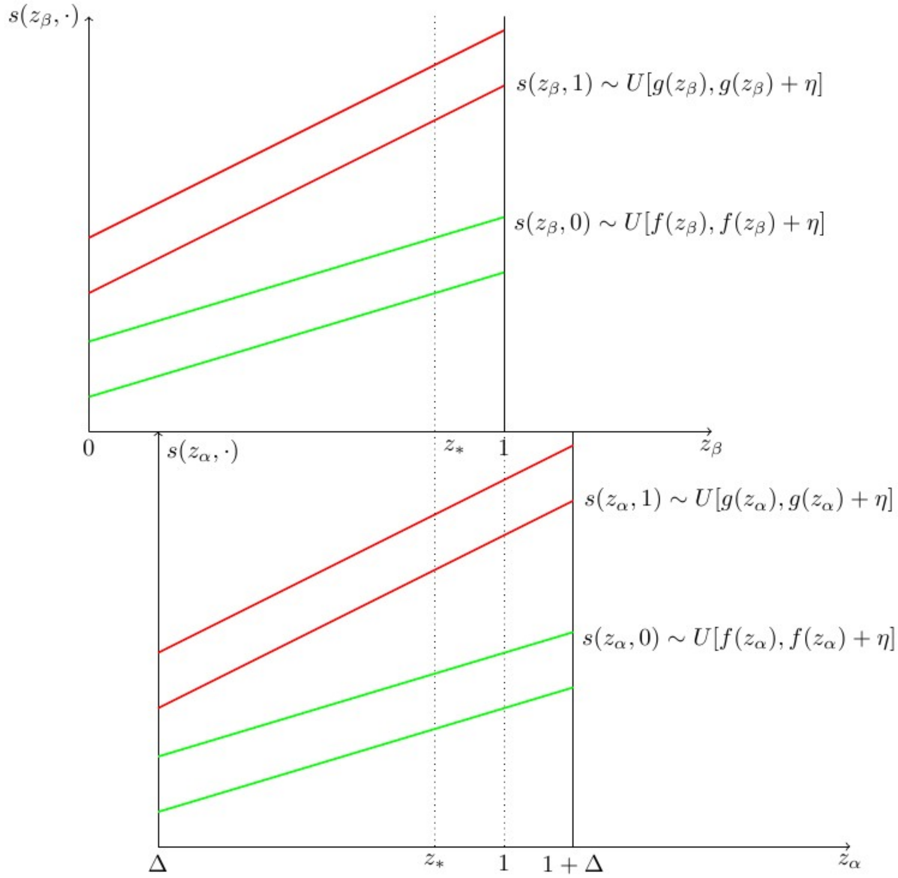

Figure 2.3: The Affirmative Action Game

As should be expected, the larger the degree of asymmetry between the two classes of players, the larger will be the difference in relative test-taking abilities between the two sections of the populace. We divide our analysis of affirmative action policies in three different classes: the "No Intervention" policy — in which the government does not intervene on behalf of the disadvantaged; the "Quota" Policy — in which a certain fraction of jobs are reserved for the β players; and the "Handicap" policy — in which the government intervenes by adding to the scores of βs, a certain fixed handicap h_0.

2.4.1 The No Intervention Policy

As the name suggests, in the No Intervention policy, the government does not do anything to interfere with the job allocation mechanism. The top measure J students, whether they be from the class α or β are handed jobs just on the basis of their realized scores. In this analysis, however, we further encounter two separate cases — one in which the degree of asymmetry is very high — and the other in which it is moderate. In either case however, the Nash Equilibrium of the induced game is a threshold equilibrium for the exact same reasons as given in the basic game.

High Asymmetry: If Δ is very high, due to extreme inequality in the test-taking skills of classes α and β, all the measure J jobs will be allocated to the top rankers in the class α, since even the highly skilled βs will not be able to compete with them. In this case, the equilibrium will be:

$$e(z_\beta) = 0 \;\; \forall\, z_\beta \in [0, 1]$$

$$e(z_\alpha) = \begin{cases} 0 & \forall\, z_\alpha < z_\alpha^* \\ 1 & \forall\, z_\alpha \geq z_\alpha^* \end{cases}$$

Fig. 2.4 shows the situation in which all jobs are allocated only to the α players.

The characterizing equations for the equilibrium will be the same two types — the critical player's condition and the measure of jobs being equal to the measure of

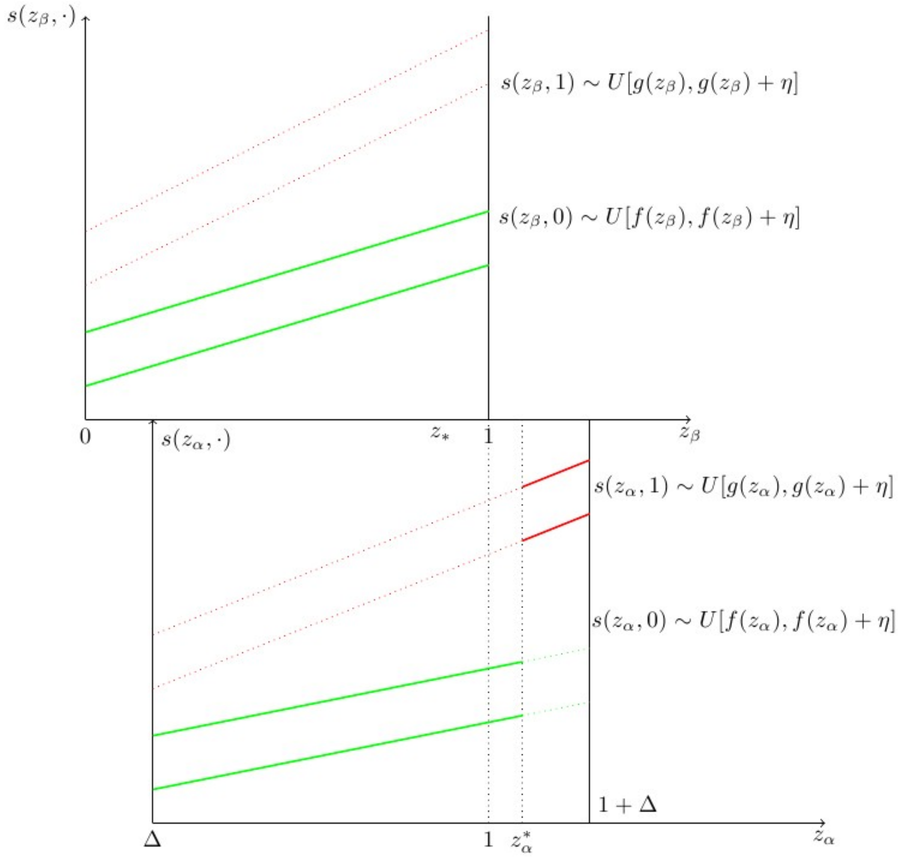

Figure 2.4: No Intervention, Large Asymmetry

winners. In this case the cutoff score $\underline{s}$ and the critical player's identity will be given by:

$$u_{z_\alpha^*}(1,\cdot) = v\,\Pr(z_\alpha^* \in W) - \delta = 0$$

$$\Rightarrow \Pr(s(z_\alpha^*, 1) \geq \underline{s}) = \frac{\delta}{v} \tag{2.1}$$

$$\int_{z_\alpha^*}^{1+\Delta} \Pr(s(z_\alpha, 1) \geq \underline{s}) dz_\alpha = J \tag{2.2}$$

Low Asymmetry: If Δ is moderate, both α and β players have a chance of winning the jobs. The highly skilled βs, situated close to $z_\beta = 1$ may now exert full effort and be allocated jobs. In this case too, the equilibrium is threshold and the characterizing equations are of the same type.

$$e(z_\beta) = \begin{cases} 0 & \forall\, z_\beta < z_* \\ 1 & \forall\, z_\beta \geq z_* \end{cases}$$

$$e(z_\alpha) = \begin{cases} 0 & \forall\, z_\alpha < z_* \\ 1 & \forall\, z_\alpha \geq z_* \end{cases}$$

We remark here that we take the degree of asymmetry as a given, exogenous factor. It may be exogenously large or small in different societies depending on several complicated factors. Ultimately the question of ascertaining its relative size is an empirical one. Obviously, the question of governments intervening to restore equity in a certain society is intimately linked to the already existing degree of asymmetry in that society, captured in our model, by the constant Δ.

$$u_{z_*}(1,\cdot) = v\,\Pr(z_* \in W) - \delta = 0$$

$$\Pr(s(z_*, 1) \geq \underline{s}) = \frac{\delta}{v} \tag{2.3}$$

$$\int_{z_*}^{1+\Delta} \Pr(s(z_\alpha, 1) \geq \underline{s}) dz_\alpha + \int_{z_*}^{1} \Pr(s(z_\beta, 1) \geq \underline{s}) dz_\beta = J \tag{2.4}$$

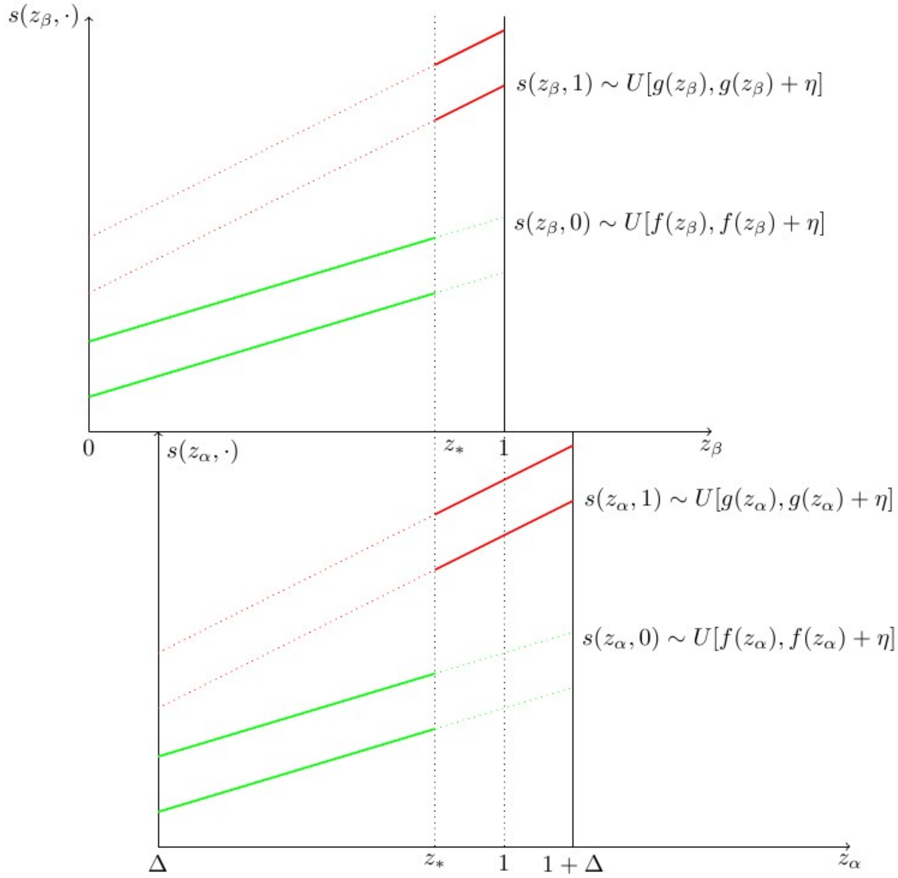

Figure 2.5: No Intervention, Low Asymmetry

2.4.2 The Quota Policy

The Quota policy, the way it is practised in India, reserves a certain fraction of jobs for the underprivileged sections of the society. For the rest of the unreserved jobs, there is open competition between *all* students. In terms of the parameters of our model, this implies that a certain measure $J_\beta < J$ of jobs are reserved for the β class of players; and for the remainder $J - J_\beta$ measure of jobs, both α and β classes of students compete, on the basis of their realized scores.

Again, our analysis will focus on two different cases — one in which the exogenous degree of asymmetry is high — and the other in which it is moderate.

High Asymmetry: If the asymmetry is high, the β students cannot compete with the αs if the government does not intervene. This was made clear in the analysis of the case of high asymmetry, no intervention above. Hence the measure of jobs that are allocated to the β class, can come *only* from the measure of jobs reserved, which is J_β. The equilibrium of this game is again threshold but a new feature of this policy is that there are separate threshold levels z_α^* and z_β^* for the two different classes of players. The cutoff scores for the two different classes of players are separate as well. However, the equations that characterize equilibrium, the identities of the indifferent players and the cutoff levels for the scores are of the same type.

$$e(z_\beta) = \begin{cases} 0 & \forall\, z_\beta < z_\beta^* \\ 1 & \forall\, z_\beta \geq z_\beta^* \end{cases}$$

$$e(z_\alpha) = \begin{cases} 0 & \forall\, z_\alpha < z_\alpha^* \\ 1 & \forall\, z_\alpha \geq z_\alpha^* \end{cases}$$

The following equations characterize equilibrium:

$$\Pr(s(z_\beta^*, 1) \geq \underline{s}^\beta) = \frac{\delta}{v} \tag{2.5}$$

$$\Pr(s(z_\alpha^*, 1) \geq \underline{s}^\alpha) = \frac{\delta}{v} \tag{2.6}$$

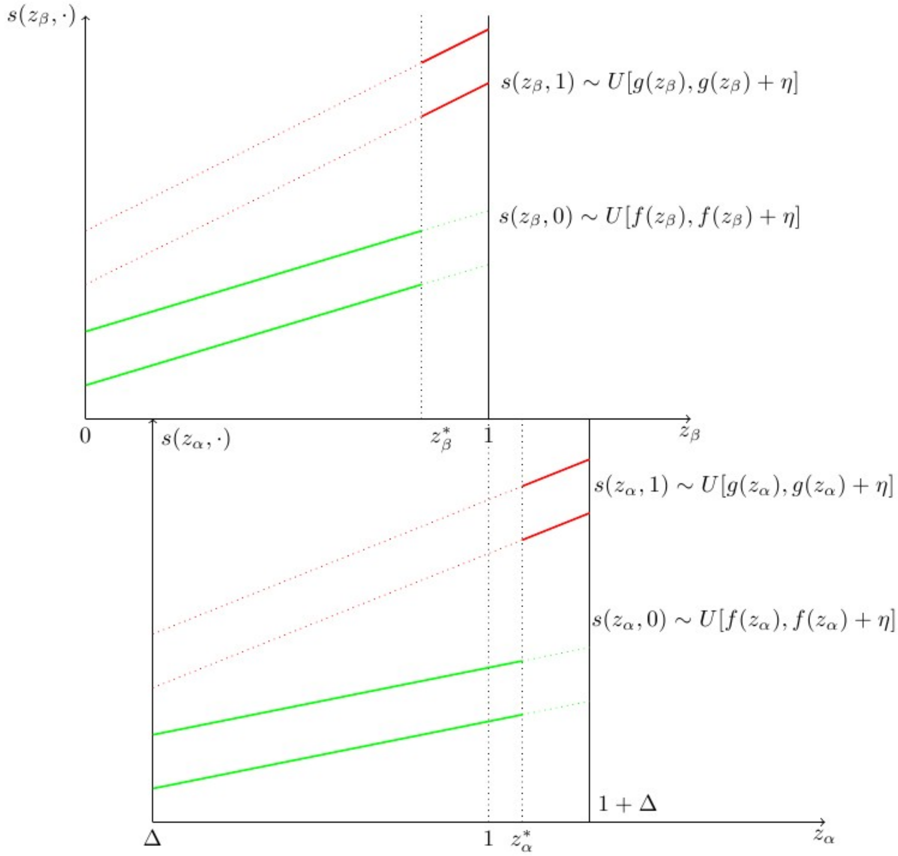

Figure 2.6: Quota Policy, Large Asymmetry

The measure of jobs going to the β players is exactly J_β, while the rest of the jobs are allocated to the α players.

$$\int_{z_\beta^*}^{1} \Pr(s(z_\beta,1) \geq \underline{s}^\beta) dz_\beta = J_\beta \tag{2.7}$$

$$\int_{z_\alpha^*}^{1+\Delta} \Pr(s(z_\alpha,1) \geq \underline{s}^\alpha) dz_\alpha = J - J_\beta \tag{2.8}$$

Low Asymmetry: When Δ is moderate, the β players get *at least* J_β measure of jobs. The reason for this is that now there are sufficiently many high skilled β players who can compete with the αs on an equal footing without the need of governmental assistance. Hence they will be able to win jobs from the "open" pool of $J - J_\beta$ while their less skilled bretheren can be allocated measure J_β jobs as before. Overall, this is detrimental to the α class of students whose claim on the measure $J - J_\beta$ is successfully challenged by the high skilled βs.

Again, the equilibrium remains threshold as before:

$$e(z_\beta) = \begin{cases} 0 & \forall\, z_\beta < z_\beta^* \\ 1 & \forall\, z_\beta \geq z_\beta^* \end{cases}$$

$$e(z_\alpha) = \begin{cases} 0 & \forall\, z_\alpha < z_\alpha^* \\ 1 & \forall\, z_\alpha \geq z_\alpha^* \end{cases}$$

As before, the critical players are different for different groups. Again, as before, there are different cutoff levels for different groups.

$$\Pr(s(z_\beta^*,1) \geq \underline{s}^\beta) = \frac{\delta}{v}$$

$$\Pr(s(z_\alpha^*,1) \geq \underline{s}^\alpha) = \frac{\delta}{v}$$

The third equation is obtained from the observation that a section of the β players get jobs from the measure J_β that is reserved for them:

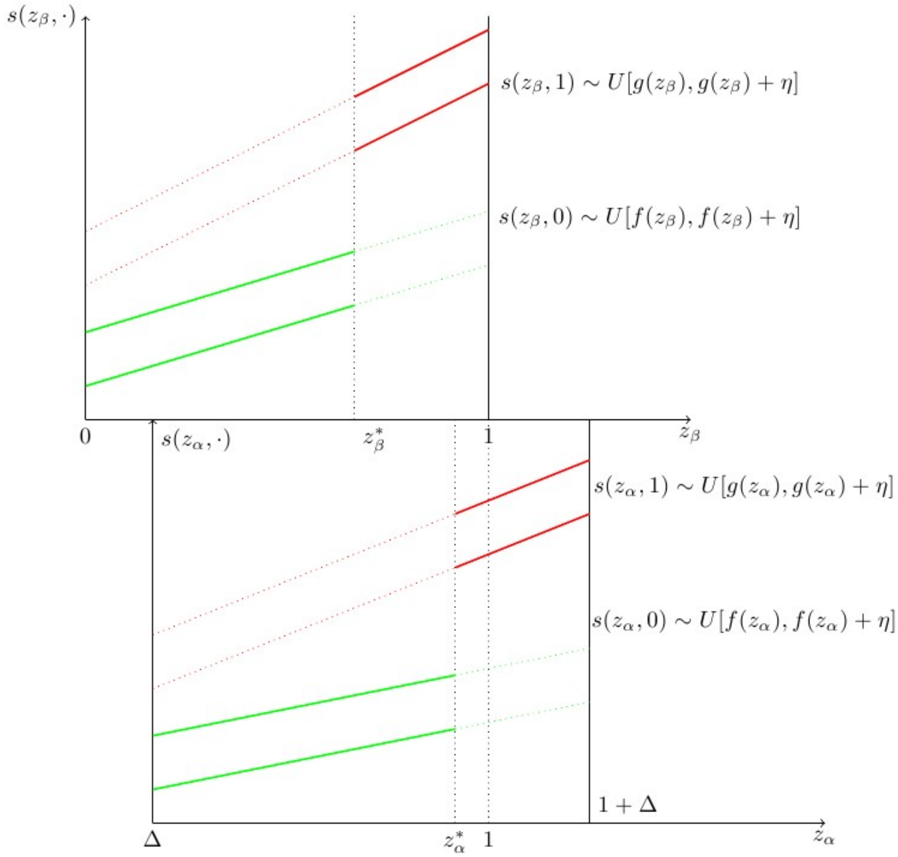

Figure 2.7: Quota Policy, Low Asymmetry

$$\int_{z_\beta^*}^{1} \Pr(s(z_\beta, 1) \in [\underline{s}^\beta, \underline{s}^\alpha))dz_\beta = J_\beta \tag{2.9}$$

While the last equation may be deduced from the fact that there is open competition for the remainder of the $J - J_\beta$ measure of jobs:

$$\int_{z_\alpha^*}^{1+\Delta} \Pr(s(z_\alpha, 1) \geq \underline{s}^\alpha)dz_\alpha + \int_{z_\beta^*}^{1} \Pr(s(z_\beta, 1) \geq \underline{s}^\alpha)dz_\beta = J - J_\beta \tag{2.10}$$

2.4.3 The Handicap Policy

The Handicap Policy is another type of affirmative action policy that is aimed to help level the playing field for the disadvantaged group of the populace. However, the method by which this is done is very different from that of the quota policy. Instead of reserving a certain fraction of jobs for the βs, the handicap policy seeks to augment the realized scores of the βs by a specific additive constant — the "handicap" — h_0. As an example, if the handicap is $h_0 = 20$ points and an α and a β score 50 and 40 points (out of say, a normalized score of 100), the final score of the two students in question will be 50 and 40+20 = 60 respectively. This is so because the score of the β player is boosted by the additive handicap while that of the α remains unchanged.

Hence the score functions for the βs in the handicap game become:

$$\tilde{s}(z_\beta, e_{z_\beta}) = \begin{cases} s(z_\beta, 0) + h_0 \sim \mathbb{U}[f(z_\beta) + h_0, f(z_\beta) + h_0 + \eta] \\ s(z_\beta, 1) + h_0 \sim \mathbb{U}[g(z_\beta) + h_0, g(z_\beta) + h_0 + \eta] \end{cases}$$

In our analysis we assume that the handicap h_0 is not "too high" to do away with uninteresting trivialities. Indeed, as an example, consider a maximum handicap of a 100 points. In this case, *all* βs, irrespective of their test-taking skills or actions taken obtain a perfect score of 100. As a result, no α gets any job and all the measure J jobs end up being allocated to the βs — all of whom are tied with a perfect score. As a result of randomly broken ties, a random collection of measure J βs then end up with jobs.

Under this assumption on the size of the handicap, we can analyze the problem of job allocation under two different scenarios — high asymmetry and low asymmetry.

High Asymmetry: If the exogenous asymmetry between the two groups is very high, even after application of the handicap, the βs will not be able to compete with the α players and all jobs will end up being allocated to them. We note that in this case the analysis is identical to the case of "No Intervention, High Δ" and the equilibrium equations for the identification of the indifferent α player and the cutoff score are found exactly in the same way as in the case referred to above. Also, no β puts in any effort since there is no chance of them winning a job even after the exertion of full effort.

Low Asymmetry: For the case of low asymmetry, the handicaps do end up benefiting the downtrodden. The equilibrium is threshold as before and the characterizing equations are:

Criticality for β players:

$$\Pr(s(z^*_\beta, 1) + h_0 \geq \underline{s}) = \frac{\delta}{v}$$

$$= \Pr(s(z^*_\beta, 1) \geq \underline{s} - h_0) = \frac{\delta}{v}$$

Criticality for α players:

$$\Pr(s(z^*_\alpha, 1) \geq \underline{s}) = \frac{\delta}{v}$$

Total measure of winners = Total measure of jobs:

$$\int_{z^*_\beta}^{1} \Pr(s(z_\beta, 1) \geq \underline{s} - h_0) dz_\beta + \int_{z^*_\alpha}^{1+\Delta} \Pr(s(z_\alpha, 1) \geq \underline{s}) dz_\alpha = J$$

2.5 Evaluation of Affirmative Action Policies

There is no unique way in which one can rank Affirmative Action policies from top to bottom. Indeed, different governments may have different preferences over the type of allocation rules they want to implement. For example, if a government wishes to not worry about social inequities and wants to allocate the measure J jobs to the best performing students, they should do away with all affirmative action policies and simply allocate the jobs to the top rankers. At the other extreme, if the only concern of the government is to allocate jobs to as many students from the disadvantaged section as possible, the best policy to follow would be to reserve all jobs for the βs, i.e., put $J_\beta = J$.

Some criteria that governments might find useful in ranking affirmative action policies can be as follows:

1 All other things being equal, the policy giving higher quality students is preferable.

2 All other things being equal, the policy under which students exert more effort is preferable.

3 All other things being equal, the policy under which representation of job winners is proportional to their proportion in the population is preferable.

$\vdots$

d ...

While in general, the space of affirmative action policies is a very complicated function space, on the basis of some d performance criteria as listed above, one can assign ordinal real numbers for each such criterion and hence locate *all* affirmative action policies, no matter how complicated, on the "performance space" $\mathbb{R}^d$.

Once all affirmative action policies have been identified with points in the $\mathbb{R}^d$ space, governments that are searching for affirmative action policies to implement may look at the coordinates[3] of each policy in performance space $\mathbb{R}^d$; and based on their individual preference ordering over each criterion (axis) choose the one they prefer most.

2.5.1 Comparing Quotas and Handicaps

Of the many different criteria on which one could rank Quotas and Handicaps, we choose to focus on three particular cases: one in which we compare the total measure of jobs allocated to the βs, as a measure of the government's inequality reducing mandate; the second in which we compare the "quality" of the job-winners by comparing the critical player in each such mechanism — clearly lower critical player thresholds mean lower minimally skilled job holders; and the third criterion that we focus on is the measure of students exerting full effort in each policy. We posit that all other things being equal, a government would favor a policy in which more students exert full effort since it implies building up of the human capital of the country.

Job Allocation to β players

Quotas: In the Quota policy the least measure of jobs that the βs can attain is J_β:

$$\int_{z_\beta^*}^{1} \Pr(s(z_\beta, 1) \in [\underline{s}^\beta, \underline{s}^\alpha))dz_\beta = J_\beta \tag{2.11}$$

However, the βs can win a further $\tilde{J}$ measure of jobs by competing for measure $J - J_\beta$ jobs.[4]

$$\tilde{J} = \int_{z_\beta^*}^{1} \Pr(s(z_\beta, 1) \geq \underline{s}^\alpha)dz_\beta \tag{2.12}$$

[3]Each government will place different policies on different points in the performance space. The coordinates refer to the ordinal score that each government puts on various criteria consistent with their preferences. For example, a government not concerned with social inequities collapses the general performance space to $\mathbb{R}^1$ in which it gives the highest ordinal score to the "No Intervention Policy" and hence chooses it for implementation.

[4]This analysis is done assuming the degree of asymmetry is not very high. The case of high Δ is a special case of the analysis of low Δ.

We can combine equations 2.11 and 2.12 to arrive at the measure of jobs that βs get in the Quota policy $\tilde{J}_{\beta}^{Q}$:

$$J \geq \tilde{J}_{\beta}^{Q} = \int_{z^*_{\beta,Q}}^{1} \Pr(s(z_{\beta}^{Q}, 1) \geq \underline{s}^{\beta}) dz_{\beta,Q} \geq J_{\beta}$$

Handicaps: In the case of high asymmetry, the measure of jobs allocated to the βs is 0. In such a case, from the point of view of jobs allocated to the disadvantaged, the Quota system does better since exactly J_{β} measure of jobs are then allocated to βs. However this is not the case in which Δ is moderate.

$$\tilde{J}_{\beta}^{H} = \int_{z^*_{\beta,H}}^{1} \Pr(s(z_{\beta}^{H}, 1) \geq \underline{s} - h_0) dz_{\beta,H}$$

Minimal Quality of Job Winners

The minimal quality of job winners can be identified with the last student who manages to win — i.e., the critical player. Hence in the Quota policy, this player is $z^*_{\beta,Q}$ while in the Handicap policy, this player is $z^*_{\beta,H}$.

Equivalence: It is easy to see that for both Quotas and Handicaps

$$\tilde{J}_{\beta}^{Q} = \tilde{J}_{\beta}^{H} \Leftrightarrow h_0 = \underline{s} - \underline{s}^{\beta} \text{ and } z^*_{\beta,Q} = z^*_{\beta,H}$$

This implies that under low asymmetry, for any job allocation outcome that Quota achieves, one can find a suitable handicap which will induce the same allocational efficiency and the same minimal quality of job winners as in the Quota mechanism.

The argument runs in the other direction too. Under low Δ, for any handicap that allocates a certain measure of jobs and a certain minimal quality of job winners, there is a quota level J_{β} that ensures that the job allocation and minimal student quality in the game induced by the Quota mechanism is the same as that in the Handicap mechanism.

Hence on the two desiderata of allocational efficiency and minimal student quality, the Quota and Handicap mechanisms are equivalent (under low asymmetry).

Measure of Players Exerting Full Effort

Governments want to maximize the measure of students exerting full effort. However, in the Quota and Handicap mechanisms, this is simply $1 - z^*_{\beta,Q} + 1 - z^*_{\alpha,Q}$ and $1 - z^*_{\beta,H} + 1 - z^*_{\alpha,H}$ respectively. Since we have seen above that for the same allocational efficiency, $z^*_{\beta,Q} = z^*_{\beta,H}$, hence

$$1 - z^*_{\beta,Q} + 1 - z^*_{\alpha,Q} = 1 - z^*_{\beta,H} + 1 - z^*_{\alpha,H}$$

or, in other words, the measure of players exerting full effort in the two policies is also the same.

Hence we conclude that on the basis of some natural efficiency desiderata, the Quota and Handicap policies are equivalent to each other.

2.6 Conclusion

In this paper we analyzed two different classes of affirmative action mechanisms in the presence of a large number of students taking a competitive examination for a highly valued, scarce set of jobs. We showed that we can analyze this situation as a game and that the threshold Nash Equilibrium of the game allocates jobs to the top rankers and also determines the minimal quality student that manages to be a job winner. We then analyzed the games induced by the Quota and the Handicap affirmative action policies and characterized their Nash Equilibria. We also described a general procedure according to which governments can decide which affirmative action policy suits their interests best given arbitrary criteria for measuring affirmative action policies' efficiency. We are, under some natural criteria, able to compare Quota and Handicap policies and conclude that if the exogenous asymmetry between the disadvantaged and advantaged groups of the population is not so high, the Quota and Handicap mechanisms produce equivalent allocational outcomes, minimal quality of job winners and equilibrium measure of students exerting full effort. In this sense, hence, the two policies are equally efficient. If however, the degree of asymmetry is high, the Quota system produces better allocational outcomes for the disadvantaged, while the Handicap system performs better as far as the minimal quality of job winners is concerned. The measure of students who exert full effort also, is better under the Quota policy.

We stress again that the matter of ascertaining the degree of asymmetry between the two groups of students is necessarily empirical. As a policy prescription, we recommend that if asymmetries are higher, the Quota system reduces inequities in the society while the Handicap system may not affect it. Governments must first determine the extant degree of asymmetry before putting one or the other policy in implementation.

Bibliography

Admati, A. and Hellwig, M. (2013). *The Bankers' New Clothes: What's Wrong With Banking and What to Do About It*. Princeton, New Jersey: Princeton University Press.

Alexander, S. S. (1961). Price Movements in Speculative Markets: Trends or Random Walks. *Industrial Management Review*, *2*, 7–25.

Artzner, P., Delbaen, F., Eber, J.-M., and Heath, D. (1998). Coherent Measures of Risk. *Mathematical Finance*, *6*(3), 203–228.

Berkowitz, J. (2001). Testing density forecasts, with applications to risk management. *Journal of Business and Economic Statistics*, *19*, 465–474.

Bianchi, M. L., Rachev, S. T., Kim, Y. S., and Fabozzi, F. J. (2010). Tempered Stable Distributions and Processes in Finance: Numerical Analysis. *Mathematical and Statistical Methods for Actuarial Sciences and Finance*, 33–42.

Brent R. Hickman (2010). Effort, Race Gaps, And Affirmative Action: A Game Theoretic Analysis Of College Admissions. Working Paper.

Christofferson, P. F. (1998). Evaluating interval forecasts. *International Economic Review*, *39*(4), 841–862.

Fama, E. F. (1963). Mandelbrot and the Stable Paretian Hypothesis. *Journal of Business*, *36*, 420–429.

Fama, E. F. (1965). The Behavior of Stock Market Prices. *Journal of Business*, *38*, 34–105.

Foster, D. P. and Hart, S. (2009). An Operational Measure of Riskiness. *Journal of Political Economy*, *117*(5), 785–814.

Haas, M. and Pigorsch, C. (2009). *Complex Systems in Finance and Econometrics*, New York: Springer, chap. Financial Economics, Fat-Tailed Distributions. 308–339.

Kim, Y. S., Giacometti, R., Rachev, S. T., Fabozzi, F. J., and Mignacca, D. (2012). Measuring Financial Risk and Portfolio Optimization with a Non-Gaussian Multivariate Model. *Annals of Operations Research*, 325–343.

Kupiec, P. H. (1995). Techniques for verifying the accuracy of risk measurement models. *Journal of Derivatives*, *6*, 6–24.

Kurosaki, T. and Kim, Y. S. (2013). Sytematic Risk Measurement in the Global Banking Stock Market With Time Series Analysis and CoVaR. *Investment Management and Financial Innovations*, *10*(1), 184–196.

Mandelbrot, B. (1963a). New Methods in Statistical Economics. *Journal of Political Economy*, *71*, 421–440.

Mandelbrot, B. (1963b). The Variation of Certain Speculative Prices. *Journal of Business*, *36*, 394–419.

Mandelbrot, B. (1967). The Variation of Some Other Speculative Prices. *Journal of Business*, *40*, 393–413.

Markowitz, H. M. (1952). Portfolio Selection. *Journal of Finance*, *7*(1), 77–91.

Markowitz, H. M. (1959). *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley and Sons.

Rachev, S. T., Menn, C., and Fabozzi, F. J. (2005a). *Fat Tailed and Skewed Asset Return Distributions*. Hoboken, New Jersey: John Wiley and Sons.

Rachev, S. T. and Mittnik, S. (2000). *Stable Paretian Models in Finance*. Hoboken, New Jersey: John Wiley and Sons.

Rachev, S. T., Stoyanov, S., Biglova, A., and Fabozzi, F. (2005b). *Data Analysis and Decision Support*, Berlin: Springer, chap. An Empirical Examination of Daily Stock Return Distributions for US Stocks.

Rachev, S. T., Stoyanov, S. V., and Fabozzi, F. J. (2008). *Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization*. Hoboken, New Jersey: John Wiley and Sons.

Riedel, F. and Hellmann, T. (2014). The Foster-Hart Measure of Riskiness for General Gambles. *Theoretical Economics*.

Taleb, N. N. (2007). *The Black Swan: The Impact of the Highly Improbable*. New York, New York: Random House.

Appendices

Table 1: List of the 29 G-SIFIs (as of November 2013)

United States	Europe	Asia
Bank of America (BAC)	Groupe BPCE	Bank of China (BOC)
Bank of New York Mellon (BK)	Barclays (BARC)	Mitsubishi UFJ FG (MUFG)
Citigroup (C)	BNP Paribas (BNP)	Mizuho FG (MHFG)
Goldman Sachs (GS)	Standard Chartered (STAN)	Sumitomo Mitsui FG (SMFG)
JP Morgan Chase (JPM)	Credit Suisse (CSGN)	Industrial and Commercial Bank of China (ICBC)
Morgan Stanley (MS)	Deutsche Bank (DBK)	
State Street (STT)	Banco Bilbao Vizcaya Argentaria (BBVA)	
Wells Fargo (WFC)	Group Crédit Agricole (ACA)	
	HSBC (HSBA)	
	ING Bank (INGA)	
	Unicredit Group (UCG)	
	Nordea (NDA)	
	Royal Bank of Scotland (RBS)	
	Santander (SAN)	
	Société Générale (GLE)	
	UBS (UBSN)	

The appendices include tables indicating the membership of G-SIFIs, as assessed by the Financial Stability Board. Also contained are graphical results that chart variation in computed risk with changes in confidence levels, definition of risk and with changes in the underlying statistical model.

.1 The Global Systemically Important Financial Institutions

The list of Global Systemically Important Financial Institutions is included as a table. This table was prepared in November 2013 by the Financial Stability Board [5]. It features 8 banks from North America, 16 banks from Europe and 5 banks from Asia.

.2 Variation of Risk with Confidence Levels

For both VaR and AVaR, computed equity risk rises with increases in confidence level.

[5] See `http://www.financialstabilityboard.org/publications/r_131111.pdf` for more details

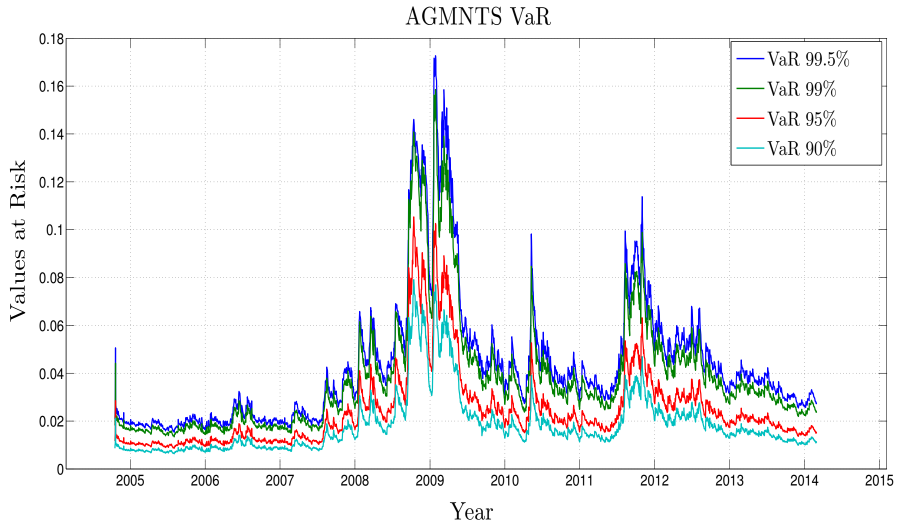

Figure 8: Variation of Value at Risk with confidence levels under the AGMNTS model

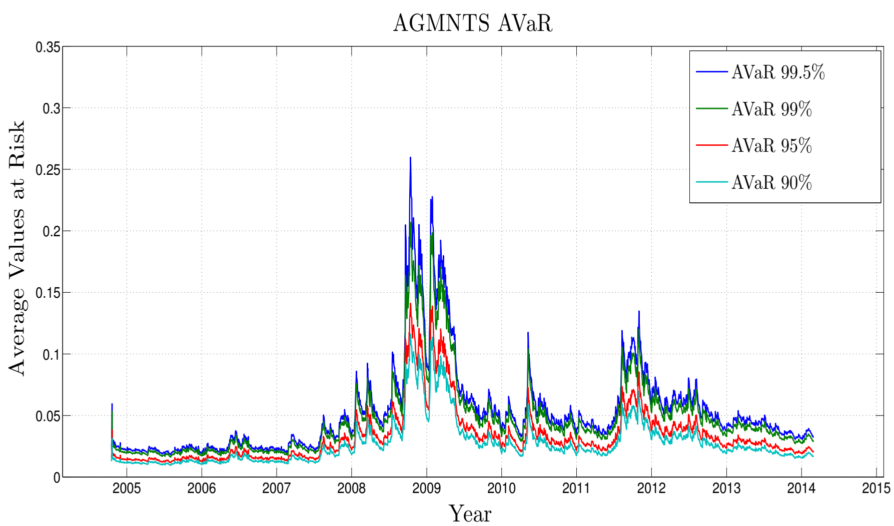

Figure 9: Variation of Average Value at Risk with confidence levels under the AGMNTS model

.3 Variation of FH Risk with the Underlying Statistical Model

As should be expected, due to the fat tailed distributional hypothesis of the AGM-NTS model, the FH equity risk under AGMNTS is much higher than that computed under the AGMNormal model. While there are qualitative similarities between the two measures, for almost all time periods, the FH risk under AGMNTS dominates that under the AGMNormal model. (We note however, that this behavior is reversed for the second and third quarter of 2012 during which FH risk under AGMNormal dominates that under AGMNTS.)

Since the Foster-Hart risk is defined only for random variables whose expectation is positive — i.e., gambles whose mean is more than 0, for the cases in which the expected portfolio returns are negative (this occurs during very heavy downturns in the financial market), the Foster-Hart risk is not defined. For such cases, we substitute the maximum loss of the portfolio for the Foster-Hart equity risk. We also use the unique, continuity property of the Foster-Hart risk to rule out the existence of outliers. We note that this technique is justified since the time series of maximum losses of the portfolio is continuous (Foster and Hart, 2009).

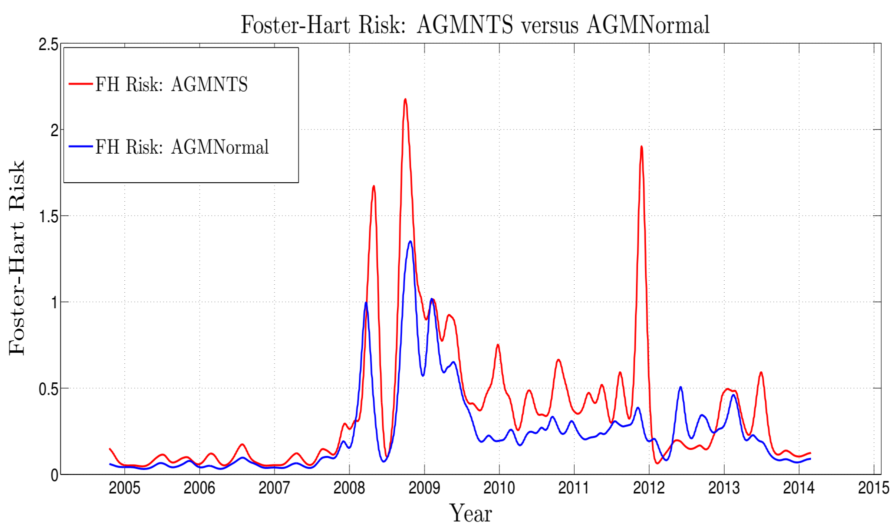

Figure 10: Variation of FH Risk with AGMNTS and AGMNormal models. All series have been smoothed by using the Hodrick-Prescott filter with $\lambda = 20000$.