Essays on Patent Licensing

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Abstract of the Dissertation

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This dissertation consists of three essays dealing with issues related to patent licensing from a game-theoretic perspective. In the first essay, Chapter 2, the problem facing an inventor who holds the patent of a technology that can potentially be used to reduce the costs of firms in a given industry is considered. The technology's ability to cut firms' costs depends on a use for it being discovered, and the inventor has the option of trying or not to discover the use before licensing the technology to the firms. In this context, two main questions are addressed. First, which alternative should the inventor choose? Second, how does this decision affect the diffusion of the technology in the industry? Interestingly, the inventor may not try to discover the use of the technology, even when trials are costless. Moreover, this decision may lead to a higher diffusion of the technology than its alternative.

The second essay, Chapter 3, considers a patent licensing model in which an outside inventor holds the patent of a cost-reducing technology that can be licensed to heterogeneous Cournot duopolists, one being more efficient in production—in the sense of operating at lower costs—than the other. Could the licensing of the technology—when carried opti-

mally from the patentee's point of view—reduce the efficiency gap between these firms? It will be seen that, under the assumptions maintained in the chapter, the answer to this question is no.

Finally, the third essay, Chapter 4, studies a model in which two patentees engage simultaneously in licensing. The technologies owned by these inventors are substitutes—i.e., both allow the production of the same good. However, one technology is at least as efficient as the other. Allowing for competition between patent owners is a major departure from the literature. What are the implications of this departure? More specifically, what is the impact of competition on a patentee's licensing behavior? Is it possible for the owner of a relatively inefficient technology to survive competition with a stronger rival patentee? In general lines, the answer to the first question is: the introduction of one competitor may lead to a significant increase in the number of licenses sold by a patentee; whereas the answer to the second question is: yes, however, with a small probability.

Each of the chapters mentioned above is an original contribution to the theoretical literature on patent licensing. The chapters are mutually independent and each is organized in the format of a research paper. In particular, each chapter contains its own introduction and conclusion. An introduction to the dissertation is provided in Chapter 1; Chapter 5 concludes the dissertation with a brief review of the main findings.

To João Manoel (*in memoriam*) & Maria Gilda.

Contents

Acknowledgments **viii**

1 Introduction **1**

1.1 A Typical Model of Patent Licensing 1

1.2 Earlier Results 2

1.3 This Dissertation 3

2 On the Licensing of a Technology with Unknown Use **5**

2.1 Introduction 5

2.2 The Model 8

2.3 Analysis 11

2.3.1 The Cournot Stage 11

2.3.2 The Auction Stage 12

2.3.3 The Inventor's Revenue Maximization Problem 14

2.3.4 The Diffusion of the Technology 15

2.4 Conclusion 15

2.5 Omitted Proofs 16

2.5.1 Proof of Proposition 1 16

2.5.2 Proof of Proposition 4 21

2.5.3 Proofs of propositions 5 and 6 24

3 Patent Licensing and Technological Catch-up in a Heterogeneous Duopoly **27**

3.1 Introduction 27

3.2 The Model 28

3.3 Proof of the Proposition 29

3.4 Concluding Remarks 31

4 Patent Licensing with Asymmetric Competing Inventors **33**

4.1 Introduction 33

4.2 The Model 34

4.3 Analysis and Main Result 35

4.4 Examples 37

4.5 Concluding Remarks 39

5 Conclusion **40**

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Chapter 1

Introduction

A patent confers to its holder property rights over a technological innovation for a limited period of time. As a monopolist, the patentee can commercially exploit the patent and, thus, collect returns to the innovative effort. In this way, the patent system aims at providing incentives to economic agents—individuals, firms, private and public research institutions, etc.—to engage in R&D activities.[1] The use of patents as means to spur technological discovery has a long history. Probably the first patent law ever enacted, according to Machlup (1958), was in the Republic of Venice in 1474. The same author notices that the Congress of the United States of America passed its first patent law in 1790.

A common strategy adopted by patentees to commercially exploit patents is licensing—whereby, in exchange for a payment, or flow of payments, the licensee is granted by the patentee the right to use the patented technology. A wealth of studies applying game-theoretic tools to investigate questions related to the practice of patent licensing has been produced in the last decades. Chapters 2, 3, and 4 of the present dissertation are contributions to this literature. To help the reader appreciate these contributions, let us next describe briefly a typical patent licensing model and some of the results found in earlier research.

1.1 A Typical Model of Patent Licensing

Consider an industry consisting of $n \geq 2$ firms facing a market demand given by $q = D(p)$, with firm $i \in \{1, 2, \ldots, n\}$ operating under a constant marginal cost technology, $c_i(q_i) =$

[1] Whether the patent system serves its purposes of encouraging innovation and disclosure of information is an interesting and important question not dealt with in this dissertation. See Boldrin and Levine (2013) and the references thereof.

$c \cdot q_i$, $c > 0$. An *outside* inventor holds the patent of a cost-reducing technology—or process innovation—that cuts the firms' marginal cost from its current level, c, to $c - \varepsilon > 0$.[2] The inventor intends to license the technology to the firms. In principle, the inventor has a number of *licensing strategies* available: it can announce a price—or fixed fee—and let any interested firm buy a license at the quoted price; it can announce an auction, in which $k \leq n$ licenses are put for sale, and let firms competitively decide how much to pay for them; it can charge, from any willing firm, output-based royalties; it can use two-part tariffs.[3]

From the above description, a game in extensive-form, having inventor and firms as players, can be naturally defined. The inventor moves first, announcing a licensing strategy. Next, firms, simultaneously and independently, choose a move regarding the decision of becoming a licensee—available moves, of course, depend on the licensing strategy selected by the inventor. The set of firms is then partitioned into a subset of licensees and another of nonlicensees. Finally, in the last stage of the game firms engage in either price or quantity competition—usually, as is the case in all chapters that follow, firms are assumed to be Cournot competitors. At any terminal node of the game tree, the payoff to the inventor is given by licensing revenues; firms' payoffs are given by the resulting oligopoly profits net of payments to the inventor, if any.

Observe that the above game is one of *imperfect* information: firms always move simultaneously. To solve it via backward induction, we must impose conditions on $P(\cdot)$—the market inverse demand, given by $p = P(q)$—to guarantee a unique equilibrium in the Cournot stage. In Chapters 2 and 4 we assume a linear demand, as in the seminal work of Kamien and Tauman (1984) and Kamien and Tauman (1986). In Chapter 3 we allow for a general demand, satisfying conditions listed in Badia et al. (2014). Some authors, as Katz and Shapiro (1986), for example, abstract from this problem by assuming that, given a subset of licensees, oligopoly profits for licensees and nonlicensees are unique.

1.2 Earlier Results

Many questions can be asked at this point. For instance, (i) What is the optimal licensing strategy? and (ii) What is the resulting diffusion of the technology?

[2] The case of an *inside* inventor has also been studied in the literature. Whereas an outside inventor's objective is to maximize licensing revenues, an inside inventor has to also take into account the negative effect of licensing on its *downstream* profits.

[3] Information on the relative relevance of these strategies in actual licensing practice can be found in Rostoker (1984) and Radauer and Dudenbostel (2013).

Kamien et al. (1992) show that, compared to fixed fee and royalty licensing, auctioning an appropriately chosen number of licenses is optimal.[4] Sen (2005a) and Sen and Tauman (2013), however, show that royalty licensing outperforms auction licensing almost surely when n—the size of the downstream industry—is sufficiently large. Sen and Tauman (2007), Giebe and Wolfstetter (2008), and Sen and Tauman (2013), consider two-part tariff licensing contracts.[5] Since two-part tariffs contain the former strategies as special cases, they yield a non-inferior performance.

It is not difficult to see that the diffusion of the technology varies with the adopted licensing strategy. For instance, an optimal licensing auction limits the number of licensees to be no more than $c/[\varepsilon\eta(c)]$, $\eta(\cdot)$ being the price elasticity of demand in the downstream market. If $c/[\varepsilon\eta(c)]$ or more firms operate under the superior technology, the remaining firms are driven out of the industry. Consequently, auction licensing, although more profitable, results in a more concentrated market structure. Consumers, however, benefit, since the market price falls to c. Other licensing strategies—i.e., royalties and two-part tariffs—yield more diffusion—indeed, either full or "almost" full diffusion—, as Sen and Tauman (2007) and Sen and Tauman (2013) have shown.

1.3 This Dissertation

Let us now describe the content of Chapters 2, 3, and 4—the core of this dissertation. With the background provided above, departures from the typical patent licensing model as well as the questions these departures give rise should be evident.

In Chapter 2, we study the problem facing an inventor who holds the patent of a technology that can *potentially* be used to reduce the costs of firms in a given industry: the technology's ability to cut firms' costs depends on a use for it being discovered. Before licensing the technology, the inventor has the option of trying or not to discover this use. Which alternative should the inventor choose? How does this decision affect the diffusion of the technology in the industry? To answer these questions we consider a game that unfolds as follows. The first move belongs to the inventor, who decides whether or not to attempt to discover the use for the technology. This attempt can result in either a success or a failure, each outcome occurring with exogenously given probability. The game then proceeds to an auction licensing stage, and, finally, to Cournot competition involving licensees and nonlicensees. We show that the answer to the above questions are intimately related to how firms interpret a failed attempt by the inventor in terms

[4]These authors also describe a licensing mechanism that approximates arbitrarily well the maximum revenue the patentee can achieve.

[5]Giebe and Wolfstetter (2008) allow the patentee to combine two-part tariffs—in which the fixed component is decided via auction—with pure royalty contracts.

of their own likelihood of discovering a use for the technology. If this interpretation is very negative, then it is optimal for the inventor to not carry a trial, even when it is costless. Moreover, this decision may lead to a higher diffusion of the technology than its alternative.

In Chapter 3, we consider a model in which an outside inventor holds the patent of a cost-reducing technology that can be licensed to *heterogeneous* Cournot duopolists, one being more efficient in production—in the sense of operating at lower costs—than the other. In this model, the game tree unfolds exactly as in the typical model described above. The central question we address is whether the licensing of the technology—when optimally carried by the patentee—could reduce the efficiency gap between the Cournot duopolists. It turns out that, under the assumptions maintained in the chapter, the answer to this question is no.

In Chapter 4 we introduce a model of competition between two patentees who engage simultaneously in licensing to perfectly competitive firms. The technologies owned by these inventors are substitutes—i.e., both allow the production of the same good. However, one technology is at least as efficient as the other. What is the impact of competition on a patentee's licensing behavior? Is it possible for the owner of a relatively inefficient technology to survive competition with a stronger rival patentee? In general lines, the answer to the first question is: the introduction of one competitor may lead to a significant increase in the diffusion of a patentee's technology; whereas the answer to the second question is: yes, however, with a small probability.

The chapters outlined above are mutually independent and each is organized in the format of a research paper. In particular, each chapter contains its own introduction—where the topic is properly motivated and the relevant literature is reviewed—and conclusion—where closing remarks and suggestions for future research may be found. Chapter 5 concludes the dissertation with a brief review of the main findings.

Chapter 2

On the Licensing of a Technology with Unknown Use

2.1 Introduction

Empirical evidence suggests that technologies are often patented before a mature stage of development. Pakes (1986), for instance, shows that in France and Germany the average return to holding a patent increases significantly in the first few years of a patent's life. Starting from relatively low levels, the profitability of a patent may increase as new uses for the patented technology are discovered. To illustrate this point, consider the case of sildenafil, popularly known as Viagra. Pfizer filed in 1992 a patent covering its use as a drug to treat cardiovascular diseases.[1] Clinical trials, however, suggested that the drug could be used to treat erectile dysfunction. A patent covering the latter use was filed in 1994.[2] Viagra, as is widely known, has been a commercial success.

In some cases, on the other hand, it may well be that a useful application is never found for a technology that happened to be patented early in its development. Indeed, Boldrin and Levine (2013) observe that the increasing number of patents issued each year in the United States is not being followed by a correspondent productivity growth in the country's economy.[3] One potential explanation for this fact is the uselessness of many patented technologies.

Our goal in this chapter is to model and study the problem of an inventor who holds the patent of a technology with unknown use and has the option of licensing it to interested

[1] The patent was published as U.S. Patent 5,250,534 in October, 1993.

[2] Published in October of 2002 as U.S. Patent 6,469,012.

[3] Precise figures on the number of patents issued yearly in the United States, as well as information on the patents referred to in footnotes 1 and 2, can be found at the United States Patent and Trademark Office's website: www.uspto.gov.

parties. Our focus is on the inventor's decision making as well as on the technology's diffusion among potential licensees. In view of the discussion in the preceding paragraphs, these are both realistic and relevant issues. Indeed, in a recent survey with European firms, *further development of technology through licensee* appeared as a reason given by firms to engage in licensing activity.[4]

To be more specific about the questions we are going to address, let us briefly describe our model and our main results. We assume that an outside inventor holds the patent of a technology that could *potentially* be used to reduce the costs of firms operating in a Cournot industry. By potentially we mean that cost reduction is conditional on a use for the technology being discovered, capturing the concept of *unknown use*.

The interaction between the inventor and the firms is modeled as a game in extensive form that unfolds as follows.[5] The inventor moves first, deciding whether to try or not to discover the use of his technology. The outcome of a trial is random and can be either a success or a failure. In the game's next stage, the inventor announces a number of licenses to be sold in an auction. Firms then decide whether to participate in the auction or not, and, in the case of participation, how much to bid for a license. If licensing follows either a failure or a non-trial by the inventor, it is then each licensee's task to try do discover the use for the technology. The outcome of each of these trials is private information to the corresponding licensee. If licensing follows a successful discovery attempt by the inventor, then no further trials are made, for the use of the technology becomes known to all players. In the game's last stage, Cournot competition among the firms takes place and payoffs are distributed.

In this chapter, we are mainly concerned with two questions. The first regards the inventor's first move in the game and can be phrased as follows: should the inventor try to discover the cost reducing use of his technology before licensing it, or should he license the technology as soon as he is granted the patent, leaving to licensees the task of discovery? The second question concerns the diffusion of the technology. In particular, is the ultimate decision taken by the inventor, identified in the answer to our first question, consistent with the highest diffusion of the technology among potential licensees?

To close our model and answer these questions, we posit the existence of a commonly known prior probability density function over the outcomes of discovery efforts by the players, describing how these outcomes are correlated. We focus on two assumptions about this density. Under the *independent discoveries* (ID) assumption, outcomes are statistically independent across players. In other words, under the ID assumption, the probability that

[4]Interestingly, in the same survey a fraction of respondents pointed *technology not developed enough* as a barrier to patent licensing. See Radauer and Dudenbostel (2013).

[5]A precise description of the game is given in section 2.2.

a discovery trial by a player results in a success is unaltered by the knowledge of other players' outcomes. Alternatively, under the *perfectly correlated discoveries*, or simply *correlated discoveries* (CD) assumption, the density assigns positive probability to only two events: either *each* outcome is a success or *each* outcome is a failure. For instance, under the CD assumption, if a player attempts to discover the use for the technology and fails, then this player assigns a posterior probability of zero to a successful attempt by some other player.

We show that under the ID assumption the inventor should try to discover the use of the technology before licensing it, whereas under the CD assumption the opposite holds. Interestingly, the latter result is true even when there are no costs associated to a discovery trial. Intuitively, in the independent discoveries scenario, a failure by the inventor does not alter the value firms attribute to the technology. Hence, the inventor can only gain by trying to discover the use before licensing: if he succeeds, he will license a technology with a definite and well known use to which the firms will attribute a high value; if he fails, the value firms attribute to the technology will be unchanged. For the correlated discoveries scenario the intuition is not so clear. Indeed, if the inventor succeeds, he again proceeds to license a more valuable technology. However, if he fails, firms' assessment of the probability of a use for the technology being discovered is updated to zero. Consequently, no firm would be willing to pay to become a licensee. The inventor's behavior, in this scenario, is then determined by the interaction of these two forces.

Consider now our second question. Since a licensee does not necessarily become a more efficient firm, we define *technology diffusion* as the expected number of licensees operating with a reduced cost. Under the ID assumption, neither of the alternatives available to the inventor in his first move leads unequivocally to the highest diffusion of the technology. Under the CD assumption, however, *not trying* to discover the use for the technology is not only the alternative chosen by the inventor, but also the one leading to highest diffusion of the technology in the industry.

Our model is close in spirit to the models appearing in Kamien and Tauman (1986), Kamien et al. (1992), and Sen and Tauman (2007), among others, in that it takes the Cournot industry structure in which the potential licensees operate explicitly into account. Kamien (1992) provides a survey of the early literature and a review of the standard patent licensing game. Our model extends the previous literature in that it allows for licensing to take place in an environment where neither the inventor nor the firms are certain about the cost reducing use of the new technology.[6]

[6]Observe that in our model information is symmetric. Many papers have studied patent licensing problems under asymmetric information. Early examples are Gallini and Wright (1990), who focus on the case where the patent holder has private information about the quality of the innovation, and Beggs (1992),

Different from previous studies, we do not consider the problem of choosing among different licensing strategies. Instead, in order to focus on the questions we pose above, we assume that the inventor has exogenously chosen to license his technology by means of an auction, as in Katz and Shapiro (1986). This assumption is justified by the fact that auction licensing usually revenue-dominates other licensing mechanisms.[7] An interesting question, that we do not address in this chapter, is how the revenue from different licensing mechanisms are ranked in our setting. Royalty licensing, for example, seems particularly appealing in the present context, since firms with failed discovery outcomes would not have to make payments to the inventor.

Finally, we notice that in our model, even though the use of the patented technology is, to some degree, unknown, the patent does give the inventor complete monopoly rights over the technology. A recent literature on *probabilistic*, or *weak*, patents considers situations where these rights are uncertain.[8]

The chapter is organized as follows. In the next section we introduce the main elements of our model, a game played by the inventor and the firms. In Section 2.3 we analyze the game. Since we focus on subgame-perfect equilibrium outcomes of the game, we carry our analysis via backward induction. Each of the steps involved in our analysis is considered in a separate subsection. In Section 2.4 we present our concluding remarks.

2.2 The Model

Consider an industry with $n \geq 2$ firms producing a homogeneous good and competing in quantities. To produce quantity q_i, firm i incurs cost $c_i(q_i) = c_H q_i$. The market inverse demand for the homogeneous good is given by $P(q) = \max\{0, a - q\}$.

An outside inventor holds the patent of a technology that could potentially reduce firms' marginal costs to $c_L < c_H$. By *potentially* we mean that any agent with access to the technology, attempting to discover its cost reducing use, will succeed with unconditional probability $\alpha \in (0, 1)$, and will fail with the remaining probability.

In particular, let $\omega_i \in \Omega_i \equiv \{\text{failure} \equiv 0, \text{success} \equiv 1\}$ denote the outcome of agent i's attempt to discover the use, and let $f(\omega_0, \omega_1, \ldots, \omega_n)$ denote the joint probability density

who assumes that the licensee is privately informed about the value of the innovation. Other examples include Macho-Stadler et al. (1996), Schmitz (2002), and Sen (2005b).

[7]See Sen (2005a) for an illuminating discussion on the comparison of revenues from different licensing strategies. Traditionally, the literature focused on the comparison of revenues from auction, fixed-fee, and royalty licensing. Sen and Tauman (2007) consider the combination of upfront fees and royalties. Giebe and Wolfstetter (2008) propose an alternative mechanism that outperforms the standard mechanisms just mentioned.

[8]See, for instance, Lemley and Shapiro (2005), Farrell and Shapiro (2008), and Amir et al. (2013).

function of outcomes by all agents, where "0" denotes the inventor. We then have,

$$\sum_{\omega_{-i} \in \Omega_{-i}} f(\omega_{-i}, \omega_i = 1) = \alpha, \quad \forall i \in I \equiv \{0, 1, \ldots, n\}.$$

We assume that $f(\cdot)$ is common knowledge among inventor and firms.

In principle, the attempt to discover the use for the technology carries some fixed cost F, which we assume to be zero throughout.

We model the interaction between inventor and firms as a game in extensive form, Γ. The inventor moves first, deciding whether to try or not try to discover the use for his technology. We label these moves t and $\sim t$, respectively. If the inventor's choice is t, then nature moves and decides with probabilities α and $1-\alpha$ for the inventor's success or failure. Each of these moves by nature is observed by all players and leads to a corresponding subgame, Γ^{t_s} and Γ^{t_f}, respectively, which we will describe shortly. If the inventor's choice is $\sim t$, then the subgame $\Gamma^{\sim t}$ is played. The structure described thus far is depicted in Figure 2.1.

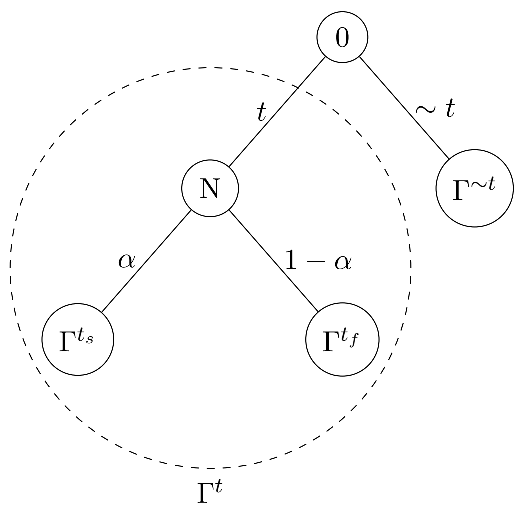

Figure 2.1: The game tree. "0" stands for inventor, "N" for nature. Notice that firms observe whether the inventor has tried or not to discover the use of the technology as well as whether he has succeeded or failed, provided his first move is t.

We next turn to the description of the games Γ^{t_s}, Γ^{t_f}, and $\Gamma^{\sim t}$, which we call the patent licensing subgames of Γ. In the first stage of each of these games the inventor announces an integer number $0 \leq k \leq n$ of licenses to be auctioned together with a minimum acceptable bid, $b \geq 0$. In the second stage, firms simultaneously offer bids in the auction. The k highest bidders win the licenses, paying the respective bids to the

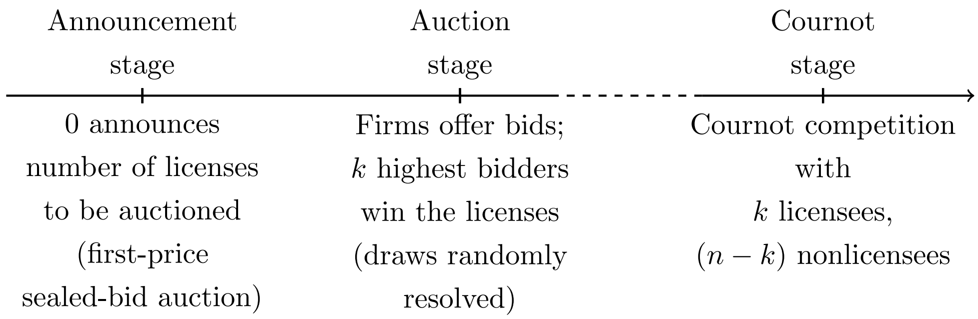

Figure 2.2: Timing in Γ^{t_s}, Γ^{t_f}, and $\Gamma^{\sim t}$. The dashed segment represents the possibility of a chance move, governed by $f(\cdot)$, deciding the outcomes of the licensees' attempts to discover the use for the technology.

inventor (we assume draws are randomly resolved).[9] The set of firms then partitions into the sets of k licensees and $n-k$ nonlicensees.

In the games Γ^{t_f} and $\Gamma^{\sim t}$ nature moves after the auction stage and selects the profile of marginal costs of the licensees, with probabilities derived from the density $f(\cdot)$. Each licensee is then privately informed of its own marginal cost. In the game Γ^{t_s} each licensee is known to operate with marginal cost c_L, since this subgame follows a successful attempt by the inventor.

In the last stage of each of these games, Cournot competition takes place. Figure 2.2 depicts the sequence of events just described.

At any terminal node of the extensive form game Γ, the inventor's payoff is given by the revenue he obtains in the auction. The firms' payoffs are given by their Cournot profits net of bid expenses (if any).[10]

Our focus is on subgame-perfect equilibrium outcomes of Γ. Clearly, these outcomes depend on assumptions about the joint density $f(\cdot)$. We consider two such assumptions.

Under the *independent discoveries* (ID) assumption

$$f(\omega) = \prod_{j=0}^{n} f_j(\omega_j), \tag{2.1}$$

where, for each $i \in I$, $f_i(\omega_i) = \alpha$ iff $\omega_i = 1$.

Alternatively, we consider the *perfectly correlated discoveries* (CD) assumption, ac-

[9] Hence, we restrict the licensing strategies of the inventor to the class of first-price sealed-bid auctions.
[10] If an attempt to discover the use carries a positive cost $F > 0$, then this cost should be subtracted from the relevant players' payoffs.

cording to which

$$f(\omega) = \begin{cases} \alpha, \text{ if } \omega_i = 1 \quad \forall i \in I, \\ 1-\alpha, \text{ if } \omega_i = 0 \quad \forall i \in I, \\ 0, \text{ otherwise.} \end{cases} \tag{2.2}$$

Hence, under the ID assumption, knowledge of the outcome of an attempt to discover the use for the technology by one player does not affect the probability distribution over the outcomes of other players. Alternatively, under the CD assumption, this knowledge implies the outcomes of other players. In particular, the *news* of a failed attempt by the inventor has an extremely negative effect on the firms' assessment of the probability that a use for the technology can be discovered.

To close this section, let us briefly interpret these assumptions. The ID assumption is arguably applicable to environments where the firms' technologies are initially heterogeneous. In other words, environments in which the firms' common marginal cost results from the use of different technologies. The CD assumption, on the other hand, fits in the description of a homogeneous environment, i.e. an environment in which the firms' common marginal cost is derived from the same technology.

2.3 Analysis

Since we focus on SPE outcomes of Γ, we carry our analysis by backward induction. Thus, we start by studying the Cournot subgames that take place in the game's last stage.

2.3.1 The Cournot Stage

Recall that in Γ^{t_s}, i.e., the licensing subgame following a successful trial by the inventor, the technology being licensed is known to reduce the licensees' marginal costs from c_H to c_L. In particular, for any number $k \geq 1$ of licensees, in the Cournot subgame of Γ^{t_s} the profile of marginal costs in the downstream industry is common knowledge among all players.

Differently, in both Γ^{t_f} and $\Gamma^{\sim t}$, the outcome of an attempt by a licensee is the licensee's private information. Hence, the Cournot games played by the firms in both Γ^{t_f} and $\Gamma^{\sim t}$ are with incomplete information. In these contingencies, firms' beliefs about the cost (type) profile of other firms are derived from $f(\cdot)$.

We denote by $\mathcal{C}^{\tau}(k)$ the Cournot game played by the downstream firms in the subgame indexed by $\tau \in \{t_f, t_s, \sim t\} \equiv T$ when there are k licensees.[11] By an equilibrium of

[11] In principle, $\mathcal{C}$ is also a function of the set of licensees. Since we only consider cases with symmetric downstream firms, we abstract from this dependence.

$\mathcal{C}^\tau(k)$ we mean either a Nash or a Bayesian equilibrium, depending on whether $\tau = t_s$ or $\tau \in T\backslash\{t_s\}$, respectively.

Proposition 1. *Suppose $f(\cdot)$ satisfies either (2.1) or (2.2). Then, for any $\tau \in T$ and $k \in \{0, \ldots, n\}$, there exists a unique equilibrium of $\mathcal{C}^\tau(k)$.*

Proof. In Section 2.5.1. □

The following remarks are now in order.

Remark 1. Observe that the ID assumption implies $\mathcal{C}^{t_f}(k) = \mathcal{C}^{\sim t}(k)$, for each integer $0 \leq k \leq n$. To see this, notice that, under ID, a failed attempt to discover the use for the technology by the inventor has no effect on the distribution of firms' outcomes $(\omega_1, \ldots, \omega_n)$. Hence, in both Cournot games firms' beliefs are derived from the same distribution, namely, the marginal distribution obtained from $f(\cdot)$ by summing ω_0 out.

Remark 2. Licensing does not take place in the game Γ^{t_f} under the CD assumption. This is so because each firm knows that attempting to discover the use for the technology will necessarily result in a failure. Therefore, no firm is willing to participate in an auction to buy a license. In this case, $\mathcal{C}^{t_f}(0)$ takes place.

2.3.2 The Auction Stage

For each $\tau \in T$ and integer $0 \leq k \leq n$, we denote by $\pi^\tau_\ell(k)$ and $\pi^\tau(k)$ the expected equilibrium profits of licensees and nonlicensees in $\mathcal{C}^\tau(k)$, as from the beginning of the auction stage. For $\tau = t_s$ these are simply the respective equilibrium profits in $\mathcal{C}^{t_s}(k)$. For $\tau \in T\backslash\{t_s\}$, $\pi^\tau_\ell(k)$ is the α-average between the Cournot equilibrium profit of a licensee who succeeds in discovering the use for the technology, and therefore produces with marginal cost c_L, and the Cournot equilibrium profit of a licensee who fails, and produces with cost c_H.[12]

In the following proposition we characterize the equilibrium bids in the auction stage of Γ^τ.

Proposition 2. *Suppose $f(\cdot)$ satisfies either (2.1) or (2.2). For any $\tau \in T$, suppose the announcement by the inventor is (k, b), with $b = 0$ if $k < n$, and $b = \pi^\tau_\ell(k) - \pi^\tau(k-1)$ if*

[12]More precisely, for $\tau \in T\backslash\{t_s\}$, denote by $\pi^\tau_{\ell,L}(k)$ and $\pi^\tau_{\ell,H}(k)$ the Cournot equilibrium profits of licensees who discover and do not discover the use for the technology, respectively. Then,

$$\pi^\tau_\ell(k) = \alpha\pi^\tau_{\ell,L}(k) + (1-\alpha)\pi^\tau_{\ell,H}(k).$$

$k = n$. Then there exists a unique symmetric equilibrium in the auction stage of Γ^τ. The symmetric equilibrium bid is given by

$$\beta^\tau(k) = \begin{cases} \pi_\ell^\tau(k) - \pi^\tau(k), & \text{if } k < n \\ \pi_\ell^\tau(k) - \pi^\tau(k-1), & \text{if } k = n. \end{cases} \tag{2.3}$$

Moreover, if the announcement (k, b) satisfies $k < n$ and $b = 0$, then at any equilibrium of the auction stage at least $k+1$ firms bid according to (2.3).

Proof. We first argue that, for each $\tau \in T$ and $0 < k \leq n-1$, all firms bidding $\beta^\tau(k)$ constitutes an equilibrium of the auction $(k, 0)$. In fact, if all firms bid $\beta^\tau(k)$, then each firm's payoff is $\pi^\tau(k)$, independent of whether it becomes a licensee or not. If a firm bids above $\beta^\tau(k)$, it certainly wins a license. However, its payoff falls below $\pi^\tau(k)$. If a firm bids below $\beta^\tau(k)$, it certainly loses the auction, obtaining a payoff equal to $\pi^\tau(k)$, its current payoff. Hence, no firm has incentive to deviate from $\beta^\tau(k)$.

To see that $\beta^\tau(k)$ is the unique symmetric equilibrium bid, suppose β is also a symmetric equilibrium of the auction $(k, 0)$. Clearly, $\beta < \beta^\tau(k)$. Under this equilibrium, each firm's expected payoff is some average between $\pi_\ell^\tau(k) - \beta$ and $\pi^\tau(k)$. For some $\delta > 0$, consider the deviation $\beta + \delta$ by i. Its payoff then becomes $\pi_\ell^\tau(k) - \beta - \delta$ with probability one, which is, for some appropriate choice of δ, greater than its current payoff. A contradiction. Thus, $\beta^\tau(k)$ must be the auction's unique symmetric equilibrium.

To see that at any equilibrium of the auction stage, with $k < n$ and $b = 0$, at least $k+1$ firms bid according to (2.3), first observe that in equilibrium no firm will place a bid greater than $\beta^\tau(k)$. Hence, suppose that at some equilibrium firms $i_1, \ldots, i_k$ place bid $\beta^\tau(k)$ and the bids of firms $i_{k+1}, \ldots, i_n$ satisfy $\beta_{i_{k+1}} \geq \cdots \geq \beta_{i_n}$. Any of the winning firms, say firm i_k, could profitably deviate to $\beta^\tau(k) - \delta$, for some $0 < \delta < \beta^\tau(k) - \beta_{i_{k+1}}$, a contradiction. It is not difficult to check that when at least $k+1$ firms place bid $\beta^\tau(k)$, no such deviation is possible. These observations prove the statement.

The case in which the announcement involves $k = n$ can be analyzed with similar reasoning and is therefore omitted. □

By the above proposition, we conclude that in the equilibrium path of Γ every firm's profit is given by $\pi^\tau(k)$, the profit of a nonlicensee in the presence of k licensees. Hence, we have the following standard result.[13]

Corollary 3. *In any equilibrium of Γ, when compared to the pre-innovation environment, all firms are worse-off.*

We next turn to the analysis of the inventor's problem.

[13] See Kamien et al. (1992).

2.3.3 The Inventor's Revenue Maximization Problem

The bid $\beta^\tau(k)$ identified in Proposition 2 is the firms' willingness to pay for a license when k firms are licensees. Paying more than this quantity for a license implies a payoff smaller than $\pi^\tau(k)$ (or $\pi^\tau(k-1)$ in case $k=n$), a payoff a firm can unilaterally guarantee whenever the inventor announces k licenses for auctioning.

Thus, given the announcement (k, b), $k \cdot \beta^\tau(k)$ is the maximum revenue the inventor can achieve in an auction in subgame Γ^τ. Furthermore, Proposition 2 asserts that the inventor can achieve this revenue provided he announces a minimum bid $b = 0$ whenever the announced k is less than n and a minimum bid $\beta^\tau(n)$ whenever $k = n$.[14] Therefore, the inventor's problem in Γ^τ is to

$$\begin{aligned} &\underset{k}{\text{maximize}} \quad k \cdot \beta^\tau(k) \\ &\text{s.t.} \quad k \in \{0, 1, \ldots, n\}, \\ &\quad\ \beta^\tau(k) \text{ given by (2.3).} \end{aligned} \tag{2.4}$$

We denote by v^τ the maximized value of the objective function in the above problem. Hence, $v^t = \alpha v^{t_s} + (1-\alpha) v^{t_f}$ denotes the expected revenue of the inventor when his choice in the first move of Γ is t, i.e. to try to discover the use for his technology.

Proposition 4. *There exists a subgame-perfect equilibrium of* Γ*. Moreover:*

1. *Suppose $f(\cdot)$ satisfies the ID assumption (2.1). Then, $v^{\sim t} \leq v^t$.*
2. *Suppose $f(\cdot)$ satisfies the CD assumption (2.2). Then, $v^{\sim t} \geq v^t$.*

Proof. In Section 2.5.2. □

Thus, the above proposition says that, under ID, choosing $\sim t$ can never outperform the choice of t; that is, not trying to discover the use for the technology in the first move leads to at most the same revenue the inventor obtains when trying to discover the use.

Under the CD assumption, on the other hand, the reverse statement holds. Thus, by not trying to discover the use for the technology, the inventor guarantees a payoff at least equal to the payoff he obtains by choosing t in Γ's first move. Observe that this is the case even when costs associated to an attempt to discover the use for the technology are absent, an assumption we made throughout our analysis.

[14] From now on, for the sake of simplicity, we focus our analysis on cases where the auction announcement by the inventor involves $k < n$. This does not affect our results.

2.3.4 The Diffusion of the Technology

We next turn to the question of the diffusion of the technology. In particular, we investigate which of the alternatives available to the inventor in the first move of Γ leads to the greatest technological diffusion.

Since the use for the technology is unknown at the beginning of the game, we do not measure diffusion by the (expected) number of licensees. Instead, we focus on the expected number of firms operating with low marginal cost, c_L. We provide the proofs for the following results in Section 2.5.3.

Proposition 5. *Suppose $f(\cdot)$ satisfies the ID assumption (2.1), and denote by $k^{\sim t}$ the solution to (2.4) when $\tau =\sim t$. Then, there exists $k_\alpha > 0$ such that the expected number of firms operating with marginal cost c_L is greater in Γ^t than in $\Gamma^{\sim t}$ if, and only if, $k_\alpha \geq k^{\sim t}$.*

The threshold k_α in the above proposition is such that when $k \geq k_\alpha$ downstream firms become licensees the nonlicensees are driven out of the industry. For $\alpha = 1$, Kamien and Tauman (1986) and Kamien et al. (1992) have proved that the optimal number of licensees chosen by the inventor never exceeds k_1. In the present case, $\alpha < 1$, the number of licensees may exceed k_α, but never exceeds $2k_\alpha + 1$, the minimal number of licensees that would drive unsuccessful licensees out of the industry. See the appendix, in particular the proof of Proposition 1 in Section 2.5.1, for details.

Proposition 6. *Suppose $f(\cdot)$ satisfies the CD assumption (2.2). Then, the expected number of firms operating with marginal cost c_L is greater in $\Gamma^{\sim t}$ than in Γ^t.*

The intuition for Proposition 6 is as follows. After choosing t, i.e., after attempting to discover the use for the technology, the inventor does not sell any licenses some of the time, for with probability $1 - \alpha$ the technology is proven to be of no use to the firms. In addition, when licensing does take place, the inventor has the incentive to restrict the number of licensees—see Section 2.5—in order to maximize revenue. Hence, following the choice of t, diffusion is seen to be low. Following the choice of $\sim t$, on the other hand, the inventor has no incentive to restrict the number of licensees, since firms place a "small" value on the technology. It is, therefore, clear that diffusion should be greater in $\Gamma^{\sim t}$, as stated in the proposition.

Taken together, propositions 5 and 6 say that often the inventor decides, in Γ's first move, for the alternative associated with the highest diffusion of the technology.

2.4 Conclusion

In this chapter we studied the problem facing an inventor who holds the patent of a technology that can potentially reduce the costs of firms operating in a given industry.

We focused on two questions, namely, whether the inventor should attempt or not to discover the use of his technology before licensing it and the impact of this decision on the diffusion of the technology among potential licensees. We showed that to answer these questions one has to consider how a failed attempt by the inventor is interpreted by the potential licensees. In particular, if a failure has a very negative effect on the probability firms attribute to a subsequent discovery, then the inventor should not attempt to discover the use, even when the attempt is costless. Moreover, the inventor's decision on whether to try or not is often in line with the alternative leading to the highest diffusion of the technology.

Some interesting questions for future investigation are: (i) whether the above results extend to environments with more general demands, (ii) whether the availability of different licensing mechanisms changes the above findings, and (iii) whether decisions of an inside inventor are consistent with those of an outside inventor.

2.5 Omitted Proofs

In the appendix we present all the proofs omitted in the main text.

2.5.1 Proof of Proposition 1

For each $\tau \in T$ and $k \in \{1, \dots, n\}$, we prove existence by calculating explicitly a symmetric equilibrium of $\mathcal{C}^{\tau}(k)$, under both ID and CD assumptions. Uniqueness is then easily established.

The equilibrium of $\mathcal{C}^{t_s}(k)$

Consider first $\tau = t_s$. The subgame Γ^{t_s} follows a successful attempt by the inventor to discover the use for the technology.[15] Therefore, for each k, $\mathcal{C}^{t_s}(k)$ is a Cournot game with complete information, in which k firms produce with marginal cost c_L and the remaining firms produce with marginal cost c_H. Hence, its equilibrium does not depend on assumptions about $f(\cdot)$. It can be easily shown that nonlicensees produce

$$q^{t_s}(k) = \varepsilon_1 \cdot \begin{cases} \frac{k_1 - k}{n+1}, \text{ if } k < k_1 \\ 0, \text{ if } k_1 \leq k, \end{cases}$$

at the unique equilibrium, whereas licensees produce

$$q_{\ell}^{t_s}(k) = \varepsilon_1 \cdot \begin{cases} \frac{k_1 - k}{n+1} + 1, \text{ if } k < k_1 \\ \frac{k_1 + 1}{k+1}, \text{ if } k_1 \leq k, \end{cases}$$

[15]The subgame Γ^{t_s} is, in fact, the license auction game analyzed in Kamien (1992).

where $k_1 = (a - c_H)/(c_H - c_L)$, is the minimum number of licensees in Γ^{t_s} that drive nonlicensees out of the industry.

For future reference, we observe that equilibrium Cournot profits, $\pi^{t_s}(k)$ and $\pi_\ell^{t_s}(k)$, are given by the square of the above quantities.

The equilibrium of $\mathcal{C}^\tau(k)$, $\tau \in \{t_f, \sim t\}$, under independent discoveries

Next, suppose $f(\cdot)$ satisfies the independent discoveries assumption (2.1). By Remark 1, $\mathcal{C}^{t_f}(k) = \mathcal{C}^{\sim t}(k)$. Thus, it is sufficient to prove the statement for either case $\tau \in \{t_f, \sim t\}$.

Recall that, for each k, $\mathcal{C}^\tau(k)$ is a Cournot game with incomplete information, described as follows. Each firm's type space consists of c_L and c_H if it is a licensee, and only c_H if it is a nonlicensee. Suppose firms $i_1, \ldots, i_k$ are the licensees. Nature moves first and selects with probability

$$f_{i_1,\ldots,i_k}(\omega_{i_1}, \ldots, \omega_{i_k}) = \sum_{\{\omega_{i_{k+1}},\ldots,\omega_{i_n}\}} f(\omega_1, \ldots, \omega_n)$$

the profile of marginal costs of licensees corresponding to outcomes $\omega_{i_1}, \ldots, \omega_{i_k}$ of their attempts to discover the use for the technology. Each licensee is then privately informed of its own marginal cost.

Given the independent discoveries assumption, conditional on its own marginal cost, the probability firm i assigns to a profile of other firms' marginal costs having j entries equal to c_L is given by $\alpha^j(1-\alpha)^{\tilde{k}-j}$, where $\tilde{k} = k$ if i is a nonlicensee and $\tilde{k} = k - 1$ if i is a licensee. In particular, given i's marginal cost, i's belief that exactly j licensees have succeeded in finding the use for the technology is given by

$$f_{i_1,\ldots,i_k}(\omega_{i_1}, \ldots, \omega_{i_k} \mid \omega_i) = \binom{\tilde{k}}{j} \alpha^j (1-\alpha)^{\tilde{k}-j}.$$

Strategies and payoffs are defined in an obvious manner and this structure is common knowledge.

We denote by $q^\tau(k)$ the Bayesian equilibrium quantity produced by nonlicensees in $\mathcal{C}^\tau(k)$. Similarly, we denote by $q_{\ell,H}^\tau(k)$ and $q_{\ell,L}^\tau(k)$ the equilibrium quantities produced by the high and low cost types, respectively, of each licensee. We next turn to the computation of these quantities.

Consider first the problem facing a nonlicensee. It has marginal cost c_H and therefore solves

$$\max_{\tilde{q} \geq 0} \quad \left[a - \sum_{j=0}^{k} \binom{k}{j} \alpha^j (1-\alpha)^{k-j} \big(j q_L + (k-j) q_H\big) - (n-k-1)q - \tilde{q} - c_H\right] \tilde{q}$$

where, for brevity, we adopt the simplified notation $q = q^\tau(k)$, $q_H = q^\tau_{\ell,H}(k)$, and $q_L = q^\tau_{\ell,L}(k)$. Assuming interior solution, one can easily derive the first order condition

$$a - c_H - (n-k+1)q - kq_H = (q_L - q_H)\sum_{j=0}^{k}\binom{k}{j}\alpha^j(1-\alpha)^{k-j}j. \tag{2.5}$$

Type c_H of each licensee firm solves

$$\max_{\tilde{q}\geq 0}\quad \Big[a - \sum_{j=0}^{k-1}\binom{k-1}{j}\alpha^j(1-\alpha)^{k-1-j}\big(jq_L + (k-1-j)q_H\big) - (n-k)q - \tilde{q} - c_H\Big]\tilde{q},$$

Again assuming interior solution, the first order condition can be written as

$$a - c_H - (n-k)q - (k+1)q_H = (q_L - q_H)\sum_{j=0}^{k-1}\binom{k-1}{j}\alpha^j(1-\alpha)^{k-1-j}j. \tag{2.6}$$

Finally, type c_L of each licensee firm solves

$$\max_{\tilde{q}\geq 0}\quad \Big[a - \sum_{j=0}^{k-1}\binom{k-1}{j}\alpha^j(1-\alpha)^{k-1-j}\big(jq_L + (k-1-j)q_H\big) - (n-k)q - \tilde{q} - c_L\Big]\tilde{q},$$

leading to the (interior) first order condition

$$a - c_L - (n-k)q - (k-1)q_H - 2q_L = (q_L - q_H)\sum_{j=0}^{k-1}\binom{k-1}{j}\alpha^j(1-\alpha)^{k-1-j}j. \tag{2.7}$$

Recall the elementary fact that

$$\sum_{j=0}^{k}\binom{k}{j}\alpha^j(1-\alpha)^{k-j}j = \alpha k. \tag{2.8}$$

Using (2.8), from (2.5) and (2.6), it easily follows that

$$q_H = q + \frac{\alpha\Delta c}{2}, \tag{2.9}$$

where $\Delta c = c_H - c_L$.

Additionally, from (2.6) and (2.7) we obtain

$$q_L = q_H + \frac{\Delta c}{2}. \tag{2.10}$$

Inserting these relations into (2.5), we obtain

$$q^\tau(k) = \frac{a - c_H - k\alpha\Delta c}{n+1}.$$

Making $\varepsilon_\alpha = \alpha \Delta c$ and $k_\alpha = (a - c_H)/\varepsilon_\alpha$, $q^\tau(k)$ can then be written as

$$q^\tau(k) = \frac{\varepsilon_\alpha(k_\alpha - k)}{n+1},$$

for $k < k_\alpha$ and zero otherwise.

Substituting for $q = q^\tau(k)$ in (2.6), we obtain

$$q^\tau_{\ell,H}(k) = \frac{\varepsilon_\alpha(k_\alpha - k)}{n+1} + \frac{\varepsilon_\alpha}{2},$$

provided $0 < q^\tau(k)$. Substituting for $q^\tau(k) = 0$ in (2.6), we get

$$q^\tau_{\ell,H}(k) = \frac{\varepsilon_\alpha(2k_\alpha + 1 - k)}{2(k+1)},$$

for $k_\alpha \le k < 2k_\alpha + 1$ and zero otherwise.

Substituting for $q = q^\tau(k)$ and $q_H = q^\tau_{\ell,H}(k)$ in (2.10), we obtain

$$q^\tau_{\ell,L}(k) = \frac{\varepsilon_\alpha(k_\alpha - k)}{n+1} + \frac{(1+\alpha)\varepsilon_\alpha}{2\alpha},$$

for $k < k_\alpha$,

$$q^\tau_{\ell,L}(k) = \frac{\varepsilon_\alpha(2k_\alpha + 1 - k)}{2(k+1)} + \frac{\varepsilon_\alpha}{2\alpha},$$

for $k_\alpha \le k < 2k_\alpha + 1$, and

$$q^\tau_{\ell,L}(k) = \frac{\varepsilon_\alpha(k_\alpha + 1/\alpha)}{2 + \alpha(k-1)},$$

for $2k_\alpha + 1 \le k$.

Summarizing, the equilibrium we just computed is given by

$$q^\tau(k) = \varepsilon_\alpha \cdot \begin{cases} \frac{k_\alpha - k}{n+1}, \text{ if } k < k_\alpha \\ 0, \text{ if } k_\alpha \le k, \end{cases}$$

$$q^\tau_{\ell,H}(k) = \varepsilon_\alpha \cdot \begin{cases} \frac{k_\alpha - k}{n+1} + \frac{1}{2}, \text{ if } k < k_\alpha \\ \frac{2k_\alpha + 1 - k}{2(k+1)}, \text{ if } k_\alpha \le k < 2k_\alpha + 1 \\ 0, \text{ if } 2k_\alpha + 1 \le k, \end{cases}$$

and

$$q^\tau_{\ell,L}(k) = \varepsilon_\alpha \cdot \begin{cases} \frac{k_\alpha - k}{n+1} + \frac{1+\alpha}{2\alpha}, \text{ if } k < k_\alpha \\ \frac{2k_\alpha + 1 - k}{2(k+1)} + \frac{1}{2\alpha}, \text{ if } k_\alpha \le k < 2k_\alpha + 1 \\ \frac{k_\alpha + 1/\alpha}{2 + \alpha(k-1)}, \text{ if } 2k_\alpha + 1 \le k. \end{cases}$$

Clearly, equilibrium is unique. As in the case for $\mathcal{C}^{t_s}(k)$, equilibrium profits in $\mathcal{C}^\tau(k)$, $\pi^\tau(k)$, $\pi^\tau_{\ell,H}(k)$, and $\pi^\tau_{\ell,L}(k)$, are also given by the square of the corresponding equilibrium quantities. This completes the proof of the proposition for the independent discoveries case.

The equilibrium of $\mathcal{C}^{\tau}(k)$, $\tau \in \{t_f, \sim t\}$, under correlated discoveries

Let us next suppose that $f(\cdot)$ satisfies the correlated discoveries assumption (2.2). As observed in Remark 2, no licensing takes place in subgame Γ^{t_f} under correlated discoveries. Hence, for given $k > 0$, it is sufficient to prove the statement of the proposition for $\tau =\sim t$.

As in the ID case, $\mathcal{C}^{\sim t}(k)$ is again a Cournot game with incomplete information. Firms' type spaces is as before, and nature moves first selecting a profile of outcomes (types) for the licensees, according to the marginal $f_{i_1,\ldots,i_k}(\cdot)$, obtained from $f(\cdot)$ by "summing out" the outcomes $\omega_{i_{k+1}}, \ldots, \omega_{i_n}$ of nonlicensees.

Once again, every firm is informed of nonlicensees' marginal costs, c_H. However, different from the ID case, licensees are also informed of each others' costs, since $f_{i_1,\ldots,i_k}(\cdot)$ assigns probability α to the event that all licensees succeed (and, therefore, are of type c_L), and probability $1-\alpha$ to the event that all licensees fail (and, therefore, are of type c_H). Thus, conditional on own marginal cost being $c \in \{c_L, c_H\}$, each licensee attributes probability one to the event that all remaining licensees have marginal cost equal to c.

Of course, nonlicensees attribute probability α, respectively $1-\alpha$, to the event that all licensees have marginal cost c_L, respectively c_H. This structure is common knowledge among the firms.

We notice that computations here are similar to those carried for the ID case.

Again we start with the problem of nonlicensees. Each of these firms has marginal cost c_H and solves

$$\max_{\tilde{q} \geq 0} \quad \big[a - k(\alpha q_L + (1-\alpha)q_H) - (n-k-1)q - \tilde{q} - c_H\big]\tilde{q}$$

where we adopted the notation introduced in the proof for the ID case.

Assuming interior solution, the first order condition for the above problem is given by

$$a - k(\alpha q_L + (1-\alpha)q_H) - (n-k+1)q - c_H = 0. \tag{2.11}$$

Let $\theta \in \{H, L\}$. Type c_θ of each licensee then solves

$$\max_{\tilde{q} \geq 0} \quad \big[a - (k-1)q_\theta - (n-k)q - \tilde{q} - c_\theta\big]\tilde{q}.$$

The first order condition for an interior solution is

$$a - (k+1)q_\theta - (n-k)q - c_\theta = 0. \tag{2.12}$$

These equations then imply

$$q_L = q_H + \frac{\Delta c}{k+1}. \tag{2.13}$$

Now, equations (2.11), (2.12) for $\theta = H$, and (2.13) give

$$q_H = q + \alpha \Delta c \frac{k}{k+1}. \tag{2.14}$$

Using these relations in (2.12), with $\theta = L$, lead to $q = q^{\sim t}(k)$ (and hence $q_H = q^{\sim t}_{\ell,H}(k)$ and $q_L = q^{\sim t}_{\ell,L}(k)$) for the case $k < k_\alpha$.

For the case $k_\alpha \leq k$, we observe that, since nonlicensees are driven out of the industry, the Cournot competition is one of complete information among homogeneous firms. Therefore, type $c\theta$ firms will produce $(a - c_\theta)/(k+1)$.

Hence, the equilibrium is given by

$$q^{\sim t}(k) = \varepsilon_\alpha \cdot \begin{cases} \frac{k_\alpha - k}{n+1}, \text{ if } k < k_\alpha \\ 0, \text{ if } k_\alpha \leq k, \end{cases}$$

$$q^{\sim t}_{\ell,H}(k) = \varepsilon_\alpha \cdot \begin{cases} \frac{k_\alpha - k}{n+1} + \frac{k}{k+1}, \text{ if } k < k_\alpha \\ \frac{\varepsilon_\alpha k_\alpha}{k+1}, \text{ if } k_\alpha \leq k, \end{cases}$$

and

$$q^{\sim t}_{\ell,L}(k) = \varepsilon_\alpha \cdot \begin{cases} \frac{k_\alpha - k}{n+1} + \frac{k+1/\alpha}{k+1}, \text{ if } k < k_\alpha \\ \frac{k_\alpha + 1/\alpha}{k+1}, \text{ if } k_\alpha \leq k. \end{cases}$$

Moreover, as before, equilibrium profits are given by the square of the equilibrium quantities.

To conclude the proof of Proposition 1, we observe that whenever $k = 0$, that is whenever licensing does not take place, we have a simple symmetric Cournot game, which has, of course, a unique equilibrium. ■

2.5.2 Proof of Proposition 4

Existence of an SPE of Γ follows from backward induction and the fact that the inventor's problem (2.4) always has a solution.

Proof of part 1

To prove this part of the proposition, first observe that, in view of Remark 1, under the ID assumption, we have $v^{t_f} = v^{\sim t}$. Thus, from the definition of v^t, it follows that $v^t = \alpha v^{t_s} + (1-\alpha) v^{\sim t}$. It is then sufficient to show that the value of the inventor's problem in Γ^{t_s} is greater than or equal to the value of the inventor's problem in $\Gamma^{\sim t}$, that is,

$$v^{t_s} \geq v^{\sim t}. \tag{2.15}$$

Recall the definition of k_α and the notation k^τ for the solution of the inventor's problem. From Proposition 2 and the quantities and profits computed in the proof of Proposition 1, it follows that

$$\beta^{t_s}(k) = \varepsilon_1^2 \cdot \begin{cases} \frac{2(k_1 - k)}{n+1} + 1, \text{ if } 1 \leq k < k_1 \\ \left(\frac{k_1+1}{k+1}\right)^2, \text{ if } k_1 \leq k. \end{cases} \tag{2.16}$$

and

$$\beta^{\sim t}(k) = \varepsilon_\alpha^2 \cdot \begin{cases} \frac{2(k_\alpha - k)}{n+1} + \frac{1+3\alpha}{4\alpha}, \text{ if } k < k_\alpha \\ \left(\frac{k_\alpha+1}{k+1}\right)^2 + \frac{1-\alpha}{4\alpha}, \text{ if } k_\alpha \le k < 2k_\alpha + 1 \\ \alpha\left(\frac{k_\alpha+1/\alpha}{2+\alpha(k-1)}\right)^2, \text{ if } 2k_\alpha + 1 \le k \le n, \end{cases} \tag{2.17}$$

To prove (2.15), we consider three cases.

Case 1 ($k^{\sim t} < k_\alpha$). Since $k^{\sim t} < k_\alpha$, we have $\alpha k^{\sim t} < k_1$. The formulas for $\beta^{t_s}(\cdot)$ and $\beta^{\sim t}(\cdot)$, in the appropriate intervals, give

$$\begin{aligned} v^{t_s} &\ge \alpha k^{\sim t} \cdot \beta^{t_s}(\alpha k^{\sim t}) \\ &= \alpha k^{\sim t} \varepsilon_1^2 \left(\frac{2(k_1 - \alpha k^{\sim t})}{n+1} + 1\right) \\ &\ge \alpha k^{\sim t} \varepsilon_1^2 \left(\frac{2(\alpha k_\alpha - \alpha k^{\sim t})}{n+1} + \frac{1+3\alpha}{4}\right) \\ &= k^{\sim t} \varepsilon_\alpha^2 \left(\frac{2(k_\alpha - k^{\sim t})}{n+1} + \frac{1+3\alpha}{4\alpha}\right) \\ &= k^{\sim t} \cdot \beta^{\sim t}(k^{\sim t}) \\ &= v^{\sim t}, \end{aligned}$$

where the first and second inequalities follow from the optimality of k^{t_s} and the fact that $1 + 3\alpha < 4$.

Case 2 ($k_\alpha \le k^{\sim t} < 2k_\alpha + 1$). Since $k_\alpha \le k^{\sim t}$, we have $k_1 \le \alpha k^{\sim t}$. Since $k^{\sim t} < 2k_\alpha + 1$, we have $1/4 < [(k_\alpha+1)/(k^{\sim t}+1)]^2$. Moreover, $[(k_\alpha+1)/(k^{\sim t}+1)]^2 \le [(k_\alpha+1/\alpha)/(k^{\sim t}+1/\alpha)]^2$. The formulas for $\beta^{t_s}(\cdot)$ and $\beta^{\sim t}(\cdot)$, in the appropriate intervals, now give

$$\begin{aligned} v^{t_s} &\ge \alpha k^{\sim t} \cdot \beta^{t_s}(\alpha k^{\sim t}) \\ &= \alpha k^{\sim t} \varepsilon_1^2 \left(\frac{k_1 + 1}{\alpha k^{\sim t} + 1}\right)^2 \\ &= \alpha k^{\sim t} \varepsilon_1^2 \left(\frac{k_\alpha + 1/\alpha}{k^{\sim t} + 1/\alpha}\right)^2 \\ &\ge \alpha k^{\sim t} \varepsilon_1^2 \left(\frac{k_\alpha + 1}{k^{\sim t} + 1}\right)^2 \\ &\ge \alpha k^{\sim t} \varepsilon_1^2 \left(\alpha\left(\frac{k_\alpha + 1}{k^{\sim t} + 1}\right)^2 + \frac{1-\alpha}{4}\right) \\ &= k^{\sim t} \varepsilon_\alpha^2 \left(\left(\frac{k_\alpha + 1}{k^{\sim t} + 1}\right)^2 + \frac{1-\alpha}{4\alpha}\right) \\ &= k^{\sim t} \cdot \beta^{\sim t}(k^{\sim t}) \\ &= v^{\sim t}. \end{aligned}$$

Case 3 ($2k_\alpha + 1 \le k^{\sim t}$). Clearly, $k_1 < \alpha k^{\sim t}$. Therefore,

$$\begin{aligned} v^{t_s} &\ge \alpha k^{\sim t} \cdot \beta^{t_s}(\alpha k^{\sim t}) \\ &= \alpha k^{\sim t} \varepsilon_1^2 \left(\frac{k_1 + 1}{\alpha k^{\sim t} + 1} \right)^2 \\ &\ge \alpha k^{\sim t} \varepsilon_1^2 \left(\frac{k_1 + 1}{2 + \alpha(k^{\sim t} - 1)} \right)^2 \\ &= \alpha k^{\sim t} \varepsilon_1^2 \left(\frac{\alpha k_\alpha + 1}{2 + \alpha(k^{\sim t} - 1)} \right)^2 \\ &= \alpha k^{\sim t} \varepsilon_\alpha^2 \left(\frac{k_\alpha + 1/\alpha}{2 + \alpha(k^{\sim t} - 1)} \right)^2 \\ &= k^{\sim t} \cdot \beta^{\sim t}(k^{\sim t}) \\ &= v^{\sim t}. \end{aligned}$$

With case 3 we complete the proof of 1.

Proof of part 2

Let us next prove part 2 of the proposition. First, recall that, by Remark 2, no licensing occurs in Γ^{t_f} under the CD assumption. Hence, we have to show that

$$v^{\sim t} \ge \alpha v^{t_s}. \tag{2.18}$$

Secondly, by Proposition 2 and the computations in the proof of Proposition 1, we have

$$\beta^{\sim t}(k) = \varepsilon_\alpha^2 \cdot \begin{cases} \frac{2(k_\alpha - k)}{n+1} + 1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{k+1}\right)^2, & \text{if } k < k_\alpha \\ \left(\frac{k_\alpha + 1}{k+1}\right)^2 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{k+1}\right)^2, & \text{if } k_\alpha \le k. \end{cases} \tag{2.19}$$

Now, since the subgame Γ^{t_s} does not depend on assumptions about $f(\cdot)$, $\beta^{t_s}(\cdot)$ is given by (2.16).

Finally, observe that $k \cdot \beta^{t_s}(k)$ is decreasing over $k \ge k_1$. Hence, it must be $k^{t_s} \le k_1$, which, in turn, holds iff $k^{t_s}/\alpha \le k_\alpha$.

Thus,

$$\begin{aligned} v^{\sim t} &\ge (k^{t_s}/\alpha) \cdot \beta^{\sim t}(k^{t_s}/\alpha) \\ &= (k^{t_s}/\alpha)\varepsilon_\alpha^2 \left(\frac{2(k_\alpha - k^{t_s}/\alpha)}{n+1} + 1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{k^{t_s}/\alpha + 1}\right)^2 \right) \\ &\ge (k^{t_s}/\alpha)\varepsilon_\alpha^2 \left(\frac{2(k_\alpha - k^{t_s}/\alpha)}{n+1} + 1 \right) \end{aligned}$$

$$
\begin{aligned}
&= \alpha k^{t_s} \varepsilon_1^2 \left(\frac{2(k_\alpha - k^{t_s}/\alpha)}{n+1} + 1 \right) \\
&= \alpha k^{t_s} \varepsilon_1^2 \left(\frac{2(k_1 - k^{t_s})/\alpha}{n+1} + 1 \right) \\
&\geq \alpha k^{t_s} \varepsilon_1^2 \left(\frac{2(k_1 - k^{t_s})}{n+1} + 1 \right) \\
&= \alpha k^{t_s} \cdot \beta^{t_s}(k^{t_s}) \\
&= \alpha v^{t_s},
\end{aligned}
$$

proving (2.18) and concluding the proof of Proposition 4. ■

2.5.3 Proofs of propositions 5 and 6

We present the proof of each of these propositions in a separate subsection.

Proof of Proposition 5

Suppose $f(\cdot)$ satisfies the ID assumption (2.1).

We first prove necessity. Assume that $k_\alpha < k^{\sim t}$. In equilibrium, the expected number of licensees operating with marginal cost c_L in the subgame $\Gamma^{\sim t}$ is given by

$$
\sum_{j=0}^{k^{\sim t}} \binom{k^{\sim t}}{j} \alpha^j (1-\alpha)^{k^{\sim t}-j} j = \alpha k^{\sim t},
$$

since, in $\Gamma^{\sim t}$, the attempts by the licensees to discover the use for the technology consist of $k^{\sim t}$ i.i.d. Bernoulli trials, with parameter α.

Additionally, it follows from Remark 1 that Γ^{t_f} and $\Gamma^{\sim t}$, under ID, have exactly the same outcome. Hence, the expected number of licensees operating with marginal cost c_L in the subgame Γ^t is given by

$$
\alpha k^{t_s} + (1-\alpha)(\alpha k^{t_f}) = \alpha k^{t_s} + (1-\alpha)(\alpha k^{\sim t}).
$$

Therefore, we have to show that

$$
\alpha \left(k^{t_s} - \alpha k^{\sim t} \right) \leq 0. \tag{2.20}
$$

But, as observed in the text, $k^{t_s} \leq k_1$. Moreover, $k_\alpha \leq k^{\sim t} \Leftrightarrow k_1 \leq \alpha k^{\sim t}$. The inequality (2.20) then follows.

We next prove sufficiency. Assume that $k_\alpha \geq k^{\sim t}$. We need to show that the reverse of (2.20) holds. Observe that, since $k_\alpha \geq k^{\sim t}$, we have

$$
k^{\sim t} = \min \left\{ k_\alpha, \frac{k_\alpha}{2} + \left(\frac{1+3\alpha}{4\alpha} \right) \frac{n+1}{4} \right\}.
$$

But,

$$k^{t_s} = \min\left\{k_1, \frac{k_1}{2} + \frac{n+1}{4}\right\}.$$

Hence, $\alpha k^{\sim t} \leq k^{t_s}$ and the reverse of inequality (2.20) obtains. ■

Proof of Proposition 6

Suppose $f(\cdot)$ satisfies the CD assumption (2.2).

We begin by noticing that, since no licensing takes place in Γ^{t_f}, $k^{t_f} = 0$. Thus, the expected number of licensees operating with marginal cost c_L in the subgame Γ^t is simply αk^{t_s}.

Now, the expected number of licensees operating with marginal cost c_L in the subgame $\Gamma^{\sim t}$ is $\alpha k^{\sim t}$. To see this, observe that, under CD, either all licensees succeed to discover the use for the technology, event that happens with probability α, or all licensees fail, event that happens with probability $1-\alpha$.

Thus, it is sufficient to show that $k^{t_s} \leq k^{\sim t}$. We consider two cases.

Case 1 ($k^{t_s} = k_1/2 + (n+1)/4$). For all $k \leq k^{t_s}$, we have

$$\begin{aligned}
k \cdot \beta^{\sim t}(k) &= k\varepsilon_\alpha^2 \left(\frac{2(k_\alpha - k)}{n+1} + 1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{k+1}\right)^2\right) \\
&= k\varepsilon_\alpha^2 \left(\frac{2(k_1 - k)}{n+1} + 1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{2k_1}{n+1} + \left(\frac{1}{k+1}\right)^2\right)\right) \\
&= k\varepsilon_\alpha^2 \left(\frac{2(k_1 - k)}{n+1} + 1\right) + k\varepsilon_\alpha^2 \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{2k_1}{n+1} + \left(\frac{1}{k+1}\right)^2\right) \\
&\leq k^{t_s}\varepsilon_\alpha^2 \left(\frac{2(k_1 - k^{t_s})}{n+1} + 1\right) + k^{t_s}\varepsilon_\alpha^2 \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{2k_1}{n+1} + \left(\frac{1}{k^{t_s}+1}\right)^2\right) \\
&\leq k^{\sim t} \cdot \beta^{\sim t}(k^{\sim t}) \\
&= v^{\sim t},
\end{aligned}$$

where the first inequality follows from the optimality of k^{t_s} and the fact that the second term in the sum is increasing over $k \leq k^{t_s}$. Hence, it must be $k^{t_s} \leq k^{\sim t}$.

Case 2 ($k^{t_s} = k_1$). Suppose $k^{\sim t} \leq k^{t_s} < k_\alpha$. Then, $k^{\sim t}$ must satisfy the first order condition

$$\beta^{\sim t}(k) + k \cdot \frac{\mathrm{d}\beta^{\sim t}}{\mathrm{d}k}(k) = 0.$$

Hence, at $k = k^{\sim t}$, it must be

$$\frac{2(k_\alpha - k^{\sim t})}{n+1} + 1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{k^{\sim t}+1}\right)^2 = k^{\sim t}\left(\frac{2}{n+1} + \left(\frac{1-\alpha}{\alpha}\right)\frac{2}{(k^{\sim t}+1)^3}\right).$$

It then follows that

$$\begin{aligned}
v^{\sim t} &= k^{\sim t} \cdot \beta^{\sim t}(k^{\sim t}) \\
&= k^{\sim t}\varepsilon_\alpha^2 \left(\frac{2(k_\alpha - k^{\sim t})}{n+1} + 1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{k^{\sim t}+1}\right)^2 \right) \\
&= k^{\sim t}\varepsilon_\alpha^2 \left(\frac{2k^{\sim t}}{n+1} + \left(\frac{1-\alpha}{\alpha}\right)\frac{2k^{\sim t}}{(k^{\sim t}+1)^3} \right) \\
&\leq k^{\sim t}\varepsilon_\alpha^2 \left(1 + \frac{1-\alpha}{\alpha}\right) \\
&= (k^{\sim t}/\alpha)\varepsilon_\alpha^2 \\
&\leq k_\alpha \varepsilon_\alpha^2 \\
&< k_\alpha \varepsilon_\alpha^2 \left(1 + \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{k_\alpha+1}\right)^2\right) \\
&= k_\alpha \cdot \beta^{\sim t}(k_\alpha),
\end{aligned}$$

where the first inequality follows from the facts that $k^{t_s} = k_1$, and thus $k_1 \leq (n+1)/2$, and $2k < (k+1)^3$. Therefore, the optimality of $k^{\sim t}$ is contradicted and we must have $k^{t_s} < k_\alpha \leq k^{\sim t}$. ■

Chapter 3

Patent Licensing and Technological Catch-up in a Heterogeneous Duopoly

3.1 Introduction

Starting with the seminal contributions of Kamien and Tauman (1984), Kamien and Tauman (1986), and Katz and Shapiro (1986), the theoretical literature on the licensing of cost-reducing technologies has flourished in the last decades. The models usually consider the interaction between an inventor, who holds the patent of a cost-reducing technology, and potential licensees, that can use the technology in the production process. With few exceptions, most papers assume that the potential licensees are *homogeneous* firms operating in some given industry.[1]

Hence, little is known theoretically about patent licensing in environments with *heterogeneous* potential licensees. In this chapter, we consider a patent licensing model in which potential licensees are heterogeneous Cournot duopolists, i.e., firms that are subject to different constant marginal costs and compete through quantities. The inventor is an outsider to the industry and holds the patent of a technology that reduces the marginal cost of firms adopting it by the same *additive* amount. The interaction between the inventor and the firms is modeled as a game in extensive form in the spirit of the auction licensing game introduced by Katz and Shapiro (1986) and reviewed by Kamien (1992).

We show that this game has no subgame-perfect equilibrium outcome in which the (*ex ante*) least efficient firm becomes the sole licensee. Therefore, unless the inefficient firm engages in R&D activity, technological catch-up in this industry is impossible, that

[1] One such exception is Stamatopoulos and Tauman (2009). See Remark 4 in Section 4.5.

is, in equilibrium either the efficient firm becomes relatively more efficient or the cost gap between the firms is unchanged. This result illustrates the interesting conclusions that may be obtained with the introduction of asymmetric potential licensees in patent licensing models and points towards the need for further research in this area.

The chapter is organized as follows. In the next section, we describe the patent licensing model and state the result referred to above. In Section 3.3 we prove the result. Section 4.5 contains the concluding remarks.

3.2 The Model

Consider a Cournot duopoly consisting of firms 1 and 2. Each firm $i \in \{1, 2\}$ produces output with a constant marginal cost technology. We denote by $c_i > 0$ the marginal cost of firm i, and we assume that $c_1 < c_2$. Firms face demand $D(p)$, with corresponding inverse demand given by $P(q) = \max\{0, \widehat{P}(q)\}$, where $\widehat{P}(\cdot)$ is a strictly decreasing, twice differentiable log-concave function and $\lim_{q\to\infty} P(q) = 0$.[2]

An outside inventor holds the patent of a technology that reduces firms' costs by ε, that is, if firm i adopts the inventor's technology, then its marginal cost becomes $c_i - \varepsilon > 0$. The inventor's objective is to maximize licensing revenues by means of an auction (k, b), where k is the number of licenses for sale in the auction and b is the minimum acceptable bid.

Let us now describe the patent licensing game Γ, involving the inventor and the two firms. The inventor moves first, announcing an auction policy (k, b). Firms then decide whether to participate in the auction or not, and, if the decision is to participate, how much to bid. Firms offering the k highest bids win the auction—provided these bids are greater than b—and a winner pays to the inventor its own bid. After the auction, Cournot competition among the firms takes place. In this stage, firm i produces with cost $c_i - \varepsilon$ if it is a licensee, and with cost c_i if it is a nonlicensee.

The payoff to the inventor is given by the revenue he obtains in the auction, which is simply the sum of winning bids. A licensee's payoff is given by its Cournot profit net of its payment to the inventor; a nonlicensee's payoff is given by its Cournot profit.

We say that *technological catch-up through licensing is possible* if there exists a subgame-perfect equilibrium (SPE) of Γ in which the least efficient firm, namely, firm 2, becomes the sole licensee. If no such an equilibrium exists, we say that technological catch-up through licensing is impossible.[3]

[2]These assumptions relax those usually made in the patent licensing literature. See, for example, Kamien et al. (1992).

[3]Observe that if either firm 1 becomes the sole licensee or both firms become licensees, then the relative

We can now state our main result. Its proof is presented in the next section.

Proposition. *Technological catch-up through licensing is impossible.*

3.3 Proof of the Proposition

We show that if an SPE outcome of Γ has a sole licensee, then this licensee must be firm 1.

Suppose firm i is the sole licensee in the Cournot stage of Γ. Under our assumptions, this Cournot subgame has a unique equilibrium.

Let $q_j(\{i\})$, for each $i, j \in \{1, 2\}$, denote the Cournot equilibrium quantity produced by firm j when firm i is the licensee.[4] Similarly, denote by $\pi_j(\{i\})$ j's equilibrium profit when i is the licensee. The following is a useful fact.

Lemma 1. *Let $p(\{i\})$ denote the Cournot equilibrium price when firm i is the sole licensee. Suppose that in this equilibrium $q_j(\{i\}) > 0$. Then, $p(\{i\})$ is the unique solution to*

$$p \cdot [1 - 1/2\eta(p)] = (c_1 + c_2 - \varepsilon)/2, \tag{3.1}$$

where $\eta(p) = -D'(p) \cdot (p/D(p))$ is the price elasticity of demand.

Proof. Equation (3.1) is obtained by adding both firms' first order conditions (for interior solution), rearranging and using the definition of $\eta(p)$. The assumptions on $P(\cdot)$ imply that $\eta(p)$ is increasing.[5] Hence, the LHS of (3.1) is increasing in p, and the unique solution to this equation must be $p(\{i\})$. $\square$

Now, given the announcement (k, b) with $k = 1$, firm i's *willingness to pay for a license* is given by

$$w_i^1 = \pi_i(\{i\}) - \pi_i(\{j\}).$$

inefficiency of firm 2, as measured by the difference between its marginal cost and that of firm 1, either increases or remains the same. Thus, in neither of these cases, firm 2 is able to catch-up with firm 1, in the sense of reducing the gap between its cost and that of firm 1.

[4] Obviously, we may have $i = j$. The curly braces in our notation stress the fact that equilibrium quantities depend on the *set* of licensees.

[5] To see this, observe first that log-concavity of the inverse demand implies that $P'(q)/P(q)$ is decreasing in q, and, therefore, increasing in p. But

$$\frac{P'(q)}{P(q)} = -\frac{1}{\eta(p)D(p)}.$$

Taking derivative w.r.t. p gives

$$\frac{1}{\eta(p)D(p)} \left(\frac{\eta'(p)}{\eta(p)} + \frac{D'(p)}{D(p)} \right) \geq 0 \iff \frac{\eta'(p)}{\eta(p)} \geq -\frac{D'(p)}{D(p)} > 0.$$

Thus, $\eta'(p) > 0$.

In words, i's willingness to pay for a license, when only one license is put for sale in the auction announced by the inventor, is the difference between i's profit as the sole licensee and i's profit as a nonlicensee when its rival j is a licensee.

Lemma 2. $w_1^1 > w_2^1$.

Proof. There are three cases to consider. These cases regard the effect of the adoption of the technology by a single firm on the industry structure. We consider each of these cases in turn.

Case 1 (*Both firms become a monopoly*). Let π_i^m denote firm i's monopoly profit when its marginal cost is $c_i - \varepsilon$. Then, for each $i \in \{1, 2\}$, we have $w_i^1 = \pi_i^m$. The result follows by observing that $\pi_1^m > \pi_2^m$.

Case 2 (*Firm* 1 *becomes a monopoly*). We have $w_1^1 = \pi_1^m - \pi_1(\{2\})$ and $w_2^1 = \pi_2(\{2\})$. Thus,

$$\begin{aligned}
w_1^1 - w_2^1 &= \pi_1^m - \pi_1(\{2\}) - \pi_2(\{2\}) \\
&= \pi_1^m - (p(\{2\}) - c_1)\, q_1(\{2\}) - (p(\{2\}) - c_2 + \varepsilon)\, q_2(\{2\}) \\
&> \pi_1^m - (p(\{2\}) - c_1 + \varepsilon)\, q_1(\{2\}) - (p(\{2\}) - c_2 + \varepsilon)\, q_2(\{2\}) \\
&= \pi_1^m - (p(\{2\}) - c_1 + \varepsilon) \cdot (D(p(\{2\})) - q_2(\{2\})) - (p(\{2\}) - c_2 + \varepsilon)\, q_2(\{2\}) \\
&= \pi_1^m - (p(\{2\}) - c_1 + \varepsilon) \cdot D(p(\{2\})) + \Delta c \cdot q_2(\{2\}) \\
&> 0,
\end{aligned}$$

where $\Delta c = c_2 - c_1$. The first inequality follows from the fact that $\varepsilon > 0$; the third equality follows from the fact that $q_1(\{2\}) + q_2(\{2\}) = D(p\{2\})$ in equilibrium; and the last inequality follows from the fact that $\pi_1^m \geq (p(\{2\}) - c_1 + \varepsilon) \cdot D(p(\{2\}))$.

Case 3 (*Neither firm becomes a monopoly*). Lemma 1 implies that, in the present case, $p(\{1\}) = p(\{2\})$. Using this fact, and noticing that it, in turn, implies that the aggregate output q satisfies $q(\{1\}) = q(\{2\})$, we have, for each $i \in \{1, 2\}$,

$$\begin{aligned}
w_i^1 &= (p(\{i\}) - c_i + \varepsilon)\, q_i(\{i\}) - (p(\{j\}) - c_i)\, q_i(\{j\}) \\
&= \varepsilon q_i(\{i\}) + (p(\{i\}) - c_i) \cdot (q_i(\{i\}) - q_i(\{j\})) \\
&= \varepsilon q_i(\{i\}) + (p(\{i\}) - c_i) \cdot \left(-\frac{(p(\{i\}) - c_i + \varepsilon)}{P'(q(\{i\}))} + \frac{p(\{j\}) - c_i}{P'(q(\{j\}))} \right) \\
&= \varepsilon \cdot (q_i(\{i\}) + q_i(\{j\}))\,,
\end{aligned}$$

where the third equality follows from firm i's first order condition for profit maximization.

Hence,

$$w_1^1 - w_2^1 = \varepsilon \cdot (q_1(\{1\}) - q_2(\{2\}) + q_1(\{2\}) - q_2(\{1\}))$$

$$= -\frac{2\varepsilon\Delta c}{P'(q\{i\})}$$
$$> 0.$$

We observe that the case in which only firm 2, as a single licensee, becomes a monopoly is not possible. Therefore, the three cases above exhaust the possibilities and the proof of the lemma is complete. □

It follows from the above lemma that whenever the inventor announces an auction policy (k, b) with $k = 1$ and $b = 0$, firm 1 wins the auction by offering a bid slightly above w_2^1. If $k = 1$ and $b > 0$, then either firm 2 is driven to not participate in the auction or it is overbid by firm 1. These observations conclude the proof of the Proposition. ■

3.4 Concluding Remarks

Remark 1 (*Alternative licensing mechanisms*). The Proposition also holds for alternative licensing mechanisms, namely, royalties, fixed fees and two-part tariffs.[6] Indeed, under a royalty policy both firms become licensees, whereas under either fixed fee or two-part tariff policies an argument in the same lines as the one provided in the previous section holds, that is, one can show that firm 1 has higher willingness to pay for a license than firm 2. ♦

Remark 2 (*Downstream Bertrand competition*). Replacing downstream Cournot competition with Bertrand competition does not alter the Proposition. In fact, with Bertrand competition, firm 1 is the sole licensee in equilibrium and the inventor obtains a payoff equal to the difference between the post-invention and pre-invention profits of firm 1. ♦

Remark 3 (*General industry size*). A similar result holds for industry sizes $n > 2$, with, say, $c_1 \leq c_2 \leq \cdots \leq c_n$. In this case, for any auction (k, b) announced by the inventor, one can show that the k highest bids must come from firms $1, \ldots, k$. ♦

Remark 4 (*Firm-specific cost reduction*). Clearly, the result depends on the assumption that adoption of the technology leads to the same *additive* cost reduction for each firm. Stamatopoulos and Tauman (2009) have studied the case in which the technology can be adopted only by firm 2, i.e., the least efficient firm, and have provided conditions under which this firm becomes a licensee. Allowing cost reduction to be firm-specific is a natural extension of the traditional patent licensing model that can lead to many interesting research questions. ♦

[6] With homogeneous firms, Kamien and Tauman (1986) have studied the royalty and fixed fee mechanisms; Sen and Tauman (2007), among others, have studied two-part tariffs.

Remark 5 (*Abstracting from downstream Cournot competition*). Let us briefly describe a patent licensing model that abstracts from the assumption that firms are Cournot competitiors.[7] Suppose that adoption of the technology by firms in the set $S \subseteq \{1,2\}$ results in a profit equal to $\pi_i(S)$ to firm $i \in \{1,2\}$. For each firm i we assume that $\pi_i(\cdot)$ satisfies

$$\pi_i(\{i\}) > \pi_i(\{1,2\}) > \pi_i(\varnothing) > \pi_i(\{j\}),$$

i.e., becoming a licensee is always profitable for firm i; being the sole licensee is the best outcome for i; and not being a licensee, when its rival j is a licensee, is the worst outcome for i.

In this framework, firm 1 is ex ante *stronger than* firm 2 if $\pi_1(\varnothing) > \pi_2(\varnothing)$.[8] Clearly, this condition says nothing about the possibility of firm 2 becoming the sole licensee. In fact, if

$$\pi_1(\{1\}) - \pi_1(\{2\}) > \pi_2(\{2\}) - \pi_2(\{1\}), \tag{3.2}$$

then firm 2 will never become the sole licensee. However, we cannot conclude from this that technological catch-up is impossible. Indeed, at this level of generality, we could have $\pi_2(\{1,2\}) > \pi_1(\{1,2\})$, so that firm 2 is stronger than firm 1 when both firms adopt the technology.

We observe that this model provides an alternative environment to study the firm-specific cost reduction case, suggested in Remark 4. ⧫

[7] Katz and Shapiro (1986) have considered such a model with homogeneous potential licensees.

[8] This, of course, corresponds to firm 1 producing with the smallest marginal cost in the Cournot formulation above.

Chapter 4

Patent Licensing with Asymmetric Competing Inventors

4.1 Introduction

A patent confers to its holder property rights over a technological innovation. As a monopolist, the patentee can commercially exploit the patent and, thus, collect returns to the innovative effort. A common commercial strategy adopted by patentees is licensing—whereby, in exchange for a payment, or flow of payments, the licensee is granted by the patentee the right to use the patented technology.

Following Kamien and Tauman (1986) and Katz and Shapiro (1986), the literature on patent licensing has taken very seriously the fact that the patentee enjoys monopoly power.[1] In addition to correctly assuming that the patentee is a monopolist, these studies assume that no close substitutes for the new technology are available—except, of course, existing "old" technologies. Obviously, under these assumptions, the patentee is free of any competitive pressures when taking decisions concerning the licensing of its technology.

In this chapter, we consider a model with two inventors. Each of these inventors holds the patent of a technology that may be used by firms operating in a perfectly competitive industry to reduce production costs. Both inventors seek to maximize licensing profits, given by the difference between licensing revenues and licensing costs. In our interpretation, licensing costs arise from the training—transfer of know-how—provided by the patentee to each of its licensees, without which a licensee is unable to adopt the technology.

Formally, we study a game in extensive-form that unfolds as follows. In the first stage

[1] Papers considering the problem of *weak patents* may be regarded as exceptions. See, for instance, Farrell and Shapiro (2008). See also Ayres and Klemperer (1999).

both inventors move independently and simultaneously, each announcing a commitment consisting of a promise to sell no more than a given number licenses, all at a given price, paid up-front as a fixed licensing fee. In the next stage, firms decide whether, and from which inventor, to buy a license. In the game's last stage, firms compete through the choice of quantities.

Under our assumptions, absent competition from a rival patentee, each inventor would license a single firm. What is the impact of competition on inventors' licensing behavior? We show that the diffusion of each technology may be substantially larger than that achieved in the absence of a rival patentee—indeed, if both technologies are equally efficient and training costs are null, then the technologies are fully diffused, i.e. "all" firms become licensees.[2]

A related question is whether an inventor owing a relatively inefficient technology would survive competition with a stronger rival patentee. Our main result, Proposition 1, answers this question. Assuming that training costs are sufficiently small, the probability that licensees of both inventors earn positive profits is greater than zero. However, as training costs become smaller, the probability that licensees of the inefficient inventor are driven out of the market converges to one.

In his survey, Kamien (1992) asserted that *licensing of competing inventions* was an *obvious* topic for future investigation. It is therefore striking that in more than twenty years very little research has been done in this front. To the best of our knowledge, the only study considering strategic competition between patentees is Arora and Fosfuri (2003). Different from our model, these authors focus on (i) inventors who are insiders to the competitive—downstream—industry, (ii) technologies that reduce marginal production costs to zero and potentially allow for production of differentiated goods, and (iii) exogenously given shares of licensing-generated profits. All in all, our model should be seen as neither an extension nor a sub-case, but rather as a complement of these authors' work.

4.2 The Model

Consider a perfectly competitive industry with firms producing a good for which the market inverse demand is given by $P(q) = \max\{0, a - q\}$ and the marginal cost of production is constant, equal to c.

There are two outside inventors, I_1 and I_2. I_j, $j = 1, 2$, holds the patent of a technology that reduces c to c_j. We assume that $c_1 \leq c_2$ and sometimes refer to I_1 as the efficient inventor—similarly, we may refer to I_2 as the inefficient inventor. Let p_j^M denote the monopoly price under I_j's technology. Throughout the chapter we maintain the assump-

[2] See Example 2.

tion that for each j, I_j's technology is *drastic* with respect to the technology currently adopted by the firms.

Assumption 1. $p_j^M \leq c$.

Adoption of I_j's technology entails a cost $f \geq 0$ incurred by the inventor. We interpret $f > 0$ as an unavoidable fixed *cost of training* to be provided by the inventor to each firm adopting its technology.

Adoption of I_j's technology is the result of a licensing agreement between this inventor and a firm. We model the interaction between the inventors and the (countably infinite many) firms as a game in extensive form, Γ^f, which we describe next.

The game Γ^f begins with both inventors moving independently and simultaneously, I_j announcing a pair (k_j, φ_j), interpreted as a commitment by I_j to sell no more than k_j licenses, each at price φ_j, paid up-front as a fixed licensing fee. In the next stage, firms decide whether, and from which inventor, to buy a license.[3] In the game's last stage, firms compete through the choice of quantities. I_j's licensees produce with cost c_j; by Assumption 1, if at least one license is sold by either inventor, nonlicensees are driven out of the industry.

At any terminal node of Γ^f payoffs are defined in an obvious manner. Let x_j denote the number of licenses sold by I_j. I_j's payoff is then given by $x_j \cdot (\varphi_j - f)$. Firms adopting I_j's technology receive payoff $\pi_j(x_1, x_2) - \varphi_j$, where $\pi_j(x_1, x_2)$ is the Cournot profit of such a firm when x_i, $i = 1, 2$, firms are licensed by I_i. All other firms obtain zero payoff.

To analyze Γ^f we focus on subgame-perfect equilibria. In the following remark we introduce some additional notation.

Remark 1. In the case $c_1 < c_2$, straightforward computation shows that $\pi_2(x_1, x_2) = 0$ whenever $x_1 \geq \bar{k} \equiv (a - c_2)/(c_2 - c_1)$. Moreover, it is easily verified that $\bar{k}$ is the smallest number with this property. Henceforth, we assume that $\bar{k} > 1$. Therefore, we do not consider the somewhat trivial case in which I_1's technology is drastically more efficient than that of I_2. ⧫

4.3 Analysis and Main Result

Under our assumptions, (Cournot) equilibrium in Γ^f's final stage is unique. Moreover, given first stage announcements, $(k_j, \varphi_j)_{j=1,2}$, a moment's reflection reveals that, in equilibrium, it must be $x_j = k_j$ and $\varphi_j = \pi_j(k_1, k_2)$. Indeed, given (k_i, φ_i), if $\pi_j(k_j, k_i) - \varphi_j < 0$,

[3] We assume that, if more than k_j firms decide to buy an I_j's license, then I_j allocates the k_j licenses randomly.

then, assuming I_i sells k_i licenses, I_j can increase its payoff by setting φ_j equal to either $\pi_j(x_j, k_i)$ or $\pi_j(k_j, k_i)$.

The observations above allow us to analyze the interaction between the two inventors as a game in normal form, G^f. In particular, in G^f, inventors simultaneously choose $k_j \in \mathbb{Z}_+ \equiv \{0, 1, 2, \dots\}$ to maximize

$$k_j \cdot (\pi_j(k_1, k_2) - f)\,.$$

Our main result focuses on equilibria of G^f when $f > 0$ is *sufficiently small*, i.e. when the relative ranking of I_j' s pure strategies, according to I_j's objective function, is the same as when $f = 0$, except when the rival inventor's strategy k_i is such that $\pi_j(1, k_i) = 0$, in which case $0 \succ_{I_j} 1 \succ_{I_j} 2 \succ_{I_j} \cdots$.[4]

Proposition 1. *Assume $c_1 < c_2$.*

1. *Suppose $f > 0$ is sufficiently small. There exists an equilibrium of G^f. Moreover, any equilibrium of this game is in mixed strategies.*
2. *Let $\sigma_1^f(\bar{k})$ be the probability with which I_1 selects $\bar{k}$ in equilibrium. For any $\varepsilon > 0$, there exists a sufficiently small $f_\varepsilon > 0$ such that $\sigma_1^f(\bar{k}) \geq 1 - \varepsilon$ whenever $f < f_\varepsilon$.*

The first assertion in part 1 of the proposition is not obvious, since each inventor can choose from an infinite set of pure strategies. Assertion 2 says that the probability that I_2's licensees will be driven out of the industry approaches 1 as f becomes smaller. However, the probability that both inventors' technologies coexist in the industry is always positive. If $c_1 = c_2$, then it can be shown that the unique equilibrium of the game is in pure strategies. Furthermore, a threshold for f, above which the inventors do not sell licenses, can be explicitly calculated.[5]

Proof of Proposition 1. It can be readily verified that

$$\pi_1(k_1, k_2) = \begin{cases} \left(\frac{a + k_2\Delta c - c_1}{k_1 + k_2 + 1}\right)^2, \text{ if } k_1 < \bar{k} \\ \left(\frac{a - c_1}{k_1 + 1}\right)^2, \text{ if } k_1 \geq \bar{k}, \end{cases} \tag{4.1}$$

and

$$\pi_2(k_1, k_2) = \begin{cases} \left(\frac{a - k_1\Delta c - c_2}{k_1 + k_2 + 1}\right)^2, \text{ if } k_1 < \bar{k} \\ 0, \text{ if } k_1 \geq \bar{k}, \end{cases} \tag{4.2}$$

where $\Delta c = c_2 - c_1$.

[4] In particular, the exception applies only to I_2, and this is so whenever $k_1 \geq \bar{k}$.

[5] See example 3 below.

Let $\beta_j^f(k_i)$ denote I_j's best-response to I_i's pure strategy k_i. Assuming $f > 0$ is sufficiently small and using (4.1) and (4.2) we obtain

$$\beta_1^f(k_2) = \begin{cases} k_2 + 1, \text{ if } k_2 < \bar{k} - 1 \\ \bar{k}, \text{ if } k_2 \geq \bar{k} - 1, \end{cases} \tag{4.3}$$

and

$$\beta_2^f(k_1) = \begin{cases} k_1 + 1, \text{ if } k_1 < \bar{k} \\ 0, \text{ if } k_1 \geq \bar{k}. \end{cases} \tag{4.4}$$

Inspection of these best-response functions shows that G^f has no equilibrium in pure strategies. Hence, if an equilibrium exists, it must be in mixed strategies.

To prove existence, first observe that $k_2 > \bar{k}$ is a strictly dominated strategy for I_2, and can, therefore, be eliminated from I_2's set of pure strategies without altering the set of equilibria of G^f. Thus, iterative elimination of strictly dominated strategies allows us to write I_j's set of pure strategies as $\{0, 1, \ldots, \bar{k}\}$. (In fact, $k_1 = 0$ can also be eliminated from I_1's set of pure strategies.) Therefore, w.o.l.g. we may view G^f as a finite game. Hence, Nash's theorem applies and the proof of 1 in the Proposition is complete.

Let us now prove part 2 of the Proposition. Consider the game G^0. It can be verified that $\beta_1^0(k_2)$ is given by (4.3) whereas

$$\beta_2^0(k_1) = \begin{cases} k_1 + 1, \text{ if } k_1 < \bar{k} \\ \mathbb{Z}_+, \text{ if } k_1 \geq \bar{k}. \end{cases}$$

Thus, any equilibrium of G^0, $\sigma^0 = (\sigma_1^0, \sigma_2^0)$, is such that I_1's strategy places probability 1 on $\bar{k}$ and I_2 plays any $k_2 \geq \bar{k} - 1$.

Now, take a sequence $\{f_n\}$, $f_n \to 0$. By upper hemicontinuity of the Nash equilibrium correspondence, it must be that the convergent sequence $\{\sigma^{f_n}\}$ converges to some σ^0. Since $\sigma_1^0(\bar{k}) = 1$, the proof is complete. □

4.4 Examples

Let us consider a few examples.

Example 1. Suppose that $a = 3$, $c_1 = 0$, and $c_2 = 1$. Hence, $\bar{k} = 2$. Moreover, for $k_1 \leq 2$, the Cournot profits are then given by

$$\pi_1(k_1, k_2) = \left(\frac{3 + k_2}{k_1 + k_2 + 1} \right)^2,$$

and

$$\pi_2(k_1, k_2) = \left(\frac{3 - k_1 - 1}{k_1 + k_2 + 1}\right)^2.$$

Thus, to analyze the interaction between the efficient and inefficient inventors, one can simply study the following 2×3 bimatrix game.

		I_2: 0	1	2
I_1	1	$\frac{9}{4} - f, 0$	$\frac{16}{9} - f, \frac{1}{9} - f$	$\frac{25}{16} - f, \frac{2}{16} - 2f$
	2	$2 - 2f, 0$	$2 - 2f, -f$	$2 - 2f, -2f$

It can be checked that, at the *unique* equilibrium of the above game, the efficient inventor—row player—mixes between both of its strategies, with $\sigma_1^f(2) = 1 - 9f$, whereas the inefficient inventor places positive probabilities only on $k_2 = 0$ and $k_2 = 1$. Indeed, it is possible to show that there is always an equilibrium of G^f in which I_1 mixes between $\bar{k} - 1$ and $\bar{k}$, and I_2 mixes between $\bar{k} - 2$ and $\bar{k} - 1$. We were unable to prove, however, that this equilibrium is unique. ⧫

Example 2 (Symmetric inventors facing zero training cost). Suppose that $c_1 = c_2$ and $f = 0$. Thus, I_j chooses k_j to maximize $k_j \cdot \pi_j(k_1, k_2)$, where $\pi_j(k_1, k_2) = [(a - c_1)/(k_1 + k_2 + 1)]^2$. A simple computation then yields

$$\beta_j^0(k_i) = k_i + 1, \quad j \neq i, \quad j = 1, 2.$$

Hence, the argument used in the proof of Proposition 1 does not apply. Specifically, one cannot eliminate any of each inventor's pure strategies. Consequently, G^0 cannot be reduced to a finite game and equilibrium existence is not guaranteed. Indeed, equilibrium does not exist in the present example, as a quick inspection of the above best-responses reveals. However, one interpretation of this fact is that, in this scenario, both inventors' technologies are fully diffused. ⧫

Example 3 (Symmetric inventors facing positive training cost). Suppose that $c_1 = c_2$ and $0 < f \leq (a - c)^2$. At the unique equilibrium of G^f, we have $k_1^* = k_2^* = k^*$, with

$$k^* = \frac{1}{2}\left(\left(\frac{(a - c)^2}{f}\right)^{\frac{1}{3}} - 1\right).$$

Clearly, technology diffusion is decreasing in f. Interestingly, one can show that inventors' equilibrium payoffs are increasing in f. Intuitively, the direct (negative) effect of an increase in f on the cost of licensing faced by each inventor is more than compensated by the indirect (positive) effect of a decrease in the rival's number of licensees. ⧫

4.5 Concluding Remarks

We conclude with a couple of remarks.

Remark 2 (What if inventors choose only $(\varphi_j)_{j=1,2}$ in Γ^f's first stage? (I)). Changing the definition of Γ^f, so that in its first stage inventors simply choose $(\varphi_j)_{j=1,2}$, changes the analysis above. In particular, we can no longer conclude that $\varphi_j = \pi_j(k_1, k_2)$ in equilibrium. To see this, note that, in equilibrium, we must have

$$\pi_j(k_1, k_2) - \varphi_j \geq 0, \quad j = 1, 2, \tag{4.5}$$

and

$$\pi_j(k_j, k_i) - \varphi_j \geq \pi_i(k_i + 1, k_j - 1) - \varphi_i, \quad j \neq i, \quad j = 1, 2. \tag{4.6}$$

Inequalities (4.5) state that a licensee cannot profitably deviate to become a nonlicensee; inequalities (4.6) state that a licensee cannot profitably deviate to become a licensee of the rival inventor—acknowledging that this deviation increases by one the number of I_i's licensees and decreases by one the number of I_j's licensees.

Now, suppose (4.5) binds for both inventors. Inequality (4.6) for $j = 1$ then reads $\pi_2(k_1 - 1, k_2 + 1) - \pi_2(k_1, k_2) \leq 0$, which clearly cannot hold, since the profit of a firm must go up when this firm is confronted by a higher proportion of inefficient competitors—and, consequently, a smaller proportion of efficient competitors. ♦

Remark 3 (What if inventors choose only $(\varphi_j)_{j=1,2}$ in Γ^f's first stage? (II)). If Γ^f is altered so that (i) in its first stage inventors choose $(\varphi_j)_{j=1,2}$ and (ii) in the second stage firms *sequentially* decide whether and from whom to buy a license, then the conclusion of Remark 2 above does not hold. In this formulation of the game, a firm's strategy depends on the history of decisions by firms moving previously to itself. Thus, a firm considering a deviation from a decision to buy a license from I_j cannot assume that the number of I_j's licensees will decrease by one, since a succeeding firm's strategy may call for *buy from* I_j if fewer than k_j firms have obtained a license before its turn to move. In that case, inequality (4.6) would read

$$\pi_j(k_j, k_i) - \varphi_j \geq \pi_i(k_i + 1, k_j) - \varphi_i, \quad j \neq i, \quad j = 1, 2,$$

which is not in contradiction with $\varphi_j = \pi_j(k_1, k_2)$. ♦

Chapter 5

Conclusion

This dissertation contains three contributions to the theoretical literature on patent licensing. In Chapter 2 we dealt with the licensing of a technology with unknown use. Many technologies are patented in association with a specific use—in fact, "being useful" is a requirement a technology has to fulfill in order to be granted a patent. This, however, does not exclude the possibility of alternative uses for these technologies being discovered *after* the respective patents are granted. Indeed, some evidence suggests this to be many times the case. The main question we addressed in the chapter is whether an inventor in possession of such a technology should invest in discovering an alternative use to it. Strikingly, the inventor should not invest under some conditions, even if investment costs are null. The chapter also presents results relating the diffusion of such a technology to the inventor's investment decisions.

In the third chapter we investigated the possibility of technological catch-up in a heterogeneous duopoly as a result of patent licensing. The main conclusion is that this is in general not possible, since there is no scenario in which the inefficient duopolist becomes the sole licensee. Of course, the result may not hold if firms are subject to general, as opposed to constant marginal cost, technologies. Indeed, licensing to firms facing nonlinear cost functions is an unexplored possibility worth of future research.

Finally, in Chapter 4 we developed and studied a model of patent licensing with competing inventors. Traditionally, researchers have modeled *competing inventors* as participants in *patent races*, whereby the winner—the inventor to first obtain an innovation—becomes a sole patentee. However, it is not uncommon to find examples of different (patented) technologies that achieve the same ends and are, therefore, substitutes. The main finding of Chapter 4 is that competition between patentees may significantly increase the diffusion of their technologies. The result is remarkable, since in the chapter we only considered a duopoly of patentees.

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