Dividend Taxes, Financial Frictions, and the U.S. Great Depression

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Abstract of the Dissertation

Dividend Taxes, Financial Frictions, and the U.S. Great Depression

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As shown by McGrattan (2012), an anticipated increase in dividend taxes plays an important role in explaining the dramatic investment decline during the U.S. Great Depression. The first chapter of this dissertation attempts to test whether this conclusion is still robust when household heterogeneity and precautionary saving motives are taken into consideration. I build an Aiyagari model with dynamic firms, dividend taxes, and labor productivity shocks that accounts for the U.S. earnings and wealth inequality using 1930s data. The conclusion is that the impact of anticipated increases of dividend taxes on the investment is very sensitive to the presence of household heterogeneity and precautionary savings motives. The predicted investment in the heterogeneous agent model is 50% smaller than in the homogeneous agent model proposed by McGrattan (2012). The decline in output and working hours accordingly becomes much less significant. Because, although the anticipated hike in dividend taxes diminishes the expected return to the investment, it reduces the value of total assets that households hold for self-insurance against the highly persistent idiosyncratic shocks. In order to retain the desired asset level, households hesitate to reduce their saving motives even at a lower capital return rate, and therefore a lower aggregate investment decline is generated.

The second chapter explores the role of working capital during the U.S. Great Depression. My hypothesis is that the scarcity of working capital contributed to the propagation of the financial distress to the real sector. When the financial crisis reduced the availability of short-term financing, an important source of working capital, firms experienced a squeeze in their disposable cash flow. As they could not adjust their financial structure promptly to ease the liquidity tension, firms were ultimately forced to cut labor inputs and production, generating a large contraction. My main quantitative experiment shows that this mechanism explains almost 50% of the working hours decline in the early 1930s, and it also predicts considerable declines in consumption and output. Moreover, when the financial meltdown

is unexpected or when the average term to maturity of long-term debt is lengthened, the mechanism described above gets aggravated. Finally, when dividend taxes increase, I find that investment does not collapse, in contrast to literature. This finding implies that the impact of capital return decline, which is caused by an anticipated dividend tax increase, could be potentially offset by financing frictions.

The final chapter studies the relationship between corporate bond default risk and economic downturn during the U.S. Great Depression, and propose that the default risk is an effective amplifier of adverse technology and financial shocks. On the one hand, the massive wave of corporate bond defaults directly idled a considerable amount of capital, which was detrimental to production, investment, and employment; On the other hand, the indebted firms were inclined to cut more investment when default risks are looming, as they were also concerned about losing ownership of firms and experiencing the costly default process. Based on the prominent work by Cooley and Quadrini (2001) and Miao and Wang (2010), I build a rational expectation DSGE model with firm default option, which generates simulated investment dynamics that are much closer to the actual 1930s data series than in the standard RBC model. The model also predicts satisfactory declines in consumption, working hours and output. Moreover, I find that the default recovery rate decline caused by adverse financial shocks explains well the increasing corporate bond yield in the early 1930s.

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Chapter 1

Dividend Taxes, Household Heterogeneity, and the U.S. Great Depression

1.1 Introduction

Recent studies question the traditional opinion [1] that fiscal policies contribute little to the dramatic economic downturn and subsequent slow recovery during the US Great Depression. McGrattan (2012) claims that the anticipated increases in capital taxes such as the dividend tax and undistributed profit tax are responsible for the investment collapse during the 1930s. This paper studies the impact of fiscal policies during the Great Depression once again, but in an incomplete market framework where households face an idiosyncratic and persistent labor income shocks. Therefore they are ex post heterogeneous in income and wealth. With all parameters calibrated to the US economy in 1929, including economic aggregates and distribution of income and wealth, the main quantitative implication of this model is that the influences of anticipated increases in dividend tax and undistributed profit tax are very

[1] For example,Cole and Ohanian (1999)

sensitive to the presence of household heterogeneity and precautionary saving motives. The anticipated increase of dividend tax and undistributed profit tax fails to bring the economy down, especially the decline of investment, as they do in the representative agent framework. The predicted decrease in investment is 50% smaller. The decreases in output and working hours are much less significant also. The reason is that although the higher dividend tax rate lowers the return of holding assets, it shrinks the total value of household savings for self-insurance at the same time. Consequently households are reluctant to reduce their asses very much, and thus a moderate decline in investment occurs instead.

There is no consensus yet about the cause of the Great Depression, although lots of researchers cover it from different perspectives. It is surprising that very few pay attention to the fiscal policy aspect regardless of the large and frequent changes in taxes and government spending of that period: The annual data from *National Income and Product Accounts*, NIPA hereafter, shows that the share of government spending in the total GDP doubled from 1929 to 1939; The absolute level of personal income tax and corporate income tax skyrocketed[2], as plenty of literature also comply with the above facts[3]. Figure 1.1 summarizes the trends of effective rates of different tax and government spending during the US Great Depression according to the relevant literatures. Thanks to the efforts to retrieve tax rates and government spending, the recent studies on the impacts of fiscal policies during Great Depression becomes feasible. Cole and Ohanian (1999) constructs a growth model with labor income tax, capital income tax and government spending. They show that the change in fiscal factors only explains 4% of the decline in outputs so they conclude that fiscal policies play little role. McGrattan (2012) challenges this conclusion by considering more types of taxes, i.e, consumption tax τ_c, property tax τ_k, dividend tax τ_d, and undistributed profit tax

[2] *Tax Foundation* provides the historical rates in personal income tax and corporate income tax. See http://www.taxfoundation.org/taxdata/show/151.html.

[3] Joines (1981) estimates the average marginal tax rates on corporate profit and labor income respectively; McGrattan (2012) constructs dividend and undistributed profit tax rates from the *Statistic of Income* and sale and property tax rates from NIPA, and also modifies corporate profit tax rate in Joines (1981) by combining the impact of undistributed profit tax.

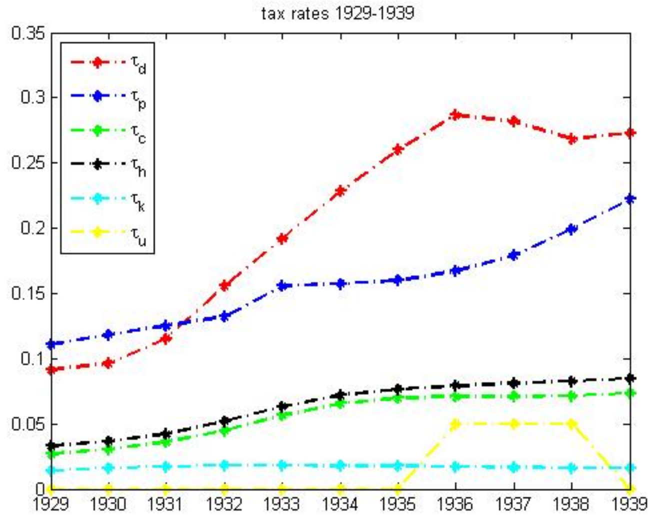

Figure 1.1: Tax rates and government spending between 1929 and 1939

τ_u, and introducing a specific anticipation pattern of tax changes, which is constructed upon the news reports during the 1930s. The simulation in her paper almost accounts for the full decline of investment in 1930s, and it also does a much better job in matching working hours and output[4].

However, the findings in McGrattan (2012) is achieved under the homogeneous agent and complete market framework. Recently heterogeneity and incomplete market have been proved to be an important aspect of taxation study. Anagnostopoulos, Carceles-Poveda, and Lin (2011) investigates the impacts of Jobs and Growth Tax Relief Reconciliation Act (JGTRRA) of 2003 with household heterogeneity. They build an Aiyagari model with dynamic firms and capital taxes, and show that the dividend tax cuts, contrary to capital gains tax cuts, lead to a decrease in investment and capital. This surprising conclusion is because the tax cuts increase the market value of existing capital and households require a higher return to hold this additional wealth. Gourio and Miao (2010) studies the long-run effect

[4]In addition, the similar methodology to study short-run tax change under forward anticipation is first discussed by Auerbach and Hines (1988), and they analyze the impact of anticipated corporate tax reform on the annual investment decision of firms and also evaluate three possible reform treatments with the simulation of a neoclassic model.

of taxation on aggregate capital accumulation. They build a dynamic general equilibrium model in which there is a continuum of firms subject to an idiosyncratic productivity shock and find that a tax cut raises aggregate output through reducing the frictions in the reallocation of capital across firms. Furthermore, Gourio and Miao (2011) shows how to solve for a transition between two steady states with dividend tax and capital gain tax regimes in the heterogeneous firm framework, and that payments, equity issuance, and aggregate investment rise immediately when the dividend and capital gains tax cuts are unexpected and permanent. By contrast, when these tax cuts are unexpected and temporary, aggregate investment falls in the short run. This fall allows firms to distribute large dividend initially in response to the temporary tax cut. The effects of a temporary dividend tax cut are very different from those of a temporary capital gains tax cut.

Inspired by the interesting relationship between capital taxation change and household or firm heterogeneity, it makes very much sense to examine the impact of dividend tax hikes during the US Great Depression again in a framework with household heterogeneity. In my model, households are ex ante identical but differs from each other in asset holdings because of the history of idiosyncratic labor income shocks. Households have the access to an asset market to trade the share of firms, where short selling is not allowed. The key feature of such incomplete market setup have been studied by Aiyagari (1994) that households make savings to insure against the income risk, and therefore their investment or saving decision could be not very sensitive to the interest rate. The quantitative results of this paper is built upon this mechanism. Although the increase of dividend tax rate reduce the interest rate in this economy, households are reluctant to reduce much investment because of strong precautionary saving motives. It consequently leads to a smaller investment decline.

This chapter is organized as follows. In section 1.2, I construct a benchmark model, in which no household heterogeneity exists at all, to confirm the results for later comparison, and also decompose the impacts of different fiscal factors to clarify the mechanism that McGrattan (2012) suggests. In section 1.3, I introduce a heterogeneous agent model and define a recursive competitive equilibrium. In section 1.4, I summarize the data source and calibration strategies. The section 1.5, 1.6 and 1.7 explain the solution method, result, and intuition respectively. The final section concludes.

1.2 Benchmark Model

1.2.1 Setup

In this economy, time is discrete in infinite horizon. Households are homogeneous and they gain utility streams from consumption c_t and leisure l_t. The discount factor is β. Their total endowment of time is normalized into one so the leisure become $1 - h_t^h$ where h_t^h represents working hours, that's the fraction of 24 hours devoted to working each day. Households own dynamic firms by holding shares s_t and hence receive the corresponding dividend payment d_t. They are able to sell or buy the shares of next period s_{t+1} but no short selling is allowed, namely $s_{t+1} \geq 0$. The government collects consumption (sale) tax at τ_c, labor income tax at τ_h and dividend tax at τ_d from households and gives the budget surplus or deficit back to households through a lump-sum transfer κ_t. The total tax payment of households is denoted by Γ^h. All the tax rates and transfer are governed by a fiscal state z_t. z_t is exogenous and perceived as a Markov process by households. The households' perception of z_t will be discussed later in later section. The problem of households can be put as follows,

$$\max_{\{c_t, s_{t+1}, h_t^h\}} E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_t) + \psi \log(l_t)] \tag{1.1}$$

$$\begin{aligned} \text{s.t.} c_t + p_t s_{t+1} &\leq (p_t + d_t) s_t + w_t h_t^h - \Gamma^h(z_t) + \kappa_t \\ l_t &= 1 - h_t^h \\ s_{t+1} &\geq 0 \\ \Gamma^h(z_t) &= \tau_d(z_t) d_t s_t + \tau_c(z_t) c_t + \tau_h(z_t) w_t h_t^h \end{aligned}$$

Thus, the first order conditions to maximize their utility stream subject to h_t^h and s_{t+1} are respectively:

$$\frac{\psi}{1 - h_t^h} = \frac{[1 - \tau_h(z_t)] w_t}{[1 + \tau_c(z_t)] c_t} \tag{1.2}$$

$$E_t \beta \frac{p_{t+1} + [1 - \tau_d(z_{t+1})] d_{t+1}}{p_t} \frac{1}{[1 + \tau_c(z_{t+1})] c_{t+1}} = \frac{1}{[1 + \tau_c(z_t)] c_t} \tag{1.3}$$

The dynamic firms in this economy are assumed to be homogeneous and possess the Cobb-Douglas technology. They own capital k_t and rent labor h_t^f from households at wage rate w_t for their production each period. They pay out the dividend d_t after investment x_t and tax liabilities Γ^f, which includes profit tax with rate τ_p, property tax with rate τ_k and undistributed profit tax with rate τ_u. The discount factor of firms Λ_t is consistent with the inter-temporal substitution of households, namely $\Lambda_t = E\beta^t \frac{[1+\tau_c(z_0)]c_0}{[1+\tau_c(z_t)]c_t}$[5]. Then the problem of dynamic firms is as follows:

$$\max_{\{h_t^f, k_{t+1}\}} \sum_{t=0}^{\infty} \Lambda_t (1 - \tau_{dt}) d_t \tag{1.4}$$

[5]It implicitly requires a no-arbitrage condition holds. See the proof in Appendix

$$\text{s.t.} d_t = f(k_t, h_t^f) - w_t h_t^f - x_t - \Gamma^f(z_t)$$

$$\Gamma^f(z_t) = \tau_p(z_t)[f(k_t, h_t^f) - w_t h_t^f - \delta k_t - \tau_k(z_t)k_t] + \tau_k(z_t)k_t + \tau_u(z_t)(k_{t+1} - k_t)$$

$$f(k_t, h_t^f) = k_t{}^\theta (A h_t^f)^{1-\theta}$$

$$x_t = k_{t+1} - (1-\delta)k_t$$

First order conditions to maximize firm value subject to k_{t+1} and h_t^f are listed below,

$$1 = \frac{\Lambda_{t+1}[1 - \tau_d(z_{t+1})]\{[1 - \tau_p(z_{t+1})][f_{k_{t+1}}(k_{t+1}, h_{t+1}^f) - \delta - \tau_k(z_{t+1})] + 1 + \tau_u(z_{t+1})\}}{\Lambda_t[1 + \tau_u(z_t)][1 - \tau_d(z_t)]} \tag{1.5}$$

$$w_t = f_{h_t^f}(k_t, h_t^f) \tag{1.6}$$

The government collects the taxation revenue $\Gamma^h(z_t)$ and $\Gamma^f(z_t)$ from households and firms, spends $g(z_t)$ and transfers the surplus or deficit κ_t to households for a balanced budget each period. The budget constraint for the government is:

$$\Gamma^h(z_t) + \Gamma^f(z_t) = g(z_t) + \kappa_t \tag{1.7}$$

There are totally three markets in this economy: common goods, labor and stock shares. The market clearing conditions are respectively,

$$c_t + x_t + g(z_t) = f(k_t, h_t) \tag{1.8}$$

$$s_{t+1} = 1 \tag{1.9}$$

$$h_t^h = h_t^f \tag{1.10}$$

1.2.2 Expectation Pattern

The exogenous process z_t are assumed to determine tax rates and government spending $\{\tau_p, \tau_d, \tau_k, \tau_c, \tau_u, \tau_h, g\}$. Besides, the anticipation of fiscal regimes z_t here are also McGrattan

(2012) does[6]. According to the perception of households, z_t can only take on 11 possible states, which correspond to the annual fiscal states in US from 1929 to 1939. However, households don't always have an accurate knowledge of the incoming fiscal state next period and only infer it with certain expectation patterns, which is abstracted by an 11-by-11 transition matrix $\Pi(z_{t+1}|z_t)$. In this way z_t is considered to follow a Markov chain by households when they make consumption or saving or working decisions. The specific transition matrix $\Pi(z_{t+1}|z_t)$ is taken from McGrattan (2012) and shown in table 1.1. Actually this kind of setup makes the roles of fiscal policies uncertainty similar to aggregate technology risk in the standard *Real Business Cycle* model and brings many advantages in computation[7]. To identify the role of this specific anticipation pattern, I also introduce another two anticipation patterns for comparisons: myopic foresight and perfect foresight. The former one supposes that households have no access to know the future fiscal regimes change and always consider that the current regime lasts for ever, whereas the alternative implies the totally opposite situation that households know exactly the tax rates and government spending next period in advance.

The foundation of this format $\Pi(z_{t+1}|z_t)$ is the news report in the 1930s. The row represents the fiscal state current period and the column shows the possible fiscal states next period. The number in each cell indicates the probability the *column* state comes next period given the *row* fiscal state current period. The z subscripted by numbers denotes 11 possible states of fiscal regimes while the *Year* denotes the year when the corresponding fiscal policy is actually carried out in US. The context in the table can be generally divided into three cases in general: First, taking the first row as an example, it's a myopic foresight: if households are in the fiscal regime of year 1929 at current period, they believe that they will stay with this regime next period; Second, taking the second row as an example, if

[6] See relevant sections in McGrattan (2012).The complicated process to generate the fiscal policies are usually discussed by political science or political economics. In this paper, we just simply assume it to be stochastic in the perception of households.

[7] See the discussion in section 2.3

	Year	Z_1 1929	Z_2 1930	Z_3 1931	Z_4 1932	Z_5 1933	Z_6 1934	Z_7 1935	Z_8 1936	Z_9 1937	Z_{10} 1938	Z_{11} 1939
Z_1	1929	1	0	0	0	0	0	0	0	0	0	0
Z_2	1930	1/3	1/3	1/3	0	0	0	0	0	0	0	0
Z_3	1931	1/3	0	1/3	1/3	0	0	0	0	0	0	0
Z_4	1932	0	0	0	0	1	0	0	0	0	0	0
Z_5	1933	0	0	0	0	0	1	0	0	0	0	0
Z_6	1934	0	0	0	0	0	0	1	0	0	0	0
Z_7	1935	0	0	0	0	0	0	1	0	0	0	0
Z_8	1936	0	0	0	0	0	0	0	1/2	1/2	0	0
Z_9	1937	0	0	0	0	0	0	0	0	1/2	1/2	0
Z_{10}	1938	0	0	0	0	0	0	0	0	0	0	1
Z_{11}	1939	0	0	0	0	0	0	0	0	0	0	1

Table 1.1: Expectation of Fiscal Policies $\Pi(z_{t+1}|z_t)$

households are in the fiscal regime of year 1930, they think that the fiscal regimes of 1929, 1930 and 1931 are equally possible to occur; Third, it's a perfect foresight: if households are in the fiscal regime of year 1932, they believe that the incoming fiscal regime would be the one of year 1933, that is agents in the economy make the correct inference of incoming fiscal policy. The anticipation pattern in table 1.1 is considered as a benchmark for more numerical experiments about anticipation, like households are completely myopic or perfectly foresighted.

1.2.3 Computation and Results

As mentioned in previous sections, the fiscal uncertainty in this paper is similar to the aggregate technological shocks in the real business cycle model, as they share the same way how they affect the real economy. As a result, I apply the similar strategy to solve stochastic growth model. z_t is considered as an aggregate "technology" shock with the transition matrix in table 1.1. Given share holdings s_t and fiscal state z_t are state variables, Solve for decision functions of households through iterating the first order conditions. Besides solving and simulating the benchmark model, two more experiments are conducted: Experiment I solves and simulates the model under perfect and myopic anticipation patterns respectively;

Experiment II solves and simulates the model with keeping *only one* tax constant under each anticipation pattern in turns. Experiment I is to explore the roles of different expectation patterns in this economy, while Experiment II is to recognize the impact of each tax under different expectation patterns.

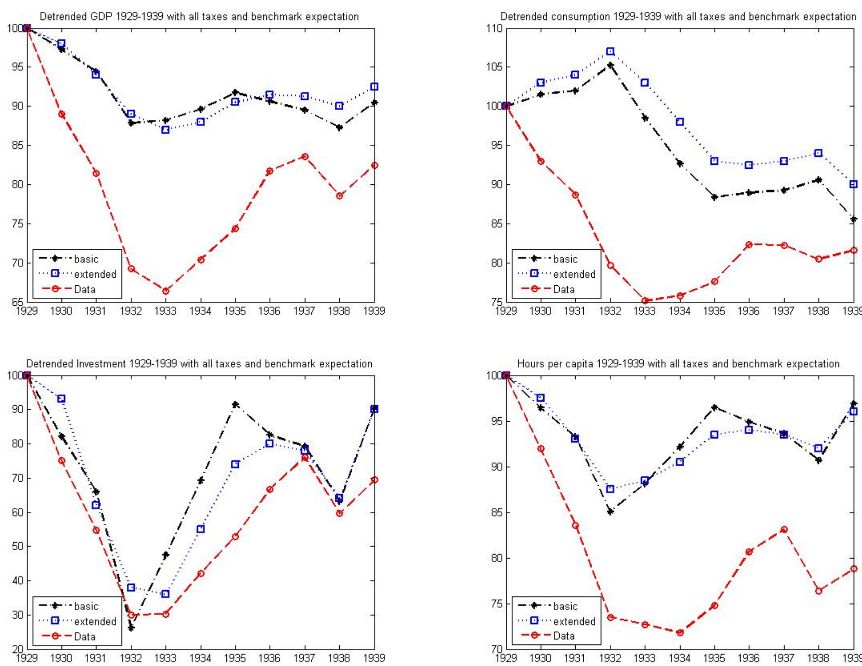

Figure 1.2: Benchmark Model vs Data

Figure 1.2 illustrates the simulation results of benchmark model along with simulation results of extended model in McGrattan (2012) and data series. It tells that the benchmark model in this paper can predict a similar aggregates trend to the one in McGrattan (2012) although it does not include the intangible capital. It implies that the intangible capital contributes little to match the data series. The simulations show a large decline at the beginning of 1930s followed by a quick recovery and then another small decline around 1937. The predicted investment and the actual investment almost overlap each other before the lowest point in 1932. Afterwards the predicted one recovers immediately and faster than its counterpart in reality. The absence of investment recovery delay has an enduring impact on the whole simulation results and makes the investment after 1932 always above the data series. The predicted output and working hours also have a significant decline but not as much as in the data. However, the model fails to capture the decline of consumption in the early 1930s too.

Figure 1.3 shows the results of experiment I. The myopic anticipation can barely produce any decline or recovery, although all fiscal policy changes are feed into the simulation. The intuition behind is very straightforward. If the households have the belief that the current fiscal regime will not change next period, it is reasonable for them to change their asset holding dramatically. On the contrary, the perfect foresight anticipation provides households an accurate knowledge of tax change next period, including the tax or undistributed profit tax increase, then they are able to take actions in advance to accommodate these changes. A proper response to the capital tax hike next period should be to cut the investment today.

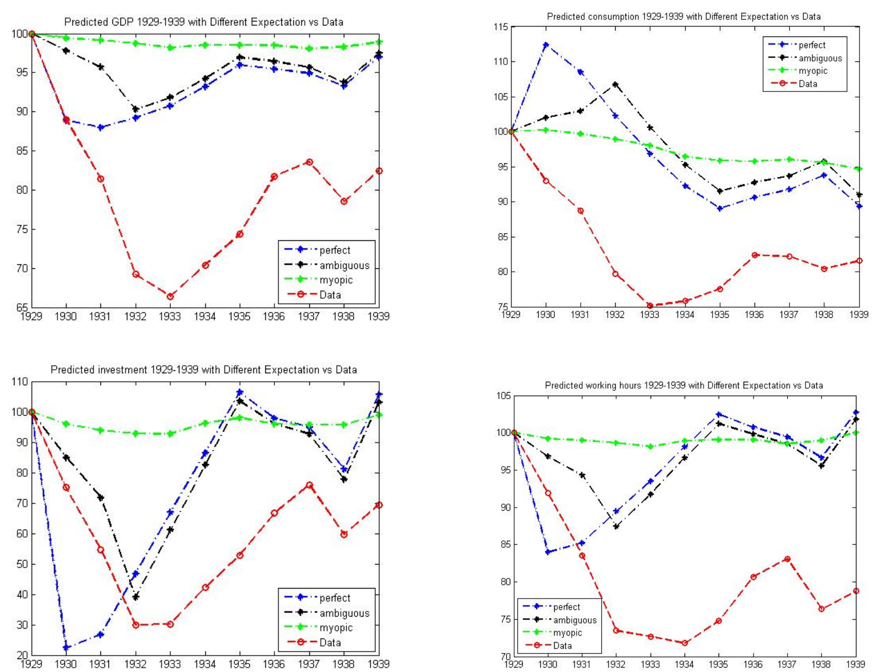

Figure 1.3: Basic Model with Different Expectation

Figure 1.4, 1.5, and 1.6 show the results of experiment II under myopic foresight, perfect foresight and benchmark anticipation in table 1.1 respectively. The experiment is made in the following way: First, given a specific anticipation pattern, solve and simulate the model with only one of six taxes constant [8]. Then, change the anticipation pattern and repeat the same procedure in the first step. Figure 1.4 illustrates the impact of each tax under the myopic foresight anticipation. It implies that no tax can generate a transition comparable to the data. All economic variables decrease along the time because of an increasing government spending. Such a result also confirms the conclusion of experiment I implicitly. Figure 1.5 illustrates the impact of each tax under perfect foresight anticipation. The influence of dividend tax and undistributed profit tax seem to be very significant: Once the dividend tax is fixed, the decline of investment at the beginning of 1930s disappears. Moreover, the decline of investment in the late 1930s will disappear if the undistributed profit tax is constant. Similarly, output and working hours have no declines when those two taxes are eliminated respectively. All the other taxes have no significant effects on any economic aggregates. However, the declines occur earlier and recover faster than the actual data series, and the simulated consumption increase at the beginning. The results from the benchmark anticipation provide the best match with data in the sense that the timing of investment is much improved. Figure 1.6 shows that the investment, output and working hours share similar trends to data series, although there are still large differences in absolute levels, especially for output and investment.

In sum, the increase of capital taxes, dividend tax or undistributed profit tax, and forward expectation patterns, perfect foresight or benchmark anticipation, together result in investment decline and therefore the decrease in output and working hours. The dividend tax and undistributed profit tax seem to contribute the most to the dramatic economic downturn

[8] Actually some alternative implementation is also feasible, that's to eliminate all taxes but the interested one. As a result of computation convenience, I do not apply this methodology. The absence of so many taxes at the same time may lead the policy functions far from the ones of the benchmark model, usually the initial guess.

and slow recovery during Great Depression. Only when households expect the increase of dividend or undistributed profit tax to increase, they cut down the investment in time. This is also the main mechanism implied by the benchmark model.

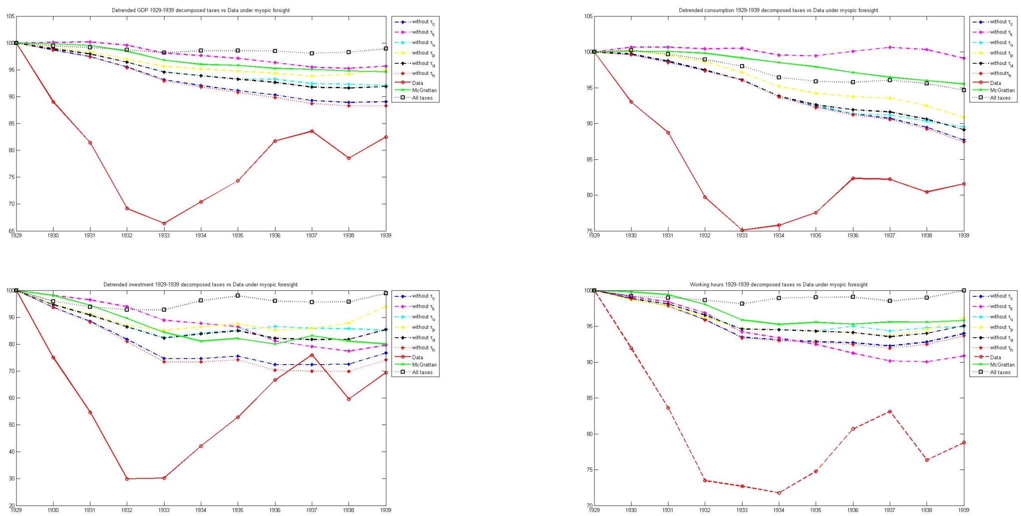

Figure 1.4: Single Tax Experiment under Myopic Expectation

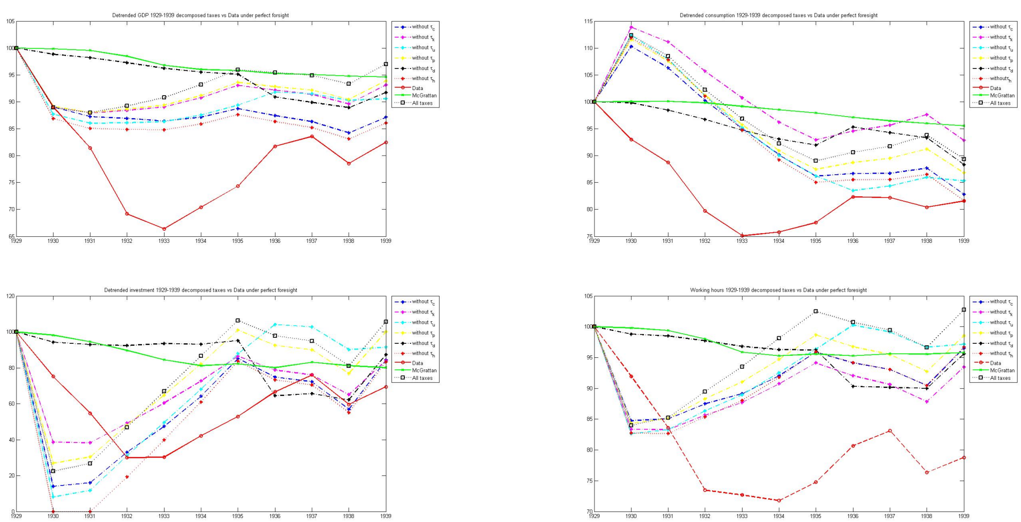

Figure 1.5: Single Tax Experiment under Perfect Expectation

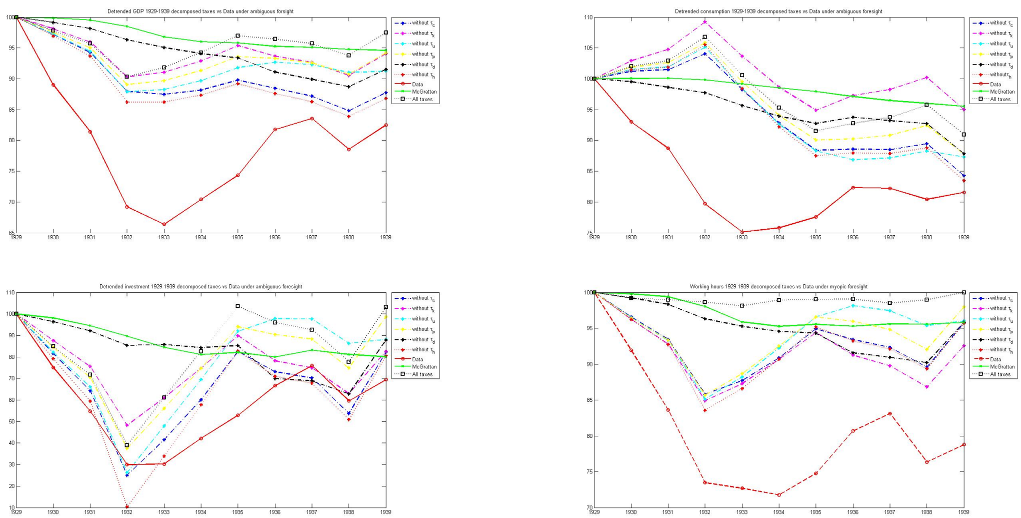

Figure 1.6: Single Tax Experiment under Benchmark Expectation

1.3 Heterogeneous Agent Model

1.3.1 Model

Assume that households are infinitely lived and in a continuum mass equal to one. They differ with regard to their share holdings s_{it} and labor productivity shocks ε_{it}. i is the index for different types of households. They make decision on their consumption c_{it}, share holding next period s_{it+1} and working hours h_{it} each period to maximize their discounted utility flow into the infinite horizon. Their discount factor is β and $\beta < 1$. As the total labor endowment of households is normalized to unity, the leisure of each period is defined as $1 - h_{it}$. All the households pay consumption tax at τ_{ct}, labor income tax at τ_{ht} and dividend tax at τ_{dt}, and also receive a lump-sum transfer κ_t. Those four fiscal factors are uniform across all households. Note, households are assumed to have a perfect knowledge of the fiscal state in the future. Therefore, their expectation is only on the labor productivity shock they might experience next period. $\Pi(\varepsilon_{t+1}|\varepsilon_t)$ is the conditional probability of the labor productivity shock next period to be ε_{t+1} given the current one ε_t. Then, the problem of the heterogeneous household i is summarized as below:

$$\max_{\{c_{it}, s_{it+1}, h_{it}\}} E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_{it}) + \psi \log(l_{it})] \tag{1.11}$$

$$\begin{aligned} \text{s.t.} c_{it} + p_t s_{it+1} &\leq (p_t + d_t) s_{it} + w_t h_{it} \varepsilon_{it} + \kappa_t - \Gamma_{it}^h \\ s_{it+1} &\geq 0 \\ l_{it} &= 1 - h_{it} \\ \Gamma_{it}^h &= \tau_{ct} c_{it} + \tau_{dt} d_t s_{it} + \tau_{ht} w_t h_{it} \varepsilon_{it} \end{aligned}$$

Then the first order conditions subject to h_{it} and s_{it+1} to maximize household utility are respectively:

$$\frac{\psi}{1 - h_{it}} = \frac{(1 - \tau_{ht}) w_t \varepsilon_{it}}{(1 + \tau_{ct}) c_{it}} \tag{1.12}$$

$$E_t \beta \frac{p_{t+1} + (1 - \tau_{dt+1}) d_{t+1}}{p_t} \frac{1}{(1 + \tau_{ct+1}) c_{it+1}} \leq \frac{1}{(1 + \tau_{ct}) c_{it}} \tag{1.13}$$

The equal sign of (13) holds only if $s_{it} > 0$. The setup of firms in this economy is similar to the one in the benchmark model except that its discount factor change according to the existence of household heterogeneity. The dynamic firms produce with their own capital k_t and the labor h_t^f rent from households. To be consistent with the heterogeneous shareholders, the discount factor $\tilde{\Lambda}_t$ is defined as,

$$\tilde{\Lambda}_t = \begin{cases} E \beta^t \frac{c_{i0}}{c_{it}} & t = 1 \ldots \infty \\ 1 & t = 0 \end{cases} \tag{1.14}$$

First order conditions for the dynamic firms to maximize their value subject to k_{t+1} and h_t are shown as below:

$$1 = \frac{\tilde{\Lambda}_{t+1}(1-\tau_{dt+1})\{(1-\tau_{pt+1})[f_{k_{t+1}}(k_{t+1}, h^f_{t+1}) - \delta - \tau_{kt+1}] + 1 + \tau_{ut+1}\}}{\tilde{\Lambda}_t(1+\tau_{ut})(1-\tau_{dt})} \tag{1.15}$$

$$w_t = f_{h^f_t}(k_t, h^f_t) \tag{1.16}$$

These two first order conditions are very important. By forward iteration, a P-K mapping, which is very important for the computation, can be proved.

$$p_t = (1-\tau_{dt})(1+\tau_{ut})k_{t+1} \tag{1.17}$$

That is, the share price is an indicator of capital stocks next period and influenced by two capital taxes, tax and undistributed tax. The P-K mapping also implies that there is a wedge between the inside-firm capital and outside-firm capital. Given the same inside-firm capital level in this economy, the increase of tax rate leads to an increase in the value of outside-firm capital, while the increase of undistributed tax rate leads to the opposite change on the outside-firm capital. Furthermore, the outside-firm capital is the asset actually traded in the economy so the change of capital taxation can cause the total volume of wealth in this economy. The detailed discussion on the P-K mapping will continue in the computation and intuition sections.

The government budget also changes as the households become heterogeneous. The total taxation revenue from households are also determined by the joint distribution of share holdings and productivity shocks among households $\Phi(s_{it}, \varepsilon_{it})$.

$$\int \Gamma^h_{it}\Phi(s_{it}, \varepsilon_{it}) + \Gamma^f_t = g_t + \kappa_t \tag{1.18}$$

Then the market clearing conditions are,

$$\int c_{it} d\Phi(s_{it}, \varepsilon_{it}) + x_t + g_t = f(k_t, h_t) \tag{1.19}$$

$$\int s_{it+1} d\Phi(s_{i,t}, \varepsilon_{it}) = 1 \tag{1.20}$$

$$\int h_{it} d\Phi(s_{i,t}, \varepsilon_{it}) = h_t^f \tag{1.21}$$

Definition 1 *The recursive competitive equilibrium in this economy can be defined as follows: Given the initial capital level* k_0*, the initial joint distribution of share holdings and productivity shock* $\Phi_0(s, \varepsilon)$*, a recursive competitive equilibrium subject to the fiscal policy* $\{\tau_p, \tau_d, \tau_k, \tau_c, \tau_u, \tau_h, g\}$*, consists of a set of laws of motion* $\{k^{'} = \Omega(k, h, \Phi), h^{'} = \Xi(k, h, \Phi), \Phi^{'} = \Upsilon(k, h, \Phi)\}$*, a price set* $\{w, p\}$*, firm choices* $\{d, h^f, k^{'}\}$*, and the individual household decision functions and value function* $\{c(s, \varepsilon), s^{'}(s, \varepsilon), h(s, \varepsilon), V(s, \varepsilon)\}$ *such that:*

- *Given the price* $\{w, p\}$ *and laws of motion* $\{k^{'} = \Omega(k, h, \Phi), h^{'} = \Xi(k, h, \Phi), \Phi^{'} = \Upsilon(k, h, \Phi)\}$*, the individual household decision functions and value function* $\{c(s, \varepsilon), s^{'}(s, \varepsilon), h(s, \varepsilon), V(s$ *solve the household optimization problem;*

$$V(s, \varepsilon) = \max_{\{c, s^{'}, h\}} \{U(s, \varepsilon) + E[V(s', \varepsilon^{'})|\varepsilon]\}$$

$$s.t. c + ps' \leq (p + d)s + wh\varepsilon + \kappa - \Gamma^h$$
$$U(s, \varepsilon) = \log(c) + \psi \log(l)$$
$$s' \geq 0$$
$$l = 1 - h$$
$$\Gamma^h = \tau_c c + \tau_d ds + \tau_h wh\varepsilon$$

- *The dynamic firms satisfy the profit maximization conditions below;*

$$p = (1 - \tau_d)(1 + \tau_u)k^{'}$$

$$w = f_{h^f}(k, h^f)$$

$$d = f(k, h^f) - wh^f - x - \Gamma^f$$

- *The government operates on a balanced budget;*

$$\int \Gamma^h(s, \varepsilon)\Phi(s, \varepsilon) + \Gamma^f = g + \kappa$$

$$\Gamma^f = \tau_p[f(k, h^f) - wh^f - \delta k - \tau_k k] + \tau_k k + \tau_u(k^{'} - k)$$

- *All the market clear each period;*

$$\int c(s, \varepsilon)d\Phi(s, \varepsilon) + x + g = f(k, h)$$

$$\int s^{'}(s, \varepsilon)d\Phi(s, \varepsilon) = 1$$

$$\int h(s, \varepsilon)d\Phi(s, \varepsilon) = h^f$$

- *The laws of motion are consistent with the individual household behavior;*

1.4 Calibration

In this paper, the US economy in 1929, the year when the US Great Depression began, is taken as the benchmark of calibration. As the fiscal policy in the experiment evolutes in the way with its counterpart in reality, the model shows the simulated response of aggregate economy to those fiscal changes. However, why is it reasonable to consider the US economy in 1929 as a steady state subject to the fiscal policy at that time? According to historic data in fiscal policies, the taxation and government spending was both very stable during 1920s, especially after *1924 Post War Reduction*[9]. In addition, the expectation of any fiscal

[9] For taxation, see *tax foundation*; For government spending, see *national income and product accounts.*

changes did not come into being until 1930 when abrupt breakout of Great Depression forced President Roosevelt to take new economic measures. Therefore, it is not unreasonable to assume that the US economy in 1929 as a steady state calibration target.

1.4.1 Economic Aggregates

To make the one-sector model comparable to the real data, the capital level in the model is calibrated to the reproducible capital stocks from all production sectors in US economy. Namely it includes private fixed assets, durable consumption goods, government fixed assets, corporate inventory, and value of lands. The data source of the first three categories is table 1 in Katz and Herman (May 1997), which is an adjusted summary of NIPA tables from Bureau of Economic Analysis. The *Statistics of Income for 1929* provides the information of inventory under the assets category of of US corporate balance sheet.[10] The land value is from the nonresidential land value in table $W-30$ of Goldsmith, Brady, and Mendershausen (1956).

The consistency in calibration also requires that the outputs of capital measured by the above strategy should be all considered as part of total products. So it is necessary to include the service flow from durable goods and government fixed assets, which is not imputed in the GDP of NIPA tables. The return to capital r is essential to infer these series. The procedure to obtain the return rate r will be discussed in later part of this section. Given r is available, I can add the term r multiplying the sum of durable goods and government fixed assets to GDP and consumption value from NIPA table respectively to impute the adjusted GDP and consumption. Moreover, as **McGrattn2012** suggests, the adjustments relevant to sale tax expenditures are also made to the consumption and investment: The sale tax on durable consumption and nondurable consumption are respectively less from the

[10]see website http://www.irs.gov/taxstats/productsandpubs/article/0,,id=125133,00.html, and download the *Statistics of Income* report from SOI Publications Archive session.

durable consumption and nondurable consumption, because the consumption expenditure in NIPA does not differentiate the price and tax. Investment is the gross investment plus consumption of durable goods in the GDP components table of NIPA. Government spending directly obtained from the government consumption in NIPA. All the series above are divided by GDP deflator and mid-year population in NIPA. They are also detrended by technology growth rate set to be 1.9%.

Cooley and Prescott (1995) has provided the methodology to calibrate the capital return r, capital income share θ and capital depreciation rate δ. I follow the standard procedures in that paper. Extract the labor income, unambiguous capital income and ambiguous capital income respectively from NIPA. With consumption of fixed assets, that's the capital depreciation if steady state assumption holds, solve for the private capital income share θ_p first by:

$$\theta_p = \frac{\text{unambiguous capital income} + \text{capital depreciation}}{GNI - \text{ambiguous capital income}} \tag{1.22}$$

Then, calculate the private capital income Y_{KP} by θ_p multiplying GNI. The return rate to capital r is determined by:

$$r = \frac{Y_{KP} - \text{capital depreciation}}{K_p} \tag{1.23}$$

Here, K_p includes private fixed assets, corporate inventory and land value. With the assumption that r is unique in the economy, I can impute the gross service flow from the durable goods Y_d and government fixed assets Y_g.[11]. Then capital income share θ is finally determined by the following formula:

[11]Estimation over the depreciation rates δ_d and δ_g are also required

$$\theta = \frac{Y_{KP} + Y_d + Yg}{GNP + Y_d + Y_g} \tag{1.24}$$

1.4.2 Income and Wealth Inequality

In this model, the earning process is responsible for the endogenous income and wealth heterogeneity. The classic methodology to calibrate the earning process has been discussed by Castáneda, Díaz-Giménez, and Ríos-Rull (2003). However, it requires a high-quality micro-level data source to detail the earning and wealth Lorenz curves.[12] Unfortunately there exists quite few micro-level data sources in early 20th century. Only a handful of literatures have disclosed very limited information in income and wealth Gini coefficients and the top group shares. Earning inequality at that time is barely exposed. Lindert (2000) does a systematical survey on income and wealth inequality of the early $20th$ century and introduces potential income and wealth inequality data sources for heterogeneous household study. In this paper, the household income and wealth inequalities are respectively taken from Goldsmith (1967) and Williamson and Lindert (1980). The former one estimates the top 20% income share and income Gini coefficient in 1929, whereas the later one displays the top 10% wealth share and wealth Gini coefficient in 1913 and 1925 from King (1915) and P. H. And Williamson (1976).[13] We infer the wealth inequalities through those two sets of statistics, because Williamson and Lindert (1980) mentions " *The period from 1860 to 1929 is thus best described as a high uneven plateau of wealth inequality. When did wealth inequality hit its historic peak? We do not yet know. We do know that there was a leveling across the 1860s. We also know that there was a leveling across the World War I decade (1912-22), which was reversed largely or entirely by* 1929.", which implies that the wealth Gini coefficients and top wealth shares in 1913 and 1925 should be very close to the ones in 1929.

[12]For instance, *Survey of Consumer Finance* used by Castáneda, Díaz-Giménez, and Ríos-Rull (2003) is eligible to calculate the quintile of earning and wealth distribution.

[13]Most of Wealth inequality literature only measure the top 0.01% top 0.5% top 1% or top 5% wealth shares. Although they are very useful information, it is very difficult to capture them by a parsimonious model with limited labor shock states.

In practice, I just take the average of these data as the proxies for wealth inequalities in 1929.

1.4.3 Parametrization

For convenience, the earning process is simply considered to have a symmetric transition matrix, which help reduce the total number of unknown parameters in the transition matrix to 3[14]. The labor productivity shock values $\{\epsilon_i\}_{i=1}^3$ add 3 more unknowns. Besides, we have another 5 aggregate parameters $\{\beta, \psi, \theta, \delta, z\}$ to settle down. Nonetheless, θ, δ and z are able to be determined by capital income share, investment-capital ratio and capital-working hours ratio respectively with no computational experiment. As a result there are totally 8 parameters to be calibrated systematically. The solution consequently requires at least 8 conditions as the calibration targets. In addition to the labor and share market clearing conditions, the remaining can be found in the normalization of labor productivity shock, the unit expected labor productivity shock at stationary equilibrium, income Gini coefficient, wealth Gini coefficient, top 20% income share and top 10% wealth share. The benchmark targets and corresponding predicted value are posted in table 1.2.The table 1.3 and 1.4 show the parameters from calibration.

target	benchmark value	predicted value
capital output ratio	3.6681	3.6681
working hours(fraction of 24 hours)	0.2892	0.2892
Wealth Gini coefficients	0.8900	0.8990
Income Gini coefficients	0.4900	0.5009
Top 10% wealth share	90.0%	87.0%
Top 20% income share	54.0%	52.5%

Table 1.2: Calibration targets

[14] See the following example for the symmetric transition matrix used for calibration computation.

$$\Pi_{ij} = \begin{pmatrix} P_{11} & 1-P_{11} & 0 \\ \frac{1-P_{22}}{2} & P_{22} & \frac{1-P_{22}}{2} \\ 0 & 1-P_{33} & P_{33} \end{pmatrix} \tag{1.25}$$

	value	$\pi(\epsilon_1\|\epsilon_i)$	$\pi(\epsilon_2\|\epsilon_i)$	$\pi(\epsilon_3\|\epsilon_i)$
ϵ_1	0.1790	0.992	0.008	0.000
ϵ_2	0.9467	0.009	0.980	0.011
ϵ_3	8.3305	0.000	0.083	0.917

Table 1.3: Earning process

parameter	value
β	0.9352
δ	0.0603
ϕ	1.9726
θ	0.4484
z	1.0499

Table 1.4: Parameters

1.5 Computation and Results

The objective of the numerical experiments in this paper is to find predicted economic trends and contrast them with the corresponding actual data series. It requires to solve out the transition from 1929 to 1939 under the changing fiscal policies then. Therefore, I use a shooting method to solve the transition between steady states in brief. To make this method feasible, the following conditions on the model must hold: First, the economy is at steady state in 1929; Second, households have a perfect foresight of the fiscal policy path from 1929 to 1939; Third, households have no knowledge of the fiscal states after 1939 and believe that the fiscal policy stays the same afterwards. Thanks to the above assumptions, the interested solution can be considered as an economic transition from the steady state under the fiscal policy of 1929 to another one under fiscal policy of 1939. The feature different from the standard shooting method is that the fiscal policies change in the first 11 periods and constant in the rest. Thus, the brief algorithm is listed as below:

- **Step 1:** Solve for the stationary equilibria under the fiscal regime of 1929 and 1939 respectively and store the invariant distributions of households, value functions, aggregate capital stocks, aggregate working hours and government transfers;

- **Step 2:** Choose the total periods of the transition between the above two steady state and make sure the first 11 periods with the fiscal policies changing each period;
- **Step 3:** Guess sequences of household distributions, aggregate capital stocks, aggregate working hours and value functions along the whole transition excluding the end period. Note, the share price, wage rate and government transfer can be imputed each period once the guess is made;
- **Step 4:** Given projected paths of share price, wage rate, government transfer and value function, do the value function iteration to update value function and decision functions each period;
- **Step 5:** Aggregate the capital stocks and working hours by updated decision functions and projected distribution to update the guess of capital stocks, working hours and distribution of households each period;
- **Step 6:** Check the convergence, otherwise go back to step 4 with the updated aggregates, value functions and distribution of households.

The benchmark model solved under the anticipation in table 1 fails to provide a good contrast to the heterogeneous agent model in this paper as a result of different anticipation patterns. A reasonable practice is to compare the solutions of the benchmark model and heterogeneous agent model both under the perfect foresight anticipation. Figure 7 shows the trends of different economic aggregates projected by benchmark model, heterogeneous agent model and actual data series respectively. The graphics of investment shows that the impacts of the tax and undistributed profit tax increase are very sensitive to the presence of household heterogeneity. The investment decline caused by the tax increase is almost 50% smaller in the heterogeneous model than in the homogeneous model. Consequently, the simulated output and working hours don't show a significant decline in the heterogeneous model as they do in the homogeneous model either. However, the counterfactual consumption increase in the early 1930s gets a little improved. My conjecture is that the smaller consumption jump mainly comes from the effect of a smaller investment drop[15]. Although the simulation under perfect foresight anticipation cannot provide a satisfactory match for the actual data series in neither benchmark model and heterogeneous model, it provides a straightforward perspective to understand the mechanism how the capital taxation affect the investment and further more economic aggregates. The absence of dramatic investment decline in the heterogeneous model with perfect foresight anticipation implies that the mechanism suggested by the benchmark model is not significant when the household heterogeneity exists. Figure 8 shows the full computational solution of capital stocks and working hours. As discussed before, the full solution is just a transition from one steady state to another with the fiscal policies changing only in the first 11 periods. Capital stocks and working hours are both consistent with such conditions: On one hand, the capital stock decreases at the beginning as a response to the tax increase, and then decreases again as a response to the undistributed profit tax increase after a small and short recovery, and finally increases to the new steady state smoothly once the fiscal policy is constant; On the other hand, the working hours show the same trend with the capital stocks but within a smaller fluctuation range.

1.6 Explanation

The disappearance of significant investment decline in the heterogeneous household model can be explained by the wealth effect discovered by Anagnostopoulos, Carceles-Poveda, and Lin (2011). When the dividend tax increases, the capital demand decrease is offset by the strong desire of households to insure against bad productivity shocks. The increase of dividend tax in a heterogeneous household economy generally causes two effects: anticipation effect and wealth effect. On one hand, the anticipation of the dividend tax increase plays the same role as it dose in homogeneous household economy. The expect return to capital

[15] If the capital stock and working hours don't fall much, then the output will more or less remain the same level as before. In this sense, a smaller decline in investment implies a larger fraction of output goes to investment and then a smaller consumption increase.

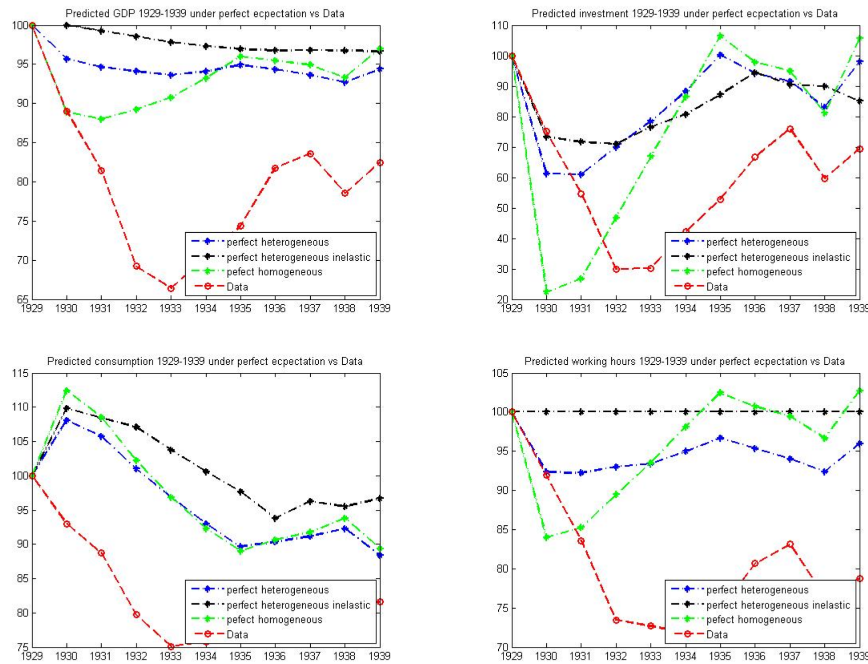

Figure 1.7: Heterogeneous Model vs Homogeneous Model under Perfect Expectation

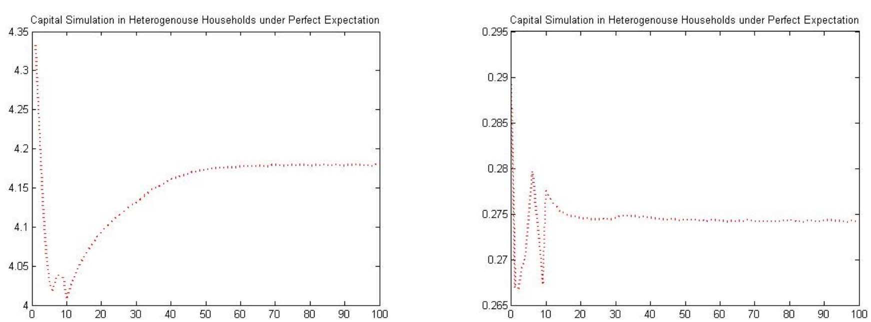

Figure 1.8: Capital and Labor Transition Path

decreases and then investment goes down accordingly, that is to reduce the inside-firm capital demand in this economy, which tends to decrease the equilibrium capital stock; On the other hand, the outside-firm capital demand or assets supply in the economy shrinks when the dividend tax increases according to the P-K mapping. The precautionary saving motive forces households to compete for scarcer assets and then households are willing to accept a lower return. If the decreasing marginal productivity and no arbitrage are presumed, the fall of return to capital leads to a higher equilibrium capital stock. In sum, these two effects above move the equilibrium capital stock in different directions. Therefore, the total impact of dividend tax increase is a quantitative issue.

In the following section these two effects will be put into concrete visual exhibition. First, for the better reference, the original model setup need be transferred to a classic heterogeneous agent model in Aiyagari (1994) by replacing $p_t s_{it+1}$ and $\frac{p_{t+1}+(1-\tau_{dt+1})d_{t+1}}{p_t}$ with a_{it+1} and r_{t+1} respectively. Then the assets demand curve A_t^h in this economy is in the similar shape to the aiyagari model, upward sloping, concave and converge to $\frac{1}{\beta}-1$. Second, capital demand curve K_t^f is obtained from equilibrium conditions. Third, construct the assets supply curve A_t^f on the basis of capital demand curve K_t^f and the P-K mapping. The total share in the economy is normalized into unity so the mapping between share price and interest rate is just the mapping between the total assets and interest rate. The typical household asset demand curve can be solved from the problem (1.26); The equation (1.27) is an implicit function of capital demand of firms; The equation (1.28) reflects the relation between assets supply (outside-firm capital demand) and (inside) capital demand.

$$\max_{\{c_{it},a_{it+1},h_{it}\}} E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_{it}) + \psi \log(1-h_{it})] \tag{1.26}$$

$$\text{s.t.} c_{it} + a_{it+1} \leq (1+r_t)a_{it} + w_t h_{it} \varepsilon_{it} + \kappa_t$$

$$a_{it+1} \geq 0$$

$$(1+r_{t+1}) = \frac{(1-\tau_{dt+1})\{(1-\tau_{pt+1})[f_{k_{t+1}}(k_{t+1},h_{t+1}) - \delta - \tau_{kt+1}] + 1 + \tau_{ut+1}\}}{(1+\tau_{ut})(1-\tau_{dt})} \tag{1.27}$$

$$A_t^h = p_t = (1-\tau_{dt})(1+\tau_{ut})k_{t+1} \tag{1.28}$$

1.6.1 Anticipation Effect

According to the decomposition results in benchmark model, only the increase of dividend tax and undistributed profit tax leads to investment decrease while all the other fiscal changes have no significant impacts. Hence, it is convenient to concentrate on the capital taxes and eliminate all the other fiscal parameters during the mechanism analysis. Then the capital demand curve K_t^f can be simplified as:

$$r_{t+1} = \frac{(1-\tau_{dt+1})(f_{k_{t+1}}(k_{t+1}, h_{t+1}) - \delta + 1)}{1-\tau_{dt}} - 1 \tag{1.29}$$

Then it is easy to observe the impact of anticipated increase of τ_{dt+1}. It pushes the capital demand curve K_t^f to left. So does the assets supply curve A_t^f. The assets market equilibrium change from A^{old} to A^{new} and arrive at a lower interest rate. Accordingly the lower interest rate drive the capital down to a new equilibrium K^{new} from K^{old}. Note, if the increase of τ_{dt+1} is not expected, then τ_{dt+1} equals to τ_{dt} in the perspective of firms, which implies the capital demand K_t^f does not receive any influence from the dividend tax change next period. So anticipation is very important for the tax change to influence the demand for capital stock next period. The anticipation effect is marked by $A.E.$ in Figure 1.9

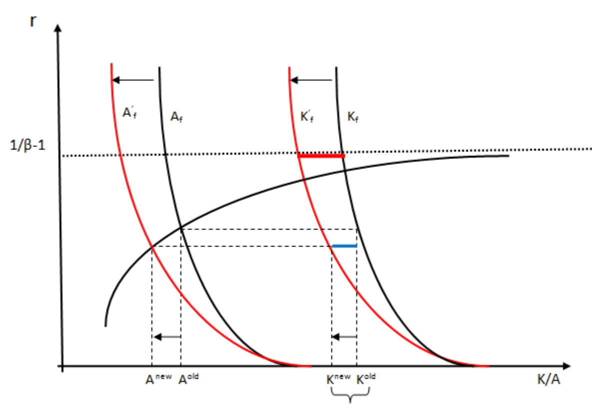

Figure 1.9: Anticipation Effect

1.6.2 Wealth Effect

The assets in this model can be considered as the outside-firm capital and also equal to the share price.[16] There is a wedge between the inside-firm and outside-firm capital, $(1 - \tau_{dt})(1 + \tau_{ut})$. When τ_{dt} increases, the wedge increase. Namely, the value of outside-firm capital decreases even given the same inside-firm capital stock. In Figure 1.10, the assets supply curve A_t^f is pushed to left with the capital demand curve K_t^f untouched. The new equilibrium interest rate becomes lower than the original one. It requires a higher equilibrium capital stock K^{new}. Note, this effect is absent in the complete market as the assets demand there is absolutely elastic. The W.E. marks the wealth effect in Figure 1.11.

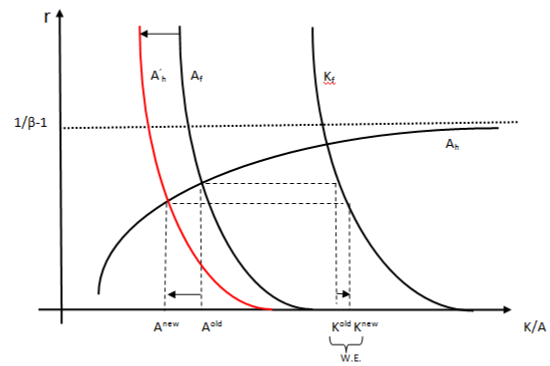

Figure 1.10: Wealth Effect

1.6.3 Total Effect

When the dividend tax increases continuously for many periods, the anticipation effect and wealth effect occur at the same time each period. For instance, the anticipated tax increase in period $t + 1$ leads to the drop of capital demand in period t. Nevertheless, the current dividend tax increase introduce the wealth effect and increases the equilibrium capital stock. In total, these two effect offset each other. However, which effect ultimately dominates

[16]Consider the total share volume in this economy always equal to one.

is a quantitative issue. In the quantitative analysis of this paper, the anticipation effect is larger than the wealth effect. The transition results show that the investment only fall a little from the steady state level after the dividend tax hikes start. Figure 1.11 illustrates the total effect in this paper. Without the wealth effect, the anticipation effect should move the assets supply curve A_f to the dot line position rather than A_f'. Then the decrease of capital stock in that case will be much larger than what the model actually produce. Note, Figure 1.11 just illustrates a specific situation in this paper. The numerical results could be different in other experiments. Suppose that the wealth effect is much larger, the result could be that the wealth effect dominates and that the capital stock increases finally.

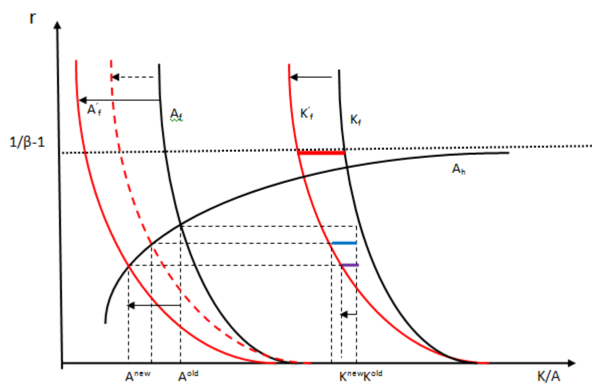

Figure 1.11: Total Effect

1.7 Conclusion

In this chapter, I first build a homogeneous agent model with disaggregated taxes, and show that the anticipated increase of dividend tax rates explains the dramatic drop of investment during the Great Depression. Subsequently, I extend the benchmark model to a heterogeneous agent environment, in which the endogenous income and wealth inequalities are calibrated to the ones in 1929. The quantitative practice implies that the impact of anticipated increase of dividend tax rate is very sensitive to the presence of household heterogeneity. Given the same capital stock level, the increase of dividend tax rate decreases the value of total assets in the economy and forces households to require a lower return for much fewer accesses to assets. Under no arbitrage condition, the lower return rate induces the firms to cut capital stock demand and then leads to an investment decline. The quantitative experiment illustrates that the heterogeneous agent model can only project half of investment decrease suggested by the benchmark model. The downturn in the working hours and output also become less significant. It seems that the role of fiscal policy during the US Great Depression is still in question.

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1.8 Appendix

1.8.1 Proof of P-K Mapping

By the first order condition subject to k_{t+1} to optimize the value of firms, we can prove that the share price is actually a function of capital in this economy.

$$(1+\tau_{ut})(1-\tau_{dt}) = \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})\{(1-\tau_{pt+1})[f_{k_{t+1}}(k_{t+1},h^f_{t+1})-\delta-\tau_{kt+1}]+1+\tau_{ut+1}\}$$

$$(1+\tau_{ut})(1-\tau_{dt})k_{t+1} = \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})\{(1-\tau_{pt+1})[f_{k_{t+1}}(k_{t+1},h^f_{t+1})-\delta-\tau_{kt+1}]+1+\tau_{ut+1}\}k_{t+1}$$

$$\begin{aligned}
&= \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})\{(1-\tau_{pt+1})[f_{k_{t+1}}k_{t+1}-\delta k_{t+1}-\tau_{kt+1}k_{t+1}]+k_{t+1}+\tau_{ut+1}k_{t+1}\}\\
&= \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})\{(1-\tau_{pt+1})[f(k_{t+1},h^f_{t+1})-\delta k_{t+1}-\tau_{kt+1}k_{t+1}]+k_{t+1}-k_{t+2}+\tau_{ut+1}(k_{t+1}-k_{t+2})\}\\
&+ \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})(1+\tau_{ut+1})k_{t+2}\\
&= \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})d_{t+1}+\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})(1+\tau_{ut+1})k_{t+2}\\
&= \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})d_{t+1}+\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\frac{\tilde{\Lambda}_{t+2}}{\tilde{\Lambda}_{t+1}}(1-\tau_{dt+2})d_{t+2}+\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\frac{\tilde{\Lambda}_{t+2}}{\tilde{\Lambda}_{t+1}}\frac{\tilde{\Lambda}_{t+3}}{\tilde{\Lambda}_{t+2}}(1-\tau_{dt+2})(1+\tau_{ut+2})k_{t+3}\\
&= \frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}(1-\tau_{dt+1})d_{t+1}+\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\frac{\tilde{\Lambda}_{t+2}}{\tilde{\Lambda}_{t+1}}(1-\tau_{dt+2})d_{t+2}+\frac{\tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t}\frac{\tilde{\Lambda}_{t+2}}{\tilde{\Lambda}_{t+1}}\frac{\tilde{\Lambda}_{t+3}}{\tilde{\Lambda}_{t+2}}(1-\tau_{dt+3})d_{t+3}+...\\
&= \sum_{j=1}^{\infty}\frac{\tilde{\Lambda}_{t+j}}{\tilde{\Lambda}_t}(1-\tau_{dt+j})d_{t+j}\\
&= p_t
\end{aligned}$$

This conclusion obviously leads to the price-capital mapping:

$$p_t = (1+\tau_{ut})(1-\tau_{dt})k_{t+1}$$

1.8.2 Algorithm to Solve the Stationary Equilibrium

- **Step 1:** Guess the aggregate capital stock and working hours at stationary equilibrium as k_{ss}^0 and h_{ss}^0

- **Step 2:** According to the production function and government budget constraint, we can solve out κ_{ss}^0 in the following way:

$$\Gamma_{ss}^0 = \tau_p[f(k_{ss}^0, h_{ss}^0) - w_{ss}^0 h_{ss}^0 - \delta k_{ss}^0] \tag{1.30}$$

$$\kappa_{ss}^0 = w_{ss}^0 h_{ss}^0 \tau_h + \Gamma_{ss}^0 - g \tag{1.31}$$

- **Step 3:** For $l = 0...L$, where l is the inner-loop iteration and L is the maximum iteration number allowed. Do the policy iteration with the following equations for all the grid points $\{s, \varepsilon\}$ with initial guess $c^0(s, \varepsilon)$:

$$\frac{1}{c^{l+1}(s,\varepsilon)} = \sum_{\varepsilon'} \Pi(\varepsilon'|\varepsilon) \frac{\hat{\beta}[1 + (1-\tau_p)(f_1^0 - \delta)]}{c^l(s', \varepsilon')} \tag{1.32}$$

$$\frac{\psi}{1 - h^l(s,\varepsilon)} = \frac{(1-\tau_h) w_{ss}^0 \varepsilon_{i,t}}{c^l(s,\varepsilon)} \tag{1.33}$$

$$c^l(s,\varepsilon) + p_{ss}^0 s' = (p_{ss}^0 + d_{ss}^0)s + (1-\tau_h) w_{ss}^0 h^l(s,\varepsilon)\varepsilon + \kappa_{ss}^0 \tag{1.34}$$

p_{ss}^0, d_{ss}^0 and w_{ss}^0 can be derived from the p-k mapping, definition and production respectively. The linear interpolation will be applied when we look for the grids of $c^l(s', \varepsilon')$. Calculate the policy function $s'^l(s,\varepsilon)$ with $c^l(s,\varepsilon)$ and $h^l(s,\varepsilon)$. If $s'^l(s,\varepsilon) \leq 0$ or $s'^l(s,\varepsilon) \geq 1$, set $s'^l(s,\varepsilon) = 0$ or $s'^l(s,\varepsilon) = 1$. Take out the intertemporal condition and re-solve out $c^l(s,\varepsilon)$ and $h^l(s,\varepsilon)$. Continue until the consumption and labor supply policy functions both converge. Store the converged $c^l(s,\varepsilon)$, $s'^l(s,\varepsilon)$ and $h^l(s,\varepsilon)$ as $c^0(s,\varepsilon)$, $s'^0(s,\varepsilon)$ and $h^0(s,\varepsilon)$ for the future steps.

- **Step 4:** Then derive the invariant distribution $\Phi^0(s,\varepsilon)$ by constructing the transition matrix of (s,ε) with $s'^0(s,\varepsilon)$ and $\pi(\varepsilon'|\varepsilon)$.

- **Step 5:** With $\Phi^0(s,\varepsilon)$ we can solve out the total demand for capital k^1_{ss} and the total labor supply h^1_{ss} as below:

$$h^1_{ss} = \int h^0(s,\varepsilon)d\Phi^0(s,\varepsilon) \tag{1.35}$$

$$k^1_{ss} = \frac{f(k^0_{ss}, h^0_{ss}) + (1-\delta)k^0_{ss} - \int c^0(s,\varepsilon)d\Phi^0(s,\varepsilon) - g}{(1+\gamma)(1+\eta)} \tag{1.36}$$

- **Step 6:** Compare $\{h^1_{ss}, k^1_{ss}\}$ with $\{h^0_{ss}, k^0_{ss}\}$. If convergence occurs, stop; Otherwise, update the guess as follows and go back to step 2:

$$k^0_{ss} = k^0_{ss} + \lambda_k(k^1_{ss} - k^0_{ss}) \tag{1.37}$$

$$h^0_{ss} = h^0_{ss} + \lambda_h(h^1_{ss} - h^0_{ss}) \tag{1.38}$$

1.8.3 Algorithm to Solve the Transition with Perfect Foresight

- **Step 1:** Choose the total number of the transition periods T. The first 11 periods represent the actual economic period $1929-1939$ with correspondent ***changing*** taxation regime of each year, and the late $(T-11)$ periods represent the transition path the economy takes from the state at the end of 11th period to the steady state under the ***fixed*** taxation regime of 1939;

- **Step 2:** As we discussed in the last section, solve the stationary equilibria under the taxation regime of 1929 and 1939 respectively and store the invariant distribution of individual states, policy functions, aggregate capital, aggregate working hours and government transfer as $\{\Phi_{initial}(s,\varepsilon), c_{initial}(s,\varepsilon), k_{initial}, h_{initial}, \kappa_{initial}\}$ and $\{\Phi_{end}(s,\varepsilon), c_{end}(s,\varepsilon), k_{end}, h_{end}, \kappa_{end}\}$;

- **Step 3:** Guess the distribution of individual states, aggregate capital stock, and aggregate working hours sequence $\{\Phi_t^0(s,\varepsilon), k_t^0, h_t^0\}_{t=1}^{T-1}$, where $k_1^0 = k_{initial}$ and $\Phi_1^0(s,\varepsilon) = \Phi_{initial}$; Also guess a sequence of policy functions for households $\{c_t^0, s_t^{\prime 0}, h_t^0\}_{t=1}^{T-1}$;

- **Step 3:** Use the previous guess $\{\Phi_t^0(s,\varepsilon), k_t^0, h_t^0\}_{t=1}^{T-1}$, government budget constraint and determination equation of government's profit tax income to solve out $\{\kappa_t^0\}_{t=1}^{T-1}$;

$$\Gamma_t^0 = \tau_{pt}[f(k_t^0, h_t^0) + (1-\delta)k_t^0 - w_t^0 h_t^0 - (1+\eta)(1+\gamma)k_{t+1}^0] \tag{1.39}$$

$$\kappa_t^0 = w_t^0 h_t^0 \tau_{ht} + \Gamma_t^0 - g_t \tag{1.40}$$

- **Step 4:** For all the periods $t = T-1...1$, do the following period by period backwards:

  - *Step 4.1:* Given $\{k_t^0, h_t^0, \kappa_t^0\}$ and $c_{t+1}^0(s,\varepsilon)$, we can update $\{c_t^0(s,\varepsilon), h_t^0(s,\varepsilon), s_t^{\prime 0}(s,\varepsilon)\}$ into $\{c_t^1(s,\varepsilon), h_t^1(s,\varepsilon), s_t^{\prime 1}(s,\varepsilon)\}$ using the following conditions:

$$\frac{1}{c_t^1(s,\varepsilon)(1+\tau_{ct})} = \sum_{\varepsilon'} \Pi(\varepsilon'|\varepsilon) \frac{\hat{\beta}[1 + (1-\tau_{pt+1})(r_{t+1}^0 - \delta)]}{c_{t+1}^0(s',\varepsilon')(1+\tau_{ct+1})} \tag{1.41}$$

$$\frac{\psi}{1 - h_t^1(s,\varepsilon)} = \frac{(1-\tau_{ht})w_t^0 \varepsilon}{c_t^1(s,\varepsilon)(1+\tau_{ct})} \tag{1.42}$$

$$(1+\tau_{ct})c_t^1(s,\varepsilon) + p_t^0 s' = (p_t^0 + d_t^0)s + (1-\tau_{ht})w_t^0 h_t^1(s,\varepsilon)\varepsilon + \kappa_t^0 \tag{1.43}$$

$$p_t^0 = (1+\gamma)(1+\eta)k_{t+1}^0 \tag{1.44}$$

$$d_t^0 = (k_t^0)^\theta (A h_t^0)^{1-\theta} + (1-\delta)k_t^0 - (1+\eta)(1+\gamma)k_{t+1}^0 - w_t^0 h_t^0 - \tau_{pt}[f(k_t^0, h_t^0) - w_t^0 h_t^0 - \delta k_t^0] \tag{1.45}$$

$$w_t^0 = (1-\theta)A(k_t^0)^{\theta}(Ah_t^0)^{-\theta} \tag{1.46}$$

$$r_t^0 = \theta(k_t^0)^{\theta-1}(Ah_t^0)^{1-\theta} \tag{1.47}$$

The linear interpolation will be applied when we look for the grid of $c_{t+1}^0(s^{'}, \varepsilon^{'})$. In addition, the solution of $s^{'}$ has to be checked for each period to guarantee $0 \leq s_t \leq 1$. If binding solutions $s' < 0$ or $s' > 1$ are found, replace the intertemporal condition with $s = 0$ or $s = 1$, solve the system again;

– *Step 4.2:* With policy functions $\{c_t^1(s,\varepsilon), h_t^1(s,\varepsilon), s'^1_t(s,\varepsilon)\}$ obtained from last step and transition matrix $\pi(\varepsilon^{'}|\varepsilon)$, we can calculate the transition matrix of individual state from period t to period $t+1$, Ω_t. Then we can update the distribution of households over shares and labor shock at period t into Φ_t^1.

$$\Phi_t^1 = {\Omega^T}_t^{-1}\Phi_{t+1}^0 \tag{1.48}$$

Here you might notice that $\Phi_T^1 = \Phi_T^0 = \Phi_{end}$. Finally, use the Φ_t^1 to solve out the aggregate capital of period $t+1$ and labor supply of period t as $\{k_{t+1}^1, h_t^1\}$ in the following way:

$$h_t^1 = \int h_t^1(s,\varepsilon)d\Phi^1(s,\varepsilon) \tag{1.49}$$

$$k_{t+1}^1 = \frac{f(k_t^0, h_t^0) + (1-\delta)k_t^0 - \int c^1(s,\varepsilon)d\Phi_t^1(s,\varepsilon) - g_t}{(1+\eta)(1+\gamma)} \tag{1.50}$$

- **Step 5:** Compare $\{\Phi_t^1, k_{t,sim}, h_t^1\}_{t=1}^{T-1}$ with $\{\Phi_t^0, k_t^0, h_t^0\}_{t=1}^{T-1}$. If convergence occurs, stop; otherwise, update the guess in the following way:

$$k_t^0 = k_t^0 + \lambda_k(k_t^1 - k_t^0) \quad \text{if} \quad t > 1 \tag{1.51}$$

$$k_1^0 = k_1^1 = k_{initial} \tag{1.52}$$

$$h_t^0 = h_t^0 + \lambda_h(h_t^1 - h_t^0) \tag{1.53}$$

Chapter 2

Financial Factors, Working Capital, and the US Great Depression

2.1 Introduction

The three important factors that were present during the U.S. Great Depression were a monetary contraction, nominal rigidities, and a financial crisis. Monetarists, e.g., Friedman and Schwartz (1971), believe that the restrictive money supply in the 1930s caused the large decline of aggregate demand. The nominal rigidity school claims that the wage and price stickiness prohibited a swift adjustment of aggregate supply, when the economy had a downturn, and thus cost more employment and output. This theory is in the meantime a promising remedy for the protracted monetary non-neutrality that plagues the monetary explanation, and has been confirmed valid by the empirical research of Eichengreen and Sachs (1985) and Bernanke and Carey (1996). The third explanation, which is also the interest of this paper, asserts that the financial crisis was responsible for the downswings of employment and output as well. The primary question of this theory is through which channel the financial turbulence caused the contraction in the real sector, which is quite challenging and still not completely settled.

In this paper, we argue that the difficulty in obtaining working capital during the U.S. Great Depression could have propagated the distress in financial sector to the labor input, and further led to the declines in other economic aggregates. The working capital refers to the amount of assets that are used to manage the short-term cash flow mismatch. Our quantitative experiments show that this channel explains almost 50% of working hours decline in the early 1930s and also predicts considerable declines in consumption, investment, and output. Moreover, we find that the *unexpected* financial meltdown and the smaller number of long-term debts approaching maturity aggravates the above mechanism, whereas the influence of capital taxation reform in the 1930s is minor. Our story could contribute a fresh and interesting perspective to the present wisdom in understanding the U.S. Great Depression.

Our paper builds a general equilibrium model with features such as dynamic firms, working capital, and an intra-period borrowing constraint associated with working capital financing. In order to circumvent the complexity in mimicking continuous working capital financing, we follow the model setup in mainstream literature, and simply assume that firms do not possess any cash or other liquid assets between periods, and that they borrow working capital to manage the cash flow mismatch at the beginning of each period. Before repaid at the end of each period and after firms realize the production revenues, the working capital is used for labor inputs, bond payments, investments and dividend payout. The intra-period borrowing contract for working capital financing is not fully enforced and requires the net-worth of firms, the total capital net the outstanding long-term debts, to serve as collateral. Hence, the financial structure of firms affects their intra-temporary financing capacity, because it affects the level of net-worth. More importantly, given the level of collateral assets, how much firms can borrow intra-temporarily is under the influence of general financial climate. The worse the general financial environment is, the tighter the collateral constraint becomes. We borrow many features of the model from Perri and Quadrini (2011) and Jermann and

Quadrini (2012). And additional attributes like dividend taxes and maturity structures are introduced, because they are found by recent literature to be either important for explaining the U.S. Great Depression or influential for determining the corporate financial structure.

The quantitative results illustrate that the difficulty in working capital financing during the U.S. Great Depression can explain 50% of the working hours decline in the early 1930s and produce moderate levels of decline in consumption, investment and output. When the financial crisis diminishes the availability of intra-period borrowing, an important source for working capital, firms have to reduce their cash flow. As they cannot adjust their financial structure promptly to ease such liquidity tension, i.e., though accumulating capital or through cutting outstanding debts very quickly, firms are ultimately forced to cut labor inputs and production. Our theory not only provides a new channel connecting the financial crisis to the real economic contraction during the U.S. Great Depression, but it also provides an alternative explanation for the sluggish labor market in the 1930s, which differs from the sticky wages and policy uncertainty theories. In addition, our model successfully replicates the general trend of important financial variables in the period $1930 - 1940$, including the ratio of newly issued long-term debt over output and the -output ratio. Additional experiments find the following. First, the unexpected financial meltdown and the fewer long-term debts approaching maturity can aggravate the mechanism we propose. This is because firms get more restrained in realigning their financial structure in these two scenarios, and they have to reduce more labor inputs and production under cash flow pressures. Precisely, if the financial crisis is not expected, firms may not reduce their outstanding debts in advance. And they may fall into a more severe disposable cash flow drought when the financial chaos arise, since larger amount of outstanding debts lowers net-worth. When fewer long-term debts approach maturity, the average term to maturity of long-term debt is lengthened. It decreases the effective long-term debt repayment each period, and equivalently reduces the marginal cost of using long-term debts. Such a change stimulates firms to keep more long-term debts, decreases their net-worth, and ultimately deteriorates the difficulty in working capital financing. Second, in contrast to the existing literature, the dividend tax hike does not lead to dramatic investment collapse in our model. The reason is that capital serves as production inputs as well as collateral for working capital financing. If firms cut too much investment, it will shrink the volume of collateral assets and deteriorate the difficulty in financing working capital in an adverse financial climate. This finding implies that the impact of capital return decline, which is caused by an anticipated dividend tax increase, could be potentially offset by financing frictions.

Our work is not the first attempt to clarify the connection between the financial chaos and real sector contraction in the 1930s. In what follows, we summarize previous studies on how financial turbulence in the U.S. Great Depression impacted the real economy, and then we briefly state their limitations that invoke this project. Fisher (1933) suggests a debt deflation theory to explain the U.S. Great Depression. He addressed that the massive liquidation debt selling in the early 1930s could have triggered a nationwide deflation. The fall of the general price level led to a significant loss in production, employment and net-worth and reinforced the motivation for another round of liquidation selling, which ultimately became a debt-deflation cycle. However, the author did not clarify why the debt deflation was detrimental to employment and production. With the introduction of imperfect information and agency costs in the capital market, the debt deflation theory has revived decades later. Bernanke and Gertler (1989) proposes an innovative framework to illustrate how debt deflation can hit the real economy: As the asset prices plummet, the debt obligation in nominal terms seriously diminishes the net-worth of borrowers and possibly makes their actions not in the best interest of creditors, e.g., more risk-taking and/or lower efforts are observed, because their net contribution to investment or production projects, usually measured by the net worth, becomes smaller. Due to the subsequent increase in lending risks, creditors also suffer more costs in acquiring inside information before signing lending contracts and monitoring borrowers afterwards. If the net-worth is lower than a certain threshold value, the agency costs become so high that borrowers cannot get any external funds at all and have to reduce their production, employment, and investment. The authors also claim that the debt-deflation redistributes wealth from borrowers to lenders and makes the invaluable investment information or production technology of borrowers under-utilized, which ultimately can devastate the efficiency of capital allocation and bring employment and production down.

Related to the previous work, Bernanke (1983) and Bernanke and James (1990) study the impact of bank panic episodes on the employment and output using U.S. domestic data and international data respectively. Their empirical findings show that bank panic during the Great Depression was mostly attributed to policy failures and bad institutions, and they conclude that they where a crucial intensifier for the economic collapse in the 1930s rather than just the passive symptom of a large recession. The financial intermediaries that were experiencing bank runs and under the threat of bank runs had to cut their lending volume and hold more liquid assets. In the meantime, massive waves of bank bankruptcies also destroyed lots of local credit supply and hurt the economy directly.

Note that the previous study of bank runs mostly concentrates on the financial sector and uncovers how banks lost their function as financial intermediaries during the Great Depression. Still, two important questions have not been well answered under this framework: First, how did the reduction of bank loans affect the production and employment exactly? Namely, how did the business sector react to the reduction of bank loans? Secondly, the bank loans were not the unique funding source for plenty of businesses. For instant, large firms were able to hold liquid assets or issue their own short-term and long-term bonds. When the businesses had plenty of alternative financing channels, how did they response to the restrictive bank loan supplies? Correspondingly, such businesses were probably exposed to financial pressures from more aspects than the limitation of bank loans. Thus, it will be interesting to establish a new theory that considers the financial structure of businesses, introduces a broader definition of financial factors, and demonstrates the reactions of business sector to the financial distress.

The imperfect information and agency cost theory might shed light on the above questions to some extent, though it is unfortunately not perfect. For example, the decline of net-worth caused by debt deflation does not apply to all businesses in the Great Depression. A considerable number of firms in the 1930s possessed a solid net-worth, including cash holdings or other assets and still experienced an austerity in spending streams, see Hunter (1982) and Hart and Mehrling (1995), which is apparently unable to be explained by the imperfect information and agency cost theory. Instead, the concerns about short-term financial management such as difficulty in obtaining working capital financing and increasing bond default risk are likely to have an impact. Introducing these types of frictions is the objective of our work.

Finally, the preceding papers on the Great Depression were mostly using reduced form estimations complemented with a supporting theoretical model.[1] Strictly speaking, their conclusions rested loosely on the proposed mechanism. So it would be good to analyze the same question using a structural model. One of the advantages of such a framework is that it allows for more quantitative analysis.

This paper tries to improve on all the three issues mentioned above. Besides, we do not intend to address the source of the financial crisis during the U.S. Great Depression nor attribute the economic downturn completely to the change in the financial environment or the difficulty in obtaining working capital financing. Instead, we are interested in the

[1] Although Bernanke, Gertler, and Gilchrist (1998) built a comprehensive structural model for financial accelerators, they focus on the general business cycle features rather than the explanation for the Great Depression.

following questions: how did the financial sector crisis propagate to the real sector through working capital financing? How much of the economic contraction, especially the working hours decline, can be explained by this channel? Furthermore, the long-term corporate bond default risk is also put aside in this paper so as to identify the impact of short-term financing.[2] The paper is organized as follows: Section 2 presents the model; Section 3 characterizes the equilibrium; Section 4 introduces the parameter values; Section 5 discusses the quantitative results. The final section concludes.

2.2 Model

We consider an infinite horizon economy in discrete time. There are two agents living forever, entrepreneurs and workers. Entrepreneurs are assumed different from workers in many aspects. First, they are more impatient and therefore require a higher return to savings. Second, entrepreneurs do not receive government transfers, if the government collects the tax revenue and makes transfers for a balanced budget. Third, they have the exclusive access to production technologies and capital, and do not supply labor. Forth, there exists an exogenous market segmentation so that workers are not allowed to buy firm shares and cannot become entrepreneurs. Because of the last two assumptions, entrepreneurs in my model are equivalent to firms.[3] So firms and entrepreneurs are interchangeable in description of our model throughout the paper. The details of two agents are given in the following subsections.

[2]The influence of long-term bond default risk has been formally discussed in another paper of mine, *Corporate Default Risk, Investment, and the U.S. Great Depression.* In that paper, we propose that the default risk is an effective amplifier of adverse technology and financial shocks. On the one hand, the massive wave of corporate bond defaults directly idled a considerable amount of capital, which was detrimental to production, investment, and employment. On the other hand, the indebted firms were inclined to cut more investment during the economic downturn, as they were also concerned about the increasing default risks besides the awful economic outlook.

[3]Such an assumption is the scheme commonly used in business cycle or assets pricing studies. See Guvenen (2000). The primary purpose is to create a bond market that has a non-zero clearing level at equilibrium.

2.2.1 Workers

A typical worker in this economy faces the following problem:

$$\max_{\{c_t^w, h_t, \Delta b_{t+1}\}} \sum_{t=0}^{\infty} \gamma^t [\log(c_t^w) + \eta \log(1 - h_t)]$$

$$\text{s.t.} c_t^w + q_t \Delta b_{t+1} = w_t h_t + \lambda b_t + Tr_t$$

$$\Delta b_{t+1} = b_{t+1} - (1 - \lambda) b_t$$

Workers gain utilities from consumption c_t^w and leisure $1 - h_t$. Their momentary utility function follows the logarithm format and discount factor over time is γ. η measures the weight of leisure in their utility. Each period workers make decision on consumption c_t^w, bond purchase Δb_{t+1} and labor supply h_t. They earn income from wage w_t and bond repayment λb_t. The setup of long-term bond b_t in this paper follows Leland. (1994). Only λ of long-term bond b_t retires every period and the remaining rolls over to next period. $1/\lambda$ denotes the average term to maturity of long-term bond. A government transfer Tr_t is also part of income. Take the first order partial derivative with respect to c_t^w, b_{t+1} and h_t respectively. After a simple manipulation, the necessary conditions to maximize the utilities of workers are transferred into:

$$\frac{w_t}{c_t^w} = \frac{\eta}{1 - h_t} \tag{2.1}$$

$$\frac{q_t}{\lambda + (1 - \lambda) q_{t+1}} = \gamma \frac{c_t^w}{c_{t+1}^w} \tag{2.2}$$

The equation (2.1) states the labor supply of workers. The equation (2.2) defines the long-term bond pricing kernel. The multiple-period bond setup makes the bond price q_t in a recursive format.

2.2.2 Entrepreneurs

As mentioned at the beginning of this section, entrepreneurs in this economy have the exclusive access to production technologies and capital. Accordingly, they make investment and production decision for the whole economy in addition to their own financial decisions such as issuing new bonds as well as distributing dividend payout. As a key feature of this model, entrepreneurs ***have to*** make a short-term (intra-period) loan to purchase working capital for the contemporaneous production, because it is assumed that entrepreneurs do not possess any disposable cash flow at the beginning of each period.[4] Prior to the realization of revenue, firms have no other funding sources but the intra-period borrowing. *Intra-period* here means that entrepreneurs borrow at the beginning of each period and repay the loan at the end of the same period. Moreover, the intra-period borrowing contract is not fully enforced and requires the net-worth of firms to serve as the collateral. Such a covenant provision eliminates the case of long-term bonds default, since the net-worth collateral implicitly ensures that the long-term bond can always get fully recovered under any scenario. It confines our attention to the short-term financing. The problem of a typical entrepreneur in this economy is as below:

[4]The surplus cash flow last period can be considered either distributed as payment or exhausted as investment and debt repayment. This restriction is not relaxed in this paper. Nonetheless, it is potentially a very interesting direction to develop this kind of model, since the recent recession show that firms appear to have a strong motivation to accumulate cash from earning in bad times.

$$\max_{\{c_t^e, h_t, b_{t+1}, k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{e\,1-\sigma} - 1}{1-\sigma}$$

$$\begin{aligned}
\text{s.t.}c_t^e &= d_t - \Gamma_{dt} \\
d_t &= F(k_t, h_t) + q_t \Delta b_{t+1} - \lambda b_t - w_t h_t - x_t \\
\Delta b_{t+1} &= b_{t+1} - (1-\lambda) b_t \\
F(k_t, h_t) &= A {k_t}^{\theta} {h_t}^{1-\theta} \\
k_{t+1} &= (1-\delta) k_t + \Psi\left(\frac{x_t}{k_t}\right) k_t \\
\Gamma_{dt} &= \tau_{dt} d_t \\
l_t &\leq \epsilon_t (k_{t+1} - q_t b_{t+1}) \\
l_t &= d_t + \lambda b_t - q_t \Delta b_{t+1} + w_t h_t + x_t
\end{aligned}$$

The momentary utility function of entrepreneurs follows the CRRA format. β is the discount factor and $\beta < \gamma$. c_t^e is the notation for the consumption of entrepreneurs, that is the dividend payout d_{t+1} net the tax liabilities in Γ_{dt}. b_t is the long-term bond. Again, the long-term debt in my model is in the multiple-period bond setup proposed by Leland. (1994). Each period firms only repay λ of their outstanding long-term debt and the newly issued long-term bond is denoted by Δb_{t+1}. The production function is Cobb-Douglas and θ is the input parameter. x_t stands for the investment. The investment bears the capital adjustment cost, $\Psi\left(\frac{x_t}{k_t}\right)$, which is an increasing and concave function. Its setup follows Jermann (1998). τ_{dt} is the tax rates on dividend, while Γ_{dt} is the corresponding tax liabilities. The way dividend taxes are levied in my model refers to McGrattan (2012).

l_t is the working capital that is used to manage the cash flow mismatch at the beginning of each period. It is used to cover the labor spending $w_t h_t$, investment expenditure x_t, net debt payment $(\lambda b_t - q_t \Delta b_{t+1})$ and dividend payout d_t. Entrepreneurs obtain l_t through intra-period borrowing against their net-worth $(k_{t+1} - q_t b_{t+1})$, namely the assets net long-term liabilities. How much firms can borrow in the short-term is also affected by the financial factor ϵ_t. After combined with the cash flow constraint, the short-term borrowing constraint for working capital financing is:

$$F(k_t, h_t) \leq \epsilon_t(k_{t+1} - q_t b_{t+1}) \tag{2.3}$$

It is necessary to stress two interesting features of entrepreneurs' problem: First, the concavity of utility function gives entrepreneurs the motivation to smooth their consumption c_t^e over time.[5] The entrepreneurs consumption c_t^e is a linear function of payout d_t, so the dividend payout d_t is equivalently required to be smoothed. Second, ϵ_t in the equation (2.3) represents the financial factor that changes over time, which can be interpreted in two ways: If there exist financial intermediaries to handle the working capital financing, ϵ_t can be taken as their health and efficiency status. The lower value of ϵ_t implies the lower efficiency. Besides, it can be regarded as an indicator of the liquidity preference of firm financial managers under various financial risks. The lower value of ϵ_t implies the milder appetite for expenditure stream and stronger motivation to accumulate earnings into cash account. The format of collateral constraint in this paper is taken from Jermann and Quadrini (2012). Take the derivative respectively to k_{t+1}, b_{t+1} and h_t, and get the necessary conditions for entrepreneurs to maximize their utilities over time as follows:

$$\frac{1-\tau_{dt}}{\Psi_{x_t} k_t} - \mu_t \epsilon_t = \beta \left(\frac{c_t^e}{c_{t+1}^e}\right)^{\sigma} \left[(1 - \tau_{dt+1} - \mu_{t+1}) F_{k_{t+1}} + \frac{(1-\tau_{dt+1})(\Psi_{k_{t+1}} k_{t+1} + \Psi_{t+1} + 1 - \delta)}{\Psi_{x_{t+1}} k_{t+1}}\right] \tag{2.4}$$

$$(1 - \tau_{dt} - \mu_t \epsilon_t) q_t = \beta \left(\frac{c_t^e}{c_{t+1}^e}\right)^{\sigma} (1 - \tau_{dt+1})[q_{t+1}(1-\lambda) + \lambda] \tag{2.5}$$

$$F_{h_t} = \frac{w_t}{1 - \frac{\mu_t}{1-\tau_{dt}}} \tag{2.6}$$

μ_t is the Lagrangian multiplier associated with short-term borrowing constraint. The detailed interpretation of these conditions are presented in the characterization of equilibrium.

[5]This feature is very important for the mechanism through which the financial factor affects the production. It will be elaborated in the next section.

2.2.3 Government

For simplicity I assume the only function of government in this economy is to tax entrepreneurs and transfer revenues to workers. τ_{dt} represents the tax rate on dividend payout, while Γ_{dt} is the corresponding tax liabilities. Then, the budget constraint of government is:

$$\Gamma_{dt} = Tr_t \tag{2.7}$$

Tr_t is the transfer to workers.

2.2.4 Market Clearing Conditions

There are totally four markets in this economy, good, labor, long-term debt and intra-period loan. The intra-period loans are assumed to be issued and repaid within the same period, so its market is not modelled explicitly for simplicity. The good market clearing condition is:

$$c_t^e + c_t^w + x_t = F(k_t, h_t) \tag{2.8}$$

2.2.5 Definition of Equilibrium

Definition 1: A competitive equilibrium is defined as a set of price $\{q_t, w_t\}_{t=0}^{\infty}$ and a set of allocation $\{c_t^w, h_t, b_t, k_t, x_t, c_t^e, d_t, Tr_t, \Gamma_{dt}, \mu_t\}_{t=0}^{\infty}$ such that: Given the price set and the exogenous sequence of $\{\epsilon_t, \tau_{dt}\}_{t=0}^{\infty}$, (i)the allocation set solves the optimization problems of workers and entrepreneurs, (ii) all markets clear, and (iii) the government runs on a balanced budget.

2.3 Characterization of Equilibrium

It is useful to analytically exposit some equilibrium properties of this model , before they are overshadowed by the complexity of computation. It provides some helpful intuitions for understanding the numerical results later on. In the following paragraphs, I will study the short-term borrowing constraint, the Lagrangian multiplier associated with this constraint, and demand functions separately. For convenience, I temporarily set the tax rate equal to zero, term to maturity equal to unity and no capital adjustment cost.

2.3.1 Financial Pressure

The key feature of this model is that firms need working capital for their contemporaneous production and that the financial factor affects how much firms get constrained by the intra-period borrowing constraint. Most fascinating results of this paper root in the relation between the financial factor and short-term borrowing constraint. The binding case offers us the most straightforward example of this mechanism. Suppose the constraint (2.3) takes the equal sign. Combined with the budget constraint of entrepreneurs, the short-term borrowing constraint is rearranged as below:

$$F(k_t, h_t) = \left(\frac{\epsilon_t}{1-\epsilon_t}\right)[(1-\delta)k_t - b_t - w_t h_t - d_t] \tag{2.9}$$

When an adverse financial factor hit the economy, the value of ϵ_t decreases and so does $\frac{\epsilon_t}{1-\epsilon_t}$. If the collateral constraint is previously binding, either h_t or d_t has to decrease so that the equation (2.9) still holds, since k_t and b_t are both predetermined. As discussed in last section, entrepreneurs have motive to smooth dividend payout d_t over periods. It is unlikely that d_t adjusts quickly enough to absorb the impact of adverse financial factor completely. Thus, h_t has to decrease as well. This process shows vividly why a decline in the financial factor ϵ_t finally influences the labor input. Note that the equation (2.9) also tells an important conclusion on the relation between the outstanding long-term debt b_t and the short-term borrowing constraint. Given anything else stays intact, the relatively higher b_t could make the short-term borrowing constraint tighter. Therefore, the dynamic of outstanding debt, namely the financial structure, is in the center of our proposed mechanism and, more interestingly, the dynamics of b_t is endogenous.

Another immediate conclusion from observing the collateral constraint (2.3) is that the decrease of ϵ_t leads to the increase of μ_t. Given the same level of net-worth $(k_{t+1} - q_t b_{t+1})$, a smaller ϵ_t definitely shrinks the short-term borrowing capacity of firms, and μ_t increases accordingly. The above statement can also be explained more quantitatively by the following equation (2.10), which is derived from the equation (2.5),

$$\mu_t = \frac{1 - \beta \left(\frac{c_t^e}{c_{t+1}^e} \right)^{\sigma} / q_t}{\epsilon_t} = \frac{1 - r_t^f / r_t^e}{\epsilon_t} \tag{2.10}$$

It provides some interesting information. How much firms are financially constrained in the short-term is affected by the equity premium $1 - r_t^f / r_t^e$ (*endogenous*), where $r^f = \frac{1}{q_t}$ and $r_t^e = \frac{c_{t+1}^e}{\beta c_t^e}$, and financial factor ϵ_t (*exogenous*). If the equity premium always stays positive, then μ_t is positive and the short-term borrowing constraint is always binding.[6] It intuitively implies that firms would like to use the external funds as long as they are relatively cheaper than their own ones. Moreover, a decline in ϵ_t pushes up μ_t and an increase in the equity premium raises μ_t. These conclusions are quite intuitive. On the one hand, a bad financial environment obviously puts more obstacles for firms to obtain the working capital. On the other hand, a larger equity premium increases the marginal benefit of borrowing in the long-term, and makes entrepreneurs more reluctant to reduce their outstanding long-term debt when the bad financial factor arises. This change will give some favor in keeping the outstanding debt. If firms do keep more outstanding debt, the resulting decline in the net-worth would further weaken the capacity of entrepreneurs in short-term borrowing.

It is necessary to point out that we do not discuss the aggregate uncertainty in this paper and that all the economic agents are assumed to possess the perfect foresight (and

[6] A set of values of β and γ can easily ensure this condition is always satisfied. The quantitative solution of the model reveals that my calibrated β and γ from the data meet this requirement.

myopic foresight in the additional experiment). Consequently, the equity premium does not represent the risk premium as usual. At steady state, it reflects the different time preferences of workers and entrepreneurs, which are captured by the different values of γ and β. In dynamics, it also receives the influence from different consumption fluctuations of workers and entrepreneurs. However, my model setup ensures that the equity premium is still counter-cyclical. Take the case in which ϵ_{t+1} decreases for an example. If workers and entrepreneurs both anticipate this change, workers will try to purchase more bonds and entrepreneurs will try to cut outstanding debts. Therefore, the market price of long-term bond q_t increases and the bond yield r^f decreases. More importantly, as the bond return r^f decrease more than the willingness of entrepreneurs thanks to the bond demand pushed by workers, the equity premium should increase.

2.3.2 Demand Functions

The equation (2.11) gives the labor demand function in the standard format. A distinct wedge $1 - \mu_t$ appears on the right-hand side and reflects the impact of the short-term borrowing constraint. As the preceding analysis of μ_t states, the adverse change in financial factor, namely the decrease of ϵ_t, increases μ_t and correspondingly shifts the labor demand downwards. This conclusion is consistent with the analysis of the short-term borrowing constraint in the previous subsection.

$$w_t = F_{h_t}(1 - \mu_t) \tag{2.11}$$

The long-term bond demand is implicitly contained in the equation (2.12), which in fact reflects entrepreneurs' tradeoff between the short-term financial pressure and the cheap external funds. The left-hand side is the marginal benefit of borrowing an extra unit of long-term bond, namely the bond sale price net its influence on the intra-period borrowing constraint. The right-hand side is the corresponding marginal cost, namely the present value of bond repayment in the future. The left-hand side is totally predetermined or exogenous for entrepreneurs, since both μ_t and ϵ_t are the function of the aggregate state $\{k_t, b_t, \epsilon_t\}$ and q_t is the price of bond. The choice of the outstanding debt b_{t+1} only affects the right-hand side, precisely $\frac{c^e_t}{c^e_{t+1}}$.

$$(1 - \mu_t \epsilon_t) q_t = \beta \left(\frac{c^e_t}{c^e_{t+1}} \right)^{\sigma} \tag{2.12}$$

Unfortunately this equation does not provide the impact of the financial factors on the long-term debt obviously, since it is heavily entangled with the general equilibrium effect. However, the short-term borrowing constraint (2.9) can provide a shortcut to understand the dynamics of outstanding debt when the financial factors change. If the financial factor ϵ decreases, firms are inclined to increase the net-worth to mitigate the negative impact on the working capital financing. Therefore, a reasonable reaction of entrepreneurs is to cut the outstanding debt.

There is an extra benefit of exploring the equation (2.12), that is to learn more about the marginal utility loss of being financially constrained, $\mu_t \epsilon_t$. After the simple rearrangement of the equation (2.12), we can obtain the following equation:

$$\mu_t \epsilon_t = 1 - \frac{\beta \left(\frac{c^e_t}{c^e_{t+1}} \right)^{\sigma}}{q_t} = 1 - \frac{r^f}{r^e} \tag{2.13}$$

The equation (2.13) says that the marginal utility loss of being financially constrained is positively correlated with equity premium. The intuition is as follows: When firms get financially constrained, they are forced to change financial structure or to reduce labor input. The equilibrium condition requires that the marginal cost of these two options are equal. In term of adjusting the financial structures, i.e., cutting the outstanding debt, the marginal cost is to lose the access to cheaper external funds, that is to give up the equity premium. Thus, the equity premium can be employed to measure the marginal cost of being financially constrained. This conclusion is extremely important for the following analysis of the investment dynamics.

After simply manipulating the equation (2.4), the capital demand is stated by the equation (2.14). It implies that the change of capital demand is determined by the cost of being financially constrained on the left-hand side and the capital return on the right-hand side. Now I just use the case when ϵ is decreasing next period to illustrate how the financial factor change affects the investment dynamics. If entrepreneurs believe that the financial constraint is going to become tighter next period, then they expect a lower return to their investment that is captured by the decrease in $(1-\mu_{t+1})$. However, the marginal cost of being financially constrained today, $\mu_t\epsilon_t$, would increase, because the counter-cyclical equity premium is going to increase when ϵ decreases. The above two effects offset each other. Finally, we will not see the investment changes dramatically.

$$1-\mu_t\epsilon_t = \frac{(1-\mu_{t+1})F_{k_{t+1}}-\delta+1}{r_t^e} \tag{2.14}$$

Intuitively, the above analysis is backed by the double roles of capital in this economy, production inputs and collateral assets. The movement involved with μ_{t+1} and $\mu_t\epsilon_t$ can be respectively interpreted as entrepreneurs' consideration of production inputs and collateral assets. When a financial meltdown is coming, the production input side of capital requires a decrease in investment, while the collateral side of capital demands resist it. The ultimate outcome depends on which impact dominates. Therefore, the movement of investment is quantitatively determined by the concrete realization of the successive financial factors along the economic transition, which has to be investigated quantitatively.

2.4 Parametrization

The quantitative exercises in this paper are confined to corporate sector. Therefore all the parameters are set to match the moments of US corporate economy in the long run.

However, it does not imply that the mechanism or the phenomenon discussed in this paper existed only in corporate sector during the U.S. Great Depression. The primary reason for this restriction is the difficulty in obtaining consistent non-corporate data at that time. The data in this paper are mainly from *National Income and Product Accounts* (*NIPA henceforth*) and *Historical Statistics of the United State*[7] (HSUS henceforth). The detailed source for each variable is listed in the Appendix A. In the following subsections, I will introduce how I construct financial factors and deal with the calibration separately. All the data series are annual and accordingly one-period in the model corresponds to one year.

2.4.1 Financial Factors

The financial factor, which affects short-term borrowing constraint for working capital financing, is denoted by ϵ_t. In order to construct the series of ϵ_t in the 1930s, this paper employs the methodology suggested by Jermann and Quadrini (2012), that is to use the corporate output as a proxy of the intra-temporal loan and also assume that the collateral constraint is always binding throughout the economic transition. Then derive the financial factor ϵ_t in the formula below:

$$\epsilon_t = \frac{\text{Corporate output in year t}}{\text{End of period capital stock in year t} - \text{End of period long-term debt stock in year t}} \tag{2.15}$$

In practice, *corporate output* is the real value added by non-financial corporate business. *End of period capital stock* is represented by the nonresidential capital stock of nonfinancial corporations. *End of period long-term debt stock* is represented by the long-term debt stock of nonfinancial corporations. Figure 2.1 shows the financial factors obtained in the above procedures. I also include the classical total productivity factor series in the top plottings for comparison. The plotting in the middle gives the retrieved financial factor series and GDP in the era $1930 - 1940$. The bottom one shows the constructed TFP, financial factors and GDP

[7]There are many editions of this book. The reference in this paper is *Historical Statistics of the United States: millennium version*

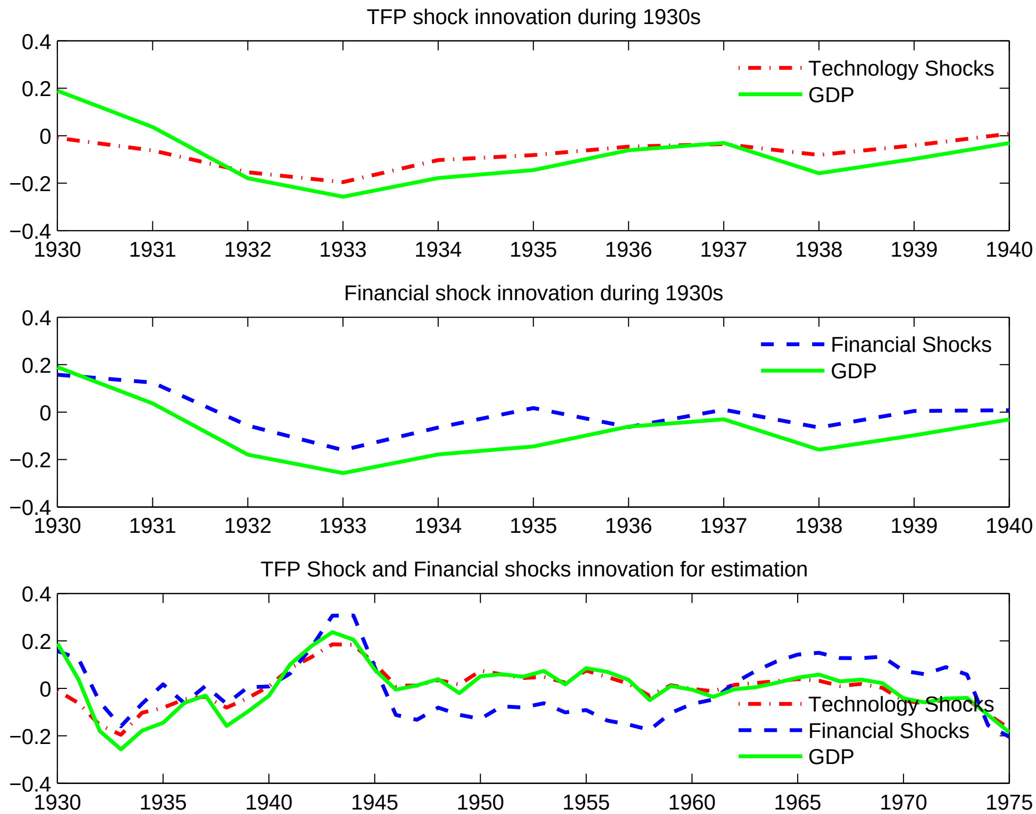

Figure 2.1: Constructed financial factors

in a wider time range $1930-1975$. All the series are transferred into the deviation from the long-term average level. We do not estimate the stochastic process of financial shocks and assume the rational expectation. It is the reason why ϵ_t is named the financial *factor* instead of the financial *shock* in this paper. Moreover, all the economic agents in this economy are assumed to possess the perfect foresight in benchmark case. Therefore, the quantitative experiment in this paper is similar to the work of Chari, Kehoe, and McGrattan (2002) on the total productivity factor (*TFP henceforth*), that is to account for the contribution of financial factor. As discussed in the introduction, such a measure of financial factor is able to reflect more financial information than the bank panics only.

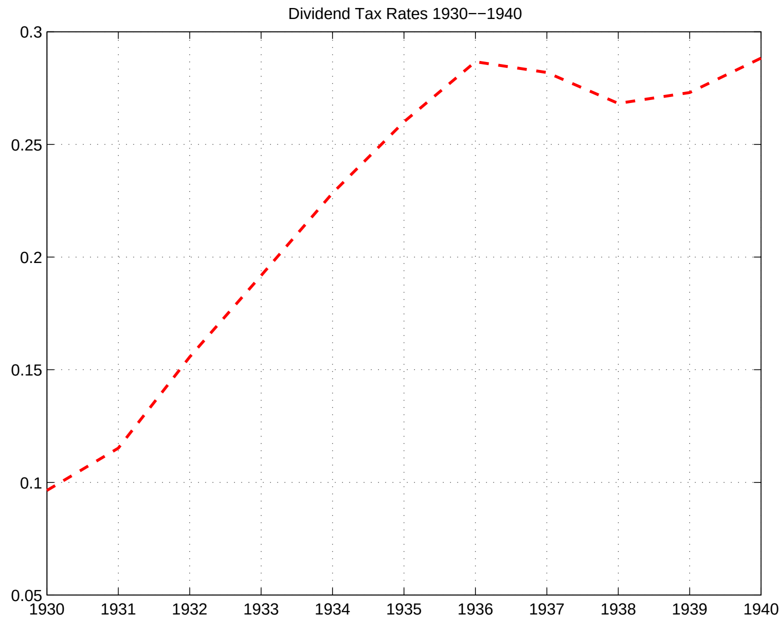

Figure 2.2: Dividend tax rates during 1930s

2.4.2 Tax Rates

The experiment involved with dividend taxes is also implemented in this paper. The increase of dividend tax τ_{dt} is considered important in explaining the U.S. Great Depression by McGrattan (2012). She discovers that the anticipated increase in dividend tax explains a large proportion of investment decline during 1930s with the help of a specific anticipation pattern. My quantitative analysis section will examine whether this conclusion is still robust in the presence of the financial turbulence and working capital financing. The dividend tax rate is taken from McGrattan (2012).[8] Figure 2.2 shows the trend of dividend tax rate in the 1930s. The dividend tax increased from 10% in 1930 to almost 30% a decade later.

2.4.3 Calibration

There are totally 8 parameters to set: $\{\theta, \delta, \beta, \gamma, \eta, \sigma, \alpha.\lambda\}$. θ is the parameter for Cobb-Douglas production function and $1 - \theta$ is set to equal the average long-term ratio of labor

[8]Her appendix provides the detailed methodology to impute the dividend tax.

Parameters		Targets 1930-1940	
θ	0.3371	Capital income share	0.37
β	0.9166	Capital-output ratio	2.98
γ	0.9891	Average equity premium	8.0%
η	2.4260	Working hours	0.3
α	2.3000	Relative investment volatility	2.4
δ	0.0629	Investment-output ratio	0.17
λ	0.1000	Average years to maturity	10
σ	5.0000	Elasticity of inter-temporal substitution	0.2

Table 2.1: Calibration

income over output. δ is the depreciation rate and targets the average long-term ratio of investment over capital. γ and β are the discount factors respectively for workers and entrepreneurs. It is not possible to identify these two parameters separately. So I assume they together match the average long-term capital-output ratio and the 1930s equity premium in Jagannathan, McGrattan, and Scherbina (2001). η is the weight of leisure in total utility of workers and ensures that the steady state working hours equals to 33% of a day. λ is set equal to 0.1 and represents that the average term to maturity is 10 years.

$$\Psi_k\left(\frac{x_t}{k_t}\right) = \frac{\bar{x}^{\alpha}}{1-\alpha} i_t^{1-\alpha} - \frac{\alpha \bar{x}}{1-\alpha} \tag{2.16}$$

σ is the elasticity of inter-temporal substitution for entrepreneurs and measures the risk aversion of entrepreneurs. Its value is taken from Jermann (1998). In the final computation experiment, we also introduce the investment adjustment cost so that the fluctuation of investment follows the classical stylized economic facts of U.S. economy. Its formation, the equation (2.16) follows the one proposed by Jermann (1998). $\bar{x}$ stands for the steady state investment level, and α is the shape parameters for the investment adjustment cost, making the volatility of investment around as 2.4 times as output. Table 2.1 lists the detailed values of parameters and their corresponding targets.

2.5 Quantitative Analysis

In this section I will first discuss the computation strategy and then the results of different quantitative experiments. The benchmark case hereafter refers to the model with term to maturity equal to ten year, the tax rates constant at the level of year 1930, and perfect foresight. The results of other experiments are going to be compared with the benchmark case so that we can identify the influence of various anticipation patterns, dividend taxes, and different maturity structures.

2.5.1 Computation

Instead of solving the policy functions and then simulating the model for limited periods, I directly solve for the transition path from the year 1930 to 1940. This solution strategy is also implemented in Atesagaoglu (2012). However, this kind of algorithm needs to impose a state the economy finally arrives at. Without loss of generality, I make it the steady state when the financial factor equals to the one in 1970, which is reasonably far enough to serve as an ending steady state. The economy is then assumed to take a large number of periods to transit from the initial state corresponding to the year 1930 to the ending steady state, during which the financial factors and dividend taxes are assumed to change in the first 10 periods as their counterparts in data did during the era $1930 - 1940$ and then stay constant at the level of the year 1970 along the remaining path. Then, I collect the equilibrium conditions of all periods and stack them together as a nonlinear equation system. Finally, apply the nonlinear solver to this system and obtain all the unknown endogenous variables. The concrete equation system are provided in Appendix Equation System. It is important to remind that this algorithm is valid only if the short-term borrowing constraint is always binding throughout the transition path, which is found to be true for all my exercises.

This computation strategy has to be modified in the exercises that are used to recognize the impact of myopic anticipations. If the agents in the economy all possess the myopic foresight, they believe the current financial factors are constant forever. This difference is going to affect the solution for the first 10 periods. The agents would consider transiting to the ending steady state under the current financial factor rather than under changing financial factor series. Particularly, in order to get the economic variables in period $T + 1$ ($T \leq 9$), I would solve for the full transition path from the Tth period state to the steady state under the financial factor of Tth period, and take the result of the second period of this path as the states for the period $T + 1$. Repeat this procedure for the first 10 periods. The computational algorithm for the remaining periods is totally the same with the one used in the benchmark case, because the financial factor stays the same anyway in these periods.

2.5.2 Benchmark

Figure 2.3 and Figure 2.4 illustrate the results of benchmark case. The solid line is data and the dash line is the result of benchmark model. In the Figure 2.3, the dynamics of all the economic aggregates seem to match data very well in timing. My model explains the recession in the early 1930s well. The working hours fall by more than 15% in the worst year 1933 and accounts for almost a half of the decline in data. This is a significant improvement compared with other new classical literature such as Cole and Ohanian (1999) and McGrattan (2012), in which their predicted working hours shows little match in either timing or magnitude. The dramatic decline in working hours is due to the mechanism discussed in Section 3. When the financial environment deteriorated, entrepreneurs failed to adjust their financial structures quickly enough because of their strong motive to smooth dividend payout over time. Then, they would rather cut the labor demand and outstanding debt at the same time to accommodate a tighter collateral constraint. The story for this economic dynamic can also be captured by the transition of the lagrangian multiplier μ_t in Figure 2.4. Moreover, the predicted decline in consumption is also significant, and around 50% of decline in data is explained by my model. It is because that working hours decline and reduction in dividend payout respectively decrease the consumption of workers and entrepreneurs.

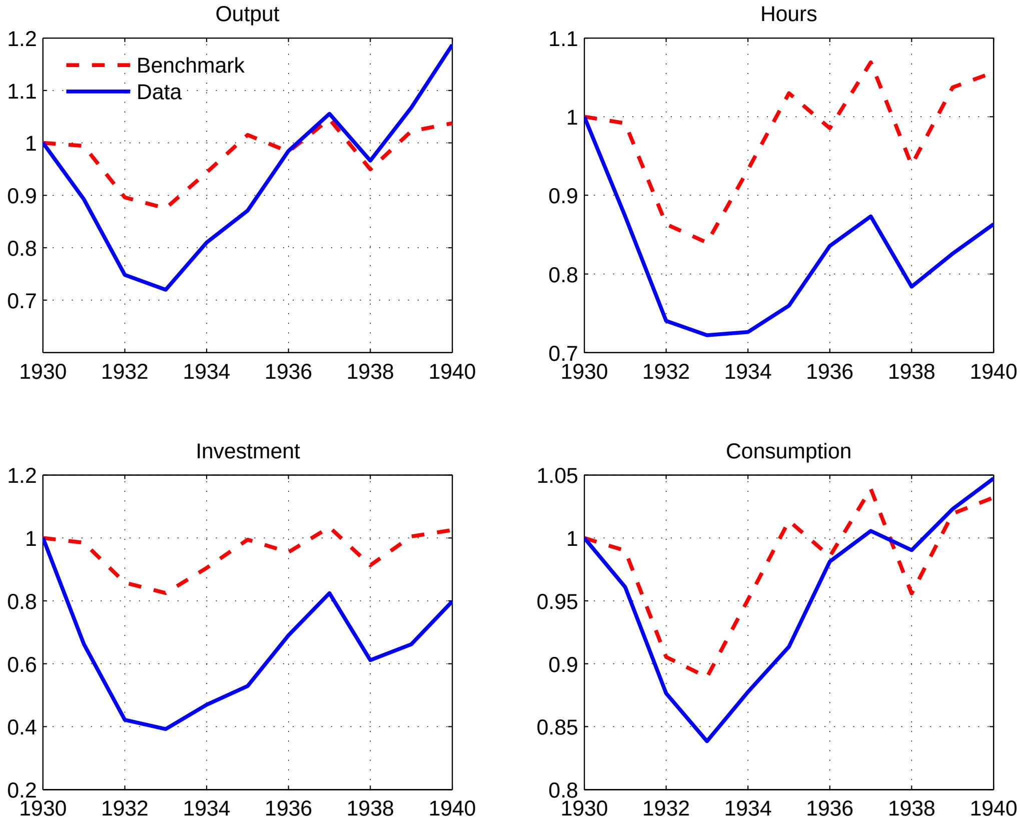

Figure 2.3: Results of the Benchmark Model: Aggregate Variables

However, the magnitude of predicted investment decline is not satisfactory. The maximum decline is only around 20% and accounts for a third of actual decline in data. The investment decreases as a result of the perceived bad economic outlook in the future. The adverse financial factor is going to restrict the productivity of capital in the future and then reduce the anticipated capital return. However, this effect is tremendously offset by the current financial distress. Firms need more collateral assets to resolve the difficulty in managing the short-term cash flow now. With a large decrease in working hours and small decrease in the investment (or capital stock), my model finally provides a moderate fall in the output. Figure 2.4 shows the predicted trends of financial and other variables. My model generally captures the decreasing trend of outstanding debt and the fluctuation of the dividend payout

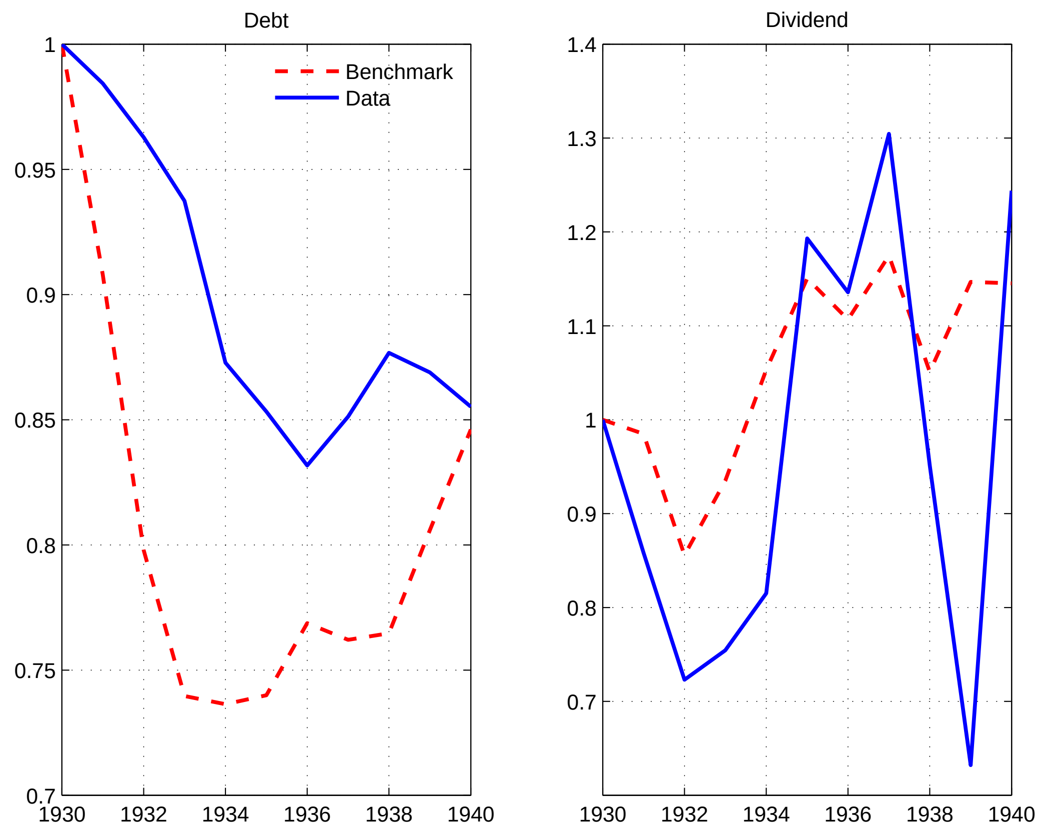

Figure 2.4: Results of the Benchmark Model: Financial Variables

in the 1930s.

2.5.3 Anticipation

In the benchmark case, I assume a perfect foresight for all the agents in this economy, that is entrepreneurs and workers know exactly the financial factors in the future. Nevertheless Jermann and Quadrini (2012) assume a rational expectation. It is important to find out whether the alternative anticipation pattern affects the quantitative results very much. For simplicity, I just take the most extreme alternative, the myopic foresight, to contrast my benchmark perfect foresight, that is entrepreneurs and workers believe that the current financial factor will stay constant for ever and the change of financial market in any period is a surprise for entrepreneurs and workers.

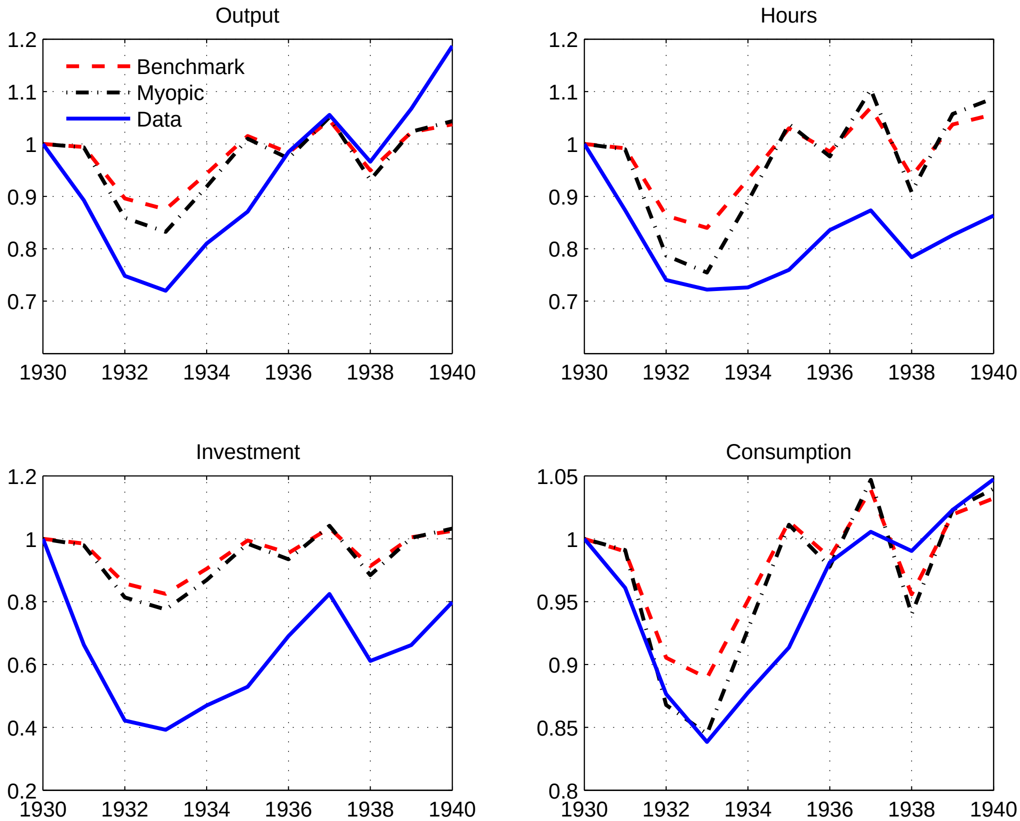

Figure 2.5: Impacts of anticipation patterns

Figure 2.5 shows that the predicted declines in all aggregates become more severe when a myopic anticipation is assumed, and match the data series much better. The predicted decline in working hours now accounts for more than 2/3 of the decline in data. The reason is the following. If entrepreneurs fail to foresee the severity of financial chaos next period, they are likely to be relatively more optimistic about the productivity and underestimate the value of consumption next period. Therefore, they might decrease less outstanding debt or even do not cut outstanding debt at all, which will hurt the short-term borrowing capacity for working capital[9] in the following period, given the same financial factor, and it aggravates the working hours decline. Due to the larger decline in working hours, the output, investment and consumption all show more declines. Therefore, the unexpected financial turbulence is much more devastating.

[9]See the equation (2.9).

2.5.4 Maturity Structure

In the benchmark case, we simply assume $\lambda = 0.1$, which indicates that the long-term debt due in 10 years (periods) on average, because there is no data source to estimate the accurate average term to maturity in the 1930s. However, Atesagaoglu (2012) shows that different long-term debt maturity structures can affect the firms' decision on financial structure. The dynamics of corporate financial structure is very crucial to the mechanism proposed by this paper. Thus, it is necessary to show the sensitivity of our quantitative results to different debt maturity structure. For simplicity, I only take one additional case to exhibit the impact of maturity structure, $\lambda = 0.2$, which represents that the long-term bond matures in 5 years on average. Usually there is a major difficulty to model the long-term bond in a tractable way, one has to follow the whole history of the debt maturity structure and confront the extremely heavy computational burden. In order to remove this obstacle, I adopt the way to model the long-term bond in Leland. (1994), Philippon (2009), and Miao and Wang (2010).

According to the plotted results in Figure 2.7, the impact of working capital financing is aggravated when the average term to maturity increases. This result is not surprising. Once the term to maturity becomes longer, smaller fraction of outstanding long-term debts are supposed to retire each period. It is equivalent to decreasing the marginal cost to borrow in the long-term. So entrepreneurs are less willing to reduce their outstanding debt when the adverse financial factors prevail. The analytical analysis of the equilibrium in the previous section shows that entrepreneurs have two ways to accommodate the effect of change in financial factor, adjusting the outstanding debt or reduce labor input. The decrease in the marginal cost to borrow makes the financial structure adjustment less in favor. Therefore, the labor demand falls by more, and the declines of aggregate economy become worse.

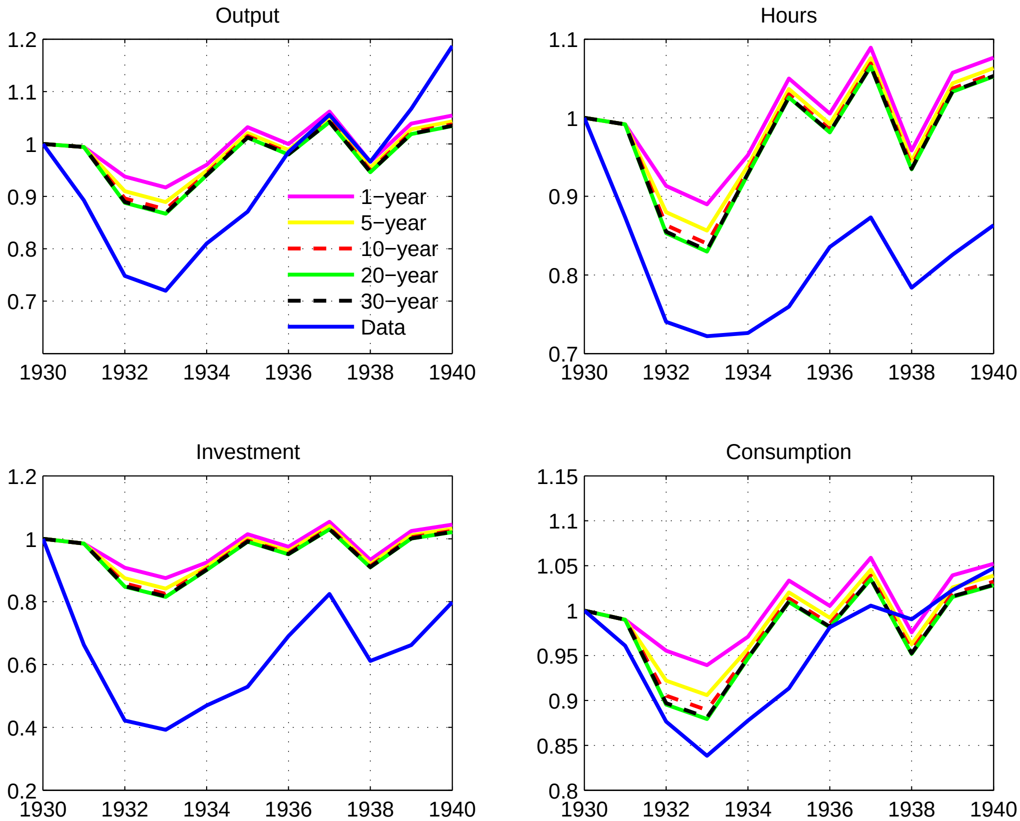

Figure 2.6: Impacts of maturity structure

2.5.5 Dividend Taxes

Although the traditional economics wisdom, e.g., Cole and Ohanian (1999), claims that the fiscal policy plays little role during the U.S. Great Depression, McGrattan (2012) recently shows that the anticipated increase in the dividend tax rate τ_{dt} can contribute a lot to the investment collapse in the 1930s. However, she demonstrates the strength of this theory using a standard RBC model and also requires a specific anticipation patterns to help in timing. The next experiment is going to investigate whether her conclusion still holds if the financial factors and working capital financing are taken into account. According to the previous analysis, the investment affects the size of collateral assets for working capital, while the dividend tax has been proved to be an important factor in determining the dynamics of investment during the U.S. Great Depression. Therefore, it is quite interesting to check whether there is some interaction between the impact of dividend tax reform and working

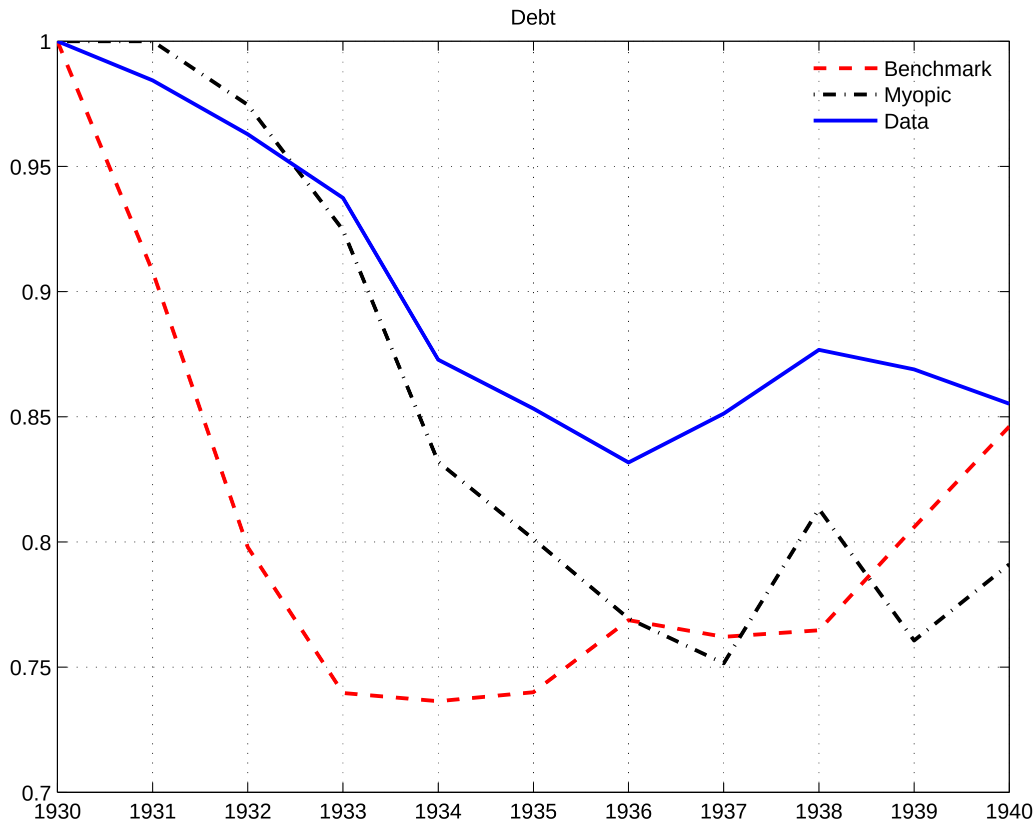

Figure 2.7: Impacts of maturity structure: outstanding debts

capital financing.

Figure 2.8 tells that the impact of the dividend tax on investment almost disappears in my model, which is contrast to the conclusion of McGrattan (2012). The solid line represents the data, the dash line represents the benchmark case and the dash-dot line represents the case with changing dividend tax rate. All the predicted aggregates in this experiment is more or less the same with the one in the benchmark case and only differs from the benchmark case by several percents. It is also due to the double roles of capital. Regardless of the anticipated decrease in the return to capital, which is caused by financial factor decline as well as the dividend tax this time, entrepreneurs are reluctant to reduce their investment too much, because it could sacrifice lots of invaluable collateral assets and aggravate the trouble in working capital financing. Then, the financial frictions that are absent in the standard

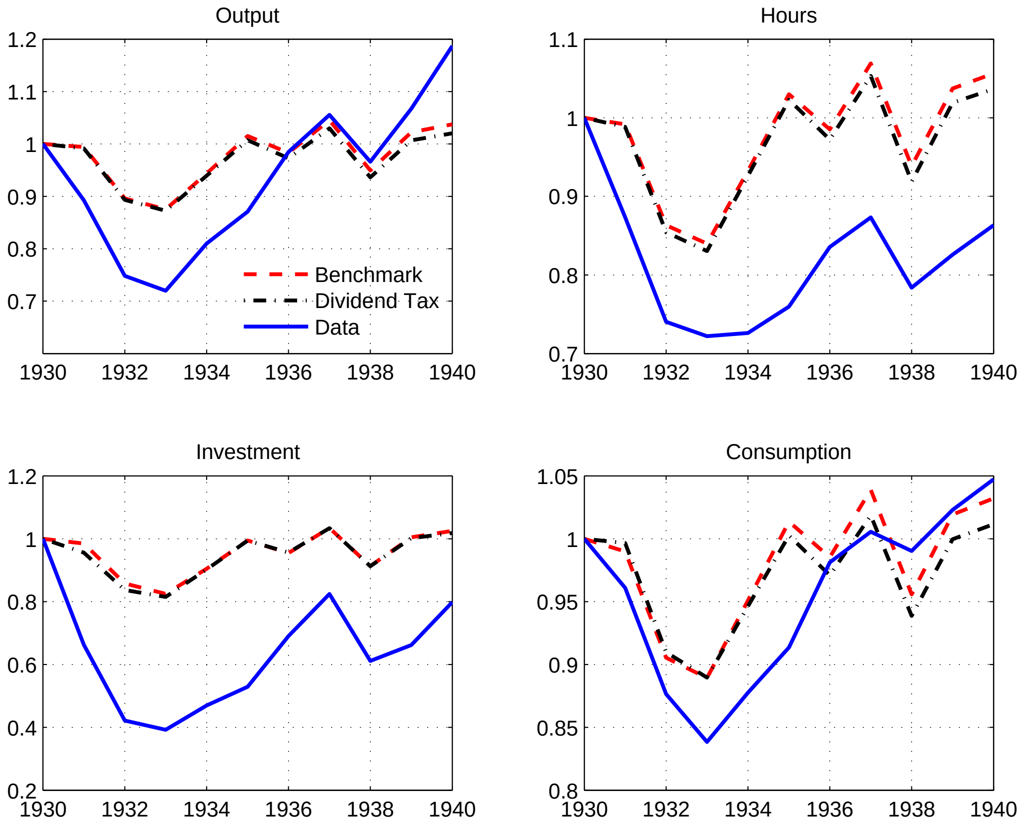

Figure 2.8: Impacts of Dividend Taxes

RBC model could undermine the impact of the mechanism proposed in McGrattan (2012).

2.6 Conclusion

This paper investigates the role of working capital for the propagation of the financial crisis to the real economy during the U.S. Great Depression. As in the recent literature on the macroeconomic effects of financial shocks (see e.g. Jermann and Quadrini (2012) and Perri and Quadrini (2011)), we refer to working capital as the amount of liquid assets that are used to manage the short-term cash flow mismatches. Following this literature, we build a general equilibrium model in which dynamic firms borrow against their net-worth to invest in working capital, while their borrowing constraint is affected by a financial shock that is obtained directly from the data. Our hypothesis is that the scarcity of working capital during the US Great Depression contributed to the propagation of the financial distress to the real sector. More precisely, when the financial crisis reduced the availability of short-term financing, an important source of working capital, firms experienced a reduction in their disposable cash flow. As they could not adjust their financial structure promptly to ease the liquidity tension, firms were ultimately forced to cut labor inputs and production, generating a larger contraction. The main quantitative experiment shows that this mechanism explains almost 50% of the working hours decline in the early 1930s, and it also predicts considerable declines in consumption and output.

Several additional experiments are implemented to identify the impact of anticipation, different long-term debt maturity structures, and increasing dividend tax rates. First, when the financial meltdown is unexpected or when the average term to maturity of long-term debt is lengthened, the mechanism described above gets aggravated. This is because firms get more restrained in realigning their financial structure under these two scenarios, implying that they have to cut more labor inputs and production in the midst of a cash flow drought. Secondly, when dividend taxes increase, we find that investment does not collapse, in contrast to the literature (see e.g. McGrattan (2012)). The reason is that capital serves as a production input as well as collateral for working capital financing. If firms cut investment too much due to the capital return decline caused by the dividend tax increase, the volume of collateral assets shrinks and the difficulty in financing working capital intensifies. This implies that the mechanism outlined by McGrattan (2012) could be potentially offset by financing frictions.

Although never discussed seriously before, the importance of our theory has actually been implied in Bernanke's works for a couple of times. For example, Bernanke (2004) says, "*a financially distressed firm may not be able to obtain working capital necessary to expand* *production, or to fund a project that would be viable under better financial conditions.*"[10] and Bernanke and Carey (1996) says, "in the spirit of the financial crisis story, it may be that "high" nominal wages had their depressing effect on output primarily by increasing financial pressures (i.e., cash-flow) on firms, rather than through the conventional labor cost channel."[11] This paper just confirms and evaluates its power quantitatively. Besides, the significant working hours decline implies that the difficulty in working capital financing represents an alternative theory that can explain the sluggish labor market in the 1930s, besides sticky wages and policy uncertainty.

Nonetheless, the model in this paper is parsimonious for now, which partly compromises its explanatory power and the potential for policy evaluation. Therefore, several extensions can be implemented. First, development of a comprehensive model incorporating both short-term financing and long-term default risk management. In this way, we will be able to capture better the simultaneous decline in both investment and working hours. Secondly, addition of cash or other liquid assets that are very critical for the short-term corporate financial management. Finally, investigation of the impact of monetary or taxation regime shifts during the U.S. Great Depression, once the model gets better equipped.

[10] See Bernanke (2004), the second paragraph on the page 25.

[11] See Bernanke and Carey (1996), the last paragraph of conclusion remarks.

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2.7 Appendix

2.7.1 Data

- Nominal nonfinancial corporate capital stock: BEA FA6.1 Line 4
- Real nonfinancial corporate capital stock: BEA FA6.2 Line 3
- Real nonfinancial corporate investment: BEA FA6.8 Line 4
- Nominal nonfinancial corporate capital depreciation: BEA FA6.4 Line 4
- Real nonfinancial corporate capital depreciation: BEA FA6.5 Line4
- Nominal nonfinancial corporate production: BEA NIPA1.14 Line 17
- Real nonfinancial corporate production: BEA NIPA1.14 Line 41
- Nominal nonfinancial corporate long-term debt: Susan B. Carter (2003) data series $cj870 - 889$
- U.S. population between 18 and 65: Susan B. Carter (2003) data series $Aa125 - 144$
- U.S. private non-farm total man-hours $1929 - 1953$: Kendrick (1961)
- U.S. private non-farm total man-hours $1948 - 1966$: Kendrick (1973)
- U.S average weekly private working hours $1964 - 1976$: BLS table ID EES00500005
- Nominal durable good consumption and services: BEA NIPA 1.1.5 Line 5 and Line 6
- GDP deflator: BEA NIPA1.1.5 Line 1
- Corporate Cash: Statistics of Income[12]
- Corporate inventories: Statistics of Income

[12] see the archive in the links http://www.irs.gov/uac/SOI-Tax-Stats-Archive—1934-to-1953-Statistics-of-Income-Report,-Part-2

2.7.2 Equation System

$$d_t + \lambda b_t - F(k_t, h_t) + x_t - q_t[b_{t+1} - (1-\lambda)b_t] = 0 \tag{2.17}$$

$$d_t - c_t^e - \Gamma_{dt} = 0 \tag{2.18}$$

$$k_{t+1} = (1-\delta)k_t + \Psi\left(\frac{x_t}{k_t}\right)k_t \tag{2.19}$$

$$F(k_t, h_t) = Ak_t^{\theta}h_t^{1-\theta} \tag{2.20}$$

$$\Gamma_{dt} = \tau_{dt}d_t \tag{2.21}$$

$$\frac{1-\tau_{dt}}{\Psi_{x_t}k_t} - \mu_t\epsilon_t = \beta\left(\frac{c_t^e}{c_{t+1}^e}\right)^{\sigma}\left[(1-\tau_{dt+1}-\mu_{t+1})F_{k_{t+1}} + \frac{(1-\tau_{dt+1})(\Psi_{k_{t+1}}k_{t+1} + \Psi_{t+1} + 1 - \delta)}{\Psi_{x_{t+1}}k_{t+1}}\right] \tag{2.22}$$

$$(1-\tau_{dt}-\mu_t\epsilon_t)q_t = \beta\left(\frac{c_t^e}{c_{t+1}^e}\right)^{\sigma}(1-\tau_{dt+1})[q_{t+1}(1-\lambda)+\lambda] \tag{2.23}$$

$$\epsilon_t(k_{t+1} - q_t b_{t+1}) - F(k_t, h_t) = 0 \tag{2.24}$$

$$c_t^w + q_t[b_{t+1} - (1-\lambda)bt] = w_t h_t + \lambda b_t + Tr_t \tag{2.25}$$

$$\frac{w_t}{c_t^w} = \frac{\eta}{1-h_t} \tag{2.26}$$

$$\frac{q_t}{(1-\lambda)q_{t+1}+\lambda} = \gamma\frac{c_t^w}{c_{t+1}^w} \tag{2.27}$$

$$Tr_t - \Gamma_{dt} = 0 \tag{2.28}$$

$$\Psi\left(\frac{x_t}{k_t}\right) = \frac{\delta^{\alpha}}{1-\alpha}\left(\frac{x_t}{k_t}\right)^{1-\alpha} - \frac{\alpha\delta}{1-\alpha} \tag{2.29}$$

Chapter 3

Default Risks, Investment, and the US Great Depression

3.1 Introduction

The U.S. Great Depression has attracted enormous economics research interest because of its mysterious, abrupt, dramatic and persistent downswings in almost all the important economic indicators. Economists traditionally look for the explanations from many perspectives such as rigid wages or prices, monetary restriction, financial turbulence, and comprehensive and intense fiscal or regulatory overhauls. Although none of them turn out to be fully responsible for the most severe recession in the U.S. history, the financial channel (*"not just the financial shocks"*) has been considered a favorite area to reveal the answers to all the puzzles, especially after the innovative works by Ben Bernanke[1]. It has been quite well accepted that "financial collapse is more than a symptom of economic decline." Much research has been devoted to establishing a connection between financial sector unease and real sec-

[1] In 1980s, he wrote a series of papers exploring the impact of financial factors during the U.S. Great Depression. Most importantly, Bernanke (1983) argues that debt deflation and simultaneous sales of financial assets hurt the balance-sheet of banks or financial institutions and make them either fail or tighten their credit supply. In the presence of financial market imperfection, which makes professional financial intermediaries the only ones to allocate capital efficiently, the destruction of financial intermediaries could reduce investment and production.

tor sloppiness in the U.S. Great Depression as well as investigating how financial concerns affected the decision of different agents. The study of the indebted business or household has won an extraordinary place among all these efforts.

The following are some prominent papers in this direction[2]: Fisher (1933) for the first time switches our focus to the debt market. He suggests that the liquidation debt selling in an over-indebted environment could trigger a nationwide deflation, as the dramatic decrease in outstanding loans reduces the velocity of money circulation. The fall of the general price level might lead to a significant loss in production, employment and net-worth, which would further reinforce the motivation for liquidation selling and therefore cause another round of liquidation debt selling. The worst scenario is "*the very effort of individuals to lessen their burden of debts increases it, because of the mass effect of stampede to liquidate in swelling each dollar owed*", namely a debt-deflation cycle. Frederic Mishikin shows that theories of consumer expenditures can postulate a link between household balance-sheet change and decrease of aggregate consumption in the 1930s. Mishkin (1978) exposits that the nondurable consumption could be reduced by the decline of household net-worth according to the permanent income hypothesis, while durable consumption could be suppressed by the households' demand on liquidity[3]. Yet it is far from a well accomplished mission to understand the impact of debt during the U.S. Great Depression. There are still many aspects for successors to fill in. First, Fisher (1933) and Mishkin (1978) focus on the decline of output, working hours, net-worth, and consumption, and omit the business investment collapse; Second, most of the early papers about financial factors during the U.S. Great Depression rest their conclusions loosely on proposed theories and fail to offer structural models for quantitative analysis. Third, the existing theories lack a deeper and closer examination of the corporate bond market despite the fact that the corporate bond market actually experienced the same significant

[2]A more comprehensive survey by Calomiris (1993) provides the summaries and comments on almost every article about the financial factors and the US Great Depression.

[3]This mechanism is named "Liquidity Hypothesis" and discussed in details by Mishkin (1976).

catastrophe and, more importantly, interacted with the real economy. Giesecke et al. (2011) shows that two corporate bond default peaks occurred during the 1930s and that the one between 1931 and 1935 was the second worst in the last 150 years. Hunter (1982) shows that the spending stream of large firms was drained away during the U.S. Great Depression because they had to raise liquidity in anticipation of a perceived default risk in the future; Hart and Mehrling (1995) proposes a hypothesis: "When a decline is under way, business men whose debts fall due in the visible future are obliged to do their best to remain liquid, which holds down business volume". In the meantime, the default risk is also found by Miao and Wang (2010) to be a powerful tool to understand the business cycles in a more general framework and wider time range. My paper is to improve on the above three fields, that is to build a well-parameterized quantitative structural model and shed some light on the relationship between business investment collapse and corporate bond default risk.

This paper builds a rational expectation DSGE model subject to the TFP and financial shock[4]. There are three agents: entrepreneurs, firms and workers. Entrepreneurs own the firms. Firms own the production capital and technology, rent labor and issue bonds. Workers buy bonds and supply labor. A credit shock identical and independent over firms and time hits the firms each period and randomly generates a financial expenditure proportional to their capital stock. Before making the investment and issuing new bonds, firms determine whether to default on their debt obligation after observing credit and financial shocks. Upon default they redeem the ownership of their firms through costly negotiation and restructure process and continue their operation. After calibrating the model to the long-run economic facts of the U.S. economy and feeding the actual TFP and financial shock series into simulation, the quantitative results demonstrate that the adverse technology shocks could be tremendously aggravated by the default pressure on the corporate sector. On the one hand, the massive wave of corporate bond defaults directly idled a considerable amount of capital,

[4]There are various ways to model and measure the financial shocks. The one I implement here is proposed by Perri and Quadrini (2011) and Jermann and Quadrini (2012).

which was detrimental to production, investment and employment. On the other hand, the indebted firms were inclined to cut more investment during the economic downturn, as they were also concerned about the increasing default risk besides the awful economic outlook. Besides, the default recovery rate decline caused by the financial turbulence fails to explain economic aggregates downswings well, but does contribute greatly to the awful risky corporate bond yield. More interestingly, the climbing of debt-capital ratio seems beyond the best interest of firms. It is a possible factor to deteriorate the recession. My model setups are basically built on Cooley and Quadrini (2001) and Miao and Wang (2010). Because the main focus of this paper is macroeconomic fluctuation, some specific features matching firm cross-sectional distribution and financial structure are eliminated while some additional properties are introduced. However, their key mechanism stays similar.

Finally, I want to remind readers that this paper does not intend to address the source or the nature of adverse technology and financial shocks during the U.S. Great Depression or to attribute the economic downturn completely to the firm default risk. Instead, I'm more interested in the following questions: how are these shocks propagated and amplified by the default risk and quantitatively how much of economic contraction, especially the investment decline, can be explained by this channel? My quantitative practice is restricted to the corporate sector. It does not mean that the mechanism proposed here is just within the corporate sector. It is because I have access only to the corporate data. The paper is organized as below: Section 2 presents the model; Section 3 characterizes the equilibrium properties; Section 4 introduces the data and parametrization; Section 5 illustrates the quantitative results and also provides some discussion. The final part concludes.

3.2 Model

I consider an infinite-horizon and discrete-time economy. Three types of agents live forever: entrepreneurs, workers and firms. Entrepreneurs exclusively own the firms. Firms have the production capital and technologies in this economy. They rent labor from workers and issue long-term bonds. Workers earn labor income from wages and receive bond payment. The detailed features of each agent are discussed in the following subsections.

3.2.1 Entrepreneurs

Entrepreneurs do not provide labor supply in this economy. Their problem is as follows:

$$\max_{\{c_t^e, s_{t+1}^j\}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{e\,1-\nu}}{1-\nu} \tag{3.1}$$

$$\text{subject to:} \quad c_t^e + \sum p_t^j s_{t+1}^j \leq \sum (p_t^j + d_t^j) s_t^j$$

c_t^e is the consumption of entrepreneurs at period t. s_t^j represents entrepreneurs' share holdings of firm j. p_t^j and d_t^j respectively stands for the share price and of firm j. The first order conditions with respect to s_{t+1}^j is:

$$\beta \left(\frac{c_{t+1}^e}{c_t^e} \right)^{-\nu} = \frac{p_t^j}{p_{t+1}^j + d_{t+1}^j} \tag{3.2}$$

Entrepreneurs are homogeneous. Therefore the value of s_t^j equals to unity at equilibrium. Namely the representative entrepreneur possesses all the firms at the same time. Thus it is easy to get $c_t^e = \sum d_t^j$ at equilibrium. Define $D_t = \sum d_t^j$. Then $c_t^e = D_t$. Consequently, the first order condition can be transferred into:

$$\beta \left(\frac{D_{t+1}}{D_t} \right)^{-\nu} = \frac{V_t^j - d_t^j}{V_{t+1}^j} \tag{3.3}$$

and

$$V_t^j = d_t^j + \beta \left(\frac{D_{t+1}}{D_t} \right)^{-\nu} V_{t+1}^j \tag{3.4}$$

where V_t^j is the firm value including the dividend payout. Therefore the discount factor for firms is $\beta \left(\frac{D_{t+1}}{D_t} \right)^{-\nu}$.

3.2.2 Firms

The value of firm j at period t is given by $V(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t)$. k_t^j, b_t^j,and z_t^j are the individual state variables and respectively stand for the capital stock, outstanding debt and credit risk. A_t and ϵ_t are the aggregate TFP and financial shocks. Firms decide to default on their long-term bond if and only if $\tilde{V}(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t) < 0$. Equation (3.5) shows that the firm value equals to 0 if default and otherwise $\tilde{V}(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t)$. The bellman equation (3.6) defines $\tilde{V}(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t)$. It consists of three parts: operating profit $\pi(k_t^j, z_t^j; A_t)$, debt payment $-(1-\lambda)\vartheta b_t^j - \lambda b_t^j$ and continuing value $J(k_t^j, b_t^j; A_t, \epsilon_t)$. I assume a multiple-period long-term debt setup following Leland. (1994): $1/\lambda$ represents the terms to maturity; Firms pay back only λ of their outstanding debt and get charged a coupon payment at the rate ϑ over the remaining $1-\lambda$ each period.

$$V(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t) = \max\{0, \tilde{V}(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t)\} \tag{3.5}$$

$$\tilde{V}(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t) = \pi(k_t^j, z_t^j; A_t) - (1-\lambda)\vartheta b_t^j - \lambda b_t^j + J(k_t^j, b_t^j; A_t, \epsilon_t) \tag{3.6}$$

$$\pi(k_t^j, z_t^j; A_t) = \max_{h_t^j}\{F(A_t, k_t^j, h_t^j) - w_t h_t^j - z_t^j k_t^j\} = (R_t - z_t^j)k_t^j \tag{3.7}$$

$$J(k_t^j, b_t^j; A_t, \epsilon_t)$$

$$= \max_{\{k_{t+1}^j, b_{t+1}^j\}} \{q_t[b_{t+1}^j - (1-\lambda)b_t^j] - x_t^j - \Gamma(k_{t+1}^j, b_{t+1}^j) + E_t \frac{\beta U'(d_{t+1})}{U'(d_t)} V(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j; A_{t+1}, \epsilon_{t+1})\}$$

$$\text{subject to: } k_{t+1}^j = (1-\delta)k_t^j + \Psi_k \left(\frac{x_t^j}{k_t^j} \right) k_t^j \tag{3.8}$$

$$d_t^j = (R_t - z_t^j)k_t^j - \lambda b_t^j + q_t \Delta b_t^j - x_t^j - \Gamma(k_{t+1}^j, b_{t+1}^j) \tag{3.9}$$

$$F(k_t, n_t) = A_t^T k_t^\theta n_t^{1-\theta} \tag{3.10}$$

The operating profit $\pi(k_t^j, z_t^j)$ is a intra-period optimization problem given the aggregate price and state variables. It is affected by the exogenous stochastic credit risk z_t^j, which can be considered as the financial cost relevant to the short-term (intra-period) finance that is not captured by this model explicitly. It is possible to solve for the labor demand of firms n_t^j first and convert the maximization format of $\pi(k_t^j, z_t^j)$ into a regular function as in equation (3.7) if I employ the homogeneity property of the Cobb-Douglas production. R_t is the marginal return to the capital. It is also a function of k_t/n_t as the wage rate w_t and also identical for different firms as a result. If firms decide not to default, they immediately issue new long-term bond $q_t[b_{t+1}^j - (1-\lambda)b_t^j]$ and make investment x_t. $\Gamma(k_{t+1}^j, b_{t+1}^j)$ represents the financial cost, including the transaction commission and financial position adjustment cost[5]. The expected firm value next period is discounted with entrepreneurs' inter-temporal marginal rate of substitute in dividend payout $\beta\left(\frac{D_{t+1}}{D_t}\right)^{-\nu}$. The feature of equation (3.8) is taken from Jermann (1998). The adjustment cost $\Psi_k(i_t)$ is increasing and concave.

Because firms default if and only if $\tilde{V}(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t) < 0$, the solution for the following equation (3.11) is the default trigger. If and only if the realization of z_t is larger than $\tilde{z}_t$, the optimal choice for the entrepreneurs is to claim default. In consequence the default probability is $\Pr\{z_t > \tilde{z}_t\}$.

$$(R_t - \bar{z}_t)k_t^j - b_t^j[(1-\lambda)\vartheta + \lambda] + J(k_t^j, b_t^j) = 0 \tag{3.11}$$

Furthermore recall that the firm value $V(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t)$ is 0 upon default and $\tilde{V}(k_t^j, b_t^j, z_t^j; A_t, \epsilon_t)$

[5]The detailed specification of financial cost and capital adjustment cost are discussed in the later subsection when the model is transferred into a solution-friendly format.

otherwise. The above information can help reduce the expected firm value next period.

$$
\begin{aligned}
E_t V(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j) &= \int_{\tilde{z}_{t+1}^j}^{z_{\max}} 0 d\Phi(z) + \int_{z_{\min}}^{\tilde{z}_{t+1}^j} \tilde{V}(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j) d\Phi(z) \\
&= k_{t+1}^j \int_{z_{\min}}^{\tilde{z}_{t+1}^j} (\tilde{z}_{t+1}^j - z) d\Phi(z)
\end{aligned}
\tag{3.12}
$$

where $\Phi(z)$ is the CDF of random variable z.

3.2.3 Workers

Workers have a instantaneous utility $U(c_t, n_t) = \log(c_t) + \eta \log(1 - n_t)$ as in King, Plosser, and Rebelo (1988)[6]. Their discount factor over time γ is assumed to be larger than the one of entrepreneurs β, which implies that workers are more patient and therefore ask for a lower return on their savings. This is the reason why entrepreneurs borrow from workers at equilibrium. Wage earnings $w_t n_t$, bond payment and financial service fees are their income sources. Their asset pricing kernel and labor supply are listed below:

$$
\Lambda_t^w = \frac{1}{c_t^w} \tag{3.13}
$$

$$
\frac{w_t}{c_t^w} = \frac{\eta}{1 - n_t} \tag{3.14}
$$

c_t^w is the consumption of workers. The workers consider the default risk when they determine the bond price. There are two possible scenarios for each bond b_{t+1}^j: On the one hand, firms j can fulfill the obligation with a probability $\Phi(\tilde{z}_{t+1})$, that is λ of the outstanding debt retires and $1 - \lambda$ stays circulating in the market and only pays the coupon payment; On the other hand, it default with a probability $1 - \Phi(\tilde{z}_{t+1})$. Under the circumstances of default, workers claim all the operating profit and takeover the ownership of the firms

[6]They find that $U(c_t, L_t) = \frac{c_t^{\iota} \upsilon(L_t)}{1-\iota}$ can capture the stylized facts of U.S. business cycles under particular assumptions, where L_t is leisure and equals to $1 - N_t$. Specifically, υ need to be increasing and concave if $0 \leq \iota < 1$; υ need to be decreasing and convex if $\iota > 1$; υ need to be increasing and concave if $\iota = 1$, that's the utility function in the same logarithm form as in this paper.

temporarily. After they reduce the debt level to $\epsilon_t b_t$ through negotiation with creditors, firms are going to redeem their ownership and restore another round of operation. The negotiation and restructure process brings cost and cut the capital equal to $(1-\epsilon)k_t$[7]. Therefore the initial state of the restructured firm j is $(\epsilon_t k_t^j, \epsilon b_t^j)$ and corresponding continuing value is $J(\epsilon_t k_{t+1}, \epsilon_t b_{t+1})$. The no arbitrage condition ensures the following bond price determination equation:

$$\begin{aligned} & q_t b_{t+1} \\ =& \gamma \frac{\Lambda_{t+1}^w}{\Lambda_t^w} \left[b_{t+1}[\lambda + (1-\lambda)(\vartheta + q_{t+1})]\Phi(\tilde{z}_{t+1}) + \int_{\tilde{z}_{t+1}}^{z_{\max}} \pi_{t+1} + J(\epsilon k_{t+1}, \epsilon b_{t+1}) d\Phi(z) \right] \end{aligned} \tag{3.15}$$

where Λ_{t+1}^w is the pricing kernel of workers. Note the second term on the righthand side could not be directly multiplied by $1-\Phi(\tilde{z}_{t+1}^j)$ as $\pi(k_{t+1}^j, z_{t+1}^j)$ is a function of $\bar{z}_{t+1}^j$.

3.2.4 Transformation and Optimization

As Miao and Wang (2010) shows, the model with the above features satisfies the linear homogeneity. Namely all the equations still hold if they are divided by the same non-zero value, say k_t^j. Such mathematical manipulation decreases the dimension of state space in the original model. I follow the same procedures they suggest and reduce the individual state set from $\{k_t^j, b_t^j, z_t^j\}$ to $\{\varpi_t^j, z_t^j\}$, where $\varpi_t = b_t/k_t$. Besides, all the firms in this economy indeed face the same problem so I just eliminate the superscript j from now on for convenience. The trigger value $\tilde{z}_t$ determination equation is now:

$$R_t - \tilde{z}_t - \varpi_t[(1-\lambda)\vartheta + \lambda] + J(\varpi_t) = 0 \tag{3.16}$$

[7]This assumption is to obtain a unchanged leverage ratio for the convenience of computation. We will discuss this issue later in the transformation and optimization subsection.

and the continuing operation value determination is into:

$$
\begin{aligned}
J(\varpi_t) = \max_{\{\varpi_{t+1}, i_t\}} \{ & q_t[\varpi_{t+1} g(i_t) - (1-\lambda)\varpi_t] - i_t - \Gamma(\varpi_{t+1}) g(i_t) \\
& + E_t \beta \left(\frac{D_t}{D_{t+1}} \right)^{\nu} g(i_t) \int_{z_{\min}}^{\tilde{z}_{t+1}} (\tilde{z}_{t+1} - z) d\Phi(z) \}
\end{aligned}
\tag{3.17}
$$

where

$$g(i_t) = 1 - \delta + \Psi_k(i_t), \quad \text{and} \quad i_t = \frac{x_t}{k_t}$$

Again,the financial cost $\Gamma(\varpi_{t+1})$ includes the commission and financial position adjustment cost. The former one is proportional to the market value of the outstanding long-term debt while the latter one is just the cost to push debt-capital ratio to deviate from the steady state as in Miao and Wang (2010).

$$\Gamma(\varpi_{t+1}) = \psi q_t \varpi_{t+1} + \frac{(\varpi_{t+1} - \bar{\varpi})^2}{2} \tag{3.18}$$

I also assume a capital adjustment cost in the following format:

$$\Psi_k(i_t) = \frac{\bar{i}^{\alpha}}{1-\alpha} i_t^{1-\alpha} - \frac{\alpha \bar{i}}{1-\alpha} \tag{3.19}$$

where $\bar{i}$ is the steady state investment-capital ratio. After transforming the individual firm problem, it is straightforward that the key for the firm optimization is to search for the optimal debt-capital ratio ϖ_{t+1} and investment rate i_t to maximize the value of function $J(\varpi_t)$, because the first two terms in the equation (3.6) are either predetermined or exogenous. Then the necessary condition with respect to i_t is:

$$\frac{1}{g'(i_t)} = \overbrace{q_t \varpi_{t+1}}^{\frac{\text{Debt Capital}}{\text{Capital}}} + \underbrace{E_t \beta \left(\frac{D_t}{D_{t+1}} \right)^{\nu} \int_{z_{\min}}^{\tilde{z}_{t+1}} (\tilde{z}_{t+1} - z_{t+1}) d\Phi(z)}_{\frac{\text{Equity Value}}{\text{Capital}}} - \Gamma(\varpi_{t+1}) \tag{3.20}$$

The term on the on the left-hand side of equation (3.20) can be considered as the inverse of $\Delta k_{t+1}/\Delta x_t$, which represents the marginal transformation rate of investment into the capital and therefore the marginal cost to increase one extra unit of capital. It equals to 1 if there was no capital adjustment. In addition, the first term on the right-hand side is the debt value over capital, the second term is the equity value over capital, and the last term is the financial cost. The first two term together work as the ratio between the firm market value and the corresponding capital. Therefore the right-hand side is a modified *Tobin's Q*. Note, the transformation rate of investment to capital changes as the investment-capital ratio changes in the presence of the capital adjustment cost. Therefore the entrepreneurs would like to adjust i_t until the transform rate equal to the marginal Tobin's Q. It is interesting to do some qualitative conjecture here. If my model here is correctly set up, the debt price and equity value should both go down as the default risk increases. If there is no large movement of outstanding debt in the opposite direction, then the consequent decline of Tobin's Q on the right-hand side drives down the investment capital rate on the left-hand side as $g(i_t)$ is concave. This mechanism is at the core of this paper in explaining the investment dynamics.

$$q_t = E_t\beta\left(\frac{D_t}{D_{t+1}}\right)^{\nu}\Phi(\bar{z}_{t+1})\frac{\partial\bar{z}_{t+1}}{\partial\varpi_{t+1}} + \Psi_b'(\varpi_{t+1}) - \frac{\partial q_t}{\partial\varpi_{t+1}}[\varpi_{t+1}(1-\psi) - (1-\lambda)\frac{\varpi_t}{g(i_t)}] \quad (3.21)$$

The equation (3.21) indicates the entrepreneurs' optimal choice in the debt-capital ratio and provides the fundamental for the financial structure decision. The left-hand side is all the benefit entrepreneurs could obtain from increasing a marginal unit of debt-capital ratio, that's the sale price of bonds, while the right-hand side offers the cost to increase a marginal unit of debt-capital ratio, including the present value of payment contingent on the non-default case, the marginal financial cost and the price fluctuation caused by the financial position change. It appears impossible to obtain any immediate qualitative results directly from the observation of equation (3.21) as a result of its complexity. So the dynamics of debt-capital movement under the current framework is a quantitative issue, which I will give a more detailed discussion in the quantitative result section.

The market clearing conditions in this model are nontrivial because of the heterogeneity among firms. The default firms and non-default firms make investment and financial decision according to identical decision rules but on different capital states. Specifically the default firms reorganize their assets and continue operation using only ϵ of their previous capital. Besides, I assume all the firms are heterogeneous only in their capital and debt size rather their debt-capital ratio. Thus the aggregate investment X_t, the aggregate capital next period K_{t+1}, capital accumulation equation, aggregate debt B_{t+1} and goods market clearing condition are:

$$X_t = \left(\Phi(\bar{z}_t) + [1 - \Phi(\bar{z}_t)]\epsilon_t\right) i_t K_t \tag{3.22}$$

$$K_{t+1} = g(i_t)(\Phi(\bar{z}) + [1 - \Phi(\bar{z})]\epsilon_t)K_t \tag{3.23}$$

$$B_{t+1} = \varpi_{t+1} K_{t+1} \tag{3.24}$$

$$Y_t = c_t^w + c_t^e + X_t \tag{3.25}$$

$$Y_t = A_t K_t^{\theta} n_t^{1-\theta} \tag{3.26}$$

Note I assume the production process happens before the default process. It does not receive any impact from the default. The capital used in production is K_t. The production function is the classic Cobb-Douglas function, where A_t^T is the technology shock that follows a vector autoregressive process in (3.27) together with ϵ_t.

$$\begin{bmatrix} A_t \\ \epsilon_t \end{bmatrix} = \Omega \begin{bmatrix} A_{t-1} \\ \epsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{a,t} \\ \varepsilon_{\epsilon,t} \end{bmatrix} \tag{3.27}$$

3.3 Data and Calibration

The data in this paper are all annual. One period in the model hence corresponds to one year in reality. Although the standard calibration is usually in the quarterly frequency, mine is a compromise because of the shortage in consistent quarterly data series during

Parameters		Targets	
θ	0.370	the ratio between labor expenditure and nonfinancial corporate output	0.37
δ	0.063	the ratio between the depreciation and nonfinancial corporation capital	0.24
η	2.170	working hours account for 1/3 of 24 hours	
α	0.230	the ratio between st.d of investment and GDP	2.04
β	0.885	the average capital-output ratio	2.00
γ	0.947	the gap between stock return and Baa corporate bond yield	5%
ϕ	0.050	the ratio between corporate long-term debt and capital	0.48
κ	0.042	accumulative annual default rate	1%
ϵ	0.625	historical default recovery ratio	62.5%
λ	0.200	literature	0.20
σ_a	0.019		
σ_ϵ	0.036		
$\Omega = \begin{bmatrix} 0.9509 & -0.0842 \\ 0.0743 & 0.7757 \end{bmatrix}$			

Table 3.1: Parameters and Targets

the inter-war era. Most of parameters are obtained by calibrating to the long-term U.S. macroeconomic facts. The others are taken from relevant literatures. The data series to abstract the long-term targets range from 1929 to 1976[8]. The following session summarize the calibration strategy. The concrete data sources are offered in the Appendix.

The parameters pinned down by the steady state targets are θ, η and δ. $1-\theta$ is set to equal to the share of labor relevant expenditure in the total nonfinancial corporate income; As the weight of leisure in the total utility, η guarantees the daily average working hours equal to 8 hours; δ is equal to the average of ratio between depreciation of non-residential capital and nonresidential capital stock of nonfinancial corporation. The relevant data are from National Income and Product Account of Bureau of Economics Analysis, NIPA henceforth.

[8] Although the data availability and quality are very limited at that time, fortunately it is still possible to impute all the required data from different sources. Such methodology is certainly plagued by some inconsistences issues but it is definitely the best I could achieve for now.

ν, λ, and ϵ are taken from literatures. I set the value of ν equal to 5 as in Jermann (1998); $1/\lambda$ represents the average years to maturity for the long-term debt. It is difficult to make any inference about the average years to maturity of U.S. long-term corporate bond. The recent study shows that the average maturity of long-term corporate debt in western economy is between 4 and 8.5 years and that the average maturity is pro-cyclical. Without loss of generalization the average years to maturity I take here is 5 years, namely $\lambda = 0.2$. ϵ is the recovery rate when a corporation files a default. Giesecke et al. (2011) says "*Hickman (1960) Table 152 implies that the average recovery rate of defaulted issues during the* $1900 - 1944$ *period is about* 62.5%". The remaining five parameters $\{\beta, \gamma, \psi, \alpha, \kappa\}$ are respectively the discount factor of entrepreneurs, the discount factor of workers, the commission rate, capital adjustment parameters and shape parameters for the distribution of z_t. They can not be identified individually. Therefore, I just let them work together to match five long-term business cycle moments: capital-output ratio, credit spread between Baa corporate bond and stock, debt-capital ratio, the relative volatility of investment to output, and the annual accumulative default probability. The cumulative density function of z is assumed to follow Miao and Wang (2010)

$$\Phi(z) = \left(z + \frac{\kappa}{1+\kappa}\right)^{\kappa} \quad z \in \left[-\frac{\kappa}{1+\kappa}, \frac{1}{1+\kappa}\right] \tag{3.28}$$

This distribution function is supported by the interval $[\frac{-\kappa}{\kappa+1}, \frac{\kappa}{\kappa+1}]$ and with the mean equal to 0.s My calibration successfully make the power format function generate a right-skewed and thin-tail density. See Figure 3.1. This feature is very important. Because it can further guarantees that only very few firms declare default and that a moderate movement of z_t around the right tail does not cause a large change of $\Phi(z)$. Solow residual series and the financial shocks series are the data to estimate the autoregressive system (3.27). I compute the Solow residual sequence in a standard approach. Take the log of the output, capital and

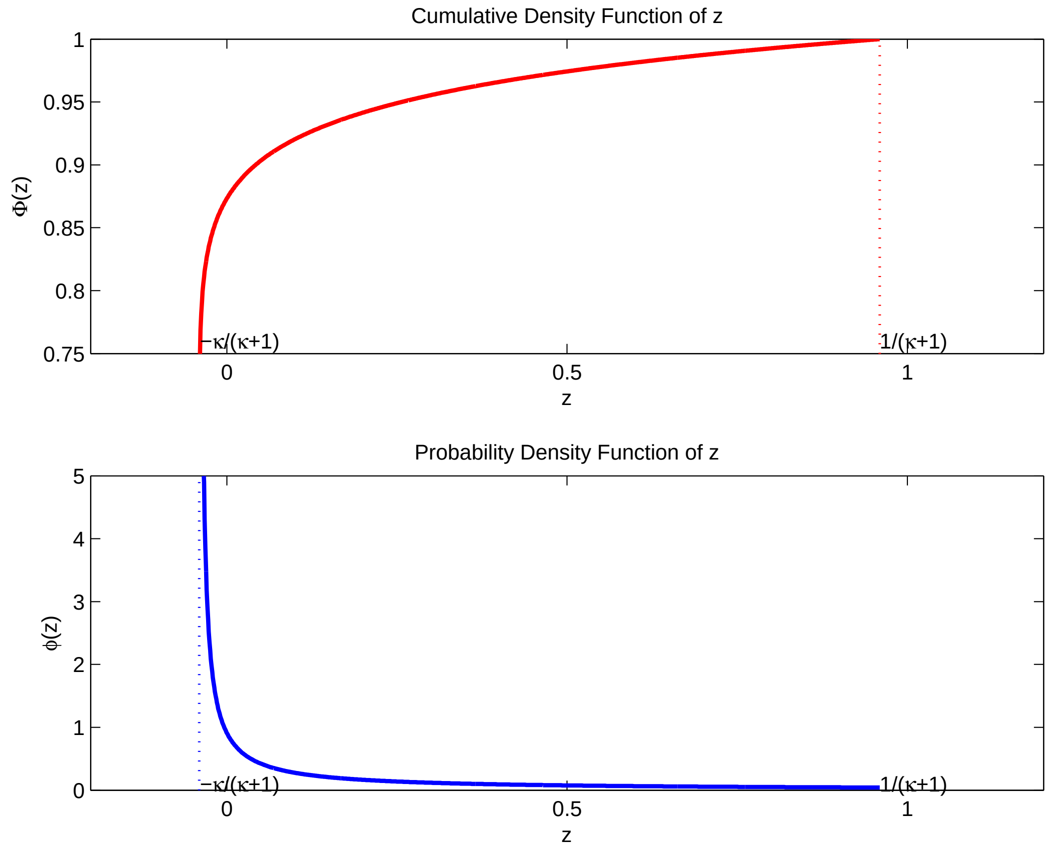

Figure 3.1: CDF and PDF of credit risk z

working hours and derive $\hat{A}_t$ the following equation:

$$\hat{A}_t = \hat{y}_t - \theta\hat{k}_t - (1-\theta)\hat{h}_t$$

For the financial shocks series, I follow the procedures proposed by Jermann and Quadrini (2012), that's

$$\epsilon_t = \frac{\text{GDP}}{\text{end of period capital} - \text{end of period debt}}$$

End of period capital is the private fixed assets of nonfinancial firms in NIPA. End of period debt is the corporate long-term debt in the *Historical Statistics of United States*. GDP is the total value added of nonfinancial corporation in NIPA. The working hours sequence is a little complicated. The series before 1963 is the weekly working hours from Kendrick (1961) and Kendrick (1973). The series after 1963 is from the BLS private average weekly hours. All the value are in the real terms and the log values are all linearly detrended before estimation.

3.4 Quantitative Analysis

In this section I will present the computation strategy, simulation results and interpretation. Although the original equation system to characterize the equilibrium has been reduced much by the variable transformation in section 2, there still exists a large state-space and especially too many lagged state variables. It is an obstacle for nonlinear methods. Therefore, I solve the model linearly instead. It is worth pointing out that some endogenous variables take negative values at steady state. So it is not possible to apply the log-linear scheme. The full equation system and details to solve for the steady state are given in the Appendix. Once the laws of motion for state variables and decision rules for the jump variables are obtained, the simulation can be done by feeding into the model the actual realization of technology and financial shocks from year 1929 to 1939. Then I compare the simulated transition path of economic aggregates and financial variables with their data counterparts. In the meantime, a standard real business cycle model with an identical capital adjustment cost and inter-temporal elasticity of substitution is solved and simulated in the same way in order to identify the amplification by the default risk. The concrete setup and solution of this benchmark model is also in the Appendix.

3.4.1 Steady State and Impulse Response

My quantitative analysis starts with the exhibition of the steady state. Two first-order partial derivatives $\frac{\partial \tilde{z}_t}{\partial \varpi_t}$ and $\frac{\partial q_t}{\partial \varpi_{t+1}}$ are included in my endogenous variable set. $\frac{\partial \tilde{z}_t}{\partial \varpi_t} < 0$ implies that a higher debt-capital ratio decreases $\tilde{z}_t$ and further increases the default probability. It is quite intuitive because firms with heavier debt burden are more vulnerable to the default risk. In addition, the interpretation of $\frac{\partial q_t}{\partial \varpi_t} < 0$ is also very straightforward. The bond price has to go down when firms are exposed to heavier debt burden and higher default probability. These two features are both consistent with our observation in the financial market. The impulse response experiments are also taken around the steady state. The figure 3.2 and 3.3 respectively show the dynamics of all important economic indicators after the negative one-percent technology or financial shock hits the steady state economy. All the movements have been measured as the percentage deviation from steady state in the graphs. In Figure 3.2, the initial deviation of output and investment are both larger than the size of the original shock, which implies that the model with default risk is able to amplify the impact of the technology shock. especially the investment decline is as three times as the one in technology decline. Consumption shows about the 80% of the decline in the technology while the working hour is less than the half. The consumption decrease is delayed by the households' motive to smooth their utility stream. A large decrease in the working hours does not occur possibly because the strong wealth effect when the wage drops[9]. The debt-capital ratio and capital are both predetermined so their responses start from zero. The capital goes down for two reason: the default-reorganization cost and lower investment. The default process make firms lose 47.5% of their capital. Nonetheless, the fraction of the firms under such effect is small. On the contrary, most of firm would response to the impulse by cutting investment. Compared with the traditional case without default risk, the default risk emerges as another propeller to the investment collapse. As the equation (3.20) shows, both debt price and equity value will go down deeper because of the increase in probability to default, which finally leads to a more severe investment decline.

However, it is surprising to find out that the debt-capital ratio is increasing instead of decreasing. A reasonable explanation is that the total outstanding debt falls more slowly than

[9]The labor income accounts for almost 70% of the total income. The wage rate drop might lead to a strong negative wealth effect and drive workers not to enjoy too much leisure. In the appendix I conduct a sensitivity analysis with a different utility function. When I introduce the utility function in Greenwood, Hercowitz, and Huffman (1988) and set the habit formation parameter equal to zero, the wealth effect of wage change is completely eliminated, The working hours decrease a lot as expected. So it is very reasonable to conjecture that the missing of working hours decline here is the result of wealth effect.

the capital. It is definitely not a result of the default process. Because the model assumes that both capital and outstanding debt will be cut in the same proportion. Therefore the default process cannot bring down the value of debt-capital ratio. Thus the fall speed discrepancy only comes from the endogenous choice of entrepreneurs. At steady state the $\psi_k(\bar{i}) = 1$, which implies one unit investment decline can decrease one unit capital decline. However, the saved one unit investment expenditure isn't completely used to buy back outstanding bond. Because firms have to pay financial position adjustment cost and also want to use part of liquidity to smooth their dividend payout. Therefore the decline of outstanding debt is smaller than one unit.

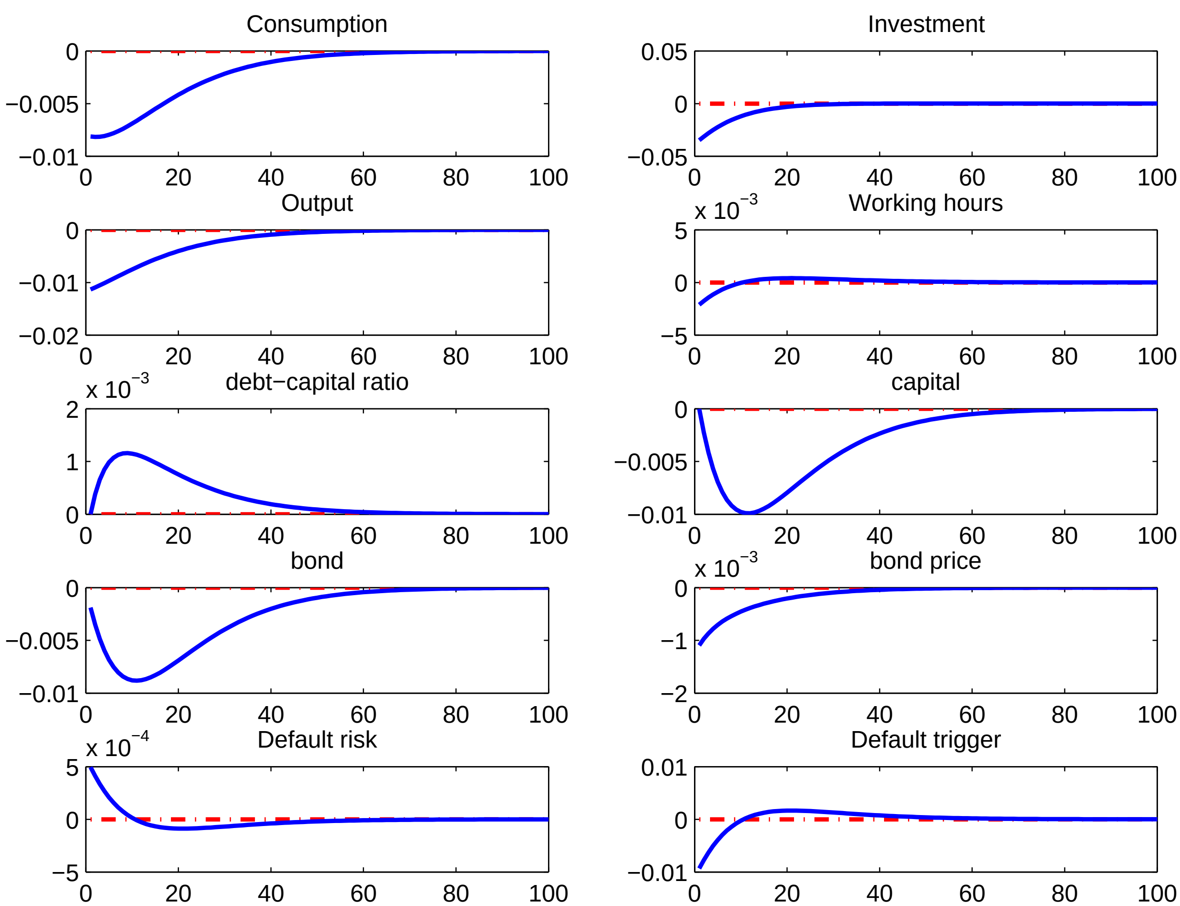

Figure 3.2: Impulse response to TFP shocks

Figure 3.3 illustrates the impulse response to the negative financial shock. All the variables move in the same direction as they are hit by the negative technology shock except the consumption. The reason why consumption increases a little bit is that the negative financial shock drives the investment down but fail to make the same amount decline in the output. Consumption has to go up. A lower recovery rate makes the entrepreneurs more aware of the default and reduce the investment to avoid the default loss. The output based on the constant technology level and slightly changed capital capital won't lead to a deep downturn. In sum, the impact of financial shock is quantitatively small. Only the impacts on investment, capital and bond price are relatively significant but still minor compared with the technology impulse response.

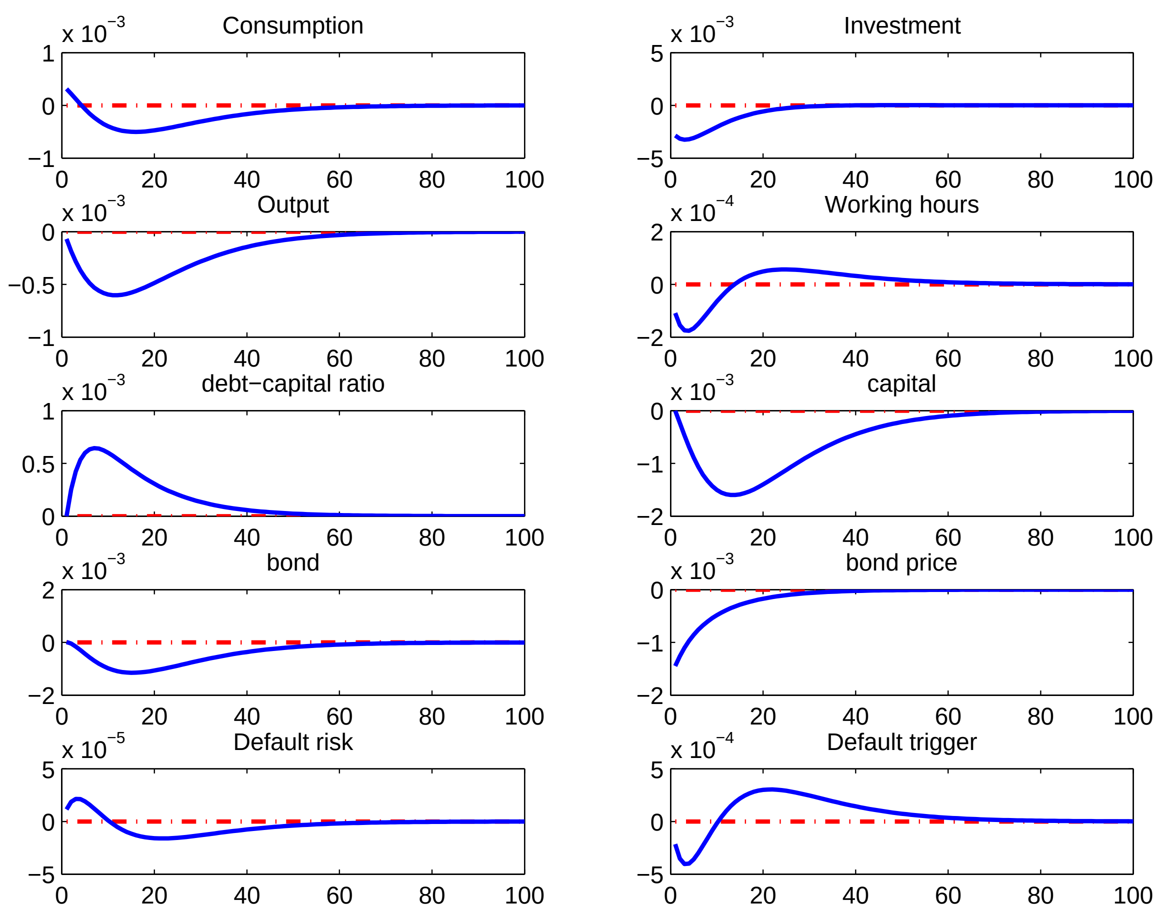

Figure 3.3: Impulse response to financial shocks

3.4.2 Simulation

Three simulation experiments are done so that we can recognize the role of different shocks during the U.S. Great Depression. First, I consider both technology and financial shocks. Figure 3.4 shows that all the simulated aggregates but working hours generate a transition comparable to data. It is necessary to emphasize that the consumption I plot here is the consumption of workers rather than the sum of workers' consumption and entrepreneurs' dividend payout, because it is more aligned with the consumption definition in the traditional literature. The black dashed line is the simulation results from the benchmark RBC model. It shows that the amplification by the default risk is large, particularly in investment. The failure in matching working hours should be attributed to the strong wealth effect that was discussed in the impulse response subsection.

$$r_d = \frac{\lambda + (1-\lambda)\vartheta}{q} + 1 - \lambda \tag{3.29}$$

In addition, the financial indicators such as the default rate, bond yield and debt-capital ratio are also within our interest. According to Giesecke et al. (2011), there are two corporate bond default peaks during the U.S. Great Depression: one is between 1931 and 1934, and the other is between 1937 and 1938. Their standard in measuring the default peaks is that the annual cumulative default rate is higher than 2%. Thus, my model successfully predicts one of the most worst default peak between 1931 and 1934. The severity, the highest annual default rate 3%, and timing are all acceptable. I fail to reproduce the second one between 1937 and 1938 but the annual default rate is very close to the 2% line. The bond yield in this model is computed following the equation (3.29) and matches well with the middle-grade corporate bond yield from Susan B. Carter (2003). Unfortunately, the predicted debt-capital ratio is very different from the actual data series. The simulated transition path completely miss the huge climbing at the beginning of 1930s and instead produces a slightly decline. It is quite surprising because intuitively firms should deplete their debt obligation to ensure their solvency. The analysis on the $\frac{\partial \tilde{z}_t}{\partial \varpi_t}$ and $\frac{\partial q_t}{\partial \varpi_{t+1}}$ also says that higher debt-capital ratio is more likely to put firms in the danger of default. In Hart and Mehrling (1995) the author mentions that lots of large utility and railway firms stuck to their finance plan before the financial turbulence and issued massive volumes of long-term debt in 1930. This is an exceptional behavior during the hard time. Another candidate explanation to the discrepancy between the data and model is that my model doesn't capture the debt adjustment cost very accurately and underestimate the difficulty in adjusting financial position during a serious recession.

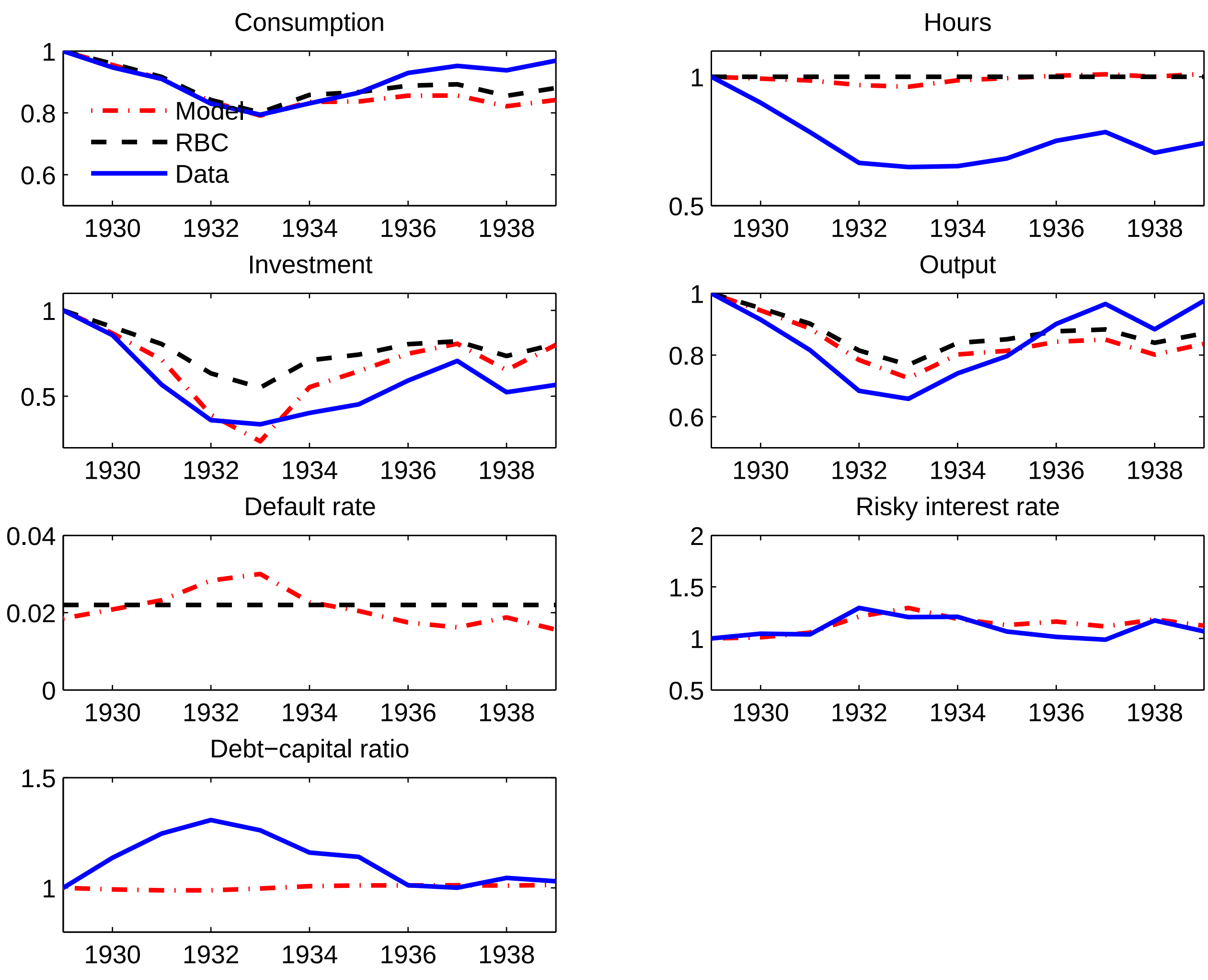

Figure 3.4: Counterfactual simulation with two shocks

Another two experiments are respectively for the financial and technology shocks. From the simulation figure 3.6, it is not difficult to figure out that all the economics aggregates and financial indicators except the bond yield fail to match with the corresponding data if only the financial shock is considered. However, the results in Figure 3.5 shows that conclusions from two-shock experiments are well preserved although the severity get slightly reduced. The bond yield is not well consistent with the actual one. This decompositions clearly demonstrate that the impact of TFP shocks is dominant and that the impact of financial shocks is minor in match the economic aggregates. However, the financial shock does help a lot in producing a jump in risky interest.

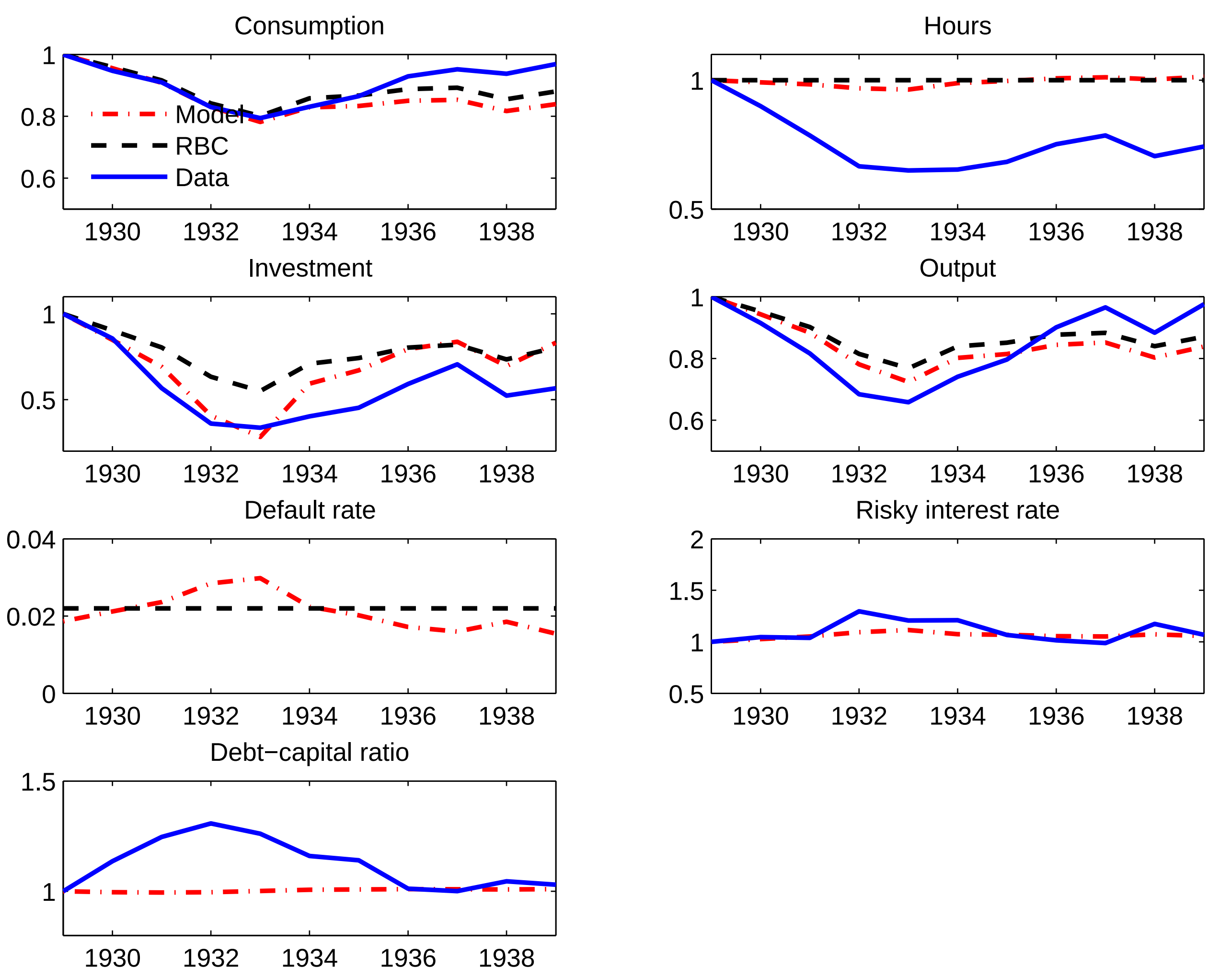

Figure 3.5: Counterfactual simulation with technology shock only

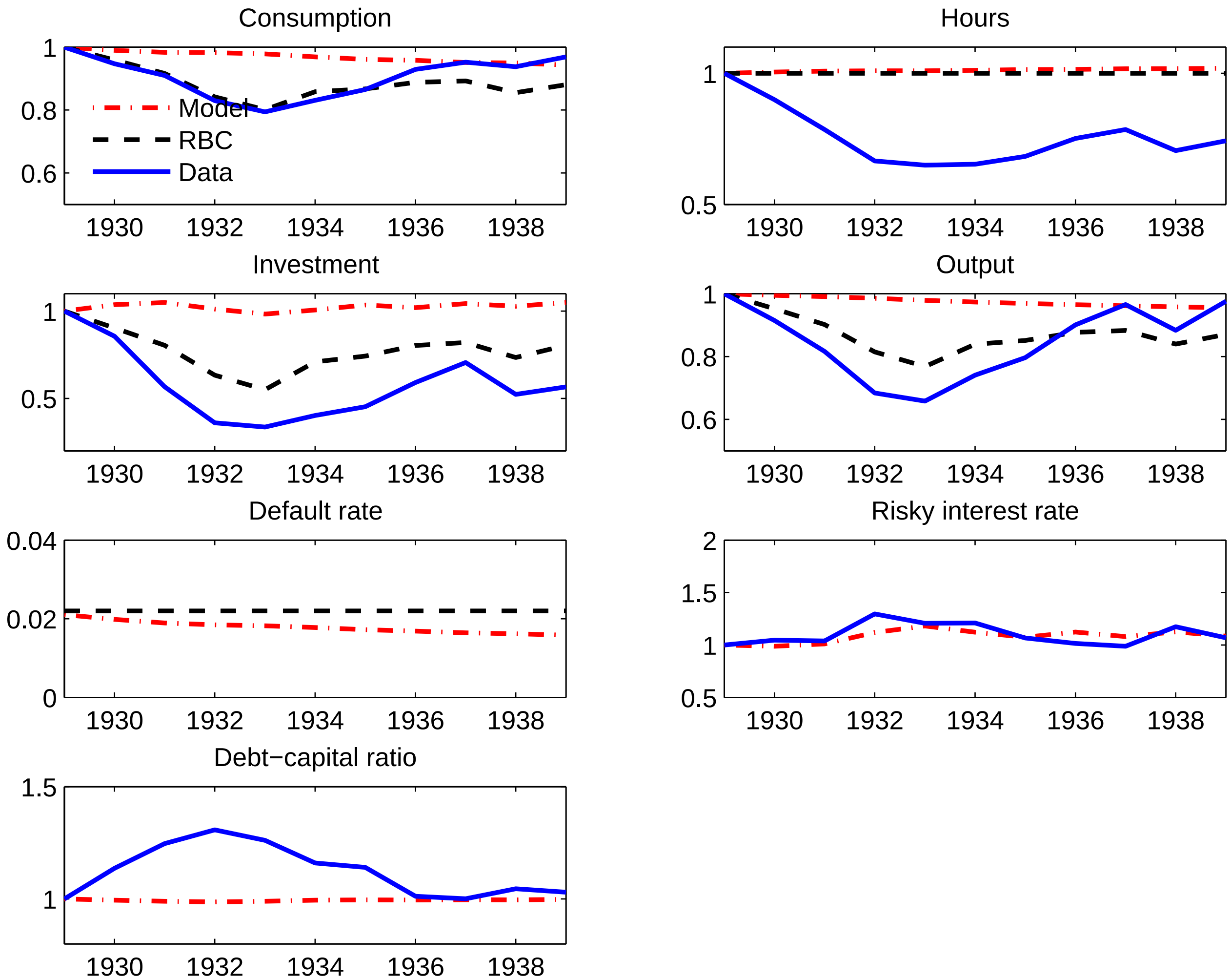

Figure 3.6: Counterfactual simulation with financial shock only

3.5 Conclusion

This paper attempts to investigate the roles of default risk during the U.S. Great Depression. My rational expectation RBC model shows that the adverse technology shocks can be amplified very much by the default risk. Intuitively, when the adverse technology shocks hit the economy, firms become more vulnerable to the credit risk and try to decrease the investment and debt heavily to maintain in solvency. Therefore the investment could get cut much more than in the case where the default risk is not considered. The simulation with TFP and financial shocks successfully explains the large decline in consumption, output and investment and nonetheless miss the working hours drop because of the strong wealth effect. In the meantime, the financial indicators such as default rate and bond yield are also very well predicted. The decomposition of simulation process tells that the effect of technology shocks dominates during the U.S. Great Depression whereas the financial shocks only play an important role in pushing up the long-term bond yield. More interestingly, the counterfactual debt-capital ratio could implies that there exists a serious obstacles for firms to unload their debt burden during the early 1930s, which therefore could be a important factor to deteriorate the economic recession.

However, the discussion is still far from ending. Jiang (2013) concludes that the adverse financial shocks could be an important aspect to understand working hours decline if the working capital or firm liquidity is correctly introduced. Besides, the monetary policy is not a negligible.factor when we discuss the corporate finance management and debt market. Thus, my future study will continue in the following directions: (1) to develop a comprehensive model that can incorporate both default risk and firm liquidity; (2) to introduce the money and cash management into the current framework.

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3.6 Appendix

3.6.1 Data

- Nominal nonfinancial corporate capital stock: BEA FA6.1 Line 4
- Real nonfinancial corporate capital stock: BEA FA6.2 Line 3
- Real nonfinancial corporate investment: BEA FA6.8 Line 4
- Nominal nonfinancial corporate capital depreciation: BEA FA6.4 Line 4
- Real nonfinancial corporate capital depreciation: BEA FA6.5 Line4
- Nominal nonfinancial corporate production: BEA NIPA1.14 Line 17
- Real nonfinancial corporate production: BEA NIPA1.14 Line 41
- Nominal nonfinancial corporate long-term debt: Susan B. Carter (2003) data series cj870 – 889
- U.S. population between 18 and 65: Susan B. Carter (2003) data series Aa125 – 144
- U.S. private non-farm total man-hours 1929 – 1953: Kendrick (1961)
- U.S. private non-farm total man-hours 1948 – 1966: Kendrick (1973)
- U.S average weekly private working hours 1964 – 1976: BLS table ID EES00500005
- Nominal durable good consumption and services: BEA NIPA 1.1.5 Line 5 and Line 6
- GDP deflator: BEA NIPA1.1.5 Line 1
- Corporate Cash: Statistics of Income[10]
- Corporate inventories: Statistics of Income

[10] see the archive in the links http://www.irs.gov/uac/SOI-Tax-Stats-Archive—1934-to-1953-Statistics-of-Income-Report,-Part-2

3.6.2 Mathematics

All the equations

$$R_t - \bar{z}_t - \varpi_t[(1-\lambda)\vartheta + \lambda] + J(\varpi_t) = 0 \tag{3.30}$$

$$-\frac{\partial \bar{z}_t}{\partial \varpi_t} - [(1-\lambda)\vartheta + \lambda] + \frac{\partial J(\varpi_t)}{\partial \varpi_t} = 0 \tag{3.31}$$

$$J(\varpi_t) = q_t[\varpi_{t+1} g(i_t) - (1-\lambda)\varpi_t] - i_t + g(i_t)\left[\beta \int_{z_{min}}^{\bar{z}_{t+1}} (\bar{z}_{t+1} - z) d\Phi(z) - \psi q_t \varpi_{t+1} - \Psi(\varpi_{t+1})\right] \tag{3.32}$$

$$\frac{\partial q_t}{\partial \varpi_{t+1}}[\varpi_{t+1} g(i_t) - (1-\lambda)\varpi_t - \psi \varpi_{t+1} g(i_t)] + q_t g(i_t)(1-\psi) + \beta g(i_t)\Phi(\bar{z}_{t+1})\frac{\partial \bar{z}_{t+1}}{\partial \varpi_{t+1}} - g(i_t)\Psi'(\varpi_{t+1}) = 0 \tag{3.33}$$

$$\frac{\partial J(\varpi_t)}{\partial \varpi_t} = -q_t(1-\lambda) \tag{3.34}$$

$$q_t \varpi_{t+1}(1-\psi)g'(i_t) - 1 + g'(i_t)\beta \frac{c_t^e}{c_{t+1}^e} \int_{z_{min}}^{\bar{z}_{t+1}} (\bar{z}_{t+1} - z_{t+1}) d\Phi(z) - g'(i_t)\Psi(\varpi_{t+1}) = 0 \tag{3.35}$$

$$\begin{aligned} q_t \varpi_{t+1} = \Lambda_{t+1}^w \{ & \Phi(\bar{z}_{t+1}) q_{t+1} \varpi_{t+1}(1-\lambda) + [\lambda + (1-\lambda)\vartheta]\varpi_{t+1} \\ & - (1-\tau_p) \int_{\bar{z}_{t+1}}^{z_{max}} (z - \bar{z}_{t+1}) d\Phi(z) + (1-\epsilon) J(\varpi_{t+1})[1 - \Phi(\bar{z}_{t+1})] \} \end{aligned} \tag{3.36}$$

$$\begin{aligned} q_t + \frac{\partial q_t}{\partial \varpi_{t+1}} \varpi_{t+1} = \Lambda_{t+1}^w \{ & \Phi(\bar{z}_{t+1}) q_{t+1}(1-\lambda) + [\lambda + (1-\lambda)\vartheta] + (1-\lambda)\phi(\bar{z}_{t+1}) q_{t+1} \varpi_{t+1} \frac{\partial z_{t+1}}{\partial \varpi_{t+1}} \\ & + (1-\tau_p)(1 - \Phi(\bar{z}_{t+1})) \frac{\partial \bar{z}_{t+1}}{\partial \varpi_{t+1}} + (1-\epsilon) J(\varpi_{t+1}) \phi(\bar{z}_{t+1}) \frac{\partial z_{t+1}}{\partial \varpi_{t+1}} \\ & - (1-\epsilon)[1 - \Phi(\bar{z}_{t+1})] \frac{\partial J(\varpi_{t+1})}{\partial \varpi_{t+1}} \} \end{aligned} \tag{3.37}$$

$$\Lambda_{t+1}^w = \gamma \frac{c_t^w}{c_{t+1}^w} \tag{3.38}$$

$$\frac{w_t}{c_t^w} = \frac{\alpha}{1 - H_t} \tag{3.39}$$

$$X_t = (\Phi(\bar{z}_t) + [1 - \Phi(\bar{z}_t)]\epsilon)\, i_t K_t \tag{3.40}$$

$$K_{t+1} = (1 - \delta + i_t)\left(\Phi(\bar{z}) + [1 - \Phi(\bar{z})]\epsilon\right) K_t \tag{3.41}$$

$$B_{t+1} = \varpi_{t+1} K_{t+1} \tag{3.42}$$

$$Y_t = A_t K_t^{\theta} H_t^{1-\theta} \tag{3.43}$$

$$Y_t = C_t^w + C_t^e + X_t \tag{3.44}$$

$$w_t = (1-\theta) A_t K_t^{\theta} H_t^{-\theta} \tag{3.45}$$

$$R_t = \theta A_t K_t^{\theta-1} H_t^{1-\theta} \tag{3.46}$$

$$\begin{aligned} C_t^e =& [(1-\tau_p)R_t + \tau_p\delta]K_t + \tau_p[(1-q_{t-1})\lambda + (1-\lambda)\vartheta]\varpi_t\Phi(\bar{z}_t)K_t - [\lambda + (1-\lambda)\vartheta]\varpi_t\Phi(\bar{z}_t)K_t \\ &+ q_t[w_{t+1}g(i_t) - (1-\lambda)\varpi_t]\Phi(\bar{z}_t)K_t - i_t\Phi(\bar{z}_t)K_t - \phi q_t\varpi_{t+1}g(i_t)]\Phi(\bar{z}_t)K_t \\ &+ \{q_t[\varpi_{t+1}g(i_t) - (1-\lambda)\varpi_t] - i_t - \phi q_t\varpi_{t+1}g(i_t)\}(1-\Phi(\bar{z}_t))\epsilon K_t \end{aligned} \tag{3.47}$$

$$\begin{aligned} &\int_{z_{min}}^{\bar{z}_{t+1}} (\bar{z}_{t+1} - z)d\Phi(z) \\ =&\bar{z}_{t+1}\int_{z_{min}}^{\bar{z}_{t+1}} d\Phi(z) - \int_{z_{min}}^{\bar{z}_{t+1}} zd\Phi(z) \\ =&\bar{z}_{t+1}\Phi(\bar{z}_{t+1}) - z\Phi(z)|_{z_{min}}^{\bar{z}_{t+1}} + \int_{z_{min}}^{\bar{z}_{t+1}} \Phi(z)dz \\ =&\bar{z}_{t+1}\Phi(\bar{z}_{t+1}) - \bar{z}_{t+1}\Phi(\bar{z}_{t+1}) + \int_{z_{min}}^{\bar{z}_{t+1}} (z + \frac{\kappa}{\kappa+1})^{\kappa}dz \\ =&\frac{(\bar{z}_{t+1} + \frac{\kappa}{\kappa+1})^{\kappa+1}}{\kappa+1} \end{aligned}$$

$$
\begin{aligned}
&(1-\tau_p)\int_{\bar{z}_{t+1}}^{z_{max}}(z-\bar{z}_{t+1})d\Phi(z)\\
=&(1-\tau_p)[\int_{\bar{z}_{t+1}}^{z_{max}}zd\Phi(z)-\int_{\bar{z}_{t+1}}^{z_{max}}\bar{z}_{t+1}d\Phi(z)]\\
=&(1-\tau_p)\{z\Phi(z)|_{\bar{z}_{t+1}}^{z_{max}}-\int_{\bar{z}_{t+1}}^{z_{max}}\Phi(z)dz-\bar{z}_{t+1}[\Phi(z_{max})-\Phi(z_{t+1})]\}\\
=&(1-\tau_p)[z_{max}-\bar{z}_{t+1}\Phi(\bar{z}_{t+1})-\int_{\bar{z}_{t+1}}^{z_{max}}\Phi(z)dz-\bar{z}_{t+1}+\bar{z}_{t+1}\Phi(\bar{z}_{t+1})]\\
=&(1-\tau_p)[(z_{max}-\bar{z}_{t+1})-\int_{\bar{z}_{t+1}}^{z_{max}}\Phi(z)dz]\\
=&(1-\tau_p)(z_{max}-\bar{z}_{t+1})-(1-\tau_p)\frac{(z+\frac{\kappa}{\kappa+1})^{\kappa+1}}{\kappa+1}|_{\bar{z}_{t+1}}^{z_{max}}\\
=&(1-\tau_p)(z_{max}-\bar{z}_{t+1})-(1-\tau_p)\frac{1}{\kappa+1}[1-(\bar{z}_{t+1}+\frac{\kappa}{\kappa+1})^{\kappa+1}]\\
=&(1-\tau_p)(\frac{1}{\kappa+1}-\bar{z}_{t+1})-(1-\tau_p)\frac{1}{\kappa+1}[1-(\bar{z}_{t+1}+\frac{\kappa}{\kappa+1})^{\kappa+1}]\\
=&(1-\tau_p)[\frac{(\bar{z}_{t+1}+\frac{\kappa}{\kappa+1})^{\kappa+1}}{\kappa+1}-\bar{z}_{t+1}]
\end{aligned}
$$

The unknowns in the above system are $\{R, \bar{z}, q, \varpi, J, z\varpi, q\varpi, J\varpi, i, \Lambda^w, c^w, w, h, X, K, B, c^e, Y\}$.

Solution for the Steady State

$$
-(1-\tau_p)\frac{\partial z_t}{\partial \varpi_t}+\tau_p[(1-q_{t-1})\lambda+(1-\lambda)\vartheta]-\tau_p\lambda\varpi_t\frac{\partial q_{t-1}}{\partial\varpi_t}-[(1-\lambda)\vartheta+\lambda]+\frac{\partial J(\varpi_t)}{\partial\varpi_t}=0
$$

$$
-(1-\tau_p)\Omega^z+(\tau_p-1)[\lambda+(1-\lambda)\vartheta]-\tau_p q\lambda-\tau_p\lambda\varpi_t\Omega^q-q(1-\lambda)=0
$$

$$
\frac{\Omega^z}{q}=\frac{1}{1-\tau_p}\left[(\tau_p-1)\frac{\lambda+(1-\lambda)\vartheta}{q}-\tau_p\lambda-\tau_p\lambda\varpi\frac{\Omega^q}{q}-(1-\lambda)\right]
$$

$$
\frac{\Omega^z}{q}=-\frac{\lambda+(1-\lambda)\vartheta}{q}-\frac{\tau_p\lambda+(1-\lambda)}{1-\tau_p}-\frac{\lambda\tau_p}{1-\tau_p}\frac{\varpi_t\Omega^q}{q} \tag{3.48}
$$

$$q_t + \frac{\partial q_t}{\partial \varpi_{t+1}}\varpi_{t+1} = \Lambda^w_{t+1}\{\Phi(\bar{z}_{t+1})q_{t+1}(1-\lambda) + [\lambda + (1-\lambda)\vartheta] + (1-\lambda)\phi(\bar{z}_{t+1})q_{t+1}\varpi_{t+1}\frac{\partial \bar{z}_{t+1}}{\partial \varpi_{t+1}}$$
$$+ (1-\tau_p)(1-\Phi(\bar{z}_{t+1}))\frac{\partial \bar{z}_{t+1}}{\partial \varpi_{t+1}} + (1-\epsilon)J(\varpi_{t+1})\phi(\bar{z}_{t+1})\frac{\partial z_{t+1}}{\partial \varpi_{t+1}}$$
$$- (1-\epsilon)[1-\Phi(\bar{z}_{t+1})]\frac{\partial J(\varpi_{t+1})}{\partial \varpi_{t+1}}\}$$

$$\begin{aligned}\frac{1}{\gamma}\left[1 + \Omega^q\frac{\varpi}{q}\right] &= \frac{\lambda + (1-\lambda)\vartheta}{q} + \Phi(\bar{z})(1-\lambda) \\ &+ (1-\lambda)\phi(\bar{z})q\varpi\frac{\Omega^z}{q} \\ &+ \{(1-\tau_p)[1-\Phi(\bar{z})] + (1-\epsilon)J(\varpi)\phi(\bar{z})\}\frac{\Omega^z}{q} \\ &+ (1-\lambda)(1-\epsilon)[1-\Phi(\bar{z})]\end{aligned} \tag{3.49}$$

$$\frac{\partial q_t}{\partial \varpi_{t+1}}[\varpi_{t+1}(1-\delta+i_t) - (1-\lambda)\varpi_t] + q_t(1-\delta+i_t) + \beta(1-\tau_p)(1-\delta+i_t)\Phi(\bar{z})\frac{\partial \bar{z}_{t+1}}{\partial \varpi_{t+1}} = 0$$

$$\Omega^q_\varpi\frac{\varpi}{q}[(1-\delta+i) - (1-\lambda)] + (1-\delta+i) + \beta(1-\tau_p)(1-\delta+i)\Phi(\bar{z})\frac{\Omega^z}{q} = 0 \tag{3.50}$$

The linear system can help us solve for

$$\frac{\Omega^z}{q} = -\frac{\lambda + (1-\lambda)\vartheta}{q} - \frac{\tau_p\lambda + (1-\lambda)}{1-\tau_p} - \frac{\tau_p\lambda}{1-\tau_p}\frac{\varpi\Omega^q}{q} \tag{3.51}$$

$$\begin{aligned}\frac{1}{\gamma}\frac{\varpi\Omega^q}{q} = \frac{\lambda + (1-\lambda)\vartheta}{q} + \Phi(\bar{z})(1-\lambda) - \frac{1}{\gamma} + (1-\lambda)(1-\epsilon)[1-\Phi(\bar{z})] \\ + \{(1-\lambda)\phi(\bar{z})\varpi q + (1-\tau_p)[1-\Phi(\bar{z})] + (1-\epsilon)J(\bar{z})\phi(\bar{z})\}\frac{\Omega^z}{q}\end{aligned} \tag{3.52}$$

$$\frac{\varpi\Omega^q}{q}[(1-\delta+i) - (1-\lambda)] + (1-\delta+i) + \beta(1-\tau_p)(1-\delta+i)\Phi(\bar{z})\frac{\Omega^z}{q} = 0 \tag{3.53}$$

within this system three variables $\{1-\delta+i, q\varpi, J\}$need to be transformed into the function of $\bar{z}$ if possible. With the manipulation of the capital evolution, FOC with respect to i and the definition of function J, we can obtain their implication.

Transformed this linear system into a more friendly format. Define $a=\frac{\Omega^z}{q}$, $b=\frac{\lambda+(1-\lambda)\vartheta}{q}$ and $c=\frac{\varpi\Omega^q}{q}$. Also

$$CC_1=-\frac{\tau_p\lambda+(1-\lambda)}{1-\tau_p} \tag{3.54}$$

$$LL_1=-\frac{\tau_p\lambda}{1-\tau_p} \tag{3.55}$$

$$LL_2=\frac{1}{\gamma} \tag{3.56}$$

$$CC_2=\Phi(\bar{z})(1-\lambda)-\frac{1}{\gamma}+(1-\lambda)(1-\epsilon)[1-\Phi(\bar{z})] \tag{3.57}$$

$$MM_2=(1-\lambda)\phi(\bar{z})\varpi q+(1-\tau_p)[1-\Phi(\bar{z})]+(1-\epsilon)J(\bar{z})\phi(\bar{z}) \tag{3.58}$$

$$LL_3=(1-\delta+i)-(1-\lambda) \tag{3.59}$$

$$CC_3=(1-\delta+i) \tag{3.60}$$

$$MM_3=\beta(1-\tau_p)(1-\delta+i)\Phi(\bar{z}) \tag{3.61}$$

$$LL_3=(1-\delta+i)-(1-\lambda) \tag{3.62}$$

The original linear system can be put into:

$$a=-b+CC_1+LL_1c$$

$$LL_2c=b+CC_2+MM_2a$$

$$LL_3c+CC_3+MM_3a=0$$

The solution to this simplified system can be obtained easily and listed as below:

$$c = \frac{CC_3 + \frac{MM_3(CC_1+CC_2)}{1-MM_2}}{LL_3 + \frac{MM_3(LL_1-LL_2)}{1-MM_2}}$$

$$a = \frac{CC_1 + CC_2 + (LL_1 - LL_2)c}{1 - MM_2}$$

$$b = LL_2c - CC_2 - MM_2a$$

The conditions or equations we might use during solving the linear system:

$$1 - \delta + i = \frac{1}{\Phi(\bar{z}) + [1 - \Phi(\bar{z})]\epsilon} \tag{3.63}$$

$$i = \frac{1}{\Phi(\bar{z}) + [1 - \Phi(\bar{z})]\epsilon} - 1 + \delta \tag{3.64}$$

$$q\varpi = 1 - \beta(1 - \tau_p) \int_{zmin}^{\bar{z}} (\bar{z} - z) d\Phi(z) \tag{3.65}$$

$$J = q\varpi[(i + 1 - \delta) - (1 - \lambda)] - i + \beta(1 - \tau_p)(i + 1 - \delta) \int_{z_{min}}^{\bar{z}} (\bar{z} - z) d\Phi(z) \tag{3.66}$$

$$\Phi(z) = (z + \frac{\kappa}{\kappa+1})^{\kappa} \left(-\frac{\kappa}{\kappa+1}, \frac{1}{\kappa+1}\right) \tag{3.67}$$

$$\phi(z) = \kappa(z + \frac{\kappa}{\kappa+1})^{\kappa-1} \tag{3.68}$$

Finally, we need a nonlinear equation to solve for the $\bar{z}$

$$\frac{1}{\gamma} = (1 - \lambda)\Phi(\bar{z}) + \frac{\lambda + (1 - \lambda)\vartheta}{q} - \frac{1}{q\varpi} \int_{\bar{z}}^{zmax} [(1 - \tau_p)(z - \bar{z}) + (1 - \epsilon)J(\bar{z})] d\Phi(z) \tag{3.69}$$

$$\int_{z_{min}}^{\bar{z}} (\bar{z} - z) d\Phi(z) = \int_{z_{min}}^{\bar{z}} \Phi(z) dz = \frac{(\bar{z} + \frac{\kappa}{\kappa+1})^{\kappa+1}}{\kappa + 1} \tag{3.70}$$

Continuing on solving for the steady state variables:

$$(R_t - \bar{z}_t)(1 - \tau_p) + \tau_p[(1 - q_{t-1})\lambda\varpi_t + (1 - \lambda)\vartheta\varpi_t + \delta] - \varpi_t[(1 - \lambda)\vartheta + \lambda] + J(\varpi_t) = 0 \tag{3.71}$$

$$(R - \bar{z})(1 - \tau_p) = -\tau_p[(1 - q)\lambda\varpi + (1 - \lambda)\vartheta\varpi + \delta] + \varpi[(1 - \lambda)\vartheta + \lambda] - J(\bar{z}) \tag{3.72}$$

$$R = \frac{-\tau_p[(1 - q)\lambda\varpi + (1 - \lambda)\vartheta\varpi + \delta] + \varpi[(1 - \lambda)\vartheta + \lambda] - J(\bar{z})}{1 - \tau_p} + \bar{z} \tag{3.73}$$